19. $\int \frac{2x^2}{\sqrt{1-4x^3}}\,dx$

20. $\int \frac{(\sqrt{x}+1)^4}{2\sqrt{x}}\,dx$

21. $\int (x^2+x)^{10}(2x+1)\,dx$

22. $\int \frac{1}{10x-3}\,dx$

23. $\int x^3(x^4+16)^6\,dx$

24. $\int \sin^{10}\theta\cos\theta\,d\theta$

25. $\int \frac{dx}{\sqrt{36-4x^2}}$

26. $\int \frac{dx}{\sqrt{1-9x^2}}$

27. $\int 6x^2 4^{x^3}\,dx$

28. $\int x^9 \sin x^{10}\,dx$

29. $\int (x^6-3x^2)^4(x^5-x)\,dx$

30. $\int \frac{dx}{1+4x^2}$

31. $\int \frac{3}{\sqrt{1-25x^2}}\,dx$

32. $\int \frac{2}{x\sqrt{4x^2-1}}\,dx,\ x>\frac{1}{2}$

33. $\int \frac{e^w}{36+e^{2w}}\,dw$

34. $\int \frac{8x+6}{2x^2+3x}\,dx$

35. $\int x\csc x^2\cot x^2\,dx$

36. $\int \sec 4w\tan 4w\,dw$

37. $\int \sec^2(10x+7)\,dx$

38. $\int \frac{\tan^{-1}w}{w^2+1}\,dw$

39. $\int 10^{4t+1}\,dt$

40. $\int (\sin^5 x+3\sin^3 x-\sin x)\cos x\,dx$

41. $\int \frac{\csc^2 x}{\cot^3 x}\,dx$

42. $\int (x^{3/2}+8)^5\sqrt{x}\,dx$

43. $\int \sin x\sec^8 x\,dx$

44. $\int \frac{e^{2x}}{e^{2x}+1}\,dx$

45–74. Definite integrals *Use a change of variables or Table 5.6 to evaluate the following definite integrals.*

45. $\int_0^{\pi/8}\cos 2x\,dx$

46. $\int_0^1 2e^{2x}\,dx$

47. $\int_0^1 2x(4-x^2)\,dx$

48. $\int_0^2 \frac{2x}{(x^2+1)^2}\,dx$

49. $\int_1^3 \frac{2^x}{2^x+4}\,dx$

50. $\int_{-2\pi}^{2\pi}\cos\frac{\theta}{8}\,d\theta$

51. $\int_0^{\pi/2}\sin^2\theta\cos\theta\,d\theta$

52. $\int_0^{\pi/4}\frac{\sin x}{\cos^2 x}\,dx$

53. $\int_{\ln\frac{\pi}{4}}^{\ln\frac{\pi}{2}} e^w\cos e^w\,dw$

54. $\int_{\pi/16}^{\pi/8} 8\csc^2 4x\,dx$

55. $\int_{-1}^{2} x^2 e^{x^3+1}\,dx$

56. $\int_0^4 \frac{p}{\sqrt{9+p^2}}\,dp$

57. $\int_{\pi/4}^{\pi/2}\frac{\cos x}{\sin^2 x}\,dx$

58. $\int_0^{\pi/4}\frac{\sin\theta}{\cos^3\theta}\,d\theta$

59. $\int_{2/(5\sqrt{3})}^{2/5}\frac{dx}{x\sqrt{25x^2-1}}$

60. $\int_0^1 \frac{v^3+1}{\sqrt{v^4+4v+4}}\,dv$

61. $\int_0^4 \frac{x}{x^2+1}\,dx$

62. $\int_0^{1/8}\frac{x}{\sqrt{1-16x^2}}\,dx$

63. $\int_{1/3}^{1/\sqrt{3}}\frac{4}{9x^2+1}\,dx$

64. $\int_0^{\ln 4}\frac{e^x}{3+2e^x}\,dx$

65. $\int_0^1 x\sqrt{1-x^2}\,dx$

66. $\int_1^{e^2}\frac{\ln p}{p}\,dp$

67. $\int_2^3 \frac{x}{\sqrt[3]{x^2-1}}\,dx$

68. $\int_0^{6/5}\frac{dx}{25x^2+36}$

69. $\int_0^2 x^3\sqrt{16-x^4}\,dx$

70. $\int_{-1}^{1}(x-1)(x^2 -$

71. $\int_{-\pi}^{0}\frac{\sin x}{2+\cos x}\,dx$

72. $\int_0^1 \frac{(v+1)(v}{2v^3+9v^2+1}$

73. $\int_1^2 \frac{4}{9x^2+6x+1}\,dx$

74. $\int_0^{\pi/4} e^{\sin^2 x}\sin 2x\,d$

75. **Average velocity** An object moves in one dimension v
velocity in m/s given by $v(t) = 8\sin\pi t + 2t$. Find i
velocity over the time interval from $t = 0$ to $t = 10$, v
measured in seconds.

T 76. **Periodic motion** An object moves along a line with a
m/s given by $v(t) = 8\cos\frac{\pi t}{6}$. Its initial position is s

a. Graph the velocity function.

b. As discussed in Chapter 6, the position of the obje
by $s(t) = \int_0^t v(y)\,dy$, for $t \geq 0$. Find the position
for $t \geq 0$.

c. What is the period of the motion—that is, starting
how long does it take the object to return to that po

77. **Population models** The population of a culture of bac
growth rate given by $p'(t) = \frac{200}{(t+1)^r}$ bacteria per h
$t \geq 0$, where $r > 1$ is a real number. In Chapter 6 it is
that the increase in the population over the time interv
given by $\int_0^t p'(s)\,ds$. (Note that the growth rate decrea
reflecting competition for space and food.)

a. Using the population model with $r = 2$, what is th
the population over the time interval $0 \leq t \leq 4$?

b. Using the population model with $r = 3$, what is th
the population over the time interval $0 \leq t \leq 6$?

c. Let ΔP be the increase in the population over a fix
interval $[0, T]$. For fixed T, does ΔP increase or de
the parameter r? Explain.

d. A lab technician measures an increase in the popul
bacteria over the 10-hr period $[0, 10]$. Estimate the
that best fits this data point.

e. Looking ahead: Use the population model in part (
the increase in population over the time interval [0
$T > 0$. If the culture is allowed to grow indefinitel
does the bacteria population increase without boun
it approach a finite limit?

78–86. Variations on the substitution method *Evaluate t integrals.*

78. $\int \frac{x}{x-2}\,dx$

79. $\int \frac{x}{\sqrt{x-4}}\,dx$

80. $\int \frac{y^2}{(y+1)^4}\,dy$

81. $\int \frac{x}{\sqrt[3]{x+4}}\,dx$

82. $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$

83. $\int x\sqrt[3]{2x+1}\,dx$

84. $\int (z+1)\sqrt{3z+2}\,dz$

85. $\int x(x+10)^9\,dx$

86. $\int_0^{\sqrt{3}} \frac{3\,dx}{9+x^2}$

87–94. Integrals with $\sin^2 x$ and $\cos^2 x$ *Evaluate the following integrals.*

87. $\int_{-\pi}^{\pi} \cos^2 x\,dx$

88. $\int \sin^2 x\,dx$

89. $\int \sin^2\left(\theta + \frac{\pi}{6}\right) d\theta$

90. $\int_0^{\pi/4} \cos^2 8\theta\,d\theta$

91. $\int_{-\pi/4}^{\pi/4} \sin^2 2\theta\,d\theta$

92. $\int x\cos^2 x^2\,dx$

93. $\int_0^{\pi/6} \frac{\sin 2y}{\sin^2 y + 2}\,dy$ (*Hint:* $\sin 2y = 2\sin y\cos y$.)

94. $\int_0^{\pi/2} \sin^4\theta\,d\theta$

95. **Explain why or why not** Determine whether the following statements are true and give an explanation or counterexample. Assume f, f', and f'' are continuous functions for all real numbers.

a. $\int f(x)f'(x)\,dx = \frac{1}{2}(f(x))^2 + C.$

b. $\int (f(x))^n f'(x)\,dx = \frac{1}{n+1}(f(x))^{n+1} + C, n \neq -1.$

c. $\int \sin 2x\,dx = 2\int \sin x\,dx.$

d. $\int (x^2+1)^9\,dx = \frac{(x^2+1)^{10}}{10} + C.$

e. $\int_a^b f'(x)f''(x)\,dx = f'(b) - f'(a).$

96–98. Areas of regions *Find the area of the following regions.*

96. The region bounded by the graph of $f(x) = \frac{x}{\sqrt{x^2-9}}$ and the x-axis between $x = 4$ and $x = 5$

97. The region bounded by the graph of $f(x) = x\sin x^2$ and the x-axis between $x = 0$ and $x = \sqrt{\pi}$

98. The region bounded by the graph of $f(x) = (x-4)^4$ and the x-axis between $x = 2$ and $x = 6$

Explorations and Challenges

99. **Morphing parabolas** The family of parabolas $y = \frac{1}{a} - \frac{x^2}{a^3}$, where $a > 0$, has the property that for $x \geq 0$, the x-intercept is $(a, 0)$ and the y-intercept is $(0, 1/a)$. Let $A(a)$ be the area of the region in the first quadrant bounded by the parabola and the x-axis. Find $A(a)$ and determine whether it is an increasing, decreasing, or constant function of a.

100. **Substitutions** Suppose f is an even function with $\int_0^8 f(x)\,dx = 9$. Evaluate each integral.

a. $\int_{-1}^{1} xf(x^2)\,dx.$

b. $\int_{-2}^{2} x^2 f(x^3)\,dx.$

101. **Substitutions** Suppose p is a nonzero real num function with $\int_0^1 f(x)\,dx = \pi$. Evaluate each i

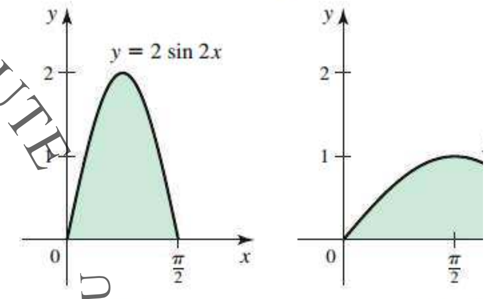
a. $\int_0^{\pi/(2p)} (\cos px)f(\sin px)\,dx$

b. $\int_{-\pi/2}^{\pi/2} (c$

102. **Average distance on a triangle** Consider the ri vertices $(0, 0)$, $(0, b)$, and $(a, 0)$, where $a > 0$ that the average vertical distance from points o hypotenuse is $b/2$, for all $a > 0$.

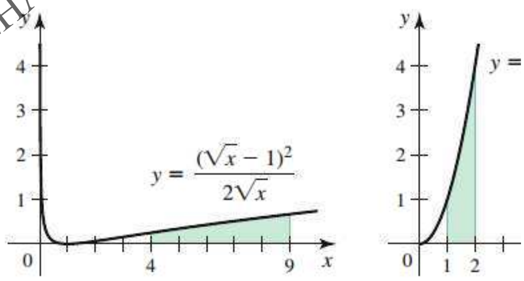
103. **Average value of sine functions** Use a graphin that the functions $f(x) = \sin kx$ have a period $k = 1, 2, 3, \ldots$. Equivalently, the first "hump" occurs on the interval $[0, \pi/k]$. Verify that the the first hump of $f(x) = \sin kx$ is independent average value?

104. **Equal areas** The area of the shaded region und $y = 2\sin 2x$ in part (a) of the figure equals the region under the curve $y = \sin x$ in part (b) of why this is true without computing areas.



(a) (b)

105. **Equal areas** The area of the shaded region und $y = \frac{(\sqrt{x}-1)^2}{2\sqrt{x}}$ on the interval $[4, 9]$ in part (figure equals the area of the shaded region unde on the interval $[1, 2]$ in part (b) of the figure. W areas, explain why.



(a)

106–108. General results *Evaluate the following in the function f is unspecified. Note that $f^{(p)}$ is the pt and f^p is the pth power of f. Assume f and its deriv ous for all real numbers.*

106. $\int (5f^3(x) + 7f^2(x) + f(x))f'(x)\,dx$

107. $\int_1^2 (5f^3(x) + 7f^2(x) + f(x))f'(x)\,dx$, where $f(2) = 5$

108. $\int (f^{(p)}(x))^n f^{(p+1)}(x)\,dx$, where p is a positive i