1.

$\int\left(\sqrt[3]{x}+\frac{12x^5}{x^{3/2}}\right)dx = \int \left(x^{\frac{1}{3}} + 12\, x^{5-\frac{3}{2}} \right) dx$

$= \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + 12\,\frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} + C$

$= \frac{3}{4}x^{\frac{4}{3}} + 12\cdot\frac{2}{9}x^{\frac{9}{2}} + C$

2.

$\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx \quad \frac{u=\sqrt{x}}{du=\frac{1}{2}x^{-\frac{1}{2}}dx} \quad \int 2\cos u \, du$

$= 2\sin u + C = 2\sin(\sqrt{x}) + C$

diff Int

3.

$\int \frac{\ln x}{x^3}\, dx$ $\quad \ln x \quad x^{-3}$

$= -\frac{x^{-2}\ln x}{2} + \int \frac{1}{x}\cdot\frac{x^{-2}}{2}dx$ $\quad \frac{1}{x} \xrightarrow{(-)} -\frac{x^{-2}}{2} = \cancel{-\frac{1}{2x^2}}$

$= -\frac{\ln x}{2x^2} + \frac{1}{2}\int x^{-3}dx$

$= -\frac{\ln x}{2x^2} - \frac{1}{4}x^{-2} + C$

4. $\int \frac{x}{x^2-3x+2}dx$ $= \int \frac{x\,dx}{(x-1)(x-2)}$

let $\frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$

$\Rightarrow x = A(x-2)+B(x-1)$ $\begin{cases} \text{let } x=1, & 1=-A \\ \text{let } x=2 & 2=B \end{cases}$

$\Rightarrow \begin{cases} A=-1 \\ B=2 \end{cases}$

$\Rightarrow \int \frac{x\,dx}{x^2-3x+2} = -\int \frac{dx}{x-1} + 2\int \frac{dx}{x-2}$

$= -\ln|x-1| + 2\ln|x-2| + C$

5. $\int \frac{3}{\sqrt{9-x^2}}dx$ $= \frac{1}{\sqrt{9}} \int \frac{3\,dx}{\sqrt{1-\frac{x^2}{9}}} = \cancel{\frac{1}{3}} \int \frac{\cancel{3}\,dx}{\sqrt{1-\left(\frac{x}{3}\right)^2}}$

$\underline{\underline{\substack{u=\frac{x}{3} \\ du=\frac{1}{3}dx}}}$ $\int \frac{3\,du}{\sqrt{1-u^2}}$

$= 3\arcsin(u) + C = 3\arcsin\left(\frac{x}{3}\right) + C$

6.

$$\int \frac{x^2+2x-3}{(x^2+1)(x-2)}dx$$

$$\frac{X^2+2X-3}{(X^2+1)(X-2)} = \frac{AX+B}{X^2+1} + \frac{C}{X-2} = \frac{(AX+B)(X-2)+C(X^2+1)}{(X^2+1)(X-2)}$$

$$\Rightarrow X^2+2X-3 = (AX+B)(X-2)+C(X^2+1)$$

i) let $X=2$. $5 = 5C \Rightarrow C=1$

ii) let $X=0$, $-3 = -2B+1 \Rightarrow B=2$

iii) let $X=1$. $0 = -(A+2)+2 \Rightarrow A=0$

$$\Rightarrow \int \frac{X^2+2X-3}{(X^2+1)(X-2)}dx = 2\int \frac{dx}{X^2+1} + \int \frac{dx}{X-2}$$

$$= 2\arctan(x) + \ln|x-2| + C$$

7.

$$\int_0^1 \frac{x}{4+x^2}\,dx \xlongequal[du=2xdx]{u=4+x^2} \frac{1}{2}\int_{4+0^2}^{4+1^2} \frac{1}{u}\,du$$

$$= \frac{1}{2}\int_4^5 \frac{1}{u}\,du = \frac{1}{2}\ln u\Big|_4^5$$

$$= \frac{1}{2}(\ln 5 - \ln 4) = \frac{1}{2}\ln\left(\frac{5}{4}\right)$$

8.

$$\int x^2 \sin(2x)\,dx$$

diff		Int
x^2		$\sin 2x$
$2x$	$(-)$	$-\frac{1}{2}\cos 2x$
2	$(+)$	$-\frac{1}{4}\sin 2x$

$$= \frac{-1}{2}x^2\cos 2x + \frac{2x}{4}\sin 2x - \frac{1}{2}\int \sin 2x\,dx$$

$$= \frac{-1}{2}x^2\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C$$

9. $\int_2^\infty \frac{1}{(x+1)^{10/9}}\,dx$

First, $\int \frac{dx}{(x+1)^{10/9}} = \int (x+1)^{-\frac{10}{9}}\,dx = \frac{(x+1)^{-\frac{10}{9}+1}}{-\frac{10}{9}+1} + C$

$= -9(x+1)^{-\frac{1}{9}} + C$

Thus, $\int_2^\infty \frac{dx}{(x+1)^{10/9}} = \lim_{t\to\infty} \int_2^t \frac{dx}{(x+1)^{10/9}}$

$= -9 \lim_{t\to\infty} (x+1)^{-\frac{1}{9}} \Big|_2^t = -9\left[\lim_{t\to\infty} \frac{1}{(t+1)^{1/9}} - \frac{1}{3^{1/9}}\right]$

$= -9\left(0 - \frac{1}{3^{1/9}}\right) = \frac{9}{3^{1/9}}$

10. $\int_0^5 \frac{10}{\sqrt{x}}\,dx$

First, $\int \frac{10}{\sqrt{x}}\,dx = 10\int x^{-\frac{1}{2}}\,dx = 10\,\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$

$= 20x^{\frac{1}{2}} + C = 20\sqrt{x} + C$

Thus, $\int_0^5 \frac{10\,dx}{\sqrt{x}} = \lim_{t\to 0^+} \int_t^5 \frac{10\,dx}{\sqrt{x}}$

$= 20 \lim_{t\to 0^+} \sqrt{x}\,\Big|_t^5 = 20\left(\sqrt{5} - \lim_{t\to 0^+} \sqrt{t}\right)$

$= 20(\sqrt{5} - 0) = 20\sqrt{5}.$