35. $\frac{1}{6}(x^2 - 8)\sqrt{x^2 + 4} + C$ **37.** $\frac{1}{x+1} + \ln|(x+1)(x^2+4)| + C$

39. $\frac{t - \ln(2 + e^t)}{2} + C$ **41.** $\frac{1}{4}(\csc 4\theta - \cot 4\theta) + C$

43. $\frac{e^x}{2}(\sin x - \cos x) + C$

45. $\ln|x| - \frac{1}{x} + \frac{1}{2}\ln(x^2 + 4x + 9) - \frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + C$

47. $\frac{\theta}{2} + \frac{1}{16}\sin 8\theta + C$ **49.** $\frac{\sec^{49} 2z}{98} + C$ **51.** $\frac{4}{15}$

53. $2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ln\left(\sqrt[6]{x} + 1\right) + C$

55. $-\frac{\sqrt{9 - y^2}}{9\sqrt{2}\,y} + C$ **57.** $\frac{\pi}{9}$ **59.** $-\operatorname{sech} x + C$ **61.** $\frac{\pi}{3}$

63. $\frac{1}{8}\ln\left|\frac{x-5}{x+3}\right| + C$ **65.** $\frac{\ln 2}{4} + \frac{\pi}{8}$ **67.** 3 **69.** $\frac{1}{3}\ln\left|\frac{x-2}{x+1}\right| + C$

71. $2(x - 2\ln|x + 2|) + C$ **73.** $e^{2t}/\left(2\sqrt{1 + e^{4t}}\right) + C$

75. a. $\sec e^x + C$ **b.** $e^x \sec e^x - \ln|\sec e^x + \tan e^x| + C$

77. $\frac{\sqrt{6}}{3}\tan^{-1}\sqrt{\frac{2x-3}{3}} + C$

79. $\frac{1}{4}\sec^3 x \tan x + \frac{3}{8}\sec x \tan x + \frac{3}{8}\ln|\sec x + \tan x| + C$

81. $2(\ln^3 2 - 3\ln^2 2 + \ln 64 - 3)$ **83.** 1 **85.** $\frac{\pi}{2}$

87. $\frac{2\pi}{\sqrt{3}}$ **89.** Converges **91.** Diverges **93.** 1.196288

95. $M(4) = 44$; $T(4) = 42$; $S(4) = \frac{124}{3}$

97. $M(40) \approx 0.398236$; $T(40) \approx 0.398771$; $S(40) \approx 0.398416$

99. 0.886227 **101.** y-axis **103.** $\pi(e - 2)$ **105.** $\frac{\pi}{2}(e^2 - 3)$

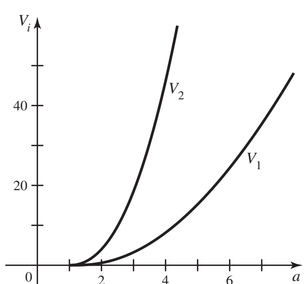
107. a. 1.603 **b.** 1.870 **c.** $b\ln b - b = a\ln a - a$ **d.** Decreasing **109.** $20/(3\pi)$ **111.** 1901 cars

113. a. $I(p) = \frac{1}{(p-1)^2}(1 - pe^{1-p})$ if $p \neq 1$, $I(1) = \frac{1}{2}$ **b.** $0, \infty$ **c.** $I(0) = 1$ **115.** 0.4054651 **117.** $n = 2$

119. a. $V_1(a) = \pi(a\ln^2 a - 2a\ln a + 2(a - 1))$

b. $V_2(a) = \frac{\pi}{2}(2a^2\ln a - a^2 + 1)$

c. $V_2(a) > V_1(a)$ for all $a > 1$



121. $a = \ln 2/(2b)$ **123.** $\ln(1 + \sqrt{2}/2)$

CHAPTER 9

Section 9.1 Exercises, pp. 604–606

1. a. 1 **b.** Linear **3.** Yes **5.** $\frac{\pi}{2} < t < \frac{3\pi}{2}$

21. $y = 3t - \frac{e^{-2t}}{2} + C$ **23.** $y = 2\ln|\sec 2x| - 3\sin x + C$

25. $y = 2t^6 + 6t^{-1} - 2t^2 + C_1 t + C_2$

27. $u = \frac{x^{11}}{2} + \frac{x^9}{2} - \frac{x^7}{2} + \frac{5}{x} + C_1 x + C_2$

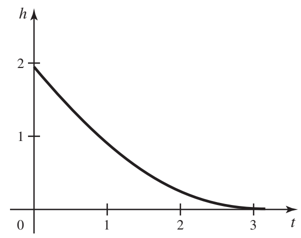
29. $u = \ln(x^2 + 4) - \tan^{-1}\frac{x}{2} + C$ **31.** $y = \sin^{-1} x + C_1 x + C_2$

33. $y = e^t + t + 3$ **35.** $y = x^3 + x^{-3} - 2, x > 0$

37. $y = -t^5 + 2t^3 + 1$ **39.** $y = e^t(t - 2) + 2(t + 1)$

41. $u = \frac{1}{4}\tan^{-1}\frac{x}{4} - 4x + 2$ **43. a.** $v(t) = -9.8t + 29.4$; $s(t) = -4.9t^2 + 29.4t + 30$; the object is above the ground for approximately $0 \le t \le 6.89$. **b.** The highest point of 74.1 m is reached at $t = 3$ s. **45.** The amount of resource is increasing for $H < 75$ and is constant if $H = 75$. If $H = 100$, the resource vanishes at approximately 28 time units.

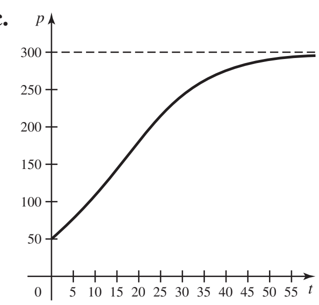
47. $h = (\sqrt{1.96} - 0.1t\sqrt{2g})^2 \approx (1.4 - 0.44t)^2$, $0 \le t \le 3.16$; the tank is empty after approximately 3.16 s.



49. a. False **b.** False **c.** True **51. c.** $y = C_1 \sin kt + C_2 \cos kt$

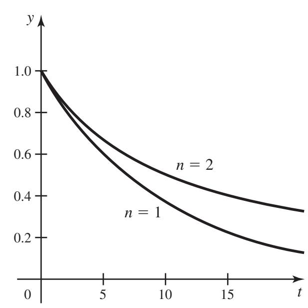
53. b. $C = \frac{K - 50}{50}$

c.



d. 300

55. c. The decay rate is greater for the $n = 1$ model.

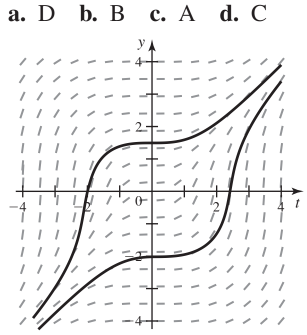


Section 9.2 Exercises, pp. 611–614

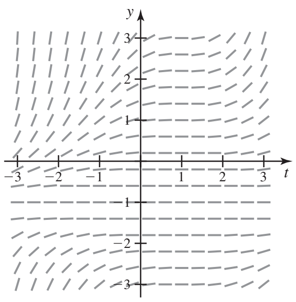
1. At selected points (t_0, y_0) in the region of interest draw a short line segment with slope $f(t_0, y_0)$. **3.** $y(3.1) \approx 1.6$

5. a. D **b.** B **c.** A **d.** C

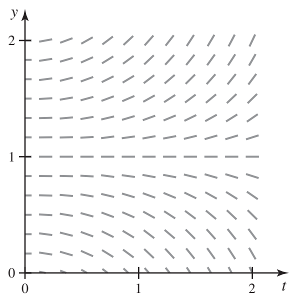
7.



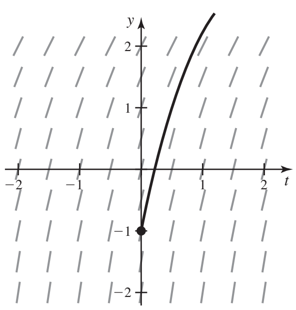
9. An initial condition of $y(0) = -1$ leads to a constant solution. For any other initial condition, the solutions are increasing over time.



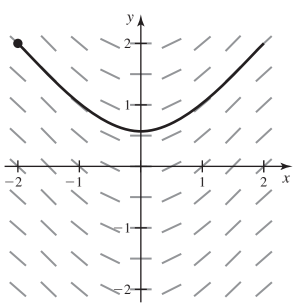
11. An initial condition of $y(0) = 1$ leads to a constant solution. Initial conditions $y(0) = A$ lead to solutions that are increasing over time if $A > 1$.



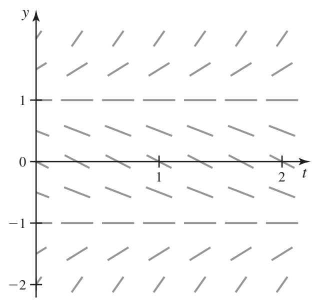
13.



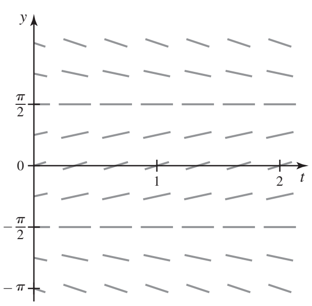
15.



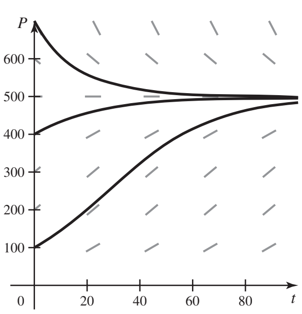
17. a. $y = 1, y = -1$
b. Solutions are increasing for $|y| > 1$ and decreasing for $|y| < 1$. **c.** Initial conditions $y(0) = A$ lead to increasing solutions if $|A| > 1$ and decreasing solutions if $|A| < 1$.
d.



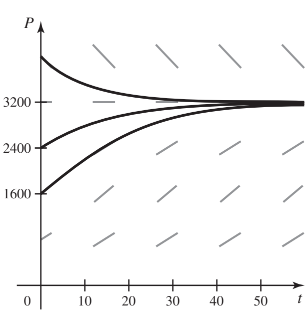
19. a. $y = \pi/2, y = -\pi/2$
b. Solutions are increasing for $|y| < \pi/2$ and decreasing for $|y| > \pi/2$. **c.** Initial conditions $y(0) = A$ lead to increasing solutions if $|A| < \pi/2$ and decreasing solutions if $\pi/2 < |A| < \pi$.
d.



21. The equilibrium solutions are $P = 0$ and $P = 500$.



23. The equilibrium solutions are $P = 0$ and $P = 3200$.



25. $y(0.5) \approx u_1 = 4; y(1) \approx u_2 = 8$
27. $y(0.1) \approx u_1 = 1.1; y(0.2) \approx u_2 = 1.19$

29. a.

Δt	approximation to $y(0.2)$	approximation to $y(0.4)$
0.20000	0.80000	0.64000
0.10000	0.81000	0.65610
0.05000	0.81451	0.66342
0.02500	0.81665	0.66692

b.

Δt	errors for $y(0.2)$	errors for $y(0.4)$
0.20000	0.01873	0.03032
0.10000	0.00873	0.01422
0.05000	0.00422	0.00690
0.02500	0.00208	0.00340

c. Time step $\Delta t = 0.025$; smaller time steps generally produce more accurate results. **d.** Halving the time steps results in approximately halving the error.

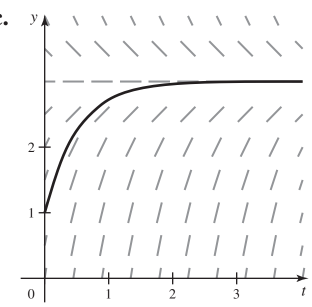
31. a.

Δt	approximation to $y(0.2)$	approximation to $y(0.4)$
0.20000	3.20000	3.36000
0.10000	3.19000	3.34390
0.05000	3.18549	3.33658
0.02500	3.18335	3.33308

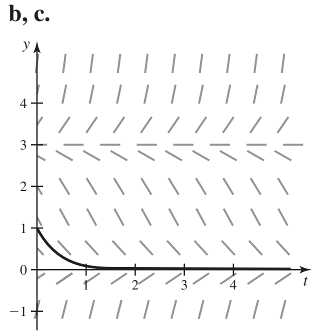
b.

Δt	errors for $y(0.2)$	errors for $y(0.4)$
0.20000	0.01873	0.03032
0.10000	0.00873	0.01422
0.05000	0.00422	0.00690
0.02500	0.00208	0.00340

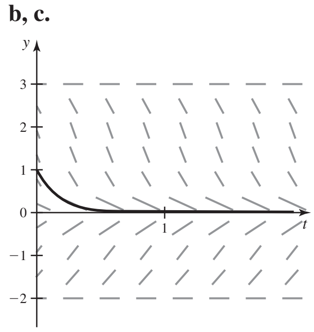
c. Time step $\Delta t = 0.025$; smaller time steps generally produce more accurate results. **d.** Halving the time steps results in approximately halving the error. **33. a.** $y(2) \approx 0.00604662$ **b.** 0.012269 **c.** $y(2) \approx 0.0115292$ **d.** Error in part (c) is approximately half of the error in part (b). **35. a.** $y(4) \approx 3.05765$ **b.** 0.0339321 **c.** $y(4) \approx 3.0739$ **d.** Error in part (c) is approximately half of the error in part (b). **37. a.** True **b.** False
39. a. $y = 3$ **b, c.**



41. a. $y = 0$ and $y = 3$
b, c.



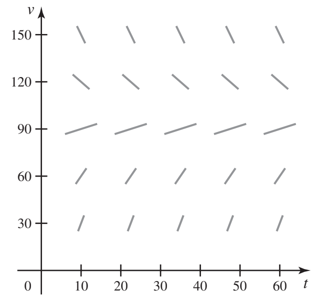
43. a. $y = -2, y = 0$, and $y = 3$
b, c.



45. a. $\Delta t = \frac{b-a}{N}$ **b.** $u_1 = A + f(a, A)\frac{b-a}{N}$

c. $u_{k+1} = u_k + f(t_k, u_k)\frac{b-a}{N}$, where $u_0 = A$ and $t_k = a + k(b-a)/N$, for $k = 0, 1, 2, \ldots, N-1$.

47. a.



b. Increasing for $A < 98$ and decreasing for $A > 98$ **c.** $v(t) = 98$

Section 9.3 Exercises, pp. 618–620

1. A first-order separable differential equation has the form $g(y)\,y'(t) = h(t)$, where the factor $g(y)$ is a function of y and $h(t)$ is a function of t. **3.** No **5.** $y = \frac{t^4}{4} + C$

7. $y = \pm\sqrt{2t^3 + C}$ **9.** $y = -2\ln\left(\frac{1}{2}\cos t + C\right)$

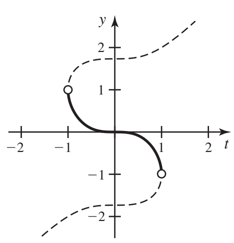
11. $y = \frac{x}{1 + Cx}$ **13.** $y = \pm\frac{1}{\sqrt{C - \cos t}}$ **15.** $u = \ln\left(\frac{e^{2x}}{2} + C\right)$

17. $y = \sqrt{t^3 + 81}$ **19.** Not separable **21.** $y(t) = -e^{e^t - 1}$

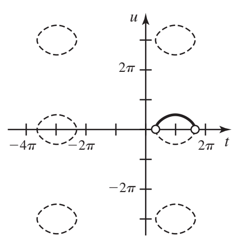
23. $y = \ln(e^x + 2)$ **25.** $y = \ln\left(\frac{\ln^4 t}{4} + 1\right)$

27. $y = \sqrt{\tan t}$, $0 < t < \pi/2$ **29.** $y = \sqrt{t^2 + 3}$ **31.** $y = \ln t + 2$

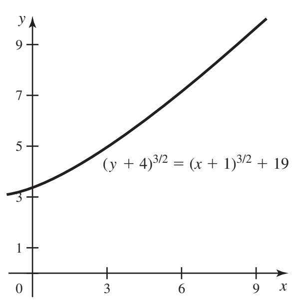
33. $y^3 - 3y = 2t^3$, $-1 < t < 1$



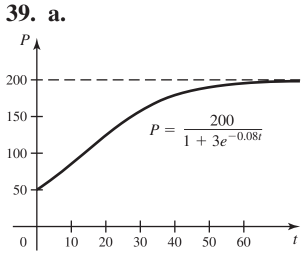
35. $\cos u = 2 - 2\sin\frac{x}{2}$, $\frac{\pi}{3} < x < \frac{5\pi}{3}$



37.



39. a.

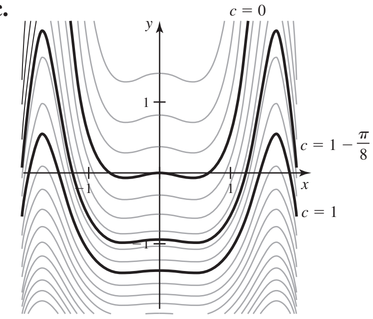


b. 200

41. a. True **b.** False **c.** True

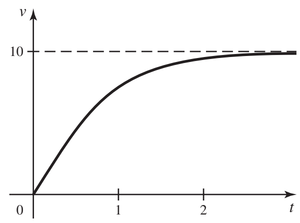
43. a. $y = -2\ln\left(\frac{x^2}{4} + \cos x^2 + C\right)$ **b.** $C = 0, 1, 1 - \frac{\pi}{8}$

c.



45. $y = kx$ **47. b.** $\sqrt{gm/k}$

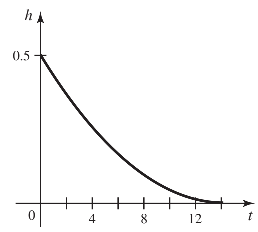
c. $v = \sqrt{\frac{g}{a}}\,\frac{Ce^{2\sqrt{ag}\,t} - 1}{Ce^{2\sqrt{ag}\,t} + 1}$, $t \ge 0$, where $a = \frac{k}{m}$



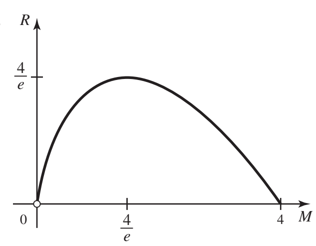
49. a. $h = \left(\sqrt{H} - \frac{kt}{2}\right)^2$, $0 \le t \le \frac{2\sqrt{H}}{k}$

b. $h = (\sqrt{0.5} - 0.05t)^2$, $0 \le t \le 14.1$

c. Approx. 14.1 s

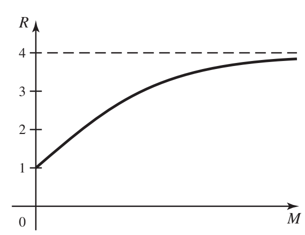


51. a.



R is positive if $0 < M < 4$; R has a maximum value when $M = \frac{4}{e}$; $\lim_{M\to 0} R(M) = 0$.

b. $M(t) = 4^{1 - e^{-t}}$, $t \ge 0$; the tumor grows quickly at first and then the rate of growth slows down; the limiting size of the tumor is 4.



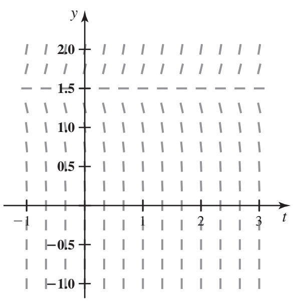
53. a. $y = \frac{1}{1 - t}$, $t < 1$ **b.** $y = \frac{1}{\sqrt{2}\sqrt{1 - t}}$, $t < 1$

c. $y = \frac{1}{(n(1 - t))^{1/n}}$, $t < 1$; as $t \to 1^-$, $y \to \infty$

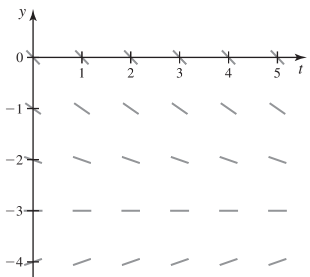
Section 9.4 Exercises, pp. 625–627

1. $y = 17e^{-10t} - 13$ **3.** $y = Ce^{-4t} + \frac{3}{2}$ **5.** $y = Ce^{3t} + \frac{4}{3}$
7. $y = Ce^{-2x} - 2$ **9.** $u = Ce^{-12t} + \frac{5}{4}$ **11.** $y = 7e^{3t} + 2$
13. $y = 4(e^{2t} - 1)$ **15.** $y = 4(2e^{3t-3} - 1)$

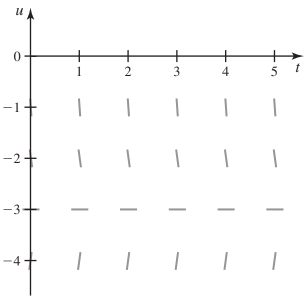
17. $y = \frac{3}{2}$; unstable



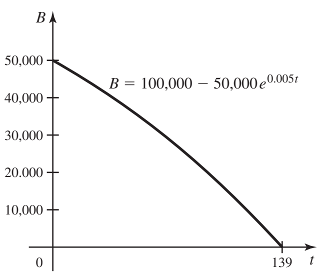
19. $y = -3$; stable



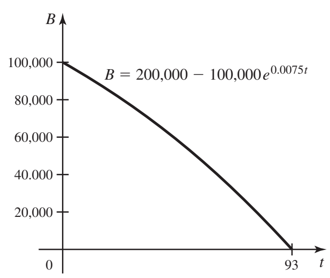
21. $u = -3$; stable



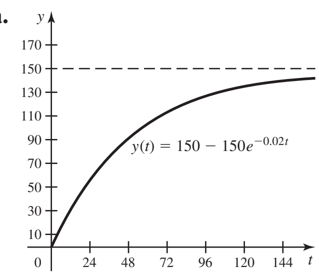
23. $B = 100{,}000 - 50{,}000e^{0.005t}$; reaches a balance of zero after approximately 139 months



25. $B = 200{,}000 - 100{,}000e^{0.0075t}$; reaches a balance of zero after approximately 93 months



27. Approx. 32 min **29.** Approx. 14 min
31. a.



b. 150 **c.** Approx. 115.1 hr

33. a. $h = 16\text{ yr}^{-1}$ **b.** 25,000 **35. a.** False **b.** True **c.** False **d.** False **37. a.** $B = 20{,}000 + 20{,}000e^{0.03t}$; the unpaid balance is growing because the monthly payment of \$600 is less than the interest on the unpaid balance. **b.** \$20,000 **c.** $\dfrac{m}{r}$

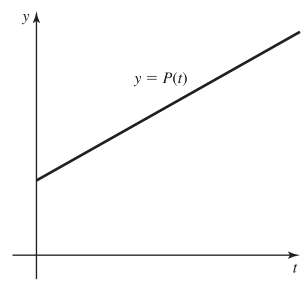
39. $y = 1 + \dfrac{t}{2} + \dfrac{5}{2t}, t > 0$ **41.** $y = \dfrac{1}{2}e^{3t} + \dfrac{7}{2}e^{t}$

45. $y(t) = \dfrac{6}{t}, t > 0$ **47.** $y = \dfrac{9t^5 + 20t^3 + 15t + 76}{15(t^2 + 1)}$

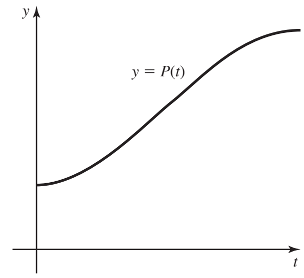
Section 9.5 Exercises, pp. 634–636

1. The growth rate function specifies the rate of growth of the population. The population is increasing when the growth rate function is positive, and the population is decreasing when the growth rate function is negative. **3.** If the growth rate function is positive (it does not matter whether it is increasing or decreasing), then the population is increasing. **5.** It is a linear, first-order differential equation. **7.** The solution curves in the *FH*-plane are closed curves that circulate around the equilibrium point.

9.



11.



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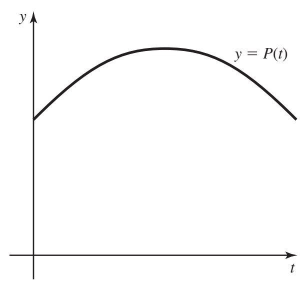
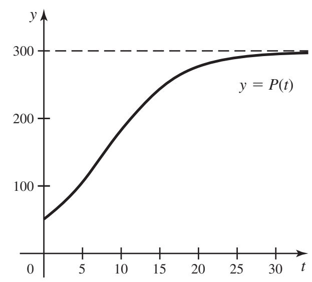
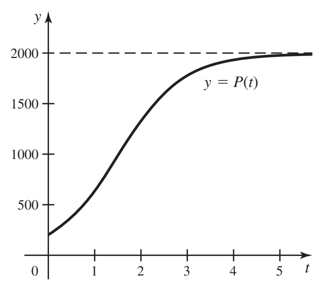


15. $P' = 0.2\,P\left(1 - \dfrac{P}{300}\right)$; $P = \dfrac{300}{5e^{-0.2t} + 1}, t \geq 0$

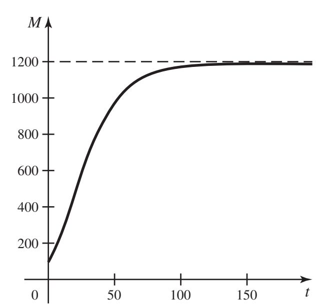


17. $P = \dfrac{2000}{9e^{-\ln(27/7)t} + 1}, t \geq 0$



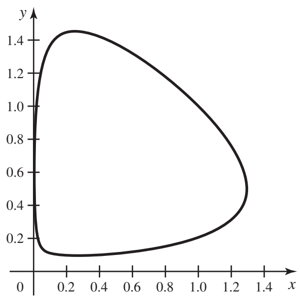
19. $M = K\left(\dfrac{M_0}{K}\right)^{e^{-rt}}, t \geq 0$

21. $M = 1200 \cdot 0.075^{\exp(-0.05t)}$, $t \geq 0$

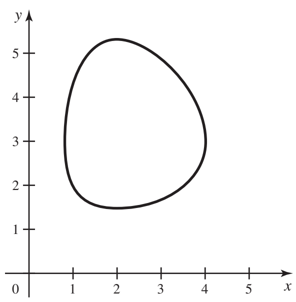


23. a. $m'(t) = -0.008t + 80, m(0) = 0$
b. $m = 10{,}000 - 10{,}000e^{-0.008t}, t \geq 0$
25. a. $m'(t) = -0.005\,t + 100, m(0) = 80{,}000$
b. $m = 60{,}000e^{-0.005t} + 20{,}000, t \geq 0$

27. a. x is the predator population; y is the prey population.
b. $x' = 0$ on the lines $x = 0$ and $y = \frac{1}{2}$; $y' = 0$ on the lines $y = 0$ and $x = \frac{1}{4}$. **c.** $(0, 0)$, $\left(\frac{1}{4}, \frac{1}{2}\right)$
d. $x' > 0$ and $y' > 0$ for $0 < x < \frac{1}{4}$, $y > \frac{1}{2}$
$x' > 0$ and $y' < 0$ for $x > \frac{1}{4}$, $y > \frac{1}{2}$
$x' < 0$ and $y' < 0$ for $x > \frac{1}{4}$, $0 < y < \frac{1}{2}$
$x' < 0$ and $y' > 0$ for $0 < x < \frac{1}{4}$, $0 < y < \frac{1}{2}$
e. The solution evolves in the clockwise direction.



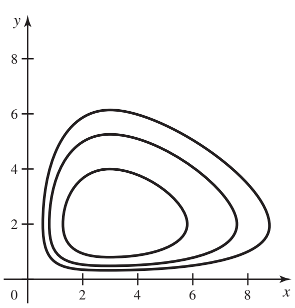
29. a. x is the predator population; y is the prey population.
b. $x' = 0$ on the lines $x = 0$ and $y = 3$; $y' = 0$ on the lines $y = 0$ and $x = 2$. **c.** $(0, 0)$, $(2, 3)$
d. $x' > 0$ and $y' > 0$ for $0 < x < 2$, $y > 3$
$x' > 0$ and $y' < 0$ for $x > 2$, $y > 3$
$x' < 0$ and $y' < 0$ for $x > 2$, $0 < y < 3$
$x' < 0$ and $y' > 0$ for $0 < x < 2$, $0 < y < 3$
e. The solution evolves in the clockwise direction.



31. a. True **b.** True **c.** True **35. c.** $\lim_{t \to \infty} m(t) = C_i V$, which is the amount of substance in the tank when the tank is filled with the inflow solution. **d.** Increasing R increases the rate at which the solution in the tank reaches the steady-state concentration.

37. a. $I = \dfrac{V}{R} e^{-t/(RC)}$ **b.** $Q = VC(1 - e^{-t/(RC)})$

39. a. $y'(x) = \dfrac{y(c - dx)}{x(-a + by)}$ **c.**

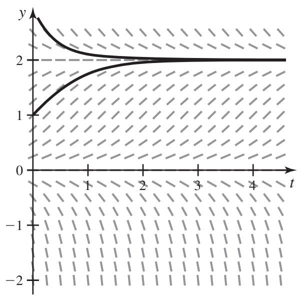


Chapter 9 Review Exercises, pp. 636–638

1. a. False **b.** False **c.** True **d.** True **e.** False
3. $y = Ce^{-2t} + 3$ **5.** $y = Ce^{t^2}$ **7.** $y = Ce^{\tan^{-1} t}$
9. $y = \tan(t^2 + t + C)$ **11.** $y = \sin t + t^2 + 1$
13. $Q = 8(1 - e^{t-1})$ **15.** $u = (3 + t^{2/3})^{3/2}$, $t > 0$

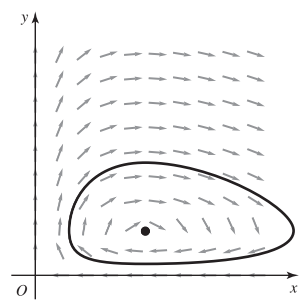
17. $s = \dfrac{t\sqrt{2}}{\sqrt{t^2 + 1}}$

19. a, b.



c. $0 < A < 2$
d. $A > 2$ or $A < 0$
e. $y = 0$ and $y = 2$

21. a. 1.05, 1.09762 **b.** 1.04939, 1.09651 **c.** 0.00217, 0.00106; the error in part (b) is smaller. **23.** $y = -3$ (unstable), $y = 0$ (stable), $y = 5$ (unstable) **25.** $y = -1$ (unstable), $y = 0$ (stable), $y = 2$ (unstable) **27. a.** 0.0713 **b.** $P = \dfrac{1600}{79e^{-0.0713t} + 1}$, $t \geq 0$
c. Approx. 61 hours **29. a.** $m = 2000(1 - e^{-0.005t})$
b. 2000 g **c.** Approx. 599 minutes **31. a.** x represents the predator. **b.** $x'(t) = 0$ when $x = 0$ and $y = 2$. $y'(t) = 0$ when $y = 0$ and $x = 5$. **c.** $(0, 0)$ and $(5, 2)$ **d.** $x' > 0$, $y' > 0$ when $0 < x < 5$ and $y > 2$; $x' > 0$, $y' < 0$ when $x > 5$ and $y > 2$; $x' < 0$, $y' < 0$ when $x > 5$ and $0 < y < 2$; $x' < 0$, $y' > 0$ when $0 < x < 5$ and $0 < y < 2$
e. Clockwise direction



33. a. $p_1 = 3$, $p_2 = -4$ **b.** $y(t) = t^3 - t^{-4}$, $t > 0$

CHAPTER 10

Section 10.1 Exercises, pp. 647–649

1. A sequence is an ordered list of numbers. Example: $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$
3. 1, 1, 2, 6, 24 **5.** $a_n = (-1)^{n+1} n$, for $n = 1, 2, 3, \ldots$; $a_n = (-1)^n (n + 1)$, for $n = 0, 1, 2, \ldots$ (Answers may vary.)

7. e **9.** 1, 5, 14, 30 **11.** $\sum_{k=1}^{\infty} 10$ (Answer is not unique.)

13. $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10{,}000}$ **15.** $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}$ **17.** $\frac{4}{3}, \frac{8}{5}, \frac{16}{9}, \frac{32}{17}$

19. 2, 1, 0, 1 **21.** 2, 4, 8, 16 **23.** 10, 18, 42, 114 **25.** $1, \dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{5}$

27. a. $\dfrac{1}{32}, \dfrac{1}{64}$ **b.** $a_1 = 1$, $a_{n+1} = \dfrac{1}{2} a_n$, for $n \geq 1$ **c.** $a_n = \dfrac{1}{2^{n-1}}$, for $n \geq 1$ **29. a.** 32, 64 **b.** $a_1 = 1$, $a_{n+1} = 2a_n$, for $n \geq 1$
c. $a_n = 2^{n-1}$, for $n \geq 1$ **31. a.** 243, 729 **b.** $a_1 = 1$, $a_{n+1} = 3a_n$, for $n \geq 1$ **c.** $a_n = 3^{n-1}$, for $n \geq 1$ **33. a.** $-5, 5$ **b.** $a_1 = -5$, $a_{n+1} = -a_n$, for $n \geq 1$ **c.** $a_n = (-1)^n \cdot 5$, for $n \geq 1$
35. 9, 99, 999, 9999; diverges **37.** $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10{,}000}$; converges to 0 **39.** 2, 4, 2, 4; diverges **41.** 2, 2, 2, 2; converges to 2
43. 54.545, 54.959, 54.996, 55.000; converges to 55