Basic Derivatives and Integrals (D_x denotes $\frac{d}{dx}$):

$$D_x(x^n) = nx^{n-1} \implies \int x^n\ dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$$

$$D_x(e^x) = e^x \implies \int e^x\ dx = e^x + C$$

$$D_x(a^x) = a^x \ln a \implies \int a^x\ dx = \frac{a^x}{\ln a} + C$$

$$D_x(\ln|x|) = \frac{1}{x} \implies \int \frac{dx}{x} = \ln|x| + C$$

$$D_x(\sin x) = \cos x \implies \int \cos x\ dx = \sin x + C$$

$$D_x(\cos x) = -\sin x \implies \int \sin x\ dx = -\cos x + C$$

$$D_x(\tan x) = \sec^2 x \implies \int \sec^2 x\ dx = \tan x + C$$

$$D_x(\sec x) = \sec x \tan x \implies \int \sec x \tan x\ dx = \sec x + C$$

$$D_x(\cot x) = -\csc^2 x \implies \int \csc^2\ dx = -\cot x + C$$

$$D_x(\csc x) = -\csc x \cot x \implies \int \csc x \cot x\ dx = -\csc x + C$$

$$D_x(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \implies \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$D_x(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \implies \int \frac{-dx}{\sqrt{1-x^2}} = \cos^{-1} x + C = -\sin^{-1} x + C$$

$$D_x(\tan^{-1} x) = \frac{1}{1+x^2} \implies \int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$D_x(\cot^{-1} x) = \frac{-1}{1+x^2} \implies \int \frac{-dx}{1+x^2} = \cot^{-1} x + C = -\tan^{-1} x + C$$

$$D_x(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}} \implies \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}|x| + C$$

$$D_x(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}} \implies \int \frac{-dx}{x\sqrt{x^2-1}} = \csc^{-1}|x| + C = -\sec^{-1}|x| + C$$

$$D_x(\ln|\sec x|) = \tan x \implies \int \tan x\ dx = \ln|\sec x| + C$$

$$D_x(\ln|\sin x|) = \cot x \implies \int \cot x\ dx = \ln|\sin x| + C$$

$$D_x(\ln|\sec x + \tan x|) = \sec x \implies \int \sec x\ dx = \ln|\sec x + \tan x| + C$$

$$D_x(-\ln|\csc x + \cot x|) = \csc x \implies \int \csc x\ dx = -\ln|\csc x + \cot x| + C$$

Trigonometric Substitutions $(c > 0)$:

(i) $\sqrt{c^2 - x^2}$. Substitute: $x = c\sin\theta$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

(ii) $\sqrt{x^2 + c^2}$. Substitute: $x = c\tan\theta$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

(iii) $\sqrt{x^2 - c^2}$. Substitute: $x = c\sec\theta$ for $0 \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$.

Trigonometric identities:

TR1. $\sin^2 x + \cos^2 x = 1$.

TR2. $\tan^2 x + 1 = \sec^2 x$.

TR3. $\cot^2 x + 1 = \csc^2 x$.

TR4. $\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$.

TR5. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$.

TR6. $\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$.

TR7. $\sin^2 x = \frac{1}{2}[1 - \cos(2x)]$, since $\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$.

TR8. $\cos x \cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$.

TR9. $\cos^2 x = \frac{1}{2}[1 + \cos(2x)]$.

TR10. $\sin x \cos y = \frac{1}{2}[\sin(x - y) + \sin(x + y)]$.

TR11. $\sin x \cos x = \frac{1}{2}[\sin(2x)]$, since $\sin(2x) = \sin(x + x) = 2\sin x \cos x$.

Classification of integrals of trigonometric functions $(\mathbb{Z} = \{0, \pm1, \pm2, \dots\})$:

Integrand	**Restriction on** m	**Restriction on** n	**Procedure**
$\sin mx \sin nx$	$\neq 0$	$\neq 0$	Use TR6
$\cos mx \cos nx$	$\neq 0$	$\neq 0$	Use TR8
$\sin mx \cos nx$	$\neq 0$	$\neq 0$	Use TR10
$\sin^m x \cos^n x$	Odd in $\mathbb{Z}_{>0}$	None	$\underbrace{-\sin^{m-1} x}_{-(1-\cos^2 x)^{\frac{m-1}{2}}} \cos^n x \; d(\cos x)$
$\sin^m x \cos^n x$	None	Odd in $\mathbb{Z}_{>0}$	$\sin^m x \underbrace{\cos^{n-1} x}_{(1-\sin^2 x)^{\frac{n-1}{2}}} d(\sin x)$
$\sin^m x \cos^n x$	Even in $\mathbb{Z}_{\geq 0}$	Even in $\mathbb{Z}_{\geq 0}$	Use TR7 & TR9 $\to \cos(2x)$
$\tan^m x \sec^n x$	None	Even in $\mathbb{Z}_{>0}$	Use: $\sec^n x \; dx =$ $(\tan^2 x + 1)^{\frac{n-2}{2}} \; d(\tan x)$
$\tan^m x \sec^n x$	in $\mathbb{Z}_{\geq 2}$	$= 0$	Reduce power using: $\tan^m x = \tan^{m-2}(\sec^2 x - 1)$ $= \tan^{m-2} \; d(\tan x) - \tan^{m-2} x = \cdots$
$\tan^m x \sec^n x$	$= 0$	Odd in $\mathbb{Z}_{>0}$	If $n = 1$, use foumula. If not, use repeated int. by parts.
$\tan^m x \sec^n x$	Odd in $\mathbb{Z}_{>0}$	Odd in $\mathbb{Z}_{>0}$	Simplify by using: $\tan^m x \sec^n x$ $= \tan^{m-1} x \sec^{n-1} x \; d\sec x$ $= (\sec^2 - 1)^{\frac{m-1}{2}} \sec^{n-1} x \; d(\sec x)$
$\tan^m x \sec^n x$	Even in $\mathbb{Z}_{>0}$	Odd in $\mathbb{Z}_{>0}$	Use: $\tan^m x = (\sec^2 x - 1)^{\frac{m}{2}}$
$\cot^m x \csc^n x$			Use: Similar to $\tan^m x \sec^n x$