Xuemao Zhang **Math141_Calculus_II**
Assignment Homework8 due 11/04/2021 at 11:59pm EDT

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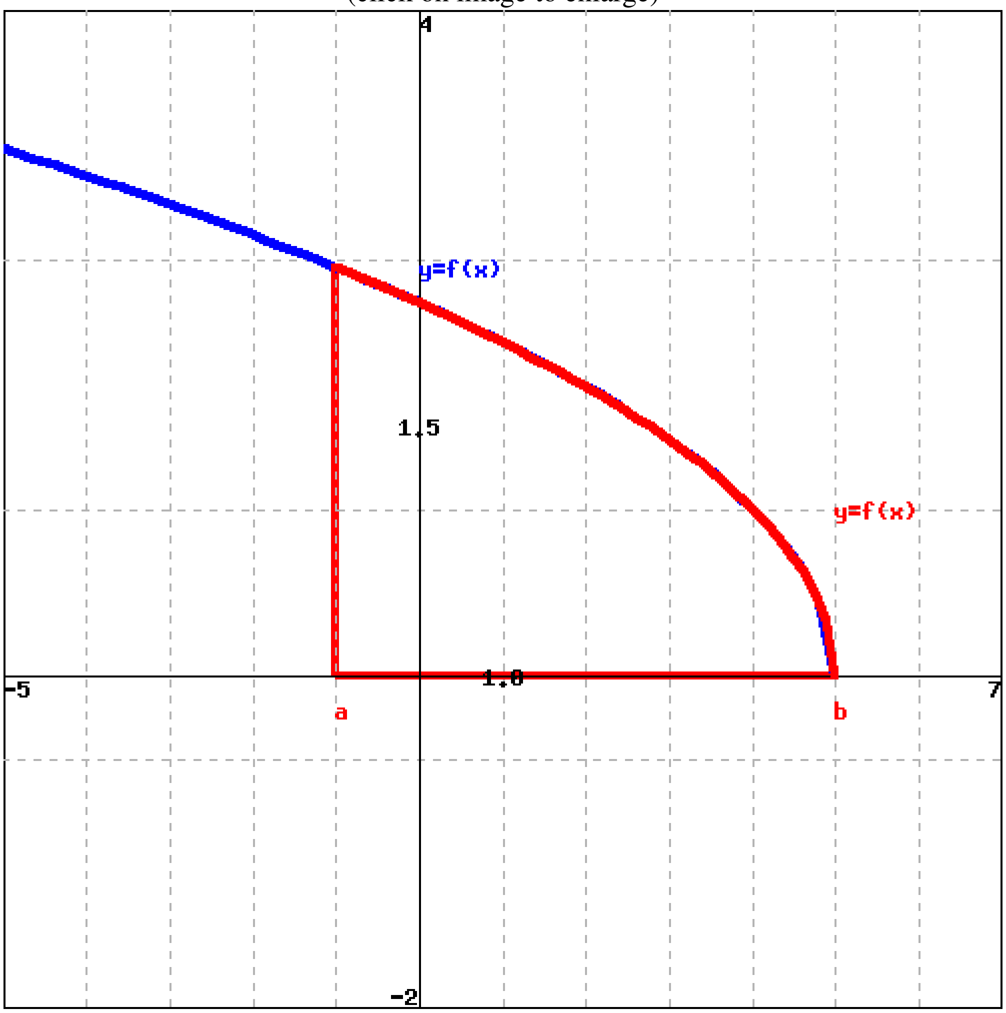
1. (1 point) `Library/Wiley/setAnton_Section_6.2/anton_6_2_Q1.pg`

Find the volume of the solid that results when the red region is revolved about the x-axis. $f(x) = \sqrt{5-x}$, $a = -1$, $b = 5$

(click on image to enlarge)



Volume = __________

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION
Area:

$$A(x) = \pi\, r(x)^2 = \pi\left(\sqrt{5-x}\right)^2 = \pi\,(5-x)$$

Volume

$$= \int_{-1}^{5} A(x)\, dx = \pi \int_{-1}^{5} [5-x]\, dx = \pi \left[5x - \frac{x^2}{2}\right]_{-1}^{5} = \pi \left[\left(\frac{25}{2}\right) - \left(-\frac{11}{2}\right)\right] = 18\pi$$

The volume of the solid revolved around the x-axis is 18π.

Correct Answers:

- `18*pi`

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2. **(1 point)** `Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/6_Applications_of_the_Integral/6.3_Volumes_of_Revolution/6.3.42.pg`

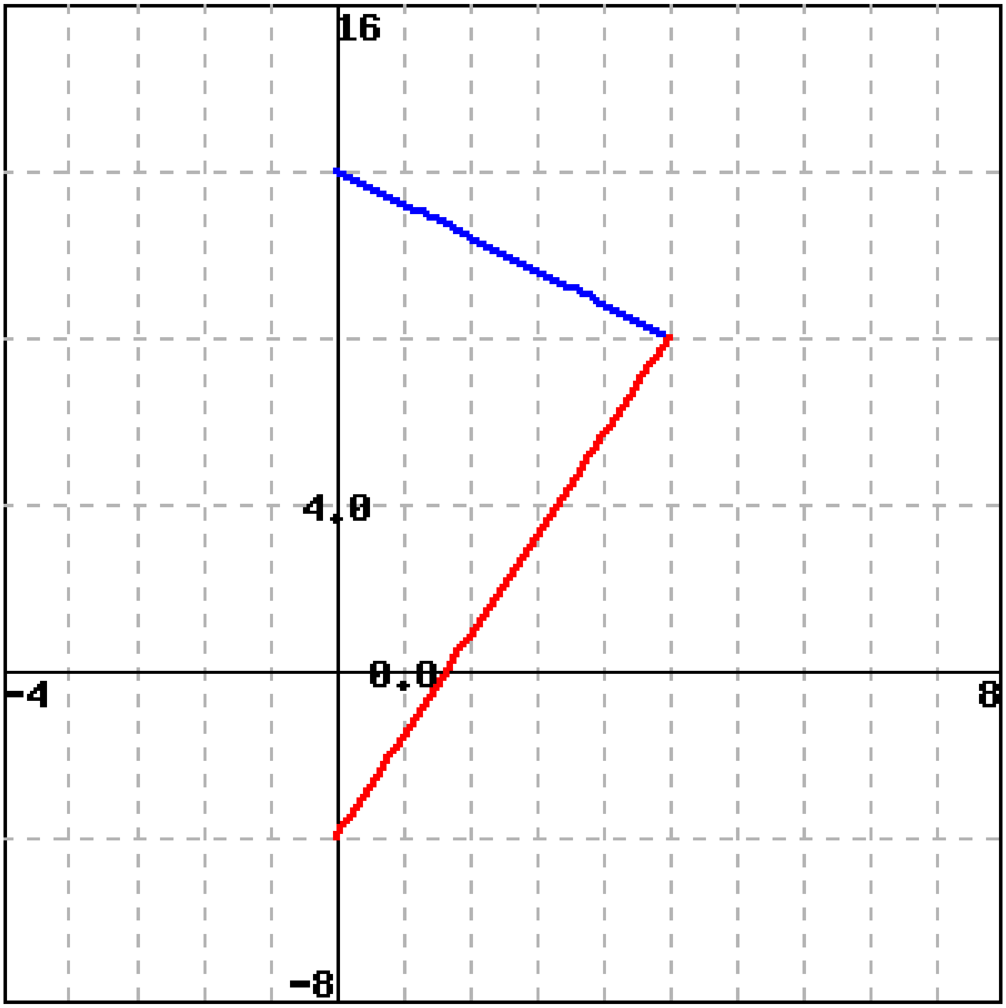
Find the volume of the solid obtained by rotating the region enclosed by the graphs of $y = 12 - x$, $y = 3x - 4$ and $x = 0$ about the y-axis.

$V =$__________

Solution: *(Instructor solution preview: show the student solution after due date.)*

Solution: Rotating the region enclosed by $y = 12 - x$, $y = 3x - 4$, and the y-axis (shown in the figure below) about the y-axis produces a solid with two different cross sections. For each $y \in [-4, 8]$, the cross section is a disk with radius $R = \frac{1}{3}(y+4)$; for each $y \in [8, 12]$, the cross section is a disk with radius $R = 12 - y$. The volume V of the solid of revolution is

$$\begin{aligned} V = \;& \pi \textstyle\int_{-4}^{8} \left(\frac{y+4}{3}\right)^2 dy + \pi \int_{8}^{12} (12-y)^2 dy \\ & = \tfrac{\pi}{9} \textstyle\int_{-4}^{8} (y+4)^2 dy + \pi \int_{8}^{12} (12-y)^2 dy \\ & = \tfrac{\pi}{9}\, \tfrac{(y+4)^3}{3}\Big|_{-4}^{8} - \pi\, \tfrac{(12-y)^3}{3}\Big|_{8}^{12} \\ & = \tfrac{\pi}{9}\left(\tfrac{12^3}{3}\right) + \pi\left(\tfrac{4^3}{3}\right) \\ & = \pi\left(4^3\right) + \pi\left(\tfrac{4^3}{3}\right) \\ & = \tfrac{4\pi}{3}\left(4^3\right) \\ & = 268.082573106329 \end{aligned}$$



Correct Answers:

- `268.082573106329`

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3. **(1 point)** `Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/6_Applications_of_the_Integral/6.3_Volumes_of_Revolution/6.3.9.pg`

Find the volume of the solid obtained by rotating the region under the graph of the function $f(x) = \frac{2}{x+1}$ about the x-axis over the interval $[0,1]$.

$V =$__________

Solution: *(Instructor solution preview: show the student solution after due date.)*

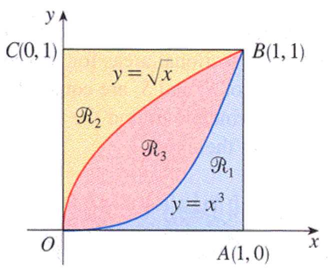
Solution: The volume of the solid of revolution is

$$\pi \int_0^1 \left(\frac{2}{x+1}\right)^2 dx = 4\pi \int_0^1 (x+1)^{-2} dx = -4\pi\,(x+1)^{-1}\Big|_0^1 = 2\pi$$

Correct Answers:

- `6.28318530717959`

4. **(1 point)** `Library/UCSB/Stewart5_6_2/Stewart5_6_2_23/Stewart5_6_2_23.pg`



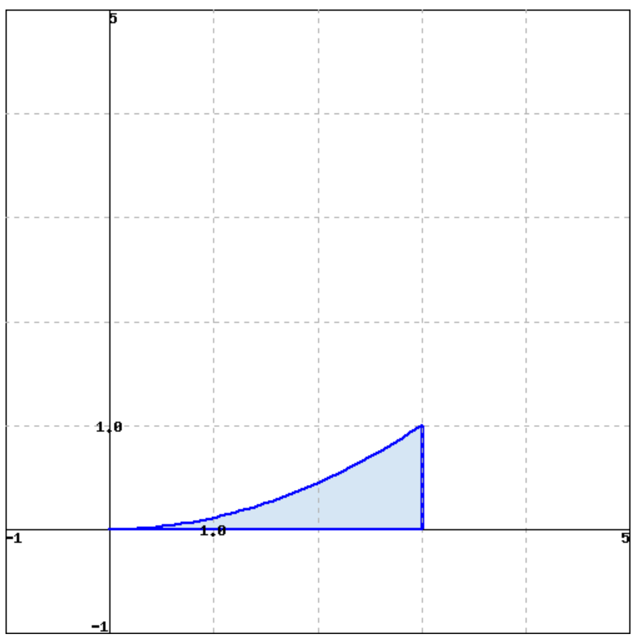
Referring to the figure above, find the volume generated by rotating the region $\mathcal{R}_2$ about the line OA.
Volume = ____________

Correct Answers:

- `pi/2`

5. **(1 point)** `Library/UMN/calculusStewartCCC/s_6_2_8.pg`

Find the volume of the solid obtained by rotating the region bounded by $y = \frac{1}{9}x^2$, $x = 3$, and $y = 0$ about the y-axis. Below is a graph of the bounded region.



Volume = __________

Note: *You can click on the graph to enlarge the image.*

Correct Answers:

- `9/2*pi`

6. **(1 point)** `Library/UCSB/Stewart5_6_2/Stewart5_6_2_10.pg`

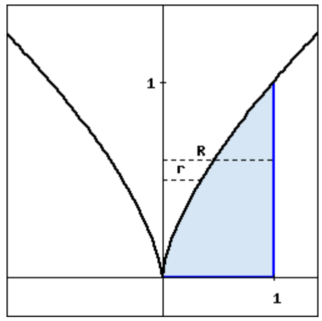
Using disks or washers, find the volume of the solid obtained by rotating the region bounded by the curves $y = x^{2/3}$, $x = 1$, and $y = 0$ about the y-axis.

Volume = ______________

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

The region to be rotated about the y-axis is shown below.



The curve $y = x^{2/3}$ intersects the vertical line $x = 1$ at $y = 1$. Also note that the right half of the curve $y = x^{2/3}$ is the curve $x = y^{3/2}$.

We use the method of slicing, that is, $V = \int_c^d A(y)\,dy$, with each slice a **washer** with thickness dy.

The area of each washer slice is $A(y) = \pi\left(R^2 - r^2\right)$ where each radius depends on the value of y and the range of y for the region is $0 \le y \le 1$.

The larger radius R is 1 for all values of y.

The smaller radius r goes from the y-axis ($x = 0$) to the curve $x = y^{3/2}$.

So $r = y^{3/2} - 0 = y^{3/2}$, and

$$
\begin{aligned}
V &= \int_c^d A(y)\,dy = \int_c^d \pi\left(R^2 - r^2\right)\,dy \\
&= \int_0^1 \pi\left(1 - \left(y^{3/2}\right)^2\right)\,dy \\
&= \int_0^1 \pi\left(1 - y^3\right)\,dy \\
&= \pi\left[y - \frac{y^4}{4}\right]_0^1 \\
&= \pi\left[1 - \frac{1}{4}\right] \\
&= \frac{3}{4}\pi \quad \text{(cubic units).}
\end{aligned}
$$

Correct Answers:

- `pi*3/4`

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7. **(1 point)** `Library/UCSB/Stewart5_6_2/Stewart5_6_2_8.pg`

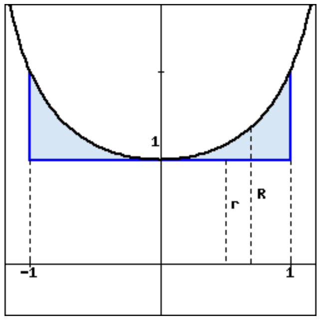
Using disks or washers, find the volume of the solid obtained by rotating the region bounded by the curves $y = \sec(x)$, $y = 1$, $x = -1$, and $x = 1$ about the x-axis.

Volume = ______________

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

The region to be rotated about the x axis is shown below.



We use the method of slicing, that is, $V = \int_a^b A(x)\,dx$, with each slice a **washer** with thickness dx.
The area of each washer slice is $A(x) = \pi(R^2 - r^2)$ where each radius depends on the value of x, and the range of x for the region is $-1 \leq x \leq 1$.
The largest radius, R, goes from the $x-$axis to the curve $y = \sec x$, and so is $R = \sec x$.
The smallest radius, r, goes from the $x-$axis to the line $y = 1$, and so is $r = 1$.
We will use symmetry and evaluate twice the volume of the solid obtained by rotating the region from $x = 0$ to $x = 1$: So

$$\begin{aligned} V &= \int_a^b A(x)\,dx = \int_a^b \pi(R^2 - r^2)\,dx \\ &= 2\int_0^1 \pi\left[\sec^2 x - 1\right]\,dx \\ &= 2\pi\Big[\tan x - x\Big]_0^1 \\ &= 2\pi(\tan 1 - 1) \end{aligned}$$

Correct Answers:

- `pi*2*(tan(1)-1)`

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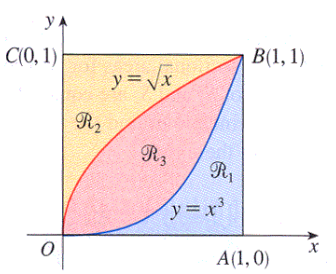
8. **(1 point)** `Library/UCSB/Stewart5_6_2/Stewart5_6_2_21/Stewart5_6_2_21.pg`



Referring to the figure above, find the volume generated by rotating the region $\mathcal{R}_1$ about the line AB.
Volume = ____________
Correct Answers:

- `pi/10`