Homework 8 Solution

Tuesday, November 2, 2021 10:56 AM

Disc method or Washer method.

i) slice perpendicular to the revolving axis

⇒ integrate wrt the variable in the revolving axis

ii) Find the inside radius and outside radius

Xuemao Zhang **Math141_Calculus_II**
Assignment Homework8 due 11/04/2021 at 11:59pm EDT

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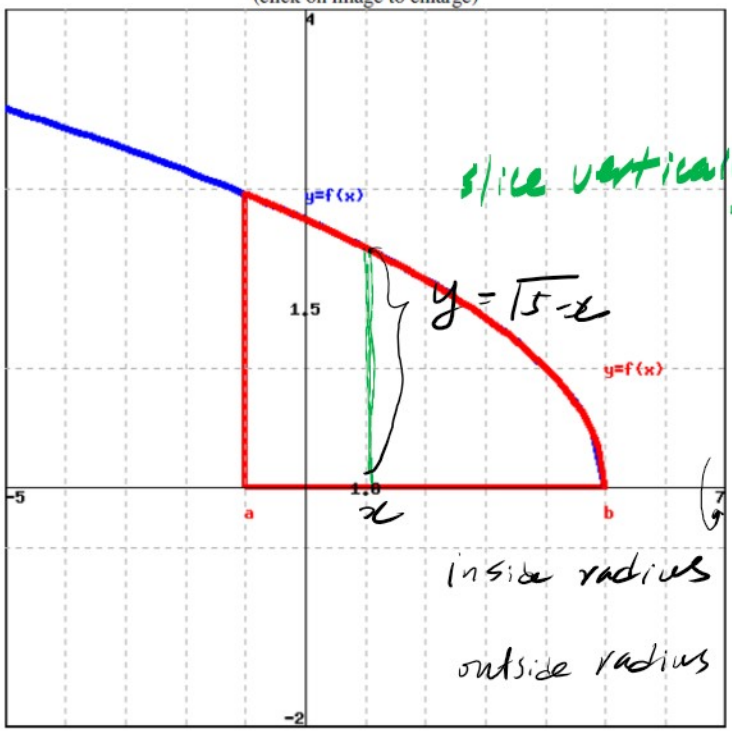
1. **(1 point)** Library/Wiley/setAnton_Section_6.2/anton_6_2_Q1.pg

Find the volume of the solid that results when the red region is revolved about the x-axis. $f(x) = \sqrt{5-x}$, $a = -1$, $b = 5$

$x=-1$, $x=5$

(click on image to enlarge)



slice vertically: integrate wrt x

$y = \sqrt{5-x}$

inside radius $= 0$

outside radius

$$V = \int_{-1}^{5} \pi r^2 \, dx = \int_{-1}^{5} \pi \left(\sqrt{5-x}\right)^2 dx$$

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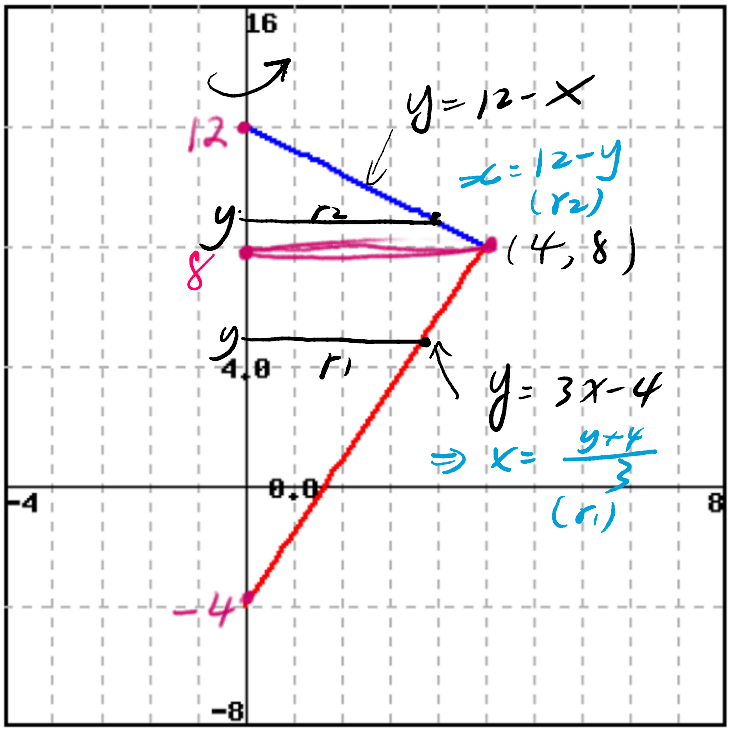
$$= \int_{-1}^{5} \pi (5-x) \, dx$$

2. **(1 point)** Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/6_Applications_of_the_Integral/6.3_Volumes_of_Revolution/6.3.42.pg

Find the volume of the solid obtained by rotating the region enclosed by the graphs of $y = 12 - x$, $y = 3x - 4$ and $x = 0$ about the y-axis.

$V =$_____

Solution: *(Instructor solution preview: show the student solution after due date.)*



Integrate wrt y

$$\begin{cases} y = 12 - x \\ y = 3x - 4 \end{cases}$$

$$\Rightarrow 12 - x = 3x - 4$$

$$16 = 4x$$

$$\Rightarrow \begin{cases} x = 4 \\ y = 8 \end{cases}$$

Correct Answers:

$$V = \int_{-4}^{12} \pi r^2 \, dy = \int_{-4}^{8} \pi r_1^2 \, dy + \int_{8}^{12} \pi r_2^2 \, dy$$

$$= \int_{-4}^{8} \pi \left(\frac{y+4}{3}\right)^2 dy + \int_{8}^{12} \pi (12-y)^2 \, dy$$

$$= \int_{-4}^{0} \pi \left(\frac{y+4}{3}\right)^2 dy + \int_{8}^{12} \pi (12-y)^2 dy$$

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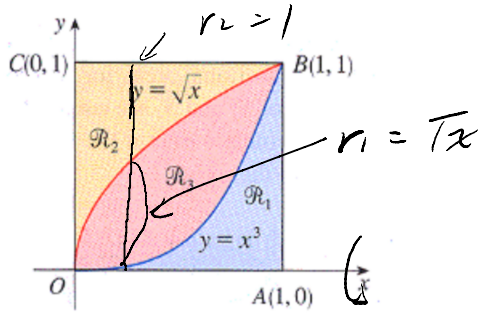
4. **(1 point)** Library/UCSB/Stewart5_6_2/Stewart5_6_2_23/Stewart5_6_2_23.pg



$r_2 = 1$

$r_1 = \sqrt{x}$

Integrate wrt $x \in [0,1]$

Referring to the figure above, find the volume generated by rotating the region $\mathcal{R}_2$ about the line OA.
Volume = _____

$$V = \pi \int_0^1 \left[r_2^2 - r_1^2 \right] dx$$

$$= \pi \int_0^1 \left[1 - (\sqrt{x})^2 \right] dx$$

$$= \pi \int_0^1 (1 - x)\, dx$$

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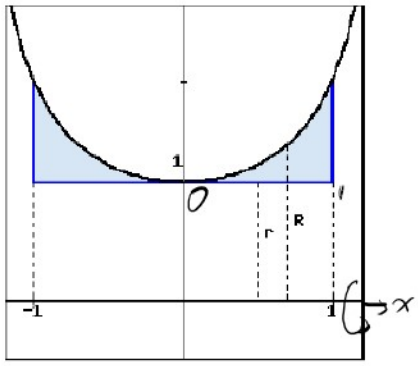
7. **(1 point)** Library/UCSB/Stewart5_6_2/Stewart5_6_2_8.pg

Using disks or washers, find the volume of the solid obtained by rotating the region bounded by the curves $y = \sec(x)$, $y = 1$, $x = -1$, and $x = 1$ about the x-axis.
Volume = _____
Solution: *(Instructor solution preview: show the student solution after due date.)*



$V = 2\int_0^1 \pi\left[\sec^2 x - 1\right] dx$

by symmetry

$r = 1,\ R = \sec x$

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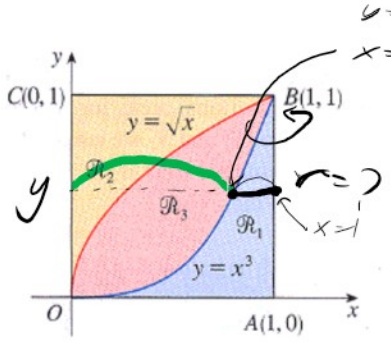
8. **(1 point)** Library/UCSB/Stewart5_6_2/Stewart5_6_2_21/Stewart5_6_2_21.pg



$\Rightarrow r = 1 - y^3$

Disc method

Integrate wrt $y \in [0, 1]$

Referring to the figure above, find the volume generated by rotating the region $\mathcal{R}_1$ about the line AB.
Volume = _____

$$V = \int_0^1 \pi r^2 \, dy$$

$$= \pi \int_0^1 \left(1 - y^{\frac{1}{3}}\right)^2 dy$$

$$= \pi \int_0^1 \left(1 + y^{\frac{2}{3}} - 2y^{\frac{1}{3}}\right) dy$$

Correct Answers:

- pi/10