Chapter 11-2 – Faster Sorting Algorithms

1. <u>Merge Sort – Divide and Conquer sorting algorithm</u>

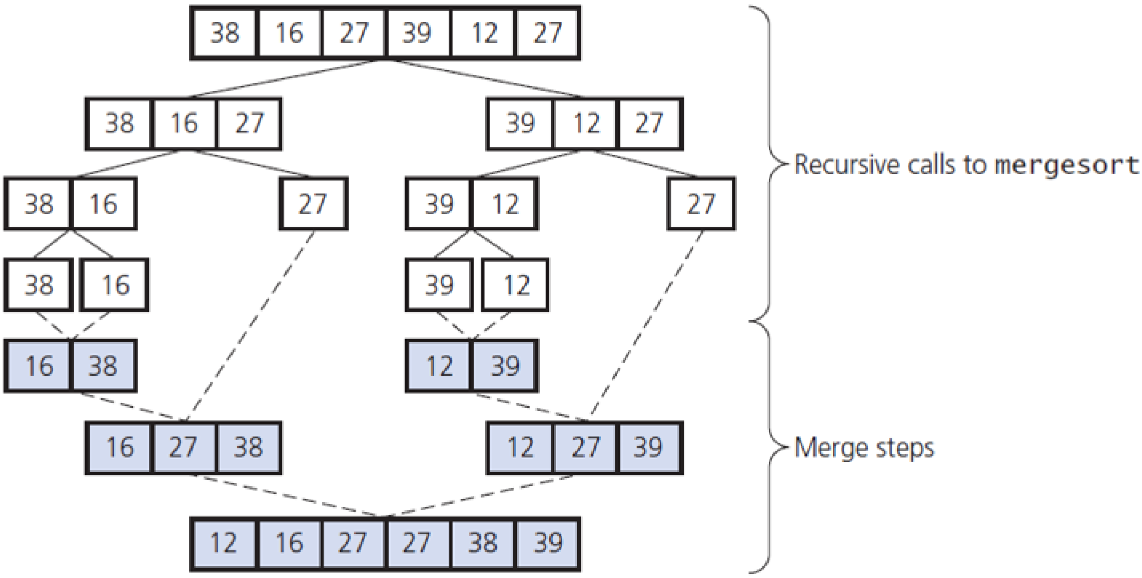
Divide: divide the n-element sequence to be sorted into two subsequences of n/2 elements each
Conquer: sort the two subsequences recursively using merge sort. IF the length of a sequence is 1, do nothing since it is already in order.
Combine: merge two sorted subsequences to produce the sorted answer.

```
// Sorts theArray[first..last] by
//   1. Sorting the first half of the array
//   2. Sorting the second half of the array
//   3. Merging the two sorted halves
mergeSort(theArray: ItemArray, first: integer, last: integer)
{
   if (first < last)
   {
      mid = (first + last) / 2        // Get midpoint

      // Sort theArray[first..mid]
      mergeSort(theArray, first, mid)

      // Sort theArray[mid+1..last]
      mergeSort(theArray, mid + 1, last)

      // Merge sorted halves theArray[first..mid] and theArray[mid+1..last]
      merge(theArray, first, mid, last)
   }
   // If first >= last, there is nothing to do
}
```

Example 1)

8	1	4	3	2

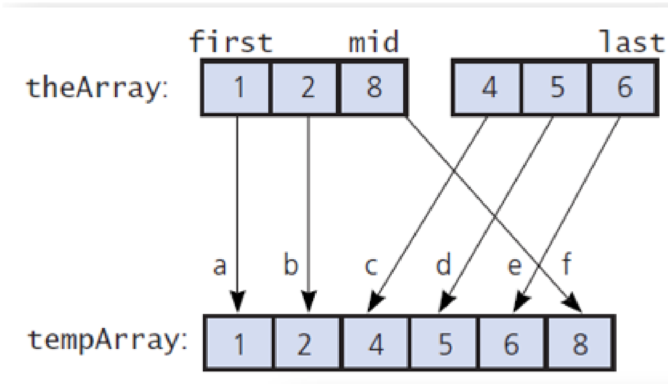
Example 2)



38 16 27 39 12 27
38 16 27
39 12 27
38 16
27
39 12
27
38
16
39
12
16 38
12 39
16 27 38
12 27 39
12 16 27 27 38 39
Recursive calls to mergesort
Merge steps

Analysis - Merge step

- worst-case instance of the merge step in a merge sort

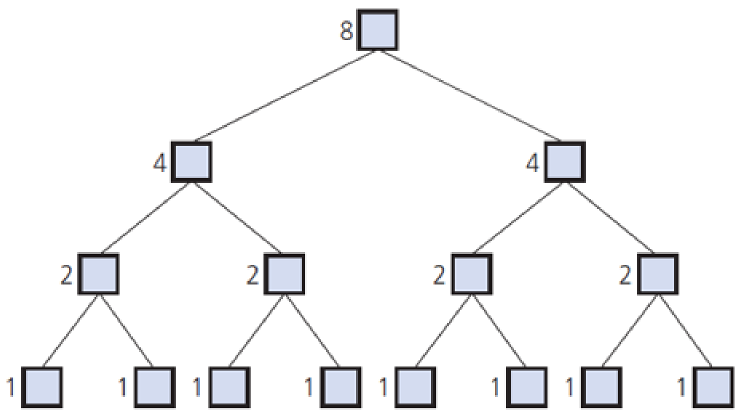


Merge the halves:

a. compare 1 < 4, move 1 to the tempArray
b. compare 2 < 4, move 2 to the tempArray
c. compare 8 > 4, move 4 to the tempArray
d. compare 8 > 5, move 5 to the tempArray
e. compare 8 > 6, move 6 to the tempArray
f. move 8 to the tempArray
g. move all values in the tempArray to the original array

Analysis – mergeSort

```
mergeSort(theArray: ItemArray, first: integer, last: integer)
{
   if (first < last)
   {
      mid = (first + last) / 2        // Get midpoint

      // Sort theArray[first..mid]
      mergeSort(theArray, first, mid)

      // Sort theArray[mid+1..last]
      mergeSort(theArray, mid + 1, last)

      // Merge sorted halves theArray[first..mid] and theArray[mid+1..last]
      merge(theArray, first, mid, last)
   }
   // If first >= last, there is nothing to do
}
```



Level 0: `mergesort` 8 items

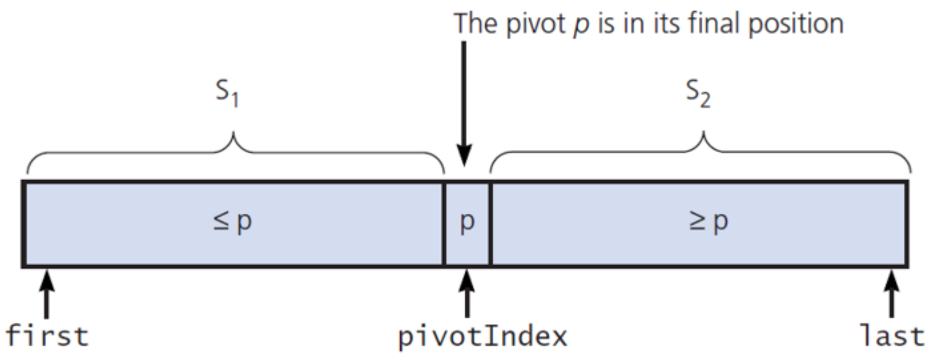
Level 1: 2 calls to `mergesort` with 4 items each

Level 2: 4 calls to `mergesort` with 2 items each

Level 3: 8 calls to `mergesort` with 1 item each

2. **<u>The Quick Sort</u>**

- Another divide-and-conquer algorithm
- Partitions an array into items that are
  - Less than or equal to the pivot and
  - Those that are greater than or equal to the pivot
- Partitioning places pivot in its correct position within the array
  - Place chosen pivot in theArray[last] before partitioning



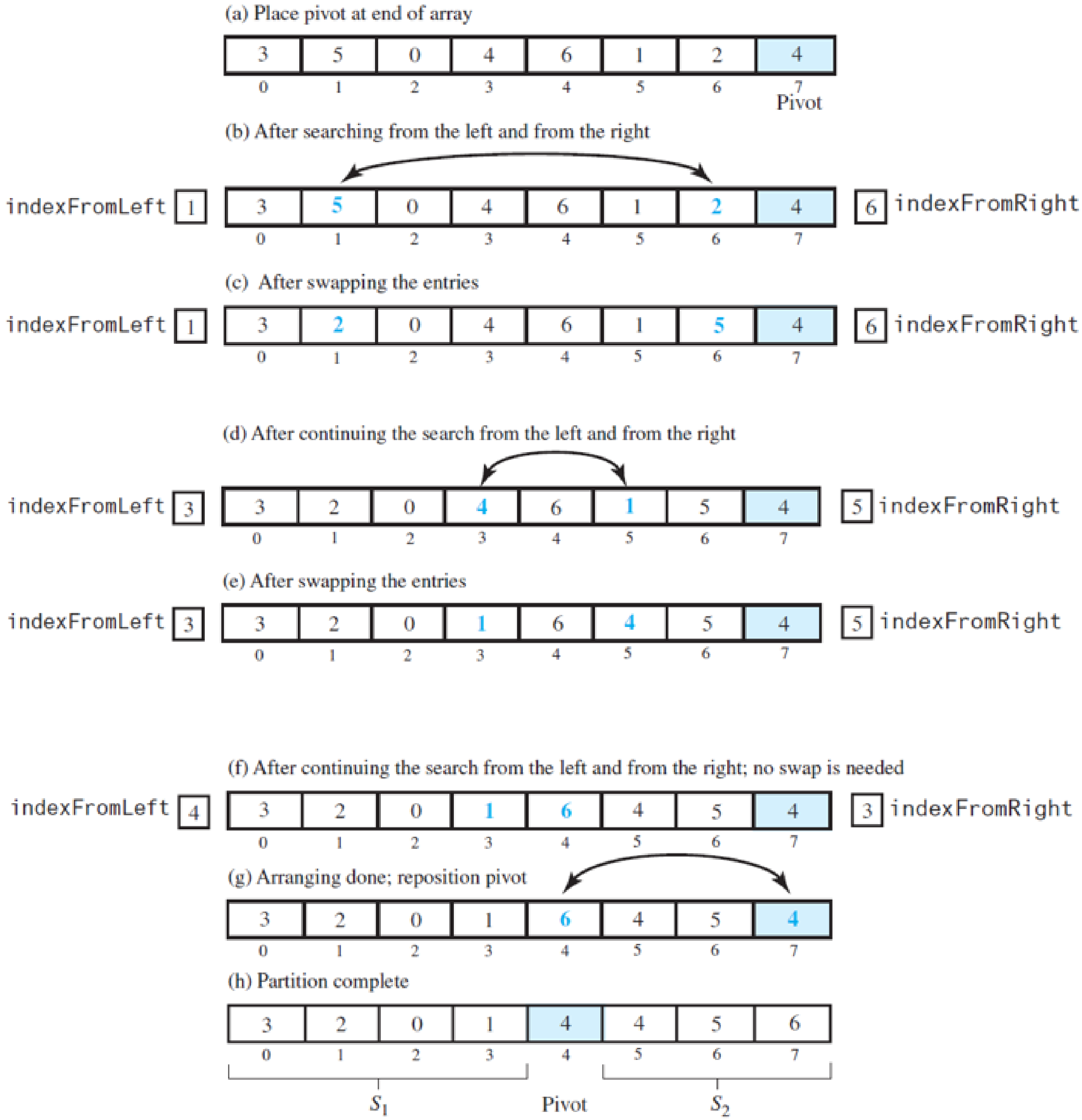

```
// Sorts theArray[first..last].
quickSort(theArray: ItemArray, first: integer, last: integer): void
{
   if (first < last)
   {
      Choose a pivot item p from theArray[first..last]
      Partition the items of theArray[first..last] about p
      // The partition is theArray[first..pivotIndex..last]

      quickSort(theArray, first, pivotIndex - 1) // Sort S1

      quickSort(theArray, pivotIndex + 1, last)  // Sort S2
   }
   // If first >= last, there is nothing to do
}
```

Example A partitioning of an array during a quick sort

3	5	0	7	6	1	2	4

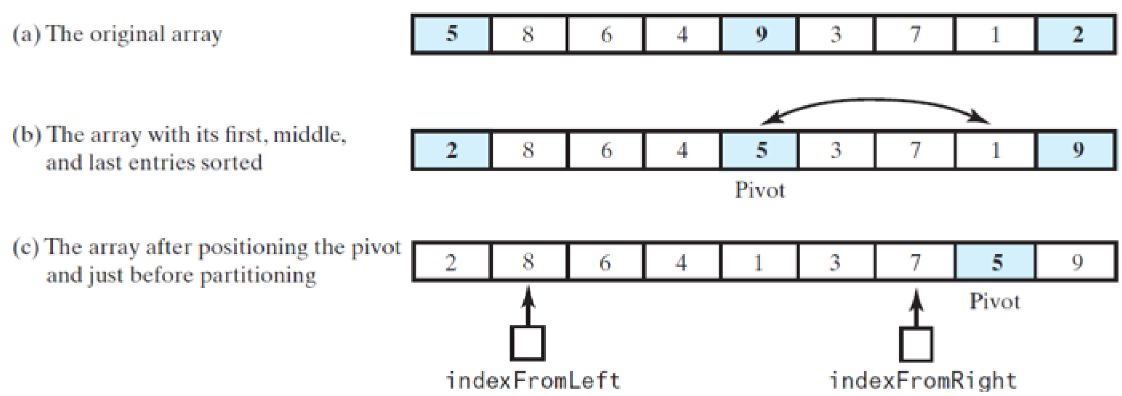
```
// Partitions theArray[first..last].
partition(theArray: ItemArray, first: integer, last: integer): integer
{
   // Choose pivot and reposition it
   mid = first + (last - first) / 2
   sortFirstMiddleLast(theArray, first, mid, last)
   Interchange theArray[mid] and theArray[last - 1]
   pivotIndex = last - 1
   pivot = theArray[pivotIndex]

   // Determine the regions S1 and S2
   indexFromLeft = first + 1
   indexFromRight = last - 2

   done = false
   while (not done)
   {
      // Locate first entry on left that is ≥ pivot
      while (theArray[indexFromLeft] < pivot)
         indexFromLeft = indexFromLeft + 1

      // Locate first entry on right that is ≤ pivot
      while (theArray[indexFromRight] > pivot)
         indexFromRight = indexFromRight - 1

      if (indexFromLeft < indexFromRight)
      {
         Interchange theArray[indexFromLeft] and theArray[indexFromRight]
         indexFromLeft = indexFromLeft + 1
         indexFromRight = indexFromRight - 1
      }
      else
         done = true
   }
   // Place pivot in proper position between S1 and S2, and mark its new location
   Interchange theArray[pivotIndex] and theArray[indexFromLeft]
   pivotIndex = indexFromLeft

   return pivotIndex
}
```

Example A partitioning of an array during a quick sort



Median-of-three pivot selection

```
// Arranges the first, middle, and last entries in an array into ascending order.
sortFirstMiddleLast(theArray: ItemArray, first: integer, mid: integer,
                    last: integer): void
{
   if (theArray[first] > theArray[mid])
      Interchange theArray[first] and theArray[mid]

   if (theArray[mid] > theArray[last])
      Interchange theArray[mid] and theArray[last]

   if (theArray[first] > theArray[mid])
      Interchange theArray[first] and theArray[mid]
}
```



2	8	6	4	1	3	7	5	9

Example Trace the quicksort's partitioning algorithm as it partitions the following array

38	16	40	39	12	27

Analysis

Partitioning

Average case

Worst case

A Comparison of sorting algorithms

	Worst case	Average case
Selection sort	n^2	n^2
Bubble sort	n^2	n^2
Insertion sort	n^2	n^2
Merge sort	$n \times \log n$	$n \times \log n$
Quick sort	n^2	$n \times \log n$