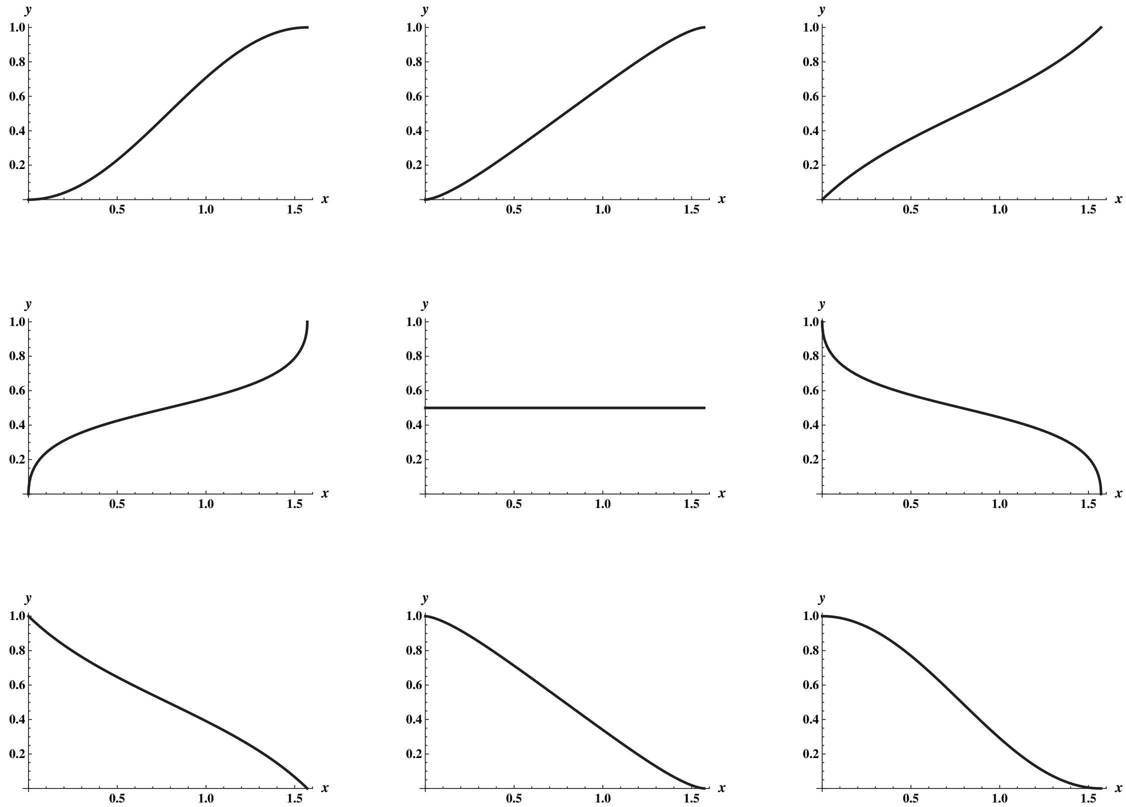
c. The values decrease as n increases.

8.7.88

a. The graphs change shape, but the area under the curve remains constant.



b. When $m = 0$, the integrand is equal to $\frac{1}{2}$ and the bounded region forms a rectangle of width $\frac{\pi}{2}$ and height $\frac{1}{2}$. For all values of m, the graph of $f(x) = \frac{1}{(1 + \tan^m x)}$ is symmetric with respect to the point $(\pi/2, 1/2)$. When $m \neq 0$, for any area located above $y = \frac{1}{2}$ and below f on one side of $x = \frac{\pi}{4}$, there is an equal amount an area below $y = \frac{1}{2}$ and above f on the other side of $x = \frac{\pi}{4}$. The result is easily verified with a computer algebra system.

8.8 Numerical Integration

8.8.1 $\Delta x = \frac{18 - 4}{28} = \frac{1}{2}$.

8.8.2 The Midpoint Rule uses the value of the function evaluated at the midpoint of each subinterval to determine the height of the approximating rectangle over each subinterval. The areas of these rectangles are added up to yield an approximation to the definite integral.

8.8.3 The Trapezoidal Rule approximates the definite integral by using a trapezoid over each subinterval rather than a rectangle.

8.8.4 It is evaluated at 1, 5, and 9, which are the midpoints of the 3 subinterval of length 4.

8.8.5 $M(4) = 2(2) + 5(2) + 6(2) + 8(2) = 42$.

8.8.6 $T(4) = \left(\frac{1}{2}(1) + 4 + 7 + 5 + \frac{1}{2}(5)\right) 2 = 38$.

8.8.7 $S(4) = (1 + 4(4) + 2(7) + 4(5) + 5)\,\frac{2}{3} = \frac{112}{3} \approx 37.33$.

8.8.8 $S(8) = ((1 + 4(2) + 2(4) + 4(5) + 2(7) + 4(6) + 2(5) + 4(8) + 5)\,\frac{1}{3} = \frac{122}{3} \approx 40.67$.

8.8.9 The endpoints of the subintervals are -1, 1, 3, 5, 7, and 9. The trapezoidal rule uses the value of f at each of these endpoints.

8.8.10 $S(4) = \dfrac{4T(4) - 6}{3} = 4.8$.

8.8.11 The absolute error is $|\pi - 3.14| \approx 0.0015926536$. The relative error is $\dfrac{|\pi - 3.14|}{\pi} \approx 5 \times 10^{-4}$.

8.8.12 The absolute error is $|\sqrt{2} - 1.414| \approx 2.14 \times 10^{-4}$. The relative error is $\dfrac{|\sqrt{2} - 1.414|}{\sqrt{2}} \approx 1.51 \times 10^{-4}$.

8.8.13 The absolute error is $|e - 2.72| \approx 0.0017181715$. The relative error is $\dfrac{|e - 2.72|}{e} \approx 6.32 \times 10^{-4}$.

8.8.14 The absolute error is $|e - 2.718| \approx 2.81 \times 10^{-4}$ and the relative error is $\dfrac{|e - 2.718|}{e} \approx 1.04 \times 10^{-4}$.

8.8.15

For $n = 1$, we have $f(6) \cdot 8 = 72 \cdot 8 = 576$.
For $n = 2$ we have $f(4) \cdot 4 + f(8) \cdot 4 = 32 \cdot 4 + 128 \cdot 4 = 640$.
For $n = 4$, we have $f(3) \cdot 2 + f(5) \cdot 2 + f(7) \cdot 2 + f(9) \cdot 2 = 18 \cdot 2 + 50 \cdot 2 + 98 \cdot 2 + 162 \cdot 2 = 656$.

8.8.16

For $n = 1$, we have $f(5) \cdot 8 = 125 \cdot 8 = 1000$.
For $n = 2$ we have $f(3) \cdot 4 + f(7) \cdot 4 = 27 \cdot 4 + 343 \cdot 4 = 1480$.
For $n = 4$, we have $f(2) \cdot 2 + f(4) \cdot 2 + f(6) \cdot 2 + f(8) \cdot 2 = 8 \cdot 2 + 64 \cdot 2 + 216 \cdot 2 + 512 \cdot 2 = 1600$.

8.8.17 We have

$$\frac{1}{6}\left(\sin(\pi/12) + \sin(\pi/4) + \sin(5\pi/12) + \sin(7\pi/12) + \sin(3\pi/4) + \sin(11\pi/12)\right) \approx 0.6439505509.$$

8.8.18 We have

$$\frac{1}{8}\left(e^{-1/16} + e^{-3/16} + e^{-5/16} + e^{-7/16} + e^{-9/16} + e^{-11/16} + e^{-13/16} + e^{-15/16}\right) \approx 0.6317092095.$$

8.8.19 For $n = 2$ we have $T(2) = \frac{4}{2}(f(2) + 2f(6) + f(10)) = 2(8 + 2(72) + 200) = 704$.

Using the results of number 15: For $n = 4$, note that $T(4) = \dfrac{T(2) + M(2)}{2} = \dfrac{704 + 640}{2} = 672$.

Using the results of number 15: For $n = 8$ we have that $T(8) = \dfrac{T(4) + M(4)}{2} = \dfrac{672 + 656}{2} = 664$.

8.8.20

$T(2) = \dfrac{T(1) + M(1)}{2}$. Using problem 16 if we compute $T(1)$.

We have $T(1) = \left(\dfrac{1}{2} + \dfrac{729}{2}\right) \cdot 8 = 2920$. Thus, $T(2) = \dfrac{2920 + 1000}{2} = 1960$.

For $n = 4$, we have $T(4) = \dfrac{T(2) + M(2)}{2} = \dfrac{1960 + 1480}{2} = 1720$.

For $n = 8$ we have $T(8) = \dfrac{T(4) + M(4)}{2} = \dfrac{1720 + 1600}{2} = 1660$.

8.8.21 We have

$$T(6) = \frac{1}{12}\left(\sin 0 + 2\sin\frac{\pi}{6} + 2\sin\frac{\pi}{3} + 2\sin\frac{\pi}{2} + 2\sin\frac{2\pi}{3} + 2\sin\frac{5\pi}{6} + \sin\pi\right)$$
$$= \frac{1}{6}\left(\frac{1}{2} + \frac{\sqrt{3}}{2} + 1 + \frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \frac{1}{6}\left(2+\sqrt{3}\right).$$

8.8.22 $T(8) = \frac{1}{16}(e^{-0} + 2e^{-1/8} + 2e^{-1/4} + 2e^{-3/8} + 2e^{-1/2} + 2e^{-5/8} + 2e^{-3/4} + 2e^{-7/8} + e^{-1}) \approx 0.6329434182.$

8.8.23 $S(4) = \frac{\pi}{12}\left(\sqrt{\sin 0} + 4\sqrt{\sin\frac{\pi}{4}} + 2\sqrt{\sin\frac{\pi}{2}} + 4\sqrt{\sin\frac{3\pi}{4}} + \sqrt{\sin\pi}\right) \approx 2.28477.$

$$S(8) \approx \frac{\pi}{24}\left(\sqrt{\sin 0} + 4\sqrt{\sin\frac{\pi}{8}} + 2\sqrt{\sin\frac{\pi}{4}} + 4\sqrt{\sin\frac{3\pi}{8}} + \right.$$
$$\left. 2\sqrt{\sin\frac{\pi}{2}} + 4\sqrt{\sin\frac{5\pi}{8}}. + 2\sqrt{\sin\frac{3\pi}{4}} + 4\sqrt{\sin\frac{7\pi}{8}} + \sqrt{\sin\pi}\right) \approx 2.35646.$$

8.8.24 $S(4) = \frac{1}{3}\left(\sqrt{4} + 4\sqrt{5} + 2\sqrt{6} + 4\sqrt{7} + \sqrt{8}\right) \approx 9.75156.$

$$S(8) \approx \frac{1}{6}\left(\sqrt{4} + 4\sqrt{4.5} + 2\sqrt{5} + 4\sqrt{5.5} + 2\sqrt{6} + 4\sqrt{6.5} + 2\sqrt{7} + 4\sqrt{7.5} + \sqrt{8}\right) \approx 9.75161.$$

8.8.25

$$S(10) = \frac{1}{6}\left(e^{-4} + 4e^{-2.25} + 2e^{-1} + 4e^{-0.25} + 2e^{0} + 4e^{-0.25} + 2e^{-1}\right.$$
$$\left. + 4e^{-2.25} + 2e^{-4} + 4e^{-6.25} + e^{-9}\right) \approx 1.7680.$$

8.8.26

$$S(8) = \frac{1}{12}\left(\cos\sqrt{2} + 4\cos\sqrt{2.25} + 2\cos\sqrt{2.5} + 4\cos\sqrt{2.75}\right.$$
$$\left. + 2\cos\sqrt{3} + 4\cos\sqrt{3.25} + 2\cos\sqrt{3.5} + 4\cos\sqrt{3.75} + \cos\sqrt{4}\right) \approx -0.30082.$$

8.8.27 The width of each subinterval is 1/25, so

$$M(25) = \frac{1}{25}(\sin\pi/50 + \sin 3\pi/50 + \sin 5\pi/50 + \cdots + \sin 49\pi/50) \approx 0.63704.$$

Because $\int_0^1 \sin\pi x\, dx = \frac{2}{\pi}$, the absolute error is $|2/\pi - M(25)| \approx 4.19 \times 10^{-4}$ and the relative error is this number divided by $2/\pi$ which is approximately 6.58×10^{-4}. The Trapezoidal Rule yields approximately 0.63578, with a relative error of ≈ 0.00132.

8.8.28 The width of each subinterval is 1/50, so

$$M(50) = \frac{1}{50}(e^{-1/100} + (e^{-1/100})^3 + (e^{-1/100})^5 + \cdots + (e^{-1/100})^{99}) \approx 0.63211.$$

$$T(50) = \frac{1}{100}(e^0 + 2e^{-1/50} + 2e^{-2/50} + \cdots + 2e^{-49/50} + e^{-1}) \approx 0.63214.$$

The actual value of the integral is $1 - \frac{1}{e}$. The absolute error for $M(50)$ is $|1 - \frac{1}{e} - .63211| \approx 1 \times 10^{-5}$, and the relative error is that number divided by $1 - \frac{1}{e}$ which is about 1.66×10^{-5}. The absolute error for $T(50)$ is $|1 - \frac{1}{e} - .63214| \approx 2.1 \times 10^{-5}$ and the relative error is that number divided by $1 - \frac{1}{e}$ which is about 3.33×10^{-5}.

8.8.29

n	$M(n)$	Absolute Error	$T(n)$	Absolute Error
4	99	1	102	2
8	99.75	0.250	100.5	0.5
16	99.9375	0.0625	100.125	0.125
32	99.984375	0.0156	100.03125	0.03125

8.8.30

n	$T(n)$	Absolute Error	$M(n)$	Absolute Error
4	6	2	3	1
8	4.5	0.5	3.75	0.25
16	4.125	0.125	3.9375	0.0625
32	4.03125	0.03125	3.984375	0.015625

8.8.31

n	$M(n)$	Absolute Error	$T(n)$	Absolute Error
4	1.50968181	9.7×10^{-3}	1.48067370	1.9×10^{-2}
8	1.50241228	2.4×10^{-3}	1.49517776	4.8×10^{-3}
16	1.50060256	6.0×10^{-4}	1.49879502	1.2×10^{-3}
32	1.50015061	1.5×10^{-4}	1.49969879	3.0×10^{-4}

8.8.32

n	$M(n)$	Absolute Error	$T(n)$	Absolute Error
4	1.004785839	4.8×10^{-3}	0.99036501	9.6×10^{-3}
8	1.001210217	1.2×10^{-3}	0.99757542	2.4×10^{-3}
16	1.000303459	3.03×10^{-4}	0.99939282	6.07×10^{-4}
32	1.000075922	7.59×10^{-5}	0.99984814	1.52×10^{-4}

8.8.33 Because the given function has odd symmetry about the midpoint of the interval $[0, \pi]$, the midpoint rule calculates to be zero for all even values of n, as does the trapezoidal rule.

8.8.34

n	$M(n)$	Absolute Error	$T(n)$	Absolute Error
4	0.27572053	0.224	1.0373146	0.54
8	0.425459016	0.075	0.65651757	0.15
16	0.47975863	0.02	0.54098829	0.041
32	0.49482934	0.0052	0.51037346	0.01

8.8.35 $T(4) = 3\,(9.1 + 2(12) + 2(26) + 2(46) + 53) = 690.3\,\text{million}\,\text{ft}^3$.
$S(4) = 2(9.1 + 4(12) + 2(26) + 4(46) + 53) = 692.2\,\text{million ft}^3$.

8.8.36 $T(6) = 2(9.1 + 2(10.5) + 2(15) + 2(26) + 2(40) + 2(49) + 53) = 686.2\,\text{million}\,\text{ft}^3$.
$S(6) = \dfrac{4}{3}\,(9.1 + 4(10.5) + 2(15) + 4(26) + 2(40) + 4(49) + 53) = 685.5\,\text{million}\,\text{ft}^3$.

8.8.37 Answers may vary.

$$\begin{aligned}\overline{T} &= \frac{1}{12}\int_0^{12} T(t)\,dt \approx \frac{1}{12}\text{Trapezoid}(12)\\ &\approx \frac{1}{24}(47 + 2(50 + 46 + 45 + 48 + 52 + 54 + 61 + 62 + 63 + 63 + 59) + 55) = 54.5.\end{aligned}$$