WAVELET ANALYSIS IN ECONOMICS: CAPM

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Abstract

The risk that affects the entire market segment is called ‘systematic risk’. In modern portfolio analysis, a common way of measuring risk is Capital Asset Pricing Model (CAPM). The beta is the common and more used principle of calculating systematic risk in both Economics and Finance. The level of systematic risk is depending on the asset expected return. However, the validity of CAPM beta is questioned by Nawazish Mirza & Ghalia Shabbir (2005). In this paper, we are going to discuss extensively the use of wavelet analysis, its benefits over Fourier and how the CAPM validity can be examined in different time scales. In order to verify the theory, we use the top 88 stock symbols in Nasdaq, we short them by their rates of return over time and then we proceed in a typical regression. Afterwards, we decompose the signals into Least Asymmetric 8 wavelets and these coefficients are being regressed again in order to check if the stock market is efficient in the short term or in the long term.

Key Words: Capital Asset Pricing Model, Wavelets, Beta, Systematic risk, R, Shiny App, Matlab

Introduction

There are two kinds of risks; the diversifiable and the non-diversifiable. The first risk, can be eliminated through investing in a plethora of assets. However, we should take as an assumption that all the investors are risk averse and utility maximizers. Moreover, Investment is an asset that is not expended today but rather is utilized as a part without bounds to accumulate wealth. In fund, a speculation is a fiscal resource acquired with the possibility that the benefit will give pay later on or will be sold at a higher cost for a benefit. As a result, investors face the challenge of getting return of their investment, which is also called risk. Hence, the expected return of an investment should be proportional to the risk. Also, it should be correlated with a relatively risk free asset, which roughly measures the opportunity cost of investing instead of expending the wealth today. The Capital Asset Pricing model is used over 30 years in the modern portfolio theory, and suggest that higher level of returns is associated with high level of risk. However, research shows that CAPM is not being able to explain actual movements of asset returns. Furthermore, we examine the period between 2006-2016, where there is a housing bubble in U.S (2006) and the global financial crisis (2008), so we expect that the returns would have drop dramatically and this may have caused a misspecification in the beta estimates. Also, Nasdaq is listed with technological companies, thus we expect high volatility in the data and higher betas, because the companies are highly affected by the Research and Development changes. Moreover, we are going to use wavelets to analyze the beta estimates in different time scales and see if there is a short term or a long term relationship in the beta estimates, because according to the CAPM assumptions low betas are translated as low returns, and high betas into high returns respectively.

Literature Review

The development of CAPM is started in 1960s with the pioneer theories developed by Sharpe (1964), Lintner (1965) and Mossin (1966). According to Michailidis et al (2006), “the CAPM predicts that expected return on an asset above the risk-free rate is linearly related to the non-diversifiable risk, which is measured by the asset’s beta”. Other economists have used different approaches of analyzing CAPM. One of them is the zero-beta estimation by Black (1972), which has the assumption of not using risk free rate for borrowing and lending. In other words, he dismisses the possibility of an investor to lend and borrow. Intertemporal Capital Asset pricing Model (ICAMP) developed by Mertons (1972), is able to analyze different equilibrium periods in the CAPM. Kraus and Litzenberger (1976) used another method of estimation by estimating its mean covariance and skewness. Next, the Consumption Capital Asset pricing Model by Breeden (1979), describes that the covariance of betas is proportional to the consumption rather than the risk-free rate. Finally, the most used theory is Ross’ (1976), which expands the arbitrage pricing theory, highly used in both economics and finance.

Although wavelet analysis is relatively new in the economics and finance, it is a common practice in other disciplines, such as data compression, filtering processes or even encryption. The foundation of wavelets is based on Fourier analysis by the mathematician Jean-Baptiste Joseph Fourier in the book “La Theorie Analytique de la Chaleur”, which was published in 1822. Fourier transform decomposes a signal into a summation of a sine cosine function. However, the use of sinusoids assumes stationarity over time, which may prove problematic in financial data, including asset prices. On the other hand, wavelet analysis allows data to be analyzed at different scales (Aktan et al, 2009).

Weedon (2003) and Graps (1995) mention that frequency and resolution cannot be captured in the Fourier, because the signals are random walk. Also, wavelets have an infinite set of possible basis functions according to B. Aktan et al (2009). Therefore, they are more powerful from a single Fourier transform in a sense, although both are able to analyze investment horizons in in various or different scales (In, Kim, Marisetty and Faff, 2008). Also, wavelets can be viewed as a sine cosine functions that fluctuates about zero (Gencay, Selcuk & Whitcher, 2002; Selcuk, 2005; Percival & Walden, 2000). According to Fernandez (2006), Cifter (2007) and Ozun (2008), a signal can be viewed as functional form grouping of wavelet functions. Finally, there are two forms of wavelets; the Continuous Wavelet Transform, or CWT and the Discrete Wavelet Transform, or DWT (Percival & Walden, 2000; Aktan et al, 2009). CWT is a function that operates in the entire real axis, while DWT operates on discrete time . Although, the first can be used in very large datasets without computational problems.

The criticism of CAPM starts from the theories developed by Mandelbrot (1963) and Fama (1965), where the returns do not have a constant variance in statistical sense (Heteroscedasticity). In other words, we can describe stock returns as a random walk. According to Nawazish Mirza and Ghalia Shabbir (2005), there are some econometric limitations with the beta estimation. First, beta estimates are based on rational expectations for an investment in a single period. However, many investors have different horizons and it is hard to estimate. Second, betas are estimated by classical linear regression models (CLRMs), therefore the returns should have been normally distributed. Finally, market portfolio has its own limitations, because it is excluding major information such as human capital, private investment and real estate.

Theoretical Background

There are two components on the measurement of risk

1. the Capital Asset Pricing Model (CAPM)
2. and the Beta (*β*)

The implementation of CAPM is very important because it represents the equilibrium that attaches on a mean variance portfolio selection under uncertainty, hence it is a relationship between an investment’s non-diversifiable (systematic) risk and its expected return. Given the general risk aversion, high expected returns are associated with high levels of risk, which is measured by beta (Michailidis et al., 2006; Aktan et al., 2009).

As CAPM is the basis in portfolio theory and it is intensively used in empirical works, a lot of research has been conducted on the ability of CAPM capturing the actual movements of asset returns, since it presupposes a number of assumptions. According to the theory, investors are rational, risk averse and utility maximizers, where utility is obtained via returns and risk is measured by standard deviations of returns. Also, there is only a single period investment in the horizon and all investors have the same expectations about uncertainty. Furthermore, CAPM assumes that capital markets are perfect, hence there are no taxes and transaction costs and investors can both lend and borrow at the risk free rate of return (Aktan et al., 2009).

The required return is defined by CAPM as follows:

(1)

Where,

is an asset *i* expected return,

is the rate on a risk-free asset,

is the expected return on the market index (benchmark),

is the estimate of risk for asset *i*.

The model derives from the idea of risk-free rate of return versus the risk premium. Risk premium, , is the amount of return in excess of the risk-free rate of return to compensate for the investment’s systematic risk as measured by beta. In this paper, we will regress formula (1) as shown in formula (2) to capture coefficients *α* and *β*, which respectively represent the existence of transaction costs and the volatility of the stocks in comparison with the market. To avoid any confusion, the majority of the literature extracts *β* directly by dividing the covariance of , by the variance of (formula 3):

(2)

(3)

Thereafter, we will examine the data in different frequencies using *wavelet analysis*. While wavelet analysis is still considered a relatively new methodology in economics and finance, frequency analysis like Fourier exists since 1822, where Jean-Baptiste Joseph Fourier showed that any periodic function (or signal) can be expressed as a summation of complex exponential functions (Fourier, 1822). It is fundamental to understand Fourier transform in order to understand how wavelets function. In order to show an elegant version of Fourier theory (Percival and Walden, 2000) we will take Euler’s relationship, where the complex exponential is defined to be a complex variable whose real and imaginary parts are and respectively:

(4)

&

(5)

It is worth noting that complex exponential functions have many similarities with ordinary exponentials. Now let’s assume that a vector is an infinite sequence of real or complex variables and it satisfies the following condition:

Then, we can use Euler’s relationship (6) for as basis for a Discrete Fourier transform (7) of vector :

(6)

(7)

Where, *f* variable is the *frequency* and is called the Fourier analysis of . Frequency is defined as the number of cycles the sinusoid goes through as a continuous variable *t* changes. Fourier transform may seem confusing at first glance but it is just a summation of sines and cosines, which transforms a given signal into a wave. Therefore, Fourier Transform converts a time series into the frequency domain. (Owens, 1997; Percival and Walden, 2000; Tangborn & National Aeronautics and Space Administration, 2010).

Fourier may have been proved a powerful tool in time series analysis, but it has a very serious drawback in financial and economic analysis. It assumes stationarity. As shown in formula (6) and (7), Fourier transform is an infinite summation of sinusoids, which are periodic functions and they describe a smooth repetitive oscillation, therefore they are inherently *non-local* (Schleicher, 2002). In other words, Fourier Transform is unable to capture the behavior of random walk variables in a certain time frame, including frequency analysis of CAPM. This gap is filled by wavelet analysis.

While Fourier transform decomposes a signal into its frequency components, wavelets are functions that decompose a signal by *localized* time and frequency. Hence, the wavelet transform contains information on both the time location and frequency of the signal, at the cost of frequency resolution. Below there are some typical properties of wavelets (Tangborn & National Aeronautics and Space Administration, 2010):

1. Orthogonality: Both wavelet transform matrix and wavelet functions can be orthogonal, which is useful for creating basis functions for computation.
2. Zero Mean: This condition forces the wavelet functions to oscillate between positive and negative.
3. Compact Support: They are efficient at representing localized data and functions.

However, we will discuss some of the properties of wavelets later, since we need to include generalized wavelet mathematical formulas and this may affect the comprehension of a wavelet transform procedure. In order to describe wavelets, we will demonstrate the simplest type of wavelet, the *Haar transform*. Let’s assume we have a signal *f* as follows:

(8)  
{\displaystyle \mathbb {Z} }  
{\displaystyle \mathbb {Z} }

Where,

(9)

Our signal *f* length is , because all wavelet transforms decompose a discrete signal into two sub-signals of half of their initial length. The first subsignal is the *trend* and the second is the *fluctuation*. Trend is a running average (10) and fluctuation the running difference (11) of *n* values. The transform can be performed in several stages or *levels* as far as our sub-signals are divisible by 2. The exponent “1” in formulas (10) and (11) denote the level.

(10)

&

(11)

Until now we have discussed the decomposition of an initial signal *f* into two sub signals *a* and *d* and the ability to decompose the signal up to several levels, but not how. In order to decompose a signal to trend and fluctuations we can use the following formulas:

(12)

&

(13)

Therefore, the first level of a Haar Transform is defined by:

or simply

Hence, the level Haar trend, fluctuations and transform of the signal *f* may be expressed as:

*Trend*: (14)

*Fluctuations*: (15)

*Transform*: (16)

It is worth mentioning that wavelet transforms have the ability to compress the initial signal, but as the levels increase and there is further compaction or *localization*, they lose their energy or *information,* which is referred as Heisenberg’s Uncertainty Principle (Heisenberg, 1927).

So far, we have discussed about the decomposition of a signal into a Haar wavelet. However, wavelets have *various transforms*. The rifest wavelet families are the asymmetric Daubechies or daublets, the symmetric Least Asymmetric or symmlets, and the symmetric coiflets. The most Discrete Wavelet Transforms (DWT) are designed by the mathematician and physicist Ingrid Daubechies and their actual difference is how the trend and the fluctuations are calculated (Daubechies, 1992). However, as we have progressed from the Haar example to any wavelet transform, it is more appropriate to refer to the trend and fluctuations as *father* (17) and *mother* (18) wavelets instead. These functions should satisfy the following condition:

(17)

&

(18)

This condition ensures the wavelets oscillatory nature and the second property of wavelets (p. 5). In order to ensure the norm that the father wavelet (17) is equal to one, a term is included (Crowley, 2005). We can recall this term in formulas (12) and (13) from our Haar example. Also, this term ensures that our wavelet will preserve its energy (Walker, 2008).

Now we are able to describe what the father and mother functions represent in any wavelet. The father wavelet, or scaling function, essentially represents the smooth, trend (low frequency) part of a signal, whereas the mother wavelets represent the detailed ((high frequency) parts by scale by noting the amount of stretching of the wavelet or ‘dilation’ (Crowley, 2005).

But still we have not discussed what distinguishes the wavelet families. It is pretty obvious that every wavelet family has its own set of wavelets and a family’s sets have something in common, otherwise there would be no reason to name them “families”. Nevertheless, all of the wavelet family “members” are classified with the same perspective. It is the amount of signal values used in order to calculate the trend and the fluctuations formulas. For example, Haar transform is also known as Daubechies 2. We can recall from our example, we are able to calculate the father (12) and mother (13) wavelets by picking the signal values two by two. If we have picked them four by four or six by six, then the wavelet transform would have been called Daubechies 4 and Daubechies 6 respectively. Therefore, as the number increases, the wavelet oscillates smoother, as the running average sets are larger, but the non-zero mean is less compact (Tangborn & National Aeronautics and Space Administration, 2010).

*Note: Orthogonality @ Wave guide p.13*

Data and Methodology

The paper empirically examines the beta coefficients of the top stocks in NASDAQ using two techniques:

1. Via regressing the CAPM model directly and
2. Via regressing the coefficients of different frequencies using wavelet analysis

Applying both methodologies on 88 stocks in a 10-year time frame might have been very time-consuming task. Hence, we designed an standalone application using the “R” programming language (R Development Core Team, 2008) and we cross validated the results in Matlab. The scripts have been included in Appendix A and B. The application is able to calculate *a,* *b* and the wavelet coefficients of any given data set. For this exact reason, we are going to present how the software operates, including the generic functions and then we will represent our data.

According to the theoretical background, CAPM model is defined as:

(1)

and it can be estimated as:

(2)

Where,

is an asset *i* expected return,

is the rate on a risk-free asset,

is the expected return on the market index (benchmark),

is the estimate of risk for asset *i*.

As we try to estimate multiple alphas and betas at the same time we have defined as a matrix where different stocks are represented in columns and their time series in rows. Benchmark and risk-free remain vectors as we have assumed that the portfolio is examined on the same market index and risk-free asset. Therefore:

(19)

(20)

(21)

Previously we defined CAPM variables , , and but as we have already mentioned (p.5, para. 3) these variables are not stationary, which may violate the assumptions of Ordinary Least Squares (OLS) regression. Therefore, we change the variables to change rates (22) in favor of non-stationarity elimination. Then we will generate the and to regress them as shown in formula (23) and (24) respectively:

(22)

(23)

&

(24)

Formula (24) represents a transformation of into a matrix, although it is a mere mathematical convention in order to rewrite the OLS regression as follows:

(25)

Where *i* are the individual stocks in the portfolio. Using the formula above, we are able to estimate the whole portfolio alphas, betas and Adjusted R Squared. It is important to remember that this estimation methodology of series of regressions (25) assumes that each stock in our portfolio has the same number of observations *t*.

In theoretical background we discussed upon various wavelets and wavelet families (p. 7). While our empirical work is conducted using Maximal Overlap Discrete Wavelet Transform of Least Asymmetric 8 with 4 levels, we have included a variety of MODWTs and levels in order to satisfy any future need. To avoid any confusion, we need to clarify that the application calculates each level’s father wavelet, but only the last father wavelet is used to estimate the betas, as formula (16) suggests from the theoretical background.

Nevertheless, in our case we estimate the coefficients using the following Least Asymmetric 8 father and mother wavelets (Schleicher, 2002):

& (26)

Where,

& (27)

While the father represents the long run, the interpretation of the different mother levels is as follows (Schleicher, 2002):

|  |  |  |  |
| --- | --- | --- | --- |
| Scale  Crystals | Monthly  frequency  resolution | Weekly  frequency  resolution | Daily  frequency  resolution |
|  | 1–2 | 1–2 | 1–2 |
|  | 2–4 | 2–4 | 2–4 |
|  | 4–8 | 4–8 | 4–8 |
|  | 8–16 = 8m–1yr4m | 8–16 = 2m–4m | 8–16 = 6w2d–12w4d |
|  | 16–32 | 16–32 | 16–32 |
|  |  |  |  |

In this paper, the data frequency is weekly. However, we chose not to use the adjusted closes prices week by week, rather than using an average of the daily adjusted closes, but this is of minor importance.

The beta estimation procedure using the wavelet coefficients is similar with the standard regression. The only difference is that we will initially calculate the wavelet coefficients of the already calculated and as shown in formulas (23) and (24) and then we regress them. We found that alphas remain almost the same () in different scales and it is very close to zero, therefore there was not any reason to include them.

The matrices (30) and (31) specify how the signal decomposition occurs in multiple levels using the mother wavelet. The rows denote the frequency, the columns the individual stocks, while the layers of a matrix denote the levels of decomposition. However, father is estimated separately, because it appears only at the last level of decomposition (see formula 16). The father coefficients are estimated as shown in matrices (32) and (33).

Later, we will use these coefficients in order to estimate beta through regression. For notational convenience we refer to and as *y* and *x* respectively, where:

(28)

&

(29)

Fluctuation Coefficient Matrices

(30)

&

(31)

Trend Coefficient Matrices

(32)

&

(33)

After getting the wavelet coefficients from both *x* and *y*, we are able to calculate the betas over the periods we have chosen, according to the levels of decomposition (see table p. 10). The new betas will be extracted by a series of regression as follows:

&

Where,

*i* is an individual stock and the process repeats for,

*l* is the decomposition level of the wavelet,

the estimations proceed for in the mother, while in the father (see formulae 28 & 29),

*j* is the wavelet family

Data Representation

So far we have discussed the general framework of our methodology. Afterwards, we represent the data we have collected for our empirical work. We have chosen the adjusted closures of 88 stocks from the top 100 NASDAQ companies (Nasdaq Stock Market). Then, the Nasdaq Composite we used as a benchmark (market). Both datasets have been extracted by Yahoo Finance (Yahoo Finance). On the other hand, the risk free asset has been gotten by the St. Luis Fred (FRED Economic Data) and it is the U.S. 3-month treasury bill.

The stocks and the benchmark have been collected in daily basis from January 2006 to January 2016 and then they have been transformed into weekly frequency by their means, while the free-risk asset has been collected directly into weekly basis. Afterwards, the returns of the stocks and the benchmark are calculated by the application using the formula 22. It’s worth mentioning that we don’t multiply by 100 to get the percentage change. In order to calculate the risk-free return we divide the initial values by 52, which are the total weeks annually and then we divide them again by 100 to adjust to the individual stocks and the benchmark.

Finally, the stock returns are shorted by the means over the period we examine. Then using these results we created 4 portfolios as shown below in table C.1 (Appendix III). This will enable us to examine the assumption of CAPM, that higher risk is related with higher returns.

First of all, the portfolios A, B, C and D are shorted from the higher to the lowest, we expect that the betas will have a downward trend to satisfy the CAPM’s assumption.

Then, we calculated the standard deviations and the median per stock and portfolio to have an overall view about our data collection. The Tables C.2 and C.3 represent them respectively (Appendix III). We are able to observe a number of discrepancies between the medians and the means in the individual stocks such as the *VRTX* stock, which has a mean of 0.0048 and a median of -0.0002. However, the discrepancies are reduced in portfolio levels. The second portfolio has the best performance, in which the difference of the mean and the median is 122 times smaller than a standard deviation, while the worst is the last with 13 times smaller. Also, the standard deviations of the portfolios do not follow any trend.

Afterwards, we proceed to the descriptive statistics of the benchmark and the risk-free returns. The medians and means have a significant difference but still not larger than a standard deviation.

|  |  |  |
| --- | --- | --- |
| **Statistics** | **Free** | **Benchmark** |
| *mean* | 0.0002116 | 0.001766545 |
| *median* | 0.0000212 | 0.004919893 |
| *min* | 0 | -0.14065975 |
| *max* | 0.0009712 | 0.070119526 |
| *St.Dev* | 0.0003449 | 0.022009755 |

Finally, we estimated the correlations between the portfolios, the benchmark and the risk-free. In order to calculate a portfolio correlation, we firstly calculated the means by the time frequency and then we calculated the correlations. Simply put, we represent the correlation of the means of each portfolio.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | ***A*** | ***B*** | ***C*** | ***D*** | ***Bench*** | ***Free*** |
| ***A*** | 1 | 0.973026 | 0.977928 | 0.97214 | 0.124713 | -0.0194 |
| ***B*** | 0.973026 | 1 | 0.973396 | 0.939154 | 0.152899 | -0.02602 |
| ***C*** | 0.977928 | 0.973396 | 1 | 0.971454 | 0.15183 | -0.03707 |
| ***D*** | 0.97214 | 0.939154 | 0.971454 | 1 | 0.156111 | -0.08054 |
| ***Bench*** | 0.124713 | 0.152899 | 0.15183 | 0.156111 | 1 | -0.03927 |
| ***Free*** | -0.0194 | -0.02602 | -0.03707 | -0.08054 | -0.03927 | 1 |

There is a very significant positive correlation between the portfolios. This may be caused by two facts. Firstly, Nasdaq companies have a certain particularity. Their very existence is dependent to the company’s Research and Development department. Also, in the majority of Nasdaq companies the R&D progression is high influenced by Computer Science progress, or electrical engineering in general. So, it is safe to assume that these companies may have a non-quantitative coherence on the medium and long run strategies, according to widely accepted patterns such as the Moore’s first and second law. However, it is important not to fall into the fallacy that strategy coherence is associated with a company’s risk aversion, aggressiveness or the terms of competition in their sectors. Secondly, stock markets experience a lot of short run small shocks, which sometimes are driven by pure noise. A reason behind these shocks may be the bandwagon effect. Since the stocks that we have selected share the market leading factor, there is no reason to decline that they satisfy the taste of a certain investor group. So, the fact that these companies may satisfy the preferences of an investor subgroup, does support the assumption that this group may have more common characteristics, such as the risk averseness, the reaction flexibility, access to information or even the resistance to a bandwagon effect. The above speculations cannot be proven, but it is not impossible that forces like these may drive the correlation to these high standards.

Data Discussion

In this section we are going to discuss the results of our study. In order to do so, we have cited two tables. The first represents the beta means of each portfolio per period, the grand means of the periods and scale and finally the CLRM mean. The second represents the beta standard deviations of each portfolio in the same manner as the first table.

Portfolio Beta Means

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | D1  Mean | D2  Mean | D3  Mean | D4  Mean | Scale  Mean | Grand Mean | OLS Mean |
| Portfolio A | 0.999558 | 1.0019283 | 0.998837 | 0.997551 | 0.891517 | 0.977878 | 0.971216 |
| Portfolio B | 1.000356 | 0.999342 | 0.999585 | 0.999139 | 1.080042 | 1.015693 | 1.088676 |
| Portfolio C | 1.000045 | 1.0006732 | 1.000711 | 0.999475 | 0.8901711 | 0.978215 | 0.95004 |
| Portfolio D | 0.999639 | 1.0014499 | 1.000532 | 1.00179 | 1.1962015 | 1.039922 | 1.006129 |

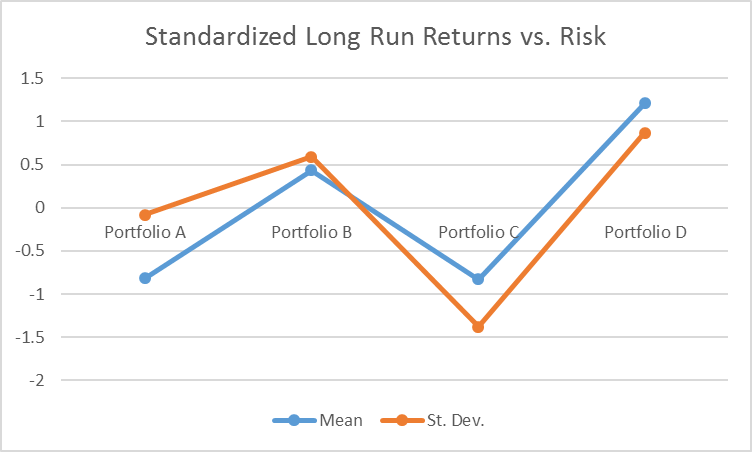
Portfolio Standard Deviations

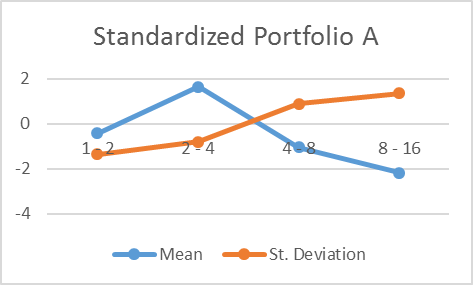
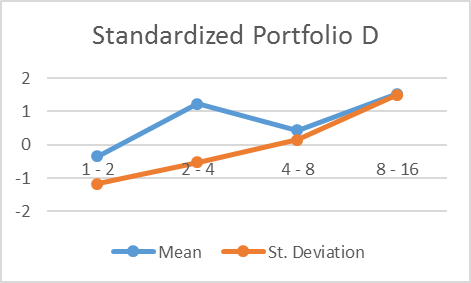
|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | D1  St. dev. | D2  St. Dev. | D3  St. Dev. | D4  St. Dev. | Scale  St. Dev. | Portfolio  St. Dev. | OLS  St. Dev. |
| Portfolio A | 0.00149 | 0.0032591 | 0.008646 | 0.010102 | 0.4929536 | 0.220765 | 0.272305 |
| Portfolio B | 0.002667 | 0.0040041 | 0.00577 | 0.010073 | 0.5884037 | 0.260342 | 0.30713 |
| Portfolio C | 0.002512 | 0.0043851 | 0.007075 | 0.009723 | 0.3076833 | 0.142224 | 0.243293 |
| Portfolio D | 0.002052 | 0.0041203 | 0.006263 | 0.010567 | 0.6280682 | 0.286695 | 0.242907 |

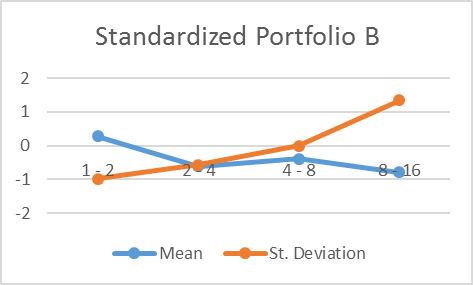
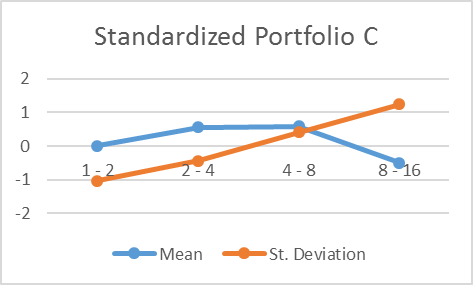
At first glance, it is clear that the CLRM does not perform very well. We expected that high betas are associated with high standard deviations, as a measurement of risk, but this is not the case. On the other hand, the grand mean of betas after the wavelet decomposition performs much better. However, the wavelet coefficient CLRM grand means are not an efficient, since they are inflated by the scale betas. Before we examine them period by period (see table p. 10), it is important to mention that the standard deviations increase over time, which was expected because uncertainty (i.e. risk) increases as the investment horizon increases. Therefore, the use of standard deviations as a measurement of risk cannot be rejected in different investments horizons.

However, the results are mixed between the periods. Except the long run – scale – means and standard deviations, we observe a general rigidity. The means are very close to 1 and the standard deviations are very small in comparison with the scale. For example, in the first period there is consistency between the betas and the standard deviations. However, this is not the case with the second period, where there is not any coherence. This incoherence continues in the third period and the last. In conclusion, the differences are so marginal that we do not have a clear view whether theory is valid or not, because the data sample contains disturbances from the housing bubble in 2006 and the financial crisis in 2008. In scale there is a completely different story.

The standard deviations skyrocket and the scale means are very volatile. There is clear consistency between the risk and betas. Although, it is not clear if there is any proportionality between them, which is very important. Since theory assumes rationality in investments, we expect to see a pattern between risk and returns. In order to capture any proportionality, we standardized both scale means and standard deviations. Below, we can see the coherence between risk, but only the second and the fourth portfolio seems to have proportional payoffs in the long run. The rest have either excess returns or risk.

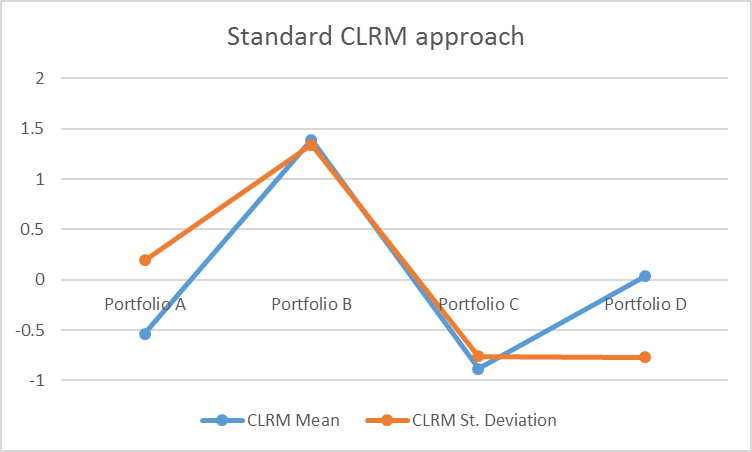


Also, we did the same the same all individual portfolios to capture the behaviors over time. As we have already mentioned, standardization tends to be inflated by outliers, such as the scale coefficients, therefore we tried to normalize instead to mitigate this effect, but with no so success. Therefore, we captured separately the fluctuations from the trend and the results over time are as follow:

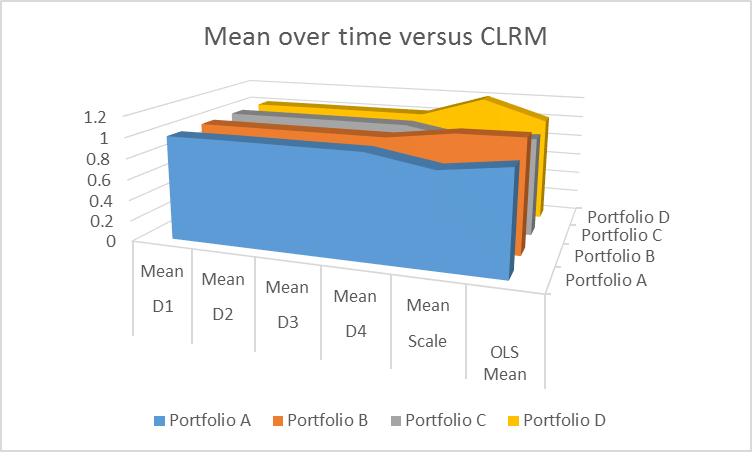


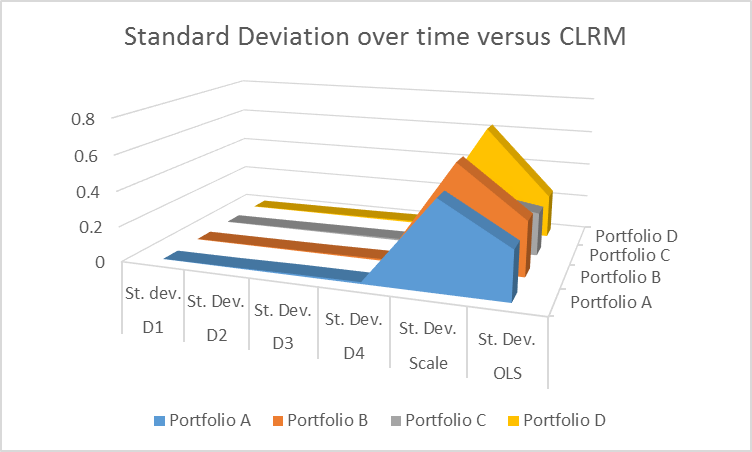
It is obvious there is not any clear pattern between risk and returns, except that the risk has an upward slope as we have already pointed before. According to our results, the only speculation we can do is that the portfolios risk and returns tend to an equilibrium roughly between the second and the third period.

Finally, it is important to compare these results with the standard CLRM approach. Below we demonstrate risk versus returns of the portfolios in the same manner as we did before.



The grand mean graph is dominated by the scale value and it is almost the same as the standardized long run returns versus risk graph. There is significant inconsistency between the wavelet and the standard OLS results in portfolio D and in some extend in portfolio C. A and B on the other hand seem to have the same standardized payoffs. But we need to keep in mind that standardized payoffs reveal patterns and trends, not actual fits between the methodologies. The actual values are represented as follow:





Conclusion

The wavelets helped us to determined how in different time periods the stock market is functioning and see if there is short term or a long-term trend. Many articles use S&P 500 and top companies that are listed there. However, using Nasdaq as a benchmark we see that the portfolios are highly correlated, for reasons we have already discussed in Data Representation section. The CAPM is a limited estimation method because some of its assumptions are weak, for instance investors’ uncertainty expectations, because a respectable portion has limited information.

Also, the fact that the assets are random walk, strengthens the lack of rational expectations theory to explain why some investors prefer the short run asset play, where random-walk behavior dominates the market. In the long run instead information mitigates in some extend the random-walk behavior and the betas are more volatile, therefore it is more open for high returns. Wavelet analysis supported that uncertainty increases over time in terms of standard deviations, but CAPM is unable to decompose the risk into equity risk and economic uncertainty. This is very important, because long run investments may include higher risk in general, but they may have a smaller proportion of non-structural exogenous risk.

Also, CAPM excludes many information such as, private investment and human capital. Moreover, we have included the financial crisis many beta estimates have disturbances. The portfolios have been shorted from the highest to the lowest return, portfolio A has a return of 0.006, B has 0.003, C has 0.002 and D 0.001. Higher beta means higher return and lower beta means lower return, however, this is not the case because the beta tables from both wavelet analysis and standard CLRM method show us that higher beta does not mean necessarily higher returns. Therefore, empirically estimated CAPM does not brink sufficient results.

Between the periods betas have a significant rigidity around 1 in comparison with the long run. However, in the scale changes dramatically, where the portfolios A and C betas fall, B remains rigid and D increases. The standard deviations are very small comparing to scale where skyrocket. Although, there is not a pattern among risk and returns, with the exception of scale risk which is positive. However, we see an equilibrium between risk and return in the second and the third period, which is not explained by CAPM.

In conclusion, the results of this paper do not support the CAPM as a tool of risk-return analysis, rather as a support that standard deviations may measure risk over time and for long run investments in some extend.

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**APPENDICES**

**APPENDIX I**

**Ui.R**

library**(**shiny**)**

library**(**wavelets, lib.loc **=** "./libraries"**)**

library**(**rsconnect, lib.loc **=** "./libraries"**)**

shinyUI**(**

fluidPage**(**

titlePanel**(**h3**(**"Maximum Overlap Discrete Wavelets Calculator"**))**,

sidebarLayout**(**

sidebarPanel**(**tags**$**head**(**

tags**$**style**(**type**=**"text/css", "select { max-width: 200px; }"**)**,

tags**$**style**(**type**=**"text/css", ".span4 { max-width: 250px; }"**)**,

tags**$**style**(**type**=**"text/css", ".well { max-width: 514px; }"**)**

**)**,

fileInput**(**"file","Upload the Stocks"**)**,

# checkboxInput(inputId = "week", "Transform Daily to Weekly", value = FALSE),

fileInput**(**"bench","Upload the Benchmark"**)**,

fileInput**(**"free","Upload the Risk Free"**)**,

helpText**(**"Accepts only '.csv' file type."**)**,

tags**$**hr**()**,

h5**(**helpText**(**"Select the table parameters"**))**,

checkboxInput**(**inputId **=** 'header', label **=** 'Header', value **=** **TRUE)**,

checkboxInput**(**inputId **=** "stringAsFactors", "string As Factors", **FALSE)**,

#br(),

tags**$**hr**()**,

### {WAVELET INPUTS} ###

selectInput**(**"filter", label **=** h5**(**helpText**(**"Select the M.O. Wavelet filter"**))**,

choices **=** list**(**"Haar" **=** "haar", "Least Assymetric 8" **=** "la8",

"Daubechies 4" **=** "d4", "Coiflet 6" **=** "c6"**)**, selected **=** "la8"**)**,

selectInput**(**"boundary", label **=** h5**(**helpText**(**"Select the M.O. Wavelet boundary"**))**,

choices **=** list**(**"Periodic" **=** "periodic", "Reflection" **=** "reflection"**)**, selected **=** "periodic"**)**,

numericInput**(**"levels", label **=** h5**(**helpText**(**"Select levels"**))**, value **=** 3, min **=** 1, max **=** 6**)**,

actionButton**(**"calc", "Calculate M.O.D.W.T", width **=** '100%'**)**,

tags**$**hr**()**,

h5**(**helpText**(**"Wavelet & Scaling Coefficient Matrices"**))**,

downloadButton**(**'downloadw', 'Download Wavelet Coefficients'**)**,

downloadButton**(**'downloadv', 'Download Scaling Coefficients'**)**,

downloadButton**(**'downloadr', 'Download Change Rate'**)**,

downloadButton**(**'downloadbeta', 'Download Betas'**)**,

downloadButton**(**'downloadwav', 'Download Wav Betas'**)**

**)**,

mainPanel**(**

uiOutput**(**"tb"**)**

**)**

**)**

**))**

**Server.R**

library**(**shiny**)**

Sys.setenv**(**R\_CMDZIP **=** 'C:/Rtools/bin/zip'**)**

shinyServer**(function(**input,output**){**

source**(**"./Functions.R", local **=** **TRUE)**

### {INPUT TABLE TAB} ###

output**$**table **<-** renderTable**({**

**if(**is.null**(**data**())){**return **()}**

shortInput**(**data**())**

**}**, rownames **=** **TRUE)**

### {RATE TABLE} ###

output**$**rate **<-** renderTable**({**

shortInput**(**rateInput**())**

**}**, rownames **=** **TRUE)**

### {INPUT SUMMARY TAB} ###

output**$**sum **<-** renderTable**({**

do.call**(**cbind, lapply**(**data**()**, summary**))**

**}**, rownames **=** **TRUE)**

### {REGRESSION TAB} ###

output**$**beta1 **<-** renderTable**({**

**if(**is.null**(**data**())** **|** is.null**(**bench**())** **|** is.null**(**free**())){**return **()}**

shortInput**(**regrInput**())**

**}**, digits **=** 4, rownames **=** **TRUE)**

### {REGRESSION TAB 2} ###

output**$**beta2 **<-** renderTable**({**

**if(**is.null**(**data**())** **|** is.null**(**bench**())** **|** is.null**(**free**())){**return **()}**

shortInput**(**regrInput2**())**

**}**, digits **=** 4, rownames **=** **TRUE)**

output**$**beta3 **<-** renderTable**({**

**if(**is.null**(**data**())** **|** is.null**(**bench**())** **|** is.null**(**free**())){**return **()}**

shortInput**(**regrInput3**())**

**}**, digits **=** 4, rownames **=** **TRUE)**

### {MODWT COEFS TAB} ###

# Title 1

output**$**tmod1 **<-** renderUI**({**

t1coefsInput**()**

**})**

# Title 2

output**$**tmod2 **<-** renderUI**({**

t2coefsInput**()**

**})**

# Table WSF

output**$**wsf **<-** renderTable**({**

**if(**is.null**(**coefs**$**wsf**)){**return**()}**

shortInput**(**coefs**$**wsf**)**

**}**, digits **=** 4, rownames **=** **TRUE)**

# Table VSF

output**$**vsf **<-** renderTable**({**

**if(**is.null**(**coefs**$**vsf**)){**return**()}**

shortInput**(**coefs**$**vsf**)**

**}**, digits **=** 4, rownames **=** **TRUE)**

# Table WBF

output**$**wbf **<-** renderTable**({**

**if(**is.null**(**coefs**$**wbf**)){**return**()}**

shortInput**(**coefs**$**wbf**)**

**}**, digits **=** 4, rownames **=** **TRUE)**

# Table VBF

output**$**vbf **<-** renderTable**({**

**if(**is.null**(**coefs**$**vbf**)){**return**()}**

shortInput**(**coefs**$**vbf**)**

**}**, digits **=** 4, rownames **=** **TRUE)**

### {MODWT PLOT TAB} ###

output**$**plot **<-** renderPlot**({**

**if(**is.null**(**data**())){**return **()}**

# input$calc

# waveInput()

plot**(**waveInput**()**, col.plot **=** "blueviolet"**)**

**})**

#height = 100%, width = 100%

### {DOWNLOADS} ###

output**$**downloadr **<-** downloadHandler**(**

filename **=** **function()** **{**

paste**(**"ChangeRate", Sys.Date**()**, '.csv', sep**=**''**)**

**}**,

content **=** **function(**file**)** **{**

write.csv**(**rateInput**()**, file**)**

**}**

**)**

output**$**downloadw **<-** downloadHandler**(**

filename **=** **function()** **{**

paste**(**input**$**filter, "L", input**$**levels, "WavCoef", Sys.Date**()**, '.csv', sep**=**''**)**

**}**,

content **=** **function(**file**)** **{**

write.csv**(**dwcoefsInput**()**, file**)**

**}**

**)**

output**$**downloadv **<-** downloadHandler**(**

filename **=** **function()** **{**

paste**(**input**$**filter, "L", input**$**levels, "ScalCoef", Sys.Date**()**, '.csv', sep**=**''**)**

**}**,

content **=** **function(**file**)** **{**

write.csv**(**dvcoefsInput**()**, file**)**

**}**

**)**

output**$**downloadbeta **<-** downloadHandler**(**

filename **=** **function()** **{**

paste**(**"BetaCoefs", Sys.Date**()**, '.csv', sep**=**''**)**

**}**,

content **=** **function(**file**)** **{**

write.csv**(**regrInput**()**, file**)**

**}**

**)**

output**$**downloadwav **<-** downloadHandler**(**

filename **=** **function()** **{**

paste**(**"WavBetaCoefs", Sys.Date**()**, '.csv', sep**=**''**)**

**}**,

content **=** **function(**file**)** **{**

write.csv**(**regrInput2**()**, file**)**

**}**

**)**

### {PATH TAB} ###

output**$**filedf **<-** renderTable**({**

**if(**is.null**(**data**())){**return **()}**

rbind**(**input**$**file, input**$**bench, input**$**free**)**

**})**

### {TAB GENERATOR} ###

output**$**tb **<-** renderUI**({**

**if(**is.null**(**data**()))**

HTML**(**'<center><img src="deree\_logo.jpg"></center>'**)**

**else**

tabsetPanel**(**tabPanel**(**"Data", tableOutput**(**"table"**))**,

tabPanel**(**"Change Rate", tableOutput**(**"rate"**))**,

tabPanel**(**"Summary", tableOutput**(**"sum"**))**,

tabPanel**(**"Beta Coefficients", tableOutput**(**"beta1"**))**,

tabPanel**(**"Beta Coefficients 2", tableOutput**(**"beta2"**))**,

tabPanel**(**"Wavelets Coefs", fluidRow**(**htmlOutput**(**"tmod1"**))**,

fluidRow**(**

column**(**width **=** 7, tableOutput**(**"wsf"**))**,

column**(**width **=** 5, tableOutput**(**"wbf"**)))**,

fluidRow**(**htmlOutput**(**"tmod2"**))**,

column**(**width **=** 7, tableOutput**(**"vsf"**))**,

column**(**width **=** 5, tableOutput**(**"vbf"**)))**,

tabPanel**(**"Wavelet Plot", plotOutput**(**"plot", width **=** 1280, height **=** 720**))**,

tabPanel**(**"About file", tableOutput**(**"filedf"**)**,

htmlOutput**(**"dcenter"**))**

**)**

**})**

**})**

**Functions.R**

### {IMPORTED DATA} ###

data **<-** reactive**({**

file1 **<-** input**$**file

**if(**is.null**(**file1**)){**return**()}**

data **<-** read.table**(**file**=**file1**$**datapath, sep**=** ',', header **=** input**$**header, stringsAsFactors **=** input**$**stringAsFactors**)**

**})**

### {BENCHMARK} ###

bench **<-** reactive**({**

file1 **<-** input**$**bench

**if(**is.null**(**file1**)){**return**()}**

data **<-** read.table**(**file**=**file1**$**datapath, sep**=** ',', header **=** input**$**header, stringsAsFactors **=** input**$**stringAsFactors**)**

**for** **(**i **in** 1**:**nrow**(**data**))** **{**

data**[**i,1**]** **<-** **(**data**[**i**+**1,1**]** **-** data**[**i,1**])/**data**[**i,1**]**

**}**

data **<-** data**[-**nrow**(**data**)**,**]**

**})**

### {FREE RISK} ###

free **<-** reactive**({**

file1 **<-** input**$**free

**if(**is.null**(**file1**)){**return**()}**

data **<-** read.table**(**file**=**file1**$**datapath, sep**=** ',', header **=** input**$**header, stringsAsFactors **=** input**$**stringAsFactors**)**

**})**

### {RATE TABLE} ###

rateInput **<-** reactive**({**

data **<-** data**()**

**for** **(**i **in** 1**:**nrow**(**data**))** **{**

**for** **(**j **in** 1**:**ncol**(**data**))** **{**

data**[**i,j**]** **<-** **(**data**[**i**+**1,j**]** **-** data**[**i,j**])/**data**[**i,j**]**

**}**

**}**

data **<-** data**[-**nrow**(**data**)**,**]**

**})**

### {SHORT OUTPUT} ###

shortInput **<-** **function(**x**)** **{**

**if(**nrow**(**x**)** **<=** 30 **&** ncol**(**x**)** **<=** 8**){**

x

**}** **else** **{**

**if(**ncol**(**x**)** **>** 8**)** **{**

x **<-** cbind**(**x**[**,1**:**3**]**, x**[**,**(**ncol**(**x**)-**2**):**ncol**(**x**)])**

**}** **else** **{}**

x **<-** rbind**(**head**(**x**)**, tail**(**x**))**

**}**

**}**

### {WAVELET HEADERS} ###

t1coefsInput **<-** reactive**({**

HTML**(**paste

**(**tags**$**br**()**, h5**(**"W: Wavelet Coefficients"**)**,

tags**$**hr**()**

**)**

**)**

**})**

t2coefsInput **<-** reactive**({**

HTML**(**paste**(**h5**(**"V: Scaling Coefficients"**)**,

tags**$**hr**())**

**)**

**})**

### {RSF/RBF} ###

rsfbf **<-** **function(**x**)** **{**

data **<-** rateInput**()**; free **<-** free**()**

free **<-** data.frame**(**free**)**

**if(**x **==** 'rsf'**)** **{**

x **<-** data.frame**(**apply**(**data**[**,**]**,2,'-',free**))**

**}** **else** **if(**x **==** 'rbf'**)** **{**

bench **<-** bench**()**

bench **<-** data.frame**(**bench**)**

x **<-** data.frame**(**apply**(**bench**[]**,2,'-',free**))**

**}** **else** **{**return**()}**

**}**

wav **<-** **function(**x, y**)** **{**

# wave <- waveInput()

data **<-** rsfbf**(**x**)**

wave **<-** wavelets**::**modwt**(**data, filter **=** input**$**filter, n.levels **=** input**$**levels, boundary **=** input**$**boundary**)**

**if(**y **==** 'w'**)** **{**

name\_list **<-** expand.grid**(**W**=**names**(**wave@W**)**,N**=**names**(**data**))**

name\_list **<-** name\_list**[**order**(**name\_list**$**W**)**,**]**

name\_list **<-** unlist**(**apply**(**name\_list,1,paste0,collapse**=**""**))**

output **<-** data.frame**(**do.call**(**cbind,wave@W**))**

names**(**output**)** **<-** name\_list

x **<-** output

**}** **else** **if(**y **==** 'v'**)** **{**

name\_list **<-** expand.grid**(**V**=**names**(**wave@V**)**,N**=**names**(**data**))**

name\_list **<-** name\_list**[**order**(**name\_list**$**V**)**,**]**

name\_list **<-** unlist**(**apply**(**name\_list,1,paste0,collapse**=**""**))**

output **<-** data.frame**(**do.call**(**cbind,wave@V**))**

names**(**output**)** **<-** name\_list

x **<-** output

**}** **else** **{**return**()}**

**}**

### {WAVELET TABLES} ###

coefs **<-** reactiveValues**(**wsf **=** **NULL**, wbf **=** **NULL**, vsf **=** **NULL**, vbf **=** **NULL)**

# WSF

wsfcoefsInput **<-** observeEvent**(**input**$**calc, **{**

coefs**$**wsf **<-** wav**(**'rsf', 'w'**)**

write.table**(**coefs**$**wsf, file **=** "./tmp/wsf.csv", col.names **=** **NA**, qmethod **=** "double", sep **=** ","**)**

**})**

# VSF

vsfcoefsInput **<-** observeEvent**(**input**$**calc, **{**

coefs**$**vsf **<-** wav**(**'rsf', 'v'**)**

write.table**(**coefs**$**vsf, file **=** "./tmp/vsf.csv", col.names **=** **NA**, qmethod **=** "double", sep **=** ","**)**

**})**

# WBF

wbfcoefsInput **<-** observeEvent**(**input**$**calc, **{**

coefs**$**wbf **<-** wav**(**'rbf', 'w'**)**

write.table**(**coefs**$**wbf, file **=** "./tmp/wbf.csv", col.names **=** **NA**, qmethod **=** "double", sep **=** ","**)**

**})**

# VBF

vbfcoefsInput **<-** observeEvent**(**input**$**calc, **{**

coefs**$**vbf **<-** wav**(**'rbf', 'v'**)**

write.table**(**coefs**$**vbf, file **=** "./tmp/vbf.csv", col.names **=** **NA**, qmethod **=** "double", sep **=** ","**)**

**})**

### {REGRESSION} ###

regrInput **<-** reactive**({**

data **<-** rateInput**()**; bench **<-** bench**()**; free **<-** free**()**

a1 **<-** vector**()**; b1 **<-** vector**()**; rs **<-** vector**()**

bench **<-** data.frame**(**bench**)**

free **<-** data.frame**(**free**)**

rsf **<-** data.frame**(**apply**(**data**[**,**]**,2,'-',free**))**

rbf **<-** data.frame**(**apply**(**bench**[]**,2,'-',free**))**

**for** **(**i **in** 1**:**ncol**(**rsf**))** **{**

mod **<-** lm**(**rsf**[**,i**]** **~** rbf**[**,1**])**

a1**[**i**]** **<-** mod**$**coefficients**[**1**]**

b1**[**i**]** **<-** mod**$**coefficients**[**2**]**

rs**[**i**]** **<-** summary**(**mod**)$**adj.r.squared

**}**

table **<-** data.frame**(**'Company\_Symbol' **=** names**(**data**)**, 'Alpha\_Coef' **=** a1, 'Beta\_Coef' **=** b1, 'Adj.R\_Squared' **=** rs**)**

**})**

### REG 3

regrInput3 **<-** reactive**({**

**})**

### {REGRESSION 2} ###

regrInput2 **<-** reactive**({**

wreghandle **<-** **function(**x, y**){**

mod **<-** lm**(**wsf**[[**x**]]** **~** wbf**[[**y**]])**

return**(**list**(**Beta\_coef **=** unname**(**mod**$**coefficients**[**2**])**,

Adj.R\_Squared **=** unname**(**summary**(**mod**)$**adj.r.squared**)))**

**}**

a1 **<-** vector**()**; b1 **<-** vector**()**; rs **<-** vector**()**

vsf **<-** data.frame**(**coefs**$**vsf**)**

vlevel **<-** **(**ncol**(**vsf**)/**input**$**levels**)\*(**input**$**levels**-**1**)** **+** 1

vsf **<-** vsf**[**,vlevel**:**ncol**(**vsf**)]**

vbf **<-** data.frame**(**coefs**$**vbf**)**

**for** **(**i **in** 1**:**ncol**(**vsf**))** **{**

mod **<-** lm**(**vsf**[**,i**]** **~** vbf**[**,input**$**levels**])**

b1**[**i**]** **<-** mod**$**coefficients**[**2**]**

rs**[**i**]** **<-** summary**(**mod**)$**adj.r.squared

**}**

vtable **<-** data.frame**(** 'Beta\_Coef' **=** b1, 'Adj.R\_Squared' **=** rs**)**

k **<-** ncol**(**wsf**)/**input**$**levels

wbenchmarknames **<-** sort**(**rep**(**names**(**wbf**)**, k**))**

wtablelist **<-** Map**(**wreghandle, names**(**wsf**)**, wbenchmarknames**)**

table **<-** do.call**(**rbind, lapply**(**wtablelist, data.frame**))**

count **<-** sort**(**rep**(**1**:**input**$**levels, nrow**(**table**)/**input**$**levels**))**

names **<-** array**(**rep**(**names**(**data**())**, input**$**levels**))**

table **<-** cbind**(**names, count, table**)**

table **<-** reshape**(**table, idvar **=** "names", timevar **=** "count", direction **=** "wide"**)**

table **<-** cbind**(**table, vtable**)**

row.names**(**table**)** **<-** names**(**data**())**; table **<-** table**[**,**-**1**]**

table

**})**

### {DOWNLOADS} ###

dwcoefsInput **<-** reactive**({**

coefs**$**w

**})**

dvcoefsInput **<-** reactive**({**

coefs**$**v

**})**

**APPENDIX II**

(Tables)

Table C.1: Stock Returns Means

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Portfolio A | | Portfolio B | | Portfolio C | | Portfolio D | |
| **Symbol** | **Change**  **Rate** | **Symbol** | **Change**  **Rate** | **Symbol** | **Change**  **Rate** | **Symbol** | **Change**  **Rate** |
| *PCLN* | 0.008631 | *ISRG* | 0.004259 | *INTU* | 0.002937 | *MCHP* | 0.00194 |
| *NFLX* | 0.008579 | *AAL* | 0.004242 | *DISCA* | 0.002933 | *AMGN* | 0.001933 |
| *REGN* | 0.008308 | *SBAC* | 0.004086 | *GOOG* | 0.00293 | *MU* | 0.00191 |
| *BIDU* | 0.008075 | *ATVI* | 0.004059 | *COST* | 0.002913 | *SIRI* | 0.001907 |
| *INCY* | 0.007823 | *EXPE* | 0.003823 | *DISH* | 0.002852 | *TXN* | 0.001756 |
| *ALXN* | 0.007664 | *CTSH* | 0.003725 | *MYL* | 0.002721 | *ADI* | 0.001738 |
| *ILMN* | 0.007453 | *SHPG* | 0.00372 | *HSIC* | 0.002697 | *VOD* | 0.001732 |
| *SWKS* | 0.006475 | *CERN* | 0.003715 | *CTXS* | 0.002567 | *ADSK* | 0.001628 |
| *MNST* | 0.006332 | *SBUX* | 0.003342 | *MDLZ* | 0.002489 | *INTC* | 0.001603 |
| *CTRP* | 0.006329 | *WDC* | 0.003335 | *FAST* | 0.002408 | *PAYX* | 0.00157 |
| *AMZN* | 0.006057 | *NVDA* | 0.003328 | *MAT* | 0.002394 | *CSCO* | 0.001485 |
| *NTES* | 0.006005 | *LBTYK* | 0.003316 | *ADBE* | 0.002361 | *EBAY* | 0.001332 |
| *AAPL* | 0.005266 | *LBTYA* | 0.003285 | *ADP* | 0.002341 | *MXIM* | 0.001311 |
| *BMRN* | 0.005244 | *CSX* | 0.003181 | *MAR* | 0.002307 | *EA* | 0.001247 |
| *DLTR* | 0.004777 | *ESRX* | 0.00318 | *LRCX* | 0.002287 | *BBBY* | 0.001061 |
| *VRTX* | 0.004549 | *STX* | 0.003135 | *MSFT* | 0.002243 | *WFM* | 0.001023 |
| *ROST* | 0.004418 | *FISV* | 0.003055 | *FOXA* | 0.002161 | *AMAT* | 0.000988 |
| *CELG* | 0.004391 | *SRCL* | 0.003008 | *WBA* | 0.002041 | *SYMC* | 0.000985 |
| *TSCO* | 0.004371 | *GOOGL* | 0.002972 | *XRAY* | 0.001983 | *QCOM* | 0.000971 |
| *ORLY* | 0.004367 | *CMCSA* | 0.002969 | *PCAR* | 0.001983 | *NTAP* | 0.000801 |
| *GILD* | 0.004303 | *AKAM* | 0.002962 | *FOX* | 0.00196 | *CA* | 0.000797 |
| *BIIB* | 0.004267 | *CHKP* | 0.002939 | *XLNX* | 0.001957 | *VIAB* | 0.000749 |
| Mean | 0.006077 | Mean | 0.003438 | Mean | 0.00243 | Mean | 0.001385 |

Table C.2: Stock Returns Standard Deviations

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Portfolio A | | Portfolio B | | Portfolio C | | Portfolio D | |
| **Symbol** | **St. Dev.** | **Symbol** | **St. Dev.** | **Symbol** | **St. Dev.** | **Symbol** | **St. Dev.** |
| *PCLN* | 0.04380527 | *ISRG* | 0.052064 | *INTU* | 0.027699 | *MCHP* | 0.0313756 |
| *NFLX* | 0.06253493 | *AAL* | 0.092154 | *DISCA* | 0.034218 | *AMGN* | 0.0282301 |
| *REGN* | 0.05724758 | *SBAC* | 0.038751 | *GOOG* | 0.034473 | *MU* | 0.0604794 |
| *BIDU* | 0.05663271 | *ATVI* | 0.036951 | *COST* | 0.023017 | *SIRI* | 0.0770035 |
| *INCY* | 0.06332661 | *EXPE* | 0.05086 | *DISH* | 0.042434 | *TXN* | 0.029145 |
| *ALXN* | 0.0389954 | *CTSH* | 0.038429 | *MYL* | 0.039963 | *ADI* | 0.0296639 |
| *ILMN* | 0.05169906 | *SHPG* | 0.032078 | *HSIC* | 0.022234 | *VOD* | 0.0273721 |
| *SWKS* | 0.05128313 | *CERN* | 0.033796 | *CTXS* | 0.039341 | *ADSK* | 0.043101 |
| *MNST* | 0.04969161 | *SBUX* | 0.034961 | *MDLZ* | 0.021477 | *INTC* | 0.0307033 |
| *CTRP* | 0.05642912 | *WDC* | 0.045625 | *FAST* | 0.034879 | *PAYX* | 0.0232532 |
| *AMZN* | 0.04466186 | *NVDA* | 0.053273 | *MAT* | 0.032941 | *CSCO* | 0.0303214 |
| *NTES* | 0.04572973 | *LBTYK* | 0.040113 | *ADBE* | 0.035772 | *EBAY* | 0.03534 |
| *AAPL* | 0.0385389 | *LBTYA* | 0.041442 | *ADP* | 0.02094 | *MXIM* | 0.0347071 |
| *BMRN* | 0.04499186 | *CSX* | 0.035301 | *MAR* | 0.035908 | *EA* | 0.0401939 |
| *DLTR* | 0.03072255 | *ESRX* | 0.033257 | *LRCX* | 0.040189 | *BBBY* | 0.0313235 |
| *VRTX* | 0.06199383 | *STX* | 0.056029 | *MSFT* | 0.028735 | *WFM* | 0.0449164 |
| *ROST* | 0.02841148 | *FISV* | 0.025515 | *FOXA* | 0.036145 | *AMAT* | 0.0339293 |
| *CELG* | 0.03505964 | *SRCL* | 0.024022 | *WBA* | 0.029552 | *SYMC* | 0.0332157 |
| *TSCO* | 0.03772713 | *GOOGL* | 0.034451 | *XRAY* | 0.025513 | *QCOM* | 0.0299408 |
| *ORLY* | 0.02812907 | *CMCSA* | 0.031053 | *PCAR* | 0.036361 | *NTAP* | 0.0402986 |
| *GILD* | 0.031426 | *AKAM* | 0.049906 | *FOX* | 0.034274 | *CA* | 0.0280378 |
| *BIIB* | 0.03706208 | *CHKP* | 0.026542 | *XLNX* | 0.031057 | *VIAB* | 0.0336224 |
| *St. Dev.* | 0.01132895 | *St. Dev.* | *0.014608* | *St. Dev.* | *0.006441* | *St. Dev.* | *0.012059* |

Table C.3: Stock Returns Medians

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Portfolio A | | Portfolio B | | Portfolio C | | Portfolio D | |
| **Symbol** | **Median** | **Symbol** | **Median** | **Symbol** | **Median** | **Symbol** | **Median** |
| *PCLN* | 0.00752724 | *ISRG* | 0.002222 | *INTU* | 0.002759 | *MCHP* | 0.0032063 |
| *NFLX* | 0.00607246 | *AAL* | 0.002673 | *DISCA* | 0.00398 | *AMGN* | 0.0029862 |
| *REGN* | 0.00584031 | *SBAC* | 0.005942 | *GOOG* | 0.004411 | *MU* | -0.000029 |
| *BIDU* | 0.00619582 | *ATVI* | 0.002901 | *COST* | 0.002388 | *SIRI* | 0.0001645 |
| *INCY* | 0.00649133 | *EXPE* | 0.003885 | *DISH* | 0.004323 | *TXN* | 0.0022956 |
| *ALXN* | 0.00668396 | *CTSH* | 0.003751 | *MYL* | 0.003097 | *ADI* | 0.0018683 |
| *ILMN* | 0.00692143 | *SHPG* | 0.003994 | *HSIC* | 0.00278 | *VOD* | 0.0038868 |
| *SWKS* | 0.00697847 | *CERN* | 0.00305 | *CTXS* | 0.002931 | *ADSK* | 0.002912 |
| *MNST* | 0.00566036 | *SBUX* | 0.003738 | *MDLZ* | 0.002572 | *INTC* | 0.0023334 |
| *CTRP* | 0.00405074 | *WDC* | 0.003505 | *FAST* | 0.003611 | *PAYX* | 0.002806 |
| *AMZN* | 0.00499534 | *NVDA* | 0.006661 | *MAT* | 0.002653 | *CSCO* | 0.0025654 |
| *NTES* | 0.00555953 | *LBTYK* | 0.003083 | *ADBE* | 0.004778 | *EBAY* | 0.0027132 |
| *AAPL* | 0.00628042 | *LBTYA* | 0.003383 | *ADP* | 0.002338 | *MXIM* | 0.0019998 |
| *BMRN* | 0.00485997 | *CSX* | 0.0041 | *MAR* | 0.003838 | *EA* | 0.0019592 |
| *DLTR* | 0.00529616 | *ESRX* | 0.003641 | *LRCX* | 0.001908 | *BBBY* | 0.0024222 |
| *VRTX* | -0.0002225 | *STX* | 0.003119 | *MSFT* | 0.003379 | *WFM* | 0.0015605 |
| *ROST* | 0.0047914 | *FISV* | 0.003169 | *FOXA* | 0.002891 | *AMAT* | 0.0035213 |
| *CELG* | 0.0041344 | *SRCL* | 0.003778 | *WBA* | 0.002262 | *SYMC* | 0.0020566 |
| *TSCO* | 0.00337409 | *GOOGL* | 0.004522 | *XRAY* | 0.002635 | *QCOM* | 0.0020089 |
| *ORLY* | 0.00393319 | *CMCSA* | 0.002858 | *PCAR* | 0.001098 | *NTAP* | 0.0020337 |
| *GILD* | 0.00375937 | *AKAM* | 0.003155 | *FOX* | 0.003035 | *CA* | 0.0015905 |
| *BIIB* | 0.0042421 | *CHKP* | 0.003611 | *XLNX* | 0.00276 | *VIAB* | 0.0023891 |
| *Median* | 0.00542784 | *Median* | *0.003558* | *Median* | *0.002835* | *Median* | *0.002314* |

Table C.4: OLS Regression Table

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Portfolio A** | | | **Portfolio B** | | | **Portfolio C** | | | **Portfolio D** | | |
| **Symbol** | **Alpha** | **Beta** | **Symbol** | **Alpha** | **Beta** | **Symbol** | **Alpha** | **Beta** | **Symbol** | **Alpha** | **Beta** |
| *PCLN* | 0.006808 | 0.988638 | *ISRG* | 0.002252 | 1.202429 | *INTU* | 0.001422 | 0.786268 | *MCHP* | 0.000239 | 0.985116 |
| *NFLX* | 0.006737 | 1.016217 | *AAL* | 0.001087 | 1.758254 | *DISCA* | 0.001246 | 0.924411 | *AMGN* | 0.000772 | 0.573293 |
| *REGN* | 0.00613 | 1.232882 | *SBAC* | 0.001983 | 1.185309 | *GOOG* | 0.001139 | 1.036989 | *MU* | -0.00086 | 1.651456 |
| *BIDU* | 0.005613 | 1.408164 | *ATVI* | 0.002534 | 0.889112 | *COST* | 0.001745 | 0.610966 | *SIRI* | -0.00048 | 1.342651 |
| *INCY* | 0.005398 | 1.410475 | *EXPE* | 0.001675 | 1.273252 | *DISH* | 0.000727 | 1.17947 | *TXN* | 6.65E-05 | 0.919108 |
| *ALXN* | 0.006166 | 0.810372 | *CTSH* | 0.001641 | 1.168381 | *MYL* | 0.000951 | 0.998251 | *ADI* | 0.000168 | 0.894233 |
| *ILMN* | 0.00598 | 0.95518 | *SHPG* | 0.002522 | 0.694741 | *HSIC* | 0.001483 | 0.656908 | *VOD* | 0.000522 | 0.632972 |
| *SWKS* | 0.004143 | 1.371908 | *CERN* | 0.002103 | 0.893548 | *CTXS* | 0.000599 | 1.107276 | *ADSK* | -0.00085 | 1.414229 |
| *MNST* | 0.005267 | 0.686016 | *SBUX* | 0.001402 | 1.069849 | *MDLZ* | 0.00151 | 0.487886 | *INTC* | -0.0002 | 0.999126 |
| *CTRP* | 0.004054 | 1.306393 | *WDC* | 0.001299 | 1.26111 | *FAST* | 0.000534 | 1.07997 | *PAYX* | 0.000136 | 0.722692 |
| *AMZN* | 0.003917 | 1.146553 | *NVDA* | 0.000882 | 1.488068 | *MAT* | 0.000908 | 0.784267 | *CSCO* | -0.00018 | 0.978797 |
| *NTES* | 0.004541 | 0.9782 | *LBTYK* | 0.001278 | 1.127953 | *ADBE* | 0.000261 | 1.196011 | *EBAY* | -0.00058 | 1.077539 |
| *AAPL* | 0.003334 | 1.186287 | *LBTYA* | 0.001145 | 1.191193 | *ADP* | 0.001042 | 0.669756 | *MXIM* | -0.00044 | 1.032598 |
| *BMRN* | 0.003469 | 1.021888 | *CSX* | 0.001205 | 1.131447 | *MAR* | 0.000159 | 1.236858 | *EA* | -0.0006 | 1.066298 |
| *DLTR* | 0.003794 | 0.501753 | *ESRX* | 0.001594 | 0.858183 | *LRCX* | 0.000117 | 1.274145 | *BBBY* | -0.00046 | 0.833157 |
| *VRTX* | 0.003128 | 0.936748 | *STX* | 0.000592 | 1.596057 | *MSFT* | 0.000675 | 0.856951 | *WFM* | -0.00074 | 0.975216 |
| *ROST* | 0.003102 | 0.728064 | *FISV* | 0.00144 | 0.871848 | *FOXA* | 0.000108 | 1.204405 | *AMAT* | -0.00081 | 1.087521 |
| *CELG* | 0.003088 | 0.729483 | *SRCL* | 0.002133 | 0.404621 | *WBA* | 0.000774 | 0.643274 | *SYMC* | -0.00054 | 0.903783 |
| *TSCO* | 0.002615 | 0.985239 | *GOOGL* | 0.001186 | 1.03392 | *XRAY* | 0.000564 | 0.786451 | *QCOM* | -0.00049 | 0.858314 |
| *ORLY* | 0.00316 | 0.621523 | *CMCSA* | 0.00138 | 0.908669 | *PCAR* | -0.00028 | 1.278056 | *NTAP* | -0.00107 | 1.139898 |
| *GILD* | 0.003135 | 0.638639 | *AKAM* | 0.000876 | 1.240648 | *FOX* | -3.09E-05 | 1.153137 | *CA* | -0.00084 | 0.931667 |
| *BIIB* | 0.002931 | 0.706137 | *CHKP* | 0.001657 | 0.702277 | *XLNX* | 0.000327 | 0.949183 | *VIAB* | -0.00124 | 1.115179 |

Table C.5.1: Wavelet Coefficient Regression

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Portfolio A** | | | | | | **Portfolio B** | | | | | |
| **Symbol** | **Beta**  **D1** | **Beta**  **D2** | **Beta**  **D3** | **Beta**  **D4** | **Beta**  **Scale** | **Symbol** | **Beta**  **D1** | **Beta**  **D2** | **Beta**  **D3** | **Beta**  **D4** | **Beta**  **Scale** |
| *PCLN* | 1.00155 | 0.99825 | 1.00479 | 1.0065 | 1.13662 | *ISRG* | 0.999565 | 1.00371 | 1.00333 | 1.006687 | 1.24772 |
| *NFLX* | 1.00264 | 1.00014 | 0.99953 | 1.00177 | 0.76201 | *AAL* | 0.999835 | 0.99412 | 1.00236 | 1.016855 | -0.2816 |
| *REGN* | 1.0008 | 1.00572 | 1.00739 | 1.02035 | 0.79819 | *SBAC* | 1.000892 | 0.99801 | 0.99828 | 1.0031 | 1.10749 |
| *BIDU* | 0.99863 | 1.00214 | 0.99122 | 0.99839 | 1.67736 | *ATVI* | 1.000321 | 0.9939 | 1.00019 | 1.004944 | 0.77712 |
| *INCY* | 0.9981 | 1.002 | 0.99816 | 0.98887 | 1.77868 | *EXPE* | 0.999985 | 1.00136 | 1.00623 | 1.01E+00 | 1.59004 |
| *ALXN* | 0.99821 | 1.00204 | 0.9982 | 0.98923 | 0.68499 | *CTSH* | 1.001635 | 0.99907 | 0.9963 | 1.003737 | 1.08464 |
| *ILMN* | 1.00126 | 1.00185 | 1.00648 | 0.99174 | 1.25181 | *SHPG* | 0.990565 | 0.99089 | 0.98156 | 0.993114 | 0.69817 |
| *SWKS* | 1.00187 | 1.00453 | 0.97269 | 0.98046 | 1.75554 | *CERN* | 0.999854 | 1.00023 | 0.99513 | 0.993729 | 0.94608 |
| *MNST* | 0.99904 | 1.00084 | 0.9994 | 0.99802 | -0.0726 | *SBUX* | 1.002946 | 1.0043 | 0.99861 | 1.008399 | 1.25761 |
| *CTRP* | 0.99782 | 1.00644 | 1.00159 | 0.99527 | 1.39843 | *WDC* | 0.998254 | 1.00123 | 1.00201 | 0.987536 | 1.91521 |
| *AMZN* | 0.9997 | 0.99626 | 1.00229 | 1.00937 | 0.99554 | *NVDA* | 1.001031 | 1.00531 | 1.00465 | 1.011456 | 1.66345 |
| *NTES* | 0.9978 | 1.00456 | 0.99608 | 0.9964 | 0.82427 | *LBTYK* | 0.998129 | 0.99784 | 1.00039 | 0.995106 | 1.31673 |
| *AAPL* | 1.00014 | 1.00121 | 0.99136 | 0.98306 | 1.42363 | *LBTYA* | 1.003111 | 0.99707 | 1.0004 | 0.99902 | 1.32719 |
| *BMRN* | 0.99944 | 0.99809 | 1.0079 | 1.00647 | 0.71129 | *CSX* | 1.001281 | 0.9969 | 1.00288 | 0.985655 | 1.08461 |
| *DLTR* | 0.99926 | 0.99801 | 0.98057 | 0.99037 | 0.3105 | *ESRX* | 1.000653 | 1.00245 | 1.00682 | 1.003464 | 0.74232 |
| *VRTX* | 1.00104 | 1.00478 | 1.00368 | 1.00142 | 0.83726 | *STX* | 1.003443 | 0.9974 | 1.00714 | 1.002634 | 2.65846 |
| *ROST* | 0.99842 | 1.0051 | 1.00778 | 1.00456 | 0.77368 | *FISV* | 1.001521 | 0.99815 | 1.00201 | 0.997915 | 0.82652 |
| *CELG* | 0.99834 | 1.00402 | 1.00048 | 0.97789 | 0.66395 | *SRCL* | 1.000786 | 0.9968 | 0.99582 | 1.003609 | 0.2557 |
| *TSCO* | 1.00011 | 1.00533 | 1.00341 | 0.99776 | 0.55534 | *GOOGL* | 1.001154 | 1.00027 | 0.99328 | 0.992169 | 1.08799 |
| *ORLY* | 1.00038 | 0.99521 | 0.99644 | 0.99931 | 0.26548 | *CMCSA* | 1.00383 | 1.00751 | 0.99409 | 0.982858 | 0.62713 |
| *GILD* | 0.99794 | 1.00544 | 1.00338 | 1.00738 | 0.44287 | *AKAM* | 1.00E+00 | 0.99662 | 0.99574 | 0.975321 | 1.10919 |
| *BIIB* | 0.99781 | 1.00047 | 1.0016 | 1.00155 | 0.63855 | *CHKP* | 0.999515 | 1.00239 | 1.00368 | 1.0063 | 0.71911 |

C.5.2: Wavelet Coefficient Regression

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Portfolio C** | | | | | | **Portfolio D** | | | | | |
| **Symbol** | **Beta**  **D1** | **Beta**  **D2** | **Beta**  **D3** | **Beta**  **D4** | **Beta**  **Scale** | **Symbol** | **Beta**  **D1** | **Beta**  **D2** | **Beta**  **D3** | **Beta**  **D4** | **Beta**  **Scale** |
| *INTU* | 0.99792 | 1.00018 | 1.00006 | 0.9961 | 0.52644 | *MCHP* | 1.00149 | 0.99957 | 0.99491 | 0.97858 | 0.928 |
| *DISCA* | 0.99923 | 0.9881 | 1.00012 | 0.9964 | 0.9842 | *AMGN* | 1.00078 | 1.00249 | 1.00196 | 0.99846 | 0.33715 |
| *GOOG* | 0.9983 | 1.00521 | 1.00062 | 1.00446 | 1.09239 | *MU* | 0.9992 | 1.00143 | 0.99882 | 0.99844 | 1.77248 |
| *COST* | 1.00249 | 0.99769 | 0.98784 | 0.98012 | 0.58734 | *SIRI* | 1.00257 | 1.00168 | 0.99651 | 1.00435 | 3.58802 |
| *DISH* | 1.00233 | 0.9968 | 0.98788 | 0.98011 | 1.55182 | *TXN* | 0.99846 | 1.00169 | 1.00446 | 0.99695 | 1.12847 |
| *MYL* | 1.00084 | 0.99981 | 0.99411 | 1.00152 | 0.8998 | *ADI* | 0.99635 | 1.00156 | 0.99355 | 1.03479 | 1.10756 |
| *HSIC* | 0.99962 | 1.00284 | 1.00283 | 0.99903 | 0.83716 | *VOD* | 1.00011 | 0.99917 | 0.99105 | 1.0022 | 0.74465 |
| *CTXS* | 0.99913 | 1.00413 | 1.0029 | 1.00107 | 0.66221 | *ADSK* | 1.00051 | 1.00334 | 1.0014 | 1.00289 | 1.6035 |
| *MDLZ* | 0.99767 | 1.00271 | 1.00712 | 1.00938 | 0.33721 | *INTC* | 1.00174 | 1.00392 | 1.00448 | 1.01672 | 1.00761 |
| *FAST* | 1.00848 | 1.00888 | 0.9825 | 0.98364 | 0.69276 | *PAYX* | 1.00166 | 0.99625 | 1.00204 | 0.99488 | 0.75727 |
| *MAT* | 1.00018 | 0.99685 | 0.99874 | 0.9987 | 0.67291 | *CSCO* | 0.99658 | 0.99895 | 0.99876 | 0.99797 | 0.88716 |
| *ADBE* | 0.99952 | 1.00519 | 1.00098 | 0.99729 | 1.51388 | *EBAY* | 1.00152 | 0.99711 | 1.00321 | 1.00332 | 1.44867 |
| *ADP* | 1.00276 | 1.00046 | 1.00442 | 1.00106 | 0.47942 | *MXIM* | 0.99842 | 1.00088 | 1.0084 | 1.01166 | 1.19998 |
| *MAR* | 0.99831 | 1.0028 | 1.01038 | 1.00246 | 1.00862 | *EA* | 0.99998 | 0.99564 | 0.99373 | 0.99728 | 1.26892 |
| *LRCX* | 0.99856 | 1.00235 | 1.00722 | 1.00296 | 1.18886 | *BBBY* | 0.99902 | 0.99438 | 0.99201 | 0.99726 | 0.72038 |
| *MSFT* | 0.99731 | 0.99433 | 1.0015 | 1.01495 | 0.88341 | *WFM* | 0.99892 | 1.00723 | 0.99623 | 0.98913 | 1.54531 |
| *FOXA* | 0.99814 | 1.0045 | 1.00981 | 0.98563 | 1.14069 | *AMAT* | 1.00037 | 1.00062 | 0.9931 | 1.0023 | 1.20728 |
| *WBA* | 0.99924 | 1.00017 | 1.00189 | 1.00817 | 0.80528 | *SYMC* | 0.99916 | 1.00037 | 1.00312 | 0.99959 | 0.90335 |
| *XRAY* | 0.99872 | 1.00014 | 1.00646 | 1.01373 | 0.72785 | *QCOM* | 1.00047 | 1.0053 | 1.0078 | 1.00104 | 0.83082 |
| *PCAR* | 1.00285 | 0.99993 | 1.00388 | 0.99913 | 0.97046 | *NTAP* | 1.00271 | 1.00704 | 1.00563 | 1.00962 | 1.25204 |
| *FOX* | 0.99952 | 0.99825 | 1.00428 | 1.00754 | 1.1465 | *CA* | 0.99533 | 1.01236 | 1.01567 | 0.99882 | 0.92543 |
| *XLNX* | 0.99987 | 1.00348 | 1.0001 | 1.005 | 0.87455 | *VIAB* | 0.99672 | 1.00093 | 1.00487 | 1.00313 | 1.15239 |