Chapter 10 Review Exercises, pp. 704–707

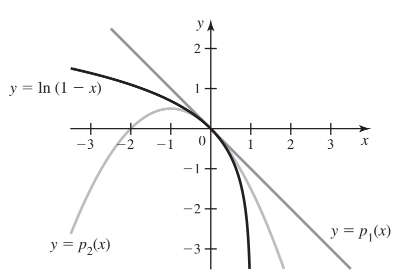
1. a. False **b.** False **c.** True **d.** False **e.** True **f.** False **g.** False **h.** True **3.** Approx. 1.25; approx. 0.05 **5.** $\lim_{k\to\infty} a_k = 0$, $\lim_{n\to\infty} S_n = 8$ **7.** $a_k = \frac{1}{k}$ **9. a.** 0 **b.** $\frac{5}{9}$ **11. a.** Yes; $\lim_{k\to\infty} a_k = 1$ **b.** No; $\lim_{k\to\infty} a_k \neq 0$ **13.** Diverges **15.** 5 **17.** 0 **19.** 0 **21.** $1/e$ **23.** Diverges **25. a.** 80, 48, 32, 24, 20 **b.** 16 **27.** Diverges **29.** Diverges **31.** Diverges **33.** $\frac{3\pi}{4}$ **35.** 3 **37.** 2/9 **39.** $\frac{311}{990}$ **41.** 200 mg **43.** Diverges **45.** Diverges **47.** Converges **49.** Converges **51.** Converges **53.** Converges **55.** Converges **57.** Diverges **59.** Converges **61.** Converges **63.** Converges **65.** Converges **67.** Converges **69.** Converges **71.** Converges **73.** Diverges **75.** Diverges **77.** Converges conditionally **79.** Converges absolutely **81.** Diverges **83.** Converges absolutely **85.** Converges absolutely **87.** Diverges **89. a.** Approx. 1.03666 **b.** 0.0004 **c.** $L_5 = 1.03685$; $U_5 = 1.03706$ **91.** 0.0067 **93.** 100 **95. a.** 803 m, 1283 m, $2000(1 - 0.95^N)$ m **b.** 2000 m **97. a.** $\frac{\pi}{2^{n-1}}$ **b.** 2π **99. a.** $T_1 = \frac{\sqrt{3}}{16}$, $T_2 = \frac{7\sqrt{3}}{64}$ **b.** $T_n = \frac{\sqrt{3}}{4}\left(1 - \left(\frac{3}{4}\right)^n\right)$ **c.** $\lim_{n\to\infty} T_n = \frac{\sqrt{3}}{4}$ **d.** 0 **101.** $\sqrt{\frac{20}{g}\left(\frac{1 + \sqrt{p}}{1 - \sqrt{p}}\right)}$ s

CHAPTER 11

Section 11.1 Exercises, pp. 718–721

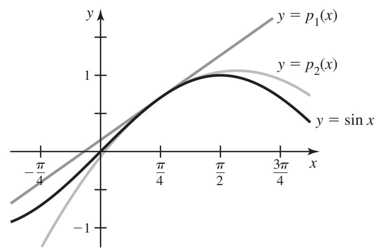
1. $f(0) = p_2(0)$, $f'(0) = p_2'(0)$, and $f''(0) = p_2''(0)$ **3.** 1, 1.05, 1.04875 **5.** $p_3(x) = 1 + x^2 + x^3$; 1.048 **7.** $p_3(x) = 1 + (x - 2) + 2(x - 2)^3$; 0.898 **9. a.** $p_1(x) = 8 + 12(x - 1)$ **b.** $p_2(x) = 8 + 12(x - 1) + 3(x - 1)^2$ **c.** 9.2; 9.23 **11. a.** $p_1(x) = 1 - 2x$ **b.** $p_2(x) = 1 - 2x + 2x^2$ **c.** 0.8, 0.82 **13. a.** $p_1(x) = 1 - x$ **b.** $p_2(x) = 1 - x + x^2$ **c.** 0.95, 0.9525 **15. a.** $p_1(x) = 2 + \frac{1}{12}(x - 8)$ **b.** $p_2(x) = 2 + \frac{1}{12}(x - 8) - \frac{1}{288}(x - 8)^2$ **c.** $1.958\overline{3}$, 1.95747 **17.** $p_1(x) = 1$, $p_2(x) = p_3(x) = 1 - 18x^2$, $p_4(x) = 1 - 18x^2 + 54x^4$ **19.** $p_3(x) = 1 - 3x + 6x^2 - 10x^3$, $p_4(x) = 1 - 3x + 6x^2 - 10x^3 + 15x^4$ **21.** $p_1(x) = 1 + 3(x - 1)$, $p_2(x) = 1 + 3(x - 1) + 3(x - 1)^2$, $p_3(x) = 1 + 3(x - 1) + 3(x - 1)^2 + (x - 1)^3$ **23.** $p_3(x) = 1 + \frac{1}{e}(x - e) - \frac{1}{2e^2}(x - e)^2 + \frac{1}{3e^3}(x - e)^3$ **25. a.** $p_1(x) = -x$, $p_2(x) = -x - \frac{x^2}{2}$

b.



27. a. $p_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$, $p_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2$

b.



29. a. 1.0247 **b.** 7.6×10^{-6} **31. a.** 0.8613 **b.** 5.4×10^{-4} **33. a.** 1.1274988 **b.** Approx. 8.85×10^{-6} (Answers may vary if intermediate calculations are rounded.) **35. a.** Approx. −0.10033333 **b.** Approx. 1.34×10^{-6} (Answers may vary if intermediate calculations are rounded.) **37. a.** 1.0295635 **b.** Approx. 4.86×10^{-7} (Answers may vary if intermediate calculations are rounded.) **39. a.** Approx. 0.52083333 **b.** Approx. 2.62×10^{-4} (Answers may vary if intermediate calculations are rounded.) **41.** $R_n(x) = \frac{\sin^{(n+1)}(c)}{(n+1)!}x^{n+1}$, for c between x and 0 **43.** $R_n(x) = \frac{(-1)^{n+1}e^{-c}}{(n+1)!}x^{n+1}$, for c between x and 0 **45.** $R_n(x) = \frac{\sin^{(n+1)}(c)}{(n+1)!}\left(x - \frac{\pi}{2}\right)^{n+1}$, for c between x and $\frac{\pi}{2}$ **47.** 2.0×10^{-5} **49.** 1.6×10^{-5} ($e^{0.25} < 2$) **51.** 2.6×10^{-4} **53.** With $n = 4$, $|\text{error}| \le 2.5 \times 10^{-3}$ **55.** With $n = 2$, $|\text{error}| \le 4.2 \times 10^{-2}$ ($e^{0.5} < 2$) **57.** With $n = 2$, $|\text{error}| \le 5.4 \times 10^{-3}$ **59.** 4 **61.** 3 **63.** 1 **65. a.** False **b.** True **c.** True **d.** True **67. a.** C **b.** E **c.** A **d.** D **e.** B **f.** F **69. a.** 0.1; 1.7×10^{-4} **b.** 0.2; 1.3×10^{-3} **71. a.** 0.995; 4.2×10^{-6} **b.** 0.98; 6.7×10^{-5} **73. a.** 1.05; 1.3×10^{-3} **b.** 1.1; 5×10^{-3} **75. a.** 1.1; 10^{-2} **b.** 1.2; 4×10^{-2}

77. a.

x	$\lvert \sec x - p_2(x) \rvert$	$\lvert \sec x - p_4(x) \rvert$
−0.2	3.39×10^{-4}	5.51×10^{-6}
−0.1	2.09×10^{-5}	8.51×10^{-8}
0.0	0	0
0.1	2.09×10^{-5}	8.51×10^{-8}
0.2	3.39×10^{-4}	5.51×10^{-6}

b. The errors decrease as $|x|$ decreases.

79. a.

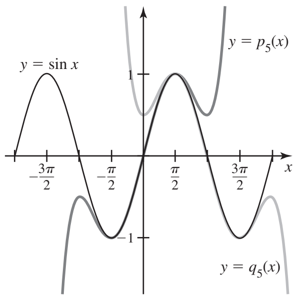
x	$\lvert e^{-x} - p_1(x) \rvert$	$\lvert e^{-x} - p_2(x) \rvert$
−0.2	2.14×10^{-2}	1.40×10^{-3}
−0.1	5.17×10^{-3}	1.71×10^{-4}
0.0	0	0
0.1	4.84×10^{-3}	1.63×10^{-4}
0.2	1.87×10^{-2}	1.27×10^{-3}

b. The errors decrease as $|x|$ decreases.

81. Centered at $x = 0$, for all n

83. a. $y = f(a) + f'(a)(x - a)$ **85. a.** $p_5(x) = x - \dfrac{x^3}{6} + \dfrac{x^5}{120}$; $q_5(x) = -(x - \pi) + \dfrac{1}{6}(x - \pi)^3 - \dfrac{1}{120}(x - \pi)^5$

b.



p_5 is a better approximation on $[-\pi, \pi/2)$; q_5 is a better approximation on $(\pi/2, 2\pi]$.

c.

x	$\lvert \sin x - p_5(x) \rvert$	$\lvert \sin x - q_5(x) \rvert$
$\pi/4$	3.6×10^{-5}	7.4×10^{-2}
$\pi/2$	4.5×10^{-3}	4.5×10^{-3}
$3\pi/4$	7.4×10^{-2}	3.6×10^{-5}
$5\pi/4$	2.3	3.6×10^{-5}
$7\pi/4$	20	7.4×10^{-2}

d. p_5 is a better approximation at $x = \pi/4$; at $x = \pi/2$ the errors are equal.

87. a. $p_1(x) = 6 + \dfrac{1}{12}(x - 36)$; $q_1(x) = 7 + \dfrac{1}{14}(x - 49)$

b.

x	$\lvert \sqrt{x} - p_1(x) \rvert$	$\lvert \sqrt{x} - q_1(x) \rvert$
37	5.7×10^{-4}	6.0×10^{-2}
39	5.0×10^{-3}	4.1×10^{-2}
41	1.4×10^{-2}	2.5×10^{-2}
43	2.6×10^{-2}	1.4×10^{-2}
45	4.2×10^{-2}	6.1×10^{-3}
47	6.1×10^{-2}	1.5×10^{-3}

c. p_1 is a better approximation at $x = 37$, 39, and 41.

Section 11.2 Exercises, pp. 729–730

1. $c_0 + c_1x + c_2x^2 + c_3x^3$ **3.** Ratio and Root Tests **5.** The radius of convergence does not change. The interval of convergence may change. **7.** $R = 10$; $[-8, 12]$ **9.** $R = \frac{1}{2}$; $\left(-\frac{1}{2}, \frac{1}{2}\right)$ **11.** $R = 0$; $\{x: x = 0\}$ **13.** $R = \infty$; $(-\infty, \infty)$ **15.** $R = 3$; $(-3, 3)$ **17.** $R = \infty$; $(-\infty, \infty)$ **19.** $R = 2$; $(-2, 2)$ **21.** $R = \infty$; $(-\infty, \infty)$ **23.** $R = 1$; $(0, 2]$ **25.** $R = \frac{1}{4}$; $\left[0, \frac{1}{2}\right]$ **27.** $R = 5$; $(-3, 7)$ **29.** $R = \infty$; $(-\infty, \infty)$ **31.** $R = \sqrt{3}$; $(-\sqrt{3}, \sqrt{3})$ **33.** $R = 1$; $(0, 2)$ **35.** $R = \infty$; $(-\infty, \infty)$ **37.** $R = e$ **39.** $R = e^4$ **41.** $\displaystyle\sum_{k=0}^{\infty}(3x)^k$; $\left(-\frac{1}{3}, \frac{1}{3}\right)$ **43.** $\displaystyle 2\sum_{k=0}^{\infty}x^{k+3}$; $(-1, 1)$ **45.** $\displaystyle 4\sum_{k=0}^{\infty}x^{k+12}$; $(-1, 1)$ **47.** $\displaystyle -\sum_{k=1}^{\infty}\frac{(3x)^k}{k}$; $\left[-\frac{1}{3}, \frac{1}{3}\right)$ **49.** $\displaystyle -2\sum_{k=1}^{\infty}\frac{x^{k+6}}{k}$; $[-1, 1)$ **51.** $\displaystyle g(x) = 2\sum_{k=1}^{\infty}k(2x)^{k-1}$; $\left(-\frac{1}{2}, \frac{1}{2}\right)$ **53.** $\displaystyle g(x) = \sum_{k=1}^{\infty}(-1)^k kx^{k-1}$; $(-1, 1)$ **55.** $\displaystyle g(x) = -\sum_{k=1}^{\infty}\frac{3^kx^k}{k}$; $\left[-\frac{1}{3}, \frac{1}{3}\right)$ **57.** $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}2kx^{2k-1}$; $(-1, 1)$ **59.** $\displaystyle\sum_{k=0}^{\infty}\left(-\frac{x}{3}\right)^k$; $(-3, 3)$ **61.** $\displaystyle\ln 2 - \frac{1}{2}\sum_{k=1}^{\infty}\frac{x^{2k}}{k\,4^k}$; $(-2, 2)$ **63. a.** True **b.** True **c.** True **d.** True **65.** $|x - a| < R$ **67.** $f(x) = \dfrac{1}{3 - \sqrt{x}}$; $1 < x < 9$ **69.** $f(x) = \dfrac{e^x}{e^x - 1}$; $0 < x < \infty$ **71.** $f(x) = \dfrac{3}{4 - x^2}$; $-2 < x < 2$ **73.** $\displaystyle\sum_{k=0}^{\infty}\frac{(-3x)^k}{k!}$; $-\infty < x < \infty$ **75.** $\displaystyle\lim_{k\to\infty}\left|\frac{c_{k+1}x^{k+1}}{c_kx^k}\right| = \lim_{k\to\infty}\left|\frac{c_{k+1}x^{k+m+1}}{c_kx^{k+m}}\right|$, so by the Ratio Test, the two series have the same radius of convergence. **77. a.** $f(x)\,g(x) = c_0d_0 + (c_0d_1 + c_1d_0)x + (c_0d_2 + c_1d_1 + c_2d_0)x^2$ **b.** $\displaystyle\sum_{k=0}^{n}c_kd_{n-k}$

Section 11.3 Exercises, pp. 740–742

1. The nth Taylor polynomial is the nth partial sum of the corresponding Taylor series. **3.** $\displaystyle\sum_{k=0}^{\infty}\frac{(x - 2)^k}{k!}$ **5.** Replace x with x^2 in the Taylor series for $f(x)$; $|x| < 1$. **7.** The Taylor series for a function f converges to f on an interval if, for all x in the interval, $\lim_{n\to\infty}R_n(x) = 0$, where $R_n(x)$ is the remainder at x. **9. a.** $1 - 2(x - 1) + 3(x - 1)^2 - 4(x - 1)^2$ **b.** $\displaystyle\sum_{k=0}^{\infty}(-1)^k(k + 1)(x - 1)^k$ **c.** $(0, 2)$ **11. a.** $1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!}$ **b.** $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^kx^k}{k!}$ **c.** $(-\infty, \infty)$ **13. a.** $2 + 6x + 12x^2 + 20x^3$ **b.** $\displaystyle\sum_{k=0}^{\infty}(k + 1)(k + 2)x^k$ **c.** $(-1, 1)$ **15. a.** $1 - x^2 + x^4 - x^6$ **b.** $\displaystyle\sum_{k=0}^{n}(-1)^kx^{2k}$ **c.** $(-1, 1)$ **17. a.** $1 + 2x + \dfrac{(2x)^2}{2!} + \dfrac{(2x)^3}{3!}$ **b.** $\displaystyle\sum_{k=0}^{\infty}\frac{(2x)^k}{k!}$ **c.** $(-\infty, \infty)$ **19. a.** $\dfrac{x}{2} - \dfrac{x^3}{3\cdot 2^3} + \dfrac{x^5}{5\cdot 2^5} - \dfrac{x^7}{7\cdot 2^7}$ **b.** $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^kx^{2k+1}}{(2k + 1)2^{2k+1}}$ **c.** $[-2, 2]$ **21. a.** $1 + (\ln 3)x + \dfrac{\ln^2 3}{2}x^2 + \dfrac{\ln^3 3}{6}x^3$ **b.** $\displaystyle\sum_{k=0}^{\infty}\frac{\ln^k 3}{k!}x^k$ **c.** $(-\infty, \infty)$ **23. a.** $1 + \dfrac{(3x)^2}{2} + \dfrac{(3x)^4}{24} + \dfrac{(3x)^6}{720}$ **b.** $\displaystyle\sum_{k=0}^{\infty}\frac{(3x)^{2k}}{(2k)!}$ **c.** $(-\infty, \infty)$ **25. a.** $(x - 3) - \dfrac{1}{2}(x - 3)^2 + \dfrac{1}{3}(x - 3)^3 - \dfrac{1}{4}(x - 3)^4$ **b.** $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}(x - 3)^k}{k}$ **c.** $(2, 4]$ **27. a.** $1 - \dfrac{(x - \pi/2)^2}{2!} + \dfrac{(x - \pi/2)^4}{4!} - \dfrac{(x - \pi/2)^6}{6!}$ **b.** $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}(x - \pi/2)^{2k}$ **29. a.** $1 - (x - 1) + (x - 1)^2 - (x - 1)^3$ **b.** $\displaystyle\sum_{k=0}^{\infty}(-1)^k(x - 1)^k$ **31. a.** $\ln 3 + \dfrac{(x - 3)}{3} - \dfrac{(x - 3)^2}{3^2\cdot 2} + \dfrac{(x - 3)^3}{3^3\cdot 3}$ **b.** $\displaystyle\ln 3 + \sum_{k=1}^{\infty}\frac{(-1)^{k+1}(x - 3)^k}{k3^k}$

33. a. $2 + 2(\ln 2)(x-1) + (\ln^2 2)(x-1)^2 + \dfrac{\ln^3 2}{3}(x-1)^3$
b. $\displaystyle\sum_{k=0}^{\infty}\frac{2(x-1)^k \ln^k 2}{k!}$ **35.** $x^2 - \dfrac{x^4}{2} + \dfrac{x^6}{3} - \dfrac{x^8}{4}$
37. $1 + 2x + 4x^2 + 8x^3$ **39.** $1 + \dfrac{x}{2} + \dfrac{x^2}{6} + \dfrac{x^3}{24}$
41. $1 - x^4 + x^8 - x^{12}$ **43.** $x^2 + \dfrac{x^6}{6} + \dfrac{x^{10}}{120} + \dfrac{x^{14}}{5040}$
45. a. $1 - 2x + 3x^2 - 4x^3$ **b.** 0.826
47. a. $1 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{7}{128}x^3$ **b.** 1.029
49. a. $1 - \frac{2}{3}x + \frac{5}{9}x^2 - \frac{40}{81}x^3$ **b.** 0.895 **51.** $1 + \dfrac{x^2}{2} - \dfrac{x^4}{8} + \dfrac{x^6}{16}$; $[-1, 1]$ **53.** $3 - \dfrac{3x}{2} - \dfrac{3x^2}{8} - \dfrac{3x^3}{16}$; $[-1, 1]$
55. $a + \dfrac{x^2}{2a} - \dfrac{x^4}{8a^3} + \dfrac{x^6}{16a^5}$; $|x| \le a$
57. $1 - 8x + 48x^2 - 256x^3$ **59.** $\dfrac{1}{16} - \dfrac{x^2}{32} + \dfrac{3x^4}{256} - \dfrac{x^6}{256}$
61. $\dfrac{1}{9} - \dfrac{2}{9}\left(\dfrac{4x}{3}\right) + \dfrac{3}{9}\left(\dfrac{4x}{3}\right)^2 - \dfrac{4}{9}\left(\dfrac{4x}{3}\right)^3$
63. $R_n(x) = \dfrac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$, where c is between 0 and x and $f^{(n+1)}(c) = \pm\sin c$ or $\pm\cos c$. Therefore, $|R_n(x)| \le \dfrac{|x|^{n+1}}{(n+1)!} \to 0$ as $n \to \infty$, for $-\infty < x < \infty$. **65.** $R_n(x) = \dfrac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$, where c is between 0 and x and $f^{(n+1)}(c) = (-1)^n e^{-c}$. Therefore, $|R_n(x)| \le \dfrac{|x|^{n+1}}{e^c(n+1)!} \to 0$ as $n \to \infty$, for $-\infty < x < \infty$.
67. a. False **b.** True **c.** False **d.** False **e.** True
69. a. $1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \dfrac{x^6}{6!}$ **b.** $R = \infty$
71. a. $1 - \frac{2}{3}x^2 + \frac{5}{9}x^4 - \frac{40}{81}x^6$ **b.** $R = 1$
73. a. $1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6$ **b.** $R = 1$
75. a. $1 - 2x^2 + 3x^4 - 4x^6$ **b.** $R = 1$ **77.** Approx. 3.9149
79. Approx. 1.8989 **85.** $\displaystyle\sum_{k=0}^{\infty}\left(\frac{x-4}{2}\right)^k$ **87.** Use three terms of the Taylor series for $\cos x$ centered at $a = \pi/4$; 0.766 **89. a.** Use three terms of the Taylor series for $\sqrt[3]{125 + x}$ centered at $a = 0$; 5.03968 **b.** Use three terms of the Taylor series for $\sqrt[3]{x}$ centered at $a = 125$; 5.03968 **c.** Yes

Section 11.4 Exercises, pp. 748–750

1. Replace f and g with their Taylor series centered at a and evaluate the limit. **3.** Substitute $x = -0.6$ into the Taylor series for e^x centered at 0. Because the resulting series is an alternating series, the error can be estimated easily. **7.** 1 **9.** $\frac{1}{2}$ **11.** 2 **13.** $\frac{2}{3}$ **15.** $\frac{1}{3}$
17. $\frac{3}{5}$ **19.** $-\frac{8}{5}$ **21.** 1 **23.** $\frac{3}{4}$ **25. a.** $1 + x + \dfrac{x^2}{2!} + \cdots + \dfrac{x^n}{n!} + \cdots$
b. e^x **c.** $-\infty < x < \infty$
27. a. $1 - x + x^2 - \cdots (-1)^{n-1}x^{n-1} + \cdots$ **b.** $\dfrac{1}{1+x}$ **c.** $|x| < 1$
29. a. $-2 + 4x - 8\cdot\dfrac{x^2}{2!} + \cdots + (-2)^n\dfrac{x^{n-1}}{(n-1)!} + \cdots$
b. $-2e^{-2x}$ **c.** $-\infty < x < \infty$ **31. a.** $1 - x^2 + x^4 - \cdots$
b. $\dfrac{1}{1+x^2}$ **c.** $-1 < x < 1$
33. a. $2 + 2t + \dfrac{2t^2}{2!} + \cdots + \dfrac{2t^n}{n!} + \cdots$ **b.** $y(t) = 2e^t$
35. a. $2 + 16t + 24t^2 + 24t^3 + \cdots + \dfrac{3^{n-1}\cdot 16}{n!}t^n + \cdots$
b. $y(t) = \frac{16}{3}e^{3t} - \frac{10}{3}$ **37.** 0.2448 **39.** 0.6958
41. 0.0600 **43.** 0.4994 **45.** $1 + 2 + \dfrac{2^2}{2!} + \dfrac{2^3}{3!}$
47. $1 - 2 + \dfrac{2}{3} - \dfrac{4}{45}$ **49.** $\frac{1}{2} - \frac{1}{8} + \frac{1}{24} - \frac{1}{64}$ **51.** $e - 1$
53. $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k+1}x^k}{k}$, for $-1 < x \le 1$; $\ln 2$ **55.** $\dfrac{2}{2-x}$ **57.** $\dfrac{4}{4+x^2}$
59. $-\ln(1-x)$ **61.** $-\dfrac{3x^2}{(3+x)^2}$ **63.** $\dfrac{6x^2}{(3-x)^3}$
65. a. False **b.** False **c.** True **67.** $\frac{a}{b}$ **69.** $e^{-1/6}$
71. $f^{(3)}(0) = 0$; $f^{(4)}(0) = 4e$ **73.** $f^{(3)}(0) = 2$; $f^{(4)}(0) = 0$
75. 2 **77.** 1.575 using four terms **79. a.** $S'(x) = \sin x^2$; $C'(x) = \cos x^2$ **b.** $\dfrac{x^3}{3} - \dfrac{x^7}{7\cdot 3!} + \dfrac{x^{11}}{11\cdot 5!} - \dfrac{x^{15}}{15\cdot 7!}$; $x - \dfrac{x^5}{5\cdot 2!} + \dfrac{x^9}{9\cdot 4!} - \dfrac{x^{13}}{13\cdot 6!}$ **c.** $S(0.05) \approx 0.00004166664807$; $C(-0.25) \approx -0.2499023614$ **d.** 1 **e.** 2
81. a. $1 - \dfrac{x^2}{4} + \dfrac{x^4}{64} - \dfrac{x^6}{2304}$ **b.** $R = \infty$, $-\infty < x < \infty$

Chapter 11 Review Exercises, pp. 750–752

1. a. True **b.** False **c.** True **d.** True **e.** True
3. $p_2(x) = 1 - \dfrac{3}{2}x^2$ **5.** $p_2(x) = 1 - (x-1) + \dfrac{3}{2}(x-1)^2$
7. $p_2(x) = 1 - \dfrac{1}{2}(x-1)^2$
9. $p_3(x) = \dfrac{5}{4} + \dfrac{3}{4}(x - \ln 2) + \dfrac{5}{8}(x - \ln 2)^2 + \dfrac{1}{8}(x - \ln 2)^3$
11. a. $p_1(x) = 1 + x$; $p_2(x) = 1 + x + \dfrac{x^2}{2}$
b.

n	$p_n(x)$	Error
1	0.92	3.1×10^{-3}
2	0.9232	8.4×10^{-5}

13. a. $p_1(x) = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}\left(x - \dfrac{\pi}{4}\right)$; $p_2(x) = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}\left(x - \dfrac{\pi}{4}\right) - \dfrac{\sqrt{2}}{4}\left(x - \dfrac{\pi}{4}\right)^2$
b.

n	$p_n(x)$	Error
1	0.5960	8.2×10^{-3}
2	0.5873	4.7×10^{-4}

15. $|R_3| < \dfrac{\pi^4}{4!}$ **17.** $R = \infty$, $(-\infty, \infty)$ **19.** $R = \infty$, $(-\infty, \infty)$
21. $R = 9$, $(-9, 9)$ **23.** $R = 2$, $[-4, 0)$ **25.** $R = \dfrac{3}{2}$, $[-2, 1]$
27. $R = \dfrac{1}{27}$ **29.** $\displaystyle\sum_{k=0}^{\infty}x^{2k}$; $(-1, 1)$ **31.** $\displaystyle\sum_{k=0}^{\infty}(-5x)^k$; $\left(-\frac{1}{5}, \frac{1}{5}\right)$

33. $\sum_{k=1}^{\infty} k(10x)^{k-1}$; $\left(-\frac{1}{10}, \frac{1}{10}\right)$ **35.** $1 + 3x + \frac{9x^2}{2!}$; $\sum_{k=0}^{\infty} \frac{(3x)^k}{k!}$

37. $-(x - \pi/2) + \frac{(x - \pi/2)^3}{3!} - \frac{(x - \pi/2)^5}{5!}$;

$\sum_{k=0}^{\infty} (-1)^{k+1} \frac{(x - \pi/2)^{2k+1}}{(2k+1)!}$

39. $4x - \frac{(4x)^3}{3} + \frac{(4x)^5}{5}$; $\sum_{k=0}^{\infty} \frac{(-1)^k (4x)^{2k+1}}{2k+1}$

41. $1 + 2(x-1)^2 + \frac{2}{3}(x-1)^4$; $\sum_{k=0}^{\infty} \frac{4^k (x-1)^{2k}}{(2k)!}$

43. $1 + \frac{x}{3} - \frac{x^2}{9} + \cdots$ **45.** $1 - \frac{3}{2}x + \frac{3}{2}x^2 - \cdots$

47. $R_n(x) = \frac{(\sinh c + \cosh c)\, x^{n+1}}{(n+1)!}$, where c is between 0 and x;

$\lim_{n\to\infty} |R_n(x)| = |\sinh c + \cosh c| \lim_{n\to\infty} \frac{|x|^{n+1}}{(n+1)!} = 0$ because

$|x|^{n+1} \ll (n+1)!$ for any fixed value of x.

49. $\frac{1}{24}$ **51.** $\frac{1}{8}$ **53.** $\frac{1}{6}$ **55.** Approx. 0.4615 **57.** Approx. 0.3819

59. $11 - \frac{1}{11} - \frac{1}{2 \cdot 11^3} - \frac{1}{2 \cdot 11^5}$ **61.** $-\frac{1}{3} + \frac{1}{3 \cdot 3^3} - \frac{1}{5 \cdot 3^5} + \frac{1}{7 \cdot 3^7}$

63. $y = 4 + 4x + \frac{4^2}{2!}x^2 + \frac{4^3}{3!}x^3 + \cdots + \frac{4^n}{n!}x^n + \cdots = 3 + e^{4x}$

65. **a.** $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$ **b.** $\sum_{k=1}^{\infty} \frac{1}{k2^k}$ **c.** $2\sum_{k=0}^{\infty} \frac{x^{2k+1}}{2k+1}$

d. $x = \frac{1}{3}$; $2\sum_{k=0}^{\infty} \frac{1}{3^{2k+1}(2k+1)}$ **e.** Series in part (d)

CHAPTER 12

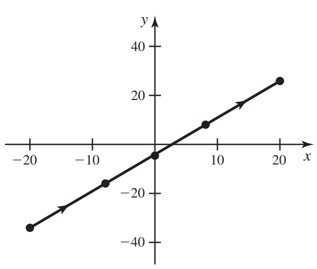
Section 12.1 Exercises, pp. 763–767

1. Plotting $\{(f(t), g(t)): a \le t \le b\}$ generates a curve in the xy-plane. **3.** $x = R\cos(\pi t/5)$, $y = -R\sin(\pi t/5)$ **5.** $x = t^2$, $y = t$, $-\infty < t < \infty$ **7.** $-\frac{1}{2}$ **9.** $x = t$, $y = t$, $0 \le t \le 6$; $x = 2t$, $y = 2t$, $0 \le t \le 3$; $x = 3t$, $y = 3t$, $0 \le t \le 2$ (answers will vary)

11. **a.**

t	-10	-4	0	4	10
x	-20	-8	0	8	20
y	-34	-16	-4	8	26

b.

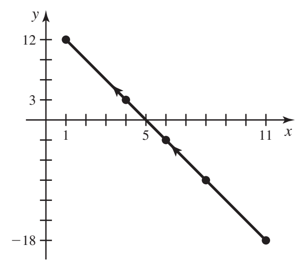


c. $y = \frac{3}{2}x - 4$
d. A line segment rising to the right as t increases

13. **a.**

t	-5	-2	0	2	5
x	11	8	6	4	1
y	-18	-9	-3	3	12

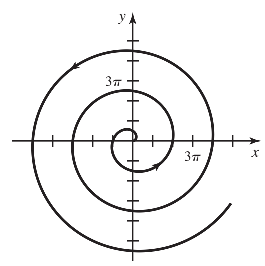
b.



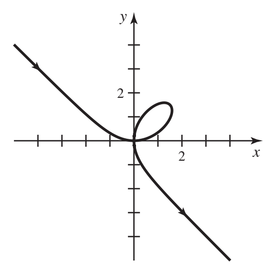
c. $y = -3x + 15$
d. A line segment rising to the left as t increases

15. **a.** $y = -x + 4$ **b.** A line segment starting at $(3, 1)$ and ending at $(4, 0)$ **17.** **a.** $y = 3x - 12$ **b.** A line segment starting at $(4, 0)$ and ending at $(8, 12)$ **19.** **a.** $x^2 + y^2 = 9$ **b.** Lower half of a circle of radius 3 centered at $(0, 0)$; starts at $(-3, 0)$ and ends at $(3, 0)$
21. **a.** $y = 1 - x^2$, $-1 \le x \le 1$ **b.** A parabola opening downward with a vertex at $(0, 1)$ starting at $(1, 0)$ and ending at $(-1, 0)$
23. **a.** $x^2 + (y-1)^2 = 1$ **b.** A circle of radius 1 centered at $(0, 1)$; generated counterclockwise, starting and ending at $(1, 1)$
25. **a.** $y = (x+1)^3$ **b.** A cubic curve rising to the right as r increases **27.** **a.** $x^2 + y^2 = 49$ **b.** A circle of radius 7 centered at $(0, 0)$; generated counterclockwise, starting and ending at $(-7, 0)$
29. **a.** $y = 1$, $-\infty < x < \infty$ **b.** A horizontal line with y-intercept 1, generated from left to right **31.** $x^2 + y^2 = 4$ **33.** $y = \sqrt{4 - x^2}$
35. $y = x^2$ **37.** $x = 4\cos t$, $y = 4\sin t$, $0 \le t \le 2\pi$
39. $x = \cos t + 2$, $y = \sin t + 3$, $0 \le t \le 2\pi$
41. $x = 2t$, $y = 8t$; $0 \le t \le 1$
43. $x = t$, $y = 2t^2 - 4$; $-1 \le t \le 5$ **45.** $x = 2$, $y = t$; $3 \le t \le 9$
47. $x = 4t - 2$, $y = -6t + 3$; $0 \le t \le 1$ and $x = t + 1$, $y = 8t - 11$; $1 \le t \le 2$
49. $x = 1 + 2t$, $y = 1 + 4t$; $-\infty < t < \infty$
51. $x = t^2$, $y = t$; $t \ge 0$
53. $x = 400\cos\left(\frac{4\pi t}{3}\right)$, $y = 400\sin\left(\frac{4\pi t}{3}\right)$; $0 \le t \le 1.5$
55. $x = 50\cos\left(\frac{\pi t}{12}\right)$, $y(t) = 50\sin\left(\frac{\pi t}{12}\right)$; $0 \le t \le 24$

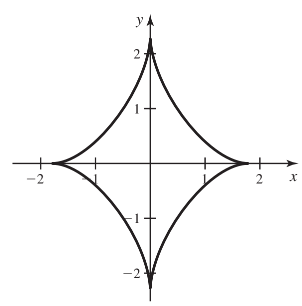
57.



59.



61.



63. Plot $x = 1 + \cos^2 t - \sin^2 t$, $y = t$.
65. Approx. 2857 m