Xuemao Zhang **Math141_Calculus_II**
Assignment Homework5 due 10/12/2021 at 11:59pm EDT

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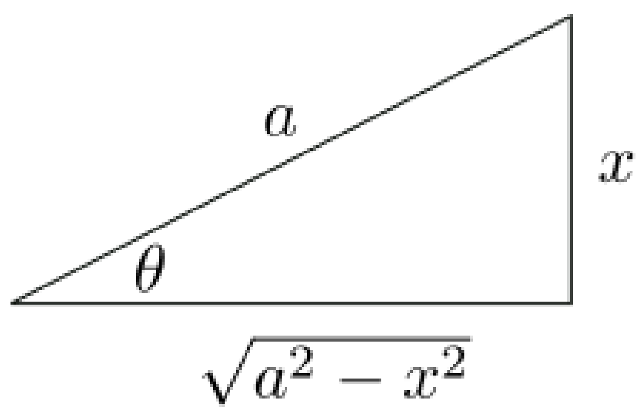
1. **(1 point)** `Library/maCalcDB/setIntegrals10InvTrig/ur_in_10_3.pg`
Evaluate the indefinite integral

$$\int \frac{x^{12}}{(25-x^2)^{15/2}}\,dx$$

________________________+C

Hint: *(Instructor hint preview: show the student hint after the following number of attempts: 1*
Make a trigonometric substitution. To attack the resulting integral, remember that $(\sin x)/(\cos x) = \tan x$ and that $1/(\cos x) = \sec x$.

Solution: *(Instructor solution preview: show the student solution after due date.)*
Use the right triangle



with $a = 5$ to create a trig substitution

$$x = 5\sin(\theta), \text{ so } dx = 5\cos(\theta)\,d\theta.$$

$$\int \frac{x^{12}}{(25-x^2)^{15/2}}\,dx = \int \frac{x^{12}}{\left(\sqrt{25-x^2}\right)^{15}}\,dx$$

$$= \int \frac{(5\sin(\theta))^{12}\cdot 5\cos(\theta)}{\left(\sqrt{5^2-5^2\sin^2\theta}\right)^{15}}\,d\theta$$

$$= \int \frac{5^{13}\sin^{12}(\theta)\cos(\theta)}{\left(\sqrt{5^2\cos^2(\theta)}\right)^{15}}\,d\theta$$

$$= \int \frac{5^{13}\sin^{12}(\theta)\cos(\theta)}{(5\cos(\theta))^{15}}\,d\theta$$

$$= \frac{1}{25}\int \tan^{12}(\theta)\sec^2(\theta)\,d\theta$$

$$= \frac{\tan^{13}(\theta)}{25\cdot 13} + C = \frac{\tan^{13}(\theta)}{325} + C$$

Now use the right triangle to find $\tan(\theta)$ in terms of x.

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{x}{\sqrt{25-x^2}},$$

so back substitution gives

$$\frac{\tan^{13}(\theta)}{325} + C = \frac{1}{325}\frac{x^{13}}{(25-x^2)^{13/2}} + C$$

That is,

$$\int \frac{x^{12}}{(25-x^2)^{15/2}}\,dx = \frac{1}{325}\frac{x^{13}}{(25-x^2)^{13/2}}$$

leaving off the arbitrary constant of integration.

Correct Answers:

- `x^12/[(25-x^2)^(15/2)]`

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2. **(1 point)** `Library/Wiley/setAnton_Section_7.1/Anton_7_1_Q23.pg`

Evaluate the integral by any method.

$\int \frac{e^{-x}}{9-e^{-2x}}\,dx =$ __________ $+C$

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

For $u = e^{-x}$ we have $-du = e^{-x}dx$ hence using partial fractions

$$\int \frac{e^{-x}}{9-e^{-2x}}\,dx = -\int \frac{1}{9-u^2}\,du = \tfrac{1}{6}\ln\left(\left|\frac{u-3}{u+3}\right|\right) + C = \tfrac{1}{6}\ln\left(\left|\frac{e^{-x}-3}{e^{-x}+3}\right|\right) + C$$

Correct Answers:

- `1/6*ln(|[e^(-x)-3]/[e^(-x)+3]|)`

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3. **(1 point)** `Library/UMN/calculusStewartET/s_7_1_34.pg`

First make a substitution and then use integration by parts to evaluate the integral

$$\int 4x^3 e^{-x^2}\,dx.$$

Answer: ____________________

Correct Answers:

- `(-2)*e^(-x^2)*(x^2+1)+C`

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4. **(1 point)** `Library/UVA-Stew5e/setUVA-Stew5e-C05S05-Substitution/5-5-41.pg`

Evaluate the indefinite integral.

$$\int \frac{5x+3}{x^2+1}dx$$

Integral = _______________

[NOTE: Remember to enter all necessary (and) !!

Enter arctan(x) for $tan^{-1}x$, arcsin(x) for $sin^{-1}x$.]

Correct Answers:

- `3 * arctan(x) + 0.5 * 5 * ln(x^2 + 1)`

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5. **(1 point)** `Library/Wiley/setAnton_Section_7.8/Anton_7_8_Q10.pg`

Evaluate the integrals that converge, enter 'DNC' if integral Does Not Converge.

$\displaystyle\int_{-\infty}^{2} \frac{3}{x^2+49}\,dx =$ __________

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Using Endpaper Integral Table entry 68.

$$\int_{-\infty}^{2} \frac{3}{x^2+49}\,dx = \lim_{a\to-\infty}\int_{a}^{2} \frac{3}{x^2+49}\,dx = \lim_{a\to-\infty}\left[\frac{3\tan^{-1}\left(\frac{x}{7}\right)}{7}\right]_a^2 = \lim_{a\to-\infty}\left[\frac{3}{7}\tan^{-1}\left(\frac{2}{7}\right) - \frac{3\tan^{-1}\left(\frac{a}{7}\right)}{7}\right] = \frac{3}{7}\tan^{-1}\left(\frac{2}{7}\right) + \frac{3\pi}{14}$$

Correct Answers:

- `3*atan(2/7)/7+3*pi/14`

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6. **(1 point)** `Library/Wiley/setAnton_Section_7.8/Anton_7_8_Q18.pg`

Evaluate the integrals that converge, enter 'DNC' if integral Does Not Converge.

$\displaystyle\int_{0}^{8} \frac{6}{\sqrt[3]{x}}\,dx =$ __________

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

$$\int_{0}^{8} \frac{6}{\sqrt[3]{x}}\,dx = \lim_{a\to0^+}\int_{a}^{8} \frac{6}{\sqrt[3]{x}}\,dx = \lim_{a\to0^+}\left[9\sqrt[3]{x^2}\right]_a^8 = 9\lim_{a\to0^+}\left[4-\sqrt[3]{a^2}\right] = 36$$

Correct Answers:

- `9*4/1`

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7. **(1 point)** `Library/Wiley/setAnton_Section_7.8/Anton_7_8_Q6.pg`

Evaluate the integrals that converge, enter 'DNC' if integral Does Not Converge.

$\int_0^{+\infty} xe^{-x^2}\,dx =$ __________

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Using Endpaper Integral Table entry 69.

$$\int_0^{+\infty} xe^{-x^2}\,dx = \lim_{a\to+\infty}\int_0^a xe^{-x^2}\,dx = \lim_{a\to+\infty}\left[-\frac{e^{-x^2}}{2}\right]_0^a = \lim_{a\to+\infty}\left[-\frac{e^{-a^2}}{2}+\tfrac{1}{2}\right] = \tfrac{1}{2}$$

Correct Answers:

- `1/2`

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8. **(1 point)** `Library/UCSB/Stewart5_7_8/Stewart5_7_8_32.pg`

Consider the integral

$$\int_0^1 \frac{-4}{\sqrt{1-x^2}}\,dx$$

If the integral is divergent, type an upper-case "D". Otherwise, evaluate the integral.

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Correct Answers:

- `-4*pi/2`

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9. **(1 point)** `Library/Michigan/Chap7Sec7/Q15.pg`

Calculate the integral, if it converges. If it diverges, enter **diverges** for your answer.

$\int_{-4}^{4} \frac{1}{v}\,dv =$ ________________

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

This integral is improper because $1/v$ is undefined at $v=0$. To evaluate it, we must split the region of integration up into two pieces, from 0 to 4 and from -4 to 0. But notice,

$$\int_0^4 \frac{1}{v}\,dv = \lim_{b\to0^+}\int_b^4 \frac{1}{v}\,dv = \lim_{b\to0^+}\left(\ln v\Big|_b^4\right) = \ln(4)-\ln b.$$

As $b\to0^+$, this goes to infinity and the integral diverges, so our original integral also diverges.

Correct Answers:

- `diverges`

10. **(1 point)** `Library/Michigan/Chap7Sec7/Q19.pg`

Calculate the integral, if it converges. If it diverges, enter **diverges** for your answer.
$\int_0^4 \frac{1}{u^2-16}\,du =$ ____________
Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION
Note that the boundary that makes this an improper integral is $x=4$. We are therefore finding

$$\int_0^4 \frac{1}{u^2-16}\,du = \lim_{a\to 4^-}\int_0^a \frac{1}{u^2-16}\,du.$$

Using partial fractions, we have

$$\frac{1}{u^2-16} = \frac{1}{8(u-4)} - \frac{1}{8(u+4)},$$

so this is

$$\lim_{a\to 4^-}\int_0^a \frac{1}{u^2-16}\,du = \lim_{a\to 4^-}\frac{1}{8}(\ln(|a-4|)-\ln(8)-\ln(4)+\ln(4)).$$

However, as $a\to 4^-$, $a-4\to 0$, and the first term goes to $-\infty$, so this integral diverges.
Correct Answers:

- `diverges`

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11. **(1 point)** `Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/7_Techniques_of_Integration/7.6_Improper_Integrals/7.6.25.pg`

Determine if the improper integral converges and, if so, evaluate it.
$\int_0^\infty \frac{dx}{7+x} =$ ________
Write F if the integral doesn't converge.

Solution: *(Instructor solution preview: show the student solution after due date.)*

Solution:
First evaluate the integral on the finite interval $[0,R]$ for $R>0$:
$\int_0^R \frac{dx}{7+x} = \ln|7+x|\big|_0^R = \ln|7+R| - \ln 7$
Now compute the limit as $R\to\infty$:
$\int_0^\infty \frac{dx}{7+x} = \lim_{R\to\infty}\int_0^R \frac{dx}{7+x} = \lim_{R\to\infty}(\ln|7+R|-\ln 7) = \infty;$
Thus, the integral doesn't converge.

Correct Answers:

- `F`

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12. **(1 point)** `Library/Utah/Quantitative_Analysis/set12_Definite_Integrals_Techniques_of_Integration/s1p12.pg`

Find what value of c does

$\int_7^\infty \frac{c}{x^3}dx = 1$?

Answer: ______________
Correct Answers:

- 98