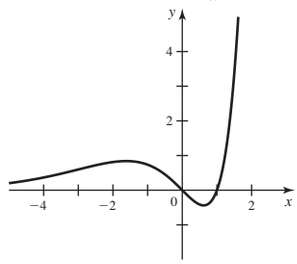
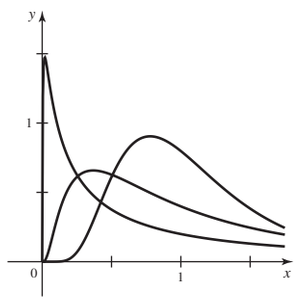
79. a. 1 **b.** 0 **c.** Undefined **d.** 1 **e.** 13/12 **f.** 40/9 **g.** $\left(\frac{e^2+1}{2e}\right)^2$ **h.** Undefined **i.** $\ln 4$ **j.** 1 **81.** $x = 0$ **83.** $x = \pm\tanh^{-1}(1/\sqrt{3}) = \pm\ln(2+\sqrt{3})/2 \approx \pm 0.658$ **85.** $\tan^{-1}(\sinh 1) - \pi/4 \approx 0.08$ **87.** Applying l'Hôpital's Rule twice brings you back to the initial limit; $\lim_{x\to\infty} \tanh x = 1$. **89.** $2/\pi$ **91.** 1 **93.** $12(3\ln(3+\sqrt{8}) - \sqrt{8}) \approx 29.5$ **95. a.** Approx. 360.8 m **b.** First 100 m: $t \approx 4.72$ s, $v_{av} \approx 21.2$ m/s; second 100 m: $t \approx 2.25$ s, $v_{av} \approx 44.5$ m/s **97. a.** $\sqrt{mg/k}$ **b.** $35\sqrt{3} \approx 60.6$ m/s **c.** $t = \sqrt{\frac{m}{kg}}\tanh^{-1} 0.95 = \frac{\ln 39}{2}\sqrt{\frac{m}{kg}}$ **d.** Approx. 736.5 m **109.** $\ln(21/4) \approx 1.66$

Chapter 7 Review Exercises, pp. 518–519

1. a. False **b.** False **c.** False **d.** True **3.** $\ln 4$ **5.** $\frac{1}{2}\ln(x^2+8x+25) + C$ **7.** $\cosh^{-1}(x/3) + C = \ln(x+\sqrt{x^2-9}) + C$ **9.** $\tanh^{-1}(1/3)/9 = (\ln 2)/18 \approx 0.0385$ **11.** $x^{3x^2+1}\left(6x\ln x + 3x + \frac{1}{x}\right)$ **13.** $\sinh^2 t + \cosh^2 t$ **15.** $3\sinh(6x-2)$ **17.** $-\csc x$ **19.** $\frac{2x}{\sqrt{x^4-1}}$ **21.** Approx. 7.3 hours **23. a.** $y(t) = 29{,}000e^{(t\ln 2)/2}$ **b.** Approx. 41,996,486 transistors (which closely approximates the actual number of transistors) **25.** 48.37 yr **27.** Local max at $x = -\frac{1}{2}(\sqrt{5}+1)$; local min at $x = \frac{1}{2}(\sqrt{5}-1)$; inflection points at $x = -3$ and $x = 0$; $\lim_{x\to-\infty} f(x) = 0$; $\lim_{x\to\infty} f(x) = \infty$



29. a.



b. $\lim_{x\to 0} f(x) = 0$ **d.** $f(x^*) = \frac{1}{\sqrt{2\pi}}\frac{e^{\sigma^2/2}}{\sigma}$ **e.** $\sigma = 1$

31. $L(x) = \frac{5}{3} + \frac{4}{3}(x - \ln 3)$; $\cosh 1 \approx 1.535$ **33. a.** $\cosh x$ **b.** $(1 - x\tanh x)\operatorname{sech} x$

CHAPTER 8

Section 8.1 Exercises, pp. 523–525

1. $u = 4 - 7x$ **3.** $\sin^2 x = \frac{1-\cos 2x}{2}$ **5.** Complete the square in $x^2 - 4x - 9$. **7.** $\frac{1}{15(3-5x)^3} + C$ **9.** $\frac{\sqrt{2}}{4}$ **11.** $\frac{1}{2}\ln^2 2x + C$ **13.** $\ln(e^x+1) + C$ **15.** $\frac{32}{3}$ **17.** $\frac{21}{110}$ **19.** $\frac{(\ln w - 1)^9}{9} + \frac{(\ln w - 1)^8}{8} + C$ **21.** $\frac{1}{2}\ln(x^2+4) + \tan^{-1}\frac{x}{2} + C$ **23.** $-\frac{1}{3}\ln|\csc(3e^x+4) + \cot(3e^x+4)| + C$ **25.** 1 **27.** $3\sqrt{1-x^2} + 2\sin^{-1}x + C$ **29.** $\ln(\sqrt{2}+1)$ **31.** $\frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right) + C$ **33.** $\frac{x^2}{2} + x + \ln(x^2+x+2) + C$ **35.** $\frac{3\pi+10}{12}$ **37.** $\sin^{-1}\left(\frac{\theta+3}{6}\right) + C$ **39.** $\tan\theta - \sec\theta + C$ **41.** $-x - \cot x - \csc x + C$ **43.** $\frac{1}{3}\ln(1+\sinh 3x) + C$ **45.** $\frac{1}{2}\ln|e^{2x}-2| + C$ **47.** $x - \ln|x+1| + C$ **49.** $\frac{4}{5}(9+\sqrt{t+1})^{3/2}(\sqrt{t+1}-6) + C$ **51.** $\frac{\ln 4 - \pi}{4}$ **53.** $\ln|\sec(e^x+1) + \tan(e^x+1)| + C$ **55.** $\frac{2\sin^3 x}{3} + C$ **57.** $2\tan^{-1}\sqrt{x} + C$ **59.** $\frac{1}{2}\ln(x^2+6x+13) - \frac{5}{2}\tan^{-1}\left(\frac{x+3}{2}\right) + C$ **61.** $-\frac{1}{e^x+1} + C$ **63.** $\frac{1}{2}$ **65. a.** False **b.** False **c.** False **d.** False **69. a.** $\frac{\tan^2 x}{2} + C$ **b.** $\frac{\sec^2 x}{2} + C$ **c.** The antiderivatives differ by a constant. **71. a.** $\frac{1}{2}(x+1)^2 - 2(x+1) + \ln|x+1| + C$ **b.** $\frac{x^2}{2} - x + \ln|x+1| + C$ **c.** The antiderivatives differ by a constant. **73.** $\frac{\ln 26}{3}$ **75.** $\frac{2}{3}(5\sqrt{5}-1)\pi$ **77.** $\pi\left(\frac{9}{2} - \frac{5\sqrt{5}}{6}\right)$ **79.** $\frac{2048 + 1763\sqrt{41}}{9375}$

Section 8.2 Exercises, pp. 529–532

1. Product Rule **3.** $\frac{x^2(2\ln x - 1)}{4} + C$ **5.** Products for which the choice for dv is easily integrated and when the resulting new integral is no more difficult than the original integral **7.** $(\tan x + 2)\ln(\tan x + 2) - \tan x + C$ **9.** $\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C$ **11.** $\frac{e^{6t}}{36}(6t-1) + C$ **13.** $\frac{x^2}{4}(2\ln 10x - 1) + C$ **15.** $(w+2)\sin 2w + \frac{1}{2}\cos 2w + C$ **17.** $\frac{3^x}{\ln 3}\left(x - \frac{1}{\ln 3}\right) + C$ **19.** $-\frac{1}{9x^9}\left(\ln x + \frac{1}{9}\right) + C$ **21.** $\frac{1}{8}\sin 2x - \frac{x}{4}\cos 2x + C$ **23.** $\frac{1}{4}(1-2x^2)\cos 2x + \frac{x}{2}\sin 2x + C$ **25.** $-e^{-t}(t^2+2t+2) + C$ **27.** $\frac{e^x}{2}(\sin x + \cos x) + C$ **29.** $-\frac{e^{-x}}{17}(\sin 4x + 4\cos 4x) + C$ **31.** $-e^{2x}\cos e^x + 2e^x\sin e^x + 2\cos e^x + C$ **33.** π **35.** $-\frac{1}{2}$ **37.** $\frac{1}{9}(5e^6+1)$ **39.** $\frac{\pi-2}{2}$ **41. a.** $x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C$

b. $\frac{1}{2}x^2\tan^{-1}x^2 - \frac{1}{4}\ln(1+x^4) + C$ **43.** $\pi(1-\ln 2)$ **45.** π

47. $\frac{2\pi}{27}(13e^6 - 1)$ **49. a.** False **b.** True **c.** True

51. Let $u = x^n$ and $dv = \cos ax\,dx$. **53.** Let $u = \ln^n x$ and $dv = dx$.

55. $\frac{x^2\sin 5x}{5} + \frac{2x\cos 5x}{25} - \frac{2\sin 5x}{125} + C$

57. $6 - 2e$ **59. a.** $\frac{2}{3}(x-2)\sqrt{x+1} + C$

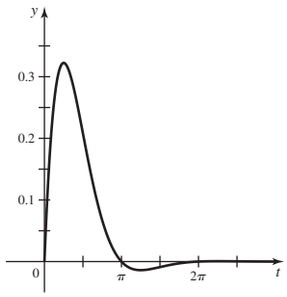
61. $\int \log_b x\,dx = \int \frac{\ln x}{\ln b}\,dx = \frac{1}{\ln b}(x\ln x - x) + C$

63. $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$

65. Let $u = x$ and $dv = f''(x)\,dx$.

67. $2e^3$ **69.** x-axis: $\pi^2/2$; y-axis: $2\pi^2$ **71.** $\pi(\pi - 2)$

75. a. $t = k\pi$, for $k = 0, 1, 2, \ldots$ **b.** $\frac{e^{-\pi}+1}{2\pi}$



c. $(-1)^n\left(\frac{e^{\pi}+1}{2\pi e^{(n+1)\pi}}\right)$

d. $a_n = a_{n-1}\cdot\frac{1}{e^{\pi}}$

77. c. $\int f(x)g(x)dx = f(x)G_1(x) - f'(x)G_2(x) + f''(x)G_3(x) - \int f'''(x)G_3(x)dx$

f and its derivatives		g and its integrals
$f(x)$	+	$g(x)$
$f'(x)$	−	$G_1(x)$
$f''(x)$	+	$G_2(x)$
$f'''(x)$	−	$G_3(x)$

d. $\int x^2e^{x/2}dx = 2x^2e^{x/2} - 8xe^{x/2} + 16e^{x/2} + C$

f and its derivatives		g and its integrals
x^2	+	$e^{x/2}$
$2x$	−	$2e^{x/2}$
2	+	$4e^{x/2}$
0	−	$8e^{x/2}$

$\frac{d^n}{dx^n}(x^2) = 0$, for $n \geq 3$, so all entries in the left column of the table beyond row three are 0, which results in no additional contribution to the antiderivative. **e.** $x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + C$; five rows are needed because $\frac{d^n}{dx^n}(x^3) = 0$, for $n \geq 4$.

f. $\frac{d^k}{dx^k}(p_n(x)) = 0$, for $k \geq n+1$

79. a. $\int e^x\cos x\,dx = e^x\sin x + e^x\cos x - \int e^x\cos x\,dx$

b. $\frac{1}{2}(e^x\sin x + e^x\cos x) + C$ **c.** $-\frac{3}{13}e^{-2x}\cos 3x - \frac{2}{13}e^{-2x}\sin 3x + C$

81. a. $I_1 = -\frac{1}{2}e^{-x^2} + C$ **b.** $I_3 = -\frac{1}{2}e^{-x^2}(x^2+1) + C$

c. $I_5 = -\frac{1}{2}e^{-x^2}(x^4 + 2x^2 + 2) + C$

Section 8.3 Exercises, pp. 536–538

1. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$; $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ **3.** Rewrite $\sin^3 x$ as $(1 - \cos^2 x)\sin x$. **5.** A reduction formula expresses an integral with a power in the integrand in terms of another integral with a smaller power in the integrand. **7.** Let $u = \tan x$.

9. $\sin x - \frac{1}{3}\sin^3 x + C$ **11.** $\frac{x}{2} - \frac{1}{12}\sin 6x + C$

13. $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$ **15.** $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$

17. $\frac{2}{3}\sin^{3/2}x - \frac{2}{7}\sin^{7/2}x + C$ **19.** $\frac{7}{24}$ **21.** $\frac{8}{45}$

23. $\frac{1}{8}x - \frac{1}{32}\sin 4x + C$ **25.** $\frac{1}{48}\sin^3 2x + \frac{1}{16}x - \frac{1}{64}\sin 4x + C$

27. $\tan x - x + C$ **29.** $-\frac{1}{3}\cot^3 x + \cot x + x + C$

31. $4\tan^5 x - \frac{20}{3}\tan^3 x + 20\tan x - 20x + C$ **33.** $\tan^{10}x + C$

35. $\frac{1}{3}\sec^3 x + C$ **37.** $\frac{1}{3}\tan^3(\ln\theta) + \tan(\ln\theta) + C$ **39.** $\ln 4$

41. $\frac{7}{6}$ **43.** $\frac{1}{8}\tan^2 4x + \frac{1}{4}\ln|\cos 4x| + C$ **45.** $\frac{2}{3}\tan^{3/2}x + C$

47. $\tan x - \cot x + C$ **49.** $\frac{1}{25}$ **51.** $-2\cot x - \frac{\cot^3 x}{3} + C$

53. $\frac{4}{3}$ **55.** $\frac{4}{3} - \ln\sqrt{3}$ **57.** $8\sqrt{2}/3$ **59.** $\sqrt{2}$ **61.** $2\sqrt{2}/3$

63. a. True **b.** False **65.** $\frac{2\pi}{35}$ **67.** $\frac{1}{8}\cos 4x - \frac{1}{20}\cos 10x + C$

69. $\frac{1}{2}\sin x - \frac{1}{10}\sin 5x + C$ **73.** $\frac{1}{2} - \ln\sqrt{2}$ **75. a.** $\frac{\pi}{2}$; $\frac{\pi}{2}$

b. $\frac{\pi}{2}$, for all n **d.** Yes **e.** $\frac{3\pi}{8}$, for all n

Section 8.4 Exercises, pp. 543–546

1. $x = 3\sec\theta$ **3.** $x = 10\sin\theta$ **5.** $\sqrt{4-x^2}/x$ **7.** $\pi/6$

9. $\frac{25\pi}{3}$ **11.** $\frac{\pi}{12}$ **13.** $\sin^{-1}\frac{x}{4} + C$ **15.** $-\frac{\sqrt{x^2+9}}{9x} + C$

17. $2 - \frac{\pi}{2}$ **19.** $\ln(\sqrt{x^2-81} + x) + C$

21. $\frac{x}{2}\sqrt{64-x^2} + 32\sin^{-1}\frac{x}{8} + C$ **23.** $\frac{x}{25\sqrt{25-x^2}} + C$

25. $-3\ln\left|\frac{\sqrt{9-x^2}+3}{x}\right| + \sqrt{9-x^2} + C$ **27.** $\sqrt{2}/6$

29. $\frac{1}{16}\left(\tan^{-1}\frac{x}{2} + \frac{2x}{x^2+4}\right) + C$

31. $8\sin^{-1}(x/4) - x\sqrt{16-x^2}/2 + C$

33. $\sqrt{x^2-9} - 3\sec^{-1}(x/3) + C$

35. $-1/\sqrt{x^2-1} - \sec^{-1}x + C$ **37.** $2 - \sqrt{2}$

39. $x/\sqrt{100-x^2} - \sin^{-1}(x/10) + C$ **41.** $x/\sqrt{1+4x^2} + C$

43. $\frac{\ln 3}{2}$ **45.** $81/(2(81-x^2)) + \ln\sqrt{81-x^2} + C$

47. $\frac{1}{16}\left(1 - \sqrt{3} - \ln(21 - 12\sqrt{3})\right)$ **49.** $\frac{1}{3} + \frac{\ln 3}{4}$

51. $\frac{x}{2}\sqrt{4+x^2} - 2\ln(x + \sqrt{4+x^2}) + C$

53. $\frac{9}{10}\cos^{-1}\frac{5}{3x} - \frac{\sqrt{9x^2-25}}{2x^2} + C$

55. $\frac{\sec^{-1}\frac{x}{10}}{2000} + \frac{\sqrt{x^2-100}}{200\,x^2} + C$

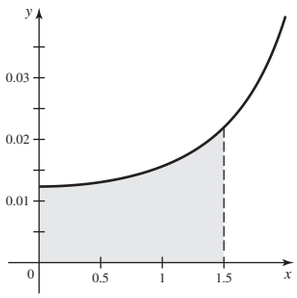
57. a. False **b.** True **c.** False **d.** False

61. $\sin^{-1}\left(\frac{x+1}{2}\right) + C$ **63.** $\frac{1}{3}\tan^{-1}\left(\frac{x+3}{3}\right) + C$

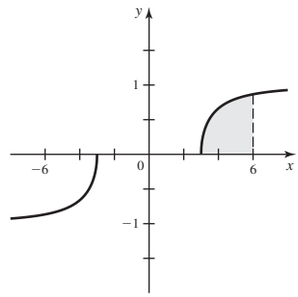
65. $\frac{\pi\sqrt{2}}{48}$ **67.** $\frac{x-4}{\sqrt{9+8x-x^2}} - \sin^{-1}\left(\frac{x-4}{5}\right) + C$

69. $\ln\left((2+\sqrt{3})(\sqrt{2}-1)\right)$

71. $\dfrac{1}{81}+\dfrac{\ln 3}{108}$



73. $3\sqrt{3}-\pi$



75. $\dfrac{3}{80}$ **77.** $\dfrac{1}{4a}\left(20a\sqrt{1+400a^2}+\ln\left(20a+\sqrt{1+400a^2}\right)\right)$

81. b. $\lim\limits_{L\to\infty}\dfrac{kQ}{a\sqrt{a^2+L^2}}=\lim\limits_{L\to\infty}2\rho k\dfrac{1}{a\sqrt{\left(\dfrac{a}{L}\right)^2+1}}=\dfrac{2\rho k}{a}$

85. a. $\dfrac{1}{\sqrt{g}}\left(\dfrac{\pi}{2}-\sin^{-1}\left(\dfrac{2\cos b-\cos a+1}{\cos a+1}\right)\right)$

b. For $b=\pi$, the descent time is $\dfrac{\pi}{\sqrt{g}}$, a constant.

Section 8.5 Exercises, pp. 554–556

1. Rational functions **3. a.** $\dfrac{A}{x-3}$ **b.** $\dfrac{A}{x-4},\dfrac{B}{(x-4)^2},\dfrac{C}{(x-4)^3}$

c. $\dfrac{Ax+B}{x^2+2x+6}$ **5.** $\dfrac{A}{x-4}+\dfrac{B}{x-5}$ **7.** $\dfrac{A}{x-5}+\dfrac{B}{(x-5)^2}$

9. $\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{x-1}+\dfrac{D}{x+2}+\dfrac{E}{x-2}$ **11.** $\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}$

13. $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{Cx+D}{x^2+x+2}+\dfrac{Ex+F}{(x^2+x+2)^2}$

15. $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{x+2}+\dfrac{D}{(x+2)^2}+\dfrac{Ex+F}{x^2+4}+\dfrac{Gx+H}{(x^2+4)^2}$

17. $\dfrac{2}{x-1}+\dfrac{3}{x-2}$ **19.** $\dfrac{1}{x-4}-\dfrac{1}{x+2}$ **21.** $2+\dfrac{3}{x+1}-\dfrac{4}{x+2}$

23. $\ln\left|\dfrac{x-1}{x+2}\right|+C$ **25.** $3\ln\left|\dfrac{x-1}{x+1}\right|+C$

27. $3\ln|x-1|-\dfrac{1}{3}\ln|3x-2|+C$ **29.** $-\ln 4$

31. $\ln\left|\dfrac{(x-2)^2(x+1)}{(x+2)^2(x-1)}\right|+C$ **33.** $3x+\ln\dfrac{(x-2)^{14}}{|x-1|}+C$

35. $\ln\left|\dfrac{x(x-2)^3}{(x+2)^3}\right|+C$ **37.** $\ln\left|\dfrac{(x-3)^{1/3}(x+1)}{(x+3)^{1/3}(x-1)}\right|^{1/16}+C$

39. $\dfrac{9}{x}+\ln\left|\dfrac{x-9}{x}\right|+C$ **41.** $\ln 2-\dfrac{3}{4}$ **43.** $-\dfrac{2}{x}+\ln\left|\dfrac{x+1}{x}\right|^2+C$

45. $\dfrac{5}{x}+\ln\left|\dfrac{x}{x+1}\right|^6+C$ **47.** $\dfrac{x^2}{2}+2\ln|x-5|-\dfrac{10}{x-5}+C$

49. $\dfrac{3}{x-1}+\ln\left|\dfrac{(x-1)^5}{x^4}\right|+C$ **51.** $\ln|x+1|+\tan^{-1}x+C$

53. $\ln(x+1)^2+\tan^{-1}(x+1)+C$

55. $\ln\left|\dfrac{(x-1)^2}{x^2+4x+5}\right|+14\tan^{-1}(x+2)+C$

57. $\dfrac{1}{2}\ln|x^2+3|-\dfrac{1}{x^2+3}+C$

59. $\dfrac{1}{2}\ln(x^2+6x+10)-3\tan^{-1}(x+3)-\dfrac{1}{x^2+6x+10}+C$

61. $\ln\left(\dfrac{x^2}{x^2+1}\right)+\dfrac{1}{x^2+1}+C$

63. $\sqrt{\dfrac{3}{7}}\tan^{-1}\left(\sqrt{\dfrac{3}{7}}x\right)-\dfrac{1}{6(3x^2+7)}+C$

65. a. False **b.** False **c.** False **d.** True **67.** $\ln 6$

69. $\left(\dfrac{24}{5}-2\ln 5\right)\pi$ **71.** $\dfrac{2}{3}\pi\ln 2$ **73.** $\ln\sqrt{\left|\dfrac{x-1}{x+1}\right|}+C$

75. $\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{(x^2+1)^2}+\dfrac{Fx+G}{x^2+x+4}+\dfrac{Hx+I}{(x^2+x+4)^2}$;
$\dfrac{1}{16x}-\dfrac{x+10}{100(x^2+1)}+\dfrac{4x+3}{50(x^2+1)^2}-\dfrac{21x-19}{400(x^2+x+4)}-$
$\dfrac{4x+1}{20(x^2+x+4)^2}$ **77.** $\ln\left|\dfrac{e^x-1}{e^x+2}\right|^{1/3}+C$

79. $\dfrac{1}{4}\ln\left(\dfrac{1+\sin t}{1-\sin t}-\dfrac{2}{1+\sin t}\right)+C$

81. $\tan^{-1}e^x-\dfrac{1}{2(e^{2x}+1)}+C$ **83.** $x-\ln(1+e^x)+C$

89. $-\cot x-\csc x+C=-\cot(x/2)+C$

91. $\dfrac{1}{\sqrt{2}}\ln\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ **93. a.** Car A **b.** Car C

c. $S_A(t)=88t-88\ln|t+1|$;
$S_B(t)=88\left(t-\ln(t+1)^2-\dfrac{1}{t+1}+1\right)$;
$S_C(t)=88(t-\tan^{-1}t)$

d. Car C **95.** Because $\dfrac{x^4(1-x)^4}{1+x^2}>0$ on $(0,1)$,
$\displaystyle\int_0^1\frac{x^4(1-x^4)}{1+x^2}\,dx>0$; therefore, $\dfrac{22}{7}>\pi$.

Section 8.6 Exercises, pp. 560–562

1. Integrate by parts. **3.** Let $x=8\sin\theta$. **5.** Use the method of partial fractions. **7.** $\dfrac{\pi}{4}$ **9.** $\dfrac{\pi}{6}$ **11.** $\dfrac{5}{4}-\dfrac{3\pi}{8}$ **13.** $-\dfrac{\sqrt{1-e^{2x}}}{e^x}+C$

15. $\dfrac{4}{\ln 2}$ **17.** $\dfrac{3e^4}{2}$ **19.** $\dfrac{16}{35}$ **21.** $\dfrac{x^{10}}{10}\ln 3x-\dfrac{x^{10}}{100}+C$

23. $\ln|\cos x+1|-\ln|\cos x|+C$

25. $\ln\left|\dfrac{x}{1+\sqrt{1-x^2}}\right|+C$ **27.** $\dfrac{3x}{8}-\dfrac{1}{2}\sin x+\dfrac{1}{16}\sin 2x+C$

29. $\ln|\sin x+\sin^2 x|+C$ **31.** $6\sin^{-1}\dfrac{x}{2}+\dfrac{3}{2}x\sqrt{4-x^2}+C$

33. $\dfrac{1}{a}\tan^{-1}\dfrac{e^x}{a}+C$ **35.** $\dfrac{11}{6}$ **37.** $\dfrac{\sqrt{3}+1}{2}$

39. $-\cos x\ln(\sin x)-\ln|\csc x+\cot x|+\cos x+C$

41. $-\dfrac{2}{5}\cot^{5/2}x-\dfrac{2}{9}\cot^{9/2}x+C$ **43.** $\dfrac{\sin^{-1}x^{10}}{10}+C$ **45.** $\ln\dfrac{4}{3}-\dfrac{1}{6}$

47. $x^2+3x+4\ln|x-2|+\ln|x+1|+C$

49. $\dfrac{\sec^{11}x}{11}-\dfrac{\sec^9 x}{9}+C$ **51.** $\dfrac{4}{7}(2^{7/4}-1)$

53. $-\dfrac{\cot^2 e^x}{2}-\ln|\sin e^x|+C$ **55.** $\ln|x^3+x|+3\tan^{-1}x+C$

57. $-2\sqrt{x}\cos\sqrt{x}+2\sin\sqrt{x}+C$ **59.** $-\dfrac{1}{x}-\tan^{-1}x+C$

61. $\dfrac{\sqrt{2}e^{\pi/4}-1}{2}$ **63.** $\dfrac{x^{a+1}}{a+1}\left(\ln x-\dfrac{1}{a+1}\right)+C$

65. $\dfrac{\pi}{18}$ **67.** $\dfrac{1}{54}(\sin^{-1}3x-3x\sqrt{1-9x^2})+C$

69. $\dfrac{1-\sqrt{1-x^2}}{x}+C$ **71.** $-2\cot x+2\csc x-x+C$

73. $\dfrac{40\sqrt{5}}{3}-\dfrac{224}{15}$ **75.** $\dfrac{7\pi^2}{144}$ **77.** $x\cos^{-1}x-\sqrt{1-x^2}+C$

79. $-\dfrac{\sin^{-1}x}{x}+\ln\left|\dfrac{x}{1+\sqrt{1-x^2}}\right|+C$

81. $\ln|x|+2\tan^{-1}x-\dfrac{3}{2(x^2+1)}+C$

83. $\dfrac{\sin^{999}e^x}{999}-\dfrac{\sin^{1001}e^x}{1001}+C$ **85. a.** True **b.** True **c.** False **d.** False **87.** $\pi(\sqrt{2}+\ln(1+\sqrt{2}))\approx 7.212$

89. $\dfrac{\pi(4\sqrt{2}+3)}{3}\approx 9.065$ **91.** $9800\pi\ln 2\approx 21{,}340.3$ J

93. $4x-2\ln(e^{2x}+2e^x+17)-\tan^{-1}\left(\dfrac{e^x+1}{4}\right)+C$

95. $\dfrac{1}{4}\ln|\tan x+1|-\dfrac{1}{4}\ln|\tan x-1|+\dfrac{x}{2}+C$

97. $x\tan^{-1}\sqrt[3]{x}-\dfrac{x^{2/3}}{2}+\dfrac{1}{2}\ln(1+x^{2/3})+C$

99. $\pi\left(\sqrt{5}-\sqrt{2}+\dfrac{1}{2}\ln\left(\dfrac{\sqrt{5}-1}{\sqrt{5}+1}\right)-\dfrac{1}{2}\ln\left(\dfrac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)\approx 3.839$

Section 8.7 Exercises, pp. 565–567

1. Substitutions, integration by parts, partial fractions **3.** The CAS may not include the constant of integration, and it may use a trigonometric identity or other algebraic simplification.

5. $-\dfrac{1}{3}\sin^3 e^x+\sin e^x+C$ **7.** $x\cos^{-1}x-\sqrt{1-x^2}+C$

9. $\ln(x+\sqrt{16+x^2})+C$ **11.** $\frac{3}{4}(2u-7\ln|7+2u|)+C$

13. $-\dfrac{1}{4}\cot 2x+C$ **15.** $\frac{1}{12}(2x-1)\sqrt{4x+1}+C$

17. $\frac{1}{3}\ln\left|x+\sqrt{x^2-\left(\frac{10}{3}\right)^2}\right|+C$ **19.** $\ln(e^x+\sqrt{4+e^{2x}})+C$

21. $-\dfrac{1}{2}\ln\left|\dfrac{2+\sin x}{\sin x}\right|+C$

23. $\dfrac{2\ln^2 x-1}{4}\sin^{-1}(\ln x)+\dfrac{\ln x\sqrt{1-\ln^2 x}}{4}+C$

25. $\dfrac{x}{16\sqrt{16+9x^2}}+C$ **27.** $-\dfrac{1}{12}\ln\left|\dfrac{12+\sqrt{144-x^2}}{x}\right|+C$

29. $2x+x\ln^2 x-2x\ln x+C$

31. $\dfrac{x+5}{2}\sqrt{x^2+10x}-\dfrac{25}{2}\ln|x+5+\sqrt{x^2+10x}|+C$

33. $\dfrac{1}{3}\tan^{-1}\left(\dfrac{x+1}{3}\right)+C$ **35.** $\ln x-\dfrac{1}{10}\ln(x^{10}+1)+C$

37. $2\ln(\sqrt{x-6}+\sqrt{x})+C$

39. $-\dfrac{\tan^{-1}x^3}{3x^3}+\ln\left|\dfrac{x}{(x^6+1)^{1/6}}\right|+C$ **41.** $4\sqrt{17}+\ln(4+\sqrt{17})$

43. $\sqrt{5}-\sqrt{2}+\ln\left(\dfrac{2+2\sqrt{2}}{1+\sqrt{5}}\right)$ **45.** $\dfrac{128\pi}{3}$ **47.** $\dfrac{\pi^2}{4}$

49. $\dfrac{(x-3)\sqrt{3+2x}}{3}+C$ **51.** $\dfrac{1}{3}\tan 3x-x+C$

53. $\dfrac{1540+243\ln 3}{8}$

55. $\dfrac{(x^2-a^2)^{3/2}}{3}-a^2\sqrt{x^2-a^2}+a^3\cos^{-1}\dfrac{a}{x}+C$ **57.** $\dfrac{\pi}{4}$

59. $-\dfrac{x}{8}(2x^2-5a^2)\sqrt{a^2-x^2}+\dfrac{3a^4}{8}\sin^{-1}\dfrac{x}{a}+C$

61. $2-\dfrac{\pi^2}{12}-\ln 4$ **63.** $\dfrac{27{,}456\sqrt{15}}{7}\approx 15{,}190.9$

65. $\dfrac{1}{8}e^{2x}(4x^3-6x^2+6x-3)+C$ **67.** $\dfrac{\tan^3 3y}{9}-\dfrac{\tan 3y}{3}+y+C$

69. $\dfrac{1}{24}(128-78\sqrt{2}-3\ln(3+2\sqrt{2}))$

71. $\dfrac{1}{a^2}(ax-b\ln|b+ax|)+C$

73. $\dfrac{1}{a^2}\left(\dfrac{(ax+b)^{n+2}}{n+2}-\dfrac{b(ax+b)^{n+1}}{n+1}\right)+C$

75. a. True **b.** True

79. $\dfrac{1}{16}\left((8x^2-1)\sin^{-1}2x+2x\sqrt{1-4x^2}\right)+C$

81. $-\dfrac{\tan^{-1}x}{x}+\ln\left(\dfrac{|x|}{\sqrt{x^2+1}}\right)+C$ **83. b.** $\dfrac{\pi}{8}\ln 2$

85. a.

θ_0	T
0.10	6.27927
0.20	6.26762
0.30	6.24854
0.40	6.22253
0.50	6.19021
0.60	6.15236
0.70	6.10979
0.80	6.06338
0.90	6.01399
1.00	5.96247

b. All are within 10%.

87. b. $\dfrac{63\pi}{512}$ **c.** Decrease

Section 8.8 Exercises, pp. 578–582

1. $\frac{1}{2}$ **3.** The Trapezoid Rule approximates areas under curves using trapezoids. **5.** 42 **7.** $\dfrac{112}{3}$ **9.** $-1, 1, 3, 5, 7, 9$

11. 1.59×10^{-3}; 5.04×10^{-4} **13.** 1.72×10^{-3}; 6.32×10^{-4}

15. 576; 640; 656 **17.** 0.643950551 **19.** 704; 672; 664

21. 0.622 **23.** 2.28476811; 2.33512377 **25.** 1.76798499

27. $M(25)\approx 0.63703884$, $T(25)\approx 0.63578179$; 6.58×10^{-4}, 1.32×10^{-3}

29.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	99	102	1.00	2.00
8	99.75	100.5	0.250	0.500
16	99.9375	100.125	6.3×10^{-2}	0.125
32	99.984375	100.03125	1.6×10^{-2}	3.1×10^{-2}

31.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	1.50968181	1.48067370	9.7×10^{-3}	1.9×10^{-2}
8	1.50241228	1.49517776	2.4×10^{-3}	4.8×10^{-3}
16	1.50060256	1.49879502	6.0×10^{-4}	1.2×10^{-3}
32	1.50015061	1.49969879	1.5×10^{-4}	3.0×10^{-4}

33.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	-1.96×10^{-16}	0	2.0×10^{-16}	0
8	7.63×10^{-17}	-1.41×10^{-16}	7.6×10^{-17}	1.4×10^{-16}
16	1.61×10^{-16}	1.09×10^{-17}	1.6×10^{-16}	1.1×10^{-17}
32	6.27×10^{-17}	-4.77×10^{-17}	6.3×10^{-17}	4.8×10^{-17}

35. $T(4) \approx 690.3$ million ft^3; $S(4) \approx 692.2$ million ft^3 (answers may vary) **37.** 54.5°F, Trapezoid Rule **39.** 35.0°F, Trapezoid Rule **41. a.** Left sum: 204.917; right sum: 261.375; Trapezoid Rule: 233.146; the approximations measure the average temperature of the curling iron on $[0, 120]$. **b.** Left sum: underestimate; right sum: overestimate; Trapezoid Rule: underestimate **c.** 305°F is the change in temperature over $[0, 120]$. **43. a.** 5907.5 **b.** 5965 **c.** 5917

45. a. $T(25) \approx 3.19623162$
$T(50) \approx 3.19495398$
b. $S(50) \approx 3.19452809$
c. $e_T(50) \approx 4.3 \times 10^{-4}$
$e_S(50) \approx 4.5 \times 10^{-8}$

47. a. $T(50) \approx 1.00008509$
$T(100) \approx 1.00002127$
b. $S(100) \approx 1.00000000$
c. $e_T(100) \approx 2.1 \times 10^{-5}$
$e_S(100) \approx 4.6 \times 10^{-9}$

49.

n	$T(n)$	$S(n)$	Error in $T(n)$	Error in $S(n)$
4	1820.0000	—	284	—
8	1607.7500	1537.0000	71.8	1
16	1553.9844	1536.0625	18.0	6.3×10^{-2}
32	1540.4990	1536.0039	4.50	3.9×10^{-3}

51.

n	$T(n)$	$S(n)$	Error in $T(n)$	Error in $S(n)$
4	0.46911538	—	5.3×10^{-2}	—
8	0.50826998	0.52132152	1.3×10^{-2}	2.9×10^{-4}
16	0.51825968	0.52158957	3.4×10^{-3}	1.7×10^{-5}
32	0.52076933	0.52160588	8.4×10^{-4}	1.1×10^{-6}

53. a. True **b.** False **c.** True

55.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	0.40635058	0.40634782	1.4×10^{-6}	1.4×10^{-6}
8	0.40634920	0.40634920	7.6×10^{-10}	7.6×10^{-10}
16	0.40634920	0.40634920	6.6×10^{-13}	6.6×10^{-13}
32	0.40634920	0.40634920	8.9×10^{-16}	7.8×10^{-16}

57.

n	$M(n)$	$T(n)$	Error in $M(n)$	Error in $T(n)$
4	4.72531819	4.72507878	1.2×10^{-4}	1.2×10^{-4}
8	4.72519850	4.72519849	9.1×10^{-9}	9.1×10^{-9}
16	4.72519850	4.72519850	0	8.9×10^{-16}
32	4.72519850	4.72519850	0	8.9×10^{-16}

63. Approximations will vary; exact value is 68.26894921 . . . .
65. a. Approx. 1.6×10^{11} barrels **b.** Approx. 6.8×10^{10} barrels
67. a. $M(50) \approx 34.4345566$

b. $f''(x) = \dfrac{3(x^4 + 4x)}{4(x^3 + 1)^{3/2}}$ **d.** $E_M \le 0.0028$

69. a. $T(40) = 0.874799972\ldots$ **b.** $f''(x) = e^x \cos e^x - e^{2x} \sin e^x$

d. $E_T \le \dfrac{1}{3200}$ **71. a.** $S(20) \approx 0.97774576$

b. $E_S \le 3.5 \times 10^{-8}$ **73.** Approximations will vary; exact value is 38.753792 . . . . **77.** Overestimate **79.** $S(20) \approx 1.00000175$

Section 8.9 Exercises, pp. 590–593

1. The interval of integration is infinite or the integrand is unbounded on the interval of integration. **3.** $\lim\limits_{b\to\infty} \int_2^b \dfrac{dx}{x^{1/5}}$ **5.** $\int_{-\infty}^{\infty} f(x)\,dx$

7. $\dfrac{1}{3}$ **9.** Diverges **11.** $\dfrac{1}{a}$ **13.** Diverges **15.** $\dfrac{\pi}{10}$

17. Diverges **19.** Diverges **21.** $\dfrac{1}{\pi}$ **23.** $\dfrac{\pi}{4}$ **25.** $\dfrac{\pi}{6}$ **27.** 0

29. $\dfrac{\pi^3}{12}$ **31.** $\ln 2$ **33.** Diverges **35.** Diverges **37.** 6

39. Diverges **41.** Diverges **43.** $2(e - 1)$ **45.** Diverges

47. $4 \cdot 10^{3/4}/3$ **49.** Diverges **51.** π **53.** -1

55. $\ln(2 + \sqrt{3})$ **57.** 2 **59.** \$41,666.67 **61.** 0.76 **63.** 20,000 hr

65. $\dfrac{\pi}{3}$ **67.** $3\pi/2$ **69.** $\pi/\ln 2$ **71.** 2π **73.** Does not exist

75. $\dfrac{72 \cdot 2^{1/3}\,\pi}{5}$ **77.** Converges **79.** Diverges **81.** Converges

83. Diverges **85.** Converges **87. a.** True **b.** False **c.** False

d. True **e.** True **89.** $1/b - 1/a$ **91. a.** $A(a, b) = \dfrac{e^{-ab}}{a}$, for $a > 0$

b. $b = g(a) = -\dfrac{1}{a} \ln 2a$ **c.** $b^* = -2/e$ **93.** π **107. a.** π

b. $\pi/(4e^2)$ **109. a.** $6.28 \times 10^7\, m$ J **b.** 11.2 km/s **c.** $\le$ 9 mm

Chapter 8 Review Exercises, pp. 593–596

1. a. True **b.** False **c.** False **d.** True **e.** False

3. $2(x - 8)\sqrt{x + 4} + C$ **5.** $\dfrac{1}{3}\sqrt{x + 2}(x - 4) + C$ **7.** $\dfrac{\pi}{4}$

9. $\dfrac{4}{105}$ **11.** $\sqrt{t - 1} - \tan^{-1}\sqrt{t - 1} + C$ **13.** $\dfrac{2}{15}(1 - e^{3\pi})$

15. $7 + \ln 40 - \ln 17$ **17.** $2 \ln|x| + 3 \tan^{-1}(x + 1) + C$

19. $\dfrac{2}{x + 3} - \dfrac{2}{(x + 3)^2} + \ln|x + 3| + C$ **21.** $\sqrt{3} - 1 - \dfrac{\pi}{12}$

23. $\dfrac{1}{5}\tan^5 t + C$ **25.** $\dfrac{\pi}{8}$ **27.** $\dfrac{\sqrt{w^2 + 2w - 8}}{9(w + 1)} + C$ **29.** $-\dfrac{\cot^5 x}{5} + C$

31. $\dfrac{x \cosh 2x}{2} - \dfrac{\sinh 2x}{4} + C$ **33.** $\dfrac{1}{15}\sec^5 3\theta - \dfrac{1}{9}\sec^3 3\theta + C$

35. $\frac{1}{6}(x^2-8)\sqrt{x^2+4}+C$ **37.** $\frac{1}{x+1}+\ln|(x+1)(x^2+4)|+C$

39. $\frac{t-\ln(2+e^t)}{2}+C$ **41.** $\frac{1}{4}(\csc 4\theta-\cot 4\theta)+C$

43. $\frac{e^x}{2}(\sin x-\cos x)+C$

45. $\ln|x|-\frac{1}{x}+\frac{1}{2}\ln(x^2+4x+9)-\frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$

47. $\frac{\theta}{2}+\frac{1}{16}\sin 8\theta+C$ **49.** $\frac{\sec^{49}2z}{98}+C$ **51.** $\frac{4}{15}$

53. $2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\ln\left(\sqrt[6]{x}+1\right)+C$

55. $-\frac{\sqrt{9-y^2}}{9\sqrt{2}\,y}+C$ **57.** $\frac{\pi}{9}$ **59.** $-\operatorname{sech} x+C$ **61.** $\frac{\pi}{3}$

63. $\frac{1}{8}\ln\left|\frac{x-5}{x+3}\right|+C$ **65.** $\frac{\ln 2}{4}+\frac{\pi}{8}$ **67.** 3 **69.** $\frac{1}{3}\ln\left|\frac{x-2}{x+1}\right|+C$

71. $2(x-2\ln|x+2|)+C$ **73.** $e^{2t}/\left(2\sqrt{1+e^{4t}}\right)+C$

75. a. $\sec e^x+C$ **b.** $e^x\sec e^x-\ln|\sec e^x+\tan e^x|+C$

77. $\frac{\sqrt{6}}{3}\tan^{-1}\sqrt{\frac{2x-3}{3}}+C$

79. $\frac{1}{4}\sec^3 x\tan x+\frac{3}{8}\sec x\tan x+\frac{3}{8}\ln|\sec x+\tan x|+C$

81. $2(\ln^3 2-3\ln^2 2+\ln 64-3)$ **83.** 1 **85.** $\frac{\pi}{2}$

87. $\frac{2\pi}{\sqrt{3}}$ **89.** Converges **91.** Diverges **93.** 1.196288

95. $M(4)=44;\ T(4)=42;\ S(4)=\frac{124}{3}$

97. $M(40)\approx 0.398236;\ T(40)\approx 0.398771;\ S(40)\approx 0.398416$

99. 0.886227 **101.** y-axis **103.** $\pi(e-2)$ **105.** $\frac{\pi}{2}(e^2-3)$

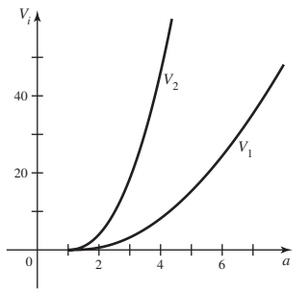
107. a. 1.603 **b.** 1.870 **c.** $b\ln b-b=a\ln a-a$
d. Decreasing **109.** $20/(3\pi)$ **111.** 1901 cars

113. a. $I(p)=\frac{1}{(p-1)^2}(1-pe^{1-p})$ if $p\neq 1$, $I(1)=\frac{1}{2}$ **b.** $0,\infty$
c. $I(0)=1$ **115.** 0.4054651 **117.** $n=2$

119. a. $V_1(a)=\pi(a\ln^2 a-2a\ln a+2(a-1))$

b. $V_2(a)=\frac{\pi}{2}(2a^2\ln a-a^2+1)$

c. $V_2(a)>V_1(a)$ for all $a>1$



121. $a=\ln 2/(2b)$ **123.** $\ln(1+\sqrt{2}/2)$

CHAPTER 9

Section 9.1 Exercises, pp. 604–606

1. a. 1 **b.** Linear **3.** Yes **5.** $\frac{\pi}{2}<t<\frac{3\pi}{2}$

21. $y=3t-\frac{e^{-2t}}{2}+C$ **23.** $y=2\ln|\sec 2x|-3\sin x+C$

25. $y=2t^6+6t^{-1}-2t^2+C_1t+C_2$

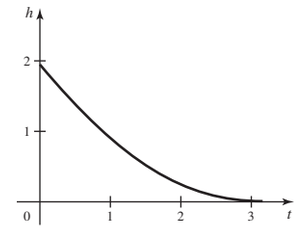
27. $u=\frac{x^{11}}{2}+\frac{x^9}{2}-\frac{x^7}{2}+\frac{5}{x}+C_1x+C_2$

29. $u=\ln(x^2+4)-\tan^{-1}\frac{x}{2}+C$ **31.** $y=\sin^{-1}x+C_1x+C_2$

33. $y=e^t+t+3$ **35.** $y=x^3+x^{-3}-2,\ x>0$

37. $y=-t^5+2t^3+1$ **39.** $y=e^t(t-2)+2(t+1)$

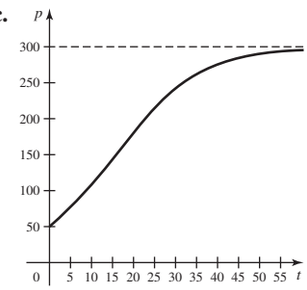
41. $u=\frac{1}{4}\tan^{-1}\frac{x}{4}-4x+2$ **43. a.** $v(t)=-9.8t+29.4$;
$s(t)=-4.9t^2+29.4t+30$; the object is above the ground for approximately $0\le t\le 6.89$. **b.** The highest point of 74.1 m is reached at $t=3$ s. **45.** The amount of resource is increasing for $H<75$ and is constant if $H=75$. If $H=100$, the resource vanishes at approximately 28 time units.
47. $h=(\sqrt{1.96}-0.1t\sqrt{2g})^2\approx(1.4-0.44t)^2$, $0\le t\le 3.16$; the tank is empty after approximately 3.16 s.



49. a. False **b.** False **c.** True **51. c.** $y=C_1\sin kt+C_2\cos kt$

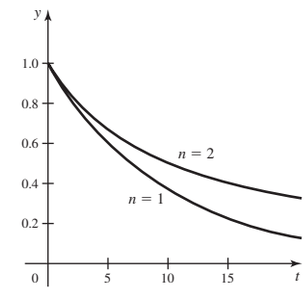
53. b. $C=\frac{K-50}{50}$

c.



d. 300

55. c. The decay rate is greater for the $n=1$ model.



Section 9.2 Exercises, pp. 611–614

1. At selected points (t_0, y_0) in the region of interest draw a short line segment with slope $f(t_0, y_0)$. **3.** $y(3.1)\approx 1.6$
5. a. D **b.** B **c.** A **d.** C
7.