Department of Mathematics East Stroudsburg University

Math 141: Calculus II

Name Solution Spring 2018

Gateway # 1

You should carry out all integration steps but do not need to simplify your answers. There is no partial credit on this test. No calculators permitted. If you use scratch paper, please turn it in with your test.

Please evaluate the following integrals.

1.

$$\int 3x^5 - x^2 + \sqrt{3} - 4x^{-3}dx$$

$$= 3\frac{x^6}{6} - \frac{x^3}{3} + \sqrt{3}x - 4\frac{x^{-3+1}}{(-3+1)} + C$$

$$= \frac{x^6}{2} - \frac{x^3}{3} + \sqrt{3}x + 2x^{-2} + C$$

2.

$$\int (x^5)\left(\sqrt[3]{x^6+1}\right) dx = \int x^5 (x^6+1)^{\frac{1}{3}} dx$$

$u = x^6+1$

$du = 6x^5 dx$

$$\frac{1}{6}\int u^{\frac{1}{3}} du = \frac{1}{6}\frac{u^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C$$

$$= \frac{1}{8}u^{\frac{4}{3}} + C = \frac{1}{8}(x^6+1)^{\frac{4}{3}} + C$$

3.

$$\int xe^x dx$$

| diff | | Int |
|---|---|---|
| $x$ | | $e^x$ |
| $1$ | $\xrightarrow{(-)}$ | $e^x$ |

$$= xe^x - \int e^x dx$$

$$= xe^x - e^x + C$$

4.
$$\int \frac{5x-2}{(x+4)(3x+1)}\,dx$$

$$\frac{5x-2}{(x+4)(3x+1)} = \frac{A}{x+4} + \frac{B}{3x+1} = \frac{A(3x+1)+B(x+4)}{(x+4)(3x+1)}$$

$$\Rightarrow 5x-2 = A(3x+1)+B(x+4)$$
$$= (3A+B)x + A + 4B \Rightarrow \begin{cases} 3A+B=5 \\ A+4B=-2 \end{cases}$$

$$\Rightarrow A = 22/11 = 2 \text{ and } B = 5-3A = -1$$

$$\Rightarrow \int \frac{5x-2}{(x+4)(3x+1)}dx = \int \frac{2}{x+4}dx - \int \frac{dx}{3x+1}$$

$u=3x+1$, $du=3dx$

$$= 2\int \frac{d(x+4)}{x+4} - \frac{1}{3}\int \frac{du}{u} = 2\ln|x+4| - \frac{1}{3}\ln|3x+1| + C$$

5.
$$\int \frac{1}{\sqrt{49-x^2}}\,dx$$

$$= \frac{1}{7}\int \frac{dx}{\sqrt{1-(\frac{x}{7})^2}}$$ let $u = \frac{x}{7}$. Then $du = \frac{1}{7}dx$.

$$= \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C$$

$$= \sin^{-1}\left(\frac{x}{7}\right) + C$$

6.

$$\int \frac{6x^2 + 2x + 8}{(x^2+2)(x+1)}dx \qquad \frac{6x^2+2x+8}{(x^2+2)(x+1)} = \frac{Ax+B}{x^2+2} + \frac{C}{x+1}$$

$$6x^2+2x+8 = (Ax+B)(x+1) + C(x^2+2) \qquad x=-1$$

$$= (A+C)x^2 + (A+B)x + B+2C \qquad \Rightarrow C=4$$

$$\Rightarrow \begin{cases} A+C=6 \\ A+B=2 \\ B+2C=8 \end{cases} \Rightarrow \begin{cases} A=2 \\ B=0 \\ C=4 \end{cases}$$

$$\Rightarrow \int \frac{6x^2+2x+8}{(x^2+2)(x+1)} = \int \frac{2x\,dx}{x^2+2} + 4\int \frac{1}{x+1}dx$$

$$\underset{du=2x\,dx}{\underline{\underline{u=x^2+2}}} \int \frac{du}{u} + 4\ln|x+1|$$

$$= \ln(x^2+2) + 4\ln|x+1| + C$$

7.

$$\int_0^1 x^2 + 7x - 3\,dx$$

$$= \left(\frac{x^3}{3} + \frac{7x^2}{2} - 3x\right)\Big|_0^1$$

$$= \frac{1}{3} + \frac{7}{2} - 3 - 0 = \frac{2+21-18}{6} = \frac{5}{6}$$

8.

$$\int x^4 \sin x \, dx$$

| diff | int |
|---|---|
| $x^4$ | $\sin x$ |
| $4x^3$ | $-\cos x$ |
| $12x^2$ | $-\sin x$ |
| $24x$ | $\cos x$ |
| $24$ | $\sin x$ |

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and $\int \sin x \, dx = -\cos x + C$

$$= -x^4\cos x + 4x^3\sin x + 12x^2\cos x - 24x\sin x - 24\cos x + C$$

9.

$$\int_1^{\infty} \frac{1}{\sqrt{x+2}} dx = \int_1^{\infty} (x+2)^{-\frac{1}{2}} \, d(x+2)$$

and $\int (x+2)^{-\frac{1}{2}} \, d(x+2) = 2\sqrt{x+2} + C$

$$\Rightarrow \int_1^{+\infty} \frac{1}{\sqrt{x+2}} dx = \lim_{t\to\infty} \int_1^t \frac{1}{\sqrt{x+2}} dx = \lim_{t\to\infty} \left[2\sqrt{x+2}\right]\Big|_1^t$$

$= +\infty$ which is divergent

10.

$$\int_3^7 \frac{1}{x-3} dx = \int_3^7 \frac{1}{x-3} \, d(x-3) = \lim_{t\to 3^+} \int_t^7 \frac{1}{x-3} \, d(x-3)$$

$$= \lim_{t\to 3^+} \ln|x-3|\Big|_t^7 = \ln 4 - \lim_{t\to 3^+} \ln|t-3|$$

$= +\infty$ Thus, $\int_3^7 \frac{1}{x-3} dx$ is divergent.