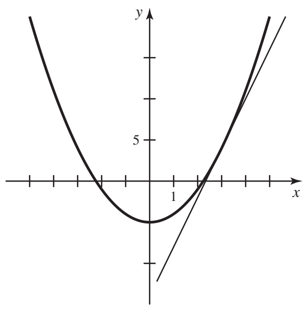
83. a. Let $f(x) = x - \cos x$; $f(0) < 0 < f\left(\frac{\pi}{2}\right)$ **b.** $x \approx 0.739$
85. a. $m(0) < 30 < m(5)$ and $m(5) > 30 > m(15)$
b. $m = 30$ when $t \approx 2.4$ hr and $t \approx 10.8$ hr **c.** No; the maximum amount is approximately $m(5.5) \approx 38.5$. **87.** $\delta = \varepsilon$
89. $\delta = \min\left\{1, \frac{\varepsilon}{15}\right\}$ **91.** $\delta = 1/\sqrt[4]{N}$

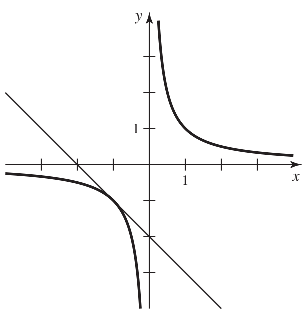
CHAPTER 3

Section 3.1 Exercises, pp. 137–140

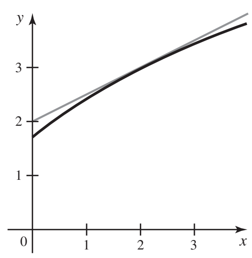
1. Given the point $(a, f(a))$ and any point $(x, f(x))$ near $(a, f(a))$, the slope of the secant line joining these points is $\frac{f(x) - f(a)}{x - a}$. The limit of this quotient as x approaches a is the slope of the tangent line at the point. **3.** The average rate of change over the interval $[a, x]$ is $\frac{f(x) - f(a)}{x - a}$. The value of $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ is the slope of the tangent line; it is also the limit of average rates of change, which is the instantaneous rate of change at $x = a$. **5.** $f'(a)$ is the slope of the tangent line at $(a, f(a))$ or the instantaneous rate of change in f at a.
7. $f(2) = 7$; $f'(2) = 4$ **9.** $y = 3x - 1$ **11.** -5 **13.** 68 ft/s
15. a. 6 **b.** $y = 6x - 14$
c.



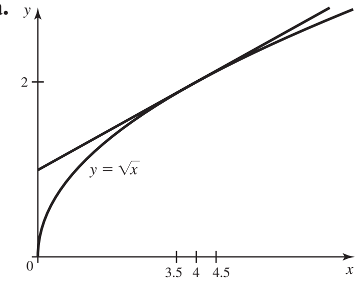
17. a. -1 **b.** $y = -x - 2$
c.



19. a. $\frac{1}{2}$ **b.** $y = \frac{1}{2}x + 2$
c.



21. a. 2 **b.** $y = 2x + 1$ **23. a.** 2 **b.** $y = 2x - 3$
25. a. 4 **b.** $y = 4x - 8$ **27. a.** 3 **b.** $y = 3x - 2$
29. a. $\frac{2}{25}$ **b.** $y = \frac{2}{25}x + \frac{7}{25}$ **31. a.** $\frac{1}{4}$ **b.** $y = \frac{1}{4}x + \frac{7}{4}$
33. a. 8 **b.** $y = 8x$ **35. a.** -14 **b.** $y = -14x - 16$
37. a. -4 **b.** $y = -4x + 3$ **39. a.** $\frac{1}{3}$ **b.** $y = \frac{1}{3}x + \frac{5}{3}$
41. a. $-\frac{1}{100}$ **b.** $y = -\frac{x}{100} + \frac{3}{20}$ **43.** $-\frac{1}{4}$ **45.** $\frac{1}{5}$ **47. a.** True **b.** False **c.** True **49.** $d'(4) = 128$ ft/s; the object falls with an instantaneous speed of 128 ft/s four seconds after being dropped.
51. $v'(3) = -4$ m/s per second; the instantaneous rate of change in the car's speed is -4 m/s^2 at $t = 3$ s.
53. a. $L'(1.5) \approx 4.3$ mm/week; the talon is growing at a rate of approximately 4.3 mm/week at $t = 1.5$ weeks (answers will vary). **b.** $L'(a) \approx 0$, for $a \geq 4$; the talon stops growing at $t = 4$ weeks. **55.** $D'(60) \approx 0.05$ hr/day; the number of daylight hours is increasing at about 0.05 hr/day, 60 days after Jan 1. $D'(170) \approx 0$ hr/day; the number of daylight hours is neither increasing nor decreasing 170 days after Jan 1. **57.** $f(x) = 5x^2$; $a = 2$; 20
59. $f(x) = x^4$; $a = 2$; 32 **61.** $f(x) = |x|$; $a = -1$; -1
63. a.



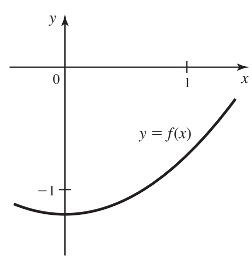
b.

h	Approximation	Error
0.1	0.25002	2.0×10^{-5}
0.01	0.25000	2.0×10^{-7}
0.001	0.25000	2.0×10^{-9}

c. Values of x on both sides of 4 are used in the formula.
d. The centered difference approximations are more accurate than the forward and backward difference approximations. **65. a.** 0.39470, 0.41545 **b.** 0.02, 0.0003

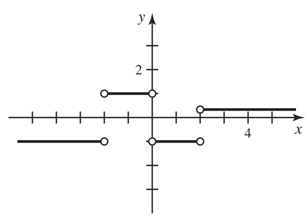
Section 3.2 Exercises, pp. 148–152

1. f' is the slope function of f. **3.** $\frac{dy}{dx}$ is the limit of $\frac{\Delta y}{\Delta x}$ as $\Delta x \to 0$.
5.



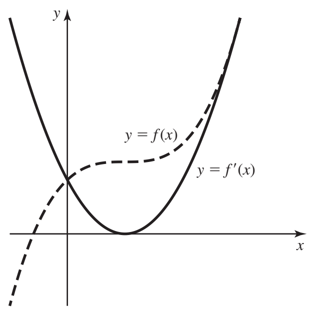
7. Yes

9. A line with a y-intercept of 1 and a slope of 3
11. $f'(x) = 7$ **13.** $\frac{dy}{dx} = 2x$; $\left.\frac{dy}{dx}\right|_{x=3} = 6$; $\left.\frac{dy}{dx}\right|_{x=-2} = -4$
15. a–C; b–C; c–A; d–B
17.

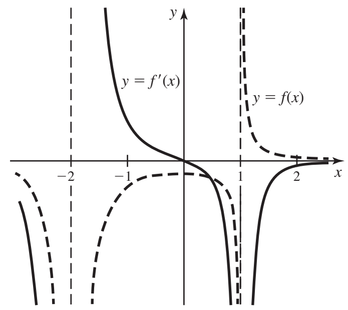


19. a. Not continuous at $x = 1$ **b.** Not differentiable at $x = 0, 1$
21. a. $f'(x) = 5$ **b.** $f'(1) = 5$; $f'(2) = 5$
23. a. $f'(x) = 8x$ **b.** $f'(2) = 16$; $f'(4) = 32$
25. a. $f'(x) = -\frac{1}{(x + 1)^2}$ **b.** $f'\left(-\frac{1}{2}\right) = -4$; $f'(5) = -\frac{1}{36}$
27. a. $f'(t) = -\frac{1}{2t^{3/2}}$ **b.** $f'(9) = -\frac{1}{54}$; $f'\left(\frac{1}{4}\right) = -4$
29. a. $f'(s) = 12s^2 + 3$ **b.** $f'(-3) = 111$; $f'(-1) = 15$
31. a. $v(t) = -32t + 100$ **b.** $v(1) = 68$ ft/s; $v(2) = 36$ ft/s
33. $\frac{dy}{dx} = \frac{1}{(x + 2)^2}$; $\left.\frac{dy}{dx}\right|_{x=2} = \frac{1}{16}$ **35. a.** $6x + 2$
b. $y = 8x - 13$ **37. a.** $\frac{3}{2\sqrt{3x + 1}}$ **b.** $y = 3x/10 + 13/5$

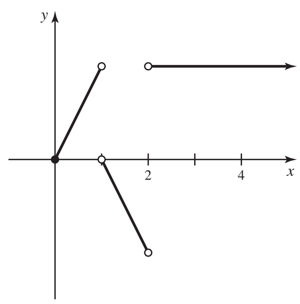
39. a. $\dfrac{6}{(3x+1)^2}$ **b.** $y = -3x/2 - 5/2$
41. a. Approximately 10 kW; approximately -5 kW
b. $t = 6, 18$ **c.** $t = 12$ **43. a.** $2ax + b$ **b.** $8x - 3$ **c.** 5
45. a. C, D **b.** A, B, E **c.** A, B, E, D, C **47.** a–D; b–C; c–B; d–A
49.



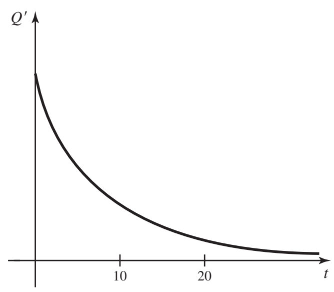
51.



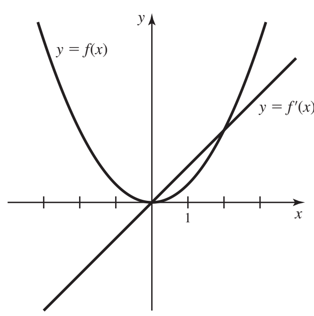
53. a. $x = 1$ **b.** $x = 1, x = 2$ **c.**



55. a. $t = 0$ **b.** Positive **c.** Decreasing
d.



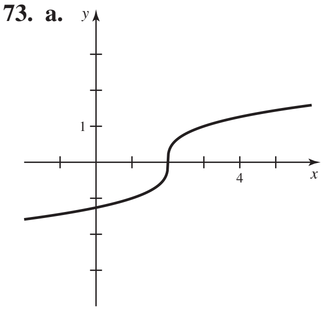
57. a. True **b.** True **c.** False **59.** $a = 4$
61. Yes



63. $y = -\dfrac{x}{3} - \dfrac{2}{3}$ **65.** $y = \dfrac{x}{2} + \dfrac{3}{2}$ **67.** $(1, 2), (5, 26)$

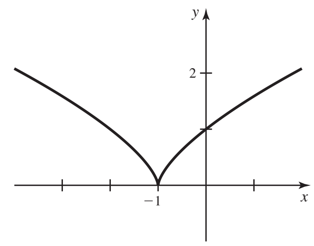
69. $(1, 1), \left(-\dfrac{1}{2}, -2\right)$ **71. b.** $f'_+(2) = 1$; $f'_-(2) = -1$
c. f is continuous but not differentiable at $x = 2$.

73. a.



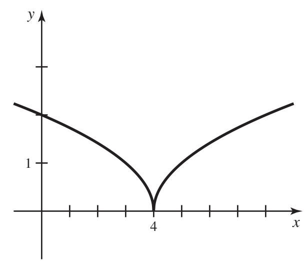
Vertical tangent line $x = 2$

b.



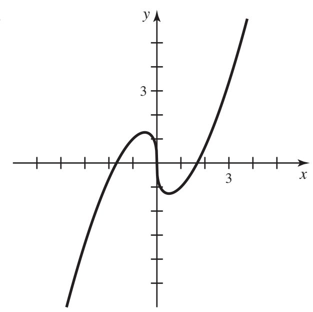
Vertical tangent line $x = -1$

c.



Vertical tangent line $x = 4$

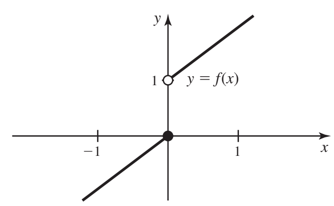
d.



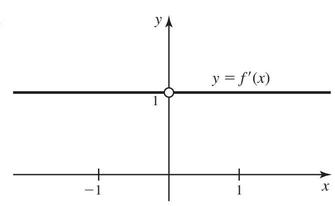
Vertical tangent line $x = 0$

75. $f'(x) = \dfrac{1}{3}x^{-2/3}$ and $\lim_{x\to 0^-} |f'(x)| = \lim_{x\to 0^+} |f'(x)| = \infty$

77. a.



b. 1 **c.** 1 **d.**



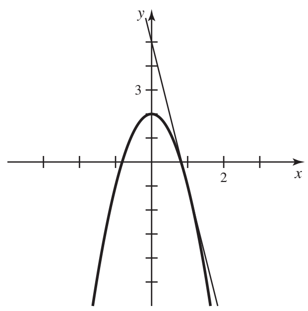
e. f is not differentiable at 0 because it is not continuous at 0.

Section 3.3 Exercises, pp. 159–162

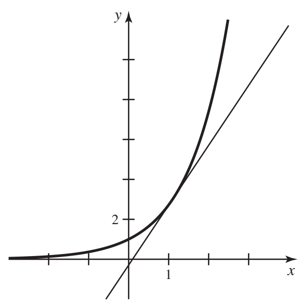
1. Using the definition can be tedious. **3.** $f(x) = e^x$ **5.** Take the product of the constant and the derivative of the function. **7.** 4
9. $-\dfrac{1}{2}$ **11.** -2 **13.** 7.5 **15.** $10t^9$; $90t^8$; $720t^7$ **17.** $\dfrac{2}{5}$ **19.** $5x^4$
21. 0 **23.** $15x^2$ **25.** t **27.** 8 **29.** $200t$ **31.** $12x^3 + 7$
33. $40x^3 - 32$ **35.** $6w^2 + 6w + 10$ **37.** $3e^x + 5$
39. $\begin{cases} 2x & \text{if } x < 0 \\ 4x + 1 & \text{if } x > 0 \end{cases}$ **41. a.** $d'(t) = 32t$; ft/s; the velocity of the stone **b.** 576 ft; approx. 131 mi/hr **43. a.** $A'(t) = -\frac{1}{25}t + 2$ measures the rate at which the city grows in mi^2/yr. **b.** 1.6 mi^2/yr **c.** 1200 people/yr

45. $w'(x) = \begin{cases} 0.4 & \text{if } 19 < x < 21 \\ 0.8 & \text{if } 21 < x < 32 \\ 1.5 & \text{if } x > 32 \end{cases}$ **47.** $18x^2 + 6x + 4$
49. $2w$, for $w \neq 0$ **51.** $4x^3 + 4x$ **53.** 1, for $x \neq 1$
55. $\dfrac{1}{2\sqrt{x}}$, for $x \neq a$ **57.** e^w

59. a. $y = -6x + 5$ **b.**



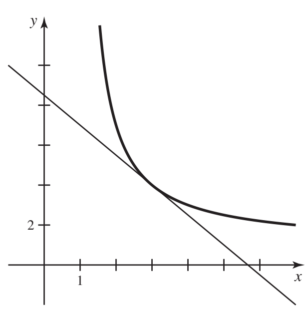
61. a. $y = 3x + 3 - 3 \ln 3$ **b.**



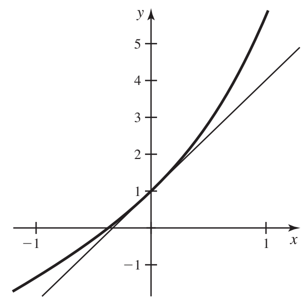
63. a. $x = 3$ **b.** $x = 4$
65. a. $(-1, 11), (2, -16)$ **b.** $(-3, -41), (4, 36)$
67. a. $(4, 4)$ **b.** $(16, 0)$ **69.** $f'(x) = 20x^3 + 30x^2 + 3$; $f''(x) = 60x^2 + 60x$; $f'''(x) = 120x + 60$
71. $f'(x) = 1$; $f''(x) = f'''(x) = 0$, for $x \neq -1$
73. a. False **b.** True **c.** False **d.** False **e.** False
75. a. $y = 7x - 1$ **b.** $y = -2x + 5$ **c.** $y = 16x + 4$
77. $b = 2, c = 3$ **79.** -10 **81.** 4 **83. a.** $f(x) = x + e^x$; $a = 0$ **b.** 2 **85. a.** $f(x) = \sqrt{x}$; $a = 9$ **b.** $\frac{1}{6}$ **87. a.** $f(x) = e^x$; $a = 3$ **b.** e^3 **89.** 3 **91.** 1 **95. d.** $\frac{n}{2} x^{n/2 - 1}$ **97. c.** $2e^{2x}$

Section 3.4 Exercises, pp. 168–170

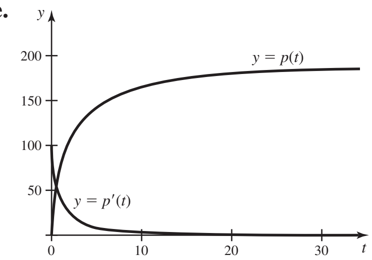
1. $\dfrac{d}{dx}(f(x)\,g(x)) = f'(x)\,g(x) + f(x)\,g'(x)$ **3.** $6x + 5$
5. $\dfrac{5}{(3x + 2)^2}$ **7. a.** $2x - 1$ **9. a.** $6x + 1$ **11. a.** $2w$, for $w \neq 0$
13. 1, for $x \neq a$ **15.** $23; \dfrac{7}{4}$ **17.** $\dfrac{2}{27}; \dfrac{3}{8}$ **19.** $36x^5 - 12x^3$
21. $\dfrac{1}{(x + 1)^2}$ **23.** $e^t t^{2/3}\left(t + \dfrac{5}{3}\right)$ **25.** $\dfrac{e^x}{(e^x + 1)^2}$ **27.** $e^{-x}(1 - x)$
29. $-\dfrac{1}{(t - 1)^2}$ **31.** $4x^3$ **33.** $e^w(w^3 + 3w^2 - 1)$ **35.** $t^2 e^t$
37. $\dfrac{e^x(x^2 - 2x - 1)}{(x^2 - 1)^2}$ **39.** $-27x^{-10}$ **41.** $6t - 42/t^8$
43. $-3/t^2 - 2/t^3$ **45.** $\dfrac{e^x(x^2 - x - 5)}{(x - 2)^2}$
47. $\dfrac{e^x(x^2 + x + 1)}{(x + 1)^2}$ **49.** $\dfrac{\sqrt{w}}{(\sqrt{w} - w)^2}$ **51.** $\dfrac{5w^{2/3}}{3(w^{5/3} + 1)^2}$
53. $8x - \dfrac{2}{(5x + 1)^2}$ **55.** $\dfrac{r - 6\sqrt{r} - 1}{2\sqrt{r}(r + 1)^2}$
57. $300x^9 + 135x^8 + 105x^6 + 120x^3 + 45x^2 + 15$ **59.** $e^x + 8x$
61. a. $y = -3x/2 + 17/2$ **b.**



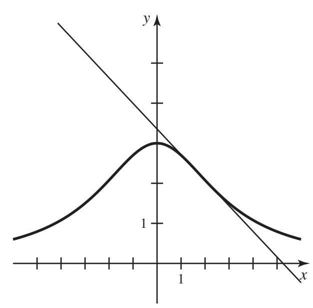
63. a. $y = 3x + 1$ **b.**



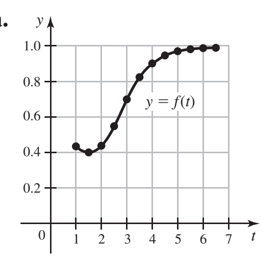
65. a. $p'(t) = \left(\dfrac{20}{t + 2}\right)^2$ **b.** $p'(5) \approx 8.16$ **c.** $t = 0$
d. $\lim_{t \to \infty} p(t) = 200$; the population approaches a steady state.
e.



67. a. $F'(x) = -\dfrac{1.8 \times 10^{10}\, Qq}{x^3}$ N/m **b.** -1.8×10^{19} N/m
c. $|F'(x)|$ decreases as x increases. **69. a.** False **b.** False
c. False **d.** False **71.** $4x - \dfrac{1}{x^2}; 2\left(\dfrac{1}{x^3} + 2\right); -\dfrac{6}{x^4}$
73. $\dfrac{x^2 + 2x - 7}{(x + 1)^2}; \dfrac{16}{(x + 1)^3}$
75. a. $y = -\dfrac{108}{169}x + \dfrac{567}{169}$ **b.**



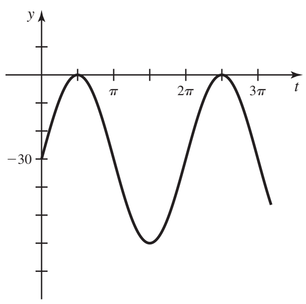
77. $-\dfrac{3}{2}$ **79.** $\dfrac{1}{9}$ **81.** $\dfrac{7}{8}$
83. a.



b. $t \approx 3$
c. $f'(3) \approx 0.28 \dfrac{\text{mm/g}}{\text{week}}$; at a young age, the bird's wings are growing quickly relative to its weight.
d. $f'(6.5) \approx 0.003 \dfrac{\text{mm/g}}{\text{week}}$; the rate of change of the ratio of wing chord length to mass is nearly 0. **85.** $\dfrac{15}{2}$ **87.** $-\dfrac{5}{2}$ **89.** 1
91. a. $y = -2x + 16$ **b.** $y = -\dfrac{5}{9}x + \dfrac{23}{9}$
93. -90 **97.** $f''g + 2f'g' + fg''$ **99. a.** $f'gh + fg'h + fgh'$ **b.** $e^x(x^2 + 4x - 1)$

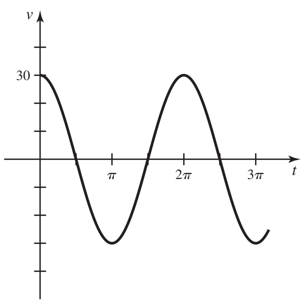
Section 3.5 Exercises, pp. 175–178

1. $\frac{\sin x}{x}$ is undefined at $x = 0$. **3.** The tangent and cotangent functions are defined as ratios of the sine and cosine functions. **5.** -1 **7.** $y = x$ **9.** $-\sin x - \cos x$ **11.** 3 **13.** $\frac{7}{3}$ **15.** 5 **17.** 7 **19.** $\frac{1}{4}$ **21.** a/b **23.** $\cos x - \sin x$ **25.** $e^{-x}(\cos x - \sin x)$ **27.** $\sin x + x \cos x$ **29.** $-\frac{1}{1 + \sin x}$ **31.** $\cos^2 x - \sin^2 x = \cos 2x$ **33.** $-2 \sin x \cos x = -\sin 2x$ **35.** $w^2 \cos w$ **37.** $x \cos 2x + \frac{1}{2} \sin 2x$ **39.** $\frac{1}{1 + \cos x}$ **41.** $\frac{2 \sin x}{(1 + \cos x)^2}$ **43.** $\sec x \tan x - \csc x \cot x$ **45.** $e^x \csc x(1 - \cot x)$ **47.** $-\frac{\csc x}{1 + \csc x}$ **49.** $\cos^2 z - \sin^2 z = \cos 2z$ **51.** $2 \sin^2 x$ **55. a.**

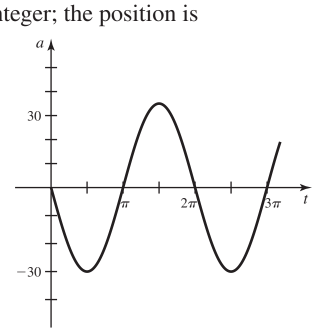


b. $v(t) = 30 \cos t$

c.

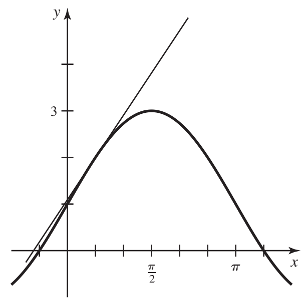


d. $v(t) = 0$, for $t = (2k + 1)\frac{\pi}{2}$, where k is any nonnegative integer; the position is $y\left((2k + 1)\frac{\pi}{2}\right) = 0$ if k is even or $y\left((2k + 1)\frac{\pi}{2}\right) = -60$ if k is odd. **e.** $v(t)$ has a maximum at $t = 2k\pi$, where k is a nonnegative integer; the position is $y(2k\pi) = -30$. **f.** $a(t) = -30 \sin t$

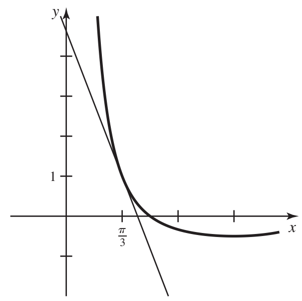


57. $2 \cos x - x \sin x$ **59.** $2e^x \cos x$ **61.** $2 \csc^2 x \cot x$ **63.** $2\,(\sec^2 x \tan x + \csc^2 x \cot x)$ **65. a.** False **b.** False **c.** True **d.** True **67.** 2 **69.** $-\frac{1}{2}$ **71.** $\frac{4}{3}$

73. a. $y = \sqrt{3}x + 2 - \frac{\pi\sqrt{3}}{6}$ **b.**

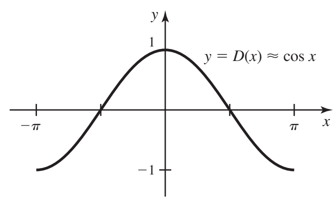


75. a. $y = -2\sqrt{3}x + \frac{2\sqrt{3}\pi}{3} + 1$ **b.**



77. $x = 7\pi/6 + 2k\pi$ and $x = 11\pi/6 + 2k\pi$, where k is an integer **85.** $a = 0$ **87. a.** $2 \sin x \cos x$ **b.** $3 \sin^2 x \cos x$ **c.** $4 \sin^3 x \cos x$ **d.** $n \sin^{n-1} x \cos x$; the conjecture is true for $n = 1$. If it holds for $n = k$, then when $n = k + 1$, we have $\frac{d}{dx}(\sin^{k+1} x) = \frac{d}{dx}(\sin^k x \cdot \sin x) = \sin^k x \cos x + \sin x \cdot k \sin^{k-1} x \cos x = (k + 1) \sin^k x \cos x$.

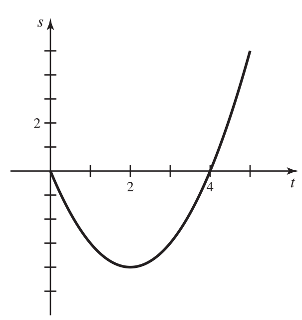
89. Because D is a difference quotient for f (and $h = 0.01$ is small), D is a good approximation to f'. Therefore, the graph of D is nearly indistinguishable from the graph of $f'(x) = \cos x$.



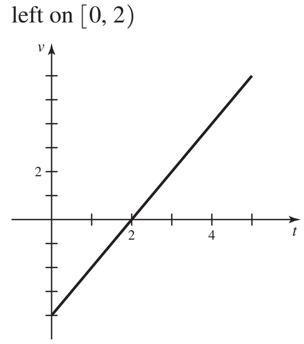
Section 3.6 Exercises, pp. 186–191

1. The average rate of change is $\frac{f(x + \Delta x) - f(x)}{\Delta x}$, whereas the instantaneous rate of change is the limit as Δx goes to zero in this quotient. **3.** Small **5.** At 15 weeks, the puppy grows at a rate of 1.75 lb/week. **7.** If the position of the object at time t is $s(t)$, then the acceleration at time t is $a(t) = d^2s/dt^2$. **9.** $v'(T) = 0.6$; the speed of sound increases by approximately 0.6 m/s for each increase of 1°C. **11. a.** 40 mi/hr **b.** 40 mi/hr; yes **c.** -60 mi/hr; -60 mi/hr; south **d.** The police car drives away from the police station going north until about 10:08, when it turns around and heads south, toward the police station. It continues south until it passes the police station at about 11:02 and keeps going south until about 11:40, when it turns around and heads north. **13.** The first 200 stoves cost, on average, \$70 to produce. When 200 stoves have already been produced, the 201st stove costs \$65 to produce.

15. a.

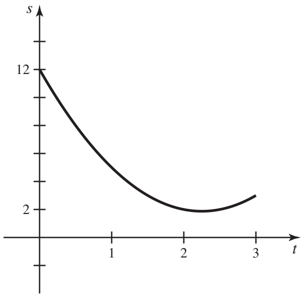


b. $v(t) = 2t - 4$; stationary at $t = 2$, to the right on $(2, 5]$, to the left on $[0, 2)$

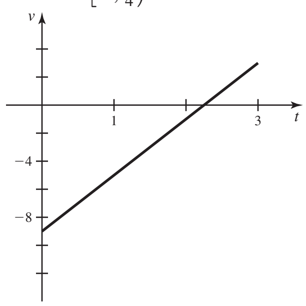


c. $v(1) = -2$ ft/s; $a(1) = 2$ ft/s^2 **d.** $a(2) = 2$ ft/s^2 **e.** $(2, 5]$

17. a.

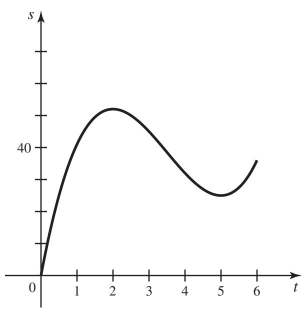


b. $v(t) = 4t - 9$; stationary at $t = \frac{9}{4}$, to the right on $\left(\frac{9}{4}, 3\right]$, to the left on $\left[0, \frac{9}{4}\right)$

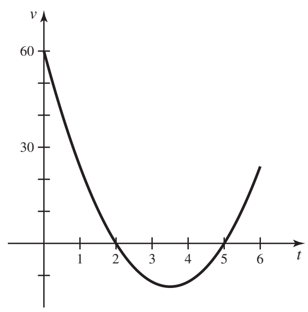


c. $v(1) = -5$ ft/s; $a(1) = 4$ ft/s^2
d. $a\left(\frac{9}{4}\right) = 4$ ft/s^2 **e.** $\left(\frac{9}{4}, 3\right]$

19. a.

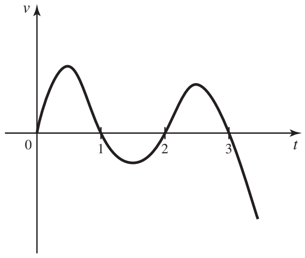


b. $v(t) = 6t^2 - 42t + 60$; stationary at $t = 2$ and $t = 5$, to the right on $[0, 2)$ and $(5, 6]$, to the left on $(2, 5)$



c. $v(1) = 24$ ft/s; $a(1) = -30$ ft/s^2 **d.** $a(2) = -18$ ft/s^2; $a(5) = 18$ ft/s^2 **e.** $\left(2, \frac{7}{2}\right), (5, 6]$ **21.** -64 ft/s; 64 ft/s
23. a. $v(t) = -32t + 32$ **b.** At $t = 1$ s **c.** 64 ft **d.** At $t = 3$ s **e.** -64 ft/s **f.** $(1, 3)$ **25. a.** $v(t) = -32t + 64$ **b.** At $t = 2$ **c.** 96 ft **d.** At $2 + \sqrt{6}$ **e.** $-32\sqrt{6}$ ft/s **f.** $(2, 2 + \sqrt{6})$
27. Approx. 90.5 ft/s **29. a.** $\overline{C}(x) = \dfrac{1000}{x} + 0.1$; $C'(x) = 0.1$
b. $\overline{C}(2000) = \$0.60/\text{item}$; $C'(2000) = \$0.10/\text{item}$
c. The average cost per item when 2000 items are produced is \$0.60/item. The cost of producing the 2001st item is \$0.10.
31. a. $\overline{C}(x) = -0.01x + 40 + 100/x$; $C'(x) = -0.02x + 40$
b. $\overline{C}(1000) = \$30.10/\text{item}$; $C'(1000) = \$20/\text{item}$
c. The average cost per item is about \$30.10 when 1000 items are produced. The cost of producing the 1001st item is \$20. **33. a.** 20 **b.** \$20 **c.** $E(p) = \frac{p}{p - 20}$ **d.** Elastic for $p > 10$; inelastic for $0 < p < 10$ **e.** 2.5% **f.** 2.5% **35. a.** False **b.** True **c.** False **d.** True **37.** 240 ft **39.** 64 ft/s **41. a.** $t = 1, 2, 3$ **b.** It is moving in the positive direction for t in $(0, 1)$ and $(2, 3)$; it is moving in the negative direction for t in $(1, 2)$ and $t > 3$.

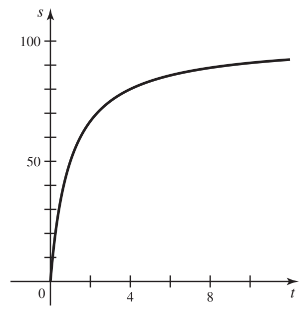
c.



d. $\left(0, \frac{1}{2}\right), \left(1, \frac{3}{2}\right), \left(2, \frac{5}{2}\right), (3, \infty)$

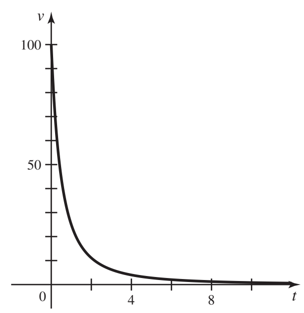
43. a. $P(x) = 0.02x^2 + 50x - 100$
b. $\dfrac{P(x)}{x} = 0.02x + 50 - \dfrac{100}{x}$; $\dfrac{dP}{dx} = 0.04x + 50$
c. $\dfrac{P(500)}{500} = 59.8$; $\dfrac{dp}{dx}(500) = 70$
d. The profit, on average, for each of the first 500 items produced is 59.8; the profit for the 501st item produced is 70.
45. a. $P(x) = 0.04x^2 + 100x - 800$
b. $\dfrac{P(x)}{x} = 0.04x + 100 - \dfrac{800}{x}$; $\dfrac{dp}{dx} = 0.08x + 100$
c. $\dfrac{P(1000)}{1000} = 139.2$; $p'(1000) = 180$
d. The average profit per item for each of the first 1000 items produced is \$139.20. The profit for the 1001st item produced is \$180.
47. About 1935; approximately 890,000 people/yr (answers will vary)

49. a.



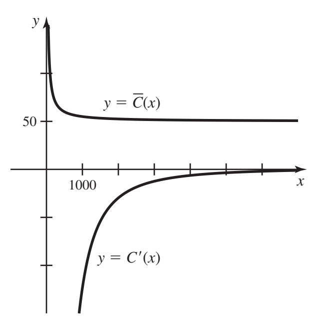
b. $v = \dfrac{100}{(t + 1)^2}$

c.



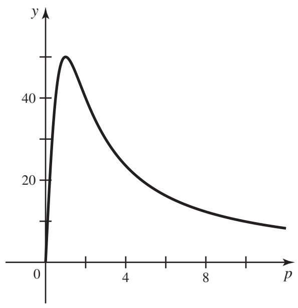
The marble moves fastest at the beginning and slows considerably over the first 5 s. It continues to slow but never actually stops.
d. $t = 4$ s **e.** $t = -1 + \sqrt{2} \approx 0.414$ s

51. a. $C'(x) = -\dfrac{125{,}000{,}000}{x^2} + 1.5$;
$$\overline{C}(x) = \frac{C(x)}{25{,}000} = 50 + \frac{5000}{x} + 0.00006x$$

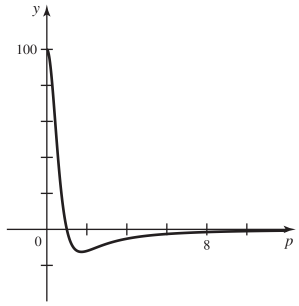


b. $C'(5000) = -3.5$; $\overline{C}(5000) = 51.3$ **c.** Marginal cost: If the batch size is increased from 5000 to 5001, then the cost of producing 25,000 gadgets will *decrease* by about \$3.50. Average cost: When batch size is 5000, it costs \$51.30 *per item* to produce all 25,000 gadgets.

53. a. $R(p) = \dfrac{100p}{p^2 + 1}$

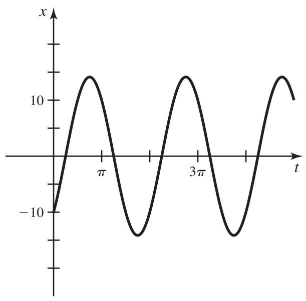


b. $R'(p) = \dfrac{100(1 - p^2)}{(p^2 + 1)^2}$



c. $p = 1$

55. a.



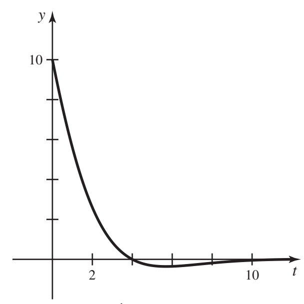
b. $dx/dt = 10 \cos t + 10 \sin t$
c. $t = 3\pi/4 + k\pi$, where k is any positive integer
d. The graph implies that the spring never stops oscillating. In reality, the weight would eventually come to rest.

57. a. Juan starts faster than Jean and opens up a big lead. Then Juan slows down while Jean speeds up. Jean catches up, and the race finishes in a tie. **b.** Same average velocity **c.** Tie **d.** At $t = 2$, $\theta'(2) = \pi/2$ rad/min; $\theta'(4) = \pi$ = Jean's greatest velocity
e. At $t = 2$, $\varphi'(2) = \pi/2$ rad/min; $\varphi'(0) = \pi$ = Juan's greatest velocity **59. a.** $v(t) = -15e^{-t}(\sin t + \cos t)$; $v(1) \approx -7.6$ m/s, $v(3) \approx 0.63$ m/s **b.** Down $(0, 2.4)$ and $(5.5, 8.6)$; up $(2.4, 5.5)$ and $(8.6, 10)$ **c.** ≈ 0.65 m/s **61. a.** $-T'(1) = -80$, $-T'(3) = 80$
b. $-T'(x) < 0$ for $0 \le x < 2$; $-T'(x) > 0$ for $2 < x \le 4$ **c.** Near $x = 0$, with $x > 0$, $-T'(x) < 0$, so heat flows toward the end of the rod. Similarly, near $x = 4$, with $x < 4$, $-T'(x) > 0$.

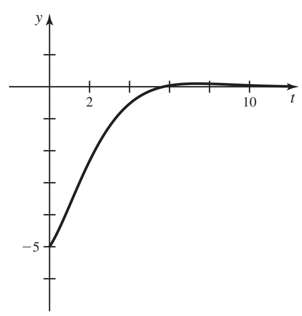
Section 3.7 Exercises, pp. 196–200

1. $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$; $\dfrac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$
3. $u = x^3 + x + 1$; $y = u^4$; $4(x^3 + x + 1)^3(3x^2 + 1)$
5. $u = \cos x$, $y = u^3$, $dy/dx = -3 \cos^2 x \sin x$;
$u = x^3$, $y = \cos u$, $dy/dx = -3x^2 \sin x^3$ **7.** $g(x)$, x **9.** $\dfrac{2}{\sqrt{4x + 1}}$
11. 50 **13.** ke^{kx} **15.** $u = 3x + 7$; $f(u) = u^{10}$; $30(3x + 7)^9$
17. $u = \sin x$; $f(u) = u^5$; $5 \sin^4 x \cos x$
19. $u = x^2 + 1$; $f(u) = \sqrt{u}$; $\dfrac{x}{\sqrt{x^2 + 1}}$
21. $u = 4x^2 + 1$; $f(u) = e^u$; $8xe^{4x^2+1}$
23. $u = 5x^2$; $f(u) = \tan u$; $10x \sec^2 5x^2$ **25. a.** 100 **b.** -100
c. -16 **d.** 40 **e.** 40 **27.** $10(6x + 7)(3x^2 + 7x)^9$
29. $\dfrac{5}{\sqrt{10x + 1}}$ **31.** $-\dfrac{315x^2}{(7x^3 + 1)^4}$ **33.** $3 \sec(3x + 1) \tan(3x + 1)$
35. $e^x \sec^2 e^x$ **37.** $(12x^2 + 3) \cos(4x^3 + 3x + 1)$
39. $\dfrac{10}{3(5x + 1)^{1/3}}$ **41.** $-\dfrac{3}{2^{7/4}x^{3/4}(4x - 3)^{5/4}}$
43. $5 \sec x (\sec x + \tan x)^5$ **45.** $25(12x^5 - 9x^2)(2x^6 - 3x^3 + 3)^{24}$

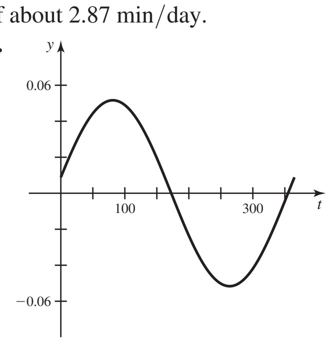
47. $9(1 + 2 \tan u)^{3.5} \sec^2 u$ **49.** $-\dfrac{\cot x \csc^2 x}{\sqrt{1 + \cot^2 x}}$
51. $\dfrac{2}{3}e^x - e^{-x}$ **53.** $e^x \cos(\sin e^x) \cos e^x$
55. $-15 \sin^4(\cos 3x)(\sin 3x)(\cos(\cos 3x))$
57. $\dfrac{2e^{2t}}{(1 + e^{2t})^2}$ **59.** $\dfrac{1}{2\sqrt{x + \sqrt{x}}}\left(1 + \dfrac{1}{2\sqrt{x}}\right)$
61. $f'(g(x^2))g'(x^2)\,2x$ **63.** $\dfrac{5x^4}{(x + 1)^6}$
65. $xe^{x^2+1}(2 \sin x^3 + 3x \cos x^3)$ **67.** $\theta(2 + 5\theta \tan 5\theta) \sec 5\theta$
69. $4((x + 2)(x^2 + 1))^3(3x + 1)(x + 1)$ **71.** $\dfrac{4x^3 - 2 \sin 2x}{5(x^4 + \cos 2x)^{4/5}}$
73. $2(p + 3)(\sin p^2 + p(p + 3) \cos p^2)$
75. $f'(x)/\left(2\sqrt{f(x)}\right)$ **77. a.** True **b.** True **c.** True
d. False **79.** -0.297 hPa/min **81.** Approx. 0.33 g/day; mass is increasing by 0.33 g/day 65 days after the diet switch.
83. a. \$297.77 **b.** \$11.85/yr **c.** $y = 11.85t + 179.27$
85. a. $x = -\frac{1}{2}$ **b.** The line tangent to the graph of $f(x)$ at $x = -\frac{1}{2}$ is horizontal. **87.** $2 \cos x^2 - 4x^2 \sin x^2$ **89.** $4e^{-2x^2}(4x^2 - 1)$
91. $y = 6x - 1$ **93. a.** $h(4) = 9$, $h'(4) = -6$ **b.** $y = -6x + 33$
95. $y = 6x + 3 - 3 \ln 3$ **97. a.** -3π **b.** -5π
99. a. $\dfrac{d^2 y}{dt^2} = -\dfrac{y_0 k}{m} \cos\left(t\sqrt{\dfrac{k}{m}}\right)$
101. a.



b. $v(t) = -5e^{-t/2}\left(\dfrac{\pi}{4} \sin \dfrac{\pi t}{8} + \cos \dfrac{\pi t}{8}\right)$

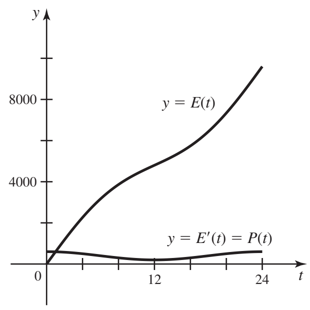


103. a. 10.88 hr **b.** $D'(t) = \dfrac{6\pi}{365} \sin\left(\dfrac{2\pi(t + 10)}{365}\right)$
c. 2.87 min/day; on March 1, the length of day is increasing at a rate of about 2.87 min/day.
d.



e. Most rapidly: approximately March 22 and September 22; least rapidly: approximately December 21 and June 21

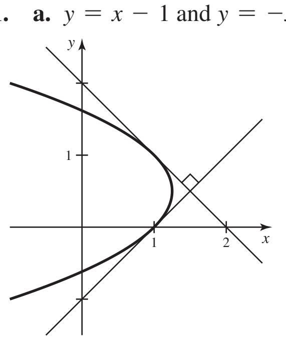
105. a. $E'(t) = 400 + 200 \cos \dfrac{\pi t}{12}$ MW
b. At noon; $E'(0) = 600$ MW **c.** At midnight; $E'(12) = 200$ MW
d.



109. a. $g(x) = (x^2 - 3)^5$; $a = 2$ **b.** 20
111. a. $g(x) = \sin x^2$; $a = \pi/2$ **b.** $\pi \cos(\pi^2/4)$ **113.** $10 f'(25)$

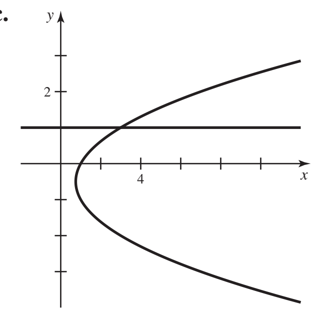
Section 3.8 Exercises, pp. 205–208

1. There may be more than one expression for y or y'.
3. When derived implicitly, dy/dx is usually given in terms of both x and y. **5.** $\dfrac{1}{2y}$ **7.** $\dfrac{1}{\cos y}$ **9. a.** $(0, 0), (0, -1), (0, 1)$
c. Slope at $(0, 0)$ is 2; slope at $(0, -1)$ and $(1, 0)$ is -1.
11. $\dfrac{d^2y}{dx^2} = -\dfrac{2}{9y^5}$ **13. a.** $-\dfrac{x^3}{y^3}$ **b.** 1 **15. a.** $\dfrac{2}{y}$ **b.** 1
17. a. $\dfrac{20x^3}{\cos y}$ **b.** -20 **19. a.** $-\dfrac{1}{\sin y}$ **b.** -1 **21. a.** $-\dfrac{y}{x}$ **b.** -7
23. a. $-\dfrac{1}{4x^{2/3}y^{1/3}}$ **b.** $-\dfrac{1}{4}$ **25. a.** $-\dfrac{3y}{x + 3y^{2/3}}$ **b.** $-\dfrac{24}{13}$
27. $\dfrac{\cos x}{1 - \cos y}$ **29.** $-\dfrac{1}{1 + \sin y}$ **31.** $\dfrac{1 - y\cos xy}{x \cos xy - 1}$ **33.** $\dfrac{1}{2y \sin y^2 + e^y}$
35. $\dfrac{3x^2(x - y)^2 + 2y}{2x}$ **37.** $\dfrac{13y - 18x^2}{21y^2 - 13x}$ **39.** $\dfrac{5\sqrt{x^4 + y^2} - 2x^3}{y - 6y^2\sqrt{x^4 + y^2}}$
41. a. $\dfrac{dK}{dL} = -\dfrac{K}{2L}$ **b.** -4 **43.** $\dfrac{dr}{dh} = \dfrac{h - 2r}{h}$; -3
45. b. $y = -5x$ **47. b.** $y = -5x/4 + 7/2$ **49. b.** $y = \dfrac{x}{2}$
51. $-\dfrac{1}{4y^3}$ **53.** $\dfrac{\sin y}{(\cos y - 1)^3}$ **55.** $\dfrac{4e^{2y}}{(1 - 2e^{2y})^3}$ **57. a.** False
b. True **c.** False **d.** False **59. a.** $\dfrac{y(3\sqrt{x} + 2y^{3/2})}{x(\sqrt{x} - 2y^{3/2})}$ **b.** -5
61. a. $y = x - 1$ and $y = -x + 2$
b.

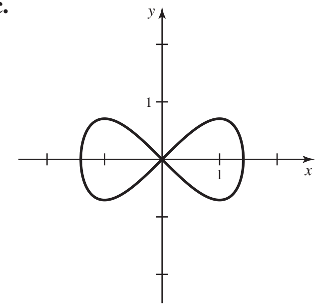


63. a. $y' = -\dfrac{2xy}{x^2 + 4}$ **b.** $y = \dfrac{1}{2}x + 2$, $y = -\dfrac{1}{2}x + 2$
c. $-\dfrac{16x}{(x^2 + 4)^2}$ **65. a.** $\left(\dfrac{5}{4}, \dfrac{1}{2}\right)$ **b.** No
67. Horizontal: $y = -6$, $y = 0$; vertical: $x = 1$, $x = 3$

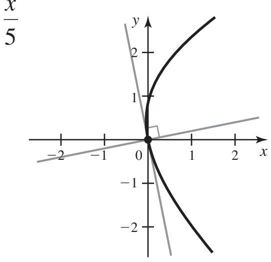
69. a. $\dfrac{dy}{dx} = 0$ on the $y = 1$ branch; $\dfrac{dy}{dx} = \dfrac{1}{2y + 1}$ on the other two branches. **b.** $f_1(x) = 1$, $f_2(x) = \dfrac{-1 + \sqrt{4x - 3}}{2}$, $f_3(x) = \dfrac{-1 - \sqrt{4x - 3}}{2}$ **c.**



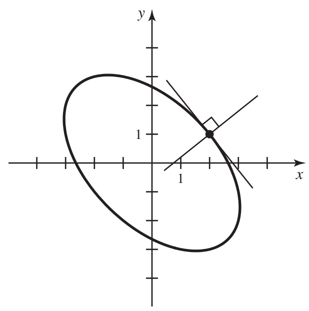
71. a. $\dfrac{dy}{dx} = \dfrac{x - x^3}{y}$ **b.** $f_1(x) = \sqrt{x^2 - \dfrac{x^4}{2}}$; $f_2(x) = -\sqrt{x^2 - \dfrac{x^4}{2}}$
c.



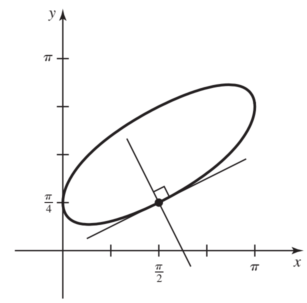
73. $y = \dfrac{x}{5}$



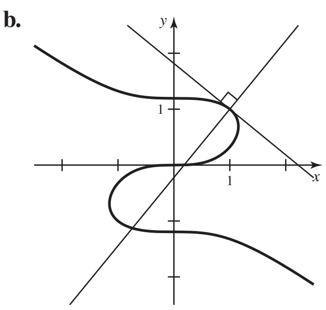
75. $y = \dfrac{4x}{5} - \dfrac{3}{5}$



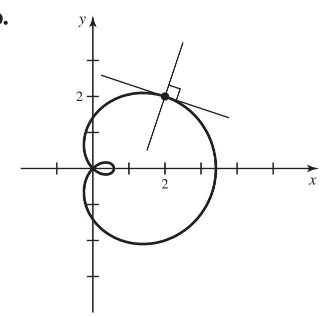
77. $y = -2x + \dfrac{5\pi}{4}$



79. a. Tangent line $y = -\dfrac{9x}{11} + \dfrac{20}{11}$; normal line $y = \dfrac{11x}{9} - \dfrac{2}{9}$
b.



81. a. Tangent line $y = -\dfrac{x}{3} + \dfrac{8}{3}$; normal line $y = 3x - 4$
b.



83. For $y = mx$, $dy/dx = m$; for $x^2 + y^2 = a^2$, $dy/dx = -x/y$.
85. For $xy = a$, $dy/dx = -y/x$; for $x^2 - y^2 = b$, $dy/dx = x/y$. Because $(-y/x) \cdot (x/y) = -1$, the families of curves form orthogonal trajectories. **87.** $\dfrac{7y^2 - 3x^2 - 4xy^2 - 4x^3}{2y(2x^2 + 2y^2 - 7x)}$

89. $\dfrac{2y^2(5+8x\sqrt{y})}{(1+2x\sqrt{y})^3}$ **91.** No horizontal tangent line; vertical tangent lines at $(2, 1)$, $(-2, 1)$ **93.** No horizontal tangent line; vertical tangent lines at $(0, 0)$, $\left(\frac{3\sqrt{3}}{2}, \sqrt{3}\right)$, $\left(-\frac{3\sqrt{3}}{2}, -\sqrt{3}\right)$

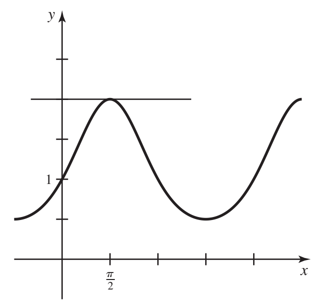
Section 3.9 Exercises, pp. 215–218

1. $x = e^y \Rightarrow 1 = e^y y'(x) \Rightarrow y'(x) = 1/e^y = 1/x$
3. $\dfrac{d}{dx}(\ln kx) = \dfrac{d}{dx}(\ln k + \ln x) = \dfrac{d}{dx}(\ln x)$ **5.** $f'(x) = \dfrac{1}{x \ln b}$; if $b = e$, then $f'(x) = \dfrac{1}{x}$. **7.** $(x^2+1)^x$ **9.** $\dfrac{x}{x^2+1}$
11. $f(x) = e^{h(x)\ln g(x)}$ **13.** $\dfrac{1+x}{x}$ **15.** $\dfrac{1}{x}$ **17.** $2/x$ **19.** $\cot x$
21. $\dfrac{4x^3}{x^4+1}$ **23.** $2/(1-x^2)$ **25.** $(x^2+1)/x + 2x\ln x$
27. $-2x\ln x^2$ or $-4x\ln x$ **29.** $1/(x\ln x)$ **31.** $\dfrac{1}{x(\ln x+1)^2}$
33. ex^{e-1} **35.** $\pi(2^x+1)^{\pi-1}2^x\ln 2$ **37.** $8^x\ln 8$ **39.** $5\cdot 4^x\ln 4$
41. $2^{3+\sin x}(\ln 2)\cos x$ **43.** $3^x\cdot x^2(x\ln 3+3)$
45. $1000(1.045)^{4t}\ln 1.045$ **47.** $\dfrac{2^x\ln 2}{(2^x+1)^2}$
49. $x^{\cos x - 1}(\cos x - x\ln x\sin x)$; $-\ln(\pi/2)$
51. $x^{\sqrt{x}}\left(\dfrac{2+\ln x}{2\sqrt{x}}\right)$; $4(2+\ln 4)$
53. $\dfrac{(\sin x)^{\ln x}(\ln(\sin x) + x(\ln x)\cot x)}{x}$; 0
55. $(4\sin x+2)^{\cos x}\left(\dfrac{2\cos^2 x}{2\sin x+1} - \sin x\ln(4\sin x+2)\right)$; 1
57. a. Approx. 28.7 s **b.** -46.512 s/1000 ft
c. $dT/da = -2.74\cdot 2^{-0.274a}\ln 2$

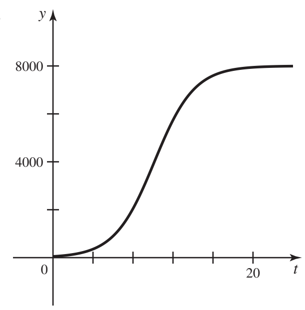
$$\text{At } a = 8, \frac{dT}{da} = -0.4156 \text{ min/1000 ft} = -24.938 \text{ s/1000 ft.}$$

If a plane travels at 30,000 feet and increases its altitude by 1000 feet, the time of useful consciousness decreases by about 25 seconds.
59. $y = x\sin 1 + 1 - \sin 1$ **61.** $y = e^{2/e}$ and $y = e^{-2/e}$
63. $\dfrac{8x}{(x^2-1)\ln 3}$ **65.** $-\sin x(\ln(\cos^2 x)+2)$
67. $-\dfrac{\ln 4}{x\ln^2 x}$ **69.** $\dfrac{12}{3x+1}$ **71.** $\dfrac{1}{2x}$
73. $\dfrac{2}{2x-1} + \dfrac{3}{x+2} + \dfrac{8}{1-4x}$ **75.** $10x^{10x}(1+\ln x)$
77. $\dfrac{(x+1)^{10}}{(2x-4)^8}\left(\dfrac{10}{x+1} - \dfrac{8}{x-2}\right)$ **79.** $2x^{\ln x - 1}\ln x$
81. $\dfrac{(x+1)^{3/2}(x-4)^{5/2}}{(5x+3)^{2/3}}\left(\dfrac{3}{2(x+1)} + \dfrac{5}{2(x-4)} - \dfrac{10}{3(5x+3)}\right)$
83. $(\sin x)^{\tan x}(1 + (\sec^2 x)\ln\sin x)$
85. $\left(1+\dfrac{1}{x}\right)^x\left(\ln\left(1+\dfrac{1}{x}\right) - \dfrac{1}{x+1}\right)$
87. a. False **b.** False **c.** False **d.** False **e.** True **f.** True
89. $-\dfrac{1}{x^2\ln 10}$ **91.** $\dfrac{2}{x}$ **93.** $3^x\ln 3$

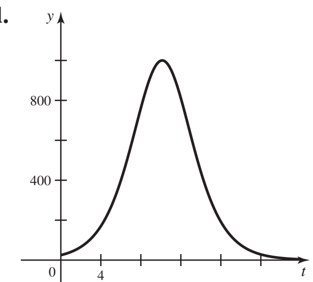
95. $y = 2$



97. a.



b. $t = 2\ln 265 \approx 11.2$ years; approx. 14.5 years
c. $P'(0) \approx 25$ fish/year; $P'(5) \approx 264$ fish/year
d.



The population is growing fastest after about 10 years.
99. b. $r(11) \approx 0.0133$; $r(21) \approx 0.0118$; the relative growth rate is decreasing. **c.** $\lim_{t\to\infty} r(t) = 0$; as the population gets close to carrying capacity, the relative growth rate approaches zero.
101. a. $A(5) = \$17{,}443$
$A(15) = \$72{,}705$
$A(25) = \$173{,}248$
$A(35) = \$356{,}178$
\$5526.20/year, \$10,054.30/year, \$18,293/year
b. $A(40) = \$497{,}873$
c. $\dfrac{dA}{dt} = 600{,}000\ln(1.005)((1.005)^{12t}) \approx (2992.5)(1.005)^{12t}$
A increases at an increasing rate.
103. $p = e^{1/e}$; (e, e) **105.** $1/e$ **107.** $27(1+\ln 3)$

Section 3.10 Exercises, pp. 225–227

1. $\dfrac{d}{dx}(\sin^{-1}x) = \dfrac{1}{\sqrt{1-x^2}}$; $\dfrac{d}{dx}(\tan^{-1}x) = \dfrac{1}{1+x^2}$; $\dfrac{d}{dx}(\sec^{-1}x) = \dfrac{1}{|x|\sqrt{x^2-1}}$ **3.** $\dfrac{1}{5}$ **5.** $\dfrac{1}{4}$ **7. a.** $\dfrac{1}{2}$ **b.** $\dfrac{2}{3}$
c. Cannot be determined **d.** $\dfrac{3}{2}$ **9.** $y = \dfrac{1}{7}x + \dfrac{13}{7}$ **11.** $\dfrac{2}{\sqrt{3}}$
13. $\dfrac{2}{\sqrt{1-4x^2}}$ **15.** $-\dfrac{4w}{\sqrt{1-4w^2}}$ **17.** $-\dfrac{2e^{-2x}}{\sqrt{1-e^{-4x}}}$
19. $\dfrac{10}{100x^2+1}$ **21.** $\dfrac{4y}{1+(2y^2-4)^2}$ **23.** $-\dfrac{1}{2\sqrt{z}(1+z)}$

25. $6x^2 \cot^{-1} x$ **27.** $\dfrac{2w^5}{1 + w^4}$ **29.** $\dfrac{1}{|x|\sqrt{x^2 - 1}}$

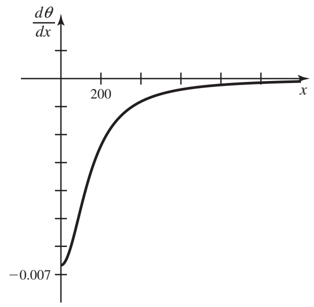
31. $-\dfrac{1}{|2u + 1|\sqrt{u^2 + u}}$ **33.** $\dfrac{2y}{(y^2 + 1)^2 + 1}$

35. $\dfrac{1}{x|\ln x|\sqrt{(\ln x)^2 - 1}}$ **37.** $-\dfrac{e^x \sec^2 e^x}{|\tan e^x|\sqrt{\tan^2 e^x - 1}}$

39. $-\dfrac{e^s}{1 + e^{2s}}$ **41.** $y = x + \dfrac{\pi}{4} - \dfrac{1}{2}$ **43.** $y = -\dfrac{4}{\sqrt{6}}x + \dfrac{\pi}{3} + \dfrac{2}{\sqrt{3}}$

45. a. Approx. -0.00055 rad/m

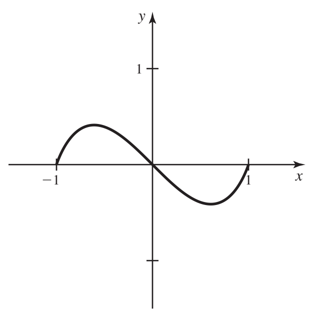
b.



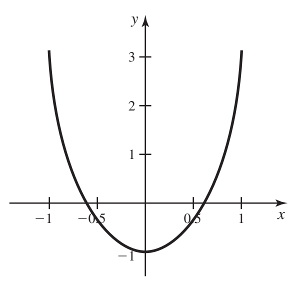
The magnitude of the change in angular size, $|d\theta/dx|$, is greatest when the boat is at the skyscraper (that is, at $x = 0$).

47. $\frac{1}{3}$ **49.** $\frac{e}{5}$ **51.** $\frac{1}{2}$ **53.** 4 **55.** $\frac{1}{12}$ **57.** $\frac{1}{4}$ **59.** $\frac{5}{4}$ **61. a.** True **b.** False **c.** True **d.** True **e.** True

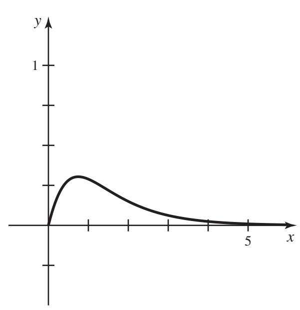
63. a.



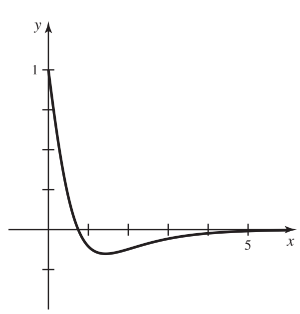
b. $f'(x) = 2x \sin^{-1} x + \dfrac{x^2 - 1}{\sqrt{1 - x^2}}$



65. a.



b. $f'(x) = \dfrac{e^{-x}}{1 + x^2} - e^{-x} \tan^{-1} x$



67. $\dfrac{1}{3}$ **69.** $1/(2\sqrt{x + 4})$ **71.** $\dfrac{1}{3x}$ **73.** $\dfrac{1}{12x \ln 10}$ **75.** $2x$

77. $-2/x^3$ **79. b.** -0.0041, -0.0289, and -0.1984

c. $\lim_{\ell \to 10^+} d\theta/d\ell = -\infty$ **d.** The length ℓ is decreasing.

81. a. $1/\sqrt{D^2 - c^2}$ **b.** $1/D$ **85.** Use the identity $\cot^{-1} x + \tan^{-1} x = \pi/2$.

Section 3.11 Exercises, pp. 231–236

1. As the side length s of a cube changes, the surface area $6s^2$ changes as well. **3.** The other two opposite sides decrease in length.

5. a. $V = 200h$; $\dfrac{dV}{dt} = 200\dfrac{dh}{dt}$ **b.** $50 \text{ ft}^3/\text{min}$

c. $\dfrac{1}{20}$ ft/min **7. a.** $\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$ **b.** $128\pi \text{ in}^3/\text{min}$

c. $\dfrac{1}{10\pi}$ in/min **9.** 59 **11. a.** $40 \text{ m}^2/\text{s}$ **b.** $80 \text{ m}^2/\text{s}$

13. a. $4 \text{ m}^2/\text{s}$ **b.** $\sqrt{2} \text{ m}^2/\text{s}$ **c.** $2\sqrt{2}$ m/s **15. a.** $\dfrac{1}{4\pi}$ cm/s

b. $\dfrac{1}{2}$ cm/s **17.** $-40\pi \text{ ft}^2/\text{min}$ **19.** $\dfrac{3}{80\pi}$ in/min

23. 720.3 mi/hr **25.** $\dfrac{3\sqrt{5}}{2}$ ft/s **27.** 57.89 ft/s **29.** 4.66 in/s

31. $\dfrac{\pi}{2} \text{ ft}^3/\text{min}$ **33.** $-75\pi \text{ cm}^3/\text{s}$ **35.** $2592\pi \text{ cm}^3/\text{s}$

37. $9\pi \text{ ft}^3/\text{min}$ **39.** $\dfrac{1}{25\pi}$ m/min **41.** $\dfrac{5}{24}$ ft/s

43. $-\dfrac{8}{3}$ ft/s, $-\dfrac{32}{3}$ ft/s **45.** $\dfrac{d\theta}{dt} = \dfrac{1}{5}$ rad/s, $\dfrac{d\theta}{dt} = \dfrac{1}{8}$ rad/s

47. -0.0201 rad/s **49.** $10 \tan 20°$ km/hr ≈ 3.6 km/hr

51. a. 187.5 ft/s **b.** 0.938 rad/s **53. a.** $P = \dfrac{1}{2}v^2\dfrac{dm}{dt}$

c. 17,388.7 W **d.** 4347.2 W **55.** 11.06 m/hr

57. $\dfrac{1}{500}$ m/min; 2000 min **59.** 0.543 rad/hr

61. $\dfrac{d\theta}{dt} = 0$ rad/s, for all $t \geq 0$ **63. a.** $-\dfrac{\sqrt{3}}{10}$ m/hr **b.** $-1 \text{ m}^2/\text{hr}$

Chapter 3 Review Exercises, pp. 236–240

1. a. False **b.** False **c.** False **d.** False **e.** True

3. $-\dfrac{2x}{(x^2 + 5)^2}$ **9.** $2x^2 + 2\pi x + 7$ **11.** $2^x \ln 2$

13. $2e^{2\theta}$ **15.** $6x^3\sqrt{1 + x^4}$ **17.** $5t^2 \cos t + 10t \sin t$

19. $-x^2e^{-x}$ **21.** $\dfrac{2 \sec 2w \tan 2w}{(\sec 2w + 1)^2}$ **23.** $3 \tan 3x$

25. $1000t(5t^2 + 10)^{99}$ **27.** $3x^2 \cot x^3$ **29.** $\dfrac{1}{t\sqrt{t^2 - 1}}$

31. $(8\theta + 12) \sec^2(\theta^2 + 3\theta + 2)$ **33.** $\dfrac{1 - 5 \ln w}{w^6}$

35. $\dfrac{32u^2 + 8u + 1}{(8u + 1)^2}$ **37.** $(\sec^2 \sin \theta) \cos \theta$

39. $-\dfrac{\cos\sqrt{\cos^2 x + 1} \cos x \sin x}{\sqrt{\cos^2 x + 1}}$ **41.** $\dfrac{e^t}{2(e^t + 1)}$

43. $2 \tan^{-1}(\cot x)$ **45.** $(2 + \ln x) \ln x$ **47.** $(2x - 1)\, 2^{x^2 - x} \ln 2$

49. $(x^2 + 1)^{\ln x}\left(\dfrac{\ln(x^2 + 1)}{x} + \dfrac{2x \ln x}{x^2 + 1}\right)$ **51.** $-\dfrac{1}{|x|\sqrt{x^2 - 1}}$

53. $6 \cot^{-1} 3x$ **55.** $1 + \csc(x - y)$

57. $\dfrac{y \cos x}{e^y - 1 - \sin x}$ **59.** $-\dfrac{xy}{x^2 + 2y^2}$

61. $\dfrac{(3x+5)^{10}\sqrt{x^2+5}}{(x^3+1)^{50}}\left(\dfrac{30}{3x+5}+\dfrac{x}{x^2+5}-\dfrac{150x^2}{x^3+1}\right)$

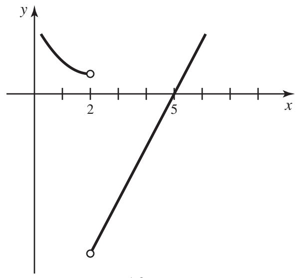
63. $\sqrt{3}+\pi/6$ **65.** 1 **67.** $2^x \ln 2(x \ln 2 + 2)$ **69.** $\dfrac{6 \ln x - 5}{x^4}$

71. $\dfrac{2(xy+y^2)}{(x+2y)^3} = \dfrac{2}{(x+2y)^3}$ **73.** $y = x$ **75.** $y = -\dfrac{4x}{5}+\dfrac{24}{5}$

77. $x^2 f'(x) + 2xf(x)$ **79.** $\dfrac{g(x)(xf'(x)+f(x)) - xf(x)g'(x)}{g^2(x)}$

81. a–D; b–C; c–B; d–A

83.



85. a. 27 **b.** $\frac{16}{27}$ **c.** 72 **d.** 1215 **e.** $\frac{1}{9}$ **87.** $\frac{6}{13}$

89. $(f^{-1})'(x) = -3/x^4$ **91. a.** $\frac{1}{4}$ **b.** 1 **c.** $\frac{1}{3}$

93. $y = 24x - 118$ **95. a.** 84 ft/s **b.** 7 s **c.** 384 ft

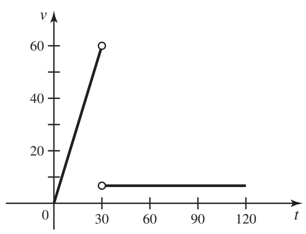
d. 96 ft/s **97. a.** \$200,366; \$21,552/yr

b. 14 yr; \$12,551/yr **99. a.** 2.70 million people/yr

b. The slope of the secant line through the two points is approximately equal to the slope of that tangent line at $t = 55$.

c. 2.217 million people/yr **101. a.** 40 m/s **b.** 20/3 m/s

c. 15 m/s **d.**



e. The skydiver deployed the parachute. **103.** $x = 4$; $x = 6$

105. $f(x) = \tan(\pi\sqrt{3x-11})$, $a = 5$; $f'(5) = 3\pi/4$

107. a. $\overline{C}(3000) = \$341.67$; $C'(3000) = \$280$ **b.** The average cost of producing the first 3000 lawn mowers is \$341.67 per mower. The cost of producing the 3001st lawn mower is \$280.

109. a. 6550 people/yr **b.** $p'(40) = 4800$ people/yr

111. 50 mi/hr **113.** $-5 \sin 65°$ ft/s ≈ -4.5 ft/s

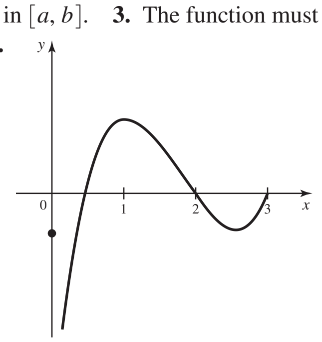
115. -0.166 rad/s **117.** 1.5 ft/s **119. a.** $(f^{-1})'(1/\sqrt{2}) = \sqrt{2}$

CHAPTER 4

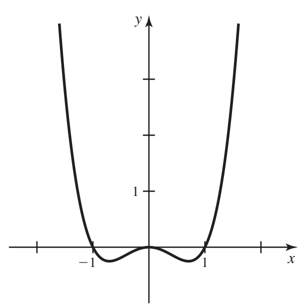
Section 4.1 Exercises, pp. 247–250

1. f has an absolute maximum at c in $[a, b]$ if $f(x) \le f(c)$ for all x in $[a, b]$. f has an absolute minimum at c in $[a, b]$ if $f(x) \ge f(c)$ for all x in $[a, b]$. **3.** The function must be continuous on a closed interval.

5.

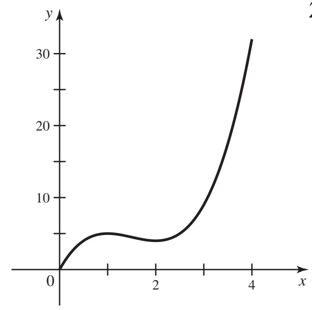


7.

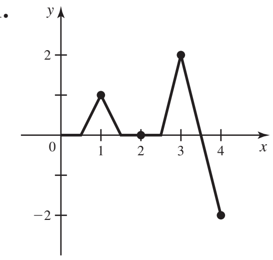


9. Evaluate the function at the critical points and at the endpoints of the interval. **11.** Abs. min at $x = c_2$; abs. max at $x = b$ **13.** Abs. min at $x = a$; no abs. max **15.** Local min at $x = q, s$; local max at $x = p, r$; abs. min at $x = a$; abs. max at $x = b$ **17.** Local max at $x = p, r$; local min at $x = q$; abs. max at $x = p$; abs. min at $x = b$

19.



21.



23. $x = \frac{2}{3}$ **25.** $x = \pm 3$ **27.** $x = -\frac{2}{3}, \frac{1}{3}$ **29.** $x = \pm\dfrac{2a}{\sqrt{3}}$

31. $t = \pm 1$ **33.** $x = 0$ **35.** $x = 1$ **37.** $x = -4, 0$

39. If $a \ge 0$, there is no critical point. If $a < 0$, $x = 2a/3$ is the only critical point. **41.** $t = \pm a$ **43.** Abs. max: -1 at $x = 3$; abs. min: -10 at $x = 0$ **45.** Abs. max: 0 at $x = 0, 3$; abs. min: -4 at $x = -1, 2$ **47.** Abs. max: 234 at $x = 3$; abs. min: -38 at $x = -1$ **49.** Abs. max: 1 at $x = 0, \pi$; abs. min: 0 at $x = \pi/2$ **51.** Abs. max: 1 at $x = \pi/6$; abs. min: -1 at $x = -\pi/6$

53. Abs. min: $(\sqrt{1/e})^{1/e}$ at $x = 1/(2e)$; abs. max: 2 at $x = 1$

55. Abs. max: $1 + \pi$ at $x = -1$; abs. min: 1 at $x = 1$

57. Abs. max: 11 at $x = 1$; abs. min: -16 at $x = 4$

59. Abs. max: 27 at $x = -3$; abs. min: $-\frac{19}{12}$ at $x = \frac{1}{2}$

61. Abs. max: $\dfrac{1}{100{,}000}$ at $x = 1$; abs. min: $-\dfrac{1}{100{,}000}$ at $x = -1$

63. Abs. max: $\sqrt{2}$ at $x = \pm\pi/4$; abs. min: 1 at $x = 0$

65. Abs. max: $27/e^3$ at $x = 3$; abs. min: $-e$ at $x = -1$

67. Abs. max: 3 at $x = \pm 1$; abs. min: 0 at $x = -2, 0, 2$

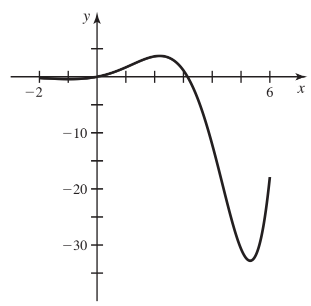
69. a. The velocity of the downstream wind v_2 is less than or equal to the velocity of the upstream wind, so $0 \le v_2 \le v_1$, or $0 \le \dfrac{v_2}{v_1} \le 1$.

b. $R(1) = 0$ **c.** $R(0) = \frac{1}{2}$ **d.** 0.593 is the maximum fraction of power that can be extracted from a wind stream by a wind turbine.

71. $t = 2$ s **73.** $t = 2$ s **75. a.** 50 **b.** 45 **77. a.** False **b.** False **c.** False **d.** True **79. a.** $x = -0.96, 2.18, 5.32$

b. Abs. max: 3.72 at $x = 2.18$; abs. min: -32.80 at $x = 5.32$

c.



81. a. $x = \tan^{-1} 2 + k\pi$, for $k = -2, -1, 0, 1$

b. $x = \tan^{-1} 2 + k\pi$, for $k = -2, 0$, correspond to local max; $x = \tan^{-1} 2 + k\pi$, for $k = -1, 1$, correspond to local min.

c. Abs. max: 2.24; abs. min: -2.24 **83. a.** $x = 5 - 4\sqrt{2}$

b. $x = 5 - 4\sqrt{2}$ corresponds to a local max. **c.** No abs. max or min

85. Abs. max: 4 at $x = -1$; abs. min: -8 at $x = 3$

87. a. $T(x) = \dfrac{\sqrt{2500 + x^2}}{2} + \dfrac{50 - x}{4}$ **b.** $x = 50/\sqrt{3}$