Partial fractions - continued

Friday, October 1, 2021 10:55 AM

Example 5.(irreducible quadrtic factors) Evaluate $\int f(x)\,dx = \int \frac{7x^2-13x+13}{(x-2)(x^2-2x+3)}\,dx$

Solution.

Check if x^2-2x+3 can be factored?

$$(-2)^2-4\cdot 1\cdot 3 = 4-12 < 0$$

$$\Rightarrow \frac{7x^2-13x+13}{(x-2)(x^2-2x+3)} = \frac{A \times (x^2-2x+3)}{x-2 \times (x^2-2x+3)} + \frac{Bx+C}{x^2-2x+3}$$

$$= \frac{A(x^2-2x+3) + (Bx+C)(x-2)}{(x-2)(x^2-2x+3)}$$

$$\Rightarrow 7x^2-13x+13 = A(x^2-2x+3) + (Bx+C)(x-2)$$

$28-26+13$

Let $x=2 \rightarrow$ $15 = A(4-4+3) = 3A \Rightarrow A=5$ (1)

Let $x=0$ $13 = 5\cdot 3 + (0+C)(-2)$

$13 = 15 - 2C \Rightarrow C = 1$ (2)

Let $x=1$ $7 = 5\times 2 + (B+1)(-1)$

$7 = 10 - B - 1 \Rightarrow B = 2$ (3)

You can double check your Answer

$$\frac{5}{x-2} + \frac{2x+1}{x^2-2x+3} \overset{?}{=} \frac{7x^2-13x+13}{(x-2)(x^2-2x+3)}$$

$$\Rightarrow \int f(x)dx = \int \frac{5}{x-2}dx + \int \frac{2x+1}{x^2-2x+3}dx$$

$u = x-2$

$(x^2-2x+3)' = 2x-2$

$$= 5\ln|x-2| + \int \frac{2x-2+2+1}{x^2-2x+3}dx$$

$$= 5\ln|x-2| + \int \frac{2x-2}{x^2-2x+3}dx + \int \frac{3}{x^2-2x+3}dx$$

$u = x^2-2x+3$

$$= 5\ln|x-2| + \ln|x^2-2x+3| + \int \frac{3}{x^2-2x+3}dx$$

$\int \frac{dx}{1+x^2} = \tan^{-1}x + C$

$$= 5\ln|x-2| + \ln|x^2-2x+3| + 3\int \frac{dx}{x^2-2x+1+2}$$

$$= 5\ln|x-2| + \ln|x^2-2x+3| + 3\int \frac{dx}{(x-1)^2+2}$$

$$= 5\ln|x-2| + \ln|x^2-2x+3| + \frac{3}{2}\int \frac{dx}{1+\left[\frac{x-1}{\sqrt{2}}\right]^2}$$

$u = \frac{x-1}{\sqrt{2}}$

$\int \frac{dx}{1+x^2} = \tan^{-1}x + C$

$du = \frac{1}{\sqrt{2}}dx \Rightarrow dx = \sqrt{2}\,du$

$$= 5\ln|x-2| + \ln|x^2-2x+3| + \frac{3}{2}\sqrt{2}\int \frac{du}{1+u^2}$$

$$= 5\ln|x-2| + \ln|x^2-2x+3| + \frac{3\sqrt{2}}{2}\arctan\left(\frac{x-1}{\sqrt{2}}\right) + C$$

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Assignment Homework4 due 10/04/2021 at 11:59pm EDT

Math141_Calculus_II

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6. **(1 point)** Library/ma123DB/set3/s7_4_19.pg

Evaluate the indefinite integral.

$$\int \frac{-4}{x^2-4x+4}dx$$

$$x^2 - 2\cdot 2\cdot x + 2^2$$

Answer: ______________ $+C$

sol. $\int \frac{-4\,dx}{x^2-4x+4} = \int \frac{-4\,dx}{(x-2)^2}$

$\underline{\underline{u=x-2}}\; -4\int u^{-2}\,du = -4\cdot \frac{u^{-2+1}}{-2+1} + C$

$= 4(x-2)^{-1} + C$

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7. **(1 point)** Library/ma123DB/set3/s7_4_31.pg

The form of the partial fraction decomposition of a rational function is given below.

$$\frac{x^2-3x-3}{(x+4)(x^2+9)} = \frac{A}{x+4} + \frac{Bx+C}{x^2+9}$$

$A =$ _____ $B =$ _____ $C =$ _____

$$= \frac{A(x^2+9) + (Bx+C)(x+4)}{(x+4)(x^2+9)}$$

Now evaluate the indefinite integral.

$$\int \frac{x^2-3x-3}{(x+4)(x^2+9)}dx = \underline{\hspace{6cm}}$$

$\Rightarrow x^2 - 3x - 3 = A(x^2+9) + (Bx+C)(x+4)$

let $x=-4$, $\rightarrow \underline{16+12-3} = 25A \Rightarrow \underline{A=1}$ ①

let $x=0 \rightarrow -3 = 9\cdot 1 + 4C$

$\Rightarrow 4C = -12 \Rightarrow \underline{C=-3}$ ②

let $x=1 \longrightarrow 1-3-3 = 10 + 5(B-3)$

$\Rightarrow -5 = 10 + 5B - 15 \Rightarrow 5B = 0 \Rightarrow B=0$ ③

$$\Rightarrow \int f(x)dx = \int \frac{1}{x+4}dx + \int \frac{-3\,dx}{x^2+9}$$

$u = x+4$

$$= \ln|x+4| - \frac{3}{9}\int \frac{dx}{1+\left(\frac{x}{3}\right)^2}$$

$u = \frac{x}{3} \Rightarrow du = \frac{1}{3}dx$

$\Rightarrow dx = 3\,du$

$$= \ln|x+4| - \int \frac{du}{1+u^2}$$

$$= \ln|x+4| - \arctan\left(\frac{x}{3}\right) + C$$

8. **(1 point)** `Library/Wiley/setAnton_Section_7.5/Anton_7_5_Q19.pg`

Evaluate the integral.

$$\int \frac{4x-1}{x^2+x-2}\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$$

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Using partial fraction,

$$\frac{4x-1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

DIY

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9. **(1 point)** `Library/Rochester/setIntegrals25RationalFunctions/S07.04.PartialFractions.PTP18.pg`

What is the correct form of the partial fraction decomposition for the following integral?

$$\int \frac{x^2+1}{(x-5)^3(x^2+9x+47)}\,dx$$

$9^2 - 4 \times 47 < 0$

$$\frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{C}{(x-5)^3} + \frac{Dx+E}{x^2+9x+47}$$

- A. $\int \left(\frac{A}{x-5} + \frac{Bx+C}{(x-5)^2} + \frac{Dx+E}{(x-5)^3} + \frac{Fx+G}{x^2+9x+47} \right) dx$
- B. $\int \left(\frac{A}{(x-5)^3} + \frac{Bx+C}{x^2+9x+47} \right) dx$
- C. $\int \left(\frac{A}{(x-5)^3} + \frac{B}{x-9} + \frac{C}{x-47} \right) dx$
- D. $\int \left(\frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{C}{(x-5)^3} + \frac{Dx+E}{x^2+9x+47} \right) dx$
- E. There is no partial fraction decomposition yet because there is cancellation.

- F. There is no partial fraction decomposition yet because long division must be done first.
- G. $\int \left(\frac{A}{(x-5)^3} + \frac{B}{x-9} + \frac{C}{(x-9)^2} + \frac{Dx+E}{x^2+1} \right) dx$
- H. There is no partial fraction decomposition because the denominator does not factor.

10. **(1 point)** `Library/UMN/calculusStewartET/s_7_4_prob04.pg`

Evaluate the integral

$$\int \frac{x+2}{x^2+4x+5}\,dx.$$

Answer: ____________________

$4^2 - 4 \times 1 \times 5$

$= 16 - 20 < 0$

$(x^2+4x+5)' = 2x+4$

$= 2(x+2)$

$u = x^2+4x+5, \quad du = 2(x+2)\,dx$

$\Rightarrow (x+2)\,dx = \frac{1}{2}\,du$

$$\Rightarrow \int \frac{x+2}{x^2+4x+5}\,dx = \frac{1}{2}\int \frac{du}{u}$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln(x^2+4x+5) + C$$