Homework 3 Solution

Friday, September 24, 2021 10:47 AM

Recall $\int f(x)\,dx$

$\left\{\begin{array}{l} f(x) \text{ is continuous} \\ dx \text{ is differential} \end{array}\right.$

For example, the substitution method

Let $u = g(x)$. $du = u'\,dx \overset{\text{or}}{=} g'(x)\,dx$

derivative of u wrt x

Verification if your answer is correct:

$$\int f(x)\,d(x) = F(x) + C$$

$$F'(x) \overset{?}{=} f(x)$$

Example. $\int \ln(x)\,dx$

$(x>0)$

diff	int
$\ln(x)$	1
$\frac{1}{x}$	x

$(-)$

$$\Rightarrow \int \ln x\,dx = x\ln x - \int \frac{x}{x}\,dx$$

$$= x\ln x - x + C$$

verification: $(x\ln x - x)'$

$= x' \ln x + x (\ln x)' - 1$

$= x' \ln x + x \cdot (\ln x)' - 1$

$= \ln x + x \cdot \frac{1}{x} - 1 = \ln x$

Xuemao Zhang **Math141_Calculus_II**
Assignment Homework3 due 09/27/2021 at 11:59pm EDT

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1. **(1 point)** `Library/ma123DB/set2/s7_2_17.pg`

Evaluate the indefinite integral.

$$\int \cos^5 x \tan^3 x \, dx$$

Answer: ______________ $+ C$

$$= \int \cos^5 x \cdot \frac{\sin^3 x}{\cos^3 x} \, dx = \int \cos^2 x \sin^3 x \, dx$$

$u = \cos x$

$du = -\sin x dx$

$\Rightarrow \sin x dx = -du$

$$\underline{\underline{\quad}} \; -\int \cos^2 x \sin^2 x \, du$$

$$= -\int u^2 (1 - u^2) \, du$$

$$= -\int (u^2 - u^4) \, du \overset{\text{or}}{=} \int (u^4 - u^2) \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$$

2. **(1 point)** `Library/Union/setIntTrigonometric/an8_3_06.pg`

Evaluate the indefinite integral.

$$\int \tan^9(x)\sec^2(x)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C.$$

Solution: *(Instructor solution preview: show the student solution after due date.)*

$\tan^m x \sec^n x$	None	Even in $\mathbb{Z}_{>0}$	Use: $\sec^n x\;dx = (\tan^2 x+1)^{\frac{n-2}{2}}\;d(\tan x)$

let $u = \tan x$, $du = \underbrace{\sec^2 x}_{u'}\,dx$

$$\Rightarrow \int \tan^9 x \sec^2 x\,dx = \int u^9\,du$$

$$= \frac{u^{10}}{10} + C = \frac{\tan^{10} x}{10} + C$$

3. **(1 point)** `Library/Rochester/setIntegrals5Trig/S07.02.TrigIntegrals.PTP17.pg`

Evaluate the indefinite integral.

$$\int \tan^2 x\,dx$$

Answer: ______________ $+C$

$1+\tan^2 x = \sec^2 x$

$\tan^m x \sec^n x$	in $\mathbb{Z}_{\geq 2}$	$n = 0$	Reduce power using: $\tan^m x = \tan^{m-2}(\sec^2 x - 1)$ $= \tan^{m-2}\;d(\tan x) - \tan^{m-2} x = \cdots$

$$\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx$$

$$= \tan x - x + C$$

4. **(1 point)** `Library/UMN/calculusStewartCCC/s_5_3_48.pg`

Find the general indefinite integral $\int \frac{\sin 2x}{\sin x}\,dx$.

suppose $\sin x \neq 0$

Answer:

4. (1 point) Library/UMN/calculusStewartCCC/s_5_5_48.pg

Find the general indefinite integral $\int \frac{\sin 2x}{\sin x}\,dx$.

suppose $\sin x \neq 0$

Answer: ____________________

Correct Answers:

- 2*sin(x)+C

$$\frac{\sin 2x}{\sin x} = \frac{2\sin x\cos x}{\sin x} = 2\cos x$$

$$\Rightarrow \int \frac{\sin 2x}{\sin x}\,dx = 2\int \cos x\,dx = 2\sin x + C$$

5. (1 point) Library/Union/setIntSubstitution/an6_3_03.pg

Evaluate the indefinite integral.

$\int \sec(10x)\tan(10x)\,dx =$ ______________________ $+C$.

Correct Answers:

$(\sec x)' = \sec x \tan x$

$u = 10x$

$du = 10\,dx \Rightarrow dx = \frac{1}{10}du$

$$\Rightarrow \int \sec(10x)\tan(10x)\,dx = \frac{1}{10}\int \sec u \tan u\,du$$

$$= \frac{1}{10}\sec(10x) + C$$

6. (1 point) Library/UMN/calculusStewartET/s_7_2_2.pg

Evaluate

$$\int \sin^8 x \cos^3 x\,dx.$$

Answer: ____________________

$\sin^m x\cos^n x$	None	n Odd in $\mathbb{Z}_{>0}$	$\sin^m x \underbrace{\cos^{n-1} x}_{(1-\sin^2 x)^{\frac{n-1}{2}}}$	$d(\sin x)$

let $u = \sin x$, $du = \cos x\,dx \Rightarrow \cos x\,dx = du$

$\Rightarrow \int \sin^8 x \cos^2 x \underline{\cos x \, dx}$

$= \int u^8 (1-u^2) \, du = \int (u^8 - u^{10}) \, du$

$= \dfrac{\sin^9 x}{9} - \dfrac{\sin^{11} x}{11} + C$

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7. **(1 point)** `Library/UCSB/Stewart5_7_2/Stewart5_7_2_20.pg`

Evaluate the integral

$$\int 4\cos^2(x)\sin(2x)\,dx$$

Note: Use an upper-case "C" for the constant of integration.

$\Rightarrow 4\int \cos^2 x \sin 2x \, dx = 8\int \cos^3 x \sin x \, dx$

$= 8\int \sin x \cos^3 x \, dx = 8\int \sin x \cos^2 x \cos x \, dx$

$\xrightarrow[du = \cos x dx]{u = \sin x} 8\int u(1-u^2) \, du$

You can try
$u = \cos x$
$du = -\sin x \, dx$

$= 8\int (u - u^3) \, du$

$= 8 \cdot \dfrac{u^2}{2} - \dfrac{8 \cdot u^4}{4} + C$

$= 4\sin^2 x - 2\sin^4 x + C$

8. **(1 point)** `Library/UCSB/Stewart5_7_2/Stewart5_7_2_21.pg`

Evaluate the integral

$$\int -3\sec^2(x)\tan(x)\,dx$$

Note: Use an upper-case "C" for the constant of integration.

$u = \tan x$

$du = \sec^2 x\,dx$ $= -3\int u\,du = -3\cdot\frac{u^2}{2} + C$

$= \frac{-3}{2}\cdot(\tan x)^2 + C$

9. **(1 point)** `Library/UCSB/Stewart5_7_2/Stewart5_7_2_29.pg`

Evaluate the integral

$$\int -3\tan^3(x)\sec(x)\,dx$$

Note: Use an upper-case "C" for the constant of integration.

$u = \sec x,\quad du = \sec x\tan x\,dx$

$\Rightarrow\ -3\int \tan^3 x\sec x\,dx$

$= -3\int \tan^2 x\,\sec x\tan x\,dx$

$= -3\int (u^2 - 1)\,du$

$1 + \tan^2 x = \sec^2 x$

$\Rightarrow \tan^2 x = \sec^2 x - 1$

$= -3\left(\frac{u^3}{3} - u\right) + C$

$= 3u - u^3 + C = 3\sec x \quad \sec^3 \quad C$

$= 3u - u^3 + C = 3\sec x - \sec^3 x + C$

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10. **(1 point)** `Library/UCSB/Stewart5_7_2/Stewart5_7_2_41.pg`

Evaluate the integral

$$\int 8\sin(5x)\sin(2x)\,dx$$

Note: Use an upper-case "C" for the constant of integration.

TR6. $\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$.

$\sin 5x \sin 2x = \frac{1}{2}\left[\cos(3x) - \cos 7x\right]$

$\Rightarrow 8\int \sin 5x \sin 2x\,dx = 4\int (\cos 3x - \cos 7x)\,dx$

$= 4\int \cos 3x\,dx - 4\int \cos 7x\,dx$

$u = 3x$, $du = 3dx$ $\qquad$ $u = 7x$, $du = 7dx$

$= \frac{4}{3}\sin(3x) - \frac{4}{7}\sin(7x) + C$

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11. **(1 point)** `Library/Indiana/Indiana_setIntegrals5Trig/ur_in_5_5.pg`

Evaluate the indefinite integral.

$\int \sin(8x)\cos(9x)\,dx =$ ________________ $+C$

Solution: *(Instructor solution preview: show the student solution after due date.)*

TR10. $\sin x \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$.

$\sin(8x)\cos(9x) = \frac{1}{2}\left[\sin(-x) + \sin(17x)\right]$

$$\sin(8x)\cos(9x) = \frac{1}{2}\left[\sin(-x) + \sin(17x)\right]$$

$$= \frac{1}{2}\left[-\sin x + \sin(17x)\right]$$

$$\Rightarrow \int \sin(8x)\cos(9x)\,dx$$

$$= -\frac{1}{2}\int \sin x\,dx + \frac{1}{2}\int \sin(\underline{17x})\,dx$$

$u = 17x$

$du = 17\,dx$

$$= \frac{1}{2}\cos x - \frac{1}{34}\cos(17x) + C$$

12. **(1 point)** Library/UCSB/Stewart5_7_2/Stewart5_7_2_43.pg

Evaluate the integral

$$\int -3\cos(7x)\cos(5x)\,dx$$

Note: Use an upper-case "C" for the constant of integration.

TR8. $\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$.

$$\cos 7x \cos 5x = \frac{1}{2}\left[\cos 2x + \cos 12x\right]$$

$$= -3 \cdot \frac{1}{2}\int (\cos \underline{2x} + \cos \underline{12x})\,dx$$

$u = 2x$ $u = 12x$

$$= \frac{-3}{2}\left[\frac{1}{2}\sin 2x + \frac{1}{12}\sin(12x)\right] + C$$

15. **(1 point)** `Library/ma123DB/set3/s7_5_48.pg`

Evaluate the indefinite integral.

$$\int x^2 \arctan(4x)\,dx$$

Answer: ______________________________ $+C$

diff		int
$\arctan 4x$		x^2
$\dfrac{1}{1+16x^2}\cdot 4$	$\xrightarrow[(-)]{}$	$\dfrac{x^3}{3}$

$$\Rightarrow \int x^2 \arctan 4x\,dx$$

$$= \frac{1}{3}x^3 \cdot \arctan 4x - \frac{4}{3}\int \frac{x^3}{16x^2+1}\,dx$$

$$\begin{array}{r|l} & \frac{1}{16}x \\ \hline 16x^2+1 & x^3 \\ & x^3 + \frac{1}{16}x \\ \hline & -\frac{1}{16}x \end{array}$$

$$= \frac{1}{3}x^3 \arctan 4x - \frac{4}{3}\int \left[\frac{1}{16}x - \frac{\frac{1}{16}x}{16x^2+1}\right] dx$$

$$= \frac{1}{3}x^3 \arctan 4x - \frac{4}{3}\cdot\frac{1}{16}\cdot\frac{x^2}{2} + \frac{4}{3}\cdot\frac{1}{16}\cdot\int \frac{x}{16x^2+1}\,dx$$

$u = 16x^2$

$du = 32x\,dx \Rightarrow x\,dx = \frac{1}{32}du$

$$= \frac{1}{3}x^3 \arctan 4x - \frac{4}{3} \cdot \frac{1}{16} \cdot \frac{x^2}{2} + \frac{4}{3} \cdot \frac{1}{16} \cdot \frac{1}{32} \int \frac{du}{u}$$

$$= \frac{1}{3}x^3 \arctan 4x - \frac{4}{3} \cdot \frac{1}{16} \cdot \frac{x^2}{2} + \frac{4}{3} \cdot \frac{1}{16} \cdot \frac{1}{32} \ln(16x^2+1) + C$$