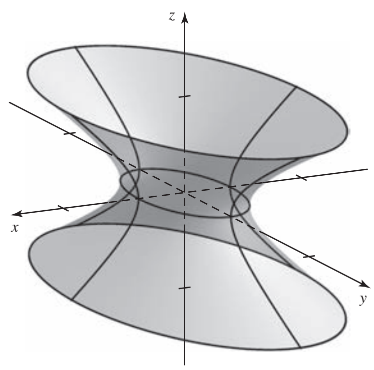
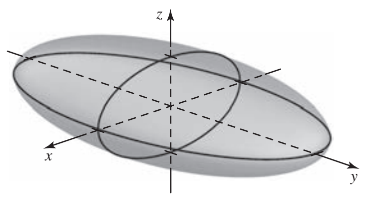
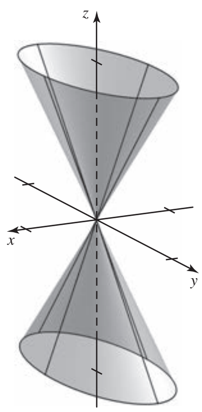
d.



71. a. Ellipsoid **b.** $\frac{x^2}{4}+\frac{y^2}{16}=4,\ \frac{x^2}{4}+z^2=4,\ \frac{y^2}{16}+z^2=4$
c. $x=\pm4,\ y=\pm8,\ z=\pm2$
d.



73. a. Elliptic cone **b.** Origin, $\frac{x^2}{9}=\frac{z^2}{64},\ \frac{y^2}{49}=\frac{z^2}{64}$ **c.** Origin
d.

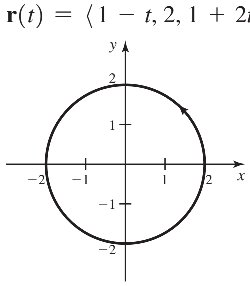


75. a. A **b.** D **c.** C **d.** B

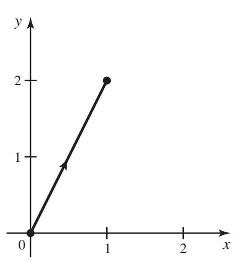
CHAPTER 14

Section 14.1 Exercises, pp. 873–875

1. One **3.** Its output is a vector.
5. $\lim_{t\to a}\mathbf{r}(t)=\langle\lim_{t\to a}f(t),\lim_{t\to a}g(t),\lim_{t\to a}h(t)\rangle$
7. $\mathbf{r}(t)=t\,\mathbf{i}+2t\,\mathbf{j}+3t\,\mathbf{k}$
9. $\mathbf{r}(t)=\langle 2+2t, 3+3t, 7-4t\rangle$
11. $\mathbf{r}(t)=\langle 3+2t, 4, 5-t\rangle$
13. $\mathbf{r}(t)=\langle 1-t, 2, 1+2t\rangle$, for $0\le t\le 1$
15.



17.



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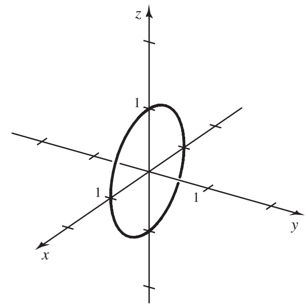
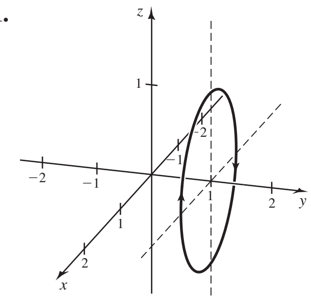
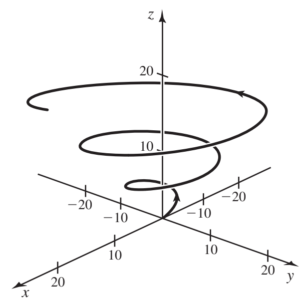


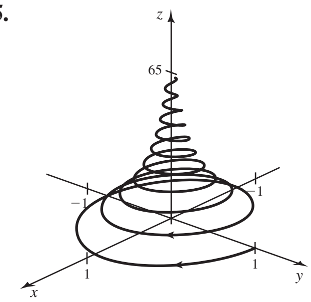
21.



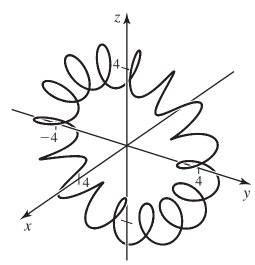
23.



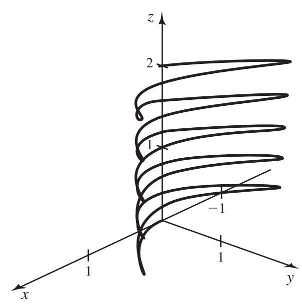
25.



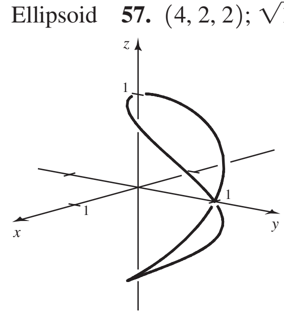
27.



29. When viewed from above, the curve is a portion of the parabola $y=x^2$.



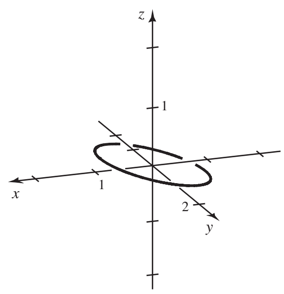
31. $-\mathbf{i}-4\mathbf{j}+\mathbf{k}$ **33.** $-2\mathbf{j}+\frac{\pi}{2}\mathbf{k}$ **35.** $\mathbf{i}$ **37. a.** True **b.** False
c. True **d.** True **39.** $\{t: |t|\le 2\}$ **41.** $\{t: 0\le t\le 2\}$
43. $(4, 8, 16)$ **45. a.** E **b.** D **c.** F **d.** C **e.** A **f.** B
47. $\mathbf{r}(t)=\langle 2\cos t, 2\sin t, 4\rangle$
49. $\mathbf{r}(t)=\langle 5\cos t, 5\sin t, 10\cos t+10\sin t\rangle$
51. a. Ball has a parabolic trajectory in the yz-plane; 1200 ft
b. Approx. 1199.7 ft **c.** 1196 ft **53.** Hyperboloid of one sheet
55. Ellipsoid **57.** $(4, 2, 2)$; $\sqrt{179}$
59.



The curve lies on the sphere $x^2+y^2+z^2=1$.

61. $\frac{2\pi}{(m,n)}$, where (m,n) = greatest common factor of m and n

63. a.



b. Curve is a tilted circle of radius 1 centered at the origin.

65. $\langle cf - ed, be - af, ad - bc \rangle$ or any scalar multiple

Section 14.2 Exercises, pp. 881–883

1. $\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$ **3.** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$

5. $\int \mathbf{r}(t)\,dt = \left(\int f(t)\,dt\right)\mathbf{i} + \left(\int g(t)\,dt\right)\mathbf{j} + \left(\int h(t)\,dt\right)\mathbf{k}$

7. $\mathbf{C} = \langle -1, -3, -10 \rangle$ **9.** $\langle -\sin t, 2t, \cos t \rangle$

11. $\left\langle 6t^2, \dfrac{3}{\sqrt{t}}, -\dfrac{3}{t^2} \right\rangle$ **13.** $e^t\,\mathbf{i} - 2e^{-t}\,\mathbf{j} - 8e^{2t}\,\mathbf{k}$

15. $\langle e^{-t}(1 - t), 1 + \ln t, \cos t - t\sin t \rangle$
17. $\langle 1, 6, 3 \rangle$ **19.** $\langle 1, 0, 0 \rangle$ **21.** $8\mathbf{i} + 9\mathbf{j} - 10\mathbf{k}$
23. $\langle 2/3, 2/3, 1/3 \rangle$

25. $\dfrac{\langle 0, -\sin 2t, 2\cos 2t \rangle}{\sqrt{1 + 3\cos^2 2t}}$ **27.** $\dfrac{t^2}{\sqrt{t^4 + 4}}\left\langle 1, 0, -\dfrac{2}{t^2} \right\rangle$

29. $\langle 0, 0, -1 \rangle$ **31.** $\left\langle \dfrac{2}{\sqrt{5}}, 0, -\dfrac{1}{\sqrt{5}} \right\rangle$

33. $\langle 30t^{14} + 24t^3, 14t^{13} - 12t^{11} + 9t^2 - 3, -96t^{11} - 24 \rangle$
35. $4t(2t^3 - 1)(t^3 - 2)\langle 3t(t^3 - 2), 1, 0 \rangle$
37. $e^t(2t^3 + 6t^2) - 2e^{-t}(t^2 - 2t - 1) - 16e^{-2t}$

39. 11 **41.** $\langle 0, 7, 1 \rangle$ **43.** $\langle 2e^{2t}, -2e^t, 0 \rangle$ **45.** $\left\langle 4, -\dfrac{2}{\sqrt{t}}, 0 \right\rangle$

47. $\langle 1 + 6t^2, 4t^3, -2 - 3t^2 \rangle$ **49.** $5te^t(t + 2) - 6t^2e^{-t}(t - 3)$

51. $-3t^2\sin t + 6t\cos t + 2\sqrt{t}\cos 2t + \dfrac{1}{2\sqrt{t}}\sin 2t$

53. $\langle 2, 0, 0 \rangle$, $\langle 0, 0, 0 \rangle$ **55.** $\langle -9\cos 3t, -16\sin 4t, -36\cos 6t \rangle$, $\langle 27\sin 3t, -64\cos 4t, 216\sin 6t \rangle$

57. $\left\langle -\dfrac{1}{4}(t + 4)^{-3/2}, -2(t + 1)^{-3}, 2e^{-t^2}(1 - 2t^2) \right\rangle$,
$\left\langle \dfrac{3}{8}(t + 4)^{-5/2}, 6(t + 1)^{-4}, -4te^{-t^2}(3 - 2t^2) \right\rangle$

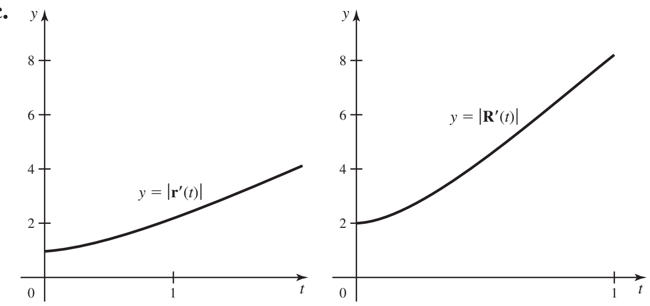
59. $\left\langle \dfrac{t^5}{5} - \dfrac{3t^2}{2}, t^2 - t, 10t \right\rangle + \mathbf{C}$

61. $\left\langle 2\sin t, -\dfrac{2}{3}\cos 3t, \dfrac{1}{2}\sin 8t \right\rangle + \mathbf{C}$

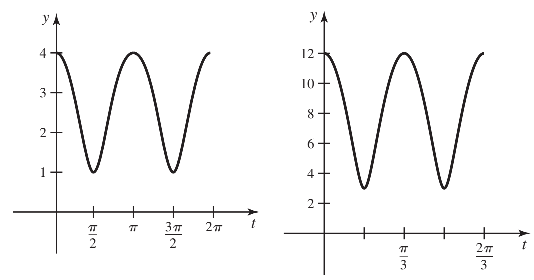
63. $\frac{1}{3}e^{3t}\,\mathbf{i} + \tan^{-1} t\,\mathbf{j} - \sqrt{2t}\,\mathbf{k} + \mathbf{C}$
65. $\mathbf{r}(t) = \langle e^t + 1, 3 - \cos t, \tan t + 2 \rangle$
67. $\mathbf{r}(t) = \langle t + 3, t^2 + 2, t^3 - 6 \rangle$
69. $\mathbf{r}(t) = \left\langle \frac{1}{2}e^{2t} + \frac{1}{2}, 2e^{-t} + t - 1, t - 2e^t + 3 \right\rangle$
71. $\langle 2, 0, 2 \rangle$ **73.** $\mathbf{i}$ **75.** $\langle 0, 0, 0 \rangle$
77. $(e^2 + 1)\langle 1, 2, -1 \rangle$ **79. a.** False **b.** True **c.** True
81. $\langle 2 - t, 3 - 2t, \pi/2 + t \rangle$ **83.** $\langle 2 + 3t, 9 + 7t, 1 + 2t \rangle$
85. $(1, 0)$ **87.** $(1, 0, 0)$ **89.** $\mathbf{r}(t) = \langle a_1 t, a_2 t, a_3 t \rangle$ or $\mathbf{r}(t) = \langle a_1 e^{kt}, a_2 e^{kt}, a_3 e^{kt} \rangle$, where a_i and k are real numbers

Section 14.3 Exercises, pp. 892–896

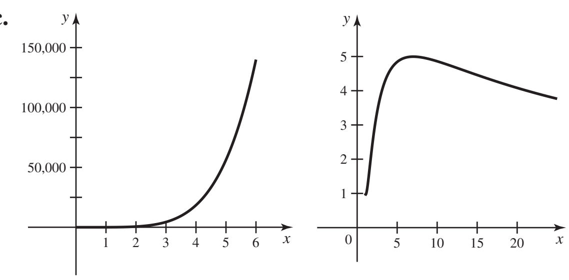
1. $\mathbf{v}(t) = \mathbf{r}'(t)$, speed $= |\mathbf{r}'(t)|$, $\mathbf{a}(t) = \mathbf{r}''(t)$ **3.** $m\mathbf{a}(t) = \mathbf{F}$
5. $\mathbf{v}(t) = \int \mathbf{a}(t)\,dt = \langle v_1(t), v_2(t) \rangle + \mathbf{C}$. Use initial conditions to find $\mathbf{C}$. **7. a.** $t = 3$ s **b.** $\mathbf{r}(t) = \langle 60t, -16t^2 + 96t + 3 \rangle$
9. a. $\langle 6t, 8t \rangle$, $10t$ **b.** $\langle 6, 8 \rangle$ **11. a.** $\mathbf{v}(t) = \langle 2, -4 \rangle$, $|\mathbf{v}(t)| = 2\sqrt{5}$ **b.** $\mathbf{a}(t) = \langle 0, 0 \rangle$ **13. a.** $\mathbf{v}(t) = \langle 8\cos t, -8\sin t \rangle$, $|\mathbf{v}(t)| = 8$ **b.** $\mathbf{a}(t) = \langle -8\sin t, -8\cos t \rangle$ **15. a.** $\langle 2t, 2t, t \rangle$, $3t$ **b.** $\langle 2, 2, 1 \rangle$ **17. a.** $\mathbf{v}(t) = \langle 1, -4, 6 \rangle$, $|\mathbf{v}(t)| = \sqrt{53}$ **b.** $\mathbf{a}(t) = \langle 0, 0, 0 \rangle$ **19. a.** $\mathbf{v}(t) = \langle 0, 2t, -e^{-t} \rangle$, $|\mathbf{v}(t)| = \sqrt{4t^2 + e^{-2t}}$ **b.** $\mathbf{a}(t) = \langle 0, 2, e^{-t} \rangle$
21. a. $[c, d] = [0, 1]$ **b.** $\langle 1, 2t \rangle$, $\langle 2, 8t \rangle$
c.



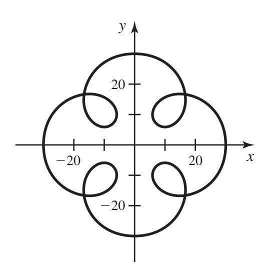
23. a. $\left[0, \frac{2\pi}{3}\right]$ **b.** $\mathbf{V_r}(t) = \langle -\sin t, 4\cos t \rangle$, $\mathbf{V_R}(t) = \langle -3\sin 3t, 12\cos 3t \rangle$
c.



25. a. $[1, e^{36}]$ **b.** $\mathbf{V_r}(t) = \langle 2t, -8t^3, 18t^5 \rangle$,
$\mathbf{V_R}(t) = \left\langle \dfrac{1}{t}, -\dfrac{4}{t}\ln t, \dfrac{9}{t}\ln^2 t \right\rangle$
c.

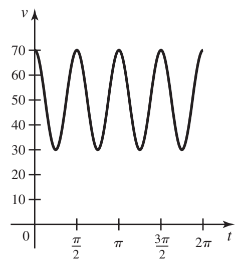


27. a.



b. $\langle -20\sin t - 50\sin 5t, 20\cos t + 50\cos 5t \rangle$

c.



d. 70 ft/s; 30 ft/s

29. $\mathbf{r}(t)$ lies on a circle of radius 8; $\langle -16 \sin 2t, 16 \cos 2t \rangle \cdot \langle 8 \cos 2t, 8 \sin 2t \rangle = 0$. **31.** $\mathbf{r}(t)$ lies on a sphere of radius 2; $\langle \cos t - \sqrt{3} \sin t, \sqrt{3} \cos t + \sin t \rangle \cdot \langle \sin t + \sqrt{3} \cos t, \sqrt{3} \sin t - \cos t \rangle = 0$. **33.** 5

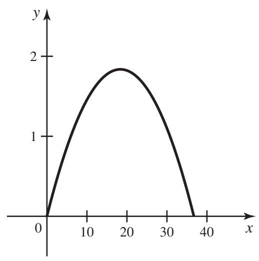
35. $\mathbf{v}(t) = \langle 2, t + 3 \rangle$, $\mathbf{r}(t) = \left\langle 2t, \dfrac{t^2}{2} + 3t \right\rangle$

37. $\mathbf{v}(t) = \langle 0, 10t + 5 \rangle$, $\mathbf{r}(t) = \langle 1, 5t^2 + 5t - 1 \rangle$

39. $\mathbf{v}(t) = \langle \sin t, -2 \cos t + 3 \rangle$, $\mathbf{r}(t) = \langle -\cos t + 2, -2 \sin t + 3t \rangle$

41. a. $\mathbf{v}(t) = \langle 30, -9.8t + 6 \rangle$, $\mathbf{r}(t) = \langle 30t, -4.9t^2 + 6t \rangle$

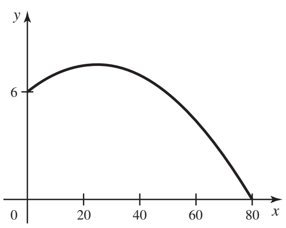
b.



c. $T \approx 1.22$ s, range ≈ 36.7 m **d.** 1.84 m

43. a. $\mathbf{v}(t) = \langle 80, 10 - 32t \rangle$, $\mathbf{r}(t) = \langle 80t, -16t^2 + 10t + 6 \rangle$

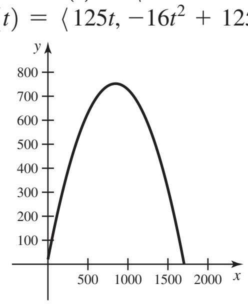
b.



c. 1 s, 80 ft **d.** Max height ≈ 7.56 ft

45. a. $\mathbf{v}(t) = \langle 125, -32t + 125\sqrt{3} \rangle$, $\mathbf{r}(t) = \langle 125t, -16t^2 + 125\sqrt{3}t + 20 \rangle$

b.



c. 13.6 s, 1702.5 ft **d.** 752.4 ft

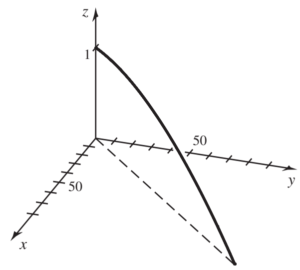
47. $\mathbf{v}(t) = \langle 1, 5, 10t \rangle$, $\mathbf{r}(t) = \langle t, 5t + 5, 5t^2 \rangle$

49. $\mathbf{v}(t) = \langle -\cos t + 1, \sin t + 2, t \rangle$,

$\mathbf{r}(t) = \left\langle -\sin t + t, -\cos t + 2t + 1, \dfrac{t^2}{2} \right\rangle$

51. a. $\mathbf{v}(t) = \langle 200, 200, -9.8t \rangle$, $\mathbf{r}(t) = \langle 200t, 200t, -4.9t^2 + 1 \rangle$

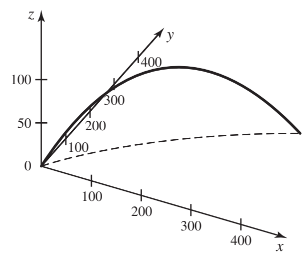
b.



c. 0.452 s, 127.8 m **d.** 1 m

53. a. $\mathbf{v}(t) = \langle 60 + 10t, 80, 80 - 32t \rangle$, $\mathbf{r}(t) = \langle 60t + 5t^2, 80t, 80t - 16t^2 + 3 \rangle$

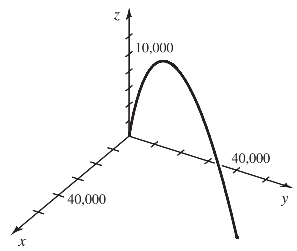
b.



c. 5.04 s, 589 ft
d. 103 ft

55. a. $\mathbf{v}(t) = \langle 300, 2.5t + 400, -9.8t + 500 \rangle$, $\mathbf{r}(t) = \langle 300t, 1.25t^2 + 400t, -4.9t^2 + 500t + 10 \rangle$

b.

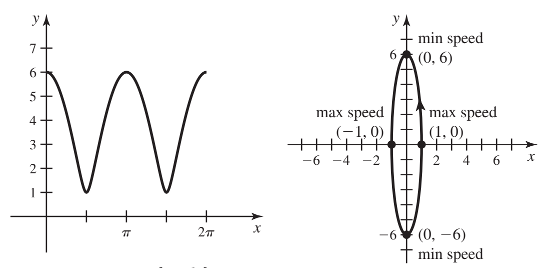


c. 102.1 s, 61,941.5 m
d. 12,765.1 m

57. a. False **b.** True **c.** False **d.** True **e.** False **f.** True **g.** True **59.** 15.3 s, 1988.3 m, 287.0 m **61.** 21.7 s, 4330.1 ft, 1875 ft **63.** Approx. 27.4° and 62.6°

65. a. $\mathbf{v}(t) = \langle -a \sin t, b \cos t \rangle$; $|\mathbf{v}(t)| = \sqrt{a^2 \sin^2 t + b^2 \cos^2 t}$

b.



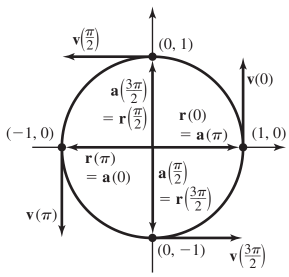
c. Yes **d.** Max $\left\{ \frac{a}{b}, \frac{b}{a} \right\}$ **67.** Approx. 23.5° or 59.6° **69.** 113.4 ft/s **71. a.** 1.2 ft, 0.46 s **b.** 0.88 ft/s **c.** 0.85 ft **d.** More curve in the second half **e.** $c = 28.17$ ft/s^2

73. $T = \dfrac{|\mathbf{v}_0| \sin \alpha + \sqrt{|\mathbf{v}_0|^2 \sin^2 \alpha + 2gy_0}}{g}$,

range $= |\mathbf{v}_0| (\cos \alpha) T$, max height $= y_0 + \dfrac{|\mathbf{v}_0|^2 \sin^2 \alpha}{2g}$

75. a. $\left[0, \dfrac{2\pi}{\omega} \right]$ **b.** $\mathbf{v}(t) = \langle -A\omega \sin \omega t, A\omega \cos \omega t \rangle$ is not constant; $|\mathbf{v}(t)| = |A\omega|$ is constant. **c.** $\mathbf{a}(t) = \langle -A\omega^2 \cos \omega t, -A\omega^2 \sin \omega t \rangle$ **d.** $\mathbf{r}$ and $\mathbf{v}$ are orthogonal; $\mathbf{r}$ and $\mathbf{a}$ are in opposite directions.

e.



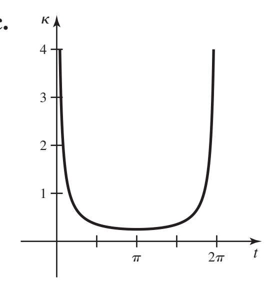
77. a. $\mathbf{r}(t) = \langle 5\sin(\pi t/6), 5\cos(\pi t/6)\rangle$
b. $\mathbf{r}(t) = \left\langle 5\sin\left(\frac{1-e^{-t}}{5}\right), 5\cos\left(\frac{1-e^{-t}}{5}\right)\right\rangle$
79. $\{(\cos t, \sin t, c\sin t): t \in \mathbb{R}\}$ satisfies the equations $x^2 + y^2 = 1$ and $z - cy = 0$ so that $\langle \cos t, \sin t, c \sin t\rangle$ lies on the intersection of a right circular cylinder and a plane, which is an ellipse.
83. a. The direction of **r** does not change. **b.** Constant in direction, not in magnitude

Section 14.4 Exercises, pp. 900–902

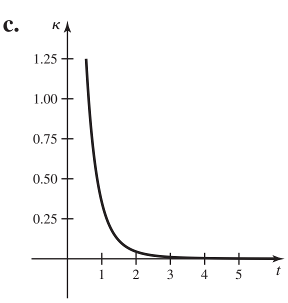
1. $\sqrt{5}(b-a)$ **3.** $\int_a^b |\mathbf{v}(t)|\,dt$ **5.** 20π **7.** If the parameter t used to describe a trajectory also measures the arc length s of the curve that is generated, we say the curve has been parameterized by its arc length.
9. 5 **11.** 3π **13.** $\frac{\pi^2}{8}$ **15.** $5\sqrt{34}$ **17.** $4\pi\sqrt{65}$ **19.** 9 **21.** $\frac{3}{2}$
23. $3t^2\sqrt{30}$; $64\sqrt{30}$ **25.** 26; 26π
27. Approx. 66,626 mi/hr **29.** 19.38
31. 32.50 **33.** Yes **35.** No; $\mathbf{r}(s) = \left\langle \frac{s}{\sqrt{5}}, \frac{2s}{\sqrt{5}}\right\rangle, 0 \le s \le 3\sqrt{5}$
37. No; $\mathbf{r}(s) = \left\langle 2\cos\frac{s}{2}, 2\sin\frac{s}{2}\right\rangle, 0 \le s \le 4\pi$
39. No; $\mathbf{r}(s) = \langle \cos s, \sin s\rangle, 0 \le s \le \pi$
41. No; $\mathbf{r}(s) = \left\langle \frac{s}{\sqrt{3}} + 1, \frac{s}{\sqrt{3}} + 1, \frac{s}{\sqrt{3}} + 1\right\rangle, s \ge 0$
43. a. True **b.** True **c.** True **d.** False **45. a.** If $a^2 = b^2 + c^2$, then $|\mathbf{r}(t)|^2 = (a\cos t)^2 + (b \sin t)^2 + (c\sin t)^2 = a^2$ so that $\mathbf{r}(t)$ is a circle centered at the origin of radius $|a|$. **b.** $2\pi a$
c. If $a^2 + c^2 + e^2 = b^2 + d^2 + f^2$ and $ab + cd + ef = 0$, then $\mathbf{r}(t)$ is a circle of radius $\sqrt{a^2 + c^2 + e^2}$ and its arc length is $2\pi\sqrt{a^2 + c^2 + e^2}$. **47. a.** $\int_a^b \sqrt{(Ah'(t))^2 + (Bh'(t))^2}\,dt$
$= \int_a^b \sqrt{(A^2 + B^2)(h'(t))^2}\,dt = \sqrt{A^2 + B^2}\int_a^b |h'(t)|\,dt$
b. $64\sqrt{29}$ **c.** $\frac{7\sqrt{29}}{4}$ **49. a.** 5.102 s
b. $\int_0^{5.102} \sqrt{400 + (25 - 9.8t)^2}\,dt$ **c.** 124.43 m **d.** 102.04 m
51. $|\mathbf{v}(t)| = \sqrt{a^2 + b^2 + c^2} = 1$, if $a^2 + b^2 + c^2 = 1$
53. $\int_a^b |\mathbf{r}'(t)|\,dt = \int_a^b \sqrt{(cf'(t))^2 + (cg'(t))^2}\,dt$
$= |c|\int_a^b \sqrt{(f'(t))^2 + (g'(t))^2}\,dt = |c|L$

Section 14.5 Exercises, pp. 913–915

1. 0 **3.** $\kappa = \frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right|$ or $\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$ **5.** $\mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$
7. These three unit vectors are mutually orthogonal at all points of the curve. **9.** The torsion measures the rate at which the curve rises or twists out of the **TN**-plane at a point. **11.** $\mathbf{T} = \frac{\langle 1, 2, 3\rangle}{\sqrt{14}}, \kappa = 0$
13. $\mathbf{T} = \frac{\langle 1, 2\cos t, -2\sin t\rangle}{\sqrt{5}}, \kappa = \frac{1}{5}$
15. $\mathbf{T} = \frac{\langle \sqrt{3}\cos t, \cos t, -2\sin t\rangle}{2}, \kappa = \frac{1}{2}$
17. $\mathbf{T} = \frac{\langle 1, 4t\rangle}{\sqrt{1 + 16t^2}}, \kappa = \frac{4}{(1 + 16t^2)^{3/2}}$
19. $\mathbf{T} = \left\langle \cos\left(\frac{\pi t^2}{2}\right), \sin\left(\frac{\pi t^2}{2}\right)\right\rangle, \kappa = \pi t$
21. $\frac{1}{3}$ **23.** $\frac{2}{(4t^2 + 1)^{3/2}}$ **25.** $\frac{2\sqrt{5}}{(20\sin^2 t + \cos^2 t)^{3/2}}$
27. $\mathbf{T} = \langle \cos t, -\sin t\rangle, \mathbf{N} = \langle -\sin t, -\cos t\rangle$
29. $\mathbf{T} = \frac{\langle t, -3, 0\rangle}{\sqrt{t^2 + 9}}, \mathbf{N} = \frac{\langle 3, t, 0\rangle}{\sqrt{t^2 + 9}}$
31. $\mathbf{T} = \langle -\sin t^2, \cos t^2\rangle$, $\mathbf{N} = \langle -\cos t^2, -\sin t^2\rangle$
33. $\mathbf{T} = \frac{\langle 2t, 1\rangle}{\sqrt{4t^2 + 1}}, \mathbf{N} = \frac{\langle 1, -2t\rangle}{\sqrt{4t^2 + 1}}$ **35.** $a_N = a_T = 0$
37. $a_T = \sqrt{3}e^t$; $a_N = \sqrt{2}e^t$ **39.** $\mathbf{a} = \frac{6t}{\sqrt{9t^2 + 4}}\mathbf{N} + \frac{18t^2 + 4}{\sqrt{9t^2 + 4}}\mathbf{T}$
41. $\mathbf{B}(t) = \langle 0, 0, -1\rangle, \tau = 0$ **43.** $\mathbf{B}(t) = \langle 0, 0, 1\rangle, \tau = 0$
45. $\mathbf{B}(t) = \frac{\langle -\sin t, \cos t, 2\rangle}{\sqrt{5}}, \tau = -\frac{1}{5}$
47. $\mathbf{B}(t) = \frac{\langle 5, 12\sin t, -12\cos t\rangle}{13}, \tau = \frac{12}{169}$ **49. a.** False
b. False **c.** False **d.** True **e.** False **f.** False **g.** False
51. $\kappa = \frac{2}{(1 + 4x^2)^{3/2}}$ **53.** $\kappa = \frac{x}{(x^2 + 1)^{3/2}}$
57. $\kappa = \frac{|ab|}{(a^2\cos^2 t + b^2 \sin^2 t)^{3/2}}$ **59.** $\kappa = \frac{2|a|}{(1 + 4a^2t^2)^{3/2}}$
61. b. $\mathbf{v}_A(t) = \langle 1, 2, 3\rangle$, $\mathbf{a}_A(t) = \langle 0, 0, 0\rangle$ and $\mathbf{v}_B(t) = \langle 2t, 4t, 6t\rangle$, $\mathbf{a}_B(t) = \langle 2, 4, 6\rangle$; A has constant velocity and zero acceleration, while B has increasing speed and constant acceleration.
c. $\mathbf{a}_A(t) = 0\mathbf{N} + 0\mathbf{T}$, $\mathbf{a}_B(t) = 0\mathbf{N} + 2\sqrt{14}\,\mathbf{T}$; both normal components are zero since the path is a straight line ($\kappa = 0$).
63. b. $\mathbf{v}_A(t) = \langle -\sin t, \cos t\rangle$, $\mathbf{a}_A(t) = \langle -\cos t, -\sin t\rangle$
$\mathbf{v}_B(t) = \langle -2t\sin t^2, 2t\cos t^2\rangle$
$\mathbf{a}_B(t) = \langle -4t^2\cos t^2 - 2\sin t^2, -4t^2\sin t^2 + 2\cos t^2\rangle$
c. $\mathbf{a}_A(t) = \mathbf{N} + 0\mathbf{T}$, $\mathbf{a}_B(t) = 4t^2\mathbf{N} + 2\mathbf{T}$; for A, the acceleration is always normal to the curve, but this is not true for B.
65. b. $\kappa = \frac{1}{2\sqrt{2(1 - \cos t)}}$ **c.**



d. Minimum curvature at $t = \pi$
67. b. $\kappa = \frac{1}{t(1 + t^2)^{3/2}}$ **c.**



d. No maximum or minimum curvature

69. $\kappa = \dfrac{e^x}{(1 + e^{2x})^{3/2}}, \left(-\dfrac{\ln 2}{2}, \dfrac{1}{\sqrt{2}}\right), \dfrac{2\sqrt{3}}{9}$

71. $\dfrac{1}{\kappa} = \dfrac{1}{2}; x^2 + \left(y - \dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

73. $\dfrac{1}{\kappa} = 4; (x - \pi)^2 + (y + 2)^2 = 16$

75. $\kappa\left(\dfrac{\pi}{2n}\right) = n^2$; κ increases as n increases.

77. **a.** Speed $= \sqrt{V_0^2 - 2V_0\,gt \sin \alpha + g^2t^2}$

b. $\kappa(t) = \dfrac{gV_0 \cos \alpha}{(V_0^2 - 2V_0\,gt \sin \alpha + g^2t^2)^{3/2}}$

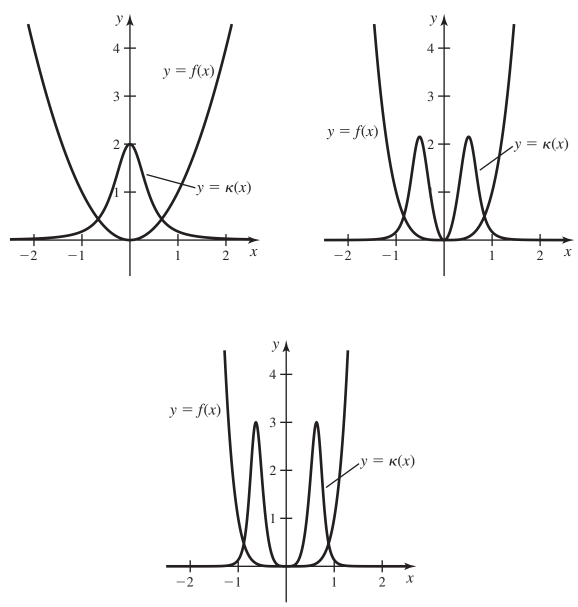
c. Speed has a minimum at $t = \dfrac{V_0 \sin \alpha}{g}$ and $\kappa(t)$ has a maximum at $t = \dfrac{V_0 \sin \alpha}{g}$. **79.** $\kappa = \dfrac{1}{|\mathbf{v}|} \cdot \left|\dfrac{d\mathbf{T}}{dt}\right|$, where $\mathbf{T} = \dfrac{\langle b, d, f\rangle}{\sqrt{b^2 + d^2 + f^2}}$ and b, d, and f are constant. Therefore, $\dfrac{d\mathbf{T}}{dt} = \mathbf{0}$ so $\kappa = 0$.

81. **a.** $\kappa_1(x) = \dfrac{2}{(1 + 4x^2)^{3/2}}$

$\kappa_2(x) = \dfrac{12x^2}{(1 + 16x^6)^{3/2}}$

$\kappa_3(x) = \dfrac{30x^4}{(1 + 36x^{10})^{3/2}}$

b.

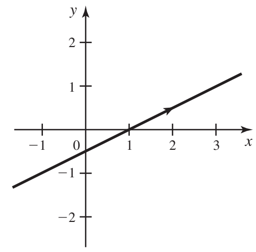


c. κ_1 has its maximum at $x = 0$, κ_2 has its maxima at $x = \pm\sqrt[6]{\frac{1}{56}}$, and κ_3 has its maxima at $x = \pm\sqrt[10]{\frac{1}{99}}$. **d.** $\lim\limits_{n\to\infty} z_n = 1$; the graphs of $y = f_n(x)$ show that as $n \to \infty$, the point corresponding to maximum curvature gets arbitrarily close to the point $(1, 0)$.

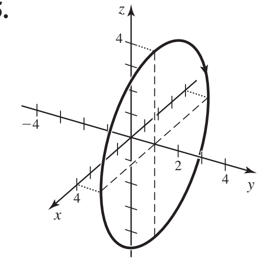
Chapter 14 Review Exercises, pp. 916–918

1. **a.** False **b.** True **c.** True **d.** True **e.** False **f.** False

3.



5.



7. $x^2 + y^2 + z^2 = 2$; $y = z$; a tilted circle of radius $\sqrt{2}$ centered at $(0, 0, 0)$ **9.** $\mathbf{r}(t) = \langle 4 + 15t, -2 - t, 3 - 5t\rangle$
11. $\mathbf{r}(t) = \langle 2, 3 \cos t, 4 \sin t\rangle$, for $0 \le t \le 2\pi$
13. $\mathbf{r}(t) = \langle \cos t, \sin t, \sin t\rangle$, for $0 \le t \le 2\pi$
15. $\mathbf{r}(t) = \langle 3 \cos t, \sin t, \sin t\rangle$, for $0 \le t \le 2\pi$
17. **a.** $\langle 1, 0\rangle$; $\langle 0, 1\rangle$ **b.** $\left\langle -\dfrac{2}{(2t + 1)^2}, \dfrac{1}{(t + 1)^2}\right\rangle$; $\langle -2, 1\rangle$

c. $\left\langle \dfrac{8}{(2t + 1)^3}, -\dfrac{2}{(t + 1)^3}\right\rangle$

d. $\left\langle \dfrac{1}{2}\ln|2t + 1|, t - \ln|t + 1|\right\rangle + \mathbf{C}$

19. **a.** $\langle 0, 3, 0\rangle$; does not exist
b. $\langle 2 \cos 2t, -12 \sin 4t, 1\rangle$; $\langle 2, 0, 1\rangle$ **c.** $\langle -4 \sin 2t, -48 \cos 4t, 0\rangle$
d. $\left\langle -\dfrac{1}{2}\cos 2t, \dfrac{3}{4}\sin 4t, \dfrac{1}{2}t^2\right\rangle + \mathbf{C}$ **21.** $2\mathbf{j} + \pi\mathbf{k}$

23. $23\mathbf{i} - 41\mathbf{k}$ **25.** $\mathbf{r}(t) = \left\langle t + 2, -\dfrac{1}{2}\cos 2t + \dfrac{5}{2}, \tan t + 2\right\rangle$

27. $\mathbf{r}(t) = \langle 4 \tan^{-1} t - \pi, t^2 + t - 2, t^3 - 1\rangle$

29. $\mathbf{T}(t) = \left\langle \dfrac{2e^t}{2e^{2t} + 1}, \dfrac{2e^{2t}}{2e^{2t} + 1}, \dfrac{1}{2e^{2t} + 1}\right\rangle$; $\left\langle \dfrac{2}{3}, \dfrac{2}{3}, \dfrac{1}{3}\right\rangle$

31. **a.** $\langle 4e^{4t}, 4e^{4t}, 2e^{4t}\rangle$; $6e^{4t}$ **b.** $\langle 16e^{4t}, 16e^{4t}, 8e^{4t}\rangle$
33. $\mathbf{v}(t) = \langle 2 + \sin t, 3 - 2 \cos t\rangle$;
$\mathbf{r}(t) = \langle 2t + 2 - \cos t, 3t + 2 - 2 \sin t\rangle$
35. **a.** $\mathbf{v}(t) = \langle 40, -32t + 40\sqrt{3}\rangle$;
$\mathbf{r}(t) = \langle 40t, -16t^2 + 40\sqrt{3}t + 3\rangle$
b. Approx. 4.37 s; approx. 174.9 ft **c.** 78 ft
37. **a.** $\mathbf{v}(t) = \langle 4t + 40, 20, 40 - 32t\rangle$;
$\mathbf{r}(t) = \langle 2t^2 + 40t, 20t, -16t^2 + 40t + 2\rangle$ **b.** 2.549 s
c. 126 ft **39.** **a.** $(116, 30)$ **b.** 39.1 ft **c.** 2.315 s
d. $\displaystyle\int_0^{2.315} \sqrt{50^2 + (-32t + 50)^2}\, dt$ **e.** 129 ft **f.** 41.4° to 79.4°
41. $(1.47, 3.15, 4.4)$ **43.** 12 **45.** Approx. 6.42
47. **a.** $\mathbf{v}(t) = \mathbf{i} + t\sqrt{2}\,\mathbf{j} + t^2\mathbf{k}$ **b.** 12
49. $\mathbf{r}(s) = \left\langle (\sqrt{1 + s} - 1)^2, \dfrac{4\sqrt{2}}{3}(\sqrt{1 + s} - 1)^{3/2}, 2(\sqrt{1 + s} - 1)\right\rangle$, for $s \ge 0$ **51.** **a.** $\mathbf{v} = \langle -6 \sin t, 3 \cos t\rangle$,

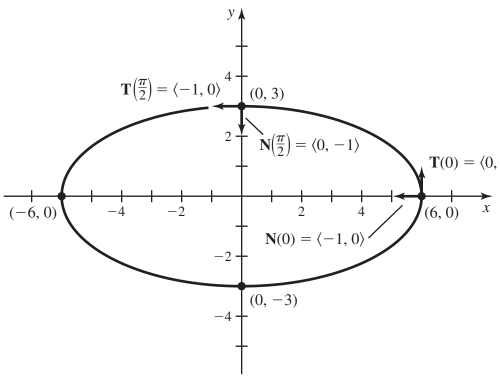
$\mathbf{T} = \dfrac{\langle -2 \sin t, \cos t\rangle}{\sqrt{1 + 3 \sin^2 t}}$ **b.** $\kappa(t) = \dfrac{2}{3(1 + 3 \sin^2 t)^{3/2}}$

c. $\mathbf{N} = \left\langle -\dfrac{\cos t}{\sqrt{1 + 3 \sin^2 t}}, -\dfrac{2 \sin t}{\sqrt{1 + 3 \sin^2 t}}\right\rangle$

d. $|\mathbf{N}| = \sqrt{\dfrac{\cos^2 t + 4 \sin^2 t}{1 + 3 \sin^2 t}} = \sqrt{\dfrac{(\cos^2 t + \sin^2 t) + 3 \sin^2 t}{1 + 3 \sin^2 t}} = 1$;

$\mathbf{T} \cdot \mathbf{N} = \dfrac{2 \sin t \cos t - 2 \sin t \cos t}{1 + 3 \sin^2 t} = 0$

e.



53. a. $\mathbf{v}(t) = \langle -\sin t, -2\sin t, \sqrt{5}\cos t\rangle$,

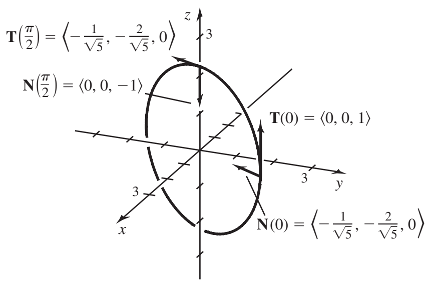
$\mathbf{T}(t) = \left\langle -\frac{1}{\sqrt{5}}\sin t, -\frac{2}{\sqrt{5}}\sin t, \cos t\right\rangle$ **b.** $\kappa(t) = \frac{1}{\sqrt{5}}$

c. $\mathbf{N}(t) = \left\langle -\frac{1}{\sqrt{5}}\cos t, -\frac{2}{\sqrt{5}}\cos t, -\sin t\right\rangle$

d. $|\mathbf{N}(t)| = \sqrt{\frac{1}{5}\cos^2 t + \frac{4}{5}\cos^2 t + \sin^2 t} = 1;$

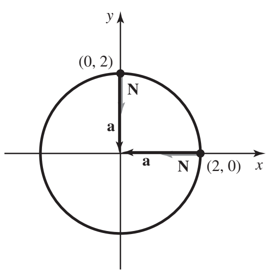
$\mathbf{T}\cdot\mathbf{N} = \left(\frac{1}{5}\cos t\sin t + \frac{4}{5}\cos t\sin t\right) - \sin t\cos t = 0$

e.



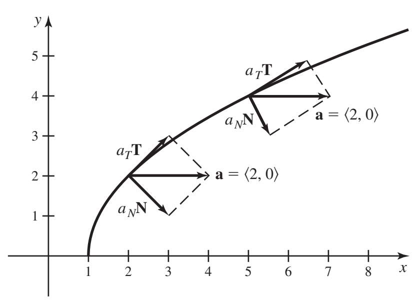
55. a. $\mathbf{a}(t) = 2\mathbf{N} + 0\mathbf{T} = 2\langle -\cos t, -\sin t\rangle$

b.



57. a. $a_T = \frac{2t}{\sqrt{t^2+1}}$ and $a_N = \frac{2}{\sqrt{t^2+1}}$

b.

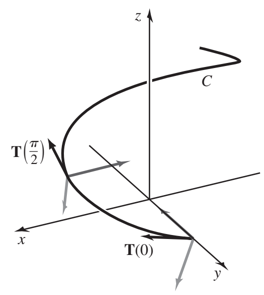


59. $\mathbf{B}(1) = \frac{\langle 3, -3, 1\rangle}{\sqrt{19}}; \tau = \frac{3}{19}$

61. a. $\mathbf{T}(t) = \frac{1}{5}\langle 3\cos t, -3\sin t, 4\rangle$

b. $\mathbf{N}(t) = \langle -\sin t, -\cos t, 0\rangle; \kappa = \frac{3}{25}$

c.



d. Yes

e. $\mathbf{B}(t) = \frac{1}{5}\langle 4\cos t, -4\sin t, -3\rangle$

f. See graph in part (c).

g. Check that $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$ have unit length and are mutually orthogonal.

h. $\tau = -\frac{4}{25}$

63. a. Consider first the case where $a_3 = b_3 = c_3 = 0$, and show that for all $s \neq t$ in I, $\mathbf{r}(t) \times \mathbf{r}(s)$ is a multiple of the constant vector $\langle b_1c_2 - b_2c_1, a_2c_1 - a_1c_2, a_1b_2 - a_2b_1\rangle$, which implies $\mathbf{r}(t) \times \mathbf{r}(s)$ is always orthogonal to the same vector, and therefore the vectors $\mathbf{r}(t)$ must all lie in the same plane. When a_3, b_3, and c_3 are not necessarily 0, the curve still lies in a plane because these constants represent a simple translation of the curve to a different location in $\mathbb{R}^3$.

b. Because the curve lies in a plane, $\mathbf{B}$ is always normal to the plane and has length 1. Therefore, $\frac{d\mathbf{B}}{ds} = \mathbf{0}$ and $\tau = 0$.

CHAPTER 15

Section 15.1 Exercises, pp. 927–930

1. Independent: x and y; dependent: z

3. $D = \{(x, y): x \neq 0 \text{ and } y \neq 0\}$ **5.** Three **7.** 3; 4

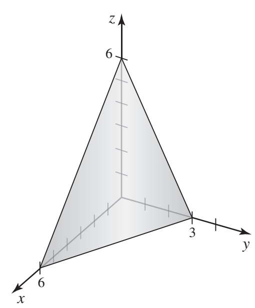
9. a. 1300 ft **b.** Katie; Katie is 100 ft higher than Zeke. **11.** Circles

13. $n = 6$ **15.** $D = \mathbb{R}^2$ **17.** $D = \{(x, y): x^2 + y^2 \leq 25\}$

19. $D = \{(x, y): y \neq 0\}$ **21.** $D = \{(x, y): y < x^2\}$

23. $D = \{(x, y): xy \geq 0, (x, y) \neq (0, 0)\}$

25. Plane; $D = \mathbb{R}^2$, $R = \mathbb{R}$



27. Hyperbolic paraboloid; $D = \mathbb{R}^2$, $R = \mathbb{R}$