MATH 141: Calculus II, **Midterm Exam 2** Practice Questions

Q1. Use the **comparison test** or **limit comparison test** to determine whether the following improper integral converges or diverges. **Show your work!**

$$\int_1^\infty \frac{x}{x^3+6}\,dx$$

Method 1

$\frac{x}{x^3+6} \le \frac{x}{x^3} = \frac{1}{x^2}$ on $[1, \infty)$ and

$\int_1^\infty \frac{1}{x^2}\,dx$ converges by the p-test

Thus $\int_1^\infty \frac{x}{x^3+6}\,dx$ converges.

Method 2

$$\lim_{x\to\infty} \frac{1/x^2}{x/(x^3+6)} = \lim_{x\to\infty} \frac{x^3+6}{x^3}$$

$$= \lim_{x\to\infty} \left(1 + \frac{6}{x^3}\right) = 1$$

and $\int_1^\infty \frac{1}{x^2}\,dx$ converges by the p-test

So, $\int_1^\infty \frac{x}{x^3+6}\,dx$ converges by the limit comparison test.

Q2. Use the **comparison test** or **limit comparison test** to determine whether the following improper integral converges or diverges. **Show your work!**

$$\int_{-2}^{5} \frac{1}{(x+2)^2}\, dx$$

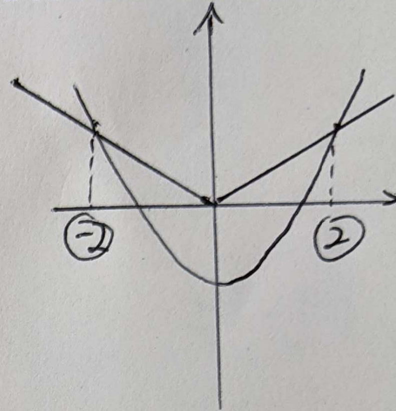
Sol. $f(x) = \dfrac{1}{(x+2)^2}$ is discontinuous at $x=-2$

$$\int_{-2}^{5} \frac{dx}{(x+2)^2} \underset{du=dx}{\overset{u=x+2}{=\!=}} \int_{0}^{7} \frac{du}{u^2}$$

which diverges by the p-test.

Thus, $\displaystyle\int_{-2}^{5} \frac{dx}{(x+2)^2}$ diverges.

Q3. Draw the region between the two curves $y = |x|$ and $y = x^2 - 2$ and find the area of the region. You are allowed to use the fnInt() function in a TI-84 calculator to find the value of the definite integral(s).



$y = |x| = \begin{cases} -x, & \text{if } x<0 \\ x, & \text{if } x \geq 0 \end{cases}$

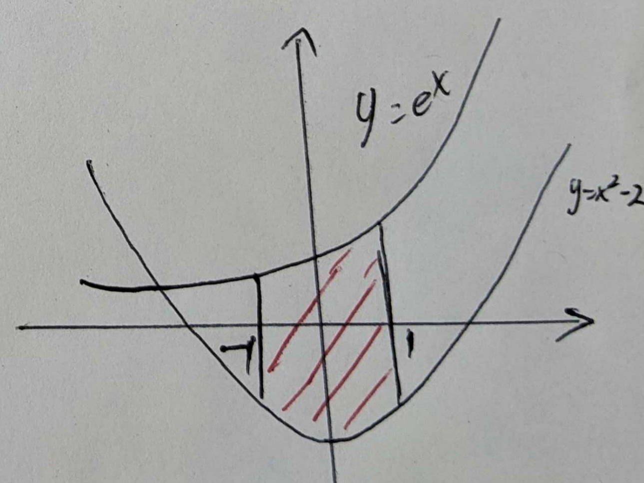
i) Let $-x = x^2-2 \Rightarrow x^2+x-2=0$

$\Rightarrow \begin{cases} (x-1)(x+2)=0 \\ x<0 \end{cases} \Rightarrow x=-2$

ii) Let $x = x^2-2 \Rightarrow \begin{cases} x^2-x-2=0 \\ x>0 \end{cases} \Rightarrow \begin{cases} (x+1)(x-2)=0 \\ x>0 \end{cases} \Rightarrow x=2$

$A = \int_{-2}^{2} \left| |x|-(x^2-2) \right| dx = \int_{-2}^{0} [-x-(x^2-2)]\,dx + \int_0^2 [x-(x^2-2)]\,dx$

Q4. Draw the region bounded by the curves $y = x^2 - 2$, $y = e^x$, $x = -1$ and $x = 1$. And find the area of the region. You are allowed to use the fnInt() function in a TI-84 calculator to find the value of the definite integral(s).



$A = \int_{-1}^{1} [e^x - (x^2-2)]\,dx$