61. $\dfrac{(3x+5)^{10}\sqrt{x^2+5}}{(x^3+1)^{50}}\left(\dfrac{30}{3x+5}+\dfrac{x}{x^2+5}-\dfrac{150x^2}{x^3+1}\right)$

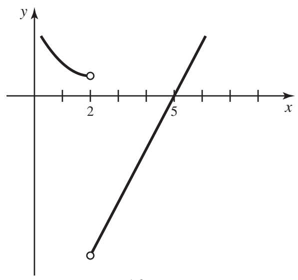
63. $\sqrt{3}+\pi/6$ **65.** 1 **67.** $2^x \ln 2(x \ln 2 + 2)$ **69.** $\dfrac{6\ln x - 5}{x^4}$

71. $\dfrac{2(xy+y^2)}{(x+2y)^3}=\dfrac{2}{(x+2y)^3}$ **73.** $y = x$ **75.** $y = -\dfrac{4x}{5}+\dfrac{24}{5}$

77. $x^2f'(x) + 2xf(x)$ **79.** $\dfrac{g(x)(xf'(x)+f(x)) - xf(x)g'(x)}{g^2(x)}$

81. a–D; b–C; c–B; d–A

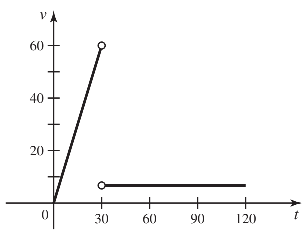
83.



85. a. 27 **b.** $\frac{16}{27}$ **c.** 72 **d.** 1215 **e.** $\frac{1}{9}$ **87.** $\frac{6}{13}$

89. $(f^{-1})'(x) = -3/x^4$ **91. a.** $\frac{1}{4}$ **b.** 1 **c.** $\frac{1}{3}$

93. $y = 24x - 118$ **95. a.** 84 ft/s **b.** 7 s **c.** 384 ft
d. 96 ft/s **97. a.** \$200,366; \$21,552/yr
b. 14 yr; \$12,551/yr **99. a.** 2.70 million people/yr
b. The slope of the secant line through the two points is approximately equal to the slope of that tangent line at $t = 55$.
c. 2.217 million people/yr **101. a.** 40 m/s **b.** 20/3 m/s
c. 15 m/s **d.**



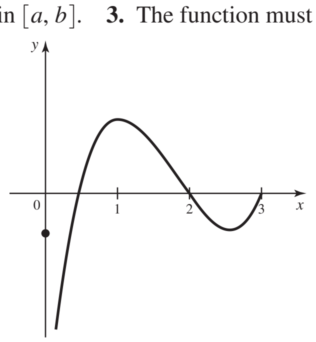
e. The skydiver deployed the parachute. **103.** $x = 4$; $x = 6$
105. $f(x) = \tan(\pi\sqrt{3x-11})$, $a = 5$; $f'(5) = 3\pi/4$
107. a. $\bar{C}(3000) = \$341.67$; $C'(3000) = \$280$ **b.** The average cost of producing the first 3000 lawn mowers is \$341.67 per mower. The cost of producing the 3001st lawn mower is \$280.
109. a. 6550 people/yr **b.** $p'(40) = 4800$ people/yr
111. 50 mi/hr **113.** $-5\sin 65°$ ft/s ≈ -4.5 ft/s
115. -0.166 rad/s **117.** 1.5 ft/s **119. a.** $(f^{-1})'(1/\sqrt{2}) = \sqrt{2}$

CHAPTER 4

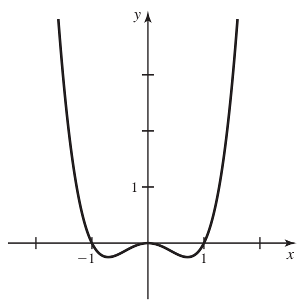
Section 4.1 Exercises, pp. 247–250

1. f has an absolute maximum at c in $[a, b]$ if $f(x) \le f(c)$ for all x in $[a, b]$. f has an absolute minimum at c in $[a, b]$ if $f(x) \ge f(c)$ for all x in $[a, b]$. **3.** The function must be continuous on a closed interval.

5.

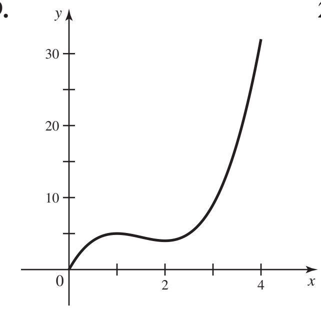


7.

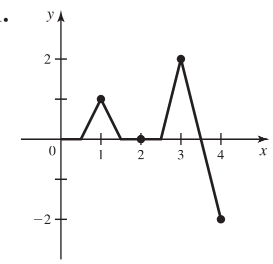


9. Evaluate the function at the critical points and at the endpoints of the interval. **11.** Abs. min at $x = c_2$; abs. max at $x = b$ **13.** Abs. min at $x = a$; no abs. max **15.** Local min at $x = q, s$; local max at $x = p, r$; abs. min at $x = a$; abs. max at $x = b$ **17.** Local max at $x = p, r$; local min at $x = q$; abs. max at $x = p$; abs. min at $x = b$

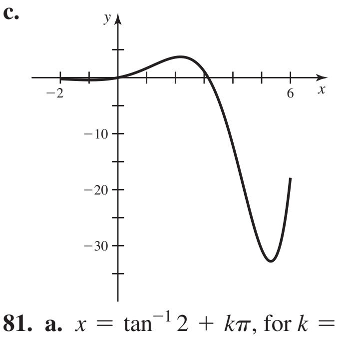
19.



21.

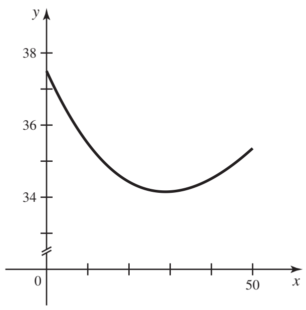


23. $x = \frac{2}{3}$ **25.** $x = \pm 3$ **27.** $x = -\frac{2}{3}, \frac{1}{3}$ **29.** $x = \pm\dfrac{2a}{\sqrt{3}}$
31. $t = \pm 1$ **33.** $x = 0$ **35.** $x = 1$ **37.** $x = -4, 0$
39. If $a \ge 0$, there is no critical point. If $a < 0$, $x = 2a/3$ is the only critical point. **41.** $t = \pm a$ **43.** Abs. max: -1 at $x = 3$; abs. min: -10 at $x = 0$ **45.** Abs. max: 0 at $x = 0, 3$; abs. min: -4 at $x = -1, 2$ **47.** Abs. max: 234 at $x = 3$; abs. min: -38 at $x = -1$ **49.** Abs. max: 1 at $x = 0, \pi$; abs. min: 0 at $x = \pi/2$ **51.** Abs. max: 1 at $x = \pi/6$; abs. min: -1 at $x = -\pi/6$
53. Abs. min: $(\sqrt{1/e})^{1/e}$ at $x = 1/(2e)$; abs. max: 2 at $x = 1$
55. Abs. max: $1 + \pi$ at $x = -1$; abs. min: 1 at $x = 1$
57. Abs. max: 11 at $x = 1$; abs. min: -16 at $x = 4$
59. Abs. max: 27 at $x = -3$; abs. min: $-\frac{19}{12}$ at $x = \frac{1}{2}$
61. Abs. max: $\dfrac{1}{100{,}000}$ at $x = 1$; abs. min: $-\dfrac{1}{100{,}000}$ at $x = -1$
63. Abs. max: $\sqrt{2}$ at $x = \pm\pi/4$; abs. min: 1 at $x = 0$
65. Abs. max: $27/e^3$ at $x = 3$; abs. min: $-e$ at $x = -1$
67. Abs. max: 3 at $x = \pm 1$; abs. min: 0 at $x = -2, 0, 2$
69. a. The velocity of the downstream wind v_2 is less than or equal to the velocity of the upstream wind, so $0 \le v_2 \le v_1$, or $0 \le \dfrac{v_2}{v_1} \le 1$.
b. $R(1) = 0$ **c.** $R(0) = \frac{1}{2}$ **d.** 0.593 is the maximum fraction of power that can be extracted from a wind stream by a wind turbine.
71. $t = 2$ s **73.** $t = 2$ s **75. a.** 50 **b.** 45 **77. a.** False
b. False **c.** False **d.** True **79. a.** $x = -0.96, 2.18, 5.32$
b. Abs. max: 3.72 at $x = 2.18$; abs. min: -32.80 at $x = 5.32$
c.



81. a. $x = \tan^{-1} 2 + k\pi$, for $k = -2, -1, 0, 1$
b. $x = \tan^{-1} 2 + k\pi$, for $k = -2, 0$, correspond to local max; $x = \tan^{-1} 2 + k\pi$, for $k = -1, 1$, correspond to local min.
c. Abs. max: 2.24; abs. min: -2.24 **83. a.** $x = 5 - 4\sqrt{2}$
b. $x = 5 - 4\sqrt{2}$ corresponds to a local max. **c.** No abs. max or min
85. Abs. max: 4 at $x = -1$; abs. min: -8 at $x = 3$
87. a. $T(x) = \dfrac{\sqrt{2500 + x^2}}{2} + \dfrac{50 - x}{4}$ **b.** $x = 50/\sqrt{3}$

c. $T(50/\sqrt{3}) \approx 34.15$, $T(0) = 37.50$, $T(50) \approx 35.36$
d.

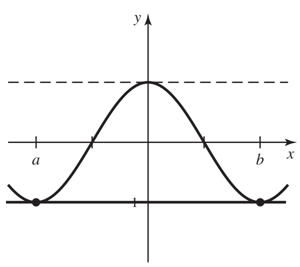


89. a. 1, 3, 0, 1 **b.** Because $h'(2) \neq 0$, h does not have a local extreme value at $x = 2$. However, g may have a local extremum at $x = 2$ (because $g'(2) = 0$). **91. a.** $f(x) - f(c) \leq 0$ for all x near c **b.** $\lim\limits_{x \to c^+} \dfrac{f(x) - f(c)}{x - c} \leq 0$ **c.** $\lim\limits_{x \to c^-} \dfrac{f(x) - f(c)}{x - c} \geq 0$

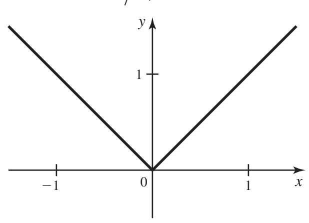
d. Because $f'(c)$ exists, $\lim\limits_{x \to c^+} \dfrac{f(x) - f(c)}{x - c} = \lim\limits_{x \to c^-} \dfrac{f(x) - f(c)}{x - c}$. By parts (b) and (c), we must have that $f'(c) = 0$.

Section 4.2 Exercises, pp. 254–257

1. If f is a continuous function on the closed interval $[a, b]$ and is differentiable on (a, b), and the slope of the secant line that joins $(a, f(a))$ to $(b, f(b))$ is zero, then there is at least one value c in (a, b) at which the slope of the line tangent to f at $(c, f(c))$ is also zero.

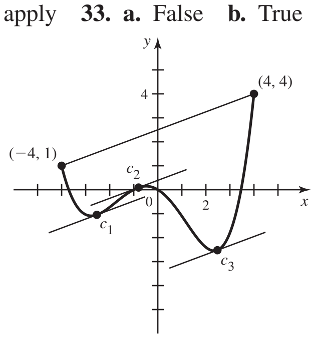


3. $f(x) = |x|$ is not differentiable at 0. **5. b.** $c = 1$
7. b. $c = \pm 2/\sqrt[4]{5} \approx \pm 1.34$
9.



11. $x = \frac{1}{3}$

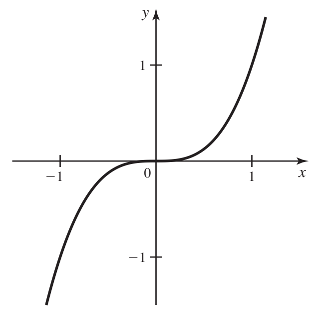
13. $x = \pi/4$ **15.** Does not apply **17.** $x = \frac{5}{3}$ **19.** Average lapse rate $= -6.3°/\text{km}$. You cannot conclude that the lapse rate at a point exceeds the threshold value. **21. a.** Yes **b.** $c = \frac{1}{2}$ **23. a.** Does not apply **25. a.** Yes **b.** $c = \ln(e - 1)$ **27. a.** Yes **b.** $c \approx \pm 0.881$ **29. a.** Yes **b.** $c = \sqrt{1 - 9/\pi^2}$ **31. a.** Does not apply **33. a.** False **b.** True **c.** False **37.** h and p
39.



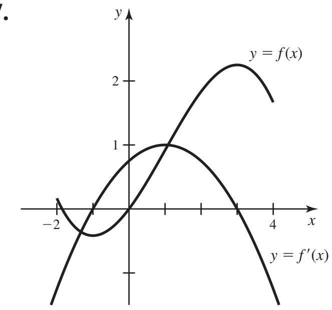
41. No such point exists; function is not continuous at 2.
43. The car's average velocity is $(30 - 0)/(28/60) = 64.3$ mi/hr. By the MVT, the car's instantaneous velocity was 64.3 mi/hr at some time.
45. Average speed $= 11.6$ mi/hr. By the MVT, the speed was exactly 11.6 mi/hr at least once. By the Intermediate Value Theorem, all speeds between 0 and 11.6 mi/hr were reached. Because the initial and final speeds were 0 mi/hr, the speed of 11 mi/hr was reached at least twice.
47. $\dfrac{f(b) - f(a)}{b - a} = A(a + b) + B$ and $f'(x) = 2Ax + B$; $2Ax + B = A(a + b) + B$ implies that $x = \dfrac{a + b}{2}$, the midpoint of $[a, b]$. **49.** $\tan^2 x$ and $\sec^2 x$ differ by a constant; in fact, $\tan^2 x - \sec^2 x = -1$. **53.** *Hint:* By the MVT, there is a value of c in $(2, 4)$ for which $\dfrac{f(4) - f(2)}{4 - 2} = f'(c)$. **57. b.** $c = \frac{1}{2}$

Section 4.3 Exercises, pp. 267–270

1. f is increasing on I if $f'(x) > 0$ for all x in I; f is decreasing on I if $f'(x) < 0$ for all x in I. **3. a.** $x = 3$ **b.** Increasing on $(3, \infty)$; decreasing on $(-\infty, 3)$
5.



7.



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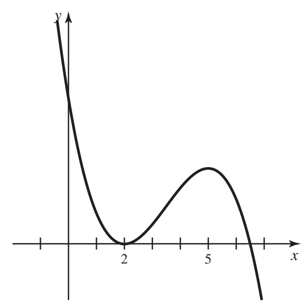

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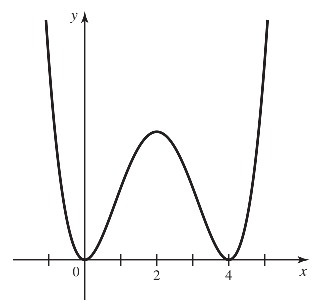
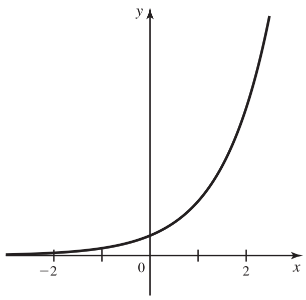


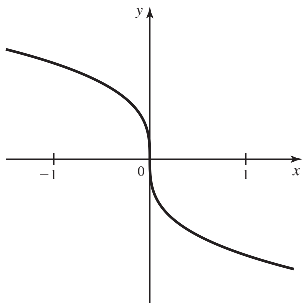
13. a. Concave up on $(-\infty, 2)$; concave down on $(2, \infty)$ **b.** Inflection point at $x = 2$ **15.** Yes; consider the graph of $y = \sqrt{x}$ on $(0, \infty)$. **17.** $f(x) = x^4$ **19.** Increasing on $(-\infty, 0)$; decreasing on $(0, \infty)$ **21.** Decreasing on $(-\infty, 1)$; increasing on $(1, \infty)$
23. Increasing on $(-\infty, 1)$ and $(4, \infty)$; decreasing on $(1, 4)$
25. Increasing on $(-\infty, 1/2)$; decreasing on $(1/2, \infty)$
27. Increasing on $(-\infty, 0)$, $(1, 2)$; decreasing on $(0, 1)$, $(2, \infty)$
29. Increasing on $\left(-\dfrac{1}{\sqrt{e}}, 0\right)$, $\left(\dfrac{1}{\sqrt{e}}, \infty\right)$; decreasing on $\left(-\infty, -\dfrac{1}{\sqrt{e}}\right)$, $\left(0, \dfrac{1}{\sqrt{e}}\right)$ **31.** Increasing on $\left(\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right)$; decreasing on $\left(0, \dfrac{\pi}{6}\right)$, $\left(\dfrac{5\pi}{6}, 2\pi\right)$ **33.** Increasing on $(-\pi, -2\pi/3)$, $(-\pi/3, 0)$, $(\pi/3, 2\pi/3)$; decreasing on $(-2\pi/3, -\pi/3)$, $(0, \pi/3)$, $(2\pi/3, \pi)$ **35.** Increasing on $(-1, 0)$, $(1, \infty)$; decreasing on $(-\infty, -1)$, $(0, 1)$ **37.** Increasing on $(-3, 1)$; decreasing on $(1, 3)$ **39.** Increasing on $(1, 4)$; decreasing on $(-\infty, 1)$, $(4, \infty)$ **41.** Increasing on $\left(-\infty, -\frac{1}{2}\right)$, $\left(0, \frac{1}{2}\right)$; decreasing on $\left(-\frac{1}{2}, 0\right)$, $\left(\frac{1}{2}, \infty\right)$ **43.** Increasing on $(-1, 1)$; decreasing on $(-\infty, -1)$, $(1, \infty)$ **45. a.** $x = 0$ **b.** Local min at $x = 0$ **c.** Abs. min: 3 at $x = 0$; abs. max: 12 at $x = -3$

47. a. $x = \pm\sqrt{2}$ **b.** Local min at $x = -\sqrt{2}$; local max at $x = \sqrt{2}$ **c.** Abs. max: 2 at $x = \sqrt{2}$; abs. min: -2 at $x = -\sqrt{2}$ **49. a.** $x = \pm\sqrt{3}$ **b.** Local min at $x = -\sqrt{3}$; local max at $x = \sqrt{3}$ **c.** Abs. max: 28 at $x = -4$; abs. min: $-6\sqrt{3}$ at $x = -\sqrt{3}$ **51. a.** $x = 2$ and $x = 0$ **b.** Local max at $x = 0$; local min at $x = 2$ **c.** Abs. min: $-10\sqrt[3]{25}$ at $x = -5$; abs. max: 0 at $x = 0, 5$ **53. a.** $x = e^{-2}$ **b.** Local min at $x = e^{-2}$ **c.** Abs. min: $-2/e$ at $x = e^{-2}$; no abs. max **55.** Abs. max: $1/e$ at $x = 1$ **57.** Abs. min: $36\sqrt[3]{\pi/6}$ at $r = \sqrt[3]{6/\pi}$

59.

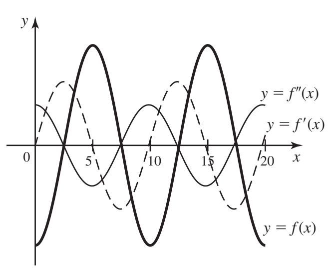


61.



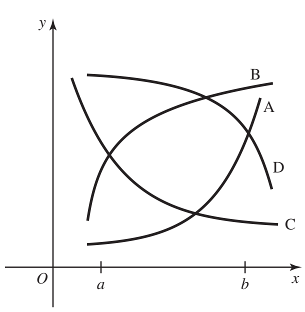
63. Concave up on $(-\infty, 0), (1, \infty)$; concave down on $(0, 1)$; inflection points at $x = 0$ and $x = 1$ **65.** Concave up on $(-\infty, 0)$, $(2, \infty)$; concave down on $(0, 2)$; inflection points at $x = 0$ and $x = 2$ **67.** Concave down on $(-\infty, 1)$; concave up on $(1, \infty)$; inflection point at $x = 1$ **69.** Concave up on $(-1/\sqrt{3}, 1/\sqrt{3})$; concave down on $(-\infty, -1/\sqrt{3}), (1/\sqrt{3}, \infty)$; inflection points at $t = \pm 1/\sqrt{3}$ **71.** Concave up on $(-\infty, -1), (1, \infty)$; concave down on $(-1, 1)$; inflection points at $x = \pm 1$ **73.** Concave up on $(0, 1)$; concave down on $(1, \infty)$; inflection point at $x = 1$ **75.** Concave up on $(0, 2), (4, \infty)$; concave down on $(-\infty, 0), (2, 4)$; inflection points at $x = 0, 2, 4$ **77.** Critical pts. $x = 0$ and $x = 2$; local max at $x = 0$, local min at $x = 2$ **79.** Critical pt. $x = 0$; local max at $x = 0$ **81.** Critical pt. $x = 6$; local min at $x = 6$ **83.** Critical pts. $x = 0$ and $x = 1$; local max at $x = 0$; local min at $x = 1$ **85.** Critical pts. $x = 0$ and $x = 2$; local min at $x = 0$; local max at $x = 2$ **87.** Critical pt. $x = e^5$; local min at $x = e^5$ **89.** Critical pts. $t = -3$ and $t = 2$; local max at $t = -3$; local min at $t = 2$ **91.** Critical pt. $x = -a$; local min at $x = -a$ **93.** Critical pts. $x = \pm\sqrt[3]{e}$; local min at $x = \pm\sqrt[3]{e}$ **95. a.** True **b.** False **c.** True **d.** False **e.** False

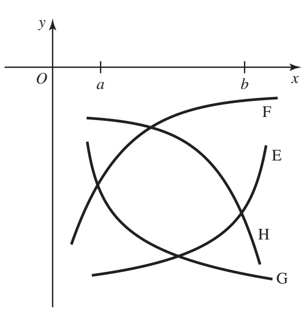
97.



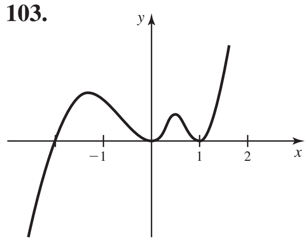
99. a–C–i, b–B–iii, c–A–ii

101.





103.



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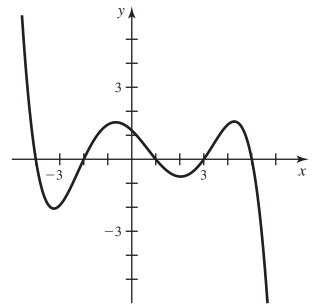
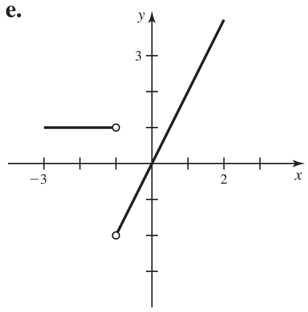


107. a. Increasing on $(-2, 2)$; decreasing on $(-3, -2)$ **b.** Critical pts. $x = -2$ and $x = 0$; local min at $x = -2$; neither a local max nor min at $x = 0$ **c.** Inflection pts. at $x = -1$ and $x = 0$ **d.** Concave up on $(-3, -1), (0, 2)$; concave down on $(-1, 0)$

e.



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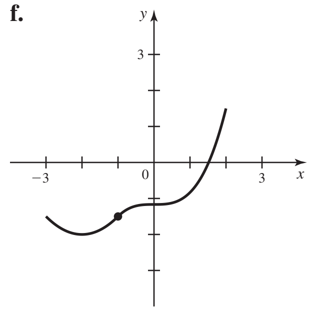


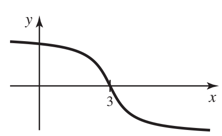
109. a. $E = \dfrac{p}{p - 50}$ **b.** -1.4% **c.** $E'(p) = -\dfrac{ab}{(a - bp)^2} < 0$, for $p \geq 0, p \neq a/b$ **d.** $E(p) = -b$, for $p \geq 0$

111. a. 300 **b.** $t = \sqrt{10}$ **c.** $t = \sqrt{b/3}$

Section 4.4 Exercises, pp. 277–280

1. We need to know on which interval(s) to graph f. **3.** No; the domain of any polynomial is $(-\infty, \infty)$; there is no vertical asymptote. Also, $\lim_{x\to\pm\infty} p(x) = \pm\infty$, where p is any polynomial; there is no horizontal asymptote. **5.** Evaluate the function at the critical points and at the endpoints. Then find the largest and smallest values among those candidates.

7.



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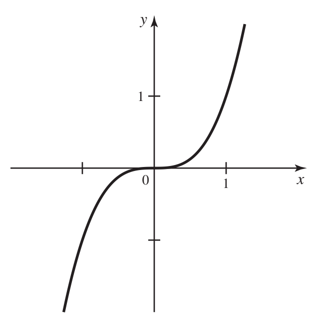

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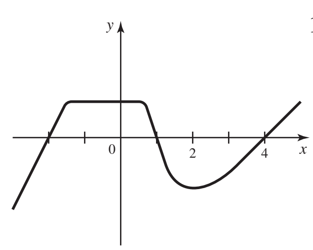

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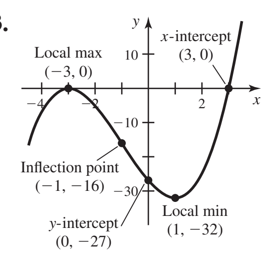
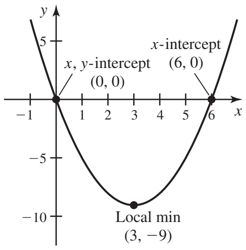
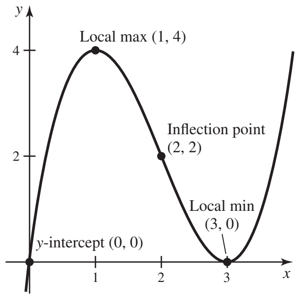


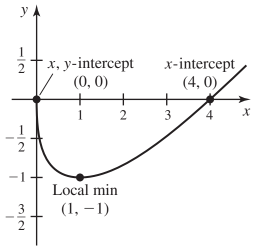
15.



17.



27.



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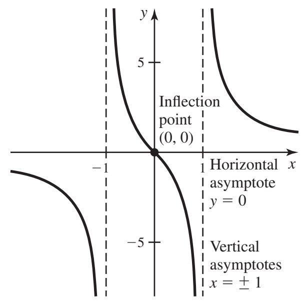

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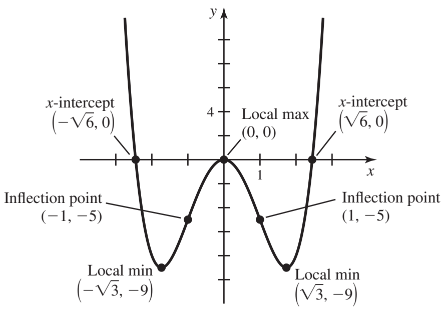

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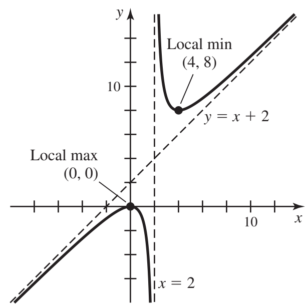

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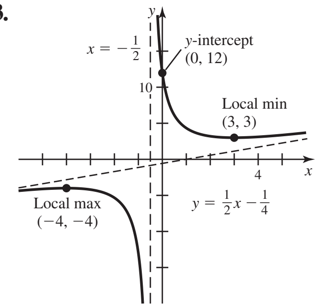

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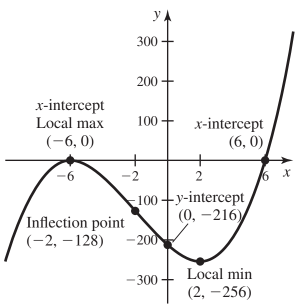

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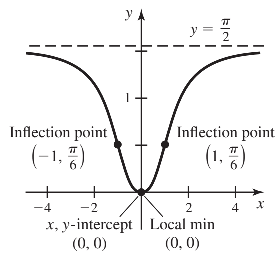

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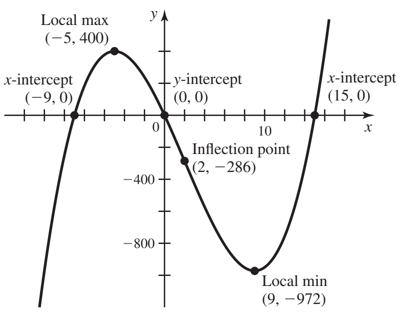

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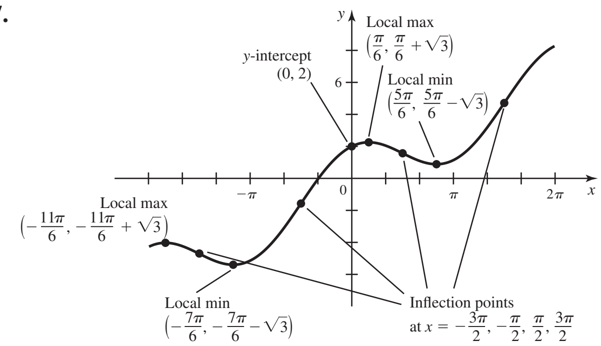

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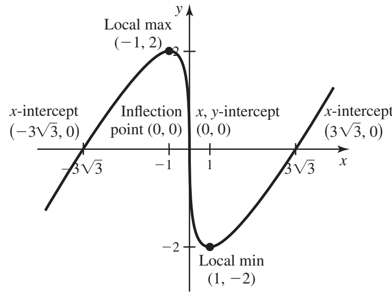

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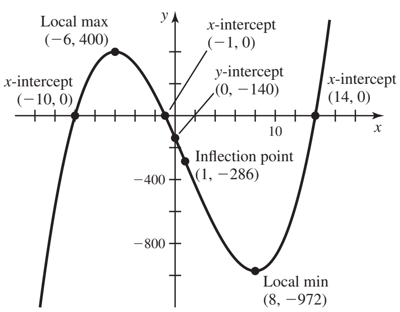

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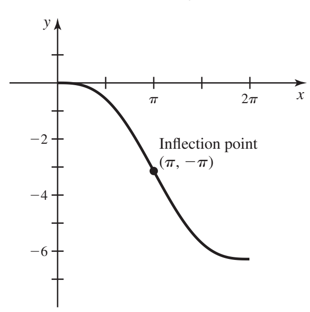

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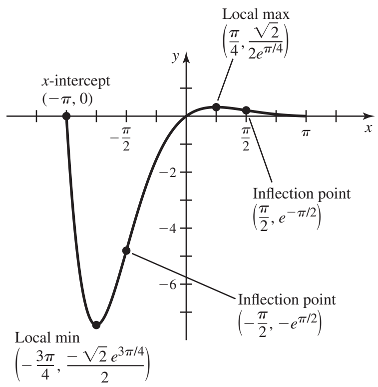

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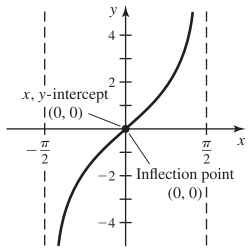
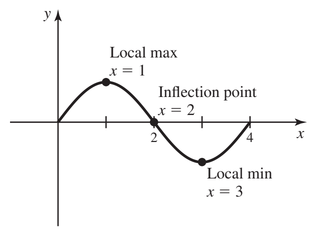


47. Critical pts. at $x = 1, 3$; local max at $x = 1$; local min at $x = 3$; inflection pt. at $x = 2$; increasing on $(0, 1)$, $(3, 4)$; decreasing on $(1, 3)$; concave up on $(2, 4)$; concave down on $(0, 2)$



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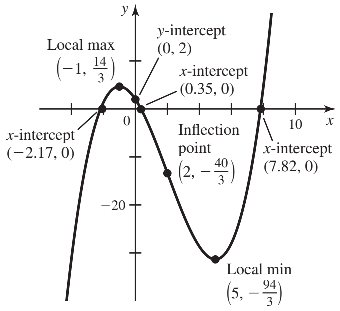

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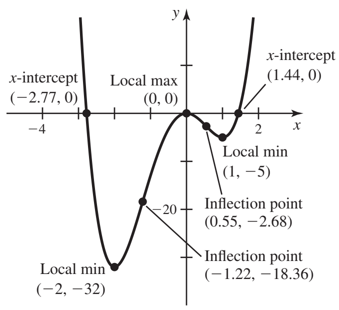

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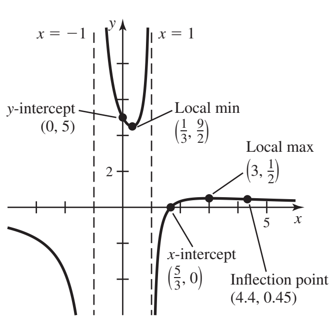
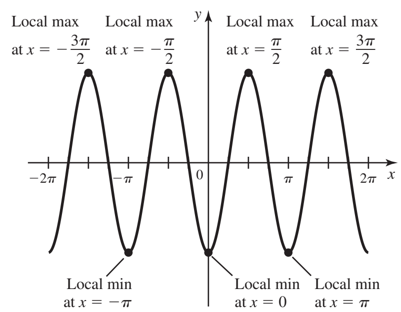


55. **a.** False **b.** False **c.** False **d.** True

57.



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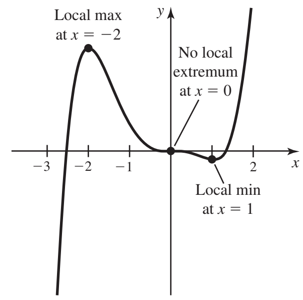
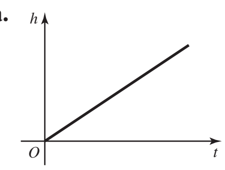
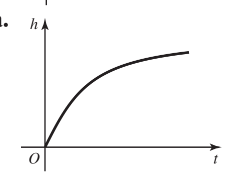


61. (A) a.



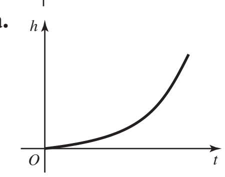
b. Water is being added at all times. **c.** No concavity **d.** h' has an abs. max at all points of $[0, 10]$.

(B) a.



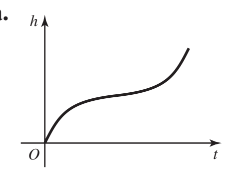
c. Concave down **d.** h' has abs. max at $t = 0$.

(C) a.



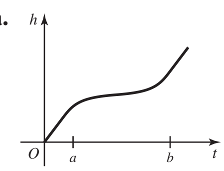
c. Concave up **d.** h' has abs. max at $t = 10$.

(D) a.



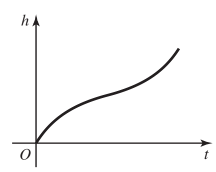
c. Concave down on $(0, 5)$, then concave up on $(5, 10)$; inflection pt. at $t = 5$ **d.** h' has abs. max at $t = 0$ and $t = 10$.

(E) a.



c. First, no concavity; then concave down, no concavity, concave up, and, finally, no concavity
d. h' has abs. max at all points of an interval $[0, a]$ and $[b, 10]$.

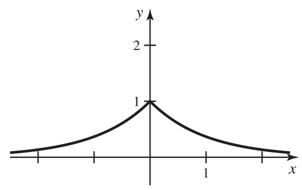
(F) a.



c. Concave down on $(0, 5)$; concave up on $(5, 10)$; inflection pt. at $t = 5$ **d.** h' has abs. max at $t = 0$ and $t = 10$.

63. Local max of $e^{1/e}$ at $x = e$

65. $f'(0)$ does not exist.



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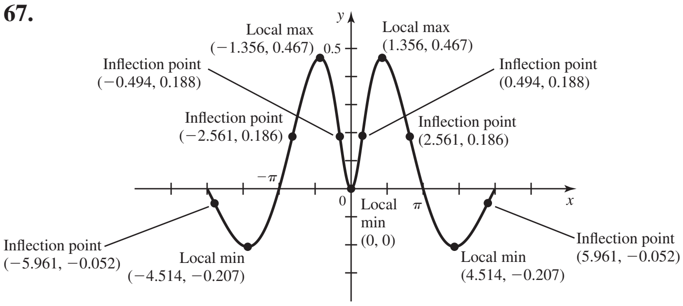

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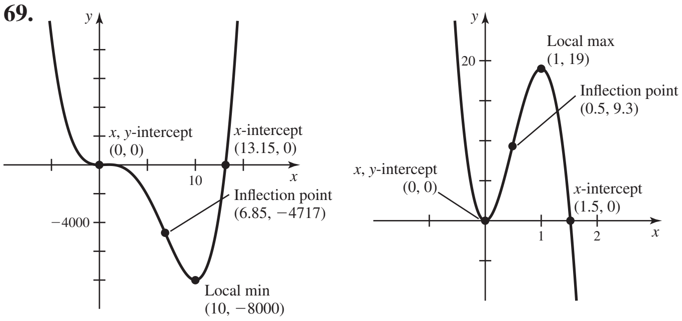

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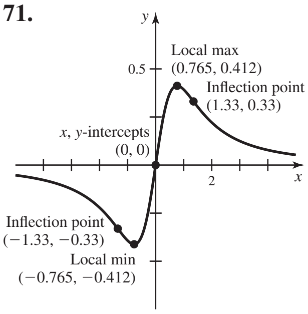

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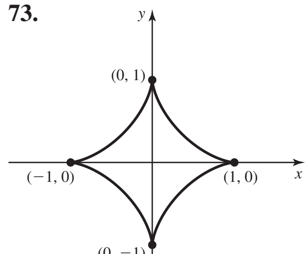
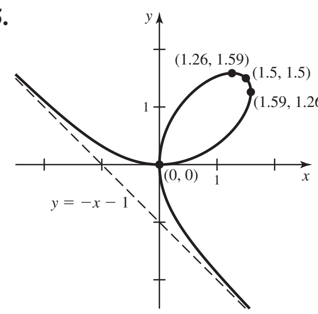
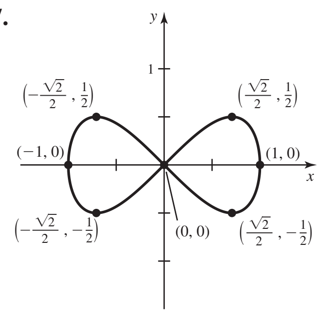


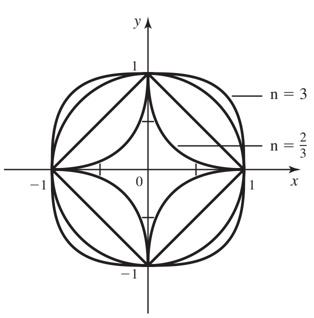
75.



77.



79.



Section 4.5 Exercises, pp. 284–291

1. Objective function, constraint(s) **3.** $Q = x^2(10 - x)$; $Q = (10 - y)^2 y$ **5. a.** $P(x) = 100x - 10x^2$ **b.** Abs. max: 250 at $x = 5$ **7.** $\frac{23}{2}$ and $\frac{23}{2}$ **9.** $5\sqrt{2}$ and $5\sqrt{2}$ **11.** Width = length = $\frac{5}{2}$ **13.** Width = length = 10 **15.** $x = \sqrt{6}, y = 2\sqrt{6}$

17. $\dfrac{10}{\sqrt{2}}$ cm by $\dfrac{5}{\sqrt{2}}$ cm **19.** Length = width = height = 2

21. $\dfrac{4}{\sqrt[3]{5}}$ ft by $\dfrac{4}{\sqrt[3]{5}}$ ft by $5^{2/3}$ ft **23.** Approx. (0.59, 0.65)

25. (5, 15), distance ≈ 47.4 **27. a.** A point $8/\sqrt{5}$ mi from the point on the shore nearest the woman in the direction of the restaurant **b.** $9/\sqrt{13}$ mi/hr **29.** A point $7\sqrt{3}/6$ mi from the point on shore nearest the island, in the direction of the power station

31. 18.2 ft **33.** $h = \dfrac{20}{\sqrt{3}}$; $r = 20\sqrt{\frac{2}{3}}$

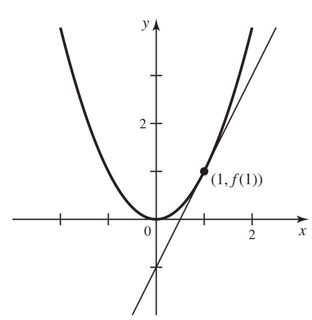
35. a. $r = \sqrt[3]{177/\pi} \approx 3.83$ cm; $h = 2\sqrt[3]{177/\pi} \approx 7.67$ cm **b.** $r = \sqrt[3]{177/2\pi} \approx 3.04$ cm; $h = 2\sqrt[3]{708/\pi} \approx 12.17$ cm; part (b) is closer to the real can. **37.** $\sqrt{15}$ m by $2\sqrt{15}$ m **39.** 12″ by 6″ by 3″; 216 in^3 **41.** Lower rectangular pane is approximately 5.6 ft wide by 2.8 ft high. **43.** $r/h = \sqrt{2}$ **45.** $r = \sqrt{2}R/\sqrt{3}$; $h = 2R/\sqrt{3}$ **47.** 3:1 **49. a.** 0, 30, 25 **b.** 42.5 mi/hr **c.** The units of $p/g(v)$ are \$/mi and so are the units of w/v. Therefore, $L\left(\dfrac{p}{g(v)} + \dfrac{w}{v}\right)$ gives the total cost of a trip of L miles. **d.** Approx. 62.9 mi/hr **e.** Neither; the zeros of $C'(v)$ are independent of L. **f.** Decreased slightly, to 62.5 mi/hr **g.** Decreased to 60.8 mi/hr **51.** $\sqrt{30} \approx 5.5$ ft **53.** The point $12/\left(\sqrt[3]{2} + 1\right) \approx 5.3$ m from the weaker source **55. b.** Because the speed of light is constant, travel time is minimized when distance is minimized. **57.** $r = h = \sqrt[3]{450/\pi}$ m **59. a.** $\dfrac{a_1 + a_2}{2}$ **b.** $\dfrac{a_1 + a_2 + a_3}{3}$ **c.** $\dfrac{a_1 + a_2 + \cdots + a_n}{n}$ **61.** $\dfrac{\pi}{3}$

63. When the seat is at its lowest point **65.** For $L \le 4r$, max at $\theta = 0$ and $\theta = 2\pi$; min at $\theta = \cos^{-1}(-L/(4r))$ and $\theta = 2\pi - \cos^{-1}(-L/(4r))$. For $L > 4r$, max at $\theta = 0$ and $\theta = 2\pi$; min at $\theta = \pi$. **67. a.** $P = 2/\sqrt{3}$ units from the midpoint of the base **69.** You can run 12 mi/hr if you run toward the point 3/16 mi ahead of the locomotive (when it passes the point nearest you). **71. a.** $r = 2R/3$; $h = \frac{1}{3}H$ **b.** $r = R/2$; $h = H/2$

73. $(1 + \sqrt{3})$ mi ≈ 2.7 mi **75. (i)** $\left(p - \frac{1}{2}, \sqrt{p - \frac{1}{2}}\right)$ **(ii)** (0, 0) **77.** Let the angle of the cuts be φ_1 and φ_2, where $\varphi_1 + \varphi_2 = \theta$. The volume of the notch is proportional to $\tan\varphi_1 + \tan\varphi_2 = \tan\varphi_1 + \tan(\theta - \varphi_1)$, which is minimized when $\varphi_1 = \varphi_2 = \dfrac{\theta}{2}$. **79.** $x \approx 38.81$, $y \approx 55.03$

Section 4.6 Exercises, pp. 298–300

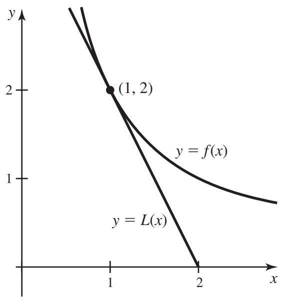
1.



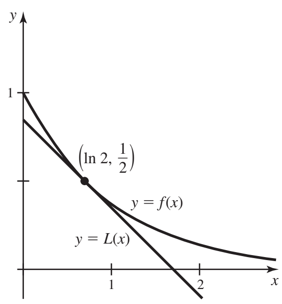
3. $f(x) \approx f(a) + f'(a)(x - a)$

5. $L(x) = 3x - 1$; 2.3 **7.** 2.7 **9.** $dy = f'(x)\,dx$

11. Approx. 25 **13.** 61 mi/hr; 61.02 mi/hr
15. $L(x) = T(0) + T'(0)(x - 0) = D - (D/60)x = D(1 - x/60)$
17. 84 min; 84.21 min **19.** $L(x) = 9x - 4$ **21.** $L(t) = t + 5$
23. $L(x) = 3x - 5$ **25. a.** $L(x) = -4x + 16$ **b.** 7.6
c. 0.13% error **27. a.** $L(x) = x$ **b.** 0.9 **c.** 40% error
29. a. $L(x) = 1$ **b.** 1 **c.** 0.005% error **31. a.** $L(x) = \frac{1}{2} - \frac{x}{48}$
b. 0.5 **c.** 0.003% error **33. a.** $L(x) = 1 - x$; **b.** $1/1.1 \approx 0.9$
c. 1% error **35. a.** $L(x) = 1 - x$ **b.** $e^{-0.03} \approx 0.97$
c. 0.05% error **37.** $a = 200$; $\frac{1}{203} \approx 0.004925$
39. $a = 144$; $\sqrt{146} \approx \frac{145}{12}$ **41.** $a = 1$; $\ln 1.05 \approx 0.05$
43. $a = 0$; $e^{0.06} \approx 1.06$ **45.** $a = 512$; $\frac{1}{\sqrt[3]{510}} \approx 0.125$
47. a. $L(x) = -2x + 4$ **b.**



c. Underestimates **d.** $f''(1) = 4 > 0$
49. a. $L(x) = -\frac{1}{2}x + \frac{1}{2}(1 + \ln 2)$
b.



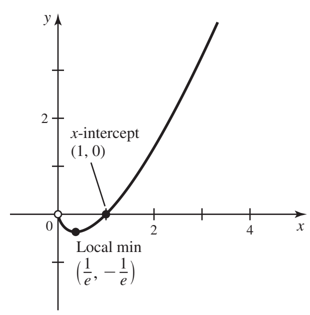
c. Underestimates **d.** $f''(\ln 2) = \frac{1}{2} > 0$
51. $E(x) \le 1$ when $-7.26 \le x \le 8.26$, which corresponds to driving times for 1 mi from about 53 s to 68 s. Therefore, $L(x)$ gives approximations to $s(x)$ that are within 1 mi/hr of the true value when you drive 1 mi in t seconds, where $53 < t < 68$.
53. a. True **b.** False **c.** True **d.** True
55. $\Delta V \approx 10\pi$ ft^3 **57.** $\Delta V \approx -40\pi$ cm^3
59. $\Delta S \approx -\frac{59\pi}{5\sqrt{34}}$ m^2 **61.** $dy = 2\,dx$ **63.** $dy = -\frac{3}{x^4}\,dx$
65. $dy = a \sin x\,dx$ **67.** $dy = (9x^2 - 4)\,dx$
69. $dy = \sec^2 x\,dx$
71. $L(x) = 2 + (x - 8)/12$

x	Linear approx.	Exact value	Percent error
8.1	$2.008\overline{3}$	2.00829885	1.7×10^{-3}
8.01	$2.0008\overline{3}$	2.000832986	1.7×10^{-5}
8.001	$2.00008\overline{3}$	2.00008333	1.7×10^{-7}
8.0001	$2.000008\overline{3}$	2.000008333	1.7×10^{-9}
7.9999	$1.999991\overline{6}$	1.999991667	1.7×10^{-9}
7.999	$1.99991\overline{6}$	1.999916663	1.7×10^{-7}
7.99	$1.9991\overline{6}$	1.999166319	1.7×10^{-5}
7.9	$1.991\overline{6}$	1.991631701	1.8×10^{-3}

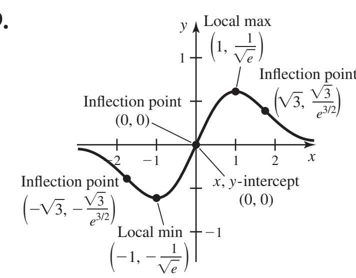
73. a. f; the rate at which f' is changing at 1 is smaller than the rate at which g' is changing at 1. The graph of f bends away from the linear function more slowly than the graph of g. **b.** The larger the value of $|f''(a)|$, the greater the deviation of the curve $y = f(x)$ from the tangent line at points near $x = a$.

Section 4.7 Exercises, pp. 310–312

1. If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, then we say $\lim_{x\to a} f(x)/g(x)$ is an indeterminate form 0/0. **3.** Take the limit of the quotient of the derivatives of the functions.
5. a. $\lim_{x\to 0}\left(x^2 \cdot \frac{1}{x^2}\right) = 1$ **b.** $\lim_{x\to 0}\left(2x^2 \cdot \frac{1}{x^2}\right) = 2$
7. If $\lim_{x\to a} f(x)g(x)$ has the indeterminate form $0 \cdot \infty$, then $\lim_{x\to a}\left(\frac{f(x)}{1/g(x)}\right)$ has the indeterminate form 0/0 or ∞/∞.
9. $\frac{1}{5}$ **11.** If $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x) = \infty$, then $f(x)^{g(x)}$ has the form 1^∞ as $x \to a$, which is meaningless; so direct substitution does not work. **13.** $\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$ **15.** $\ln x, x^3, 2^x, x^x$
17. -1 **19.** $\frac{3}{4}$ **21.** $\frac{1}{2}$ **23.** $\frac{1}{2}$ **25.** $\frac{1}{e}$ **27.** -1 **29.** $\frac{12}{5}$
31. 4 **33.** $\frac{9}{16}$ **35.** $\frac{1}{2}$ **37.** $-\frac{2}{3}$ **39.** $\frac{1}{24}$ **41.** 1 **43.** 4
45. 1 **47.** $-\frac{1}{2}$ **49.** $\frac{1}{\pi^2}$ **51.** $\frac{1}{3}$ **53.** 1 **55.** $\frac{7}{6}$ **57.** 1
59. $-\frac{1}{2}$ **61.** 0 **63.** -8 **65.** 0 **67.** $\frac{1}{2}$ **69.** $\frac{\ln 3}{\ln 2}$
71. $-\frac{9}{4}$ **73.** $\frac{1}{6}$ **75.** 1 **77.** 1 **79.** e **81.** e^{a+1} **83.** e
85. b. $\lim_{m\to\infty}(1 + r/m)^m = \lim_{m\to\infty}\left(1 + \frac{1}{(m/r)}\right)^{(m/r)r} = e^r$
87. 3 **89.** $\frac{1}{2}$ **91.** e **93.** $\ln a - \ln b$ **95.** $e^{0.01x}$ **97.** Comparable growth rates **99.** x^x **101.** 1.00001^x **103.** e^{x^2} **105. a.** False **b.** False **c.** False **d.** False **e.** True **f.** True
107.



109.



111. $\sqrt{a/c}$ **113.** $\lim_{x\to\infty} \frac{x^p}{b^x} = \lim_{t\to\infty} \frac{\ln^p t}{t \ln^p b} = 0$, where $t = b^x$
115. Show $\lim_{x\to\infty} \frac{\log_a x}{\log_b x} = \frac{\ln b}{\ln a} \ne 0$. **117.** 1/3 **121. a.** $b > e$
b. e^{ax} grows faster than e^x as $x \to \infty$, for $a > 1$; e^{ax} grows slower than e^x as $x \to \infty$, for $0 < a < 1$.

Section 4.8 Exercises, pp. 318–321

1. Newton's method generates a sequence of x-intercepts of lines tangent to the graph of f to approximate the roots of f.
3. $x_1 = 2, x_2 = 1, x_3 = 0$ **5.** $x_1 = 0.75$ **7.** Generally, if two successive Newton approximations agree in their first p digits, then those approximations have p digits of accuracy. The method is terminated when the desired accuracy is reached.

9. $x_{n+1} = x_n - \dfrac{x_n^2 - 6}{2x_n} = \dfrac{x_n^2 + 6}{2x_n}$; $x_1 = 2.5, x_2 = 2.45$

11. $x_{n+1} = x_n - \dfrac{e^{-x_n} - x_n}{e^{-x_n} - 1}$; $x_1 = 0.564382, x_2 = 0.567142$

13.

n	x_n
0	3
1	3.1667
2	3.16228
3	3.16228

$r \approx 3.16228$

15.

n	x_n
0	0.5
1	0.51096
2	0.51097
3	0.51097

$r \approx 0.51097$

17.

n	x_n
0	1.2
1	1.16935
2	1.16561
3	1.16556
4	1.16556

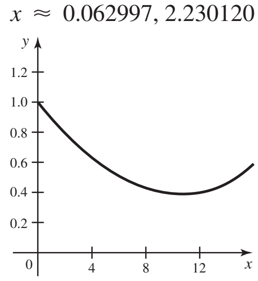
$r \approx 1.16556$

19.

n	x_n
0	0.75
1	0.73915
2	0.73909
3	0.73909

$r \approx 0.73909$

21. $x \approx -0.335408, 1.333057$ **23.** $x \approx 0.179295$
25. $x \approx 0.620723, 3.03645$ **27.** $x \approx 0, 1.895494, -1.895494$
29. $x \approx -2.114908, 0.254102, 1.860806$
31. $x \approx 0.062997, 2.230120$
33.



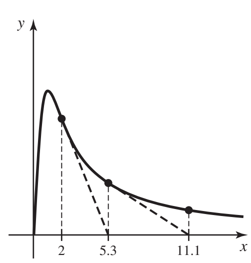
The tumor decreases in size and then starts growing again. It decreases to half its size after about 6.4 days.

35. b. $r \approx 7.3\%$ **37. a.** $t = \pi/4$ **b.** $t \approx 1.33897$
c. $t \approx 2.35619$ **d.** $t \approx 2.90977$
39. $p(x) = x^4 - 7$; $r \approx 1.62658$
41. $p(x) = x^3 + 9$; $r \approx -2.08008$ **43.** $x \approx 2.798386$
45. $x \approx -0.666667, 1.5, 1.666667$ **47. a.** True **b.** False
c. False **49.** $x \approx 0.739085$ **51.** $x = 0$ and $x \approx 1.047198$

53. a.

n	x_n
0	2
1	5.33333
2	11.0553
3	22.2931

b.

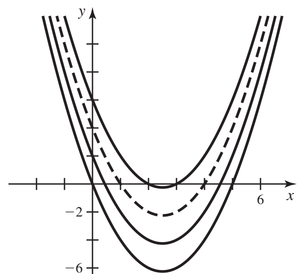


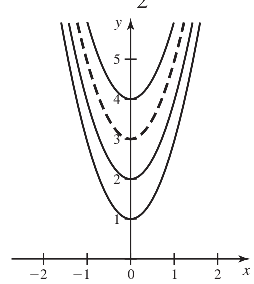
c. The tangent lines intersect the x-axis farther and farther away from the root r. **55. b.** $x \approx 0.142857$ is approximately $\frac{1}{7}$.
57. $\lambda = 1.29011, 2.37305, 3.40918$

Section 4.9 Exercises, pp. 331–334

1. The derivative, an antiderivative **3.** $x + C$, where C is an arbitrary constant **5.** $\dfrac{x^{p+1}}{p+1} + C$, where $p \neq -1$ **7.** $\ln|x| + C$

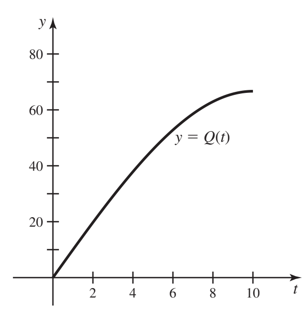
9. 0 **11.** $x^5 + C$ **13.** $-2\cos x + x + C$ **15.** $3\tan x + C$
17. $y^{-2} + C$ **19.** $e^x + C$ **21.** $\tan^{-1}s + C$ **23.** $\frac{1}{2}x^6 - \frac{1}{2}x^{10} + C$
25. $\frac{8}{3}x^{3/2} - 8x^{1/2} + C$ **27.** $\frac{25}{3}s^3 + 15s^2 + 9s + C$
29. $\frac{9}{4}x^{4/3} + 6x^{2/3} + 6x + C$ **31.** $-x^3 + \frac{11}{2}x^2 + 4x + C$
33. $-x^{-3} + 2x + 3x^{-1} + C$ **35.** $x^4 - 3x^2 + C$
37. $\dfrac{1}{2}x^2 + 6x + C$ **39.** $-\cot\theta + 2\theta^3/3 - 3\theta^2/2 + C$
41. $-2\cot y - 3\csc y + C$ **43.** $\tan x - x + C$
45. $\tan\theta + \sec\theta + C$ **47.** $t^3 - 2\cot t + C$
49. $\sec\theta + \tan\theta + \theta + C$ **51.** $\frac{1}{2}\ln|y| + C$ **53.** $3\sin^{-1}x + C$
55. $4\sec^{-1}|x| + C$ **57.** $\frac{1}{6}\sec^{-1}|x| + C$ **59.** $t + \ln|t| + C$
61. $e^{x+2} + C$ **63.** $e^w - 4w + C$ **65.** $\ln|x| + 2\sqrt{x} + C$
67. $\frac{4}{15}x^{15/2} - \frac{24}{11}x^{11/6} + C$ **69.** $\dfrac{1}{6}x^6 - \dfrac{2}{3}x^3 + x + 1$
71. $2x^4 - \cos x + 3$ **73.** $\sec v + 1, -\pi/2 < v < \pi/2$
75. $y^3 + 5\ln y + 2, y > 0$ **77.** $f(x) = x^2 - 3x + 4$
79. $g(x) = \dfrac{7}{8}x^8 - \dfrac{x^2}{2} + \dfrac{13}{8}$ **81.** $f(u) = 4(\sin u + \cos u) - 4$
83. $y(t) = 3\ln t + 6t + 2, t > 0$
85. $y(\theta) = \sqrt{2}\sin\theta + \tan\theta + 1, -\pi/2 < \theta < \pi/2$
87. $f(x) = x^2 - 5x + 4$ **89.** $f(x) = \dfrac{3}{2}x^2 - \cos x + 4$





91. $s(t) = t^2 + 4t$ **93.** $s(t) = \frac{4}{3}t^{3/2} + 1$
95. $s(t) = 2t^3 + 2t^2 - 10t$ **97.** $s(t) = -16t^2 + 20t$
99. $s(t) = \frac{1}{30}t^3 + 1$ **101.** $s(t) = t^2 + 4t - 3\sin t + 10$
103. 200 ft **105.** Runner A overtakes runner B at $t = \pi/2$
107. a. $v(t) = -9.8t + 30$ **b.** $s(t) = -4.9t^2 + 30t$
c. Approx. 45.92 m at time $t \approx 3.06$ **d.** $t \approx 6.12$
109. a. $v(t) = -9.8t + 10$ **b.** $s(t) = -4.9t^2 + 10t + 400$
c. Approx. 405.10 m at time $t \approx 1.02$ **d.** $t \approx 10.11$
111. a. True **b.** False **c.** True **d.** False
e. False **113.** $F(x) = -\cos x + 3x + 3 - 3\pi$
115. $F(x) = 2x^8 + x^4 + 2x + 1$
117. a. $Q(t) = 10t - t^3/30$ gal

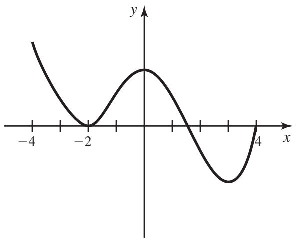
b.



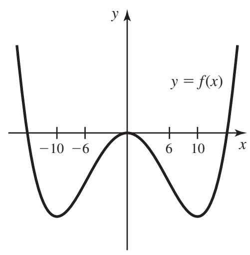
c. $\dfrac{200}{3}$ gal

Chapter 4 Review Exercises, pp. 334–337

1. a. False **b.** False **c.** True **d.** True **e.** True **f.** False
3.

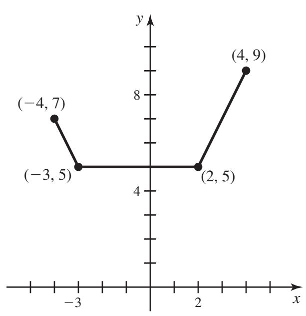


5. a. $x = 0, \pm 10$; increasing on $(-10, 0)$ and $(10, \infty)$, decreasing on $(-\infty, -10)$ and $(0, 10)$ **b.** $x = \pm 6$; concave up on $(-\infty, -6)$ and $(6, \infty)$, concave down on $(-6, 6)$ **c.** Local min at $x = -10, 10$; local max at $x = 0$ **d.**



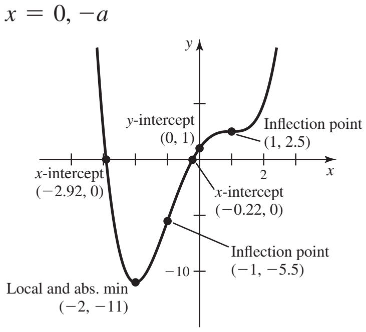
7. Critical pts. $x = 0, \pm 1$; abs. max: 33 at $x = \pm 2$; abs. min: 6 at $x = \pm 1$ **9.** $x = 3$ and $x = -2$; no abs. max or min
11. Critical pt. $x = 1$; abs. max: $\ln 2$ at $x = 0, 2$; abs. min: 0 at $x = 1$
13. Critical pts. $x = \frac{2\pi}{3}, \frac{4\pi}{3}$; abs. max: $\frac{3\sqrt{3}}{8}$ at $x = \frac{4\pi}{3}$; abs. min: $-\frac{3\sqrt{3}}{8}$ at $x = \frac{2\pi}{3}$ **15.** Critical pt. $x = 1/e$; abs. min: $10 - 2/e$ at $x = 1/e$

17.

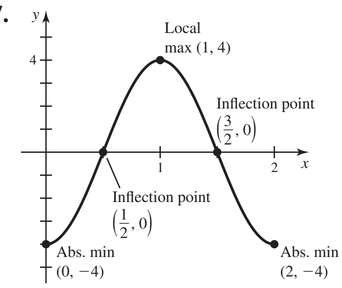


Critical pts.: all x in the interval $[-3, 2]$; abs. max: 9 at $x = 4$; abs. and local min: 5 for x in $[-3, 2]$; local max: 5 for x in $(-3, 2)$

19. a. Increasing on $(-\infty, -1)$ and $(1, \infty)$; decreasing on $(-1, 1)$ **b.** Concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$
21. Inflection pt. $x = 0$ **23.** Critical pts. $x = -a, a/2$; inflection pts. $x = 0, -a$
25.



27.



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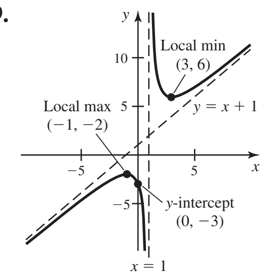

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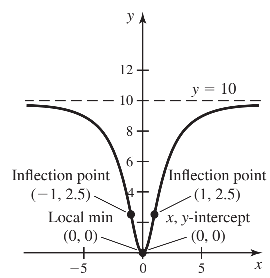

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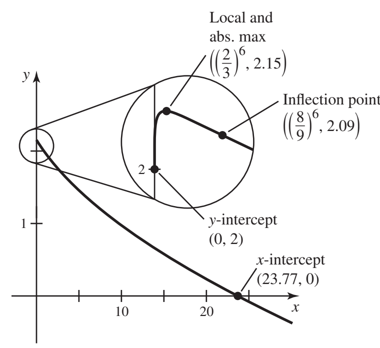

35. Approx. 2.5″ by 3.5″ by 9.5″ **37.** Approx. 59 m from the loudest speaker **39.** $r = 4\sqrt{6}/3$; $h = 4\sqrt{3}/3$ **41.** $x = 7, y = 14$
43. $p = q = 5\sqrt{2}$ **45. a.** $L(x) = \frac{2}{9}x + 3$ **b.** $\frac{85}{9} \approx 9.44$; overestimate **47.** $f(x) = 1/x^2$; $a = 4$; $1/4.2^2 \approx 0.05625$
49. $\Delta h \approx -112$ ft **51.** $c = 2.5$ **53. a.** $\frac{100}{9}$ cells/week
b. $t = 2$ weeks **55.** $-0.434259, 0.767592, 1$ **57.** $0, \pm 0.948683$
59. 0 **61.** 0 **63.** 12 **65.** $\frac{2}{3}$ **67.** ∞ **69.** 0 **71.** 1 **73.** 0
75. 1 **77.** 1 **79.** $1/e^3$ **81.** 1 **83.** $x^{1/2}$ **85.** $\sqrt{x}$ **87.** 3^x
89. Comparable growth rates **91.** $\frac{4}{3}x^3 + 2x^2 + x + C$
93. $-\frac{1}{x} + \frac{4}{3}x^{-3/2} + C$ **95.** $\theta + 3\sin\theta + C$ **97.** $\tan x + C$
99. $12\ln|x| + C$ **101.** $\tan^{-1} x + C$ **103.** $\frac{4}{7}x^{7/4} + \frac{2}{7}x^{7/2} + C$
105. $f(t) = -\cos t + t^2 + 6$ **107.** $h(x) = \frac{1}{3}x^3 - x - \tan^{-1} x + \frac{\pi}{4}$
109. $v(t) = -9.8t + 120$; $s(t) = -4.9t^2 + 120t + 125$. The rocket reaches a height of 859.69 m at time $t \approx 12.24$ s and then falls to the ground, hitting at time $t \approx 25.49$ s. **111. a.** $v(t) = 64 - 32t$
b. $s(t) = -16t^2 + 64t + 128$ **c.** $t = 2$; 192 ft
d. $-64\sqrt{3}$ ft/s ≈ -110.9 ft/s **113.** 1; 1
115. $\lim_{x\to 0^+} f(x) = 1$; $\lim_{x\to 0^+} g(x) = 0$