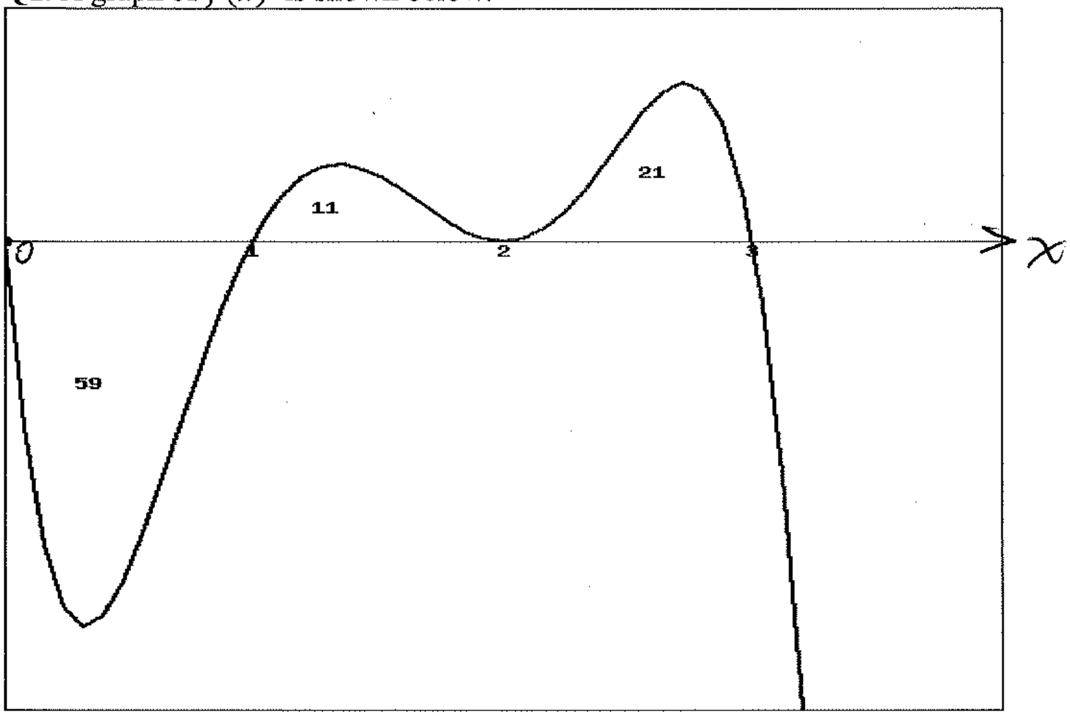
MATH 141: Calculus II, **Midterm Exam 1** Practice Questions Solution

**Q1.** A graph of $f(x)$ is shown below.




The numbers shown represent the geometric area of each region. Evaluate the following definite integrals.

(1) $\int_0^1 f(x)dx = -59$

(2) $\int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx = -48$

$= -59 + 11$

(3) $\int_0^3 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^3 f(x)dx$

$= -59 + 11 + 21 = -27$

(4) $\int_1^2 -5f(x)dx$

$= -5\int_1^2 f(x)dx = -5 \cdot 11 = -55$

**Q2.** Find the following definite integrals

(1) $\int_4^9 (5 + x\sqrt{x})dx$

First, $\int (5 + x\sqrt{x})\,dx = 5x + \int x^{\frac{3}{2}}dx = 5x + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}$

$= 5x + \frac{2}{5}x^{\frac{5}{2}} + C$

$\Rightarrow \int_4^9 (5 + x\sqrt{x})\,dx = \left(5x + \frac{2}{5}x^{\frac{5}{2}}\right)\Big|_4^9$

$= (45 + \frac{2}{5}(9^{\frac{1}{2}})^5] - [20 + \frac{2}{5}(4^{\frac{1}{2}})^5]$

$= 45 + \frac{2}{5}\cdot 3^5 - 20 - \frac{2}{5}\cdot 2^5 = 25 + \frac{2}{5}(3^5 - 2^5)$

(2) $\int_0^{49\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}dx$

$u = \sqrt{x} = x^{\frac{1}{2}}, \quad \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \Rightarrow \frac{dx}{\sqrt{x}} = 2du$

$\Rightarrow \int_0^{49\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}dx = 2\int_{x=0}^{x=49\pi^2} \sin u\, du = -2\cos u\Big|_{x=0}^{x=49\pi^2}$

$= -2\cos\sqrt{x}\Big|_0^{49\pi^2} = -2[\cos 7\pi - \cos 0] = \cancel{-2(0-1) = 2}$

$= -2(-1-1) = 4$

(3) $\int_0^{\pi} e^{\sin x}\cos x\, dx$

$u = \sin x, \quad \frac{du}{dx} = (\sin x)' = \cos x \Rightarrow du = \cos x\, dx$

$\Rightarrow \int_0^{\pi} e^{\sin x}\cos x\, dx = \int_{x=0}^{x=\pi} e^u\, du$

$= e^{\sin x}\Big|_0^{\pi} = e^{\sin\pi} - e^{\sin 0} = e^0 - e^0 = 0$

**Q3.** Find the following indefinite integrals.

(1) $\int \frac{\cos(\ln x)}{x} dx$

Sol. $u = \ln x$. Then $\frac{du}{dx} = (\ln x)' = \frac{1}{x}$ and $\frac{dx}{x} = du$

$\Rightarrow \int \frac{\cos(\ln x)\,dx}{x}$

$= \int \cos u \, du = \sin u + C$

$= \sin(\ln x) + C$

(2) $\int \frac{\sin(\frac{5}{x})}{10x^2} dx$

Sol. let $u = \frac{5}{x}$. Then $\frac{du}{dx} = u' = \left(\frac{5}{x}\right)' = -5\frac{1}{x^2}$

So, $\frac{dx}{x^2} = \frac{du}{-5}$

$\Rightarrow \int \frac{\sin(\frac{5}{x})}{10x^2} dx = \frac{1}{-5} \int \frac{\sin(u)}{10} du$

$= \frac{-1}{50} \int \sin u \, du = \frac{-1}{50}(-\cos u) + C$

$= \frac{\cos u}{50} + C = \frac{\cos(\frac{5}{x})}{50} + C$

**Q4.** If $f(t)$ is continuous and $\int_0^{81} f(t)dt = -10$, find the integral $\int_0^9 f(9t)dt$ .

Sol. $\int_0^9 f(9t)\,dt$ $\quad \dfrac{u = 9t}{du = 9dt}$ and $u = 0$ when $t = 0$, $u = 81$ when $t = 9$

$$\frac{1}{9}\int_0^{81} f(u)\,du = \frac{1}{9}(-10) = -\frac{10}{9}$$

**Q5.** Let $f(x) = 2 + \frac{1}{x}$. Find the average value $f(x)$ $on$ [1,2].

Sol. $\int_1^2 f(x)\,dx = \int_1^2 (2 + \frac{1}{x})\,dx$

$= (2x + \ln x)\Big|_1^2 = [(4 + \ln 2) - (2 + \ln 1)]$

$= 2 + \ln 2$

so, the average value of $f(x)$ on $[1, 2]$

is $\dfrac{\int_1^2 f(x)\,dx}{2 - 1} = (2 + \ln 2)/1 = 2 + \ln 2$