1.

$$\int \left(x^{100} + \frac{\sqrt{x} - x^{3/2}}{x} \right) dx = \int \left(x^{100} + x^{\frac{1}{2}-1} - x^{\frac{3}{2}-1} \right) dx$$

$$= \int \left(x^{100} + x^{-\frac{1}{2}} - x^{\frac{1}{2}} \right) dx$$

$$= \frac{x^{101}}{101} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{x^{101}}{101} + 2x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$$

2.

$$\int \tan x \; sec^3 x \, dx$$

$$(u = \sec x, \quad du = \sec x \tan x \, dx)$$

$$= \int \tan x \sec x \sec^2 x \, dx = \int u^2 \, du$$

$$= \frac{u^3}{3} + C = \frac{\sec^3 x}{3} + C$$

3. Integration parts

$$\int x e^{3x} \, dx$$

diff	int
x	e^{3x}
1 $\xrightarrow{(-)}$	$\frac{1}{3}e^{3x}$

$$= \frac{x}{3} e^{3x} - \frac{1}{3} \int e^{3x} dx$$

$$= \frac{x}{3} e^{3x} - \frac{1}{9} e^{3x} + C$$

4.

$$\int \frac{100}{x^2-1}\,dx$$

$$\frac{100}{X^2-1} = \frac{100}{(X-1)(X+1)} = \frac{A}{X-1} + \frac{B}{X+1}$$

$$= \frac{A(X+1)+B(X-1)}{(X-1)(X+1)}$$

$$\Rightarrow 100 = A(X+1) + B(X-1)$$

$$\begin{cases} \text{let } X=-1 \Rightarrow 100 = -2B \Rightarrow B=-50 \\ \text{let } X=1 \Rightarrow 100 = 2A \Rightarrow A = 50 \end{cases}$$

$$\Rightarrow \int \frac{100\,dx}{X^2-1} = 50\int \left(\frac{1}{X-1} - \frac{1}{X+1}\right) dX$$

$$= 50\left(\ln|X-1| - \ln|X+1|\right) + C$$

5.

$$\int \frac{10}{\sqrt{10-100x^2}}\,dx = \int \frac{10\,dx}{\sqrt{10}\,\sqrt{1-10X^2}} = \sqrt{10}\int \frac{dx}{\sqrt{1-(\sqrt{10}\,x)^2}}$$

$$\underline{\underline{u = \sqrt{10}\,x}}$$

$$du = \sqrt{10}\,dx$$

$$\sqrt{10}\cdot\frac{1}{\sqrt{10}}\int \frac{du}{\sqrt{1-u^2}} = \int \frac{du}{\sqrt{1-u^2}}$$

$$= \sin^{-1}(u) + C = \sin^{-1}(\sqrt{10}\,x) + C$$

$$\frac{2x^2+6x+6}{(x^2+1)(3x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{3x+2}$$

6.

$$\int \frac{2x^2+6x+6}{(x^2+1)(3x+2)}\,dx$$

$$= \frac{(Ax+B)(3x+2)+C(x^2+1)}{(x^2+1)(3x+2)}$$

$$\Rightarrow 2x^2+6x+6 = (Ax+B)(3x+2)+C(x^2+1)$$

$$= 3Ax^2+2Ax+3Bx+2B+Cx^2+C$$

$$= (3A+C)x^2+(2A+3B)x+2B+C$$

$$\Rightarrow \begin{cases} 2 = 3A+C & ① \\ 6 = 2A+3B & ② \\ 6 = 2B+C & ③ \end{cases}$$

$$① - ③: \begin{cases} -4 = 3A-2B & ④ \\ 6 = 2A+3B & ② \end{cases}$$

$$④\times 3 + ②\times 2: \quad 0 = 9A+4A \Rightarrow A = 0$$

$$\Rightarrow C = 2-3A = 2$$

$$3B = 6 \Rightarrow B = 2$$

$$\Rightarrow \int \frac{2x^2+6x+6}{(x^2+1)(3x+2)}\,dx = 2\int \frac{dx}{x^2+1} + 2\int \frac{dx}{3x+2}$$

$u = 3x+2$, $du = 3dx$

$$= 2\tan^{-1}x + 2\cdot\frac{1}{3}\int\frac{du}{u}$$

$$= 2\tan^{-1}x + \frac{2}{3}\ln|3x+2| + C$$

7.

$$\int_0^1 \frac{6x}{(x^2+1)^5}\,dx$$

First, $\displaystyle\int \frac{6x}{(x^2+1)^5}\,dx \overset{u=x^2+1}{\underset{du=2x\,dx}{=}} \int \frac{3\,du}{u^5}$

$$= \int 3u^{-5}\,du = \frac{3u^{-5+1}}{-5+1} + C = \frac{-3}{4}u^{-4} + C$$

$$= \frac{-3}{4}(x^2+1)^{-4} + C$$

$$\Rightarrow \int_0^1 \frac{6x\,dx}{(x^2+1)^5} = -\frac{3}{4}\left.\frac{1}{(x^2+1)^4}\right|_0^1 = -\frac{3}{4}\left[\frac{1}{2^4} - 1\right]$$

$$= \frac{-3}{4}\left(\frac{1}{16}-1\right) = \frac{3}{4}\cdot\frac{15}{16} = \frac{45}{64}$$

Integration by Parts

8.

$$\int x^2 \cos x \, dx$$

diff		Int
x^2		$\cos x$
$2x$	$(-)$	$\sin x$
2	$(+)$	$-\cos x$ stop!

$$= x^2 \sin x + 2x\cos x - 2\int \cos x \, dx$$

$$= x^2 \sin x + 2x \cos x - 2\sin x + C$$

9.

$$\int_1^\infty \frac{2}{(x+1)^{3/2}} dx$$

First, $\displaystyle\int \frac{2dx}{(x+1)^{3/2}} = 2\int (x+1)^{-\frac{3}{2}} d(x+1)$

$$= 2 \, \frac{(x+1)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C = -4(x+1)^{-\frac{1}{2}} + C$$

$$\Rightarrow \int_1^\infty \frac{2dx}{(x+1)^{3/2}} = \lim_{t\to\infty} \int_1^t \frac{2dx}{(x+1)^{3/2}} = -4 \lim_{t\to\infty} (x+1)^{-\frac{1}{2}} \Big|_1^t$$

$$= -4\left[\lim_{t\to\infty} \frac{1}{\sqrt{t+1}} - \frac{1}{\sqrt{1+1}}\right] = -4\left(0 - \frac{1}{\sqrt{2}}\right)$$

$$= 4/\sqrt{2} = 2\sqrt{2}$$

10.

$$\int_1^2 \frac{1}{(x-1)^{1/2}} dx$$

First $\displaystyle\int (x-1)^{-\frac{1}{2}} dx = \int (x-1)^{-\frac{1}{2}} d(x-1) = \frac{(x-1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$$= 2(x-1)^{\frac{1}{2}} + C$$

$$= \lim_{t\to 1^+} \int_t^2 \frac{dx}{(x-1)^{\frac{1}{2}}} = 2\lim_{t\to 1^+} (x-1)^{\frac{1}{2}} \Big|_t^2$$

$$= 2\left[\cancel{\lim_{t\to 1^+}} (2-1)^{\frac{1}{2}} - \lim_{t\to 1^+} (t-1)^{\frac{1}{2}}\right]$$

$$= 2(1-0) = 2$$