1. (a) $Fv((\lambda x.\lambda y.x)(\lambda z.y))$

$= Fv(\lambda x.\lambda y.x) \cup Fv(\lambda z.y)$

$= (Fv(\lambda y.x) - \{x\}) \cup \{y\}$

$= (\{x\} - \{x\}) \cup \{y\}$

$= \{y\}$

(b) $(\lambda x.\lambda y.x)(\lambda z.y)$

$= (\lambda x.\lambda t.x/y)x)(\lambda z.y)$

$= (\lambda x.\lambda t.x)(\lambda z.y) = \{\lambda z.y/x\}(\lambda t\ x)$

$= \lambda t.\lambda z.y$

(c) Wrong Step:

$(\lambda x.\lambda y.x)(\lambda z.y)$

$= \{\lambda z.y/x\}(\lambda y.x)$

$= \lambda y.\lambda z.y$

In the wrong steps, the result is totally different with (b), the inner y is bound. However, it's suppose to be free!

$$
\begin{aligned}
&2\ (\lambda f.\lambda x.\ f(f(fx)))\ (\lambda y.\ y+2)\ 2 \\
&= [\lambda y.\ y+2\ /f]\ (\lambda x\ f(f(fx)))\ 2 \\
&= (\lambda x.\ (\lambda y.\ y+2)((\lambda y.\ y+2)((\lambda y.\ y+2)\ x)))\ 2 \\
&= [2/\lambda x]\ (\lambda y.\ y+2)\ ((\lambda y.\ y+2)\ ((\lambda y.\ y+2)\ x)) \\
&= ((\lambda y.\ y+2)\ ((\lambda y.\ y+2)\ ((\lambda y.\ y+2)\ 2))) \\
&= 8
\end{aligned}
$$

3. $((\lambda f.\lambda g.f(g\,2))(\lambda x.x+5))(\lambda y.2-y)$

$=(([(\lambda x.x+5)/\lambda f](\lambda g.f(g\,2)))(\lambda y.2-y)$

$=(\lambda g.(\lambda x.x+5)(g\,2))(\lambda y.2-y)$

$=[(\lambda y.2-y)/g]((\lambda x.x+5)(g\,2))$

$=(\lambda x.x+5)((\lambda y.2-y)\,2)$

$=(\lambda x.x+5)\,0$

$=5$