Homework 4 - continued

Monday, October 4, 2021 10:59 AM

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Math141_Calculus_II

Assignment Homework4 due 10/04/2021 at 11:59pm EDT

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1. **(1 point)** `Library/UCSB/Stewart5_7_3/Stewart5_7_3_9.pg`

Evaluate the integral

$$\int \frac{-7}{\sqrt{x^2+16}}\,dx \qquad x^2+y^2$$

Note: Use an upper-case "C" for the constant of integration.

let $x = 4\tan\theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$x^2+16 = 16(1+\tan^2\theta) = 16\sec^2\theta$

$dx = 4\sec^2\theta \, d\theta$

$$\Rightarrow \int \frac{-7\,dx}{\sqrt{x^2+16}} = -7\int \frac{4\sec^2\theta\,d\theta}{4\sec\theta}$$

$$= -7\int \sec\theta\,d\theta$$

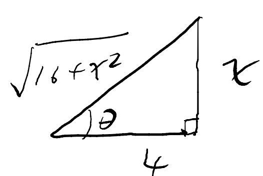
$$= -7\ln|\sec\theta + \tan\theta| + C$$

$x = 4\tan\theta$

$\Rightarrow \tan\theta = \frac{x}{4}$

$$= -7\ln\left|\frac{\sqrt{16+x^2}}{4} + \frac{x}{4}\right| + C$$

You can stop here



$$= -7\ln\left|\frac{\sqrt{16+x^2}+x}{4}\right| + C$$

$\cos\theta = \frac{4}{\sqrt{16+x^2}}$

$$= -7\left(\ln\left|\sqrt{16+x^2}+x\right| - \ln 4\right) + C$$

$$= -7 \ln\left|\sqrt{16+x^2}+x\right| + C$$

$$= -7 \ln\left(\sqrt{16+x^2}+x\right)+C \quad \text{since } \sqrt{16+x^2} \geq |x|$$

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2. **(1 point)** `Library/CSUOhio/calculus/trigonometric_substitution/trigSub10.pg`

Evaluate the indefinite integral.

$$\int \frac{dx}{(16-x^2)^{3/2}} \qquad 4^2 - x^2$$

Sol. Let $x = 4\sin\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$(16-x^2)^{\frac{3}{2}} = (16-16\sin^2\theta)^{\frac{3}{2}}$$

$$= (16\cos^2\theta)^{\frac{3}{2}} = 4^3\cos^3\theta$$

$$dx = 4\cos\theta\, d\theta$$

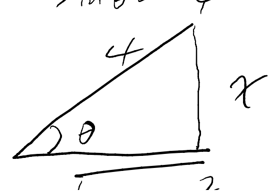
$$\Rightarrow \int \frac{dx}{(16-x^2)^{3/2}} = \int \frac{4\cos\theta\, d\theta}{4^3\cos^3\theta}$$

$$= \frac{1}{16}\int \frac{1}{\cos^2\theta}\, d\theta = \frac{1}{16}\int \sec^2\theta\, d\theta$$

$$= \frac{1}{16}\tan\theta + C$$

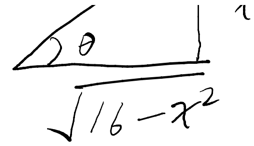
$x = 4\sin\theta$

$\sin\theta = \frac{x}{4}$



$$= \frac{1}{16}\cdot\frac{x}{\sqrt{16-x^2}} + C$$

$16 \quad \sqrt{16-x^2} \quad + C$



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3. **(1 point)** `Library/CSUOhio/calculus/trigonometric_substitution/trigSub24.pg`

Evaluate the indefinite integral.

$$\int \frac{\sqrt{x^2-4}}{x}dx \qquad x^2 - 2^2$$

Let $x = 2\sec\theta$, $\quad 0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$

$\sqrt{x^2-4} = \sqrt{4\sec^2\theta - 4} = 2\tan\theta$

$dx = 2\sec\theta\tan\theta\, d\theta$

$\Rightarrow \int \frac{\sqrt{x^2-4}}{x}\,dx = \int \frac{2\tan\theta}{\cancel{2\sec\theta}}\, \cancel{2\sec\theta}\tan\theta\, d\theta$

$= 2\int \tan^2\theta\, d\theta = 2\int (\sec^2\theta - 1)\, d\theta$

$= 2(\tan\theta - \theta) + C$

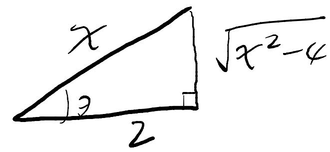
$x = 2\sec\theta$

$\Rightarrow \sec\theta = \frac{x}{2}$

$\cos\theta = \frac{2}{x}$

$= 2\left(\frac{\sqrt{x^2-4}}{2} - \arccos\frac{2}{x}\right) + C$

$$= \sqrt{x^2-4} - 2\arccos\frac{2}{x} + C$$



4. **(1 point)** `Library/CSUOhio/calculus/trigonometric_substitution/trigSub21.pg`

Evaluate the indefinite integral.

$$\int \frac{\sqrt{4-x^2}}{x^2}\,dx \qquad 2^2 - x^2$$

Similar to Qustion 2

Let $x = 2\sin\theta$, $dx = 2\cos\theta\,d\theta$

$$\sqrt{4-x^2} = \sqrt{4(1-\sin^2\theta)} = 2\cos\theta$$

$$\Rightarrow \int \frac{\sqrt{4-x^2}}{x^2}\,dx = \int \frac{\cancel{2}\cos\theta \cdot \cancel{2}\cos\theta\,d\theta}{\cancel{4}\sin^2\theta}$$

$$= \int \frac{\cos^2\theta}{\sin^2\theta}\,d\theta = \int \frac{1-\sin^2\theta}{\sin^2\theta}\,d\theta$$

$$\int \cot^2\theta\,d\theta$$

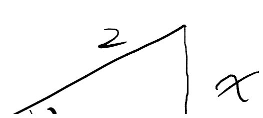
$$= \int (\csc^2\theta - 1)\,d\theta$$

$$= \int \csc^2\theta\,d\theta - \theta$$

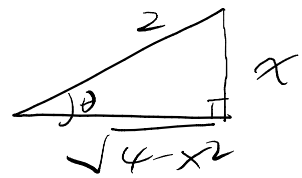
$$= -\cot\theta - \theta + C$$

$x = 2\sin\theta \Rightarrow \sin\theta = \frac{x}{2}$

$$= -\frac{\sqrt{4-x^2}}{x} - \arcsin\left(\frac{x}{2}\right) + C$$



$= -\frac{\sqrt{\ \ }}{x} - \arcsin\left(\frac{x}{2}\right) + C$



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5. **(1 point)** `Library/UCSB/Stewart5_7_5/Stewart5_7_5_33.pg`

Evaluate the integral

$$\int 5\sqrt{3-2x-x^2}\,dx$$

Note: Use an upper-case "C" for the constant of integration.

풀이. $3-2x-x^2 = -x^2-2x+3$

$= -\left[x^2 + \underset{2\cdot 1}{2x} - 3\right]$

$= -\left(\underline{x^2+2x+1}-1-3\right)$

$= -\left[(x+1)^2 - 2^2\right]$

$= 2^2 - (x+1)^2$

let $\underline{x+1 = 2\sin\theta}$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$dx = d(x+1) = 2\cos\theta\, d\theta$

$\Rightarrow \int 5\sqrt{3-2x-x^2}\,dx = 5\int\sqrt{4\cos^2\theta}\cdot 2\cos\theta\, d\theta$

$$= 20\int \cos^2\theta \, d\theta$$

$$= \frac{20}{2}\int (1+\cos 2\theta)\, d\theta$$

$u = 2\theta \Rightarrow du = 2\,d\theta$

$$= 10\theta + 10\cdot\int \cos 2\theta \, d\theta$$

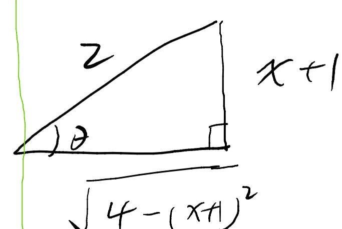
$$= 10\theta + \frac{10}{2}\int \cos(2\theta)\, d(2\theta)$$

$$= 10\theta + 5\sin(2\theta) + C$$

$$= 10\theta + 10\sin\theta\cos\theta + C$$

$x+1 = 2\sin\theta$

$\sin\theta = \frac{x+1}{2}$



$$= 10\left[\arcsin\frac{x+1}{2} + \frac{x+1}{2}\cdot\frac{\sqrt{4-(x+1)^2}}{2}\right] + C$$

Results for this submission

Entered	Answer Preview	Result
10*[asin((x+1)/2)+(x+1)*([sqrt(4-[(x+1)^2])]/4)]+C	$10\left(\sin^{-1}\left(\frac{x+1}{2}\right)+(x+1)\frac{\sqrt{4-(x+1)^2}}{4}\right)+C$	correct

The answer above is correct.

(1 point) **Library/UCSB/Stewart5_7_5/Stewart5_7_5_33.pg**

Evaluate the integral

$$\int 5\sqrt{3-2x-x^2}\,dx$$

Note: Use an upper-case "C" for the constant of integration.

10(asin((x+1)/2) + (x+1)(sqrt(4-(x+1)^2)/4))+C