Homework 2 Solution

Monday, September 20, 2021 10:55 AM

1. **(1 point)** Library/UCSB/Stewart5_7_5/Stewart5_7_5_65.pg

Evaluate the integral

$$\int \frac{10}{\sqrt{x+1}+\sqrt{x}}\,dx$$

Note: Use an upper-case "C" for the constant of integration.

sol. $\dfrac{10}{\sqrt{x+1}+\sqrt{x}} \cdot \dfrac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}} = \dfrac{10(\sqrt{x+1}-\sqrt{x})}{1}$

$$\Rightarrow \int \frac{10\,dx}{\sqrt{x+1}+\sqrt{x}} = 10\int(\sqrt{x+1}-\sqrt{x})\,dx$$

$$= 10\cdot\frac{(x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 10\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{20}{3}(x+1)^{\frac{3}{2}} - \frac{20}{3}x^{\frac{3}{2}} + C$$

2. **(1 point)** Library/Union/setIntByParts/sc5_6_01.pg

Evaluate the indefinite integral.

$\int xe^{3x}\,dx =$ ______________ $+C.$

Correct Answers:

- 1/3*[x*e^(3*x)-1/3*e^(3*x)]

diff	int
x	e^{3x}
1	$\frac{1}{3}e^{3x}$

$(-)$

due to substitution $u=3x$

$$= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\,dx$$

$$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

Evaluate the following integral:

$$\int^{2} \frac{3\ln(x)}{\ldots}\,dx$$

$\int_1 \qquad x^2 \qquad dx$

Correct Answers:

- `3*(1/2-1/2*ln(2))`

diff	int
$\ln x$	x^{-2}
$\frac{1}{x}$ $\xrightarrow[(-)]{}$	$-x^{-1}$

$$\Rightarrow \int \frac{3\ln x}{x^2}\,dx = 3\left[-\frac{\ln x}{x} + \int x^{-2}\,dx\right]$$

$$= 3\left[-\frac{\ln x}{x} - x^{-1}\right] + C$$

$$= -3\,\frac{1+\ln x}{x}$$

$$\Rightarrow \int_1^2 \frac{3\ln x}{x^2}\,dx$$

$$= -3\cdot\frac{1+\ln x}{x}\Big|_1^2 = -3\left[\frac{1+\ln 2}{2} - 1\right]$$

$$= 3 - \frac{3(1+\ln 2)}{2}$$

$$= -3\left[-\frac{1}{2} + \frac{\ln 2}{2}\right] = 3\left[\frac{1}{2} - \frac{\ln 2}{2}\right]$$

---

4. **(1 point)** `Library/UCSB/Stewart5_7_1/Stewart5_7_1_22.pg`

Evaluate the following integral:

$$\int_1^4 1\sqrt{t}\ln(t)\,dt$$

Correct Answers:

- `1*(16/3*ln(4)-28/9)`

diff	int
$\ln t$	$t^{\frac{1}{2}}$

$$\frac{1}{t} \xrightarrow{(-)} \frac{2}{3} t^{\frac{3}{2}}$$

$$\Rightarrow \int \sqrt{t} \ln t \, dt = \frac{2}{3} \ln t \; t^{\frac{3}{2}} - \frac{2}{3} \int t^{\frac{1}{2}} \, dt$$

$$= \frac{2}{3} \ln t \; t^{\frac{3}{2}} - \frac{2}{3} \cdot \frac{2}{3} t^{\frac{3}{2}} + C$$

$$= \frac{2}{3} t^{\frac{3}{2}} \left(\ln t - \frac{2}{3} \right) + C$$

$$\Rightarrow \int_1^4 \sqrt{t} \ln t \, dt = \frac{2}{3} \left[4^{\frac{3}{2}} \left(\ln 4 - \frac{2}{3} \right) - \left(0 - \frac{2}{3} \right) \right]$$

$$= \frac{2}{3} \left[8 \left(\ln 4 - \frac{2}{3} \right) + \frac{2}{3} \right]$$

$$= \frac{2}{3} \left[8 \ln 4 - \frac{14}{3} \right]$$

---

5. **(1 point)** `Library/UCSB/Stewart5_7_1/Stewart5_7_1_23.pg`

Evaluate the integral

$$\int_0^1 \frac{-8y}{e^{2y}} \, dy$$

Correct Answers:

- `-3/4*exp(-2)*-8+1/4*-8`

diff   int

y e^{-2y} $u = -2y$, $du = -2 \, dy$

$$1 \xrightarrow{(-)} -\frac{1}{2} e^{-2y}$$

$$\Rightarrow \int \frac{y}{e^{2y}} \, dy = -\frac{1}{2} y e^{-2y} + \frac{1}{2} \int e^{-2y} \, dy$$

$$= -\frac{1}{2} y e^{-2y} + \frac{1}{2} \left(-\frac{1}{2} \right) e^{-2y} + C$$

$$= e^{-2y}\left(-\frac{y}{2}-\frac{1}{4}\right)+C$$

$$\Rightarrow \int_0^1 \frac{-8y}{e^{2y}}\,dy = \left(e^{-2y}\left(\frac{-y}{2}-\frac{1}{4}\right)\Big|_0^1\right)\times(-8)$$

$$= \left(e^{-2}\left(-\frac{3}{4}\right)-\left(-\frac{1}{4}\right)\right)(-8)$$

$$= 6e^{-2}-2$$

---

6. (1 point) `Library/UCSB/Stewart5_7_4/Stewart5_7_4_50.pg`

Use integration by parts and the technique of partial fractions to evaluate the integral

$$\int -3x\,\underline{\arctan}(x)\,dx$$

← do not type tan^(-1) x !!!

Note: Use an upper-case "C" for the constant of integration.

____

diff		int
$\arctan x$		$-3x$
$\frac{1}{1+x^2}$	$\xrightarrow{(-)}$	$\frac{-3x^2}{2}$

$$\Rightarrow \int -3x\arctan x\,dx$$

$$= -\frac{3}{2}x^2\arctan x + \frac{3}{2}\int \frac{x^2+1-1}{1+x^2}\,dx$$

$$= -\frac{3}{2}x^2\arctan x + \frac{3}{2}\left[\int 1\,dx - \int \frac{1}{1+x^2}\,dx\right]$$

$$= -\frac{3}{2}x^2 \arctan x + \frac{3x}{2} - \frac{3}{2}\arctan x + C$$

---

7. **(1 point)** Library/UCSB/Stewart5_7_1/Stewart5_7_1_33.pg

First make a substitution and then use integration by parts to evaluate the integral

$$\int -7\sin(\sqrt{x})\,dx \quad = -7\int \sin\sqrt{x}\,dx$$

Note: Use an upper-case "C" for the constant of integration.

$$u = \sqrt{x} = x^{\frac{1}{2}},\quad du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2}\frac{dx}{\sqrt{x}}$$

$$= \frac{1}{2}\frac{dx}{u}$$

$$\Rightarrow \quad dx = 2u\,du$$

$$\int \sin\sqrt{x}\,dx \;\underset{=}{\overset{u=\sqrt{x}}{=}}\; 2\int u\sin u\,du$$

diff	int
u	$\sin u$
1	$-\cos u$

$(-)$

$$= 2\left[-u\cos u + \int \cos u\,du\right]$$

$$= 2(-u\cos u + \sin u) + C$$

$$= 2(-\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x}) + C$$

---

8. **(1 point)** Library/UCSB/Stewart5_7_2/Stewart5_7_2_54.pg

The integral

$$\int -7\sin(x)\cos(x)\,dx$$

can be evaluated in four different ways:

(1) The substitution $u = \cos(x)$
(2) The substitution $u = \sin(x)$
(3) The identity $\sin(2x) = 2\sin(x)\cos(x)$
(4) Integration by parts

Use any of these methods to evaluate the integral.
Note: Use an upper-case "C" for the constant of integration.

Correct Answers:

---

9. **(1 point)** `Library/UCSB/Stewart5_7_1/Stewart5_7_1_34.pg`

First make a substitution and then use integration by parts to evaluate the integral

$$\int_1^4 10e^{\sqrt{x}}\,dx$$

Correct Answers:

- `10*2*exp(2)`

$u = \sqrt{x}, \Rightarrow u^2 = x$

$\frac{du^2}{du} \cdot \frac{du}{dx} = 1 \Rightarrow 2u\frac{du}{dx} = 1$

chain Rule

$\Rightarrow dx = 2u\,du$

$$\int e^{\sqrt{x}}\,dx \overset{u=\sqrt{x}}{=\!=} 2\int u e^u\,du$$

diff int

u e^u

1 $\xrightarrow{(-)}$ e^u

$= 2[ue^u - e^u) + C$

$= 2(\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}}) + C$

---

10. **(1 point)** `Library/Michigan/Chap7Sec2/Q38.pg`

For each of the following integrals, indicate whether integration by substitution or integration by parts is more appropriate, or if neither method is appropriate. Do not evaluate the integrals.

1. $\int x \sin x\,dx$

- A. substitution
- ~~B. integration by parts~~
- C. neither

2. $\int \frac{x^4}{1+x^5}\,dx$

- A. substitution $u = 1 + x^5$
- B. neither

- B. neither
- C. integration by parts

3. $\int x^4 e^{x^5} dx$

$u = x^5$

- A. substitution
- B. integration by parts
- C. neither

4. $\int x^4 \cos(x^5)\, dx$

$u = x^5$

- A. substitution
- B. neither
- C. integration by parts

5. $\int \frac{1}{\sqrt{2x+1}}\, dx$

$\int (2x+1)^{-\frac{1}{2}}\, dx$

$u = 2x+1$

- A. substitution
- B. integration by parts
- C. neither

(Note that because this is multiple choice, you will not be able to see which parts of the problem you got correct.)

Solution: *(Instructor solution preview: show the student solution after due date.)*