

1.

$$(a) \text{FV}((\lambda x. \lambda y. x) (\lambda z. y))$$

$$= \text{FV}(\lambda x. \lambda y. x) \cup \text{FV}(\lambda z. y)$$

$$= (\text{FV}(\lambda y. x) - \{x\}) \cup (\text{FV}(y) - \{z\})$$

$$= (\text{FV}(x) - \{y\} - \{x\}) \cup (\{y\} - \{z\})$$

$$= (\{x\} - \{y\} - \{x\}) \cup (\{y\} - \{z\})$$

$$= \{\} \cup \{y\}$$

$$= \{y\}$$

$$(b) (\lambda x. \lambda y. x) (\lambda z. y)$$

$$\rightarrow (\lambda x. \lambda q. x) (\lambda z. y)$$

$$\rightarrow \lambda q. \lambda z. y$$

(c) If we do not rename bound variables, the free variable y gets captured by the lambda abstraction with parameter y as follows:

$$(\lambda x. \lambda y. x) (\lambda z. y)$$

$$\rightarrow (\lambda y. (\lambda z. y))$$

$$\rightarrow \lambda y. \lambda z. y$$

2.

$$(\lambda f. \lambda x. f (f (f x))) (\lambda y. y + 2) 2$$

$$\rightarrow (\lambda x. (\lambda y. y + 2) ((\lambda y. y + 2) ((\lambda y. y + 2) x))) 2$$

$$\rightarrow (\lambda y. y + 2) ((\lambda y. y + 2) ((\lambda y. y + 2) 2))$$

$$\rightarrow (\lambda y. y + 2) ((\lambda y. y + 2) 4)$$

$$\rightarrow (\lambda y. y + 2) 6$$

$$\rightarrow 8$$

3.

$$((\lambda f. \lambda g. f (g 2)) (\lambda x. x + 5)) (\lambda y. 2 - y)$$

$$\rightarrow (\lambda g. (\lambda x. x + 5) (g 2)) (\lambda y. 2 - y)$$

$$\rightarrow (\lambda x. x + 5) ((\lambda y. 2 - y) 2)$$

$$\rightarrow (\lambda x. x + 5) (2 - 2)$$

$$\rightarrow (\lambda x. x + 5) 0$$

$$\rightarrow 0 + 5$$

$$\rightarrow 5$$