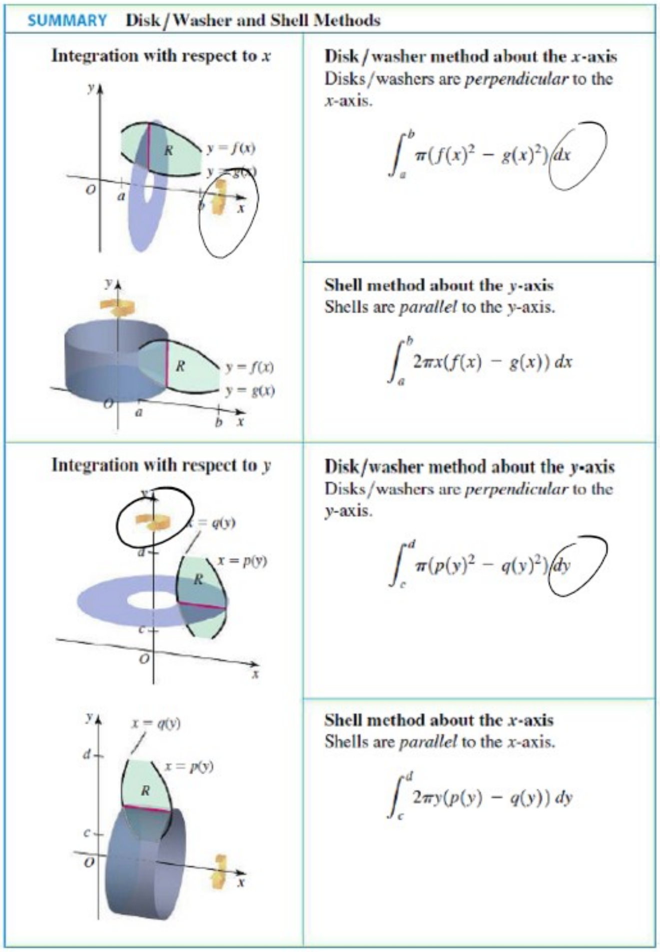
Homework 9 Solution

Monday, November 8, 2021 11:04 AM



SUMMARY Disk / Washer and Shell Methods

Integration with respect to x	
	Disk / washer method about the x-axis Disks / washers are *perpendicular* to the x-axis. $$\int_a^b \pi(f(x)^2 - g(x)^2)\,dx$$
	Shell method about the y-axis Shells are *parallel* to the y-axis. $$\int_a^b 2\pi x(f(x) - g(x))\,dx$$
Integration with respect to y	**Disk/washer method about the y-axis** Disks / washers are *perpendicular* to the y-axis. $$\int_c^d \pi(p(y)^2 - q(y)^2)\,dy$$
	Shell method about the x-axis Shells are *parallel* to the x-axis. $$\int_c^d 2\pi y(p(y) - q(y))\,dy$$

Washer method:

rotate a region about x-axis,

i) Integrate wrt x

ii) radius by y-coordinates (function of x)

shell method:

rotate a region about x-axis

i) Integrate wrt y

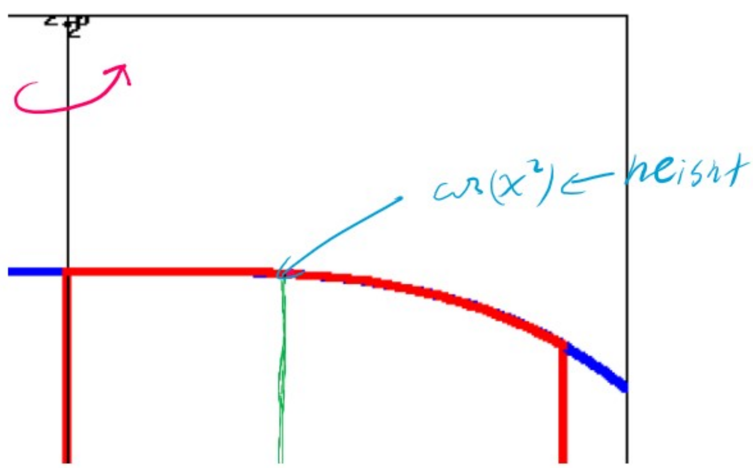
ii) heights by x-coordinates (function of y)

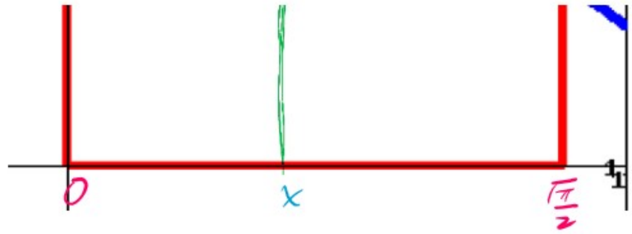
1. **(2 points)** Library/Wiley/setAnton_Section_6.3/anton_6_3_Q8.pg

Use cylindrical shells to find the volume of the solid generated when the region enclosed by the given curves is revolved about the y-axis.

$y = \cos(x^2)$, $x = 0$, $x = \frac{\sqrt{\pi}}{2}$, $y = 0$
Volume = __________

$$V = \int_0^{\frac{\sqrt{\pi}}{2}} 2\pi x \cdot \cos(x^2)\, dx$$





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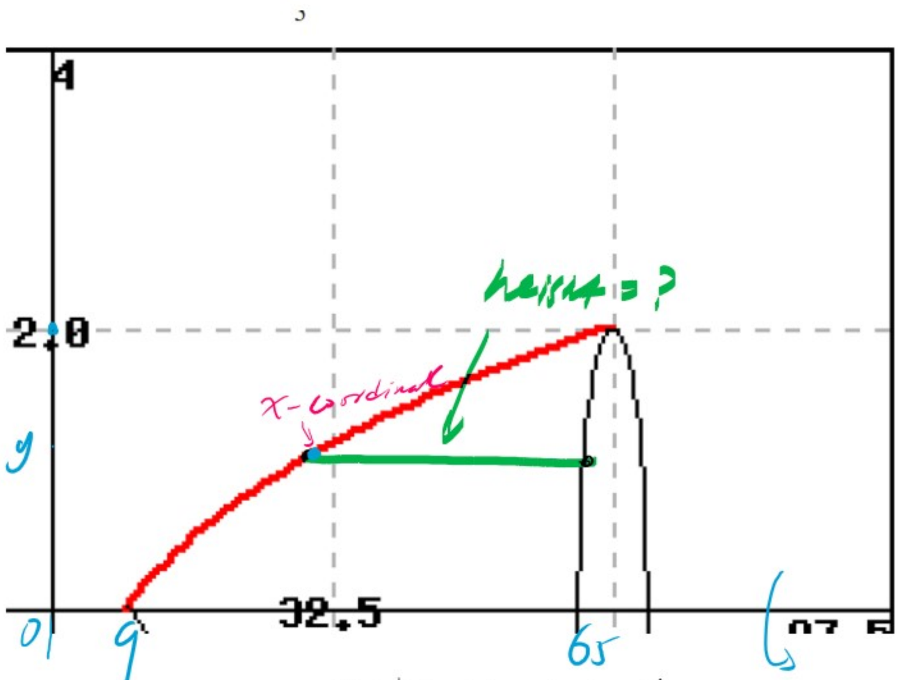
2. **(2 points)** Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/6_Applications_of_the_Integral/6.4_The_Method_of_Cylindrical_Shells/6.4.26.pg

Use the Shell Method to calculate the volume of rotation, V, about the x-axis for the region underneath the graph of $y = (x-1)^{\frac{1}{3}} - 2$ where $9 \le x \le 65$. $y \in [0, 2]$

$V =$__________

Solution: *(Instructor solution preview: show the student solution after due date.)*



Integrate wrt y

$$V = \int_0^2 2\pi y \cdot \text{height} \cdot dy$$

$$\left(\text{height} = 65 - [(y+2)^3 + 1] \right)$$

$$y = (x-1)^{\frac{1}{3}} - 2$$

$$y + 2 = (x-1)^{\frac{1}{3}}$$

$$(y+2)^3 = x - 1$$

$$\Rightarrow x = (y+2)^3 + 1$$

$$= \int_0^2 2\pi y \left[64 - (y+2)^3 \right] dy$$

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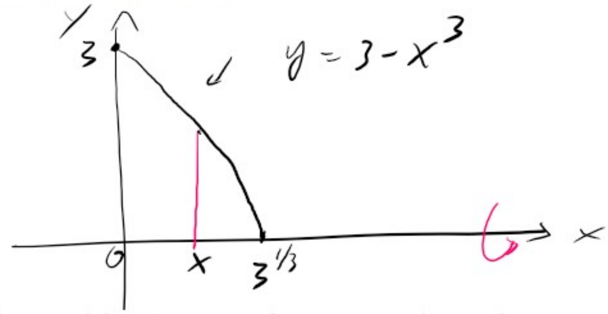
3. **(4 points)** `Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/6_Applications_of_the_Integral/6.4_The_Method_of_Cylindrical_Shells/6.4.27.pg`

Use both the Shell and Disk Methods to calculate the volume of the solid obtained by rotating the region under the graph of $f(x) = 3 - x^3$ for $0 \le x \le 3^{\frac{1}{3}}$ about the x-axis and the y-axis.

$y \in [0, 3]$



Using the disk method, the volume D_x of the solid obtained by rotating the region about the x-axis is $\int_0^a g(x)dx$ (this is the initial integral when you setup the problem), where
a = $3^{\frac{1}{3}}$

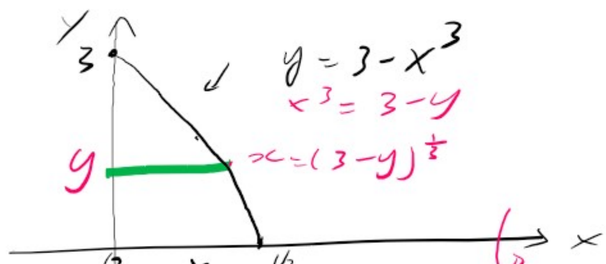
integrate wrt $x \in [0, 3^{\frac{1}{3}}]$

$$V = \int_0^{3^{\frac{1}{3}}} \pi \cdot (3 - x^3)^2 \, dx$$

shell method:

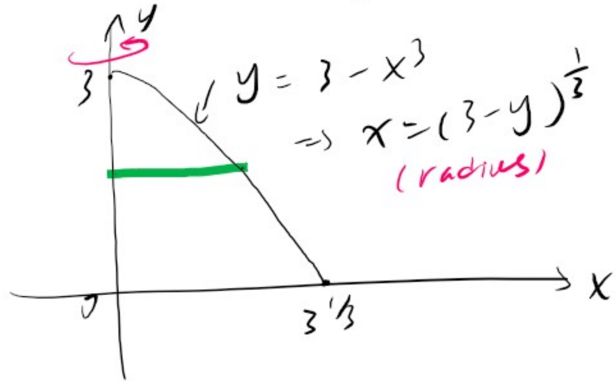
Integrate wrt $y \in [0, 3]$

height $= (3-y)^{\frac{1}{3}}$



Using the shell method, the volume S_x of the solid obtained by rotating the region about the x-axis is $\int_0^b h(y)dy$ (this is the initial integral when you setup the problem), where
b= 3
h(y)=____________________
S_x=__________

$$V = \int_0^3 2\pi y \cdot (3-y)^{\frac{1}{3}} \, dy$$



Integrate wrt y

Using the disk method, the volume D_y of the solid obtained by rotating the region about the y-axis is $\int_0^A G(y)dy$ (this is the initial integral when you setup the problem), where

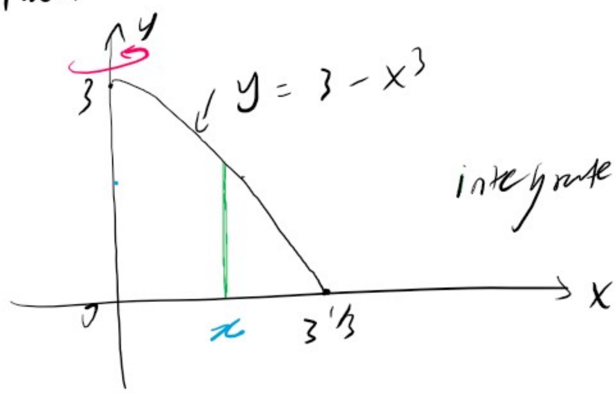
A = 3

G(y)=____

D_y=____________

$$V = \int_0^3 \pi \cdot \left((3-y)^{\frac{1}{3}} \right)^2 \, dy$$

shell method



integrate wrt $x \in [0, 3^{\frac{1}{3}}]$

$$V = \int_0^{3^{\frac{1}{3}}} 2\pi x \cdot \text{height} \cdot dx$$

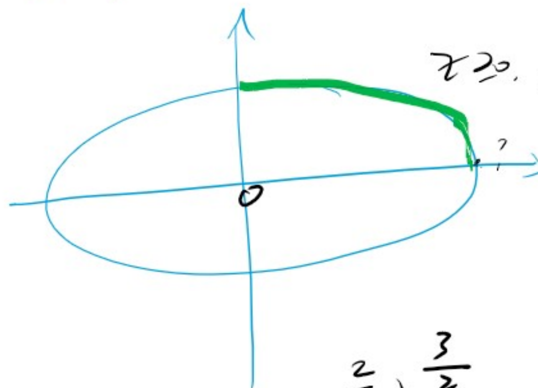
$$= \int_0^{3^{\frac{1}{3}}} 2\pi x \cdot (3 - x^3) \, dx$$

4. **(2 points)** Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/8_Further_Applications_of_the_Integral_and_Taylor_Polynomials/8.1_Arc_Length_and_Surface_Area/8.1.15.pg

Calculate the length of the astroid of $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 3$.

$s =$ ____________

$$\left(x^{\frac{1}{3}}\right)^2 + \left(y^{\frac{1}{3}}\right)^2 = 3$$



In the 1st quadrant,

$$y^{\frac{2}{3}} = 3 - x^{\frac{2}{3}} \Rightarrow y = \left(3 - x^{\frac{2}{3}}\right)^{\frac{3}{2}},$$

$x \geq 0$

when $y = 0$,

$3 - x^{\frac{2}{3}} = 0$

$\Rightarrow 3 = x^{\frac{2}{3}}$

$\Rightarrow x = 3^{\frac{3}{2}}$

$$\boxed{L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx.}$$

$a = 0,\ b = 3^{\frac{3}{2}}$

$$y = \left(3 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

$$\Rightarrow y' = \frac{3}{2}\left(3 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} \cdot \left(0 - \frac{2}{3}x^{-\frac{1}{3}}\right)$$

$$= -\left(3 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} \cdot x^{-\frac{1}{3}}$$

$$\Rightarrow (y')^2 = (3 - x^{\frac{2}{3}}) \cdot x^{-\frac{2}{3}}$$

$$= 3x^{-\frac{2}{3}} - 1$$

$$\Rightarrow \quad 1 + (y')^2 = 3x^{-\frac{2}{3}}$$

$$\Rightarrow \quad l = 4\int_0^{3^{\frac{3}{2}}} \sqrt{3x^{-\frac{2}{3}}}\, dx$$

by symmetry.

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5. **(2 points)** Library/UCSB/Stewart5_8_1/Stewart5_8_1_4.pg

Find the exact length of the curve

$$y = \frac{x^3}{6} + \frac{1}{2x}, \quad \frac{1}{2} \le x \le 1.$$

Arc length = ________________

$$y' = \frac{3x^2}{6} + 2(-1)x^{-2}$$

$$= \frac{x^2}{2} - \frac{1}{2x^2}$$

$$\Rightarrow L = \int_{\frac{1}{2}}^{1} \sqrt{1 + (y')^2}\, dx$$

$$= \int_{}^{1}$$

$$= \int_{\frac{1}{2}}^{1} \sqrt{1 + \left(\frac{x^2}{2}\right)^2 + \left(\frac{1}{2x^2}\right)^2 - \frac{2}{4}} \; dx$$

$$= \int_{\frac{1}{2}}^{1} \sqrt{\left(\frac{x^2}{2}\right)^2 + \left(\frac{1}{2x^2}\right)^2 + \frac{1}{2}} \; dx$$

$$= \int_{\frac{1}{2}}^{1} \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2} \; dx$$

$$= \int_{\frac{1}{2}}^{1} \left(\frac{x^2}{2} + \frac{1}{2x^2}\right) dx$$