SECTION 8.8 EXERCISES

Getting Started

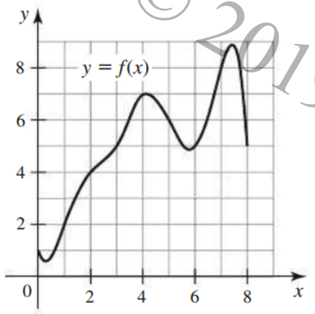
1. If the interval $[4, 18]$ is partitioned into $n = 28$ subintervals of equal length, what is Δx?

2. Explain geometrically how the Midpoint Rule is used to approximate a definite integral.

3. Explain geometrically how the Trapezoid Rule is used to approximate a definite integral.

4. If the Midpoint Rule is used on the interval $[-1, 11]$ with $n = 3$ subintervals, at what x-coordinates is the integrand evaluated?

5–8. *Compute the following estimates of $\int_0^8 f(x)\,dx$ using the graph in the figure.*



5. $M(4)$ **6.** $T(4)$ **7.** $S(4)$ **8.** $S(8)$

9. If the Trapezoid Rule is used on the interval $[-1, 9]$ with $n = 5$ subintervals, at what x-coordinates is the integrand evaluated?

10. Suppose two Trapezoidal Rule approximations of $\int_a^b f(x)\,dx$ are $T(2) = 6$ and $T(4) = 5.1$. Find the Simpson's Rule approximation $S(4)$.

T 11–14. *Compute the absolute and relative errors in using c to approximate x.*

11. $x = \pi$; $c = 3.14$ **12.** $x = \sqrt{2}$; $c = 1.414$

13. $x = e$; $c = 2.72$ **14.** $x = e$; $c = 2.718$

Practice Exercises

T 15–18. Midpoint Rule approximations *Find the indicated Midpoint Rule approximations to the following integrals.*

15. $\int_2^{10} 2x^2\,dx$ using $n = 1, 2,$ and 4 subintervals

16. $\int_1^9 x^3\,dx$ using $n = 1, 2,$ and 4 subintervals

17. $\int_0^1 \sin \pi x\,dx$ using $n = 6$ subintervals

18. $\int_0^1 e^{-x}\,dx$ using $n = 8$ subintervals

T 19–22. Trapezoid Rule approximations *Find the indicated Trapezoid Rule approximations to the following integrals.*

19. $\int_2^{10} 2x^2\,dx$ using $n = 2, 4,$ and 8 subintervals

20. $\int_1^9 x^3\,dx$ using $n = 2, 4,$ and 8 subintervals

21. $\int_0^1 \sin \pi x\,dx$ using $n = 6$ subintervals

22. $\int_0^1 e^{-x}\,dx$ using $n = 8$ subintervals

T 23–26. Simpson's Rule approximations *Find the indicated Simpson's Rule approximations to the following integrals.*

23. $\int_0^{\pi} \sqrt{\sin x}\,dx$ using $n = 4$ and $n = 6$ subintervals

24. $\int_4^8 \sqrt{x}\,dx$ using $n = 4$ and $n = 8$ subintervals

25. $\int_{-2}^{3} e^{-x^2}\,dx$ using $n = 10$ subintervals

26. $\int_2^4 \cos \sqrt{x}\,dx$ using $n = 8$ subintervals

T 27. Midpoint Rule, Trapezoid Rule, and relative error Find the Midpoint and Trapezoid Rule approximations to $\int_0^1 \sin \pi x\,dx$ using $n = 25$ subintervals. Compute the relative error of each approximation.

T 28. Midpoint Rule, Trapezoid Rule, and relative error Find the Midpoint and Trapezoid Rule approximations to $\int_0^1 e^{-x}\,dx$ using $n = 50$ subintervals. Compute the relative error of each approximation.

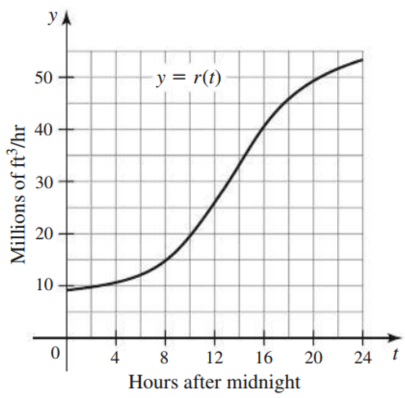
T 29–34. Comparing the Midpoint and Trapezoid Rules *Apply the Midpoint and Trapezoid Rules to the following integrals. Make a table similar to Table 8.5 showing the approximations and errors for $n = 4, 8, 16,$ and 32. The exact values of the integrals are given for computing the error.*

29. $\int_1^5 (3x^2 - 2x)\,dx = 100$ **30.** $\int_{-2}^{6} \left(\frac{x^3}{16} - x\right) dx = 4$

31. $\int_0^{\pi/4} 3 \sin 2x\,dx = \frac{3}{2}$ **32.** $\int_1^e \ln x\,dx = 1$

33. $\int_0^{\pi} \sin x \cos 3x\,dx = 0$ **34.** $\int_0^8 e^{-2x}\,dx = \frac{1 - e^{-16}}{2}$

35–36. River flow rates *The following figure shows the discharge rate $r(t)$ of the Snoqualmie River near Carnation, Washington, starting on a February day when the air temperature was rising. The variable t is the number of hours after midnight, $r(t)$ is given in millions of cubic feet per hour, and $\int_0^{24} r(t)\,dt$ equals the total amount of water that flows by the town of Carnation over a 24-hour period. Estimate $\int_0^{24} r(t)\,dt$ using the Trapezoidal Rule and Simpson's Rule with the following values of n.*



35. $n = 4$ **36.** $n = 6$