## CSE 461: Programming Language Concepts – Homework 5 Prof. G. Tan Spring 2018

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1.
     (a) FV((\lambda x. \lambda y. x) (\lambda z. y))
           =FV(\lambda x. \lambda y. x) \cup FV(\lambda z. y)
           =(FV(\lambda y. x) - \{x\}) \cup (FV(y) - \{z\})
           =(FV(x) - \{y\} - \{x\}) \cup (\{y\} - \{z\})
           = (\{x\} - \{y\} - \{x\}) \cup (\{y\} - \{z\})
           = \{\} \cup \{y\}
           = \{y\}
      (b) (\lambda x. \lambda y. x) (\lambda z. y)
           \rightarrow (\lambda x. \lambda q. x)(\lambda z. y)
           → λq. λz. y
      (c) If we do not rename bound variables, the free variable y gets captured by the
           lambda abstraction with parameter y as follows:
           (\lambda x. \lambda y. x) (\lambda z. y)
           → (\(\lambda\)y. (\(\lambda\)z. y))
           → λy. λz. y
2.
           (\lambda f. \lambda x. f(f(fx)))(\lambda y. y + 2) 2
           \rightarrow (\lambda x. (\lambda y. y + 2) ((\lambda y. y + 2) ((\lambda y. y + 2) x))) 2
           \rightarrow (\lambda y. y + 2) ((\lambda y. y + 2) ((\lambda y. y + 2) 2))
           \rightarrow (\lambda y. y + 2) ( (\lambda y. y + 2) 4)
           \rightarrow (\lambda y. y + 2) 6
           → 8
3.
           ((\lambda f. \lambda g. f (g 2)) (\lambda x. x + 5)) (\lambda y. 2 - y)
           \rightarrow (\lambda g. (\lambda x. x + 5) (g 2)) (\lambda y. 2 - y)
           \rightarrow (\lambda x. x + 5) ((\lambda y. 2 - y) 2)
           \rightarrow (\lambda x. x + 5) (2-2)
```

 $\rightarrow$   $(\lambda x. x + 5) 0$ 

→ 0+5→ 5