Xuemao Zhang **Math141_Calculus_II**
Assignment Homework6 due 10/20/2021 at 11:59pm EDT

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1. **(2 points)** `Library/Wiley/setAnton_Section_7.8/Anton_7_8_Q2.pg`

In each part, determine all values of p for which the integral is improper. Enter in interval notation or "none" if there are no relevant values of p.

(a) $\displaystyle\int_1^5 \frac{dx}{x^p}$

p values that make integral improper ______

(b) $\displaystyle\int_2^3 \frac{dx}{x-p}$

p values that make integral improper ______

(c) $\displaystyle\int_{-3}^1 e^{-px}dx$

p values that make integral improper ______

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

(a) $\displaystyle\int_1^5 \frac{dx}{x^p}$, the only possible singularity is at $x = 0$ which is not in the range of integration.

(b) $\displaystyle\int_2^3 \frac{dx}{x-p}$, will have singularities whenever $p \in [2,3]$.

(c) $\displaystyle\int_{-3}^1 e^{-px}dx$, never has any singularities.

Correct Answers:

- `none`
- `[2,3]`
- `none`

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2. **(2 points)** `Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/7_Techniques_of_Integration/7.6_Improper_Integrals/7.6.63.pg`

Determine if the improper integral converges and, if so, evaluate it.

$\int_5^\infty \frac{dx}{\sqrt{x}-2}$.

- A. 0
- B. 5
- C. Diverges
- D. 1

Solution: *(Instructor solution preview: show the student solution after due date.)*

Solution:

Since $\sqrt{x} \geq \sqrt{x}-2$, we have (for $x > 5$)

$\frac{1}{\sqrt{x}} \leq \frac{1}{\sqrt{x}-2}$.

The integral $\int_1^\infty \frac{dx}{\sqrt{x}} = \int_1^\infty \frac{dx}{x^{\frac{1}{2}}}$ diverges because $\frac{1}{2} < 1$. Since the function $x^{-\frac{1}{2}}$ is continuous (and therefore finite) on [1,5], we also know that $\int_5^\infty \frac{dx}{x^{\frac{1}{2}}}$ diverges. Therefore, by the comparison test,

$\int_5^\infty \frac{dx}{\sqrt{x}-2}$ also diverges.

Correct Answers:

- `C`

3. **(2 points)** `Library/UCSB/Stewart5_7_8/Stewart5_7_8_48.pg`

Let $g(x) = \dfrac{1}{\sqrt{x}-1}$.

(a) Use a calculator or computer algebra system to evaluate $\int_2^t g(x)\,dx$ for t = 5, 10, 100, 1000, and 10000. Make sure each answer is correct to three decimal places.

t = 5: ______

t = 10: ______

t = 100: ______

t = 1000: ______

t = 10000: ______

(b) Use the Comparison Theorem to determine whether $\int_2^\infty g(x)\,dx$ is convergent or divergent.

[?] 1. $\int_2^\infty g(x)\,dx$

Correct Answers:

- 3.830326716
- 6.801199648
- 23.32876922
- 69.02336139
- 208.1245598
- D

4. **(2 points)** `Library/UCSB/Stewart5_7_8/Stewart5_7_8_52.pg`

Use the Comparison Theorem to determine whether the following integral is convergent or divergent.

[?] 1. $\int_1^\infty \dfrac{1x}{\sqrt{1+x^6}}\,dx$

Correct Answers:

- C

5. **(2 points)** `Library/Rochester/setIntegrals18Improper/S07.08.ImproperIntegrals.PTP18.pg`

The improper integral $\int_{-\infty}^{\infty} x\,dx$ is

- A. divergent since $\int_{-\infty}^{0} x\,dx$ is convergent and $\int_0^\infty x\,dx$ is divergent.
- B. divergent by comparison to $\int_{-\infty}^{\infty} xe^{-x}\,dx$.
- C. convergent since it equals $\lim_{t\to\infty}\int_{-t}^{t} x\,dx = \lim_{t\to\infty}\left(\dfrac{t^2}{2} - \dfrac{(-t)^2}{2}\right) = 0$.
- D. divergent by comparison to $\int_{-\infty}^{\infty} \sqrt{x}\,dx$.
- E. divergent since both integrals $\int_{-\infty}^{0} x\,dx = -\infty$ and $\int_0^\infty x\,dx = +\infty$ are divergent.
- F. convergent since the area to the left of $x=0$ cancels with the area to the right of $x=0$.
- G. convergent since it equals $\lim_{a\to-\infty}\int_a^0 x\,dx + \lim_{b\to\infty}\int_0^b x\,dx = -\infty+\infty = 0$.

Correct Answers:

- E

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6. **(2 points)** `Library/Michigan/Chap7Sec8/Q13.pg`

For each of the following improper integrals, carefully use the comparison test to decide if the integral converges or diverges. Give a reasonable "best" comparison function that you use in the comparison (by "best", we mean that the comparison function has known integral convergence properties, and is a reasonable upper or lower bound for the integrand we are evaluating).

1. $\int_5^9 \dfrac{6}{\sqrt{t-5}}\,dt$
This integral

- A. converges
- B. diverges

2. $\int_{-4}^5 \dfrac{dt}{(t+4)^2}$
This integral

- A. converges
- B. diverges

3. $\int_6^\infty \dfrac{d\theta}{\sqrt{\theta^3+4}}$
This integral

- A. converges
- B. diverges

4. $\int_4^\infty \dfrac{dz}{e^z+5^z}$
This integral

- A. converges
- B. diverges

5. $\int_4^\infty \dfrac{3+\sin z}{z}\,dz$
This integral

- A. converges
- B. diverges

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

1. Note that with the substitution $w=t-5$ we have $\int_5^9 \dfrac{6}{\sqrt{t-5}}\,dt = 6\displaystyle\int_0^4 \frac{1}{\sqrt{w}}\,dw$. We know that $\int_0^4 \frac{1}{x^{1/2}}\,dx$ converges because $p=1/2<1$, so this integral converges.

2. By substituting $w=t+4$, we have $\int_{-4}^5 \dfrac{dt}{(t+4)^2} = \displaystyle\int_0^9 \frac{dw}{w^2}$. We know that $\int_0^9 \frac{1}{x^2}\,dx$ diverges because $p=2\geq 1$, so this integral diverges.

3. Note that $\frac{1}{\sqrt{\theta^3+4}} < \frac{1}{\sqrt{\theta^3}} = \frac{1}{\theta^{3/2}}$. Therefore, because $\int_6^\infty \frac{d\theta}{\theta^{3/2}}$ converges (because $p=3/2>1$), we know that $\int_6^\infty \dfrac{d\theta}{\sqrt{\theta^3+4}}$ converges.

4. Here, we have $\frac{1}{e^z+5^z} < \frac{1}{e^z}$, so, because $\int_4^\infty e^{-z}\,dz$ converges, we know that $\int_4^\infty \dfrac{dz}{e^z+5^z}$ does also.

5. We know that $3+\sin\alpha \geq 2$, so $\frac{3+\sin\alpha}{\alpha} \geq \frac{2}{\alpha}$. Then, because $2\int_4^\infty \frac{1}{\alpha}\,d\alpha$ diverges, we know that $\int_4^\infty \frac{3+\sin z}{z}\,dz$ diverges as well.

Correct Answers:

- A
- B
- A
- A
- B

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7. **(2 points)** `Library/Rochester/setIntegrals18Improper/S07.08.ImproperIntegrals.PTP17.pg`

For each of the improper integrals below, if the comparison test applies, enter either A or B followed by one letter from C to K that best applies, and if the comparison test does not apply, enter only L. For example, one possible answer is BF, and another one is L.

Hint: $0 < e^{-x} \leq 1$ for $x \geq 1$.

___1. $\displaystyle\int_1^\infty \frac{6+\sin(x)}{\sqrt{x-0.7}}\,dx$

___2. $\displaystyle\int_1^\infty \frac{1}{x^2+5}\,dx$

___3. $\displaystyle\int_1^\infty \frac{x}{\sqrt{x^6+5}}\,dx$

___4. $\displaystyle\int_1^\infty \frac{e^{-x}}{x^2}\,dx$

___5. $\displaystyle\int_1^\infty \frac{\cos^2(x)}{x^2+5}\,dx$

A. The integral is convergent

B. The integral is divergent

C. by comparison to $\displaystyle\int_1^\infty \frac{1}{x^2-5}\,dx$.

D. by comparison to $\displaystyle\int_1^\infty \frac{1}{x^2+5}\,dx$.

E. by comparison to $\displaystyle\int_1^\infty \frac{\cos^2(x)}{x^2}\,dx$.

F. by comparison to $\displaystyle\int_1^\infty \frac{e^x}{x^2}\,dx$.

G. by comparison to $\displaystyle\int_1^\infty \frac{-e^{-x}}{2x}\,dx$.

H. by comparison to $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$.

I. by comparison to $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^5}}\,dx$.

J. by comparison to $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$.

K. by comparison to $\displaystyle\int_1^\infty \frac{1}{x^3}\,dx$.

L. The comparison test does not apply.

Correct Answers:

- BH
- AJ

- AJ
- AJ
- AJ