

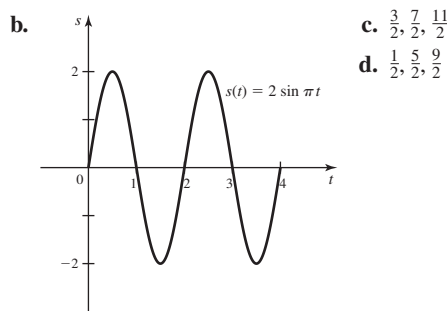
CHAPTER 6

Section 6.1 Exercises, pp. 410–416

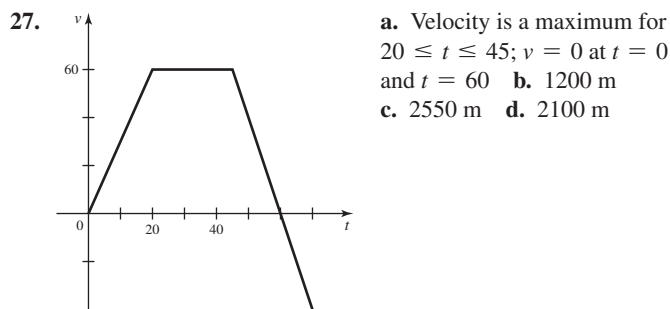
1. The position $s(t)$ is the location of the object relative to the origin. The displacement is the change in position between time $t = a$ and $t = b$. The distance traveled between $t = a$ and $t = b$ is $\int_a^b |v(t)| dt$, where $v(t)$ is the velocity at time t . 3. The displacement between $t = a$ and $t = b$ is $\int_a^b v(t) dt$. 5. $Q(t) = Q(0) + \int_0^t Q'(x) dx$
7. a. $[0, 1)$, $(3, 5)$ b. -4 mi c. 26 mi d. 6 mi e. 6 mi on the positive side of the initial position 9. a. 3 b. $\frac{13}{3}$ c. 3

$$d. s(t) = \begin{cases} -\frac{t^2}{2} + 2t & \text{if } 0 \leq t \leq 3 \\ \frac{3t^2}{2} - 10t + 18 & \text{if } 3 < t \leq 4 \\ -t^2 + 10t - 22 & \text{if } 4 < t \leq 5 \end{cases}$$

11. a. 3 m b. 3 m; 0 m; 3 m; 0 m c. 12 m
13. a. Positive direction for $2 < t \leq 3$; negative direction for $0 < t < 2$ b. 0 m c. 8 m 15. a. Positive direction for $0 \leq t < 2$ and $4 < t \leq 5$; negative direction for $2 < t < 4$
- b. 20 m c. 28 m 17. a. $s(t) = 2 - \cos t$ 19. a. $s(t) = 6t - t^2$
21. a. $s(t) = 9t - \frac{t^3}{3} - 2$ 23. a. $s(t) = 2 \sin \pi t$



25. a. $s(t) = 10t(48 - t^2)$ b. 880 mi c. $\frac{2720\sqrt{6}}{9} \approx 740.29$ mi



29. $v(t) = -32t + 70$; $s(t) = -16t^2 + 70t + 10$
31. $v(t) = -9.8t + 20$; $s(t) = -4.9t^2 + 20t$
33. $v(t) = -\frac{1}{200}t^2 + 10$; $s(t) = -\frac{1}{600}t^3 + 10t$
35. $v(t) = \frac{1}{2} \sin 2t + 5$; $s(t) = -\frac{1}{4} \cos 2t + 5t + \frac{29}{4}$
37. a. $s(t) = 44t^2$ b. 704 ft c. $\sqrt{30} \approx 5.477$ s

d. $\frac{5\sqrt{33}}{11} \approx 2.611$ s e. Approx. 180.023 ft

39. 6.154 mi; 1.465 mi 41. a. 2639 people
b. $P(t) = 250 + 20t^{3/2} + 30t$ people 43. a. 1897 cells;
1900 cells b. $N(t) = 1900 - 400e^{-0.25t}$ 45. a. 27,250 barrels

- b. 31,000 barrels c. 4000 barrels 47. a. $\frac{10^7(1 - e^{-kt})}{k}$

b. $\frac{10^7}{k}$ = total number of barrels of oil extracted if the nation extracts the oil indefinitely, and it has at least $\frac{10^7}{k}$ barrels of oil in reserve.

c. $k = \frac{1}{200} = 0.005$ d. Approx. 138.6 yr

49. a. $\frac{120}{\pi} + 40 \approx 78.20$ m³

b. $Q(t) = 20 \left(t + \frac{12}{\pi} \sin \left(\frac{\pi}{12} t \right) \right)$ c. Approx. 122.6 hr

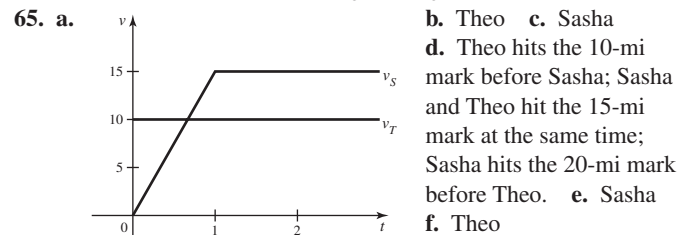
51. a. $V(t) = 5 + \cos \frac{\pi t}{2}$ b. 15 breaths/min c. 2 L, 6 L

53. a. 7200 MWh or 2.592×10^{13} J b. 16,000 kg; 5,840,000 kg

c. 450 g; 164,250 g d. About 1500 turbines 55. a. \$96,875

b. \$86,875 57. a. \$69,583.33 b. \$139,583.33 59. a. False

b. True c. True d. True 61. $\frac{2}{3}$ 63. $\frac{25}{3}$

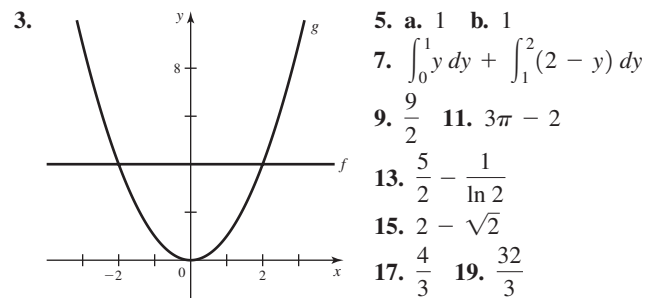


67. Approx. 11:23 A.M.

69. $\int_a^b f'(x) dx = f(b) - f(a) = g(b) - g(a) = \int_a^b g'(x) dx$

Section 6.2 Exercises, pp. 420–425

1. $\int_a^b (f(x) - g(x)) dx + \int_b^c (g(x) - f(x)) dx$



21. 9 23. $\frac{81}{2}$ 25. 2 27. $\frac{8\sqrt{2} - 7}{6}$ 29. $\frac{125}{2}$

31. a. $\int_0^1 (\sqrt{x} - x^3) dx$ b. $\int_0^1 (\sqrt[3]{y} - y^2) dy$

33. $\frac{19}{6} \approx 3.17$ km; the faster runner jogged approximately 3.17 km farther than the slower runner. 35. a. 7 b. 4 37. 25 39. $\frac{81}{32}$

41. $\pi - 2$ 43. $\frac{1}{2} + \ln 2$ 45. $\frac{7}{3}$ 47. 3 49. $\frac{64}{5}$ 51. $\ln 2$

53. $\frac{5}{24}$ 55. $\frac{63}{4}$ 57. $\frac{9}{2}$ 59. $\frac{32}{3}$ 61. $\frac{15}{8} - 2 \ln 2$ 63. $\frac{17}{3}$

65. a. False b. False c. True 67. $\frac{4}{9}$ 69. $\frac{n-1}{2(n+1)}$

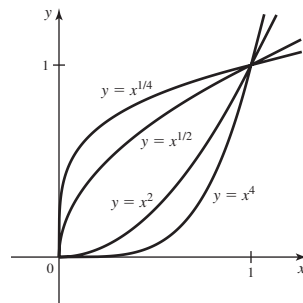
71. $A_n = \frac{n-1}{n+1}$; $\lim_{n \rightarrow \infty} A_n = 1$; the region approximates a square

with side length of 1. 73. $k = 1 - \frac{1}{\sqrt{2}}$ 75. a. The lowest $p\%$ of households own exactly $p\%$ of the wealth for $0 \leq p \leq 100$.

b. The function must be one-to-one and its graph must lie below $y = x$ because the poorest $p\%$ cannot own more than $p\%$ of the wealth. c. $p = 1.1$ is most equitable; $p = 4$ is least equitable.

e. $G(p) = \frac{p-1}{p+1}$ f. $0 \leq G \leq 1$ for $p \geq 1$ g. $\frac{5}{18}$

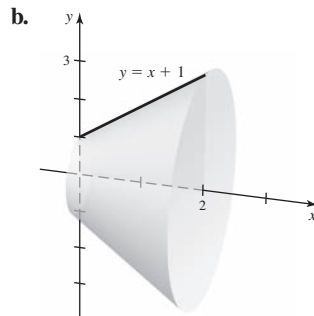
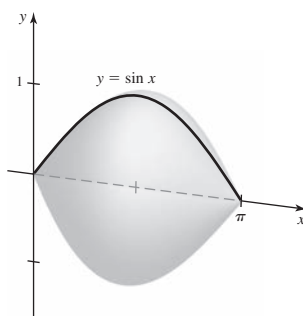
77. a.



b. $A_n(x)$ is the net area of the region between the graphs of f and g from 0 to x . c. $x = n^{n/(n^2-1)}$; the root decreases with n .

Section 6.3 Exercises, pp. 434–439

1. $A(x)$ is the area of the cross section through the solid at the point x .
 3. a. $3 - x$ b. $\int_0^2 (3 - x) dx$ 5. a. $\sqrt{\cos x}$ b. $\pi \cos x$
 c. $\int_0^{\pi/2} \pi \cos x dx$ 7. a. $\sqrt{x} + 1$ b. 1 c. $\pi((\sqrt{x} + 1)^2 - 1)$
 d. $\int_0^4 \pi((\sqrt{x} + 1)^2 - 1) dx$ 9. a. \sqrt{x} b. πx c. $\int_0^4 \pi x dx$
 11. $\frac{4}{3}$ 13. 1 15. $\frac{\pi}{3}$ 17. 36π 19. $\frac{15\pi}{32}$ 21. $\frac{\pi^2}{2}$ 23. $\frac{32\pi}{3}$
 25. $\frac{5\pi}{6}$ 27. $\frac{2\pi}{5}$ 29. $\frac{\pi^2}{4}$ 31. $\frac{\pi^2}{2}$ 33. $\frac{\pi(\pi - 2)}{8}$
 35. $\frac{4\pi - \pi^2}{4}$ 37. $\frac{128\pi}{5}$ 39. $\pi \ln 3$ 41. $\frac{\pi}{2}(e^4 - 1)$
 43. $\frac{49\pi}{2}$ 45. Volumes are equal. 47. x -axis 49. $\frac{\pi}{2}$
 51. $\pi\sqrt{3}$ 53. $\frac{\pi}{6}$ 55. $2\pi(8 + \pi)$ 57. $(6\sqrt{3} - 2\pi)\pi$
 59. 4π 61. a. False b. True c. True
 63. Volume (S) = $8\pi a^{5/2}/15$; volume (T) = $\pi a^{5/2}/3$
 65. a.



67. Left: 166π ; right: 309π ; midpoint: 219π 69. a. $\frac{1}{3}V_C$ b. $\frac{2}{3}V_C$
 71. $24\pi^2$ 73. b. $V = \pi r^2 h$

Section 6.4 Exercises, pp. 447–451

1. $\int_a^b 2\pi x(f(x) - g(x)) dx$ 3. $x; y$ 5. a. x b. $2 - x^2 - x$
 c. $\int_0^1 2\pi x(2 - x^2 - x) dx$ 7. a. $2 - y$ b. $4 - (2 - y)^2 = 4y - y^2$
 c. $\int_0^2 2\pi(2 - y)(4y - y^2) dy$ 9. $\frac{\pi}{6}$ 11. π 13. 8π 15. $\frac{32\pi}{3}$
 17. π 19. $\frac{\pi}{2}$ 21. $\frac{81\pi}{2}$ 23. $\frac{2\pi}{3}$ 25. $\frac{3\pi}{10}$ 27. 90π

29. $2\pi e(e - 1)$ 31. π 33. $\frac{\pi}{5}$ 35. $\frac{4\pi}{15}$ 37. 500π
 39. $\frac{11\pi}{6}$ 41. $\frac{5\pi}{6}$ 43. $\frac{23\pi}{15}$ 45. $\frac{52\pi}{15}$ 47. $\frac{36\pi}{5}$
 51. a. $4\pi \int_1^5 x\sqrt{4 - (x - 3)^2} dx$ b. $12\pi \int_{-2}^2 \sqrt{4 - y^2} dy$
 c. $24\pi^2$ 53. $\frac{\pi}{9}$ 55. $\frac{16\pi}{3}$ 57. $\frac{608\pi}{3}$ 59. $\pi(\sqrt{e} - 1)^2$
 61. $\frac{5\pi}{6}$ 63. a. True b. False c. True 65. 24π 67. 54π
 69. a. $V_1 = \frac{\pi}{15}(3a^2 + 10a + 15)$; $V_2 = \frac{\pi}{2}(a + 2)$
 b. $V(S_1) = V(S_2)$, for $a = 0$ and $a = -\frac{5}{6}$ 71. 10π
 73. a. $27\sqrt{3}\pi r^3/8$ b. $54\sqrt{2}/(3 + \sqrt{2})^3$ c. $500\pi/3$

Section 6.5 Exercises, pp. 455–457

1. Determine whether f has a continuous derivative on $[a, b]$.
 If so, calculate $f'(x)$ and evaluate the integral $\int_a^b \sqrt{1 + f'(x)^2} dx$.
 3. $\int_{-2}^5 \sqrt{1 + 9x^4} dx$ 5. $\int_0^2 \sqrt{1 + 4e^{-4x}} dx$ 7. $4\sqrt{5}$
 9. $8\sqrt{65}$ 11. 168 13. $\frac{4}{3}$ 15. $\frac{123}{32}$ 17. $\frac{123}{32}$ 19. $7\sqrt{5}$
 21. a. $\int_{-1}^1 \sqrt{1 + 4x^2} dx$ b. 2.96 23. a. $\int_1^4 \sqrt{1 + \frac{1}{x^2}} dx$ b. 3.34
 25. a. $\int_3^4 \sqrt{\frac{4y - 7}{4y - 8}} dy$ b. 1.08 27. a. $\int_0^\pi \sqrt{1 + 4\sin^2 2x} dx$
 b. 5.27 29. a. $\int_1^{10} \sqrt{1 + 1/x^4} dx$ b. 9.15
 31. Approx. 1326 m 33. a. False b. True c. False
 35. a. $f(x) = \pm 4x^3/3 + C$ b. $f(x) = \pm 3 \sin 2x + C$
 37. $y = 1 - x^2$ 39. a. $L/2$ b. L/c

Section 6.6 Exercises, pp. 463–465

1. 15π 3. Evaluate $\int_a^b 2\pi f(x)\sqrt{1 + f'(x)^2} dx$ 5. a. $4\sqrt{2}\pi$
 7. $156\sqrt{10}\pi$ 9. $\frac{2912\pi}{3}$ 11. $\frac{\pi}{9}(17^{3/2} - 1)$ 13. 2π
 15. $15\sqrt{17}\pi$ 17. $\frac{\pi}{8}(16 + e^8 - e^{-8})$ 19. 96π
 21. $\frac{9\pi}{125} m^3$ 23. a. False b. False c. True d. False
 25. a. $\int_0^{\pi/2} 2\pi(\cos x)\sqrt{1 + \sin^2 x} dx$ b. Approx. 7.21
 27. a. $\int_0^{\pi/4} 2\pi(\tan x)\sqrt{1 + \sec^4 x} dx$ b. Approx. 3.84
 29. $\frac{12\pi a^2}{5}$ 31. $\frac{53\pi}{9}$ 33. $\frac{275\pi}{32}$ 35. $\frac{48,143\pi}{48}$ 39. a. $\frac{6}{a}$ b. $\frac{3}{a}$
 c. $\frac{3}{2a} + \frac{3a}{2\sqrt{a^2 - 1}} \sin^{-1}\left(\frac{\sqrt{a^2 - 1}}{a}\right)$ d. The sphere e. A sphere
 41. a. $c^2 A$ b. A

Section 6.7 Exercises, pp. 473–477

1. 150 g 3. 25 J 5. Horizontal cross sections of water at various locations in the tank are lifted different distances. 7. $39,200 \text{ N/m}^2$
 9. $\int_5^{10} 25\pi \rho g(15 - y) dy$ 11. $\int_0^{10} 25\pi \rho g(10 - y) dy$ 13. $\pi + 2$
 15. 3 17. $(2\sqrt{2} - 1)/3$ 19. 10 21. 9 J 23. a. $k = 150$
 b. 12 J c. 6.75 J d. 9 J 25. a. 112.5 J b. 12.5 J

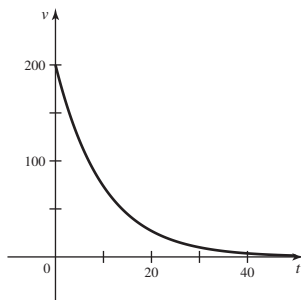
27. a. 31.25 J b. 312.5 J 29. a. 625 J b. 391 J
 31. a. 22,050 J b. 36,750 J 33. 3675 J 35. 1.15×10^7 J
 37. 3.94×10^6 J 39. a. 66,150 J b. No 41. a. 2.10×10^8 J
 b. 3.78×10^8 J 43. a. 32,667 J b. Yes 45. 7.70×10^3 J
 47. 1.47×10^7 N 49. 2.94×10^7 N 51. 6533 N 53. 6737.5 N
 55. 8×10^5 N 57. a. True b. True c. True d. False
 59. a. Compared to a linear spring, $F(x) = 16x$, the restoring force is less for large displacements. b. 17.87 J c. 31.6 J 61. 1,381,800 J
 63. 0.28 J 65. a. Yes b. 4.296 m 67. Left: 16,730 N; right: 14,700 N 69. a. 8.87×10^9 J
 b. $500 GMx/(R(x+R)) = (2 \times 10^{17})x/(R(x+R))$ J
 c. GmM/R d. $v = \sqrt{2GM/R}$

Chapter 6 Review Exercises, pp. 478–482

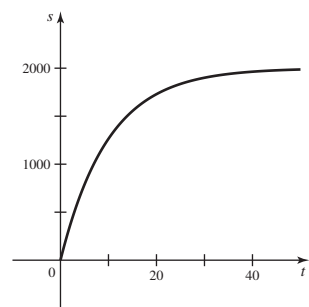
1. a. True b. True c. True 3. a. Positive direction for $0 \leq t < \frac{1}{2}$ and $2 < t \leq 3$; negative direction for $\frac{1}{2} < t < 2$
 b. 9 m c. 22.5 m d. $s(t) = 4t^3 - 15t^2 + 12t + 1$
 5. $s(t) = 20t - 5t^2$; displacement = $20t - 5t^2$;
 $D(t) = \begin{cases} 20t - 5t^2 & \text{if } 0 \leq t < 2 \\ 5t^2 - 20t + 40 & \text{if } 2 \leq t \leq 4 \end{cases}$
 7. a. $v(t) = -\frac{8}{\pi} \cos \frac{\pi t}{4}$; $s(t) = -\frac{32}{\pi^2} \sin \frac{\pi t}{4}$ b. Min value = $-\frac{32}{\pi^2}$;
 max value = $\frac{32}{\pi^2}$ c. 0; 0 9. a. $R(t) = 3t^{4/3}$

b. $R(t) = \begin{cases} 3t^{4/3} & \text{if } 0 \leq t \leq 8 \\ 2t + 32 & \text{if } t > 8 \end{cases}$ c. $t = 59$ min

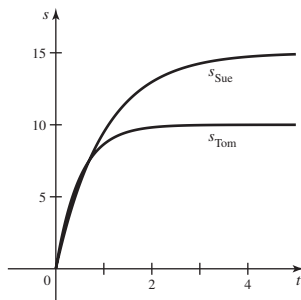
11. a. b. $10 \ln 4 \approx 13.86$ s



- c. $s(t) = 2000(1 - e^{-t/10})$ d. No



13. a. $s_{\text{Tom}}(t) = -10e^{-2t} + 10$
 $s_{\text{Sue}}(t) = -15e^{-t} + 15$



- b. $t = 0$ and $t = \ln 2$ c. Sue 15. $1 - \frac{\pi}{4}$ 17. $e - 2$ 19. $\frac{7}{3}$

21. 8 23. 1 25. $\frac{1}{3}$ 27. $R_1: \frac{7}{6}; R_2: \frac{10}{3}; R_3: 4\sqrt{3} - \frac{10}{3}$ 29. $\frac{11\pi}{15}$
 31. $\frac{14\pi}{3}$ 33. $\int_1^3 2\pi(3-x)(2\sqrt{x}-3+x) dx$ 35. $\frac{7}{3}$ 37. $\frac{31\pi}{5}$
 39. $R_1: \sqrt{3}; R_2: \frac{4\pi}{3} - \sqrt{3}$ 41. $\frac{1}{3}$ 43. $\frac{5}{6}$ 45. $\frac{8}{15}$ 47. $\frac{8\pi}{5}$
 49. $\pi(e-1)^2$ 51. π 53. $\frac{512\pi}{15}$ 55. About $y = -2$; 80π ;
 about $x = -2$; 112π 57. $c = 5$ 59. 1 61. $2\sqrt{3} - \frac{4}{3}$

63. $\int_2^4 \sqrt{4x^2 + 8x + 5} dx \approx 16.127$

65. $\sqrt{b^2 + 1} - \sqrt{2} + \ln \left(\frac{(\sqrt{b^2 + 1} - 1)(1 + \sqrt{2})}{b} \right)$; $b \approx 2.715$

67. a. 9π b. $\frac{9\pi}{2}$ 69. a. $\frac{263,439\pi}{4096}$ b. $\frac{483}{64}$ c. $\frac{\pi}{8}(84 + \ln 2)$

d. $\frac{264,341\pi}{18,432}$ 71. $\left(450 - \frac{450}{e}\right)g$ 73. a. 562.5 J b. 56.25 J

75. a. 980 J b. 627.2 J 77. a. 1,411,200 J b. 940,800 J
 79. a. 1,477,805 J b. The work required to pump out the top 3 m of water is 1,015,991 J, and the work required to pump out the bottom 3 m of water is 461,814 J. More work is required to pump out the top 3 m of water. 81. 4,987,592 J 83. 5716.7 N 85. 5.2×10^7 N

CHAPTER 7

Section 7.1 Exercises, pp. 490–492

1. $D = (0, \infty)$, $R = (-\infty, \infty)$ 3. $\frac{4^x}{\ln 4} + C$
 5. $e^{x \ln 3}$, $e^{\pi \ln x}$, $e^{(\sin x)(\ln x)}$ 7. $3(\ln x + 1)$ 9. $\frac{\cos(\ln x)}{x}$, $x > 0$
 11. $-\frac{5}{x(\ln 2x)^6}$ 13. $4^{2x+1}x^{4x}(1 + \ln 2x)$ 15. $(\ln 2)2^{x^2+1}x$
 17. $2(x+1)^{2x} \left(\frac{x}{x+1} + \ln(x+1) \right)$
 19. $y^{\sin y} \left(\cos y \ln y + \frac{\sin y}{y} \right)$ 21. $-20xe^{-10x^2}$ 23. $x^{2x}(2 \ln x + 2)$
 25. $-(1/x)^x(1 + \ln x)$ 27. $\left(-\frac{4}{x+4} + \ln \left(\frac{x+4}{x} \right) \right) \left(1 + \frac{4}{x} \right)^x$
 29. $6(1 - \ln 2)$ 31. $\frac{3}{8}$ 33. $\frac{1}{2} \ln(4 + e^{2x}) + C$ 35. $\frac{1}{\ln 2} - \frac{1}{\ln 3}$
 37. $4 - \frac{4}{e^2}$ 39. $2e^{\sqrt{x}} + C$ 41. $\ln|e^x - e^{-x}| + C$ 43. $\frac{99}{10 \ln 10}$
 45. 3 47. $\frac{6^{x^3+8}}{3 \ln 6} + C$ 49. $\frac{1}{6}e^{3x^2+1} + C$ 51. $-\frac{1}{9^x \ln 9} + C$
 53. $\frac{10^{x^3}}{3 \ln 10} + C$ 55. $\frac{3 \cdot 3^{\ln 2} - 1}{\ln 3}$ 57. $\frac{32}{3}$ 59. $\frac{1}{3} \ln \frac{65}{16}$

61. $2e^{5+\sqrt{x}} + C$ 63.

h	$(1+2h)^{1/h}$	h	$(1+2h)^{1/h}$
10^{-1}	6.1917	-10^{-1}	9.3132
10^{-2}	7.2446	-10^{-2}	7.5404
10^{-3}	7.3743	-10^{-3}	7.4039
10^{-4}	7.3876	-10^{-4}	7.3905
10^{-5}	7.3889	-10^{-5}	7.3892
10^{-6}	7.3890	-10^{-6}	7.3891

$\lim_{h \rightarrow 0} (1+2h)^{1/h} = e^2$