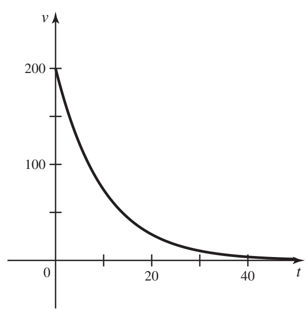
27. a. 31.25 J **b.** 312.5 J **29. a.** 625 J **b.** 391 J **31. a.** 22,050 J **b.** 36,750 J **33.** 3675 J **35.** 1.15×10^7 J **37** 3.94×10^6 J **39. a.** $66{,}150\pi$ J **b.** No **41. a.** 2.10×10^8 J **b.** 3.78×10^8 J **43. a.** 32,667 J **b.** Yes **45.** 7.70×10^3 J **47.** 1.47×10^7 N **49.** 2.94×10^7 N **51.** 6533 N **53.** 6737.5 N **55.** 8×10^5 N **57. a.** True **b.** True **c.** True **d.** False **59. a.** Compared to a linear spring, $F(x) = 16x$, the restoring force is less for large displacements. **b.** 17.87 J **c.** 31.6 J **61.** 1,381,800 J **63.** 0.28 J **65. a.** Yes **b.** 4.296 m **67.** Left: 16,730 N; right: 14,700 N **69. a.** 8.87×10^9 J **b.** $500\,GMx/(R(x+R)) = (2 \times 10^{17})x/(R(x+R))$ J **c.** GMm/R **d.** $v = \sqrt{2GM/R}$

Chapter 6 Review Exercises, pp. 478–482

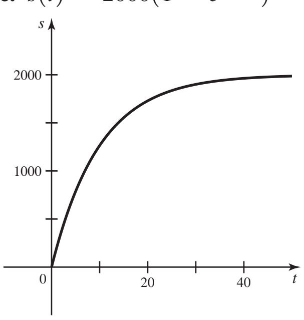
1. a. True **b.** True **c.** True **3. a.** Positive direction for $0 \le t < \frac{1}{2}$ and $2 < t \le 3$; negative direction for $\frac{1}{2} < t < 2$ **b.** 9 m **c.** 22.5 m **d.** $s(t) = 4t^3 - 15t^2 + 12t + 1$ **5.** $s(t) = 20t - 5t^2$; displacement $= 20t - 5t^2$;
$D(t) = \begin{cases} 20t - 5t^2 & \text{if } 0 \le t < 2 \\ 5t^2 - 20t + 40 & \text{if } 2 \le t \le 4 \end{cases}$
7. a. $v(t) = -\dfrac{8}{\pi}\cos\dfrac{\pi t}{4}$; $s(t) = -\dfrac{32}{\pi^2}\sin\dfrac{\pi t}{4}$ **b.** Min value $= -\dfrac{32}{\pi^2}$; max value $= \dfrac{32}{\pi^2}$ **c.** 0; 0 **9. a.** $R(t) = 3t^{4/3}$
b. $R(t) = \begin{cases} 3t^{4/3} & \text{if } 0 \le t \le 8 \\ 2t + 32 & \text{if } t > 8 \end{cases}$ **c.** $t = 59$ min

11. a.

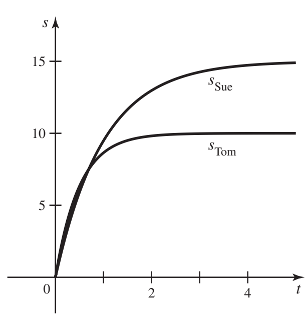


b. $10 \ln 4 \approx 13.86$ s

c. $s(t) = 2000(1 - e^{-t/10})$ **d.** No



13. a. $s_{\text{Tom}}(t) = -10e^{-2t} + 10$
$s_{\text{Sue}}(t) = -15e^{-t} + 15$



b. $t = 0$ and $t = \ln 2$ **c.** Sue **15.** $1 - \dfrac{\pi}{4}$ **17.** $e - 2$ **19.** $\dfrac{7}{3}$

21. 8 **23.** 1 **25.** $\dfrac{1}{3}$ **27.** R_1: $\dfrac{7}{6}$; R_2: $\dfrac{10}{3}$; R_3: $4\sqrt{3} - \dfrac{10}{3}$ **29.** $\dfrac{11\pi}{15}$ **31.** $\dfrac{14\pi}{3}$ **33.** $\displaystyle\int_1^3 2\pi(3 - x)(2\sqrt{x} - 3 + x)\,dx$ **35.** $\dfrac{7}{3}$ **37.** $\dfrac{31\pi}{5}$ **39.** R_1: $\sqrt{3}$; R_2: $\dfrac{4\pi}{3} - \sqrt{3}$ **41.** $\dfrac{1}{3}$ **43.** $\dfrac{5}{6}$ **45.** $\dfrac{8}{15}$ **47.** $\dfrac{8\pi}{5}$ **49.** $\pi(e - 1)^2$ **51.** π **53.** $\dfrac{512\pi}{15}$ **55.** About $y = -2$: 80π; about $x = -2$: 112π **57.** $c = 5$ **59.** 1 **61.** $2\sqrt{3} - \dfrac{4}{3}$
63. $\displaystyle\int_2^4 \sqrt{4x^2 + 8x + 5}\,dx \approx 16.127$
65. $\sqrt{b^2 + 1} - \sqrt{2} + \ln\left(\dfrac{(\sqrt{b^2 + 1} - 1)(1 + \sqrt{2})}{b}\right)$; $b \approx 2.715$
67. a. 9π **b.** $\dfrac{9\pi}{2}$ **69. a.** $\dfrac{263{,}439\pi}{4096}$ **b.** $\dfrac{483}{64}$ **c.** $\dfrac{\pi}{8}(84 + \ln 2)$ **d.** $\dfrac{264{,}341\pi}{18{,}432}$ **71.** $\left(450 - \dfrac{450}{e}\right)$ g **73. a.** 562.5 J **b.** 56.25 J
75. a. 980 J **b.** 627.2 J **77. a.** 1,411,200 J **b.** 940,800 J **79. a.** 1,477,805 J **b.** The work required to pump out the top 3 m of water is 1,015,991 J, and the work required to pump out the bottom 3 m of water is 461,814 J. More work is required to pump out the top 3 m of water. **81.** 4,987,592 J **83.** 5716.7 N **85.** 5.2×10^7 N

CHAPTER 7

Section 7.1 Exercises, pp. 490–492

1. $D = (0, \infty), R = (-\infty, \infty)$ **3.** $\dfrac{4^x}{\ln 4} + C$
5. $e^{x \ln 3}, e^{\pi \ln x}, e^{(\sin x)(\ln x)}$ **7.** $3(\ln x + 1)$ **9.** $\dfrac{\cos(\ln x)}{x}, x > 0$
11. $-\dfrac{5}{x(\ln 2x)^6}$ **13.** $4^{2x+1}x^{4x}(1 + \ln 2x)$ **15.** $(\ln 2)\,2^{x^2+1}x$
17. $2(x+1)^{2x}\left(\dfrac{x}{x+1} + \ln(x+1)\right)$
19. $y^{\sin y}\left(\cos y \ln y + \dfrac{\sin y}{y}\right)$ **21.** $-20xe^{-10x^2}$ **23.** $x^{2x}(2\ln x + 2)$
25. $-(1/x)^x(1 + \ln x)$ **27.** $\left(-\dfrac{4}{x+4} + \ln\left(\dfrac{x+4}{x}\right)\right)\left(1 + \dfrac{4}{x}\right)^x$
29. $6(1 - \ln 2)$ **31.** $\dfrac{3}{8}$ **33.** $\dfrac{1}{2}\ln(4 + e^{2x}) + C$ **35.** $\dfrac{1}{\ln 2} - \dfrac{1}{\ln 3}$
37. $4 - \dfrac{4}{e^2}$ **39.** $2e^{\sqrt{x}} + C$ **41.** $\ln|e^x - e^{-x}| + C$ **43.** $\dfrac{99}{10\ln 10}$
45. 3 **47.** $\dfrac{6^{x^3+8}}{3\ln 6} + C$ **49.** $\dfrac{1}{6}e^{3x^2+1} + C$ **51.** $-\dfrac{1}{9^x \ln 9} + C$
53. $\dfrac{10^{x^3}}{3\ln 10} + C$ **55.** $\dfrac{3 \cdot 3^{\ln 2} - 1}{\ln 3}$ **57.** $\dfrac{32}{3}$ **59.** $\dfrac{1}{3}\ln\dfrac{65}{16}$
61. $2e^{5+\sqrt{x}} + C$ **63.**

h	$(1 + 2h)^{1/h}$	h	$(1 + 2h)^{1/h}$
10^{-1}	6.1917	-10^{-1}	9.3132
10^{-2}	7.2446	-10^{-2}	7.5404
10^{-3}	7.3743	-10^{-3}	7.4039
10^{-4}	7.3876	-10^{-4}	7.3905
10^{-5}	7.3889	-10^{-5}	7.3892
10^{-6}	7.3890	-10^{-6}	7.3891

$\lim_{h \to 0}(1 + 2h)^{1/h} = e^2$

65.

x	$\frac{2^x - 1}{x}$	x	$\frac{2^x - 1}{x}$
10^{-1}	0.71773	-10^{-1}	0.66967
10^{-2}	0.69556	-10^{-2}	0.69075
10^{-3}	0.69339	-10^{-3}	0.69291
10^{-4}	0.69317	-10^{-4}	0.69312
10^{-5}	0.69315	-10^{-5}	0.69314
10^{-6}	0.69315	-10^{-6}	0.69315

$\lim_{x\to 0} \frac{2^x - 1}{x} = \ln 2$

67. a. True **b.** False **c.** False **d.** False **e.** True

69. $\frac{\ln p}{p - 1}, 0$ **71. a.** No **b.** No

75. $\ln 2 = \int_1^2 \frac{dt}{t} < L_2 = \frac{5}{6} < 1$

$\ln 3 = \int_1^3 \frac{dt}{t} > R_7$

$= 2\left(\frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \frac{1}{15} + \frac{1}{17} + \frac{1}{19} + \frac{1}{21}\right) > 1$

Section 7.2 Exercises, pp. 499–501

1. The relative growth is constant. **3.** The time it takes a function to double in value **5.** $T_2 = (\ln 2)/k$ **7.** $\frac{\ln 2}{20} \approx 0.03466$

9. Compound interest, world population **11.** $\ln 1.11 \approx 0.1044$.

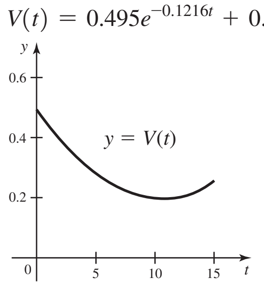
13. $\frac{df}{dt} = 10.5; \frac{dg}{dt} \cdot \frac{1}{g} = \frac{1}{10}$

15. a. $\ln 1.024 \approx 0.02372$; $y(t) = 90{,}000\, e^{t \ln 1.024}$ **b.** 2028

17. a. $\frac{\ln 1.1}{10} \approx 0.009531$; $y(t) = 50{,}000\, e^{t \ln 1.1/10}$ **b.** 60,500

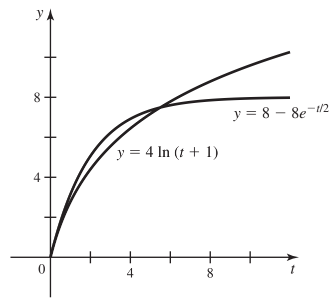
19. a. $\ln 1.016 \approx 0.01587$; $y(t) = 100\, e^{t \ln 1.016}$ **b.** \$126.88

21. 3.71% **23. a.** 88.1 years; 423.4 million
b. 99.4 years; 412.2 million **25.** 28.7 million **27.** 2026
29. $a(t) = 20e^{(t/36)\ln 0.5}$ mg with $t = 0$ at midnight; 15.87 mg; 119.6 hr $\approx$ 5 days **31.** 1.798 million; the downward turn in the population size may be temporary. **33.** 18,928 ft; 125,754 ft
35. 1.055 billion yr **37.** 6.2 hours **39.** 2 dollars **41.** 1044 days
43. a. False **b.** False **c.** True **d.** True **e.** True
45. a. $V_1(t) = 0.495e^{-0.1216t}$ **b.** $V_2(t) = 0.005e^{0.239t}$
c. $V(t) = 0.495e^{-0.1216t} + 0.005e^{0.239t}$
d.



The tumor initially shrinks significantly in size but eventually starts growing again. **e.** 10.9 days; give a second treatment just before the end of the 10th day after the first treatment.

47. a. Bob; Abe **b.** $y = 4 \ln (t + 1)$ and $y = 8 - 8e^{-t/2}$; Bob



49. 10.034%; no **51.** 1.3 s

53. $k = \ln (1 + r)$; $r = 2^{1/T_2} - 1$; $T_2 = (\ln 2)/k$

Section 7.3 Exercises, pp. 513–517

1. $\cosh x = \frac{e^x + e^{-x}}{2}$; $\sinh x = \frac{e^x - e^{-x}}{2}$ **3.** $\cosh^2 x - \sinh^2 x = 1$

5. $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ **7.** Evaluate $\sinh^{-1} \frac{1}{5}$.

9. $\int \frac{dx}{16 - x^2} = \frac{1}{4} \coth^{-1} \frac{x}{4} + C$ when $|x| > 4$; the values in the interval of integration $6 \le x \le 8$ satisfy $|x| > 4$.
23. $2 \cosh x \sinh x$ **25.** $2 \tanh x \operatorname{sech}^2 x$ **27.** $-2 \tanh 2x$
29. $2x(3x \sinh 3x + \cosh 3x)\cosh 3x$ **31.** $4/\sqrt{16x^2 - 1}$
33. $2v/\sqrt{v^4 + 1}$ **35.** $\sinh^{-1} x$ **37.** $(\sinh 2x)/2 + C$
39. $\ln(1 + \cosh x) + C$ **41.** $x - \tanh x + C$
43. $(\cosh^4 3 - 1)/12 \approx 856$ **45.** $\ln(5/4)$

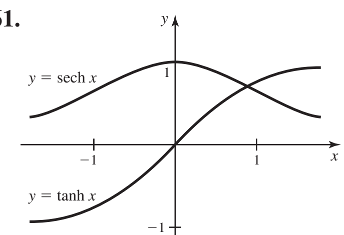
47. $\frac{1}{2\sqrt{2}} \coth^{-1}\left(\frac{x}{2\sqrt{2}}\right) + C$ **49.** $\tanh^{-1}(e^x/6)/6 + C$

51. $-\operatorname{sech}^{-1}(x^4/2)/8 + C$ **53.** $-\operatorname{csch} z + C$

55. $\ln \sqrt{3} \cdot \ln(4/3) \approx 0.158$ **57.** $\frac{x^2 + 1}{2x} + C$

59. a. The values of $y = \coth x$ are close to 1 on $[5, 10]$.
b. $\ln(\sinh 10) - \ln(\sinh 5) \approx 5.0000454$; $|\text{error}| \approx 0.0000454$

61.



a. $x = \sinh^{-1} 1 = \ln(1 + \sqrt{2})$
b. $\pi/4 - \ln \sqrt{2} \approx 0.44$

63. $\sinh^{-1} 2 = \ln(2 + \sqrt{5})$ **65.** $-(\ln 5)/3 \approx -0.54$

67. $3 \ln\left(\frac{\sqrt{5} + 2}{\sqrt{2} + 1}\right) = 3(\sinh^{-1} 2 - \sinh^{-1} 1)$

69. $\frac{1}{15}\left(17 - \frac{8}{\ln(5/3)}\right) \approx 0.09$

71. a. Sag $= f(50) - f(0) = a(\cosh(50/a) - 1) = 10$; now divide by a. **b.** $t \approx 0.08$ **c.** $a = 10/t \approx 125$; $L = 250 \sinh(2/5) \approx 102.7$ ft **73.** $\lambda \approx 32.81$ m
75. b. When $d/\lambda < 0.05$, $2\pi d/\lambda$ is small. Because $\tanh x \approx x$ for small values of x, $\tanh(2\pi d/\lambda) \approx 2\pi d/\lambda$; therefore,

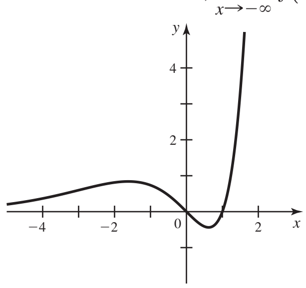
$$v = \sqrt{\frac{g\lambda}{2\pi} \tanh\left(\frac{2\pi d}{\lambda}\right)} \approx \sqrt{\frac{g\lambda}{2\pi} \cdot \frac{2\pi d}{\lambda}} = \sqrt{gd}.$$

c. $v = \sqrt{gd}$ is a function of depth alone; when depth d decreases, v also decreases. **77. a.** False **b.** False **c.** True **d.** False

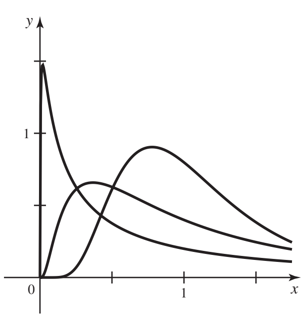
79. a. 1 **b.** 0 **c.** Undefined **d.** 1 **e.** 13/12 **f.** 40/9 **g.** $\left(\frac{e^2+1}{2e}\right)^2$ **h.** Undefined **i.** ln 4 **j.** 1 **81.** $x = 0$
83. $x = \pm\tanh^{-1}(1/\sqrt{3}) = \pm\ln(2+\sqrt{3})/2 \approx \pm 0.658$
85. $\tan^{-1}(\sinh 1) - \pi/4 \approx 0.08$ **87.** Applying l'Hôpital's Rule twice brings you back to the initial limit; $\lim_{x\to\infty} \tanh x = 1$.
89. $2/\pi$ **91.** 1 **93.** $12(3\ln(3+\sqrt{8}) - \sqrt{8}) \approx 29.5$
95. a. Approx. 360.8 m **b.** First 100 m: $t \approx 4.72$ s, $v_{\text{av}} \approx 21.2$ m/s; second 100 m: $t \approx 2.25$ s, $v_{\text{av}} \approx 44.5$ m/s **97. a.** $\sqrt{mg/k}$ **b.** $35\sqrt{3} \approx 60.6$ m/s **c.** $t = \sqrt{\frac{m}{kg}}\tanh^{-1} 0.95 = \frac{\ln 39}{2}\sqrt{\frac{m}{kg}}$
d. Approx. 736.5 m **109.** $\ln(21/4) \approx 1.66$

Chapter 7 Review Exercises, pp. 518–519

1. a. False **b.** False **c.** False **d.** True **3.** ln 4
5. $\frac{1}{2}\ln(x^2 + 8x + 25) + C$
7. $\cosh^{-1}(x/3) + C = \ln(x + \sqrt{x^2 - 9}) + C$
9. $\tanh^{-1}(1/3)/9 = (\ln 2)/18 \approx 0.0385$
11. $x^{3x^2+1}\left(6x\ln x + 3x + \frac{1}{x}\right)$ **13.** $\sinh^2 t + \cosh^2 t$
15. $3\sinh(6x - 2)$ **17.** $-\csc x$ **19.** $\frac{2x}{\sqrt{x^4 - 1}}$
21. Approx. 7.3 hours **23. a.** $y(t) = 29{,}000e^{(t \ln 2)/2}$
b. Approx. 41,996,486 transistors (which closely approximates the actual number of transistors) **25.** 48.37 yr
27. Local max at $x = -\frac{1}{2}(\sqrt{5}+1)$; local min at $x = \frac{1}{2}(\sqrt{5}-1)$; inflection points at $x = -3$ and $x = 0$; $\lim_{x\to-\infty} f(x) = 0$; $\lim_{x\to\infty} f(x) = \infty$



29. a.



b. $\lim_{x\to 0} f(x) = 0$
d. $f(x^*) = \frac{1}{\sqrt{2\pi}}\frac{e^{\sigma^2/2}}{\sigma}$
e. $\sigma = 1$

31. $L(x) = \frac{5}{3} + \frac{4}{3}(x - \ln 3)$; $\cosh 1 \approx 1.535$
33. a. $\cosh x$ **b.** $(1 - x\tanh x)\operatorname{sech} x$

CHAPTER 8

Section 8.1 Exercises, pp. 523–525

1. $u = 4 - 7x$ **3.** $\sin^2 x = \frac{1 - \cos 2x}{2}$ **5.** Complete the square in $x^2 - 4x - 9$. **7.** $\frac{1}{15(3-5x)^3} + C$ **9.** $\frac{\sqrt{2}}{4}$ **11.** $\frac{1}{2}\ln^2 2x + C$
13. $\ln(e^x + 1) + C$ **15.** $\frac{32}{3}$ **17.** $\frac{21}{110}$
19. $\frac{(\ln w - 1)^9}{9} + \frac{(\ln w - 1)^8}{8} + C$
21. $\frac{1}{2}\ln(x^2+4) + \tan^{-1}\frac{x}{2} + C$
23. $-\frac{1}{3}\ln|\csc(3e^x + 4) + \cot(3e^x + 4)| + C$ **25.** 1
27. $3\sqrt{1 - x^2} + 2\sin^{-1} x + C$ **29.** $\ln(\sqrt{2} + 1)$
31. $\frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right) + C$ **33.** $\frac{x^2}{2} + x + \ln(x^2 + x + 2) + C$
35. $\frac{3\pi + 10}{12}$ **37.** $\sin^{-1}\left(\frac{\theta+3}{6}\right) + C$ **39.** $\tan\theta - \sec\theta + C$
41. $-x - \cot x - \csc x + C$ **43.** $\frac{1}{3}\ln(1 + \sinh 3x) + C$
45. $\frac{1}{2}\ln|e^{2x} - 2| + C$ **47.** $x - \ln|x+1| + C$
49. $\frac{4}{5}(9 + \sqrt{t+1})^{3/2}(\sqrt{t+1} - 6) + C$ **51.** $\frac{\ln 4 - \pi}{4}$
53. $\ln|\sec(e^x + 1) + \tan(e^x + 1)| + C$
55. $\frac{2\sin^3 x}{3} + C$ **57.** $2\tan^{-1}\sqrt{x} + C$
59. $\frac{1}{2}\ln(x^2 + 6x + 13) - \frac{5}{2}\tan^{-1}\left(\frac{x+3}{2}\right) + C$
61. $-\frac{1}{e^x + 1} + C$ **63.** $\frac{1}{2}$ **65. a.** False **b.** False **c.** False **d.** False **69. a.** $\frac{\tan^2 x}{2} + C$ **b.** $\frac{\sec^2 x}{2} + C$ **c.** The antiderivatives differ by a constant. **71. a.** $\frac{1}{2}(x+1)^2 - 2(x+1) + \ln|x+1| + C$
b. $\frac{x^2}{2} - x + \ln|x+1| + C$ **c.** The antiderivatives differ by a constant. **73.** $\frac{\ln 26}{3}$ **75.** $\frac{2}{3}(5\sqrt{5} - 1)\pi$
77. $\pi\left(\frac{9}{2} - \frac{5\sqrt{5}}{6}\right)$ **79.** $\frac{2048 + 1763\sqrt{41}}{9375}$

Section 8.2 Exercises, pp. 529–532

1. Product Rule **3.** $\frac{x^2(2\ln x - 1)}{4} + C$ **5.** Products for which the choice for dv is easily integrated and when the resulting new integral is no more difficult than the original integral
7. $(\tan x + 2)\ln(\tan x + 2) - \tan x + C$
9. $\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C$ **11.** $\frac{e^{6t}}{36}(6t - 1) + C$
13. $\frac{x^2}{4}(2\ln 10x - 1) + C$ **15.** $(w+2)\sin 2w + \frac{1}{2}\cos 2w + C$
17. $\frac{3^x}{\ln 3}\left(x - \frac{1}{\ln 3}\right) + C$ **19.** $-\frac{1}{9x^9}\left(\ln x + \frac{1}{9}\right) + C$
21. $\frac{1}{8}\sin 2x - \frac{x}{4}\cos 2x + C$ **23.** $\frac{1}{4}(1 - 2x^2)\cos 2x + \frac{x}{2}\sin 2x + C$
25. $-e^{-t}(t^2 + 2t + 2) + C$ **27.** $\frac{e^x}{2}(\sin x + \cos x) + C$
29. $-\frac{e^{-x}}{17}(\sin 4x + 4\cos 4x) + C$
31. $-e^{2x}\cos e^x + 2e^x\sin e^x + 2\cos e^x + C$ **33.** π **35.** $-\frac{1}{2}$
37. $\frac{1}{9}(5e^6 + 1)$ **39.** $\frac{\pi - 2}{2}$ **41. a.** $x\tan^{-1} x - \frac{1}{2}\ln(1 + x^2) + C$