East Stroudsburg University
Department of Mathematics

MATH 141 – Calculus II – Fall 2021
Exam 1

Date of Examination: Tuesday, September 21, 2021
Time of Examination: 11:00 AM – 12:00 PM
Instructor: Xuemao Zhang

Name: (Last) ______________ **(First)** Solution .

READ THESE INSTRUCTIONS

1. You have **60 minutes** to complete this test. Candidates must NOT start writing their answers until told to do so.

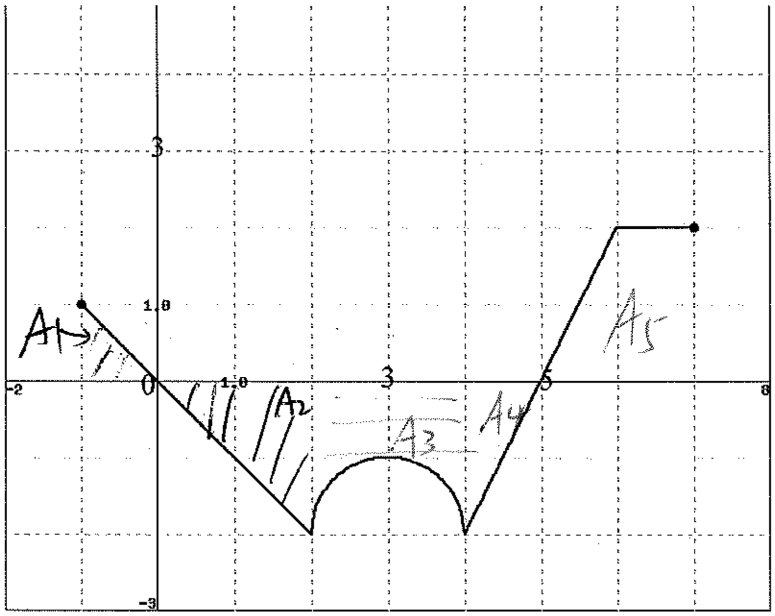
2. This test is entirely closed-book and closed-notes. The cheat sheet for the Gateway Exam will be distributed to the class. You may use a calculator for some questions. However no examination aids other than those specified are permitted.

3. No hints will be given during the exam.

4. There are **4** pages in this examination, excluding this cover page.

5. Good luck!

Question 1 (9 marks). A graph of $f(x)$ is shown below.



Evaluate each integral by interpreting it in terms of areas.

(1) $\int_{-1}^{2} f(x)dx = A_1 - A_2 = \frac{1}{2}\times 1 - \frac{1}{2}\times 2\times 2$

$= \frac{1}{2} - 2 = \frac{-3}{2}$

(2) $\int_{2}^{4} f(x)dx = -A_3 = -\left(2\times 2 - \frac{1}{2}\cdot \pi \cdot 1^2\right)$

$= -\left(4 - \frac{\pi}{2}\right) = \frac{\pi}{2} - 4$

(3) $\int_{-1}^{7} f(x)dx = A_1 + A_5 - (A_2 + A_3 + A_4)$

$= \frac{1}{2} + (2\times 2 - 1\times 2\times \frac{1}{2}) - (2 + 4 - \frac{\pi}{2} + 1)$

$= \frac{1}{2} + 3 - 7 + \frac{\pi}{2} = \frac{\pi}{2} - \frac{7}{2}$

Question 2 (12 marks). Find the following definite integrals.

(1) $\int_0^2 x^3(\sqrt{x} + \sqrt[5]{x})dx$

$$\int x^3(\sqrt{x}+\sqrt[5]{x})dx = \int (x^{\frac{7}{2}} + x^{\frac{16}{5}})dx = \frac{2}{9}x^{\frac{9}{2}} + \frac{5}{21}x^{\frac{21}{5}} + C$$

$$\Rightarrow \int_0^2 x^3(\sqrt{x}+\sqrt[5]{x})dx = \frac{2}{9}x^{\frac{9}{2}} + \frac{5}{21}x^{\frac{21}{5}}\Big|_0^2$$

$$= \frac{2}{9}\cdot 2^{\frac{9}{2}} + \frac{5}{21}\, 2^{\frac{21}{5}}$$

(2) $\int_{-5}^6 (6x - e^x)dx$

$$\int (6x - e^x)\,dx = 6\cdot\frac{x^2}{2} - e^x + C$$

$$\Rightarrow \int_{-5}^6 (6x-e^x)dx = (3x^2 - e^x)\Big|_{-5}^6$$

$$= (3\cdot 6^2 - e^6) - (3\cdot 5^2 - e^{-5})$$

$$= 33 - e^6 + e^{-5}$$

(3) $\int_1^{\sqrt{10}} \frac{6}{1+x^2}dx$

$$\int \frac{6}{1+x^2}dx = 6\arctan x + C$$

$$\Rightarrow \int_1^{\sqrt{10}} \frac{6}{1+x^2}dx = 6\arctan x\Big|_1^{\sqrt{10}}$$

$$= 6(\arctan(\sqrt{10}) - \text{artan}\, 1)$$

$$= 6(\arctan\sqrt{10} - \frac{\pi}{4})$$

Question 3 (8 marks). Find the following indefinite integrals using the substitution method.

(1) $\int \frac{x^5}{\sqrt{x^6+6}} dx$

Let $u = x^6+6$, Then $du = 6x^5 dx$

and $x^5 dx = \frac{1}{6} du$

$$\Rightarrow \int \frac{x^5}{\sqrt{x^6+6}} dx = \frac{1}{6}\int \frac{du}{\sqrt{u}} = \frac{1}{6}\int u^{-\frac{1}{2}} du$$

$$= \frac{1}{6} \cdot \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{1}{6} \cdot 2 \cdot u^{\frac{1}{2}} + C$$

$$= \frac{1}{3}\sqrt{x^6+6} + C$$

(2) Suppose $x > 0$, find $\int \frac{ln(x^5)}{x} dx$

$$\int \frac{\ln(x^5)}{x} dx = 5\int \frac{\ln x}{x} dx$$

$$\xlongequal[du = \frac{1}{x}dx]{u = \ln x} 5\int u \, du = 5 \cdot \frac{u^2}{2} + C$$

$$= \frac{5}{2}(\ln x)^2 + C$$

Question 4 (6 marks).

If $f(t)$ is continuous and $\int_1^8 f(t)dt = 12$, find the integral $\int_1^2 t^2 f(t^3)dt$.

$$\int_1^2 t^2 f(t^3)\,dt \;\; \frac{u = t^3}{\substack{du = 3t^2 dt \\ t^2 dt = \frac{1}{3} du}} \;\; \frac{1}{3}\int_1^8 f(u)\,du$$

$$= \frac{1}{3} \cdot 12 = 4$$

Question 5 (6 marks). A car drives down a road in such a way that its velocity (in m/s) at t (seconds) is

$$v(t) = 2t^{\frac{1}{2}} + 1$$

Find the car's average velocity (in m/s) between t =1 and t =7.

Sol. $\int v(t)\,dt = \int (2t^{\frac{1}{2}}+1)\,dt = 2 \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + t + C$

$$= \frac{4}{3} t^{\frac{3}{2}} + t + C$$

$\Rightarrow$ Average velocity of the car is

$$\frac{\int_1^7 v(t)\,dt}{7-1} = \frac{1}{6}\int_1^7 v(t)\,dt = \frac{1}{6}\cdot\left[\frac{4}{3}t^{\frac{3}{2}} + t\right]_1^7$$

$$= \frac{1}{6}\left[\frac{4}{3}\cdot 7^{\frac{3}{2}} + 7 - \frac{4}{3} - 1\right]$$

$$= \frac{1}{6}\left(\frac{4}{3}\cdot 7^{\frac{3}{2}} + \frac{14}{3}\right)$$