Homework 7 Solution

Tuesday, October 26, 2021 10:54 AM

Xuemao Zhang **Math141_Calculus_II**

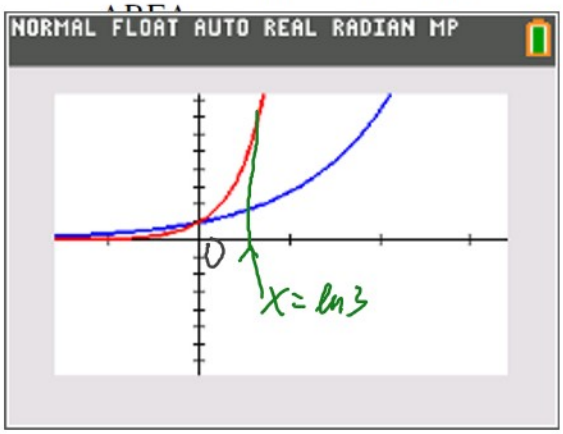
Assignment Homework7 due 10/27/2021 at 11:59pm EDT

---

1. **(1 point)** Library/Wiley/setAnton_Section_6.1/anton_6_1_Q13.pg

Sketch the region enclosed by the curves and find its area.

$y = e^x,\ y = e^{3x},\ x = 0,\ x = \ln 3$



$$A = \int_0^{\ln 3} [\text{top} - \text{bottom}]\, dx$$

$$= \int_0^{\ln 3} [e^{3x} - e^{x}]\, dx$$

---

2. **(1 point)** Library/Wiley/setAnton_Section_6.1/anton_6_1_Q18.pg

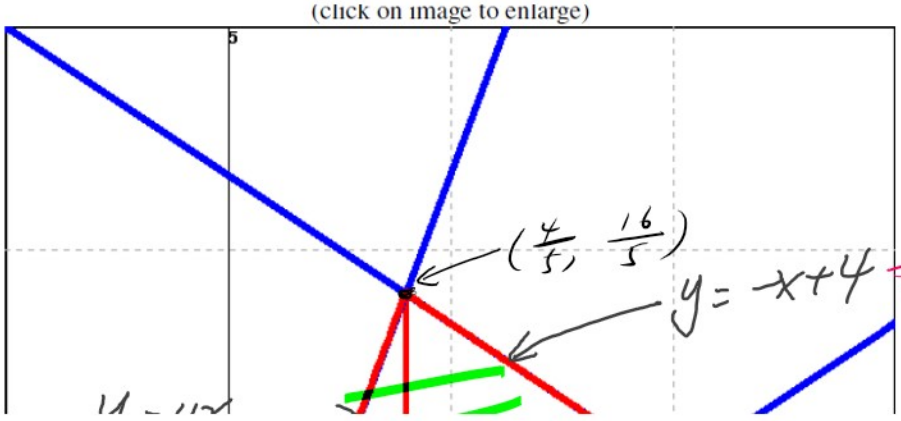
Sketch the region enclosed by the curves and find its area.

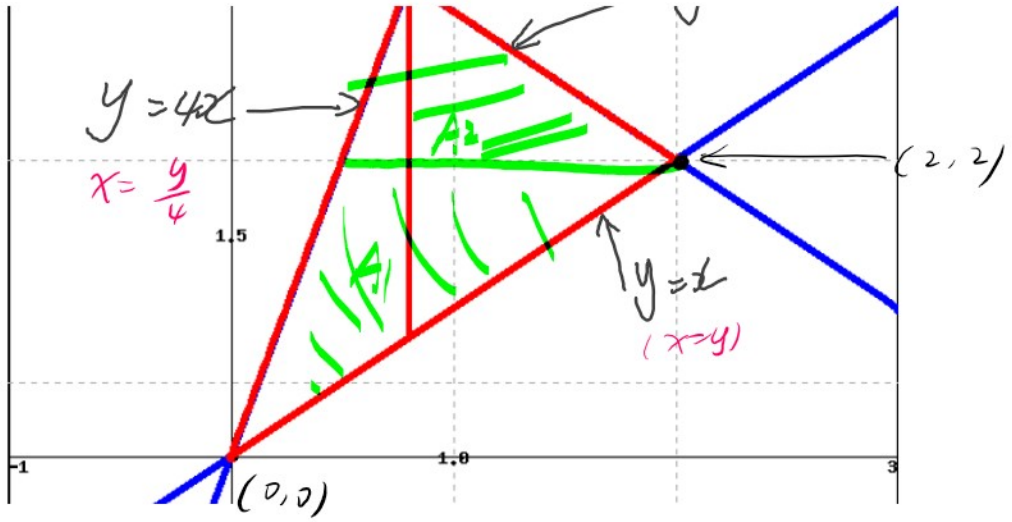
$y = x,\ y = 4x,\ y = -x + 4$

AREA = _____

Solution: *(Instructor solution preview: show the student solution after due date.)*

(click on image to enlarge)





$$\begin{cases} y = 4x \\ y = -x + 4 \end{cases} \Rightarrow 4x = -x + 4 \Rightarrow 5x = 4$$

$$\Rightarrow \begin{cases} x = 4/5 \\ y = \frac{16}{5} \end{cases}$$

$$\begin{cases} y = x \\ y = -x + 4 \end{cases} \Rightarrow x = -x + 4 \Rightarrow 2x = 4$$

$$\Rightarrow \begin{cases} x = 2 \\ y = 2 \end{cases}$$

Method 1 :

$$A = \int_0^{4/5} [\text{top} - \text{bottom}]\, dx + \int_{\frac{4}{5}}^{2} [\text{top} - \text{bottom}]\, dx$$

$$= \int_0^{4/5} [4x - x]\, dx + \int_{\frac{4}{5}}^{2} [-x + 4 - x]\, dx$$

Method 2

$$A = \int_0^{2} [\text{right} - \text{left}]\, dy + \int_{2}^{\frac{16}{5}} [\text{right} - \text{left}]\, dy$$

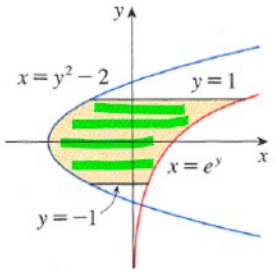
$$A = \int_0^2 [\text{right} - \text{left}]\,dy + \int_2^{\frac{16}{5}} [\text{right} - \text{left}]\,dy$$

$$= \int_0^2 \left[y - \frac{y}{4} \right] dy + \int_2^{\frac{16}{5}} \left[-y + 4 - \frac{y}{4} \right] dy$$

---

6. **(1 point)** Library/UCSB/Stewart5_6_1/Stewart5_6_1_3/Stewart5_6_1_3.pg

Find the area of the shaded region below.



integrate in terms of y

Area = _____

Correct Answers:

- exp(1)-1/exp(1)+10/3

$$\int_{-1}^{1} [\text{right} - \text{left}]\,dy$$

$$= \int_{-1}^{1} \left[e^y - (y^2 - 2) \right] dy$$

---

8. **(1 point)** Library/UMN/calculusStewartCCC/s_6_1_9.pg

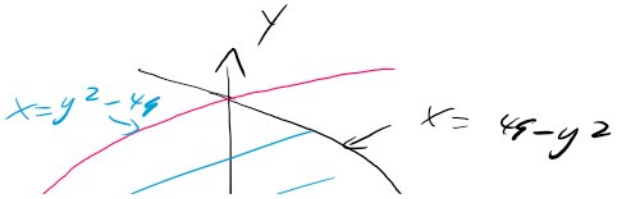
Sketch the region enclosed by the curves $x = 49 - y^2$ and $x = y^2 - 49$. Decide whether to integrate with respect to x or y. Then find the area of the region.

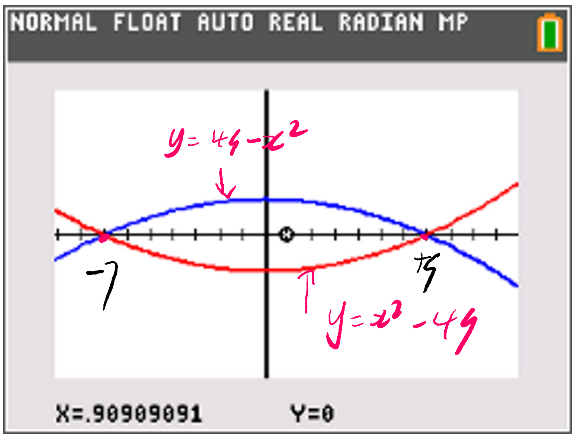
output output input input

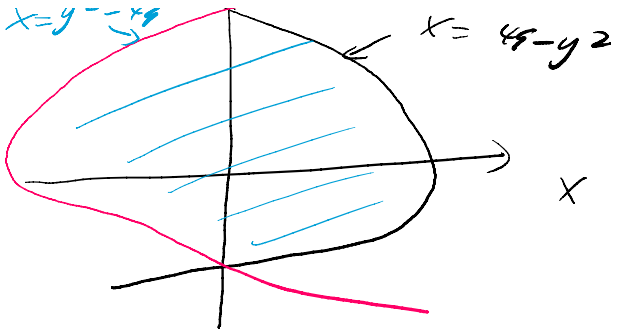
Area = _____

Region B: $y = 49 - x^2$

$y = x^2 - 49$







The two regions have same areas!

$$\begin{cases} y = 49 - x^2 \\ y = x^2 - 49 \end{cases} \Rightarrow 49 - x^2 = x^2 - 49$$

$$\Rightarrow 2x^2 = 2 \times 49$$

$$\Rightarrow x^2 = 49 \Rightarrow x = \pm 7$$

$$\Rightarrow A = \int_{-7}^{7} [49 - x^2 - (x^2 - 49)]\, dx$$