Xuemao Zhang **Math141_Calculus_II**
Assignment Homework4 due 10/04/2021 at 11:59pm EDT

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1. **(1 point)** `Library/UCSB/Stewart5_7_3/Stewart5_7_3_9.pg`

Evaluate the integral

$$\int \frac{-7}{\sqrt{x^2+16}}\,dx$$

Note: Use an upper-case "C" for the constant of integration.

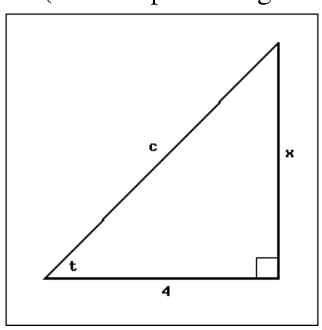
Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

We use the trigonometric substitution: $x = 4\tan\theta$ where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.
Then $dx = 4\sec^2\theta\,d\theta$ and
$\sqrt{x^2+16} = \sqrt{16\tan^2\theta+16} = 4\sqrt{1+\tan^2\theta} = 4\sqrt{\sec^2\theta} = 4\sec\theta$. So

$$\begin{aligned}\int \frac{-7}{\sqrt{x^2+16}}\,dx &= -7\int \frac{1}{4\sec\theta}\left(4\sec^2\theta\right)d\theta\\ &= -7\int \sec\theta\,d\theta\\ &= -7\ln|\sec\theta+\tan\theta|+C\end{aligned}$$

We now need to return to the original variable, x. From the original substitution, $x = 4\tan\theta \implies \tan\theta = \frac{x}{4}$. If we interpret θ as being an angle in a right triangle, and label the side opposite θ as x and the side adjacent to θ as 4, we get a triangle as shown below (with t representing the angle θ).



Using the Pythagorean theorem, we solve for the hypotenuse and get $c = \sqrt{x^2+16}$. So $\cos\theta = \frac{4}{\sqrt{x^2+16}}$, and $\sec\theta = \frac{\sqrt{x^2+16}}{4}$ (for all values of θ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$) .
Therefore, the indefinite integral continues as

$$\begin{aligned}-7\int \frac{1}{\sqrt{x^2+16}}\,dx &= -7\ln|\sec\theta+\tan\theta|+C\\ &= -7\ln\left(\frac{\sqrt{x^2+16}}{4}+\frac{x}{4}\right)+C\\ &= -7\ln\left(\frac{\sqrt{x^2+16}+x}{4}\right)+C\\ &= -7\ln\left(\sqrt{x^2+16}+x\right)+7\ln 4+C\\ &= -7\ln\left(\sqrt{x^2+16}+x\right)+C\end{aligned}$$

Correct Answers:

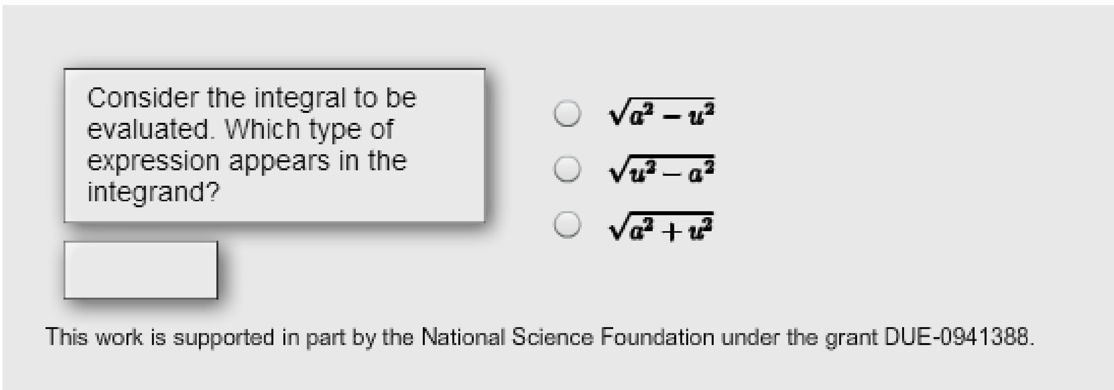
- `-7*ln(x+sqrt(x^2+16))+C+c`

2. **(1 point)** `Library/CSUOhio/calculus/trigonometric_substitution/trigSub10.pg`

Evaluate the indefinite integral.

$$\int \frac{dx}{(16-x^2)^{3/2}}$$

Hi xzhang2, If you don't get this in 5 tries I'll give you a hint with an applet to help you out. (Instructor hint preview: show the student hint after the following number of attempts: 5



Follow the step-by-step questions in the hint in the online version of this problem.

Solution: *(Instructor solution preview: show the student solution after due date.)*

Solution:

To evaluate this integral use a trigonometric substitution. For this problem use the tan substitution.

$$x = 4\sin(\theta)$$

Before proceeding note that $\sin\theta = \frac{x}{4}$, and $\cos\theta = \frac{\sqrt{16-x^2}}{4}$. To see this, label a right triangle so that the sine is $x/4$. We will have the opposite side with length x, and the hypotenuse with length 4, so the adjacent side has length $\sqrt{16-x^2}$.

With the substitution

$$x = 4\sin\theta$$
$$dx = 4\cos\theta\, d\theta$$

Therefore:

$$\begin{aligned}\int \frac{dx}{(16-x^2)^{3/2}} &= \int \frac{4\cos\theta}{(16-16\sin^2\theta)^{3/2}}\, d\theta \\ &= \int \frac{4\cos\theta}{64\cos^3\theta}\, d\theta \\ &= \frac{1}{16}\int \sec^2\theta\, d\theta\end{aligned}$$

$$= \frac{1}{16}\tan\theta + C$$

Substituting back in terms of θ yields:

$$= \frac{1}{16}\tan\theta + C = \frac{x}{16\sqrt{16-x^2}} + C$$

so

$$\int \frac{dx}{(16-x^2)^{3/2}} = \frac{x}{16\sqrt{16-x^2}} + C$$

Correct Answers:

- `x/[16*(16-x^2)^0.5]+C`

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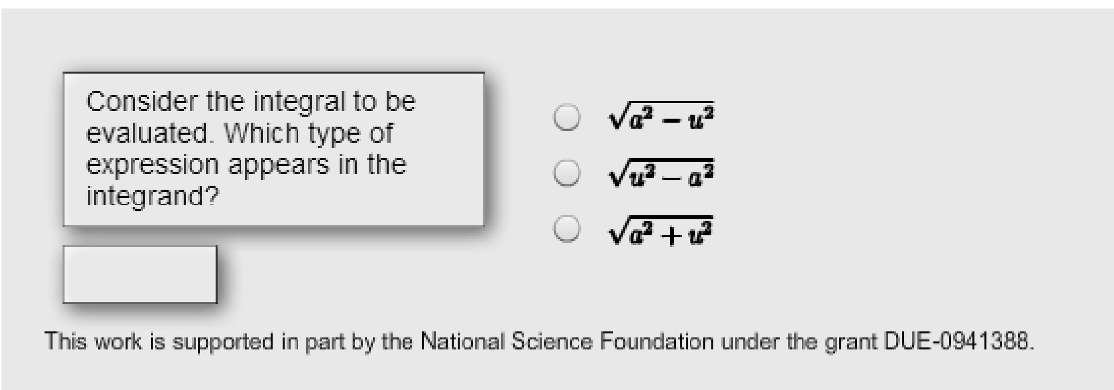
3. (1 point) `Library/CSUOhio/calculus/trigonometric_substitution/trigSub24.pg`

Evaluate the indefinite integral.

$$\int \frac{\sqrt{x^2-4}}{x}\,dx$$

Hi xzhang2, If you don't get this in 5 tries I'll give you a hint with an applet to help you out. (Instructor hint preview: show the student hint after the following number of attempts: 5



Follow the step-by-step questions in the hint in the online version of this problem.

Solution: *(Instructor solution preview: show the student solution after due date.)*

Solution:

To evaluate this integral use a trigonometric substitution. For this problem use the secant substitution.

$$x = 2\sec\theta$$

We are motivated by the trigonometric identity

$$\sec^2\theta - 1 = \tan^2\theta.$$

With the substitution $x = 2\sec\theta$, $\sqrt{x^2-4} = \sqrt{4\sec^2\theta-4} = 2\tan\theta$ for $x > 2$, where $0 \le \theta < \pi/2$ and $\sqrt{x^2-4} = \sqrt{4\sec^2\theta-4} = -2\tan\theta$ for $x < -2$, where $\pi/2 < \theta \le \pi$. Note that $\sec\theta = \frac{x}{2}$, and $\sin\theta = \frac{\sqrt{x^2-4}}{x}$. To see this, label a right triangle so that the secant is $x/2$. We will have the adjacent side of length 2, and the hypotenuse with length x, so the opposite side has length $\sqrt{x^2-4}$.

With the substitution

$$x = 2\sec\theta$$

$$dx = 2\sec\theta\tan\theta\, d\theta$$

Therefore:

$$\int \frac{\sqrt{x^2-4}}{x} dx = \int 2\sec\theta\tan\theta \frac{\sqrt{4\sec^2\theta - 4}}{2\sec\theta}\, d\theta$$

$$= \int 2\tan^2\theta\, d\theta$$

$$= \int 2(\sec^2\theta - 1)\, d\theta$$

$$= 2\tan\theta - 2\theta + C$$

Substituting back in terms of x:

$$2\tan\theta - 2\theta + C$$

$$= \sqrt{x^2-4} - 2\sec^{-1}\left(\frac{x}{2}\right) + C$$

so

$$\int \frac{\sqrt{x^2-4}}{x} dx = \sqrt{x^2-4} - 2\sec^{-1}\left(\frac{x}{2}\right) + C$$

Correct Answers:

- `sqrt(x^2-4)-2*atan([sqrt(x^2-4)]/2)+C`

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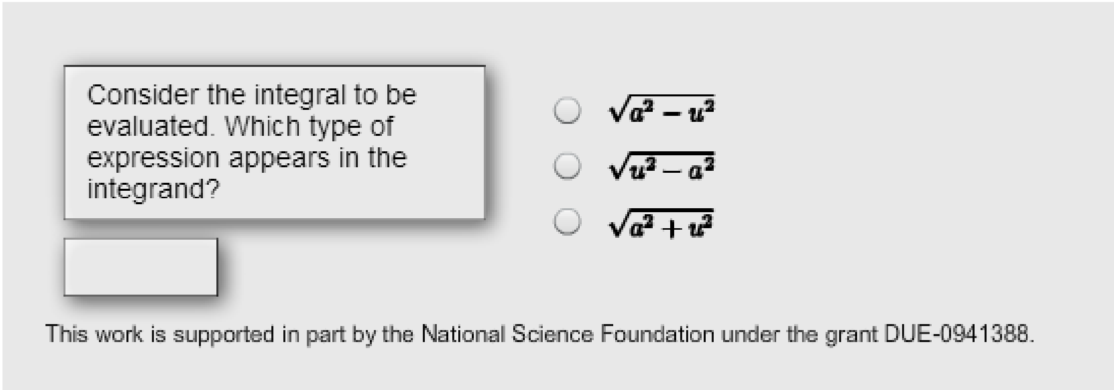
4. **(1 point)** `Library/CSUOhio/calculus/trigonometric_substitution/trigSub21.pg`

Evaluate the indefinite integral.

$$\int \frac{\sqrt{4-x^2}}{x^2} dx$$

Hi xzhang2, If you don't get this in 5 tries I'll give you a hint with an applet to help you out.
(Instructor hint preview: show the student hint after the following number of attempts: 5



Follow the step-by-step questions in the hint in the online version of this problem.

Solution: *(Instructor solution preview: show the student solution after due date.)*

Solution:

To evaluate this integral use a trigonometric substitution. For this problem use the sine substitution.

$$x = 2\sin(\theta)$$

Before proceeding note that $\sin\theta = \frac{x}{2}$, and $\cos\theta = \frac{\sqrt{4-x^2}}{2}$. To see this, label a right triangle so that the sine is $x/2$. We will have the opposite side with length x, and the hypotenuse with length 2, so the adjacent side has length $\sqrt{4-x^2}$.

With the substitution

$$x = 2\sin\theta$$
$$dx = 2\cos\theta\, d\theta$$

Therefore:

$$\begin{aligned}
\int \frac{\sqrt{4-x^2}}{x^2} dx &= \int \frac{2\cos\theta\sqrt{4-4\sin^2\theta}}{4\sin^2\theta}\, d\theta \\
&= \int \frac{\cos^2\theta}{\sin^2\theta}\, d\theta \\
&= \int \cot^2\theta\, d\theta \\
&= \int \csc^2\theta - 1\, d\theta \\
&= -\cot\theta - \theta + C
\end{aligned}$$

Substituting back in terms of x yields:

$$-\cot\theta - \theta + C = -\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\left(\frac{x}{2}\right) + C$$

so

$$\int \frac{\sqrt{4-x^2}}{x^2} dx = -\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\left(\frac{x}{2}\right) + C$$

Correct Answers:

- -[sqrt(4-x^2)]/x-asin(x/2)+C

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5. **(1 point)** Library/UCSB/Stewart5_7_5/Stewart5_7_5_33.pg

Evaluate the integral

$$\int 5\sqrt{3-2x-x^2}\,dx$$

Note: Use an upper-case "C" for the constant of integration.

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Correct Answers:

- 5*(-1/4*(-2*x-2)*(3-2*x-x^2)^(1/2)+2*asin(1/2*x+1/2))+C+c

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6. **(1 point)** Library/ma123DB/set3/s7_4_19.pg

Evaluate the indefinite integral.

$$\int \frac{-4}{x^2-4x+4}dx$$

Answer: ______________ $+C$

Correct Answers:

- 4/(x+-2)

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7. **(1 point)** Library/ma123DB/set3/s7_4_31.pg

The form of the partial fraction decomposition of a rational function is given below.

$$\frac{x^2-3x-3}{(x+4)(x^2+9)} = \frac{A}{x+4} + \frac{Bx+C}{x^2+9}$$

$A =$ _____ $B =$ _____ $C =$ _____

Now evaluate the indefinite integral.

$$\int \frac{x^2-3x-3}{(x+4)(x^2+9)}dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Multiplying by the least common denominator gives

$$x^2-3x-3 = A(x^2+9)+(Bx+C)(x+4)$$

Rearranging terms on the right hand side, yields

$$x^2-3x-3 = (A+B)x^2+(4B+C)x+9A+4C$$

Now we equate the coefficients:

$$\begin{aligned} A+B &= 1 \\ 4B+C &= -3 \\ 9A+4C &= -3 \end{aligned}$$

Solving the system gives $A=1,\ B=0$ and $C=-3$ so the partial fraction decomposition is

$$\frac{x^2-3x-3}{(x+4)(x^2+9)} = \frac{1}{x+4} + \frac{-3}{x^2+9}$$

The definite integral is then

$$\int \frac{x^2-3x-3}{(x+4)(x^2+9)} dx = \ln(|x+4|) + \frac{-3\tan^{-1}\left(\frac{x}{3}\right)}{3} + C$$

Correct Answers:

- `1`
- `0`
- `-3`
- `ln(|x+4|)-3*atan(x/3)/3+C`

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8. (1 point) `Library/Wiley/setAnton_Section_7.5/Anton_7_5_Q19.pg`

Evaluate the integral.

$\int \frac{4x-1}{x^2+x-2}\, dx =$ ____________________ $+C$

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Using partial fraction,

$$\frac{4x-1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

this gives

$$\begin{cases} A+B=4 \\ 2A-B=-1 \end{cases}$$

giving

$$A=1,\ B=3$$

therefore

$$\int \frac{4x-1}{x^2+x-2}\, dx = \int \left(\frac{1}{x-1} + \frac{3}{x+2}\right) dx = \ln(|x-1|) + 3\ln(|x+2|) + C = \ln\left(\left|(x-1)(x+2)^3\right|\right) + C$$

Correct Answers:

- `ln(|(x-1)*(x+2)^3|)`

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9. (1 point) `Library/Rochester/setIntegrals25RationalFunctions/S07.04.PartialFractions.PTP18.pg`

What is the correct form of the partial fraction decomposition for the following integral?

$$\int \frac{x^2+1}{(x-5)^3(x^2+9x+47)} dx$$

- A. $\int \left(\frac{A}{x-5} + \frac{Bx+C}{(x-5)^2} + \frac{Dx+E}{(x-5)^3} + \frac{Fx+G}{x^2+9x+47}\right) dx$
- B. $\int \left(\frac{A}{(x-5)^3} + \frac{Bx+C}{x^2+9x+47}\right) dx$
- C. $\int \left(\frac{A}{(x-5)^3} + \frac{B}{x-9} + \frac{C}{x-47}\right) dx$
- D. $\int \left(\frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{C}{(x-5)^3} + \frac{Dx+E}{x^2+9x+47}\right) dx$
- E. There is no partial fraction decomposition yet because there is cancellation.

- F. There is no partial fraction decomposition yet because long division must be done first.
- G. $\int \left(\frac{A}{(x-5)^3} + \frac{B}{x-9} + \frac{C}{(x-9)^2} + \frac{Dx+E}{x^2+1} \right) dx$
- H. There is no partial fraction decomposition because the denominator does not factor.

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Note that $x^2+9x+47$ is an irreducible quadratic since $b^2-4ac=(9)^2-4(47)=-107<0$.
Since the denominator factors in the linear term $x-5$ repeated three times, and in an irreducible quadratic, the correct form of the partial fraction is:

$$\int \left(\frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{C}{(x-5)^3} + \frac{Dx+E}{x^2+9x+47} \right) dx$$

Thus the correct answer is **D .**

Correct Answers:

- `D`

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10. **(1 point)** `Library/UMN/calculusStewartET/s_7_4_prob04.pg`

Evaluate the integral

$$\int \frac{x+2}{x^2+4x+5}\,dx.$$

Answer: ____________________

Correct Answers:

- `0.5*ln(x^2+4*x+5)+C`