f. Vertical lines become parabolas opening downward with vertices on the positive y-axis, and horizontal lines become parabolas opening upward with vertices on the negative y-axis. **59. a.** S is stretched in the positive u- and v-directions but not in the w-direction. The amount of stretching increases with u and v. **b.** $J(u, v, w) = ad$

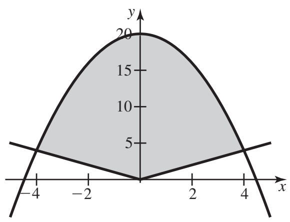
c. Volume $= ad$ **d.** $\left(\frac{a+b+c}{2}, \frac{d+e}{2}, \frac{1}{2}\right)$

Chapter 16 Review Exercises, pp. 1084–1088

1. a. False **b.** True **c.** False **d.** False **3.** $\frac{26}{3}$

5. $\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y)\, dx\, dy$ **7.** $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x, y)\, dy\, dx$

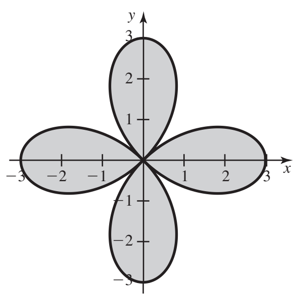
9. $\frac{304}{3}$



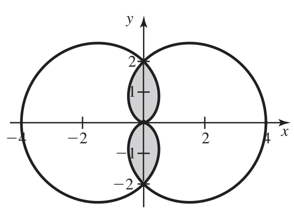
11. $\frac{\sqrt{17} - \sqrt{2}}{2}$

13. 8π **15.** $\frac{2}{7\pi^2}$ **17.** $\frac{1}{5}$

19. $\frac{9\pi}{2}$



21. $6\pi - 16$



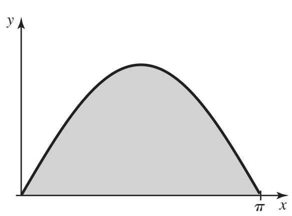
23. 2 **25.** $\int_0^1 \int_{2y}^{2} \int_0^{\sqrt{z^2-4y^2}/2} f(x, y, z)\, dx\, dz\, dy$ **27.** $\pi - \frac{4}{3}$

29. $8 \sin^2 2 = 4(1 - \cos 4)$ **31.** $\frac{848}{9}$ **33.** $\frac{8}{15}$ **35.** $\frac{16}{3}$

37. $\frac{128}{3}$ **39.** $\frac{\pi}{6} - \frac{\sqrt{3}}{2} + \frac{1}{2}$ **41.** $\frac{1}{3}$ **43.** $\frac{1}{3}$ **45.** π

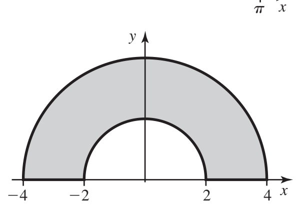
47. 4π **49.** $\frac{28\pi}{3}$ **51.** $\frac{2048\pi}{105}$

53.



$(\bar{x}, \bar{y}) = \left(\frac{\pi}{2}, \frac{\pi}{8}\right)$

55.



$(\bar{x}, \bar{y}) = \left(0, \frac{56}{9\pi}\right)$

57. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 24)$ **59.** $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{63}{10}\right)$

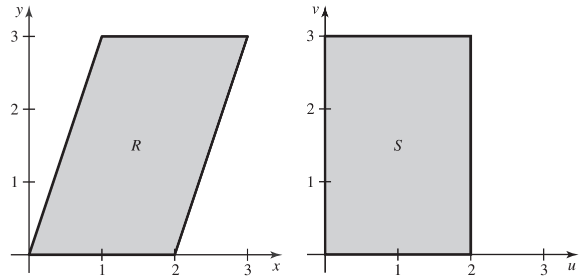
61. $\frac{h}{3}$ **63.** $\left(\frac{4\sqrt{2}a}{3\pi}, \frac{4(2-\sqrt{2})a}{3\pi}\right)$ **65. a.** $\frac{4\pi}{3}$ **b.** $\frac{16Q}{3}$

67. $R = \{(x, y): 0 \le x \le 1, 0 \le y \le 1\}$

69. The parallelogram with vertices $(0, 0)$, $(3, 1)$, $(4, 4)$, and $(1, 3)$

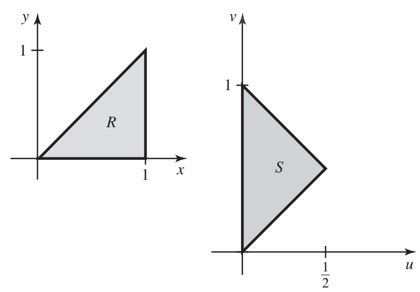
71. 10 **73.** 6

75. a.



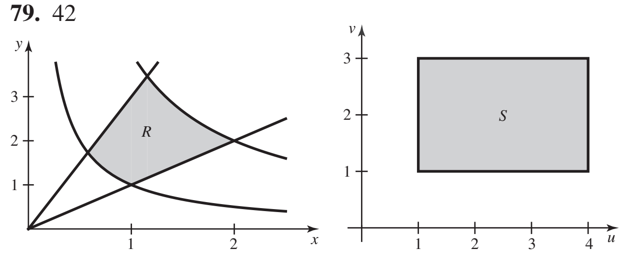
b. $0 \le u \le 2, 0 \le v \le 3$ **c.** $J(u, v) = 1$ **d.** $\frac{63}{2}$

77. a.



b. $u \le v \le 1 - u, 0 \le u \le \frac{1}{2}$ **c.** $J(u, v) = 2$ **d.** $\frac{1}{60}$

79. 42



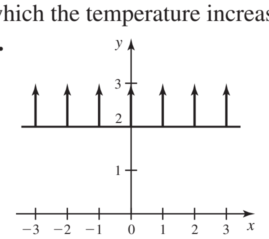
81. $-\frac{7}{16}$

CHAPTER 17

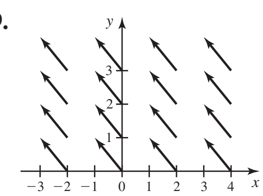
Section 17.1 Exercises, pp. 1096–1098

1. $\mathbf{F} = \langle f, g, h \rangle$ evaluated at (x, y, z) is the velocity vector of an air particle at (x, y, z) at a fixed point in time. **3.** At selected points (a, b), plot the vector $\langle f(a, b), g(a, b) \rangle$. **5.** It shows the direction in which the temperature increases the fastest and the amount of increase.

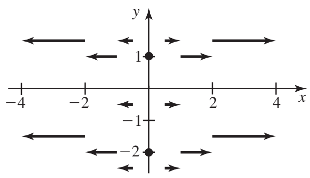
7.



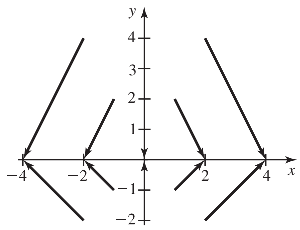
9.



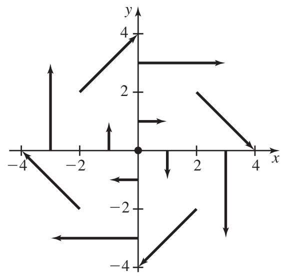
11.



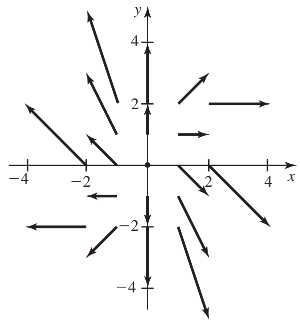
13.



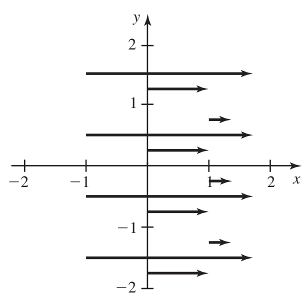
15.



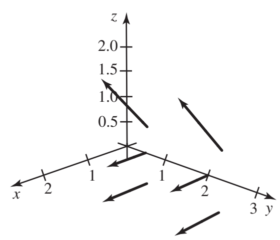
17.



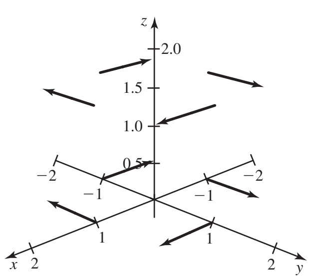
19.



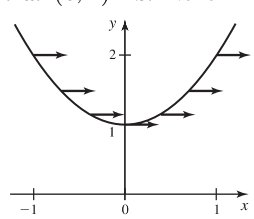
21.



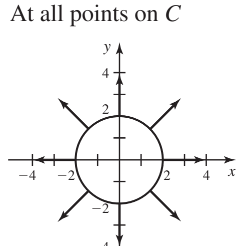
23.



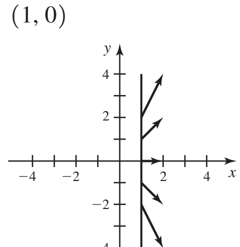
25. a. $(0, 1)$ **b.** None
c.



27. a. None
b. At all points on C
c.



29. a. None
b. $(1, 0)$
c.



31. $\mathbf{F} = \langle -y, x \rangle$ or $\mathbf{F} = \langle -1, 1 \rangle$

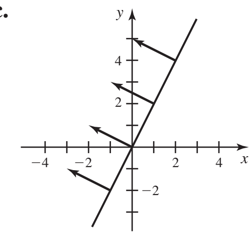
33. $\mathbf{F}(x, y) = \dfrac{\langle x, y \rangle}{\sqrt{x^2 + y^2}} = \dfrac{\mathbf{r}}{|\mathbf{r}|}, \mathbf{F}(0, 0) = \mathbf{0}$

35. $\nabla\varphi(x, y) = \langle 2xy - y^2, x^2 - 2xy \rangle$

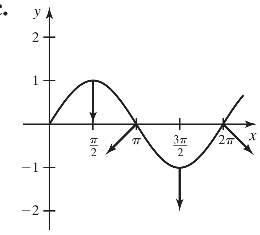
37. $\nabla\varphi(x, y) = \langle 1/y, -x/y^2 \rangle$ **39.** $\nabla\varphi(x, y, z) = \langle x, y, z \rangle = \mathbf{r}$

41. $\nabla\varphi(x, y, z) = -(x^2 + y^2 + z^2)^{-3/2} \langle x, y, z \rangle = -\dfrac{\mathbf{r}}{|\mathbf{r}|^3}$

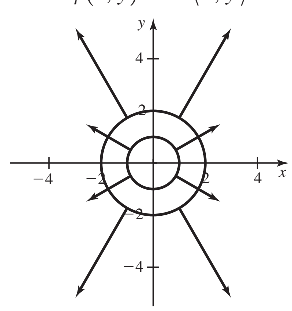
43. a. $\mathbf{F} = \langle -2, 1 \rangle$
b. $\mathbf{F}(-1, -2) = \mathbf{F}(0, 0) = \mathbf{F}(1, 2) = \mathbf{F}(2, 4) = \langle -2, 1 \rangle$
c.



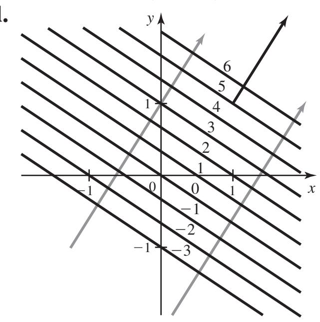
45. a. $\mathbf{F} = \langle \cos x, -1 \rangle$
b. $\mathbf{F}\left(\dfrac{\pi}{2}, 1\right) = \langle 0, -1 \rangle; \mathbf{F}(\pi, 0) = \langle -1, -1 \rangle;$
$\mathbf{F}\left(\dfrac{3\pi}{2}, -1\right) = \langle 0, -1 \rangle; \mathbf{F}(2\pi, 0) = \langle 1, -1 \rangle$
c.



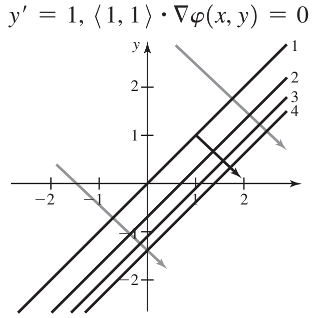
47. $\nabla\varphi(x, y) = 2\langle x, y \rangle$



49. a. $\nabla\varphi(x, y) = \langle 2, 3 \rangle$
b. $y' = -2/3, \left\langle 1, -\frac{2}{3} \right\rangle \cdot \nabla\varphi(1, 1) = 0$
c. $y' = -2/3, \left\langle 1, -\frac{2}{3} \right\rangle \cdot \nabla\varphi(x, y) = 0$
d.

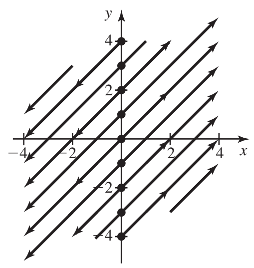


51. a. $\nabla\varphi(x, y) = \langle e^{x-y}, -e^{x-y} \rangle = e^{x-y} \langle 1, -1 \rangle$
b. $y' = 1, \langle 1, 1 \rangle \cdot \nabla\varphi(1, 1) = 0$
c. $y' = 1, \langle 1, 1 \rangle \cdot \nabla\varphi(x, y) = 0$
d.



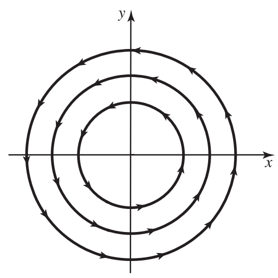
53. a. True **b.** False **c.** True **55. a.** $\mathbf{E} = \dfrac{c}{x^2 + y^2} \langle x, y \rangle$
b. $|\mathbf{E}| = \left| \dfrac{c}{|\mathbf{r}|^2} \mathbf{r} \right| = \dfrac{c}{r}$ **c.** *Hint:* The equipotential curves are circles centered at the origin. **57.** The slope of the streamline at (x, y) is $y'(x)$, which equals the slope of the vector $\mathbf{F}(x, y)$, which is g/f. Therefore, $y'(x) = g/f$.

59.



$y = x + C$

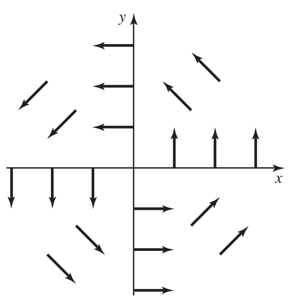
61.



$x^2 + y^2 = C$

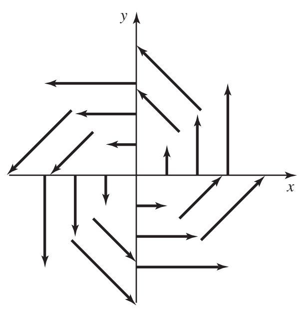
63. For $\theta = 0$: $\mathbf{u}_r = \mathbf{i}$ and $\mathbf{u}_\theta = \mathbf{j}$
for $\theta = \frac{\pi}{2}$: $\mathbf{u}_r = \mathbf{j}$ and $\mathbf{u}_\theta = -\mathbf{i}$
for $\theta = \pi$: $\mathbf{u}_r = -\mathbf{i}$ and $\mathbf{u}_\theta = -\mathbf{j}$
for $\theta = \frac{3\pi}{2}$: $\mathbf{u}_r = -\mathbf{j}$ and $\mathbf{u}_\theta = \mathbf{i}$

65.



$\mathbf{F} = \dfrac{1}{\sqrt{x^2 + y^2}} \langle -y, x \rangle$

67.



$\mathbf{F} = r\,\mathbf{u}_\theta$

Section 17.2 Exercises, pp. 1110–1114

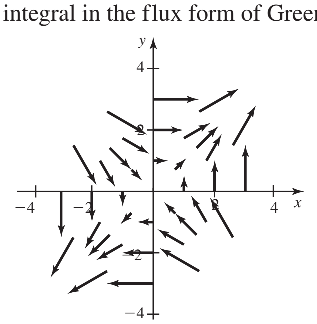
1. A line integral is taken along a curve; an ordinary single-variable integral is taken along an interval. **3.** $\displaystyle\int_{\pi/2}^{\pi} \frac{1}{t} \cos t \sqrt{\sin^2 t + 1}\, dt$
5. $\mathbf{r}(t) = \langle 1 + 4t, 2 + 2t, 3 - 3t \rangle$, for $0 \le t \le 1$
7. $\mathbf{r}(t) = \langle t^2 + 1, t \rangle$, for $2 \le t \le 4$
9. **a.** $\displaystyle\int_0^2 (t + 6t^5)\,dt$ **b.** 66 **11.** $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ and $\displaystyle\int_C f\,dx + g\,dy + h\,dz$
13. 7 **15.** Take the line integral of $\mathbf{F} \cdot \mathbf{T}$ along the curve with arc length as the parameter. **17.** 0 **19.** 100 **21.** 8 **23.** $-40\pi^2$
25. 128π **27.** $\dfrac{\sqrt{2}}{2} \ln 10$ **29.** $\dfrac{112}{9}$ **31.** 8 **33.** 414 **35.** 409.5
37. $\dfrac{15}{2}$ **39.** $\sqrt{101}$ **41.** $\dfrac{17}{2}$ **43.** 49 **45.** $\dfrac{3}{4\sqrt{10}}$ **47.** **a.** Negative
b. Positive **49.** 0 **51.** 16 **53.** 0 **55.** $\dfrac{3\sqrt{3}}{10}$ **57.** **b.** 0
59. **a.** Negative **b.** -4π **61.** **a.** True **b.** True **c.** True **d.** True
63. **a.** Both paths require the same work: $W = 28{,}200$.
b. Both paths require the same work: $W = 28{,}200$.
65. **a.** $\dfrac{5\sqrt{5} - 1}{12}$ **b.** $\dfrac{5\sqrt{5} - 1}{12}$ **c.** The results are identical.
67. The work equals zero for all three paths.
69. $8\pi(48 + 7\pi - 128\pi^2) \approx -29{,}991.4$ **71.** 2π **73.** **a.** 4 **b.** -4
e. 0 **75.** *Hint:* Show that $\displaystyle\int_C \mathbf{F} \cdot \mathbf{T}\, ds = \pi r^2 (c - b)$.
77. *Hint:* Show that $\displaystyle\int_C \mathbf{F} \cdot \mathbf{n}\, ds = \pi r^2 (a + d)$. **79.** **a.** $\ln a$ **b.** No
c. $\dfrac{1}{6}\left(1 - \dfrac{1}{a^2}\right)$ **d.** Yes **e.** $W = \dfrac{3^{1-p/2}}{2 - p}(a^{2-p} - 1)$, for $p \ne 2$; otherwise, $W = \ln a$. **f.** $p > 2$ **81.** ab

Section 17.3 Exercises, pp. 1121–1123

1. A simple curve has no self-intersections; the initial and terminal points of a closed curve are identical. **3.** Test for equality of partial derivatives as given in Theorem 17.3. **5.** Integrate f with respect to x and make the constant of integration a function of y to obtain $\varphi = \int f\,dx + h(y)$; finally, set $\frac{\partial\varphi}{\partial y} = g$ to determine h. **7.** 0
9. Conservative **11.** Not conservative **13.** Conservative
15. Conservative **17.** $\varphi(x, y) = \frac{1}{2}(x^2 + y^2)$ **19.** Not conservative
21. $\varphi(x, y) = \sqrt{x^2 + y^2}$ **23.** $\varphi(x, y, z) = xz + y$
25. Not conservative **27.** $\varphi(x, y, z) = xy + yz + zx$
29. $\varphi(x, y) = \sqrt{x^2 + y^2 + z^2}$ **31.** **a, b.** 0 **33.** **a, b.** 2 **35.** 3
37. -10 **39.** 24 **41.** $-\frac{1}{2}$ **43.** $-\pi^2$ **45.** 0 **47.** 0 **49.** 0
51. -5 **53.** 0 **55.** 1 **57.** **a.** False **b.** True **c.** True **d.** True
e. True **59.** 10 **61.** 25 **63.** **a.** Negative **b.** Positive **c.** No
67. **a.** Compare partial derivatives.
b. $\varphi(x, y, z) = \dfrac{GMm}{\sqrt{x^2 + y^2 + z^2}} = \dfrac{GMm}{|\mathbf{r}|}$
c. $\varphi(B) - \varphi(A) = GMm\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right)$ **d.** No
69. **a.** $\dfrac{\partial}{\partial y}\left(\dfrac{-y}{(x^2 + y^2)^{p/2}}\right) = \dfrac{-x^2 + (p - 1)y^2}{(x^2 + y^2)^{1+p/2}}$ and
$\dfrac{\partial}{\partial x}\left(\dfrac{x}{(x^2 + y^2)^{p/2}}\right) = \dfrac{(1 - p)x^2 + y^2}{(x^2 + y^2)^{1+p/2}}$
b. The two partial derivatives in (a) are equal if $p = 2$.
c. $\varphi(x, y) = \tan^{-1}(y/x)$ **73.** $\varphi(x, y) = \frac{1}{2}(x^2 + y^2)$
75. $\varphi(x, y) = \frac{1}{2}(x^4 + x^2y^2 + y^4)$

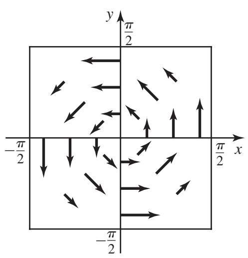
Section 17.4 Exercises, pp. 1133–1136

1. In both forms, the integral of a *derivative* is computed from boundary data. **3.** Area $= \frac{1}{2}\oint_C x\,dy - y\,dx$, where C encloses the region **5.** The integral in the flux form of Green's Theorem vanishes.
7. $\mathbf{F} = \langle y, x \rangle$

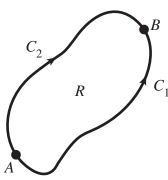


9. **a.** 0 **b.** 2 **c.** Yes **d.** No **11.** **a.** -4 **b.** 0 **c.** No **d.** Yes
13. **a.** y^2 **b.** $12x^2y + 2xy$ **c.** No **d.** No **15.** **a.** 1; no
b. $\mathbf{r}_1(t) = \langle t, t^2 \rangle$, for $0 \le t \le 1$, and $\mathbf{r}_2(t) = \langle 1 - t, 1 - t \rangle$, for $0 \le t \le 1$ (answers may vary) **c.** Both integrals equal $\dfrac{1}{6}$. **d.** 0
17. **a.** -4 **b.** Both integrals equal -8. **19.** **a.** $4x$ **b.** $\dfrac{16}{3}$
21. 25π **23.** 16π **25.** 32 **27.** **a.** 2 **b.** Both integrals equal 8π.
29. **a.** $2y$ **b.** $\dfrac{16}{15}$ **31.** 104 **33.** $\dfrac{31 - 3e^4}{6}$ **35.** 6 **37.** $\dfrac{8}{3}$
39. $8 - \dfrac{\pi}{2}$ **41.** **a.** 0 **b.** 3π **43.** **a.** 0 **b.** $-\dfrac{15\pi}{2}$ **45.** **a.** 0
b. 2π **47.** **a.** $\dfrac{16}{3}$ **b.** 0 **49.** **a.** True **b.** False **c.** True

51. Note: $\dfrac{\partial f}{\partial y} = 0 = \dfrac{\partial g}{\partial x}$ **53.** The integral becomes $\iint_R 2\,dA$.
55. a. $f_x = g_y = 0$ **b.** $\psi(x, y) = -2x + 4y$
57. a. $f_x = e^{-x}\sin y = -g_y$ **b.** $\psi(x, y) = e^{-x}\cos y$
59. a. *Hint:* $f_x = e^x\cos y$, $f_y = -e^x\sin y$,
$g_x = -e^x\sin y$, $g_y = -e^x\cos y$
b. $\varphi(x, y) = e^x\cos y$, $\psi(x, y) = e^x\sin y$
61. a. *Hint:* $f_x = -\dfrac{y}{x^2 + y^2}$, $f_y = \dfrac{x}{x^2 + y^2}$,
$g_x = \dfrac{x}{x^2 + y^2}$, $g_y = \dfrac{y}{x^2 + y^2}$
b. $\varphi(x, y) = x\tan^{-1}\dfrac{y}{x} + \dfrac{y}{2}\ln(x^2 + y^2) - y$,
$\psi(x, y) = y\tan^{-1}\dfrac{y}{x} - \dfrac{x}{2}\ln(x^2 + y^2) + x$
63. a.



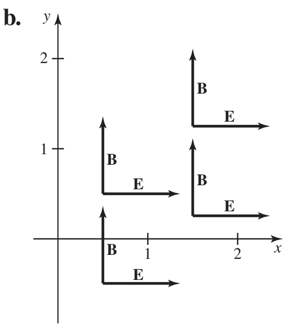
$\mathbf{F} = \langle -4\cos x\sin y, 4\sin x\cos y\rangle$ **b.** Yes, the divergence equals zero. **c.** No, the two-dimensional curl equals $8\cos x\cos y$. **d.** 0 **e.** 32
67. c. The vector field is undefined at the origin.
69.



Basic ideas: Let C_1 and C_2 be two smooth simple curves from A to B.
$$\int_{C_1}\mathbf{F}\cdot\mathbf{n}\,ds - \int_{C_2}\mathbf{F}\cdot\mathbf{n}\,ds = \oint_C \mathbf{F}\cdot\mathbf{n}\,ds = \iint_R (f_x + g_y)\,dA = 0$$
and $\displaystyle\int_{C_1}\mathbf{F}\cdot\mathbf{n}\,ds = \int_{C_1}\psi_x\,dx + \psi_y\,dy = \int_{C_1} d\psi = \psi(B) - \psi(A)$
71. Use $\nabla\varphi\cdot\nabla\psi = \langle f, g\rangle\cdot\langle -g, f\rangle = 0$.

Section 17.5 Exercises, pp. 1143–1146

1. Compute $f_x + g_y + h_z$. **3.** There is no source or sink. **5.** It indicates the axis and the angular speed of the circulation at a point. **7.** 0 **9.** 3 **11.** 0 **13.** $2(x + y + z)$
15. $\dfrac{x^2 + y^2 + 3}{(1 + x^2 + y^2)^2}$ **17.** $\dfrac{1}{|\mathbf{r}|^2}$ **19.** $-\dfrac{1}{|\mathbf{r}|^4}$ **21. a.** Positive for both points **b.** div $\mathbf{F} = 2$ **c.** Outward everywhere **d.** Positive
23. a. curl $\mathbf{F} = 2\mathbf{i}$ **b.** $|\text{curl }\mathbf{F}| = 2$
25. a. curl $\mathbf{F} = 2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ **b.** $|\text{curl }\mathbf{F}| = 2\sqrt{3}$ **27.** $3y\,\mathbf{k}$
29. $-4z\,\mathbf{j}$ **31. 0** **33. 0** **35.** Follows from partial differentiation of $\dfrac{1}{(x^2 + y^2 + z^2)^{3/2}}$ **37.** Combine Exercise 36 with Theorem 17.10.
39. a. False **b.** False **c.** False **d.** False **e.** False **41. a.** No **b.** No **c.** Yes, scalar function **d.** No **e.** No **f.** No **g.** Yes, vector field **h.** No **i.** Yes, vector field **43. a.** At $(0, 1, 1)$, $\mathbf{F}$ points in the positive x-direction; at $(1, 1, 0)$, $\mathbf{F}$ points in the negative z-direction; at $(0, 1, -1)$, $\mathbf{F}$ points in the negative x-direction; and at $(-1, 1, 0)$, $\mathbf{F}$ points in the positive z-direction. These vectors circle the y-axis in the counterclockwise direction looking along $\mathbf{a}$ from head to tail. **b.** The argument in part (a) can be repeated in any plane perpendicular to the y-axis to show that the vectors of $\mathbf{F}$ circle the y-axis in the counterclockwise direction looking along $\mathbf{a}$ from head to tail. Alternatively, computing the cross product, we find that $\mathbf{F} = \mathbf{a}\times\mathbf{r} = \langle z, 0, -x\rangle$, which is a rotation field in any plane perpendicular to $\mathbf{a}$. **45.** Compute an explicit expression for $\mathbf{a}\times\mathbf{r}$ and then take the required partial derivatives. **47.** div $\mathbf{F}$ has a maximum value of 6 at $(1, 1, 1)$, $(1, -1, -1)$, $(-1, 1, 1)$, and $(-1, -1, -1)$. **49.** $\mathbf{n} = \langle a, b, 2a + b\rangle$, where a and b are real numbers **51.** $\mathbf{F} = \frac{1}{2}(y^2 + z^2)\,\mathbf{i}$; no **53. a.** The wheel does not spin. **b.** Clockwise, looking in the positive y-direction **c.** The wheel does not spin. **55.** $\omega = \dfrac{10}{\sqrt{3}}$, or $\dfrac{5}{\sqrt{3}\pi} \approx 0.9189$ revolution per unit time
57. $\mathbf{F} = -200ke^{-x^2+y^2+z^2}(-x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k})$
$\nabla\cdot\mathbf{F} = -200k(1 + 2(x^2 + y^2 + z^2))e^{-x^2+y^2+z^2}$
59. a. $\mathbf{F} = -\dfrac{GMm\mathbf{r}}{|\mathbf{r}|^3}$ **b.** See Theorem 17.11.
61.
$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$
$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$
$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$
63. a. Use $\nabla\times\mathbf{B} = -Ak\cos(kz - \omega t)\,\mathbf{i}$ and $\dfrac{\partial\mathbf{E}}{\partial t} = -A\omega\cos(kz - \omega t)\,\mathbf{i}$. **b.**



Section 17.6 Exercises, pp. 1159–1161

1. $\mathbf{r}(u, v) = \langle a\cos u, a\sin u, v\rangle$, $0 \le u \le 2\pi$, $0 \le v \le h$
3. $\mathbf{r}(u, v) = \langle a\sin u\cos v, a\sin u\sin v, a\cos u\rangle$, $0 \le u \le \pi$, $0 \le v \le 2\pi$ **5.** Use the parameterization from Exercise 3 and compute $\displaystyle\int_0^{\pi}\int_0^{2\pi} f(a\sin u\cos v, a\sin u\sin v, a\cos u)\,a^2\sin u\,dv\,du$.
7. The normal vectors point outward. **9.** $\langle u, v, \frac{1}{3}(16 - 2u + 4v)\rangle$, $|u| < \infty$, $|v| < \infty$ **11.** $\langle v\cos u, v\sin u, v\rangle$, $0 \le u \le 2\pi$, $2 \le v \le 8$ **13.** $\langle 3\cos u, 3\sin u, v\rangle$, $0 \le u \le \dfrac{\pi}{2}$, $0 \le v \le 3$
15. The plane $z = 2x + 3y - 1$ **17.** Part of the upper half of the cone $z^2 = 16x^2 + 16y^2$ of height 12 and radius 3 (with $y \ge 0$)
19. 28π **21.** $16\sqrt{3}$ **23.** $\pi r\sqrt{r^2 + h^2}$ **25.** 1728π **27.** 0
29. 12 **31.** $4\pi\sqrt{5}$ **33.** $\dfrac{(65\sqrt{65} - 1)\pi}{24}$ **35.** $\dfrac{2\sqrt{3}}{3}$ **37.** $\dfrac{1250\pi}{3}$
39. $e - 1$ **41.** $\dfrac{1}{4\pi}$ **43.** -8 **45.** 0 **47.** 4π **49. a.** True **b.** False **c.** True **d.** True **51.** $8\pi(4\sqrt{17} + \ln(\sqrt{17} + 4))$
53. $8\pi a$ **55.** 8 **57. a.** 0 **b.** 0; the flow is tangent to the surface (radial flow). **59.** $2\pi ah$ **61.** $-400\left(e - \dfrac{1}{e}\right)^2$ **63.** $8\pi a$

65. a. $4\pi(b^3 - a^3)$ **b.** The net flux is zero. **67.** $\left(0, 0, \frac{2}{3}h\right)$

69. $\left(0, 0, \frac{7}{6}\right)$ **73.** Flux $= \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_R dA$

Section 17.7 Exercises, pp. 1169–1171

1. The integral measures the circulation along the closed curve C. **3.** Under certain conditions, the accumulated rotation of the vector field over the surface S equals the net circulation on the boundary of S. **5.** Both integrals equal -2π. **7.** Both integrals equal zero. **9.** Both integrals equal -18π. **11.** -24π **13.** $-\frac{128}{3}$ **15.** 15π **17.** 0 **19.** 0 **21.** -2π **23.** -4π **25.** $\nabla \times \mathbf{v} = \langle 1, 0, 0 \rangle$; a paddle wheel with its axis aligned with the x-axis will spin with maximum angular speed counterclockwise (looking in the negative x-direction) at all points. **27.** $\nabla \times \mathbf{v} = \langle 0, -2, 0 \rangle$; a paddle wheel with its axis aligned with the y-axis will spin with maximum angular speed clockwise (looking in the negative y-direction) at all points. **29. a.** False **b.** False **c.** True **d.** True **31.** 0 **33.** 0 **35.** 2π **37.** $\pi(\cos\varphi - \sin\varphi)$; maximum for $\varphi = 0$ **39.** The circulation is 48π; it depends on the radius of the circle but not on the center. **41. a.** The normal vectors point toward the z-axis on the curved surface of S and in the direction of $\langle 0, 1, 0 \rangle$ on the flat surface of S. **b.** 2π **c.** 2π **43.** The integral is π for all a. **45. a.** 0 **b.** 0 **47. b.** 2π for any circle of radius r centered at the origin **c. F** is not differentiable along the z-axis.

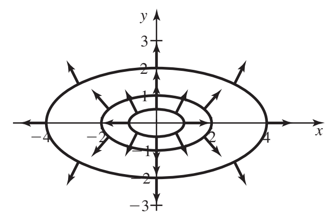
49. Apply the Chain Rule. **51.** $\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z}\right) dA$

Section 17.8 Exercises, pp. 1179–1182

1. The surface integral measures the flow across the boundary. **3.** The flux across the boundary equals the cumulative expansion or contraction of the vector field inside the region. **5.** 32π **7.** The outward fluxes are equal. **9.** Both integrals equal 96π. **11.** Both integrals equal zero. **13.** 0 **15.** 0 **17.** $16\sqrt{6}\pi$ **19.** $\frac{2}{3}$ **21.** $-\frac{128}{3}\pi$ **23.** 24π **25.** -224π **27.** 12π **29.** 20 **31. a.** False **b.** False **c.** True **33.** 0 **35.** $\frac{3}{2}$ **37. b.** The net flux between the two spheres is $4\pi(a^2 - \varepsilon^2)$. **39. b.** Use $\nabla \cdot \mathbf{E} = 0$. **c.** The flux across S is the sum of the contributions from the individual charges. **d.** For an arbitrary volume, we find $\frac{1}{\varepsilon_0}\iiint_D q(x, y, z)\, dV = \iint_S \mathbf{E} \cdot \mathbf{n}\, dS = \iiint_D \nabla \cdot \mathbf{E}\, dV$. **e.** Use $\nabla^2\varphi = \nabla \cdot \nabla\varphi$. **41.** 0 **43.** $e^{-1} - 1$ **45.** $800\pi a^3 e^{-a^2}$

Chapter 17 Review Exercises, pp. 1182–1184

1. a. False **b.** True **c.** False **d.** False **e.** True **3.** $\nabla\varphi = \langle 2x, 8y \rangle$



5. $-\frac{\mathbf{r}}{|\mathbf{r}|^3}$ **7.** $\mathbf{n} = \frac{1}{2}\langle x, y \rangle$ **9.** $\frac{7}{8}(e^{48} - 1)$ **11.** Both integrals equal zero. **13.** 0 **15.** The circulation is -4π; the outward flux is zero. **17.** The circulation is zero; the outward flux is 2π.

19. $\frac{4v_0 L^3}{3}$ **21.** $\varphi(x, y, z) = xy + yz^2$ **23.** $\varphi(x, y, z) = xye^z$ **25.** 0 for both methods **27. a.** $-\pi$ **b. F** is not conservative. **29.** 0 **31.** $\frac{20}{3}$ **33.** 8π **35.** The circulation is zero; the outward flux equals 2π. **37. a.** $b = c$ **b.** $a = -d$ **c.** $a = -d$ and $b = c$ **39.** $\nabla \cdot \mathbf{F} = 4\sqrt{x^2 + y^2 + z^2} = 4|\mathbf{r}|$, $\nabla \times \mathbf{F} = \mathbf{0}$, $\nabla \cdot \mathbf{F} \neq 0$; irrotational but not source free **41.** $\nabla \cdot \mathbf{F} = 2y + 12xz^2$, $\nabla \times \mathbf{F} = \mathbf{0}$, $\nabla \cdot \mathbf{F} \neq 0$; irrotational but not source free

43. a. -1 **b.** 0 **c.** $\mathbf{n} = \frac{1}{\sqrt{3}}\langle -1, 1, 1 \rangle$ **45.** 18π **47.** $4\sqrt{3}$

49. $\frac{8\sqrt{3}}{3}$ **51.** 8π **53.** $4\pi a^2$ **55. a.** Use $x = y = 0$ to confirm the highest point; use $z = 0$ to confirm the base. **b.** The hemisphere S has the greater surface area—$2\pi a^2$ for S versus $\frac{5\sqrt{5} - 1}{6}\pi a^2$ for T.

57. 0 **59.** 99π **61.** 0 **63.** $\frac{972}{5}\pi$ **65.** $\frac{124}{5}\pi$ **67.** $\frac{32}{3}$