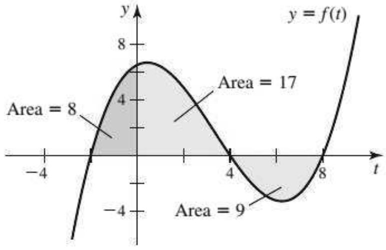
Practice Exercises

13. Area functions The graph of f is shown in the figure. Let $A(x) = \int_{-2}^{x} f(t)\,dt$ and $F(x) = \int_{4}^{x} f(t)\,dt$ be two area functions for f. Evaluate the following area functions.

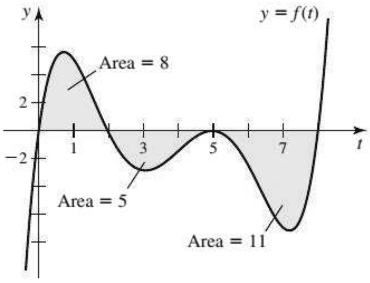
a. $A(-2)$ **b.** $F(8)$ **c.** $A(4)$ **d.** $F(4)$ **e.** $A(8)$



14. Area functions The graph of f is shown in the figure. Let $A(x) = \int_{0}^{x} f(t)\,dt$ and $F(x) = \int_{2}^{x} f(t)\,dt$ be two area functions for f. Evaluate the following area functions.

a. $A(2)$ **b.** $F(5)$ **c.** $A(0)$ **d.** $F(8)$

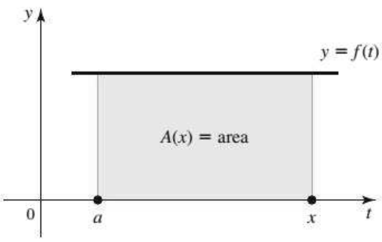
e. $A(8)$ **f.** $A(5)$ **g.** $F(2)$



15–16. Area functions for constant functions *Consider the following functions f and real numbers a (see figure).*

a. *Find and graph the area function $A(x) = \int_{a}^{x} f(t)\,dt$ for f.*

b. *Verify that $A'(x) = f(x)$.*



15. $f(t) = 5, a = 0$ **16.** $f(t) = 5, a = -5$

17. Area functions for the same linear function Let $f(t) = t$ and consider the two area functions $A(x) = \int_{0}^{x} f(t)\,dt$ and $F(x) = \int_{2}^{x} f(t)\,dt$.

a. Evaluate $A(2)$ and $A(4)$. Then use geometry to find an expression for $A(x)$, for $x \geq 0$.

b. Evaluate $F(4)$ and $F(6)$. Then use geometry to find an expression for $F(x)$, for $x \geq 2$.

c. Show that $A(x) - F(x)$ is a constant and that $A'(x) = F'(x) = f(x)$.

18. Area functions for the same linear function Let $f(t) = 2t - 2$ and consider the two area functions $A(x) = \int_{1}^{x} f(t)\,dt$ and $F(x) = \int_{4}^{x} f(t)\,dt$.

a. Evaluate $A(2)$ and $A(3)$. Then use geometry to find an expression for $A(x)$, for $x \geq 1$.

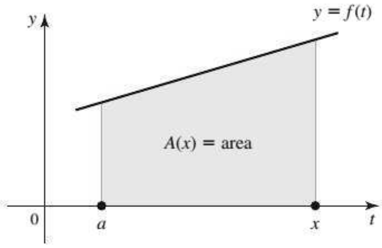
b. Evaluate $F(5)$ and $F(6)$. Then use geometry to find an expression for $F(x)$, for $x \geq 4$.

c. Show that $A(x) - F(x)$ is a constant and that $A'(x) = F'(x) = f(x)$.

19–22. Area functions for linear functions *Consider the following functions f and real numbers a (see figure).*

a. *Find and graph the area function $A(x) = \int_{a}^{x} f(t)\,dt$.*

b. *Verify that $A'(x) = f(x)$.*

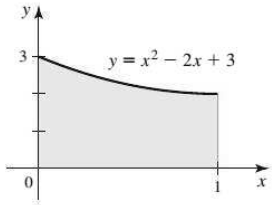


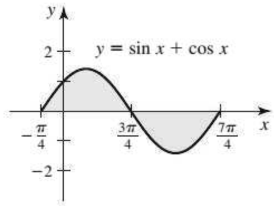
19. $f(t) = t + 5, a = -5$ **20.** $f(t) = 2t + 5, a = 0$

21. $f(t) = 3t + 1, a = 2$ **22.** $f(t) = 4t + 2, a = 0$

23–24. Definite integrals *Evaluate the following integrals using the Fundamental Theorem of Calculus. Explain why your result is consistent with the figure.*

23. $\int_{0}^{1} (x^2 - 2x + 3)\,dx$ **24.** $\int_{-\pi/4}^{7\pi/4} (\sin x + \cos x)\,dx$





25–28. Definite integrals *Evaluate the following integrals using the Fundamental Theorem of Calculus. Sketch the graph of the integrand and shade the region whose net area you have found.*

25. $\int_{-2}^{3} (x^2 - x - 6)\,dx$ **26.** $\int_{0}^{1} (x - \sqrt{x})\,dx$

27. $\int_{0}^{5} (x^2 - 9)\,dx$ **28.** $\int_{1/2}^{2} \left(1 - \frac{1}{x^2}\right) dx$

29–62. Definite integrals *Evaluate the following integrals using the Fundamental Theorem of Calculus.*

29. $\int_{0}^{2} 4x^3\,dx$ **30.** $\int_{0}^{2} (3x^2 + 2x)\,dx$ **31.** $\int_{1}^{8} 8x^{1/3}\,dx$

32. $\int_{1}^{16} x^{-5/4}\,dx$ **33.** $\int_{0}^{1} (x + \sqrt{x})\,dx$ **34.** $\int_{0}^{\pi/4} 2\cos x\,dx$

35. $\int_{1}^{9} \frac{2}{\sqrt{x}}\,dx$ **36.** $\int_{4}^{9} \frac{2 + \sqrt{t}}{\sqrt{t}}\,dt$ **37.** $\int_{-2}^{2} (x^2 - 4)\,dx$

38. $\int_{0}^{\ln 8} e^x\,dx$ **39.** $\int_{1/2}^{1} (t^{-3} - 8)\,dt$ **40.** $\int_{0}^{4} t(t - 2)(t - 4)\,dt$

41. $\displaystyle\int_1^4 (1-x)(x-4)\,dx$

42. $\displaystyle\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$

43. $\displaystyle\int_{-2}^{-1} x^{-3}\,dx$

44. $\displaystyle\int_0^{\pi} (1-\sin x)\,dx$

45. $\displaystyle\int_0^{\pi/4} \sec^2\theta\,d\theta$

46. $\displaystyle\int_{-\pi/2}^{\pi/2} (\cos x - 1)\,dx$

47. $\displaystyle\int_1^2 \frac{3}{t}\,dt$

48. $\displaystyle\int_4^9 \frac{x-\sqrt{x}}{x^2}\,dx$

49. $\displaystyle\int_1^8 \sqrt[3]{y}\,dy$

50. $\displaystyle\frac{1}{2}\int_0^{\ln 2} e^x\,dx$

51. $\displaystyle\int_1^4 \frac{x-2}{\sqrt{x}}\,dx$

52. $\displaystyle\int_1^2 \frac{2s^2-4}{s^3}\,ds$

53. $\displaystyle\int_0^{\pi/3} \sec x\tan x\,dx$

54. $\displaystyle\int_{\pi/4}^{\pi/2} \csc^2\theta\,d\theta$

55. $\displaystyle\int_{\pi/4}^{3\pi/4} (\cot^2 x + 1)\,dx$

56. $\displaystyle\int_0^1 10e^{x+3}\,dx$

57. $\displaystyle\int_1^{\sqrt{3}} \frac{1}{1+x^2}\,dx$

58. $\displaystyle\int_0^{\pi/4} \sec x(\sec x + \cos x)\,dx$

59. $\displaystyle\int_1^2 \frac{z^2+4}{z}\,dz$

60. $\displaystyle\int_{\sqrt{2}}^2 \frac{dx}{x\sqrt{x^2-1}}$

61. $\displaystyle\int_0^{\pi} f(x)\,dx$, where $f(x) = \begin{cases} \sin x + 1 & \text{if } x \le \pi/2 \\ 2\cos x + 2 & \text{if } x > \pi/2 \end{cases}$

62. $\displaystyle\int_1^3 g(x)\,dx$, where $g(x) = \begin{cases} 3x^2 + 4x + 1 & \text{if } x \le 2 \\ 2x + 5 & \text{if } x > 2 \end{cases}$

63–66. Area *Find (i) the net area and (ii) the area of the following regions. Graph the function and indicate the region in question.*

63. The region bounded by $y = x^{1/2}$ and the x-axis between $x = 1$ and $x = 4$

64. The region above the x-axis bounded by $y = 4 - x^2$

65. The region below the x-axis bounded by $y = x^4 - 16$

66. The region bounded by $y = 6\cos x$ and the x-axis between $x = -\pi/2$ and $x = \pi$

67–72. Areas of regions *Find the area of the region bounded by the graph of f and the x-axis on the given interval.*

67. $f(x) = x^2 - 25$ on $[2, 4]$

68. $f(x) = x^3 - 1$ on $[-1, 2]$

69. $f(x) = \dfrac{1}{x}$ on $[-2, -1]$

70. $f(x) = x(x+1)(x-2)$ on $[-1, 2]$

71. $f(x) = \sin x$ on $\left[-\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right]$

72. $f(x) = \cos x$ on $\left[\dfrac{\pi}{2}, \pi\right]$

73–86. Derivatives of integrals *Simplify the following expressions.*

73. $\displaystyle\frac{d}{dx}\int_3^x (t^2 + t + 1)\,dt$

74. $\displaystyle\frac{d}{dx}\int_1^x e^{t^2}\,dt$

75. $\displaystyle\frac{d}{dx}\int_x^1 \sqrt{t^4+1}\,dt$

76. $\displaystyle\frac{d}{dx}\int_x^0 \frac{dp}{p^2+1}$

77. $\displaystyle\frac{d}{dx}\int_2^{x^3} \frac{dp}{p^2}$

78. $\displaystyle\frac{d}{dx}\int_0^{x^2} \frac{dt}{t^2+4}$

79. $\displaystyle\frac{d}{dx}\int_0^{\cos x} (t^4 + 6)\,dt$

80. $\displaystyle\frac{d}{dw}\int_0^{\sqrt{w}} \ln(x^2+1)\,dx$

81. $\displaystyle\frac{d}{dz}\int_{\sin z}^{10} \frac{dt}{t^4+1}$

82. $\displaystyle\frac{d}{dy}\int_{y^3}^{10} \sqrt{x^6+1}\,dx$

83. $\displaystyle\frac{d}{dt}\left(\int_1^t \frac{3}{x}\,dx - \int_{t^2}^1 \frac{3}{x}\,dx\right)$

84. $\displaystyle\frac{d}{dt}\left(\int_0^t \frac{dx}{1+x^2} + \int_1^{1/t} \frac{dx}{1+x^2}\right)$

85. $\displaystyle\frac{d}{dx}\int_{-x}^x \sqrt{1+t^2}\,dt$

(*Hint:* $\displaystyle\int_{-x}^x \sqrt{1+t^2}\,dt = \int_{-x}^0 \sqrt{1+t^2}\,dt + \int_{-x}^x \sqrt{1+t^2}\,dt$.)

86. $\displaystyle\frac{d}{dx}\int_{e^x}^{e^{2x}} \ln t^2\,dt$

87. **Matching functions with area functions** Match the functions f, whose graphs are given in a–d, with the area functions $A(x) = \int_0^x f(t)\,dt$, whose graphs are given in A–D.