Assignment Homework1 due 09/13/2021 at 11:59pm EDT

1. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/5_The_Integral/5.2_The_Definite_Integral/5.2.29.pg

Enter T or F depending on whether the statement is true or false. (You must enter T or F – True and False will not work.)

___1. The sign of the integral $\int_{-10}^{5} x^2 dx$ is negative.

Solution: (Instructor solution preview: show the student solution after due date.)

Solution:

The integrand is always positive. The integral must therefore be positive, since the signed area has only positive part.

Correct Answers:

F

2. (1 point) Library/Valdosta/APEX_Calculus/5.2/APEX_5.2_18-21.pg

Let

$$\int_0^2 f(x) dx = 3, \quad \int_0^3 f(x) dx = 12, \quad \int_0^2 g(x) dx = -1, \quad \int_2^3 g(x) dx = 9,$$

Use these values to evaluate the given definite integrals.

a)
$$\int_0^2 (f(x) + g(x)) dx =$$

b)
$$\int_0^3 (f(x) - g(x)) dx =$$

c)
$$\int_{2}^{3} (3f(x) + 2g(x)) dx =$$

d) Find the value a such that $\int_0^3 (af(x) + g(x)) dx = 0.$

Solution: (Instructor solution preview: show the student solution after due date.)

Solution:

a)

$$\int_0^2 (f(x) + g(x)) dx = \int_0^2 f(x) dx + \int_0^2 g(x) dx = 3 - 1 = 2.$$

b) First find the integral of g from 0 to 3.

$$\int_0^3 g(x) \, dx = \int_0^2 g(x) \, dx + \int_2^3 g(x) \, dx = -1 + 9 = 8.$$

Then

$$\int_0^3 (f(x) - g(x)) \, dx = \int_0^3 f(x) \, dx - \int_0^3 g(x) \, dx = 12 - 8 = 4.$$

c) First find the integral of f from 2 to 3.

$$\int_0^3 f(x) dx = \int_0^2 f(x) dx + \int_2^3 f(x) dx$$

$$12 = 3 + \int_2^3 f(x) dx$$

$$\int_2^3 f(x) dx = 9$$

Then

$$\int_{2}^{3} (3f(x) + 2g(x)) dx = 3 \int_{2}^{3} f(x) dx + 2 \int_{2}^{3} g(x) dx$$
$$= 3(9) + 2(9) = 45.$$

d) Substitute known values for the integrals.

$$\int_{0}^{3} (af(x) + g(x)) dx = 0$$

$$a \int_{0}^{3} f(x) dx + \int_{0}^{3} g(x) dx = 0$$

$$a(12) + 8 = 0$$

$$a = \frac{-8}{12}$$

Correct Answers:

- -0.666666666666667

3. (1 point) Library/UCSB/Stewart5_5_2/Stewart5_5_2_35.pg

Evaluate the following integral by interpreting it in terms of areas:

$$\int_0^3 \left(\frac{1}{2}x - 1\right) dx$$

Value of integral = Correct Answers:

- -.75
- 4. (1 point) Library/UCSB/Stewart5_5_2/Stewart5_5_2_68.pg

Which of the following correctly expresses the limit $\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+(i/n)^2}$, as a definite integral?

• A.
$$\int_0^1 \frac{2}{1+x^4} dx$$

• B.
$$\int_0^1 \frac{1}{1+x^3} dx$$

• C.
$$\int_0^1 \frac{1}{1+x^2} dx$$

• C.
$$\int_0^1 \frac{1+x^3}{1+x^2} dx$$

• D. $\int_0^1 \frac{1}{1+x^4} dx$
• E. $\int_0^1 \frac{2}{1+x^2} dx$

• E.
$$\int_0^1 \frac{2}{1+x^2} dx$$

• F.
$$\int_0^1 \frac{1+x^3}{1+x^3} dx$$

5. (1 point) Library/FortLewis/Calc1/05-02-Definite-integral/HGM5-05-02-Definite-integral-27.pg

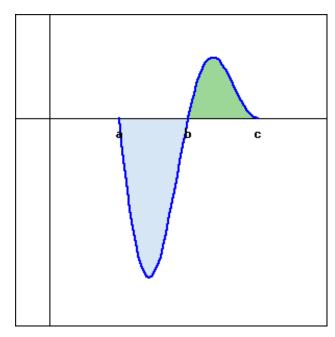
Suppose the region on the left in the figure (with blue shading) has area is 12, and the region on the right (with green shading) has area 4. Using the graph of f(x) in the figure, find the following integrals.

$$\int_{a}^{b} f(x) dx = \underline{\qquad}$$

$$\int_{b}^{c} f(x) dx = \underline{\qquad}$$

$$\int_{a}^{c} f(x) dx = \underline{\qquad}$$

$$\int_{a}^{c} |f(x)| dx = \underline{\qquad}$$



Graph of y = f(x)

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

We know that the value of the definite integral is the area between the graph of y = f(x) and the x-axis, with area below the axis being counted as negative area. Therefore

 $\int_a^b f(x)dx = -12$, $\int_b^c f(x)dx = 4$, $\int_a^c f(x)dx = -12 + 4 = -8$, and $\int_a^c |f(x)|dx = |-12| + |4| = 16$. Correct Answers:

- −12
- 4
- −8
- 16

6. (1 point) Library/Wiley/setAnton_Section_5.6/Anton_5_6_Q46.pg Sketch the region below the curve $y = 8x - x^2$ and above the *x*-axis and find its area.

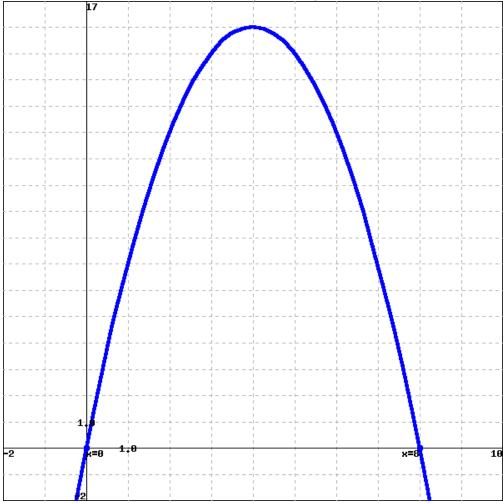
Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

f(x) = x(8-x) = 0 when x = 0, x = 8 from analyzing the sign of f and plotting the graph we see that the required x-interval is (0,8).

Hence, AREA =
$$\int_0^8 (8x - x^2) dx = \left[4x^2 - \frac{x^3}{3} \right]_0^8 = \frac{256}{3} - 0 = \frac{256}{3}$$

(click on image to enlarge)



$$y = f(x)$$

Correct Answers:

• 256/3

Use the Fundamental Theorem of Calculus to evaluate (if it exists)

$$\int_0^1 \frac{-7}{t^2 + 1} \, dt.$$

If the integral does not exist, type "DNE" as your answer.

Correct Answers:

- -7*pi/4
- **8.** (1 point) Library/UCSB/Stewart5_5_3/Stewart5_5_3_42.pg

Use the Fundamental Theorem of Calculus to evaluate (if it exists)

$$\int_{-\pi}^{\pi} f(x) \, dx,$$

where

$$f(x) = \begin{cases} 6x & \text{if } -\pi \le x \le 0\\ -4\sin(x) & \text{if } 0 < x \le \pi \end{cases}$$

If the integral does not exist, type "DNE" as your answer.

Correct Answers:

- -1/2*6*pi^2+2*-4
- **9.** (1 point) Library/ma122DB/set12/s5_5_77.pg

If f is continuous and $\int_0^{10} f(x) dx = -16$, evaluate $\int_0^2 f(5x) dx$.

Answer:

Solution: (Instructor solution preview: show the student solution after due date.)

Let u = 5x. Then du = 5dx so

$$\int_{0}^{2} f(5x) dx = \int_{0}^{10} f(u) \left(\frac{1}{5} du\right)$$
$$= \frac{1}{5} \int_{0}^{10} f(u) du$$
$$= \frac{-16}{5}$$

Correct Answers:

- −16/5
- 10. (1 point) Library/Wiley/setAnton_Section_5.3/Anton_5_3_Q16.pg

Evaluate the integral using an appropriate substitution.

$$\int x^3 \sqrt{x^4 - 9} \, dx = \underline{\qquad} + C$$

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

For $u = x^4 - 9$ we have $\frac{1}{4}du = x^3dx$ and hence;

$$\int x^3 \sqrt{x^4 - 9} \, dx = \frac{1}{4} \int \sqrt{u} \, du = \frac{1}{4} \cdot \frac{2}{3} \sqrt{u^3} + C = \frac{1}{6} \sqrt{(x^4 - 9)^3} + C$$

Correct Answers:

• $1/6*sqrt((x^4-9)^3)$

11. (1 point) Library/Valdosta/APEX_Calculus/6.1/APEX_6.1_14.pg

Evaluate the indefinite integral using Substitution. (use C for the constant of integration.)

$$\int \frac{9\ln x}{x} dx = \underline{\hspace{1cm}}$$

Solution: (Instructor solution preview: show the student solution after due date.) Substitute $u = \ln x$. Then $du = \frac{1}{x} dx$.

$$\int \frac{9 \ln x}{x} dx = 9 \int \ln(x) \cdot \frac{1}{x} dx$$
$$= 9 \int u du$$
$$= 9 \cdot \frac{u^2}{2} + C$$
$$= \frac{9(\ln x)^2}{2} + C.$$

Correct Answers:

- $9*[ln(x)]^2/2+C$
- 12. (1 point) Library/Valdosta/APEX_Calculus/6.1/APEX_6.1_24.pg

Evaluate the indefinite integral using Substitution. (use C for the constant of integration.)

$$\int e^{9x+7} dx = \underline{\hspace{1cm}}$$

Solution: (Instructor solution preview: show the student solution after due date.) Substitute u = 9x + 7. Then du = 9 dx.

$$\int e^{9x+7} dx = \frac{1}{9} \int e^{u} du$$
$$= \frac{1}{9} e^{u} + C$$
$$= \frac{1}{9} e^{9x+7} + C.$$

Correct Answers:

- $e^{(9*x+7)/9+C}$
- 13. (1 point) Library/Valdosta/APEX_Calculus/6.1/APEX_6.1_30.pg

Evaluate the indefinite integral using Substitution. (use C for the constant of integration.)

$$\int 9^{6x} dx = \underline{\hspace{1cm}}$$

Solution: (Instructor solution preview: show the student solution after due date.) Substitute u = 6x. Then du = 6dx.

$$\int 9^{6x} dx = \frac{1}{6} \int 9^u du$$
$$= \frac{1}{6} \cdot \frac{9^u}{\ln 9} + C$$
$$= \frac{9^{6x}}{6 \ln 9} + C.$$

Correct Answers:

• 9^(6*x)/13.1833+C

14. (1 point) Library/Valdosta/APEX_Calculus/6.1/APEX_6.1_59.pg

Evaluate the indefinite integral. (use *C* for the constant of integration.)

$$\int \frac{2x-1}{x^2 - 1x - 30} \, dx = \underline{\hspace{1cm}}$$

Solution: (Instructor solution preview: show the student solution after due date.)

Substitute $u = x^2 - 1x - 30$. Then du = 2x - 1 dx.

$$\int \frac{2x-1}{x^2-1x-30} dx = \int \frac{du}{u}$$

$$= \ln u + C$$

$$= \ln |x^2 - 1x - 30| + C.$$

Correct Answers:

• $\ln(x^2+(-1)*x+(-30))+C$

15. (1 point) Library/ma122DB/set12/s5_5_29.pg

Evaluate the indefinite integral.

$$\int 4\sin^5 x \cos x \, dx$$

Answer: _____ + *C*

Correct Answers:

•
$$4 / 6 * (\sin(x))^6$$

16. (1 point) Library/Wiley/setAnton_Section_7.1/Anton_7_1_Q5.pg

Evaluate the integral by any method.

$$\int \frac{\sin(8x)}{2 + \cos(8x)} \, dx = \underline{\qquad} + C$$

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

For $u = 2 + \cos(8x)$ we have $-\frac{1}{8}du = \sin(8x) dx$ hence;

$$\int \frac{\sin(8x)}{2 + \cos(8x)} \, dx = -\frac{1}{8} \int \frac{1}{u} \, du = -\frac{1}{8} \ln(|u|) + C = -\frac{1}{8} \ln(|2 + \cos(8x)|) + C$$

Correct Answers:

• -1/8*ln(|2+cos(8*x)|)

17. (1 point) Library/Wiley/setAnton_Section_5.3/Anton_5_3_Q20.pg

Evaluate the integral using an appropriate substitution.

$$\int \sec^2(9x) \ dx = \underline{\qquad} + C$$

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

For u = 9x we have $\frac{1}{9}du = dx$ and hence;

$$\int \sec^2(9x) \ dx = \frac{1}{9} \int \sec^2(u) \ du = \frac{1}{9} \tan(u) + C = \frac{1}{9} \tan(9x) + C$$

Correct Answers:

• 1/9*tan(9*x)

18. (1 point) Library/Utah/AP_Calculus_I/set8_Exponentials_and_Logarithms/1220s8p5.pg

$$\int x^4 \sin(x^5) \, dx = \underline{\qquad} +C.$$

Hint: Use substitution.

Solution: (Instructor solution preview: show the student solution after due date.)

Solution: substituting $u = x^5$ the integral evaluates to

$$-\frac{1}{5}\cos(x^5)$$

Correct Answers:

• -0.2 * cos(x**(5))

19. (1 point) Library/UMN/calculusStewartET/s_7_2_prob05.pg

Evaluate

$$\int \tan^7 x \sec^2 x \, dx.$$

Answer: _

Correct Answers:

• $0.125*[tan(x)]^8+C$

20. (1 point) Library/UMN/calculusStewartCCC/s_5_5_24.pg

Evaluate the indefinite integral

$$\int \frac{\sin(\ln x)}{x} \, dx.$$

Answer: _

Correct Answers:

• $-[\cos(\ln(x))]+C$

21. (1 point) Library/UCSB/Stewart5_7_2/Stewart5_7_2_21.pg

Evaluate the integral

$$\int 2\sec^2(x)\tan(x)\,dx$$

Note: Use an upper-case "C" for the constant of integration.

Correct Answers:

• 1/2*2*sec(x)^2+C+c

22. (1 point) Library/UCSB/Stewart5_5_5/Stewart5_5_5_4.pg

Evaluate the following integral by making the given substitution:

$$\int \frac{-8\sin(\sqrt{x})}{\sqrt{x}} dx, \quad u = \sqrt{x}$$

Note: Any arbitrary constants used must be an upper-case "C".

Correct Answers:

• -2*-8*cos(sgrt(x))+C+c

23. (1 point) Library/Michigan/Chap7Sec1/Q59.pg

Use the Fundamental Theorem of Calculus to find

$$\int_1^8 \frac{\sin(\sqrt[3]{x})}{\sqrt[3]{x^2}} dx = \underline{\qquad}$$

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

We use substitution with $w = \sqrt[3]{x}$, so that $dw = (\frac{1}{3})(\frac{1}{x^{2/3}})dx = \frac{1}{3\sqrt[3]{x^2}}dx$. Then w(1) = 1 and w(8) = 2, so that

$$\int_{1}^{8} \frac{\sin(\sqrt[3]{x})}{\sqrt[3]{x^2}} dx = \int_{1}^{2} 3\sin(w) dw = -3\cos(w) \Big|_{1}^{2} = -3(\cos(2) - \cos(1)).$$

Correct Answers:

• $-3*(\cos(2) - \cos(1))$

24. (1 point) Library/ma122DB/set13/s6_5_1.pg

Find the average value of $f(x) = x^3$ on the interval [3,4].

Answer: _____

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

Average value =
$$\frac{1}{4-3} \int_3^4 x^3 dx = \left[\frac{x^4}{4}\right]_3^4 = \left(\frac{4^4}{4} - \frac{3^4}{4}\right) = \frac{175}{4}$$

Correct Answers:

(4⁽³⁺¹⁾ - (3⁽³⁺¹⁾))/((3+1)*(4-(3)))

25. (1 point) Library/Wiley/setAnton_Section_5.8/Anton_5_8_Q8.pg

Find the average value of the function $f(x) = 6e^x$ on the interval $[-8, \ln 2]$.

 $f_{\text{ave}} = \underline{\hspace{1cm}}$

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

$$f_{\text{ave}} = \frac{6}{\ln 2 + 8} \int_{-8}^{\ln 2} e^x \, dx = \frac{6}{\ln 2 + 8} \left[e^x \right]_{-8}^{\ln 2} = \frac{6}{\ln 2 + 8} (2 - e^{-8})$$

Correct Answers:

• $6/[\ln(2)+8]*[2-e^{(-8)}]$

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