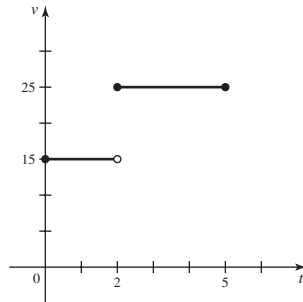


CHAPTER 5

Section 5.1 Exercises, pp. 347–352

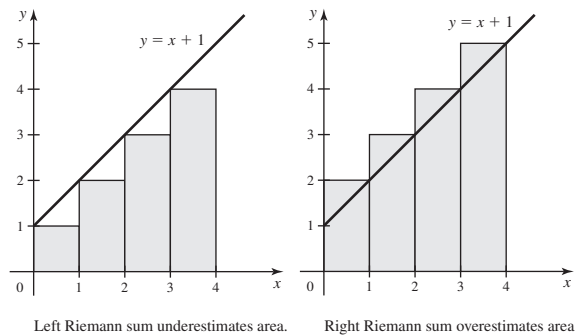
1. Displacement = 105 m



3. a. 440 ft b. 400 ft 5. a. 340 ft b. 330 ft

7. Subdivide the interval $[0, \pi/2]$ into several subintervals, which will be the bases of rectangles that fit under the curve. The heights of the rectangles are computed by taking the value of $\cos x$ at the right-hand endpoint of each base. We calculate the area of each rectangle and add them to get a lower bound on the area. 9. Left sum: 34; right sum: 24 11. 0.5; 1, 1.5, 2, 2.5, 3; 1, 1.5, 2, 2.5; 1.5, 2, 2.5, 3; 1.25, 1.75, 2.25, 2.75 13. Underestimate; the rectangles all fit under the curve. 15. a. 67 ft b. 67.75 ft 17. 40 m 19. 2.78 m 21. 148.96 mi 23. 20; 25

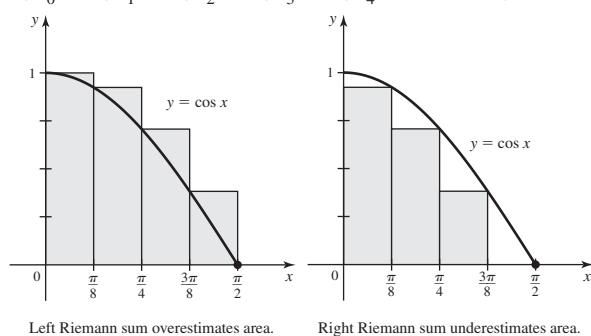
25. a. c.



Left Riemann sum underestimates area. Right Riemann sum overestimates area.

- b.
- $\Delta x = 1$
- ;
- $x_0 = 0$
- ,
- $x_1 = 1$
- ,
- $x_2 = 2$
- ,
- $x_3 = 3$
- ,
- $x_4 = 4$
- d. 10, 14

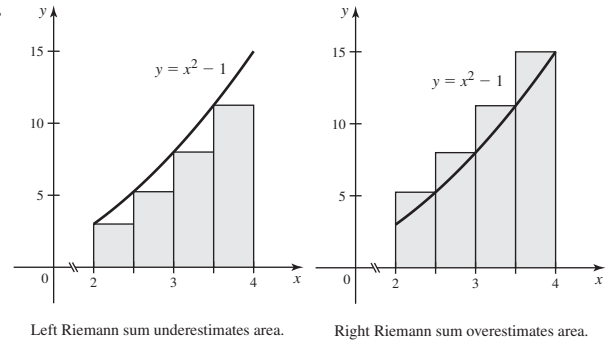
27. a. c.



Left Riemann sum overestimates area. Right Riemann sum underestimates area.

- b.
- $\Delta x = \pi/8$
- ;
- $0, \pi/8, \pi/4, 3\pi/8, \pi/2$
- d. 1.18; 0.79

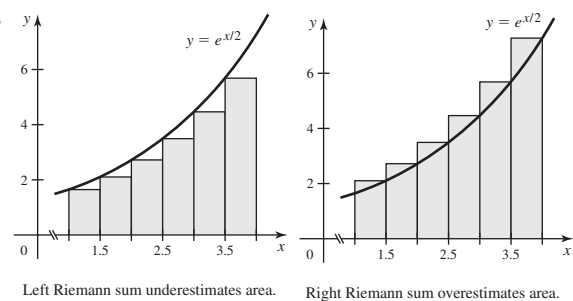
29. a. c.



Left Riemann sum underestimates area. Right Riemann sum overestimates area.

- b.
- $\Delta x = 0.5$
- ; 2, 2.5, 3, 3.5, 4 d. 13.75; 19.75

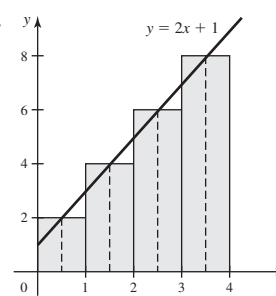
31. a. c.



Left Riemann sum underestimates area. Right Riemann sum overestimates area.

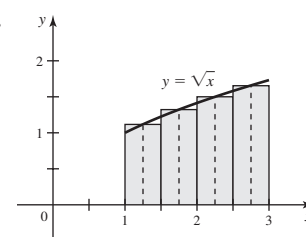
- b. $\Delta x = 0.5$; $x_0 = 1$, $x_1 = 1.5$, $x_2 = 2$, $x_3 = 2.5$, $x_4 = 3$, $x_5 = 3.5$, $x_6 = 4$ d. 10.11, 12.98 33. 670 35. a. 10,500 m; 10,350 m
b. Left Riemann sum c. Increase the number of subintervals in the partition.

37. a. c.



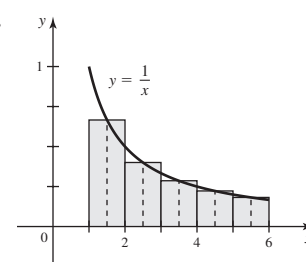
- b. $\Delta x = 1$; 0, 1, 2, 3, 4
d. 20

39. a. c.



- b. $\Delta x = \frac{1}{2}$; $1, \frac{3}{2}, 2, \frac{5}{2}, 3$
d. 2.80

41. a. c.



- b. $\Delta x = 1$; 1, 2, 3, 4, 5, 6
d. 1.76

43. 5.5, 3.5 45. b. 110, 117.5 47. a. $\sum_{k=1}^5 k$ b. $\sum_{k=1}^6 (k+3)$

c. $\sum_{k=1}^4 k^2$ d. $\sum_{k=1}^4 \frac{1}{k}$ 49. a. 55 b. 48 c. 30 d. 60 e. 6

f. 6 g. 85 h. 0 51. a. Left: $\frac{3}{10} \sum_{k=1}^{40} \sqrt{\frac{k-1}{10}} \approx 15.6809$;

right: $\frac{3}{10} \sum_{k=1}^{40} \sqrt{\frac{k}{10}} \approx 16.2809$; midpoint: $\frac{3}{10} \sum_{k=1}^{40} \sqrt{\frac{k-0.5}{10}} \approx 16.0055$

b. 16 53. a. Left: $\frac{1}{25} \sum_{k=1}^{75} \left(\left(2 + \frac{k-1}{25} \right)^2 - 1 \right) \approx 35.5808$;

right: $\frac{1}{25} \sum_{k=1}^{75} \left(\left(2 + \frac{k}{25} \right)^2 - 1 \right) \approx 36.4208$;

midpoint: $\frac{1}{25} \sum_{k=1}^{75} \left(\left(2 + \frac{k-0.5}{25} \right)^2 - 1 \right) \approx 35.9996$ b. 36

55.

n	Right Riemann sum
10	21.96
30	21.9956
60	21.9989
80	21.9994

The sums approach 22.

57.

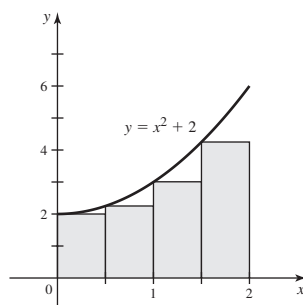
n	Right Riemann sum
10	3.14159
30	3.14159
60	3.14159
80	3.14159

The sums approach π .

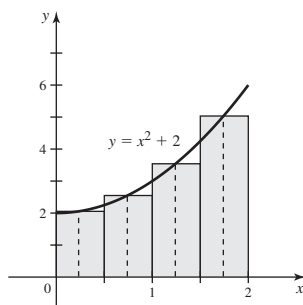
59. a. True b. False c. True 61. $\sum_{k=1}^{50} \left(\frac{4k}{50} + 1 \right) \cdot \frac{4}{50} = 12.16$

63. $\sum_{k=1}^{32} \left(3 + \frac{2k-1}{8} \right)^3 \cdot \frac{1}{4} \approx 3639.1$ 65. $[1, 5]; 4$ 67. $[2, 6]; 4$

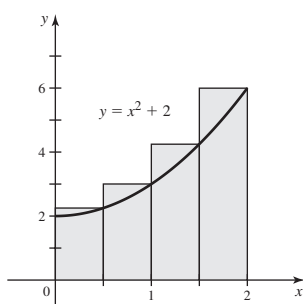
69. a. Left Riemann sum is $\frac{23}{4} = 5.75$.



b. Midpoint Riemann sum is $\frac{53}{8} = 6.625$.



c. Right Riemann sum is $\frac{31}{4} = 7.75$.



71. a. The object is speeding up on the interval $(0, 1)$, moving at a constant rate on $(1, 3)$, slowing down on $(3, 5)$, and moving at a constant rate on $(5, 6)$.

b. 30 m c. 50 m d. $s(t) = 80 + 10t$

73. a. 14.5 g b. 29.5 g c. 44 g d. $x = \frac{19}{3}$ cm

75. $s(t) = \begin{cases} 30t & \text{if } 0 \leq t \leq 2 \\ 50t - 40 & \text{if } 2 < t \leq 2.5 \\ 44t - 25 & \text{if } 2.5 < t \leq 3 \end{cases}$

77.

n	Midpoint Riemann sum
16	4.7257
32	4.7437
64	4.7485

The sums approach 4.75.

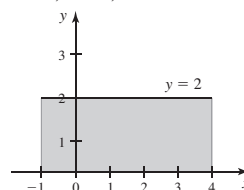
81. Underestimates for decreasing functions, independent of concavity; overestimates for increasing functions, independent of concavity

Section 5.2 Exercises, pp. 364–367

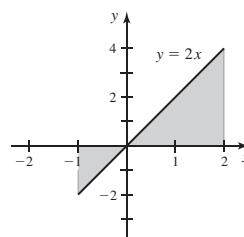
1. The difference between the area bounded by the curve above the x -axis and the area bounded by the curve below the x -axis 3. 60; 0

5. $-12; -18; -16$

7. $\int_{-1}^4 2 \, dx = 10$

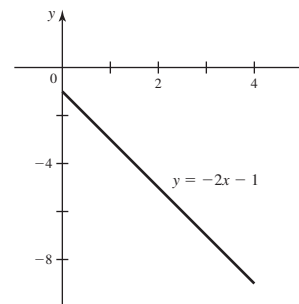


9. $\int_{-1}^2 2x \, dx = 3$

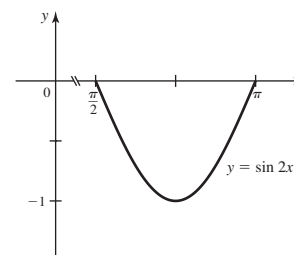


11. Both integrals equal 0. 13. The length of the interval $[a, a]$ is $a - a = 0$, so the net area is 0. 15. $\frac{a^2}{2}$

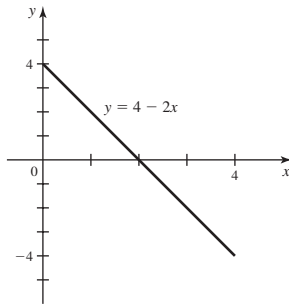
17. a. $y = -2x - 1$ b. $-16, -24, -20$



19. a. $y = \sin 2x$ b. $-0.948, -0.948, -1.026$

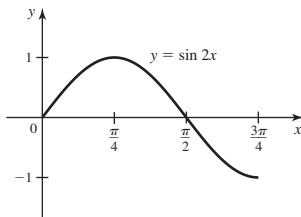


21. a.



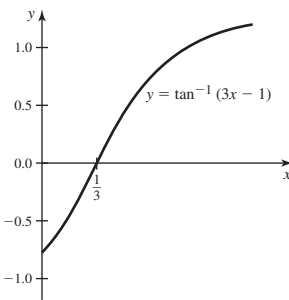
b. 4, -4, 0 c. Positive contributions on $[0, 2]$; negative contributions on $(2, 4]$

23. a.



b. 0.735, 0.146, 0.530
c. Positive contributions on $(0, \pi/2)$; negative contributions on $(\pi/2, 3\pi/4]$

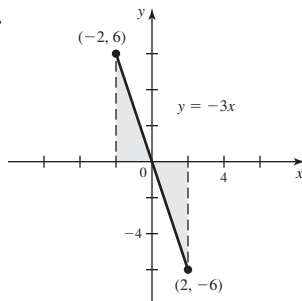
25. a.



b. 0.082; 0.555; 0.326

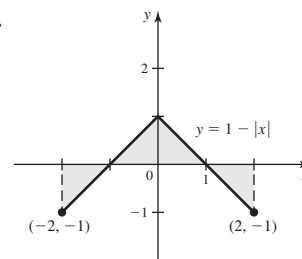
c. Positive contributions on $(\frac{1}{3}, 1]$; negative contributions on $[0, \frac{1}{3})$

27.



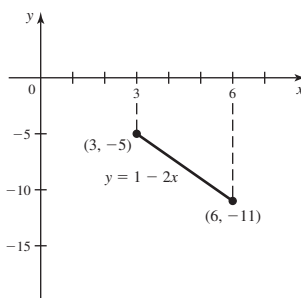
The area is 12; the net area is 0.

29.



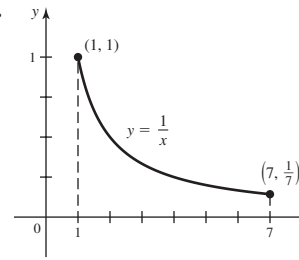
The area is 2; the net area is 0.

31. a.



b. $\Delta x = \frac{1}{2}$; 3, 3.5, 4, 4.5, 5, 5.5, 6 c. -22.5; -25.5
d. The left Riemann sum overestimates the integral; the right Riemann sum underestimates the integral.

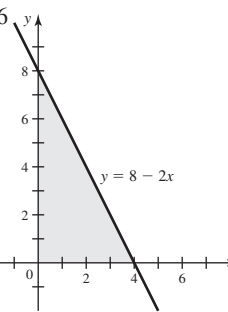
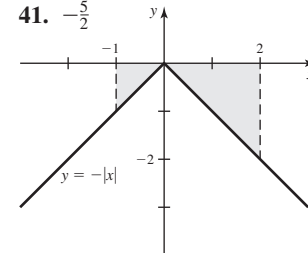
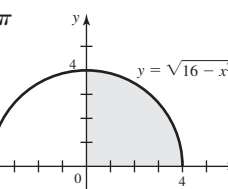
33. a.



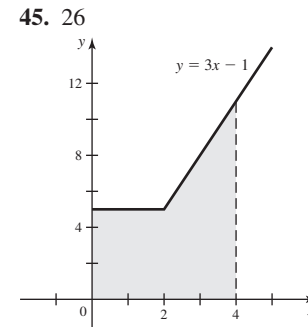
b. $\Delta x = 1$; 1, 2, 3, 4, 5, 6, 7
c. $\frac{49}{20}, \frac{223}{140}$ d. The left Riemann sum overestimates the integral; the right Riemann sum underestimates the integral.

35. $\int_0^2 (x^2 + 1) dx$ 37. $\int_1^2 x \ln x dx$

39. 16


41. $-\frac{5}{2}$

43. 4π


45. 26


47. π 49. -2π 51. a. -32 b. $-\frac{32}{3}$ c. -64 d. Not possible

53. a. 10 b. -3 c. -16 d. 3 55. a. 15 b. 5 c. 3

d. -2 e. 24 f. -10 57. a. $\frac{3}{2}$ b. $-\frac{3}{4}$ 59. 16 61. 6

63. 32 65. -16 67. $\frac{\pi}{4} + 2$ 69. a. True b. True c. True

d. False e. False

71. a. Left: $\sum_{k=1}^n \left(\left(\frac{k-1}{n} \right)^2 + 1 \right) \cdot \frac{1}{n}$;

right: $\sum_{k=1}^n \left(\left(\frac{k}{n} \right)^2 + 1 \right) \cdot \frac{1}{n}$

b.

n	Left Riemann sum	Right Riemann sum
20	1.30875	1.35875
50	1.3234	1.3434
100	1.32835	1.33835

Estimate: $\frac{4}{3}$

73. a. Left: $\sum_{k=1}^n \cos^{-1} \left(\frac{k-1}{n} \right) \frac{1}{n}$;

right: $\sum_{k=1}^n \cos^{-1} \left(\frac{k}{n} \right) \frac{1}{n}$

b.

n	Left Riemann sum	Right Riemann sum
20	1.03619	0.95765
50	1.01491	0.983494
100	1.00757	0.99186

Estimate: 1

75. a. $\sum_{k=1}^n 2\sqrt{1 + \left(k - \frac{1}{2}\right)\frac{3}{n}} \cdot \frac{3}{n}$

b.

n	Midpoint Riemann sum
20	9.33380
50	9.33341
100	9.33335

Estimate: $\frac{28}{3}$

77. a. $\frac{4}{n} \sum_{k=1}^n \left(4\left(k - \frac{1}{2}\right)\frac{4}{n} - \left(\left(k - \frac{1}{2}\right)\frac{4}{n} \right)^2 \right)$

b.

n	Midpoint Riemann sum
20	10.6800
50	10.6688
100	10.6672

Estimate: $\frac{32}{3}$

79. 6 81. 104 83. 18 85. 2 87. $25\pi/2$ 89. 25 91. 35

95. For any such partition on $[0, 1]$, the grid points are $x_k = k/n$, for $k = 0, 1, \dots, n$. That is, x_k is rational for each k so that $f(x_k) = 1$, for $k = 0, 1, \dots, n$. Therefore, the left, right, and midpoint Riemann sums are $\sum_{k=1}^n 1 \cdot (1/n) = 1$.

Section 5.3 Exercises, pp. 377–381

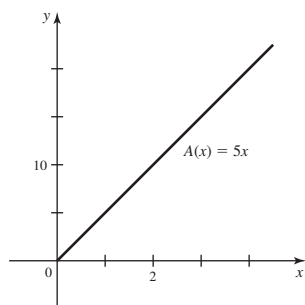
1. A is an antiderivative of f ; $A'(x) = f(x)$.

3. $\int_a^b f(x) dx = F(b) - F(a)$, where F is any antiderivative of f .

5. Increasing 7. The derivative of the integral of f is f , or $\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$. 9. $f(x), 0$ 11. 16 13. a. 0 b. -9

c. 25 d. 0 e. 16

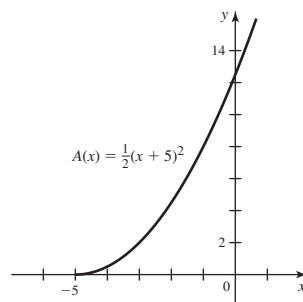
15. a. $A(x) = 5x$



b. $A'(x) = 5$

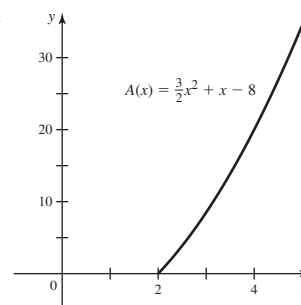
17. a. $A(2) = 2, A(4) = 8; A(x) = \frac{1}{2}x^2$ b. $F(4) = 6, F(6) = 16; F(x) = \frac{1}{2}x^2 - 2$ c. $A(x) - F(x) = \frac{1}{2}x^2 - (\frac{1}{2}x^2 - 2) = 2$

19. a.



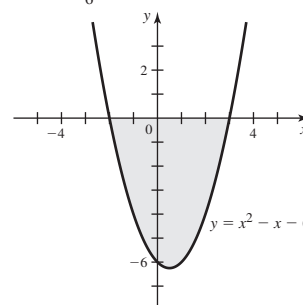
b. $A'(x) = \left(\frac{1}{2}(x+5)^2 \right)' = x+5 = f(x)$

21. a.

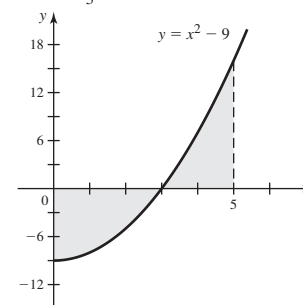


b. $A'(x) = \left(\frac{3}{2}x^2 + x - 8 \right)' = 3x + 1 = f(x)$ 23. $\frac{7}{3}$

25. $-\frac{125}{6}$



27. $-\frac{10}{3}$

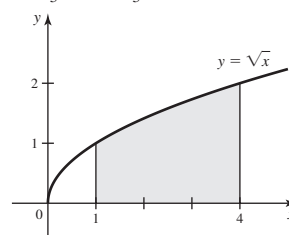


29. 16 31. 90 33. $\frac{7}{6}$ 35. 8 37. $-\frac{32}{3}$ 39. $-\frac{5}{2}$ 41. $\frac{9}{2}$ 43. $-\frac{3}{8}$

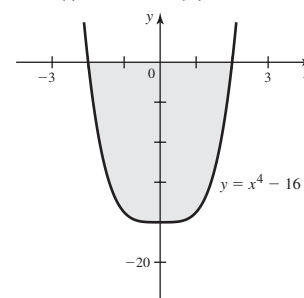
45. 1 47. $3 \ln 2$ 49. $\frac{45}{4}$ 51. $\frac{2}{3}$ 53. 1 55. 2 57. $\frac{\pi}{12}$

59. $\frac{3}{2} + 4 \ln 2$ 61. $\frac{3\pi}{2} - 1$

63. (i) $\frac{14}{3}$ (ii) $\frac{14}{3}$



65. (i) -51.2 (ii) 51.2

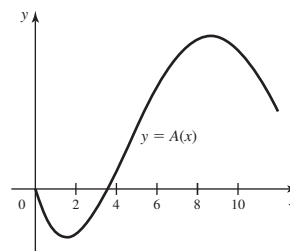


67. $\frac{94}{3}$ 69. $\ln 2$ 71. 2 73. $x^2 + x + 1$ 75. $-\sqrt{x^4 + 1}$

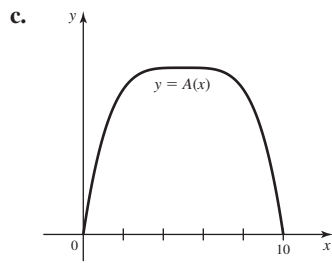
77. $3/x^4$ 79. $-(\cos^4 x + 6) \sin x$ 81. $-\frac{\cos z}{\sin^4 z + 1}$

83. $\frac{9}{t}$ 85. $2\sqrt{1+x^2}$ 87. a-C, b-B, c-D, d-A

89. a. $x = 0, x \approx 3.5$ b. Local min at $x \approx 1.5$; local max at $x \approx 8.5$ c.

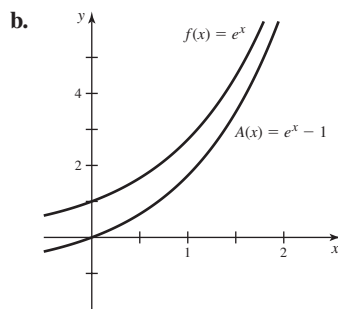


91. a. $x = 0, 10$ b. Local max at $x = 5$



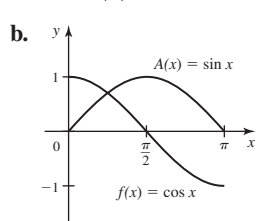
93. $-\pi, -\pi + \frac{9}{2}, -\pi + 9, 5 - \pi$

95. a. $A(x) = e^x - 1$



- c. $A(\ln 2) = 1; A(\ln 4) = 3$

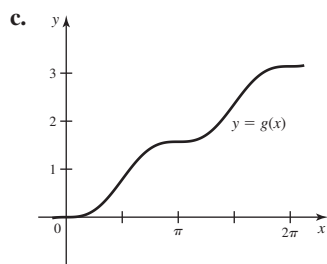
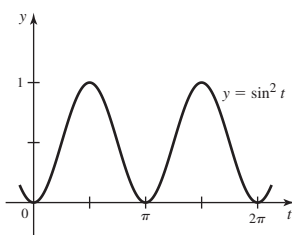
97. a. $A(x) = \sin x$



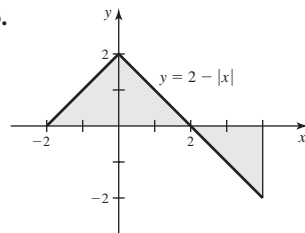
- c. $A\left(\frac{\pi}{2}\right) = 1; A(\pi) = 0$

99. Critical pts. $x = 0, 3$, and 4 ; increasing on $(-\infty, 0)$, $(0, 3)$, and $(4, \infty)$; decreasing on $(3, 4)$

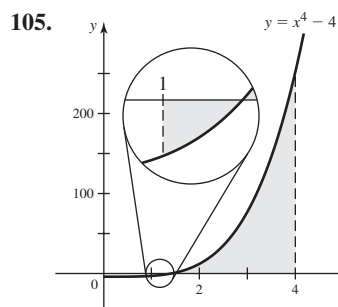
101. a. b. $g'(x) = \sin^2 x$



103.



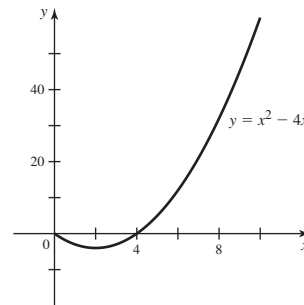
Area = 6



Area ≈ 194.05

107. a. True b. True
c. False d. True
109. 3

111. a. b. $b = 6$ c. $b = \frac{3a}{2}$



113. $f(x) = -2 \sin x + 3$ 115. $\pi/2 \approx 1.57$

$$117. (S'(x))^2 + \left(\frac{S''(x)}{2x}\right)^2 = (\sin x^2)^2 + \left(\frac{2x \cos x^2}{2x}\right)^2 \\ = \sin^2 x^2 + \cos^2 x^2 = 1$$

119. c. The summation relationship is a discrete analog of the Fundamental Theorem. Summing the difference quotient and integrating the derivative over the relevant interval give the difference of the function values at the endpoints.

Section 5.4 Exercises, pp. 385–387

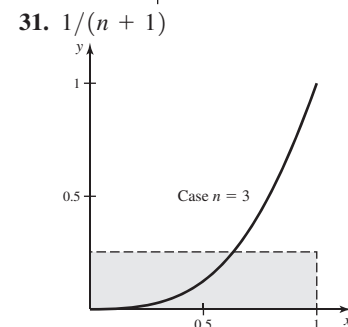
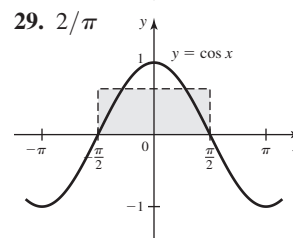
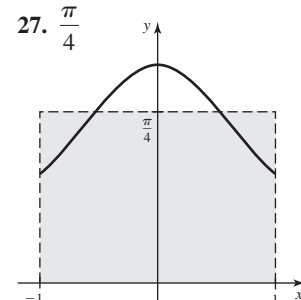
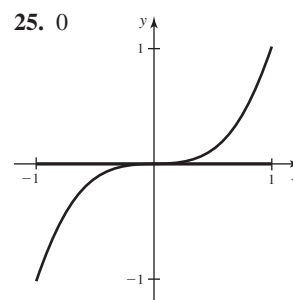
1. If f is odd, the regions between f and the positive x -axis and between f and the negative x -axis are reflections of each other through the origin. Therefore, on $[-a, a]$, the areas cancel each other.

3. a. 9 b. 0 5. $3x^3$ and x are odd functions. 7. Even; even

9. If f is continuous on $[a, b]$, then there is a c in (a, b) such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx. \quad 11. 0 \quad 13. \frac{1000}{3} \quad 15. \frac{16}{3} \quad 17. -\frac{88}{3}$$

19. 0 21. 2 23. 0



33. 2000 35. 21 m/s 37. $20/\pi$ 39. 2 41. $a/\sqrt{3}$

43. $c = \pm \frac{1}{2}$ 45. a. True b. True c. True d. False

47. 420 ft 49. $f(g(-x)) = f(g(x)) \Rightarrow$ the integrand is even;

$$\int_{-a}^a f(g(x)) dx = 2 \int_0^a f(g(x)) dx \quad 51. p(g(-x)) = p(g(x)) \Rightarrow$$

the integrand is even; $\int_{-a}^a p(g(x)) dx = 2 \int_0^a p(g(x)) dx$

53. a. $a/6$ b. $(3 \pm \sqrt{3})/6$, independent of a

57.

Even	Even
Even	Odd

Section 5.5 Exercises, pp. 395–3981. The Chain Rule 3. $u = g(x)$ 5. The lower bound a becomes $g(a)$ and the upper bound b becomes $g(b)$. 7. $\frac{(x^2 + 1)^5}{5} + C$ 9. $\frac{1}{4} \sin^4 x + C$ 11. $\frac{(x + 1)^{13}}{13} + C$ 13. $\frac{(2x + 1)^{3/2}}{3} + C$ 15. a. $\frac{1}{10} e^{10x} + C$ b. $\frac{1}{5} \sec 5x + C$ c. $-\frac{1}{7} \cos 7x + C$ d. $7 \sin \frac{x}{7} + C$ e. $\frac{1}{27} \tan^{-1} \frac{x}{3} + C$ f. $\sin^{-1} \frac{x}{6} + C$ 17. $\frac{(x^2 - 1)^{100}}{100} + C$ 19. $-\frac{(1 - 4x^3)^{1/2}}{3} + C$ 21. $\frac{(x^2 + x)^{11}}{11} + C$ 23. $\frac{(x^4 + 16)^7}{28} + C$ 25. $\frac{1}{2} \sin^{-1} \frac{x}{3} + C$ 27. $\frac{4x^3}{\ln 2} + C$ 29. $\frac{(x^6 - 3x^2)^5}{30} + C$ 31. $\frac{3}{5} \sin^{-1} 5x + C$ 33. $\frac{1}{6} \tan^{-1} \frac{e^w}{6} + C$ 35. $-\frac{1}{2} \csc x^2 + C$ 37. $\frac{1}{10} \tan(10x + 7) + C$ 39. $\frac{10^{4x+1}}{4 \ln 10} + C$ 41. $\frac{1}{2} \tan^2 x + C$ 43. $\frac{1}{7} \sec^7 x + C$ 45. $\frac{\sqrt{2}}{4}$ 47. $\frac{7}{2}$ 49. 1 51. $\frac{1}{3}$ 53. $\frac{2 - \sqrt{2}}{2}$ 55. $(e^9 - 1)/3$ 57. $\sqrt{2} - 1$ 59. $\frac{\pi}{6}$ 61. $\frac{1}{2} \ln 17$ 63. $\frac{\pi}{9}$ 65. $\frac{1}{3}$ 67. $\frac{3}{4} (4 - 3^{2/3})$ 69. $\frac{32}{3}$ 71. $-\ln 3$ 73. $\frac{1}{7}$ 75. 10 m/s 77. a. 160 b. $\frac{4800}{49} \approx 98$ c. $\Delta p = \int_0^T \frac{200}{(t + 1)^r} dt$;decreases as r increases d. $r \approx 1.28$ e. As $t \rightarrow \infty$, thepopulation approaches 100. 79. $\frac{2}{3} (x - 4)^{1/2} (x + 8) + C$ 81. $\frac{3}{5} (x + 4)^{2/3} (x - 6) + C$ 83. $\frac{3}{112} (2x + 1)^{4/3} (8x - 3) + C$ 85. $\frac{(x + 10)^{10} (x - 1)}{11} + C$ 87. π 89. $\frac{\theta}{2} - \frac{1}{4} \sin \left(\frac{6\theta + \pi}{3} \right) + C$ 91. $\frac{\pi}{4}$ 93. $\ln \frac{9}{8}$ 95. a. Trueb. True c. False d. False e. False 97. 1 99. $\frac{2}{3}$; constant101. a. π/p b. 0 103. $2/\pi$ 105. One area is $\int_4^9 \frac{(\sqrt{x} - 1)^2}{2\sqrt{x}} dx$.Changing variables by letting $u = \sqrt{x} - 1$ yields $\int_1^2 u^2 du$, which is theother area. 107. 7297/12 109. $\frac{2}{15} (3 - 2a)(1 + a)^{3/2} + \frac{4}{15} a^{5/2}$ 111. $\frac{1}{3} \sec^3 \theta + C$ 113. a. $I = \frac{1}{8} x - \frac{1}{32} \sin 4x + C$ b. $I = \frac{1}{8} x - \frac{1}{32} \sin 4x + C$ 117. $\frac{4}{3} (-2 + \sqrt{1 + x}) \sqrt{1 + \sqrt{1 + x}} + C$ 119. $-4 + \sqrt{17}$ **Chapter 5 Review Exercises, pp. 398–402**

1. a. True b. False c. True d. True e. False

f. True g. True

3. a.  b. 75 c. The area is the distance the diver ascends.

5. 9.34; 10.28; 9.82

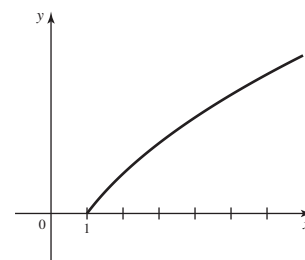
n	Midpoint Riemann sum
10	114.167
30	114.022
60	114.006

$$\int_1^{25} \sqrt{2x - 1} dx = 114$$

9. a. $1((3 \cdot 2 - 2) + (3 \cdot 3 - 2) + (3 \cdot 4 - 2)) = 21$ b. $\sum_{k=1}^n \frac{3}{n} \left(3 \left(1 + \frac{3k}{n} \right) - 2 \right)$ c. $\frac{33}{2}$ 11. $-\frac{16}{3}$ 13. 56

15. a. 20 b. 0 c. 80 d. 10 e. 0 17. 18 19. 10

21. Not enough information 23. a. 8.5 b. -4.5 c. 0 d. 11.5

25. 4π 27. A: $\int_0^x f(t) dt$; B: $f(x)$; C: $f'(x)$ 29. $\sqrt{1 + x^4 + x^6}$ 31. $-\sin x^6$ 33. $\frac{2}{x^{10} + 1}$ 35. Increasing on $(3, 6)$; decreasingon $(-\infty, 3)$ and $(6, \infty)$ 39. $\frac{212}{5}$ 41. $x^9 - x^7 + C$ 43. $\frac{7}{6}$ 45. $\frac{4}{\sqrt{3}}$ 47. $\frac{\pi}{12}$ 49. $-\frac{4}{3 \sin^{3/4} x} + C$ 51. $\frac{1}{3} \sin x^3 + C$ 53. $\frac{1}{28} \tan^{-1} \left(\frac{\sin 7w}{4} \right) + C$ 55. $\frac{1}{\ln 2}$ 57. 78 59. $\frac{5}{6} e^2 (e^3 - 1)$ 61. $e^{e^x} + C$ 63. $\frac{1}{2} \sin^{-1} 2x + C$ 65. $\pi + \frac{3\sqrt{3}}{4}$ 67. $\frac{\pi}{2}$ 69. $\frac{1}{3} \ln \frac{9}{2}$ 71. 0 73. $\cos \frac{1}{x} + C$ 75. $\ln |\tan^{-1} x| + C$ 77. $(x + 3)^{11} \left(\frac{11x - 3}{132} \right) + C$ 79. 1 81. $\frac{\pi}{12}$ 83. 0 85. 48 87. $\frac{256}{3}$ 89. 8 91. $-\frac{4}{15}, \frac{4}{15}$ 93. Approx. 431.5 ft 95. Displacement = 0; distance = $20/\pi$ 97. $\frac{3}{2 \ln 2}$ 99. a. $5/2, c = 3.5$ b. $3, c = 3$ and $c = 5$ 101. 24103. $f(1) = 0$; $f'(x) > 0$ on $[1, \infty)$; $f''(x) < 0$ on $[1, \infty)$ 105. a. $\frac{3}{2}, \frac{5}{6}$ b. x c. $\frac{1}{2} x^2$ d. $-1, \frac{1}{2}$ e. 1, 1 f. $\frac{3}{2}$ 107. e^4 113. a. Increasing on $(-\infty, 1)$ and $(2, \infty)$; decreasing on $(1, 2)$ b. Concave up on $(\frac{13}{8}, \infty)$; concave down on $(-\infty, \frac{13}{8})$ c. Local max at $x = 1$; local min at $x = 2$ d. Inflection point at $x = \frac{13}{8}$ 115. Differentiating the first equation gives thesecond equation; no. 117. $\sqrt[4]{12}$