1.

$$\int\left(x^{\frac{1}{3}}+\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx = \int\left(x^{\frac{1}{3}}+x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx$$

$$= \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{3}{4}x^{\frac{4}{3}} + \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$$

2.

$$\int \tan x\, sec^2 x\, dx \qquad u = \tan x \Rightarrow du = \sec^2 x\, dx$$

$$= \int u\, du = \frac{u^2}{2} + C$$

$$= \frac{1}{2}(\tan x)^2 + C$$

3.

$$\int x^5 \ln x \;\; dx$$

$$\begin{array}{cc} \text{diff} & \text{Int} \\ \cancel{x^5}\ln x & x^5 \\ \frac{1}{x} & \xrightarrow{(-)} \frac{x^6}{6} \end{array}$$

$$= \frac{1}{6}x^6 \ln x - \frac{1}{6}\int \frac{1}{x}\cdot x^6\, dx$$

$$= \frac{1}{6}x^6 \ln x - \frac{1}{6}\int x^5\, dx$$

$$= \frac{1}{6}x^6 \ln x - \frac{x^6}{36} + C$$

4.

$$\int \frac{5x}{x^2 - x - 6} dx$$

$$\frac{5x}{x^2-x-6} = \frac{5x}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$= \frac{A(x+2) + B(x-3)}{x^2-x-6}$$

at $x=3 \Rightarrow 15 = 5A \Rightarrow A = 3$

at $x=-2 \Rightarrow -10 = -5B \Rightarrow B = 2$

$$\Rightarrow \int \frac{5x}{x^2-x-6} dx = 3\int \frac{dx}{x-3} + 2\int \frac{dx}{x+2}$$

$$= 3 \ln|x-3| + 2 \ln|x+2| + C$$

5.

$$\int \frac{1}{\sqrt{8-2x^2}} dx = \frac{1}{\sqrt{8}} \int \frac{dx}{\sqrt{1-\frac{x^2}{4}}} = \frac{1}{\sqrt{8}} \int \frac{dx}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$$

$$\underset{dx=2du}{\overset{u=\frac{x}{2}}{=}} \frac{1}{2\sqrt{2}} \int \frac{2\,du}{\sqrt{1-u^2}}$$

$$= \frac{1}{\sqrt{2}} \arcsin(u) + C = \frac{1}{\sqrt{2}} \arcsin\left(\frac{x}{2}\right) + C$$

6.

$$\int \frac{4x^2+x+6}{(x^2+2)(x+1)}\,dx$$

$$\frac{4x^2+x+6}{(x^2+2)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2}$$

$$= \frac{A(x^2+2)+(Bx+C)(x+1)}{(x+1)(x^2+2)}$$

$$\Rightarrow 4x^2+x+6 = A(x^2+2)+(Bx+C)(x+1)$$

let $x=-1$, $4-1+6=3A \Rightarrow A=3$

let $x=0$, $6=2A+C=6+C \Rightarrow C=0$

let $x=1$, $4+1+6=3A+2B=9+2B \Rightarrow B=1$

$$\Rightarrow \int \frac{4x^2+x+6}{(x^2+2)(x+1)}\,dx = 3\int \frac{dx}{x+1} + \int \frac{x\,dx}{x^2+2}$$

$$\overset{u=x^2+2}{\underset{du=2x\,dx}{=}} 3\ln|x+1| + \frac{1}{2}\int \frac{du}{u}$$

$$= 3\ln|x+1| + \frac{1}{2}\ln(x^2+2) + C$$

7.

$$\int_1^2 \frac{4x+3}{2x^2+3x}\,dx \qquad \frac{u = 2x^2+3x}{du = (4x+3)\,dx} \qquad \int_{x=1}^{x=2} \frac{du}{u} = \ln|u| \Big|_{x=1}^{x=2}$$

$$= \ln|2x^2+3x| \Big|_1^2$$

$$= \ln 14 - \ln 5 = \ln \frac{14}{5}$$

8.

$$\int x^2 3^x\,dx$$

diff		Int
x^2		3^x
	$\xrightarrow{(+)}$	
$2x$		$\frac{3^x}{\ln 3}$
	$\xrightarrow{(-)}$	
2	$\xrightarrow{(+)}$	$\frac{3^x}{(\ln 3)^2}$

$$= \frac{1}{\ln 3} x^2 3^x - \frac{2}{(\ln 3)^2} x 3^x$$

$$+ 2\int \frac{3^x}{(\ln 3)^2}\,dx$$

$$= \frac{1}{\ln 3} x^2 3^x - \frac{2}{(\ln 3)^2} x 3^x + \frac{2}{(\ln 3)^3} 3^x + C$$

9.

$$\int_2^{\infty} \frac{1}{(2x+1)^2}\,dx$$

$$\int \frac{1}{(2x+1)^2}\,dx \ \underset{du=2dx}{\overset{u=2x+1}{=\!=}} \ \frac{1}{2}\int u^{-2}\,du$$

$$= \frac{-1}{2}u^{-1} + C = \frac{-1}{2}\frac{1}{2x+1} + C$$

$$\Rightarrow \int_2^{\infty} \frac{dx}{(2x+1)^2} = \lim_{t\to\infty}\int_2^t \frac{dx}{(2x+1)^2} = -\frac{1}{2}\lim_{t\to\infty}\frac{1}{2x+1}\Big|_2^t$$

$$= \frac{-1}{2}\left[\lim_{t\to\infty}\frac{1}{2t+1} - \frac{1}{5}\right]$$

$$= \frac{-1}{2}\left(0 - \frac{1}{5}\right) = \frac{1}{10}$$

10.

$$\int_{1/2}^{1} \frac{1}{\sqrt{2x-1}}\,dx$$

$\frac{1}{\sqrt{2x-1}}$ is discontinuous at $x = \frac{1}{2}$

and $\displaystyle\int \frac{dx}{\sqrt{2x-1}} \ \underset{du=2dx}{\overset{u=2x-1}{=\!=}} \ \frac{1}{2}\int (2x-1)^{-\frac{1}{2}}\,du = \frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$$= \frac{1}{2}\cdot 2\sqrt{u} + C = \sqrt{2x-1} + C$$

$$\Rightarrow \int_{1/2}^{1} \frac{1}{\sqrt{2x-1}}\,dx = \lim_{t\to 1/2^+}\int_t^1 \frac{dx}{\sqrt{2x-1}}$$

$$= \lim_{t\to\frac{1}{2}^+}\sqrt{2x-1}\Big|_t^1 = \lim_{t\to\frac{1}{2}^+}\left(\sqrt{1} - \sqrt{2t-1}\right)$$

$$= 1 - \lim_{t\to\frac{1}{2}^+}\sqrt{2t-1} = 1 - 0 = 1$$