Chapter 10-3 – Run-Time Analysis 3 & The Seven functions

The Seven Functions used in this semester

1. The Constant Function
   - The simplest function we can think of is the constant function.

   f(n) = c, for some fixed constant c, such c = 5, c = 27, or c = 2^10.
   - For any argument n, the constant function f(n) assigns the value c. That is, it doesn't matter what the value of n is, f(n) is always be equal to the constant value c.

2. The Logarithm Function

   f(n) = $\log_b n$, for some constant b > 1.

   This function is defined as follows:

   x = $\log_b n$ *if and only if* $b^x = n$.

   By definition, $\log_b 1 = 0$. The value b is known as the base of the algorithm.

   - Logarithm Rules

     1. Inverse properties: $\log_a a^x = x$ and $a^{(\log_a x)} = x$
     2. Product: $\log_a (xy) = \log_a x + \log_a y$
     3. Quotient: $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
     4. Power: $\log_a (x^p) = p \log_a x$
     5. Change of base formula: $\log_a x = \frac{\log_b x}{\log_b a}$

3. The Linear Function
   f(n) = n.
   Given an input n, the linear function f assigns the value n itself.

4. The N- Log–N Function
   f(n) = n log n.
   The function assigns to an input n the value of n times the logarithm base 2 of n.
   This function grows a little faster than the linear function and a lot slower than the quadratic function.

5. The Quadratic Function
   f(n) = n^2.
   - Given an input value n, the function f assigns the product of n with itself.
   - For any integer n ≥ 1, *we have*

     $\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

6. The Cubic Function and Other Polynomials
   f(n) = n^3.
   - The cubic function assigns to an input value n the product of n with itself three times.
   - Polynomials

     A **polynomial function** is a function of the form

     $f(n) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$

     where $a_n, a_{n-1}, \cdots, a_2, a_1, a_0$ are constants, called the **coefficients** of the polynomial, and $a_n \neq 0$. Integer n, which indicates the highest power in the polynomial, is called the **degree** of the polynomial.

7. The Exponential Function
   f(n) = b^n,
   where b is a positive constant, called the base and the argument n is the exponent.
   - That is, function f(n) assigns to the input argument n the value obtained by multiplying the base b by itself n times.
   - In algorithm analysis, the most common base for the exponential function is b = 2.

   - Geometric Sums

For any integer n≥ 0 *and any real number a such that* $a > o$ *and* $a \neq 1$, *consider the summation*

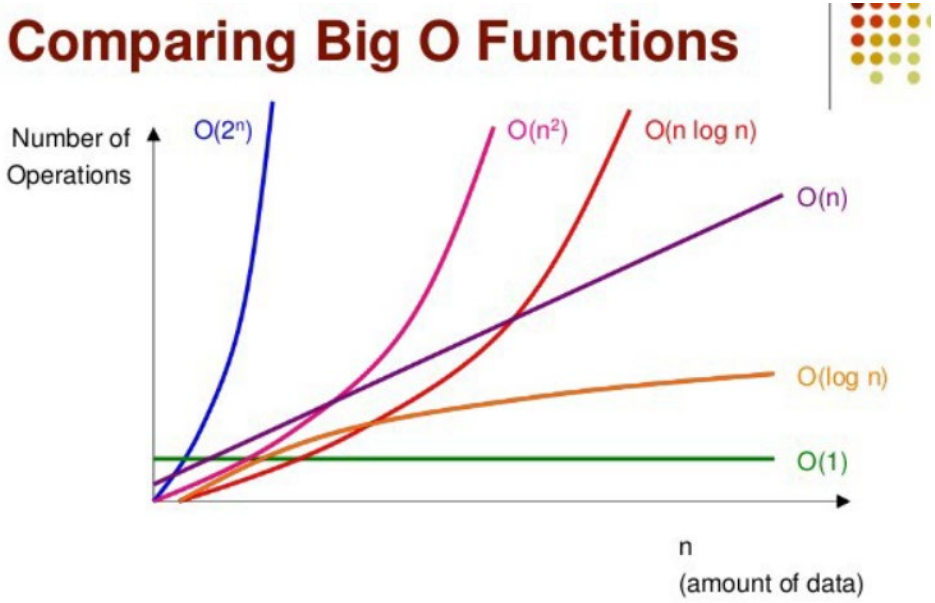
$\sum_{i=0}^{n} a^i = 1 + a + a^2 + a^3 + a^n$

$= \frac{a^{n+1} - 1}{a - 1}$ (if a > 1)

$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

Comparing Growth Rates

Input size (n)	**Constant Function f(n) = 1**	**Logarithmic Function f(n) = log n**	**Linear Function f(n) = n**	**n-log-n function f(n) = n logn**	**Quadratic Function f(n) = n^2**	**Cubic Function f(n) = n^3**	**Exponential Function f(n) = 2^n**
n = 2							
n = 2^2							
n = 2^3							
n = 2^4							
n = 2^{10}							


Comparing Big O Functions
Number of Operations
O(2^n)
O(n^2)
O(n log n)
O(n)
O(log n)
O(1)
n
(amount of data)