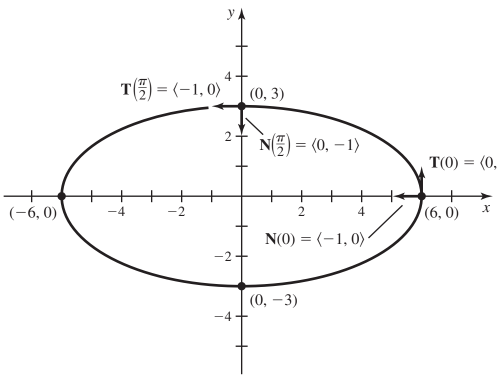
e.



53. a. $\mathbf{v}(t) = \langle -\sin t, -2\sin t, \sqrt{5}\cos t\rangle$,

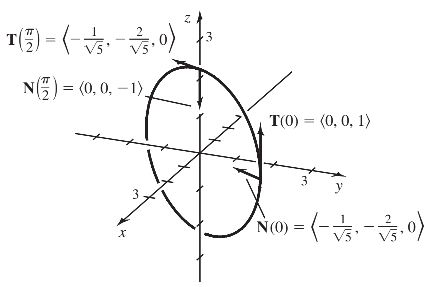
$\mathbf{T}(t) = \left\langle -\frac{1}{\sqrt{5}}\sin t, -\frac{2}{\sqrt{5}}\sin t, \cos t\right\rangle$ **b.** $\kappa(t) = \frac{1}{\sqrt{5}}$

c. $\mathbf{N}(t) = \left\langle -\frac{1}{\sqrt{5}}\cos t, -\frac{2}{\sqrt{5}}\cos t, -\sin t\right\rangle$

d. $|\mathbf{N}(t)| = \sqrt{\frac{1}{5}\cos^2 t + \frac{4}{5}\cos^2 t + \sin^2 t} = 1;$

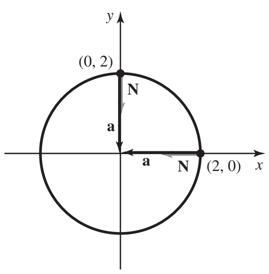
$\mathbf{T}\cdot\mathbf{N} = \left(\frac{1}{5}\cos t\sin t + \frac{4}{5}\cos t \sin t\right) - \sin t\cos t = 0$

e.



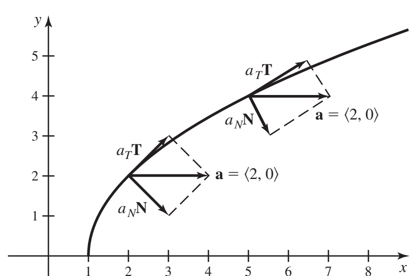
55. a. $\mathbf{a}(t) = 2\mathbf{N} + 0\mathbf{T} = 2\langle -\cos t, -\sin t\rangle$

b.



57. a. $a_T = \frac{2t}{\sqrt{t^2 + 1}}$ and $a_N = \frac{2}{\sqrt{t^2+1}}$

b.

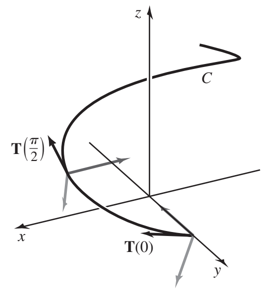


59. $\mathbf{B}(1) = \frac{\langle 3, -3, 1\rangle}{\sqrt{19}}; \tau = \frac{3}{19}$

61. a. $\mathbf{T}(t) = \frac{1}{5}\langle 3\cos t, -3\sin t, 4\rangle$

b. $\mathbf{N}(t) = \langle -\sin t, -\cos t, 0\rangle; \kappa = \frac{3}{25}$

c.



d. Yes

e. $\mathbf{B}(t) = \frac{1}{5}\langle 4\cos t, -4\sin t, -3\rangle$

f. See graph in part (c).

g. Check that **T**, **N**, and **B** have unit length and are mutually orthogonal.

h. $\tau = -\frac{4}{25}$

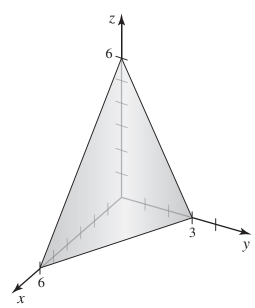
63. a. Consider first the case where $a_3 = b_3 = c_3 = 0$, and show that for all $s \neq t$ in I, $\mathbf{r}(t) \times \mathbf{r}(s)$ is a multiple of the constant vector $\langle b_1c_2 - b_2c_1, a_2c_1 - a_1c_2, a_1b_2 - a_2b_1\rangle$, which implies $\mathbf{r}(t) \times \mathbf{r}(s)$ is always orthogonal to the same vector, and therefore the vectors $\mathbf{r}(t)$ must all lie in the same plane. When a_3, b_3, and c_3 are not necessarily 0, the curve still lies in a plane because these constants represent a simple translation of the curve to a different location in $\mathbb{R}^3$.

b. Because the curve lies in a plane, **B** is always normal to the plane and has length 1. Therefore, $\frac{d\mathbf{B}}{ds} = \mathbf{0}$ and $\tau = 0$.

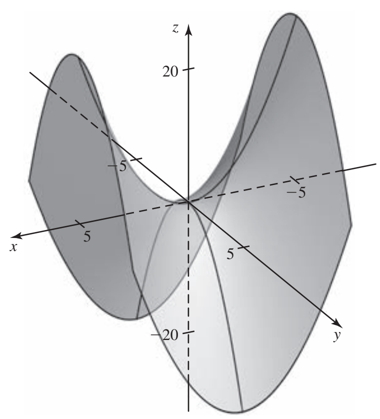
CHAPTER 15

Section 15.1 Exercises, pp. 927–930

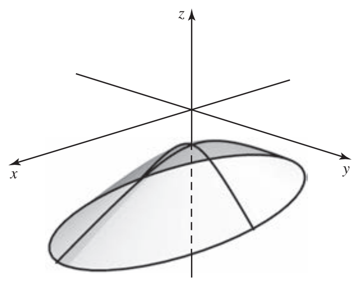
1. Independent: x and y; dependent: z
3. $D = \{(x, y): x \neq 0 \text{ and } y \neq 0\}$ **5.** Three **7.** 3; 4
9. a. 1300 ft **b.** Katie; Katie is 100 ft higher than Zeke. **11.** Circles
13. $n = 6$ **15.** $D = \mathbb{R}^2$ **17.** $D = \{(x, y): x^2 + y^2 \leq 25\}$
19. $D = \{(x, y): y \neq 0\}$ **21.** $D = \{(x, y): y < x^2\}$
23. $D = \{(x, y): xy \geq 0, (x, y) \neq (0, 0)\}$
25. Plane; $D = \mathbb{R}^2$, $R = \mathbb{R}$



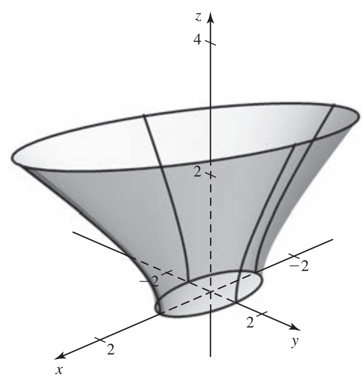
27. Hyperbolic paraboloid; $D = \mathbb{R}^2$, $R = \mathbb{R}$



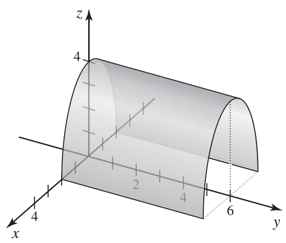
29. Lower part of a hyperboloid of two sheets;
$D = \mathbb{R}^2, R = (-\infty, -1]$



31. Upper half of a hyperboloid of one sheet;
$D = \{(x, y): x^2 + y^2 \geq 1\}, R = [0, \infty)$

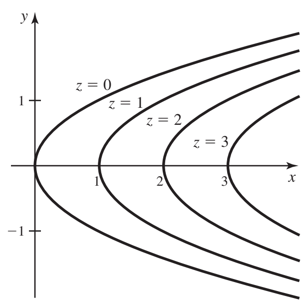


33. Upper half of an elliptical cylinder;
$D = \{(x, y): -2 \leq x \leq 2, -\infty < y < \infty\}, R = [0, 4]$

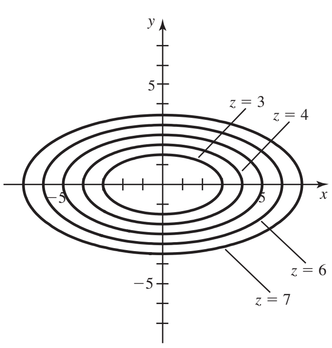


35. a. A **b.** D **c.** B **d.** C

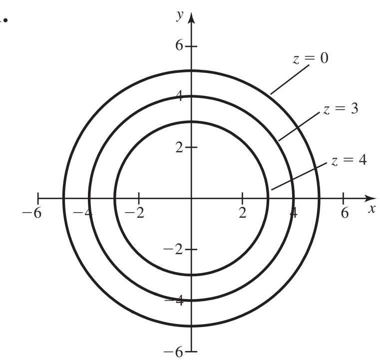
37.



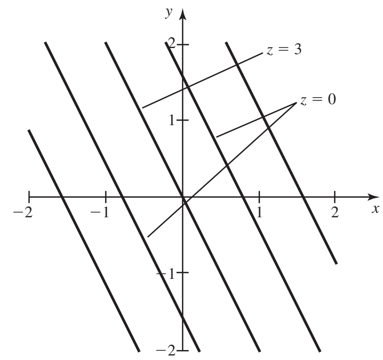
39.



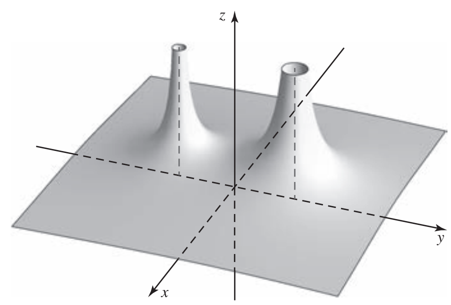
41.



43.

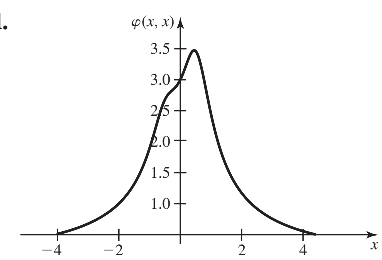


45. a.

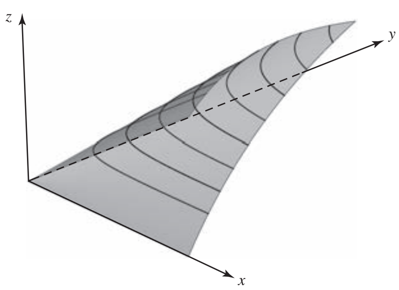


b. $\mathbb{R}^2$ excluding the points $(0, 1)$ and $(0, -1)$

c. $\varphi(2, 3)$ is greater. **d.**

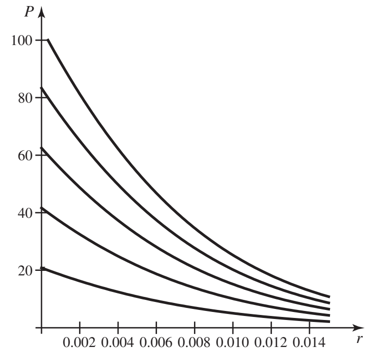


47. a.



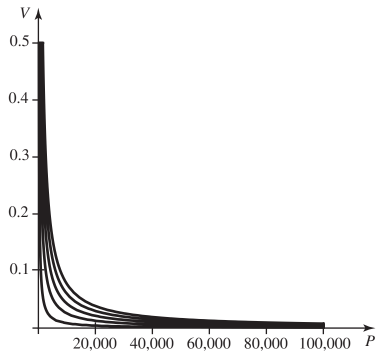
b. $R(10, 10) = 5$
c. $R(x, y) = R(y, x)$

49. a. $P = \dfrac{20{,}000r}{(1+r)^{240}-1}$ **b.** $P = \dfrac{Br}{(1+r)^{240}-1}$, with $B = 5000, 10{,}000, 15{,}000, 25{,}000$

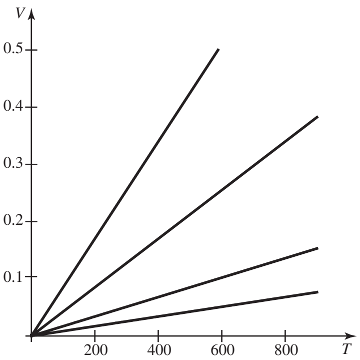


51. $D = \{(x, y, z): x \neq z\}$; all points not on the plane $x = z$
53. $D = \{(x, y, z): y \geq z\}$; all points on or below the plane $y = z$
55. $D = \{(x, y, z): x^2 \leq y\}$; all points on the side of the vertical cylinder $y = x^2$ that contains the positive y-axis **57. a.** False **b.** False **c.** True

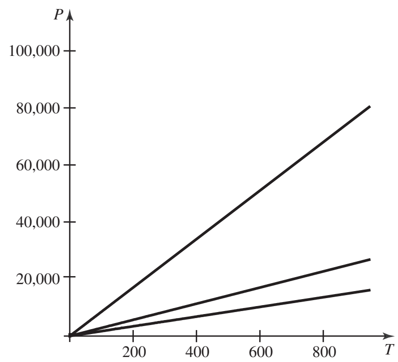
59. a.



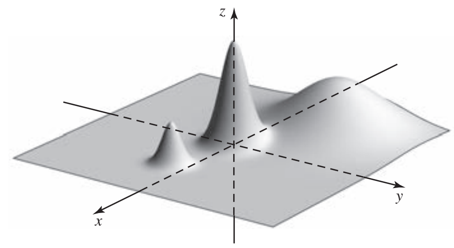
b.



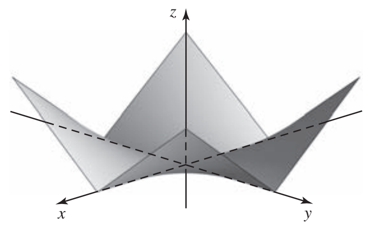
c.



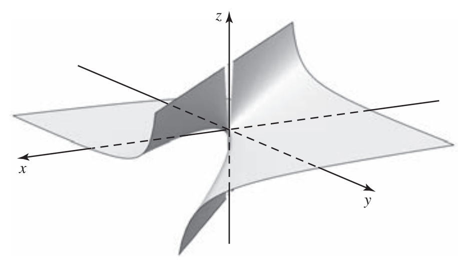
61. a.



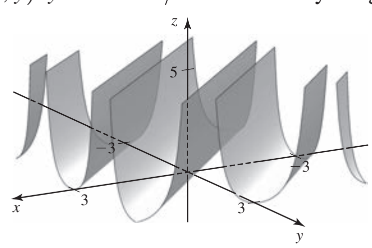
b. $(0, 0), (-5, 3), (4, -1)$
c. $f(0, 0) = 10.17, f(-5, 3) = 5.00, f(4, -1) = 4.00$
63. a. $D = \mathbb{R}^2, R = [0, \infty)$
b.



65. a. $D = \{(x, y): x \neq y\}, R = \mathbb{R}$
b.



67. a. $D = \{(x, y): y \neq x + \pi/2 + n\pi$ for any integer $n\}$, $R = [0, \infty)$ **b.**



69. Peak at the origin **71.** Depression at $(1, 0)$ **73.** The level curves are $ax + by = d - cz_0$, where z_0 is a constant, which are lines with slope $-a/b$ if $b \neq 0$ or vertical lines if $b = 0$.
75. $z = x^2 + y^2 - C$; paraboloids with vertices at $(0, 0, -C)$
77. $x^2 + 2z^2 = C$; elliptic cylinders parallel to the y-axis
79. $D = \{(x, y): x - 1 \leq y \leq x + 1\}$
81. $D = \{(x, y, z): (x \leq z \text{ and } y \geq -z) \text{ or } (x \geq z \text{ and } y \leq -z)\}$

Section 15.2 Exercises, pp. 937–939

1. The values of $f(x, y)$ are arbitrarily close to L for all (x, y) sufficiently close to (a, b). **3.** Because polynomials of n variables are continuous on all of $\mathbb{R}^n$, limits of polynomials can be evaluated with direct substitution. **5.** If the function approaches different values along different paths, the limit does not exist. **7.** f must be defined, the limit must exist, and the limit must equal the function value.
9. At any point where the denominator is nonzero **11.** 10 **13.** 101
15. 27 **17.** $1/(2\pi)$ **19.** 2 **21.** 6 **23.** -1 **25.** 2
27. $1/(2\sqrt{2}) = \sqrt{2}/4$ **29.** $L = 1$ along $y = 0$, and $L = -1$ along $x = 0$ **31.** $L = 1$ along $x = 0$, and $L = -2$ along $y = 0$
33. $L = 2$ along $y = x$, and $L = 0$ along $y = -x$ **35.** $\mathbb{R}^2$
37. All points except $(0, 0)$ **39.** $\{(x, y): x \neq 0\}$ **41.** All points except $(0, 0)$ **43.** $\mathbb{R}^2$ **45.** $\mathbb{R}^2$ **47.** $\mathbb{R}^2$ **49.** All points except $(0, 0)$ **51.** $\mathbb{R}^2$ **53.** $\mathbb{R}^2$ **55.** 6 **57.** -1 **59.** 2 **61. a.** False

b. False **c.** True **d.** False **63.** $\frac{1}{2}$ **65.** 0 **67.** Does not exist **69.** $\frac{1}{4}$ **71.** 1 **73.** 1 **75.** 5 **77.** $b = 1$ **79.** 0 **81.** 1 **85.** 0

Section 15.3 Exercises, pp. 948–951

1. $f_x(a, b)$ is the slope of the surface in the direction parallel to the positive x-axis, $f_y(a, b)$ is the slope of the surface in the direction parallel to the positive y-axis, both taken at (a, b). **3. a.** Negative **b.** Negative **c.** Negative **d.** Positive **5.** $f_x(x, y) = 6xy$; $f_y(x, y) = 3x^2$ **7.** $f_{xy} = 0 = f_{yx}$ **9.** $f_x(x, y, z) = y + z$; $f_y(x, y, z) = x + z$; $f_z(x, y, z) = x + y$

11. $f_x(x, y) = 5y$; $f_y(x, y) = 5x$ **13.** $f_x(x, y) = \dfrac{1}{y}$; $f_y(x, y) = -\dfrac{x}{y^2}$

15. $f_x(x, y) = e^y$; $f_y(x, y) = xe^y$

17. $f_x(x, y) = 2xye^{x^2y}$; $f_y(x, y) = x^2e^{x^2y}$

19. $f_w(w, z) = \dfrac{z^2 - w^2}{(w^2 + z^2)^2}$; $f_z(w, z) = -\dfrac{2wz}{(w^2 + z^2)^2}$

21. $f_x(x, y) = \cos xy - xy \sin xy$; $f_y(x, y) = -x^2 \sin xy$

23. $s_y(y, z) = z^3 \sec^2 yz$; $s_z(y, z) = 2z \tan yz + yz^2 \sec^2 yz$

25. $G_s(s, t) = \dfrac{\sqrt{st}(t - s)}{2s(s + t)^2}$; $G_t(s, t) = \dfrac{\sqrt{st}(s - t)}{2t(s + t)^2}$

27. $f_x(x, y) = 2yx^{2y-1}$; $f_y(x, y) = 2x^{2y} \ln x$

29. $h_x(x, y) = \dfrac{\sqrt{x^2 - 4y} - x}{\sqrt{x^2 - 4y}}$; $h_y(x, y) = \dfrac{2}{\sqrt{x^2 - 4y}}$

31. $f_x(x, y) = -e^{x^2}$; $f_y(x, y) = 3y^2e^{y^6}$

33. $f_x(x, y) = -\dfrac{2x}{1 + (x^2 + y^2)^2}$; $f_y(x, y) = -\dfrac{2y}{1 + (x^2 + y^2)^2}$

35. $h_x(x, y) = (1 + 2y)^x \ln(1 + 2y)$; $h_y(x, y) = 2x(1 + 2y)^{x-1}$

37. $f_x(x, y) = -h(x)$; $f_y(x, y) = h(y)$

39. $h_{xx}(x, y) = 6x$; $h_{xy}(x, y) = 2y = h_{yx}(x, y)$; $h_{yy}(x, y) = 2x$

41. $f_{xx}(x, y) = -16y^3 \sin 4x$; $f_{xy}(x, y) = 12y^2 \cos 4x = f_{yx}(x, y)$; $f_{yy}(x, y) = 6y \sin 4x$

43. $p_{uu}(u, v) = \dfrac{-2u^2 + 2v^2 + 8}{(u^2 + v^2 + 4)^2}$;

$p_{uv}(u, v) = -\dfrac{4uv}{(u^2 + v^2 + 4)^2} = p_{vu}(u, v)$;

$p_{vv}(u, v) = \dfrac{2u^2 - 2v^2 + 8}{(u^2 + v^2 + 4)^2}$

45. $F_{rr}(r, s) = 0$; $F_{rs}(r, s) = e^s = F_{sr}(r, s)$; $F_{ss}(r, s) = re^s$

47. $f_{xx}(x, y) = \dfrac{6xy^2(1 - 2x^6y^4)}{(1 + x^6y^4)^2}$;

$f_{xy}(x, y) = \dfrac{6x^2y(1 - x^6y^4)}{(1 + x^6y^4)^2} = f_{yx}(x, y)$;

$f_{yy}(x, y) = \dfrac{2x^3(1 - 3x^6y^4)}{(1 + x^6y^4)^2}$

49. $f_{xy} = e^{x+y} = f_{yx}$ **51.** $f_{xy} = -(xy \cos xy + \sin xy) = f_{yx}$

53. $f_{xy}(x, y) = -72y^2(2x - y^3)^2 = f_{yx}(x, y)$

55. $h_x(x, y, z) = h_y(x, y, z) = h_z(x, y, z) = -\sin(x + y + z)$

57. $F_u(u, v, w) = \dfrac{1}{v + w}$; $F_v(u, v, w) = F_w(u, v, w) = -\dfrac{u}{(v + w)^2}$

59. $G_r(r, s, t) = \dfrac{s^3t^5}{2\sqrt{rs^3t^5}}$;

$G_s(r, s, t) = \dfrac{3rs^2t^5}{2\sqrt{rs^3t^5}}$;

$G_t(r, s, t) = \dfrac{5rs^3t^4}{2\sqrt{rs^3t^5}}$

61. $h_w(w, x, y, z) = \dfrac{z}{xy}$; $h_x(w, x, y, z) = -\dfrac{wz}{x^2y}$;

$h_y(w, x, y, z) = -\dfrac{wz}{xy^2}$; $h_z(w, x, y, z) = \dfrac{w}{xy}$

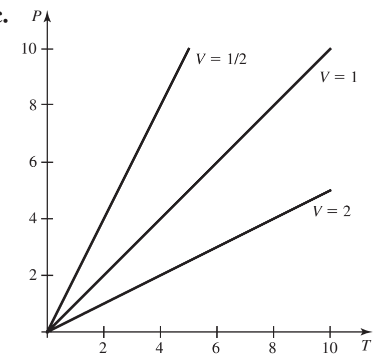
63. b. $g_x(x, y, z) = -\dfrac{8z}{3(2x - y + z)^2}$;

$g_y(x, y, z) = \dfrac{4z}{3(2x - y + z)^2}$;

$g_z(x, y, z) = \dfrac{4(2x - y)}{3(2x - y + z)^2}$

65. 1.41 **67.** 1.55 (answer will vary) **69. a.** $\dfrac{\partial V}{\partial P} = -\dfrac{kT}{P^2}$; volume decreases with pressure at fixed temperature. **b.** $\dfrac{\partial V}{\partial T} = \dfrac{k}{P}$; volume increases with temperature at fixed pressure.

c.



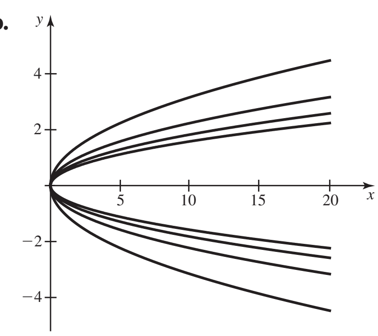
71. a. $\dfrac{\partial R}{\partial R_1} = \dfrac{R_2^2}{(R_1 + R_2)^2}$; $\dfrac{\partial R}{\partial R_2} = \dfrac{R_1^2}{(R_1 + R_2)^2}$

b. $\dfrac{\partial R}{\partial R_1} = \dfrac{R^2}{R_1^2}$; $\dfrac{\partial R}{\partial R_2} = \dfrac{R^2}{R_2^2}$ **c.** Increase **d.** Decrease

73. $u_t = -16e^{-4t} \cos 2x = u_{xx}$ **75.** $u_t = -a^2Ae^{-a^2t} \cos ax = u_{xx}$ **77. a.** No **b.** No **c.** $f_x(0, 0) = f_y(0, 0) = 0$ **d.** f_x and f_y are not continuous at $(0, 0)$. **79. a.** False **b.** False **c.** True

81. a. $z_x(x, y) = \dfrac{1}{y^2}$; $z_y(x, y) = -\dfrac{2x}{y^3}$

b.



c. z increases as x increases. **d.** z increases as y increases when $y < 0$, z is undefined for $y = 0$, and z decreases as y increases for $y > 0$.

83. a. $\varphi_x(x, y) = -\dfrac{2x}{(x^2 + (y - 1)^2)^{3/2}} - \dfrac{x}{(x^2 + (y + 1)^2)^{3/2}}$;

$\varphi_y(x, y) = -\dfrac{2(y - 1)}{(x^2 + (y - 1)^2)^{3/2}} - \dfrac{y + 1}{(x^2 + (y + 1)^2)^{3/2}}$

b. They both approach zero. **c.** $\varphi_x(0, y) = 0$

d. $\varphi_y(x, 0) = \dfrac{1}{(x^2 + 1)^{3/2}}$

87. $\dfrac{\partial^2 u}{\partial t^2} = -4c^2 \cos(2(x + ct)) = c^2 \dfrac{\partial^2 u}{\partial x^2}$

89. $\dfrac{\partial^2 u}{\partial t^2} = c^2 A f''(x + ct) + c^2 B g''(x - ct) = c^2 \dfrac{\partial^2 u}{\partial x^2}$

91. $u_{xx} = 6x$; $u_{yy} = -6x$

93. $u_{xx} = \dfrac{2(x-1)y}{((x-1)^2 + y^2)^2} - \dfrac{2(x+1)y}{((x+1)^2 + y^2)^2}$;

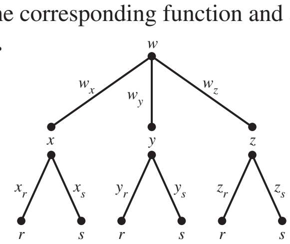
$u_{yy} = -\dfrac{2(x-1)y}{((x-1)^2 + y^2)^2} + \dfrac{2(x+1)y}{((x+1)^2 + y^2)^2}$

95. $\varepsilon_1 = \Delta y, \varepsilon_2 = 0$ or $\varepsilon_1 = 0, \varepsilon_2 = \Delta x$ **97. a.** f is continuous at $(0, 0)$. **b.** f is not differentiable at $(0, 0)$. **c.** $f_x(0, 0) = f_y(0, 0) = 0$ **d.** f_x and f_y are not continuous at $(0, 0)$. **e.** Theorem 15.5 does not apply because f_x and f_y are not continuous at $(0, 0)$; Theorem 15.6 does not apply because f is not differentiable at $(0, 0)$. **99.** $f_x(x, y) = yh(xy)$; $f_y(x, y) = xh(xy)$

Section 15.4 Exercises, pp. 957–961

1. One dependent, two intermediate, and one independent variable
3. Multiply each of the partial derivatives of w by the t-derivative of the corresponding function and add all these expressions.

5.



7. $4t^3 + 3t^2$

9. $z'(t) = 2t \sin 4t^3 + 12t^4 \cos 4t^3$

11. $w'(t) = -\sin t \sin 3t^4 + 12t^3 \cos t \cos 3t^4$

13. $z'(t) = 20(\sin^2 t + 2(3t + 4)^5)^9 (\sin t \cos t + 15(3t + 4)^4)$

15. $w'(t) = 20t^4 \sin(t + 1) + 4t^5 \cos(t + 1)$

17. $V'(t) = e^t((2t + 5)\sin t + (2t + 3)\cos t)$

19. $z_s = 2(s - t)\sin t^2$; $z_t = 2(s - t)(t(s - t)\cos t^2 - \sin t^2)$

21. $z_s = 2s - 3s^2 - 2st + t^2$; $z_t = -s^2 - 2t + 2st + 3t^2$

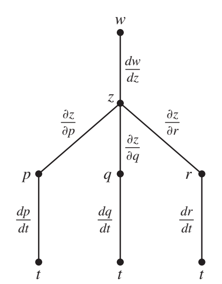
23. $z_s = (t + 1)e^{st+s+t}$; $z_t = (s + 1)e^{st+s+t}$

25. $w_s = -\dfrac{2t(t+1)}{(st + s - t)^2}$; $w_t = \dfrac{2s}{(st + s - t)^2}$

27. a. $V'(t) = 2\pi r(t)h(t)r'(t) + \pi r(t)^2 h'(t)$ **b.** $V'(t) = 0$ **c.** The volume remains constant.

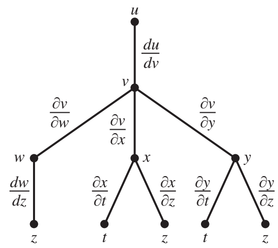
29. $z'(t) = -\dfrac{2t + 2}{(t + 2t)} - \dfrac{3t^2}{(t^3 - 2)}$

31.



$\dfrac{dw}{dt} = \dfrac{dw}{dz}\dfrac{\partial z}{\partial p}\dfrac{dp}{dt} + \dfrac{dw}{dz}\dfrac{\partial z}{\partial q}\dfrac{dq}{dt} + \dfrac{dw}{dz}\dfrac{\partial z}{\partial r}\dfrac{dr}{dt}$

33.



$\dfrac{\partial u}{\partial z} = \dfrac{du}{dv}\left(\dfrac{\partial v}{\partial w}\dfrac{dw}{dz} + \dfrac{\partial v}{\partial x}\dfrac{\partial x}{\partial z} + \dfrac{\partial v}{\partial y}\dfrac{\partial y}{\partial z}\right)$

35. $\dfrac{dy}{dx} = \dfrac{x}{2y}$ **37.** $\dfrac{dy}{dx} = -\dfrac{y}{x}$ **39.** $\dfrac{dy}{dx} = -\dfrac{x + y}{2y^3 + x}$

41. $\dfrac{\partial s}{\partial x} = \dfrac{2x}{\sqrt{x^2 + y^2}}$; $\dfrac{\partial s}{\partial y} = \dfrac{2y}{\sqrt{x^2 + y^2}}$

43. $f_{ss} = 2(3s + t)$; $f_{st} = 2(s - t)$; $f_{tt} = -2(s + 3t)$

45. $f_{ss} = \dfrac{4t^2(-3s^2 + t^2)}{(s^2 + t^2)^3}$; $f_{st} = \dfrac{8st(s^2 - t^2)}{(s^2 + t^2)^3}$; $f_{tt} = -\dfrac{4(s^4 - 3s^2t^2)}{(s^2 + t^2)^3}$

47. $f''(s) = 4\left(\dfrac{6}{s^4} - \dfrac{2}{s^3} - 1 - 9s + 9s^2\right)$ **49. a.** False **b.** False

51. $w'(t) = 0$ **53.** $\dfrac{\partial z}{\partial x} = -\dfrac{z^2}{x^2}$ **55. a.** $w'(t) = af_x + bf_y + cf_z$
b. $w'(t) = ayz + bxz + cxy = 3abct^2$

c. $w'(t) = \sqrt{a^2 + b^2 + c^2}\,\dfrac{t}{|t|}$

d. $w''(t) = a^2 f_{xx} + b^2 f_{yy} + c^2 f_{zz} + 2ab f_{xy} + 2ac f_{xz} + 2bc f_{yz}$

57. $\dfrac{\partial z}{\partial x} = -\dfrac{y + z}{x + y}$; $\dfrac{\partial z}{\partial y} = -\dfrac{x + z}{x + y}$ **59.** $\dfrac{\partial z}{\partial x} = -\dfrac{yz + 1}{xy - 1}$;

$\dfrac{\partial z}{\partial y} = -\dfrac{xz + 1}{xy - 1}$ **61. a.** $z'(t) = -2x \sin t + 8y \cos t = 3 \sin 2t$
b. $0 < t < \pi/2$ and $\pi < t < 3\pi/2$

63. a. $z'(t) = \dfrac{(x + y)e^{-t}}{\sqrt{1 - x^2 - y^2}} = \dfrac{2e^{-2t}}{\sqrt{1 - 2e^{-2t}}}$ **b.** All $t \geq \frac{1}{2}\ln 2$

65. $E'(t) = mx'x'' + my'y'' + mgy' = 0$
67. a. The volume increases. **b.** The volume decreases.

69. a. $\dfrac{\partial P}{\partial V} = -\dfrac{P}{V}$; $\dfrac{\partial T}{\partial P} = \dfrac{V}{k}$; $\dfrac{\partial V}{\partial T} = \dfrac{k}{P}$ **b.** Follows directly from part (a)

71. a. $w'(t) = \dfrac{2t(t^2 + 1)\cos 2t - (t^2 - 1)\sin 2t}{2(t^2 + 1)^2}$

b. Max value of $t \approx 0.838$, $(x, y, z) \approx (0.669, 0.743, 0.838)$

73. a. $z_x = \dfrac{x}{r} z_r - \dfrac{y}{r^2} z_\theta$; $z_y = \dfrac{y}{r} z_r + \dfrac{x}{r^2} z_\theta$

b. $z_{xx} = \dfrac{x^2}{r^2} z_{rr} + \dfrac{y^2}{r^4} z_{\theta\theta} - \dfrac{2xy}{r^3} z_{r\theta} + \dfrac{y^2}{r^3} z_r + \dfrac{2xy}{r^4} z_\theta$

c. $z_{yy} = \dfrac{y^2}{r^2} z_{rr} + \dfrac{x^2}{r^4} z_{\theta\theta} + \dfrac{2xy}{r^3} z_{r\theta} + \dfrac{x^2}{r^3} z_r - \dfrac{2xy}{r^4} z_\theta$

75. a. $\left(\dfrac{\partial z}{\partial x}\right)_y = -\dfrac{F_x}{F_z}$ **b.** $\left(\dfrac{\partial y}{\partial z}\right)_x = -\dfrac{F_z}{F_y}$; $\left(\dfrac{\partial x}{\partial y}\right)_z = -\dfrac{F_y}{F_x}$

d. $\left(\dfrac{\partial w}{\partial x}\right)_{y,z}\left(\dfrac{\partial z}{\partial w}\right)_{x,y}\left(\dfrac{\partial y}{\partial z}\right)_{x,w}\left(\dfrac{\partial x}{\partial y}\right)_{z,w} = 1$

77. a. $\left(\dfrac{\partial w}{\partial x}\right)_y = f_x + f_z \dfrac{dz}{dx} = 18$ **b.** $\left(\dfrac{\partial w}{\partial x}\right)_z = f_x + f_y \dfrac{dy}{dx} = 8$

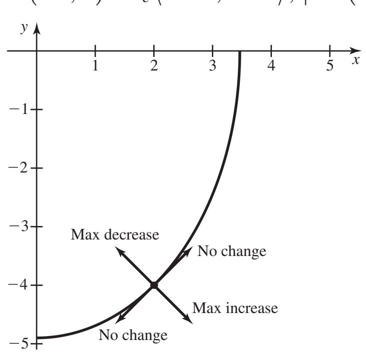
d. $\left(\dfrac{\partial w}{\partial y}\right)_x = -5$; $\left(\dfrac{\partial w}{\partial y}\right)_z = 4$; $\left(\dfrac{\partial w}{\partial z}\right)_x = \dfrac{5}{2}$; $\left(\dfrac{\partial w}{\partial z}\right)_y = \dfrac{9}{2}$

Section 15.5 Exercises, pp. 970–973

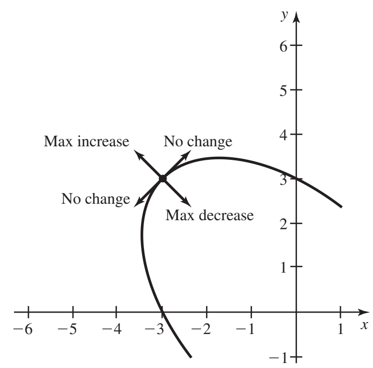
1. Form the dot product between the unit direction vector **u** and the gradient of the function. **3.** Direction of steepest ascent
5. The gradient is orthogonal to the level curves of f.
7. -2 **9.** $-7; 0$
11. a.

	$(a,b) = (2,0)$	$(a,b) = (0,2)$	$(a,b) = (1,1)$
$\mathbf{u} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$	$-\sqrt{2}$	$-2\sqrt{2}$	$-3\sqrt{2}/2$
$\mathbf{v} = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$	$\sqrt{2}$	$-2\sqrt{2}$	$-\sqrt{2}/2$
$\mathbf{w} = \left\langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$	$\sqrt{2}$	$2\sqrt{2}$	$3\sqrt{2}/2$

b. The function is decreasing at $(2, 0)$ in the direction of **u** and increasing at $(2, 0)$ in the directions of **v** and **w**.
13. $\nabla f(x, y) = \langle 6x, -10y \rangle, \nabla f(2, -1) = \langle 12, 10 \rangle$
15. $\nabla g(x, y) = \langle 2(x - 4xy - 4y^2), -4x(x + 4y) \rangle$, $\nabla g(-1, 2) = \langle -18, 28 \rangle$ **17.** $\nabla f(x, y) = e^{2xy} \langle 1 + 2xy, 2x^2 \rangle$, $\nabla f(1, 0) = \langle 1, 2 \rangle$ **19.** $\nabla F(x, y) = -2e^{-x^2-2y^2} \langle x, 2y \rangle$, $\nabla F(-1, 2) = 2e^{-9} \langle 1, -4 \rangle$ **21.** -6 **23.** $\frac{27}{2} - 6\sqrt{3}$
25. $-\dfrac{2}{\sqrt{5}}$ **27.** -2 **29.** 0 **31. a.** Direction of steepest ascent: $\dfrac{1}{\sqrt{65}}\langle 1, 8 \rangle$; direction of steepest descent: $-\dfrac{1}{\sqrt{65}}\langle 1, 8 \rangle$
b. $\langle -8, 1 \rangle$ **33. a.** Direction of steepest ascent: $\dfrac{1}{\sqrt{5}}\langle -2, 1 \rangle$; direction of steepest descent: $\dfrac{1}{\sqrt{5}}\langle 2, -1 \rangle$ **b.** $\langle 1, 2 \rangle$
35. a. Direction of steepest ascent: $\dfrac{1}{\sqrt{2}}\langle 1, -1 \rangle$; direction of steepest descent: $\dfrac{1}{\sqrt{2}}\langle -1, 1 \rangle$ **b.** $\langle 1, 1 \rangle$
37. a. $\nabla f(3, 2) = -12\mathbf{i} - 12\mathbf{j}$
b. Direction of max increase: $\theta = \dfrac{5\pi}{4}$; direction of max decrease: $\theta = \dfrac{\pi}{4}$; directions of no change: $\theta = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$
c. $g(\theta) = -12 \cos \theta - 12 \sin \theta$ **d.** $\theta = \frac{5\pi}{4}, g\left(\frac{5\pi}{4}\right) = 12\sqrt{2}$
e. $\nabla f(3, 2) = 12\sqrt{2} \left\langle \cos \frac{5\pi}{4}, \sin \frac{5\pi}{4} \right\rangle, |\nabla f(3, 2)| = 12\sqrt{2}$
39. a. $\nabla f(\sqrt{3}, 1) = \frac{\sqrt{6}}{6}\langle \sqrt{3}, 1 \rangle$ **b.** Direction of max increase: $\theta = \frac{\pi}{6}$; direction of max decrease: $\theta = \frac{7\pi}{6}$; directions of no change: $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$ **c.** $g(\theta) = \frac{\sqrt{2}}{2} \cos \theta + \frac{\sqrt{6}}{6} \sin \theta$ **d.** $\theta = \frac{\pi}{6}, g\left(\frac{\pi}{6}\right) = \frac{\sqrt{6}}{3}$
e. $\nabla f(\sqrt{3}, 1) = \frac{\sqrt{6}}{3}\langle \cos \frac{\pi}{6}, \sin \frac{\pi}{6} \rangle, |\nabla f(\sqrt{3}, 1)| = \frac{\sqrt{6}}{3}$
41. a. $\nabla F(-1, 0) = \frac{2}{e}\,\mathbf{i}$ **b.** Direction of max increase: $\theta = 0$; direction of max decrease: $\theta = \pi$; directions of no change: $\theta = \pm\frac{\pi}{2}$
c. $g(\theta) = \frac{2}{e} \cos \theta$ **d.** $\theta = 0, g(0) = \frac{2}{e}$
e. $\nabla F(-1, 0) = \frac{2}{e}\langle \cos 0, \sin 0 \rangle, |\nabla F(-1, 0)| = \frac{2}{e}$
43.

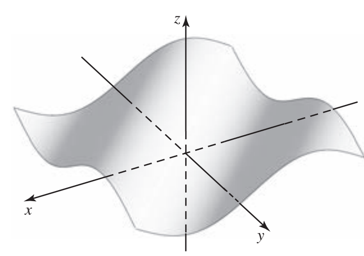


45.



47. $y' = 0$
49. Vertical tangent
51. $y' = -2/\sqrt{3}$
53. Vertical tangent

55. a. $\nabla f = \langle 1, 0 \rangle$ **b.** $x = 4 - t, y = 4, t \geq 0$
c. $\mathbf{r}(t) = \langle 4 - t, 4, 8 - t \rangle$, for $t \geq 0$
57. a. $\nabla f = \langle -2x, -4y \rangle$ **b.** $y = x^2, x \geq 1$
c. $\mathbf{r}(t) = \langle t, t^2, 4 - t^2 - 2t^4 \rangle$, for $t \geq 1$
59. a. $\nabla f(x, y, z) = 2x\,\mathbf{i} + 4y\,\mathbf{j} + 8z\,\mathbf{k}, \nabla f(1, 0, 4) = 2\mathbf{i} + 32\mathbf{k}$
b. $\dfrac{1}{\sqrt{257}}(\mathbf{i} + 16\mathbf{k})$ **c.** $2\sqrt{257}$ **d.** $17\sqrt{2}$
61. a. $\nabla f(x, y, z) = 4yz\,\mathbf{i} + 4xz\,\mathbf{j} + 4xy\,\mathbf{k}$, $\nabla f(1,-1,-1) = 4\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$ **b.** $\dfrac{1}{\sqrt{3}}(\mathbf{i} - \mathbf{j} - \mathbf{k})$ **c.** $4\sqrt{3}$
d. $\dfrac{4}{\sqrt{3}}$ **63. a.** $\nabla f(x, y, z) = \cos (x + 2y - z)(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$, $\nabla f\left(\dfrac{\pi}{6}, \dfrac{\pi}{6}, -\dfrac{\pi}{6}\right) = -\dfrac{1}{2}\mathbf{i} - \mathbf{j} + \dfrac{1}{2}\mathbf{k}$ **b.** $\dfrac{1}{\sqrt{6}}(-\mathbf{i} - 2\mathbf{j} + \mathbf{k})$
c. $\sqrt{6}/2$ **d.** $-\dfrac{1}{2}$
65. a. $\nabla f(x, y, z) = \dfrac{2}{1 + x^2 + y^2 + z^2}(x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k})$, $\nabla f(1, 1, -1) = \dfrac{1}{2}\mathbf{i} + \dfrac{1}{2}\mathbf{j} - \dfrac{1}{2}\mathbf{k}$ **b.** $\dfrac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} - \mathbf{k})$
c. $\dfrac{\sqrt{3}}{2}$ **d.** $\dfrac{5}{6}$ **67. a.** False **b.** False **c.** False **d.** True
69. $\pm\dfrac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$ **71.** $\pm\dfrac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$
73. $x = x_0 + at, y = y_0 + bt$ **75. a.** $\nabla f(x, y, z) = \langle 2x, 2y, 2z \rangle$, $\nabla f(1, 1, 1) = \langle 2, 2, 2 \rangle$ **b.** $x + y + z = 3$
77. a. $\nabla f(x, y, z) = e^{x+y-z}\langle 1, 1, -1 \rangle, \nabla f(1, 1, 2) = \langle 1, 1, -1 \rangle$
b. $x + y - z = 0$
79. a.



b. $\mathbf{v} = \pm\langle 1, 1 \rangle$
c. $\mathbf{v} = \pm\langle 1, -1 \rangle$

83. $\langle u, v \rangle = \langle \pi \cos \pi x \sin 2\pi y, 2\pi \sin \pi x \cos 2\pi y \rangle$
87. $\nabla f(x, y) = \dfrac{1}{(x^2 + y^2)^2}\langle y^2 - x^2 - 2xy, x^2 - y^2 - 2xy \rangle$
89. $\nabla f(x, y, z) = -\dfrac{1}{\sqrt{25 - x^2 - y^2 - z^2}}\langle x, y, z \rangle$
91.
$$\nabla f(x, y, z) = \frac{(y + xz)\langle 1, z, y \rangle - (x + yz)\langle z, 1, x \rangle}{(y + xz)^2} = \frac{1}{(y + xz)^2}\langle y(1 - z^2), x(z^2 - 1), y^2 - x^2 \rangle$$

Section 15.6 Exercises, pp. 980–983

1. The gradient of f is a multiple of $\mathbf{n}$.
3. $F_x(a,b,c)(x-a) + F_y(a,b,c)(y-b) + F_z(a,b,c)(z-c) = 0$
5. Multiply the change in x by $f_x(a,b)$ and the change in y by $f_y(a,b)$, and add both terms to f. **7.** $dz = f_x(x,y)\,dx + f_y(x,y)\,dy$
9. $z = 5x - 3y + 5$ **11.** $3x - y + 6z = 4$ **13.** $2x + y + z = 4$; $4x + y + z = 7$ **15.** $x + y + z = 6$; $3x + 4y + z = 12$
17. $z = -8x - 4y + 16$ and $z = 4x + 2y + 7$ **19.** $z = y + 1$ and $z = x + 1$ **21.** $x + \frac{1}{2}y + \sqrt{3}z = 2 + \frac{\sqrt{3}\pi}{6}$ and $\frac{1}{2}x + y + \sqrt{3}z = \frac{5\sqrt{3}\pi}{6} - 2$ **23.** $\frac{1}{2}x + \frac{2}{3}y + 2\sqrt{3}z = -2$ and $x - 2y + 2\sqrt{14}z = 2$ **25.** $z = 8x - 4y - 4$ and $z = -x - y - 1$
27. $z = \frac{7}{25}x - \frac{1}{25}y - \frac{2}{5}$ and $z = -\frac{7}{25}x + \frac{1}{25}y + \frac{6}{5}$
29. $z = \frac{1}{2}x + \frac{1}{2}y + \frac{\pi}{4} - 1$
31. $\frac{1}{6}(x - \pi) + \frac{\pi}{6}(y - 1) + \pi\left(z - \frac{1}{6}\right) = 0$
33. a. $L(x,y) = 4x + y - 6$ **b.** $L(2.1, 2.99) = 5.39$
35. a. $L(x,y) = -6x - 4y + 7$ **b.** $L(3.1, -1.04) = -7.44$
37. a. $L(x,y,z) = x + y + 2z$ **b.** $L(0.1, -0.2, 0.2) = 0.3$
39. $dz = -6dx - 5dy = -0.1$ **41.** $dz = dx + dy = 0.05$
43. a. The surface area decreases. **b.** Impossible to say **c.** $\Delta S \approx 53.3$ **d.** $\Delta S \approx 33.95$ **e.** $R\,dR = r\,dr$ **45.** $\frac{\Delta A}{A} \approx 3.5\%$
47. $dw = (y^2 + 2xz)\,dx + (2xy + z^2)\,dy + (x^2 + 2yz)\,dz$
49. $dw = \frac{dx}{y+z} - \frac{u+x}{(y+z)^2}dy - \frac{u+x}{(y+z)^2}dz + \frac{du}{y+z}$
51. a. $\Delta c \approx 0.035$ **b.** When $\theta = \frac{\pi}{20}$ **53. a.** True **b.** True **c.** False **55.** $(1, -1, 1)$ and $(1, -1, -1)$
57. Points with $x = 0, \pm\frac{\pi}{2}, \pm\pi$ and $y = \pm\frac{\pi}{2}$, or points with $x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$ and $y = 0, \pm\pi$
59. a. $\Delta S \approx 0.749$ **b.** More sensitive to changes in r
61. a. $\Delta A \approx \frac{2}{1225} = 0.00163$ **b.** No. The batting average increases more if the batter gets a hit than it decreases if he fails to get a hit. **c.** Yes. The answer depends on whether A is less than 0.500 or greater than 0.500. **63. a.** $\Delta V \approx \frac{21}{5000} = 0.0042$ **b.** $\frac{\Delta V}{V} \approx -4\%$ **c.** $2p$
65. a. $f_r = n(1-r)^{n-1}$, $f_n = -(1-r)^n \ln(1-r)$ **b.** $\Delta P \approx 0.027$ **c.** $\Delta P \approx 2 \times 10^{-20}$ **67.** $\Delta R \approx 7/540 \approx 0.013\Omega$
69. a. Apply the Chain Rule. **b.** Follows directly from (a) **c.** $d(\ln(xy)) = \frac{dx}{x} + \frac{dy}{y}$ **d.** $d(\ln(x/y)) = \frac{dx}{x} - \frac{dy}{y}$ **e.** $\frac{df}{f} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2} + \cdots + \frac{dx_n}{x_n}$

Section 15.7 Exercises, pp. 993–996

1. The local maximum occurs at the highest point on the surface; you cannot get to a higher point in any direction. **3.** The partial derivatives are both zero or do not exist. **5.** The discriminant is a determinant; it is defined as $D(a,b) = f_{xx}(a,b)\,f_{yy}(a,b) - f_{xy}^2(a,b)$.
7. f has an absolute minimum value on R at (a,b) if $f(a,b) \le f(x,y)$ for all (x,y) in R. **9.** Saddle point **11.** Local min **13.** $(0,0)$
15. $(0,1), (0,-1)$ **17.** $(0,0), (2,2)$, and $(-2,-2)$
19. $(0,2), (\pm 1, 2)$ **21.** $(3,0), (-15,6)$ **23.** Saddle point at $(0,0)$
25. Local min at $(0,0)$ **27.** Saddle point at $(0,0)$; local min at $(1,1)$ and $(-1,-1)$ **29.** $(0,0)$; Second Derivative Test is inconclusive; absolute min of 4 at $(0,0)$ **31.** Local min at $(2,0)$
33. Saddle point at $(0,0)$; local max at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$; local min at $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
35. Local min at $(-1,0)$; local max at $(1,0)$ **37.** Saddle point at $(0,1)$; local min at $(\pm 2, 0)$ **39.** Saddle point at $(0,0)$ **41.** Saddle point **43.** Height $= 32$ in, base is 16 in $\times$ 16 in; volume is 8192 in^3
45. 2 m $\times$ 2 m $\times$ 1 m **47.** Absolute min: $0 = f(0,1)$; absolute max: $9 = f(0,-2)$ **49.** Absolute min: $4 = f(0,0)$; absolute max: $7 = f(\pm 1, \pm 1)$ **51.** Absolute min: $0 = f(1,0)$; absolute max: $3 = f(1,1) = f(1,-1)$ **53.** Absolute min: $1 = f(1,-2) = f(1,0)$; absolute max: $4 = f(1,-1)$
55. Absolute min: $0 = f(0,0)$; absolute max: $\frac{7}{8} = f\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$
57. a. 1.83; 82.6° **b.** 14% **59.** Absolute min: $-4 = f(0,0)$; no absolute max on R **61.** Absolute max: $2 = f(0,0)$; no absolute min on R **63.** $P\left(\frac{4}{3}, \frac{2}{3}, \frac{4}{3}\right)$ **65.** $(3,4,5), (3,4,-5)$
67. a. True **b.** False **c.** True **d.** True
69. Local min at $(0.3, -0.3)$; saddle point at $(0,0)$
71. a.–d. $x = y = z = \frac{200}{3}$
73. a. $P\left(1, \frac{1}{3}\right)$ **b.** $P\left(\frac{1}{3}(x_1 + x_2 + x_3), \frac{1}{3}(y_1 + y_2 + y_3)\right)$ **c.** $P(\bar{x}, \bar{y})$, where $\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k$ and $\bar{y} = \frac{1}{n}\sum_{k=1}^{n} y_k$ **d.** $d(x,y) = \sqrt{x^2 + y^2} + \sqrt{(x-2)^2 + y^2} + \sqrt{(x-1)^2 + (y-1)^2}$. Absolute max: $1 + \sqrt{3} = f\left(1, \frac{1}{\sqrt{3}}\right)$
77. $y = \frac{22}{13}x + \frac{46}{13}$ **79.** $a = b = c = 3$
81. a. $\nabla d_1(x,y) = \frac{x - x_1}{d_1(x,y)}\mathbf{i} + \frac{y - y_1}{d_1(x,y)}\mathbf{j}$ **b.** $\nabla d_2(x,y) = \frac{x - x_2}{d_2(x,y)}\mathbf{i} + \frac{y - y_2}{d_2(x,y)}\mathbf{j}$; $\nabla d_3(x,y) = \frac{x - x_3}{d_3(x,y)}\mathbf{i} + \frac{y - y_3}{d_3(x,y)}\mathbf{j}$ **c.** Follows from $\nabla f = \nabla d_1 + \nabla d_2 + \nabla d_3$ **d.** Three unit vectors add to zero. **e.** P is the vertex at the large angle. **f.** $P(0.255457, 0.304504)$ **83. a.** Local max at $(1,0), (-1,0)$ **b.** Local max at $(1,0)$ and $(-1,0)$ **85.** $\frac{abc\sqrt{3}}{2}$

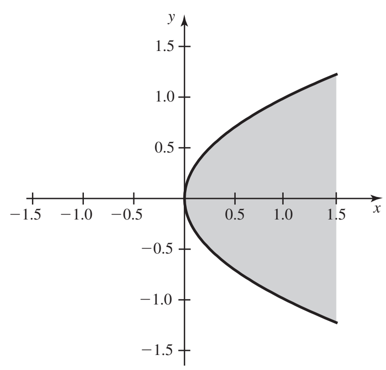
Section 15.8 Exercises, pp. 1002–1004

1. The level curve of f is tangent to the curve $g = 0$ at the optimal point; therefore, the gradients are parallel.
3. $1 = 2\lambda x$, $4 = 2\lambda y$, $x^2 + y^2 - 1 = 0$
5. Abs. min: 1; abs. max: 8 **7.** Abs. min: $-2\sqrt{5}$ at $\left(-\frac{2}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right)$; abs. max: $2\sqrt{5}$ at $\left(\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$
9. Abs. min: -2 at $(-1,-1)$; abs. max: 2 at $(1,1)$
11. Abs. min: -3 at $(-\sqrt{3}, \sqrt{3})$ and $(\sqrt{3}, -\sqrt{3})$; abs. max: 9 at $(3,3)$ and $(-3,-3)$
13. Abs. min: e^{-9} at $(-3,3)$ and $(3,-3)$; abs. max: e^3 at $(\sqrt{3}, \sqrt{3})$ and $(-\sqrt{3}, -\sqrt{3})$ **15.** Abs. min: 9 at $(0,3)$; abs. max: 34 at $(-\sqrt{15}, -2)$ and $(\sqrt{15}, -2)$ **17.** Abs. min: $-2\sqrt{11}$ at $\left(-\frac{2}{\sqrt{11}}, -\frac{6}{\sqrt{11}}, \frac{2}{\sqrt{11}}\right)$; abs. max: $2\sqrt{11}$ at $\left(\frac{2}{\sqrt{11}}, \frac{6}{\sqrt{11}}, -\frac{2}{\sqrt{11}}\right)$

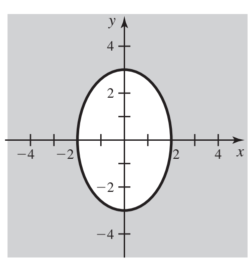
19. Abs. min: $-\frac{\sqrt{5}}{2}$ at $\left(-\frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right)$; abs. max: $\frac{\sqrt{5}}{2}$ at $\left(\frac{\sqrt{5}}{2}, 0, \frac{1}{2}\right)$
21. Abs. min: -5 at $(-2, -2, -1)$; abs. max: 5 at $(2, 2, 1)$
23. Abs. min: -10 at $(-5, 0, 0)$; abs. max: $\frac{29}{2}$ at $\left(2, 0, \pm\sqrt{\frac{21}{2}}\right)$
25. Abs. min: $-\sqrt{3}$ at $\left(0, -\frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right)$; abs. max: $\frac{7}{4}$ at $\left(\frac{1}{2}, \frac{1}{2}, 1\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}, 1\right)$ **27.** 18 in $\times$ 18 in $\times$ 36 in **29.** Abs. min: 0.6731; abs. max: 1.1230 **31.** 2×1 **33.** $\left(-\frac{3}{17}, \frac{29}{17}, -3\right)$
35. Abs. min: $\sqrt{38 - 6\sqrt{29}}$ (or $\sqrt{29} - 3$); abs. max: $\sqrt{38 + 6\sqrt{29}}$ (or $\sqrt{29} + 3$) **37.** $\ell = 3$ and $g = \frac{3}{2}$; $U = 15\sqrt{2}$
39. $\ell = \frac{16}{5}$ and $g = 1$; $U = 20.287$ **41. a.** True **b.** False
43. $\frac{\sqrt{6}}{3}$ m $\times$ $\frac{\sqrt{6}}{3}$ m $\times$ $\frac{\sqrt{6}}{6}$ m **45.** $2 \times 1 \times \frac{2}{3}$ **47.** $P\left(\frac{4}{3}, \frac{2}{3}, \frac{4}{3}\right)$
49. Abs. min: 1; abs. max: 9 **51.** Abs. min: 0; abs. max: 3
53. $K = 7.5$ and $L = 5$ **57.** Abs. max: 8 **59.** Abs. max: $\sqrt{a_1^2 + a_2^2 + a_3^2 + \cdots + a_n^2}$ **61. a.** Gradients are perpendicular to level surfaces. **b.** If the gradient were not in the plane spanned by ∇g and ∇h, f could be increased (decreased) by moving the point slightly. **c.** ∇f is a linear combination of ∇g and ∇h, since it belongs to the plane spanned by these two vectors. **d.** The gradient condition from part (c), as well as the constraints, must be satisfied.
63. Abs. min: $2 - 4\sqrt{2}$; abs. max: $2 + 4\sqrt{2}$
65. a. $y + 1 = \lambda y, x + 1 = \lambda x, xy = 4$ **c.** Abs. min of 108 over the curve C_1 **d.** Abs. max of 100 over the curve C_2
e. The constraint curve is unbounded, so there is no guarantee that an abs. min or max occurs over the curve $xy = 4$.

Chapter 15 Review Exercises, pp. 1005–1007

1. a. True **b.** False **c.** False **d.** False
3. $D = \{(x, y): x \geq y^2\}$

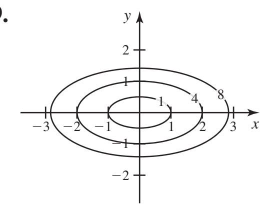


5. $D = \left\{(x, y): \frac{x^2}{4} + \frac{y^2}{9} \geq 1\right\}$



7. $D = \{(x, y): x^2 + y^2 \geq 1\}$; $R = (-\infty, 0]$; lower half of the hyperboloid of one sheet $x^2 + y^2 - z^2 = 1$

9.



11. 2 **13.** Does not exist
15. $\frac{2}{3}$ **17.** 4
19. $\{(x, y): y > x^2 + 1\}$
21. $f_x = 6xy^5$; $f_y = 15x^2y^4$

23. $f_x = \frac{2xy^2}{(x^2 + y^2)^2}$; $f_y = -\frac{2x^2y}{(x^2 + y^2)^2}$
25. $f_x = y(1 + xy)e^{xy}$; $f_y = x(1 + xy)e^{xy}$ **27.** $f_{xx} = 4y^2e^{2xy}$; $f_{xy} = 2e^{2xy}(2xy + 1) = f_{yx}$; $f_{yy} = 4x^2e^{2xy}$
29. $\frac{\partial^2 u}{\partial x^2} = 6y = -\frac{\partial^2 u}{\partial y^2}$ **31. a.** V increases with R if r is fixed, $V_R > 0$; V decreases if r increases and R is fixed, $V_r < 0$.
b. $V_r = -4\pi r^2$; $V_R = 4\pi R^2$ **c.** The volume increases more if R is increased. **33.** $4t + 2 \ln 5$
35. $w_r = \frac{3r + s}{r(r + s)}$; $w_s = \frac{r + 3s}{s(r + s)}$; $w_t = \frac{1}{t}$
37. $\frac{dy}{dx} = -\frac{2xy}{2y^2 + (x^2 + y^2) \ln (x^2 + y^2)}$
39. a. $z'(t) = -24 \sin t \cos t = -12 \sin 2t$
b. $z'(t) > 0$ for $\frac{\pi}{2} < t < \pi$ and $\frac{3\pi}{2} < t < 2\pi$
41. a.

	$(a, b) = (0, 0)$	$(a, b) = (2, 0)$	$(a, b) = (1, 1)$
$\mathbf{u} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$	0	$4\sqrt{2}$	$-2\sqrt{2}$
$\mathbf{v} = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$	0	$-4\sqrt{2}$	$-6\sqrt{2}$
$\mathbf{w} = \left\langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$	0	$-4\sqrt{2}$	$2\sqrt{2}$

b. The function is increasing at $(2, 0)$ in the direction of $\mathbf{u}$ and decreasing at $(2, 0)$ in the directions of $\mathbf{v}$ and $\mathbf{w}$.
43. $\nabla g = \langle 2xy^3, 3x^2y^2 \rangle$; $\nabla g(-1, 1) = \langle -2, 3 \rangle$; $D_{\mathbf{u}}\, g(-1, 1) = 2$
45. $\nabla h = \left\langle \frac{x}{\sqrt{2 + x^2 + 2y^2}}, \frac{2y}{\sqrt{2 + x^2 + 2y^2}} \right\rangle$; $\nabla h(2, 1) = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$; $D_{\mathbf{u}}\, h(2, 1) = \frac{7\sqrt{2}}{10}$
47. $\nabla f = \langle y \cos xy, x \cos xy, -\sin z \rangle$; $\nabla f(1, \pi, 0) = \langle -\pi, -1, 0 \rangle$; $D_{\mathbf{u}}\, f(1, \pi, 0) = -\frac{1}{7}(3 + 2\pi)$
49. a. Direction of steepest ascent: $\mathbf{u} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$; direction of steepest descent: $\mathbf{u} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$
b. No change: $\mathbf{u} = \pm\left(\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}\right)$
51. Tangent line is vertical; $\nabla f(2, 0) = -8\mathbf{i}$
53. $E = \frac{kx}{x^2 + y^2}\mathbf{i} + \frac{ky}{x^2 + y^2}\mathbf{j}$
55. $y = 2$ and $12x + 3y - 2z = 12$
57. $x + 2y + 3z = 6$ and $x - 2y + 3z = 6$
59. $x + y - z = 0$ and $x + y - z = 0$
61. a. $L(x, y) = x + 5y$ **b.** $L(1.95, 0.05) = 2.2$ **63.** Approx. -4%
65. a. $\Delta V \approx -0.1\pi \text{ m}^3$ **b.** $\Delta S \approx -0.05\pi \text{ m}^2$
67. Saddle point at $(0, 0)$; local min at $(2, -2)$

69. Saddle points at $(0, 0)$ and $(-2, 2)$; local max at $(0, 2)$; local min at $(-2, 0)$ **71.** Abs. min: $-1 = f(1, 1) = f(-1, -1)$; abs. max: $49 = f(2, -2) = f(-2, 2)$

73. Abs. min: $-\dfrac{1}{2} = f\left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$; abs. max: $\dfrac{1}{2} = f\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

75. Abs. min: $\dfrac{23}{2} = f\left(\dfrac{1}{3}, \dfrac{5}{6}\right)$ abs. max: $\dfrac{29}{2} = f\left(\dfrac{5}{3}, \dfrac{7}{6}\right)$;

77. Abs. min: $-\sqrt{6} = f\left(-\dfrac{\sqrt{6}}{6}, -\dfrac{\sqrt{6}}{3}, \dfrac{\sqrt{6}}{6}\right)$;

abs. max: $\sqrt{6} = f\left(\dfrac{\sqrt{6}}{6}, \dfrac{\sqrt{6}}{3}, -\dfrac{\sqrt{6}}{6}\right)$

79. $\dfrac{2a^2}{\sqrt{a^2 + b^2}}$ by $\dfrac{2b^2}{\sqrt{a^2 + b^2}}$

81. $x = \dfrac{1}{2} + \dfrac{\sqrt{10}}{20}, y = \dfrac{3}{2} + \dfrac{3\sqrt{10}}{20} = 3x, z = \dfrac{1}{2} + \dfrac{\sqrt{10}}{2} = \sqrt{10}x$

83. $(1, 2, 5)$

CHAPTER 16

Section 16.1 Exercises, pp. 1015–1017

1. $\int_0^2\int_1^3 xy\,dy\,dx$ or $\int_1^3\int_0^2 xy\,dx\,dy$ **3.** $\int_{-2}^4\int_1^5 f(x, y)\,dy\,dx$ or $\int_1^5\int_{-2}^4 f(x, y)\,dx\,dy$ **5.** 48 **7.** 4 **9.** $\dfrac{32}{3}$ **11.** 4 **13.** $\dfrac{224}{9}$

15. $10 - 2e$ **17.** $\dfrac{1}{2}$ **19.** $e^2 + 3$ **21.** $\dfrac{1}{2}$ **23.** $10\sqrt{5} - 4\sqrt{2} - 14$

25. $\dfrac{117}{2}$ **27.** $\dfrac{\pi^2}{4} + 1$ **29.** $\dfrac{4}{3}$ **31.** $\dfrac{9 - e^2}{2}$ **33.** $\dfrac{4}{11}$ **35.** $\dfrac{1}{4}$

37. 136 **39.** 3 **41.** $e^2 - 3$ **43.** $e^{16} - 17$ **45.** $\ln\dfrac{5}{3}$ **47.** $\dfrac{1}{2\ln 2}$

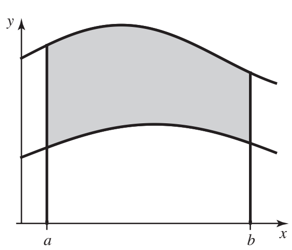
49. $\dfrac{8}{3}$ **51. a.** True **b.** False **c.** True **53. a.** 1475 **b.** The sum of products of population densities and areas is a Riemann sum.
55. $\int_c^d\int_a^b f(x)\,dy\,dx = (c - d)\int_a^b f(x)\,dx$. The integral is the area of the cross section of S. **57.** $a = \pi/6, 5\pi/6$ **59.** $a = \sqrt{6}$

61. a. $\frac{1}{2}\pi^2 + \pi$ **b.** $\frac{1}{2}\pi^2 + \pi$ **c.** $\frac{1}{2}\pi^2 + 2$

63. $f(a, b) - f(a, 0) - f(0, b) + f(0, 0)$

Section 16.2 Exercises, pp. 1024–1027

1.

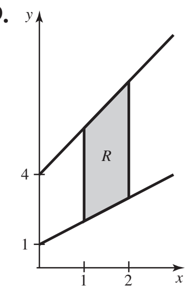


3. $dx\,dy$ **5.** $\int_0^1\int_{x^2}^{\sqrt{x}} f(x, y)\,dy\,dx$ **7.** 4 **9.** $\int_0^2\int_{x^3}^{4x} f(x, y)\,dy\,dx$

11. 2 **13.** $\dfrac{8}{3}$ **15.** 0 **17.** $e - 1$ **19.** $\dfrac{\ln^3 2}{6}$

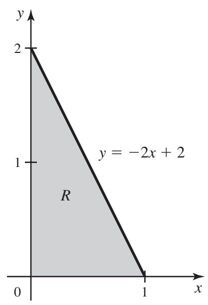
21. 2 **23.** $\dfrac{\pi}{2} - 1$ **25.** 0 **27.** $\pi - 1$

29.



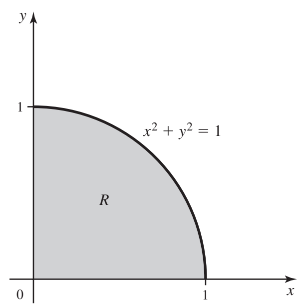
$\int_1^2\int_{x+1}^{2x+4} f(x, y)\,dy\,dx$

31.



$\int_0^1\int_0^{-2x+2} f(x, y)\,dy\,dx$

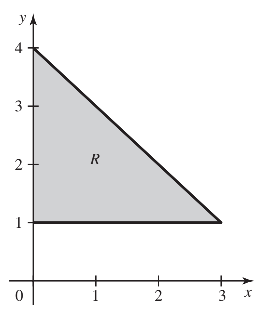
33.



$\int_0^1\int_0^{\sqrt{1-x^2}} f(x, y)\,dy\,dx$

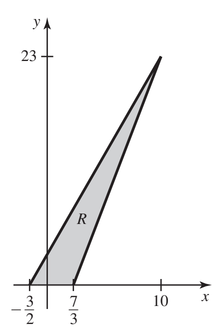
35. $\int_0^{18}\int_{y/2}^{(y+9)/3} f(x, y)\,dx\,dy$

37.



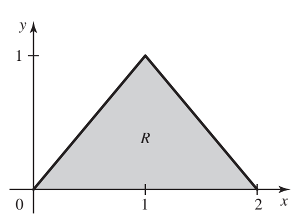
$\int_1^4\int_0^{4-y} f(x, y)\,dx\,dy$

39.



$\int_0^{23}\int_{(y-3)/2}^{(y+7)/3} f(x, y)\,dx\,dy$

41.



$\int_0^1\int_y^{2-y} f(x, y)\,dx\,dy$