CSE 461: Programming Languages Concepts

Prof. G. Tan
Spring 2018

Homework 5: **Due on Mar 30th before class (12:20pm) in Canvas.**

We will allow late homework for up to only one day with 10% penalty this time, as I want to release a solution soon after the due date to help students prepare for the second midterm.

Submission: Please submit your homework via Canvas. It's okay if you submit a scanned version of your on-paper answers, but please make sure your scanned version is legible.

1. (5 points) For the lambda-calculus term $(\lambda x.\ \lambda y.\ x)\ (\lambda z.\ y)$.

   (a) (2 point) Calculate its free variables using the `FV` function. Show the steps.

   (b) (2 point) Use lambda calculus reduction to reduce the expression to a normal form. Begin by renaming bound variables and show every step.

   (c) (1 point) Describe what would go wrong if you did not rename bound variables.

2. (3 points) Reduce the following lambda-calculus term to the normal form. Show all intermediate steps, with one beta reduction at a time. In the reduction, assume that you are supplied with extra rules that allow you to reduce the addition of two natural numbers into the corresponding results.

$$(\lambda f.\ \lambda x.\ f\ (f\ (f\ x)))\ (\lambda y.\ y + 2)\ 2$$

3. (3 points) The Algol-like program fragment

```
function f(x)
    return x+5
end;
function g(y)
    return 2-y
end;
f(g(2));
```

can be written as the following lambda expression:

$$\Big((\lambda f.\ \lambda g.\ f\ (g\ 2))\ (\lambda x.\ x + 5)\Big)\ (\lambda y.\ 2 - y)$$

Reduce the expression to a normal form. In the reduction, assume that you are supplied with extra rules that allow you to reduce the addition or subtraction of two natural numbers into the corresponding results.