DIPARTIMENTO DI ELETTRONICA,
INFORMAZIONE E BIOINGEGNERIA
Politecnico di Milano

Machine Learning (Code: 097683)
September 11, 2017

Name:
Surname:
Student ID:
Row: Column:

Time: 2 hours 30 minutes Prof. Marcello Restelli Maximum Marks: 34

- The following exam is composed of **10 exercises** (one per page). The first page needs to be filled with your **name, surname and student ID**. The following pages should be used **only in the large squares** present on each page. Any solution provided outside these spaces will not be considered for the final mark.
- During this exam you are **not allowed to use electronic devices** like laptops, smartphones, tablets and/or similar. As well, you are not allowed to bring with you any kind of note, book, written scheme and/or similar. You are also not allowed to communicate with other students during the exam.
- The first reported violation of the above mentioned rules will be annotated on the exam and will be considered for the final mark decision. The second reported violation of the above mentioned rules will imply the immediate expulsion of the student from the exam room and the **annulment of the exam**.
- You are allowed to write the exam either with a pen (black or blue) or a pencil. It is your responsibility to provide a readable solution. We will not be held accountable for accidental partial or total cancellation of the exam.
- The exam can be written either in **English** or **Italian**.
- You are allowed to withdraw from the exam at any time without any penalty. You are allowed to leave the room not early than half the time of the duration of the exam. You are not allowed to keep the text of the exam with you while leaving the room.
- **Three of the points will be given on the basis on how quick you are in solving the exam. If you finish earlier than 45 min before the end of the exam you will get 3 points, if you finish earlier than 30 min you will get 2 points and if you finish earlier than 15 min you will get 1 point (the points cannot be accumulated).**

Ex. 1	Ex. 2	Ex. 3	Ex. 4	Ex. 5	Ex. 6	Ex. 7	Ex. 8	Ex. 9	Ex. 10	Time	Tot.
/ 5	/ 5	/ 5	/ 2	/ 2	/ 2	/ 2	/ 2	/ 3	/ 3	/ 3	/ 34

Exercise 1 (5 marks)

Describe the supervised learning technique called **ridge regression** for regression problems.

Exercise 2 (5 marks)

Describe the differences existing between the **Montecarlo** and the **Temporal Difference** methods in the modelfree estimation of a value function for a given policy.

Exercise 3 (5 marks)

Describe the difference between **on-policy** and **off-policy** reinforcement learning techniques. Make an example of an on-policy algorithm and an example of an off-policy algorithm.

Exercise 4 (2 marks)

Which of the following are the benefits/drawbacks of the sparsity imposed by the Lasso?

1. Using the Lasso penalty increases the variance of the fits.
2. It does variable selection implicitly.
3. Using the Lasso penalty increases the bias of the fits.
4. Sparse models are generally easier to interpret.

Provide motivation for your answer.

1. NONE OF THEM: regularization decreases the variance in a model.
2. BENEFIT: since the solution of the Lasso optimization constraints some coefficients to zero it selects automatically those features which are more relevant to the problem.
3. DRAWBACK: even if by regularizing we are decreasing the variance, at the same time we are also likely to introduce some bias due to the fact that the model space is too simple.
4. BENEFIT: having less features which are relevant, it is easier to understand the dynamics of the problem we are solving.

Exercise 5 (2 marks)

Comment the following statements about GPs. Motivate your answers.

1. Differently from linear models, we are considering different variance for the noise in each point of the input space.
2. The specific GP formulation allows one to use them only for classification problems.
3. Any finite subset of the points in the output space predicted by a GP follows a Gaussian multivariate distribution.
4. If we have few samples in a portion of the input space it is likely that the GP will have high uncertainty in that region.

1. TRUE: they provide a way of computing the expected value and the variance of each point in the input space, which could be different from point to point.
2. FALSE: in their original formulation the GP have been designed for regression, though they can be used also for other tasks.
3. TRUE: from the definition of GP.
4. TRUE: since a GP bases its shape from the existing data and from the covariance structure, if we are in a region of the space where we do not have samples, they will provide an uncertain prediction.

Exercise 6 (2 marks)

Tell if the following statements about the perceptron algorithm for classification are true or false and motivate your answers.

1. Shuffling the initial data is not crucial for the perceptron optimization procedure.
2. There exist multiple solutions to the minimization of the perceptron loss.
3. We are guaranteed that the overall loss of the method is decreasing over time.
4. The choice of a proper learning rate α might speed up the learning process.

1. FALSE: the solution of the problem depends on the order we present the points.
2. TRUE: for instance if we have two linearly separable classes we might end our optimization process in any one of the lines dividing the classes.
3. FALSE: we are only assured that the loss of the currently considered point is decreasing.
4. FALSE: its value does not influence the solution, usually we set $\alpha = 1$.

Exercise 7 (2 marks)

The client you are working for, Apple & Co., asked you to classify the quality of some fruits (i.e., 1-st quality and 2-nd quality) by basing on their characteristics (i.e., color and weight). You decided to use a linear SVM to solve the problem. After some time, the same client asks you to provide new solutions to improve the capabilities of the classifier you proposed. Comment the following options and tell if they are promising for increasing the testing performance (accuracy) of the SVM.

1. Enhance the training set by getting data points whose values of the input are far from the boundary of the current SVM.
2. Buy a new server in order to be able to apply a kernel on the previous SVM.
3. Enhance the training set by using new data whose input are near to the margins of the current SVM.
4. Introduce new input variables (e.g., diameter, density) and train the SVM on a new dataset containing this information.

1. FALSE: points which are far from the separating hyperplane are not likely to change the result of the SVM training process.
2. FALSE: the use of kernel does not require more computational power than the linear one.
3. TRUE: the points are likely to become the new support vectors and modify the SVM separating surface.
4. TRUE: as long as they are meaningful for the problem it is always a good idea to use new input variables.

Exercise 8 (2 marks)

Consider a regression model $\hat{t}_n = \phi(x_n)^T \mathbf{w}$, where $\phi(\cdot)$ is a basis function, and a dataset $(\mathbf{x}, \mathbf{t})$. Tell if one should consider the Least Square (LS) method to get an optimal parameter $\mathbf{w}$ for each one of the following 4 different situations. Motivate your answer.

1. The loss function is $L(\mathbf{w}|\mathbf{x}, \mathbf{t}) = ||\phi(\mathbf{x}_n)^T \mathbf{w} - \mathbf{t}||_2^2 + C||\mathbf{w}||_1^2$;
2. Large number of parameters;
3. The loss function is $L(\mathbf{w}|\mathbf{x}, \mathbf{t}) = ||\phi(\mathbf{x}_n)^T \mathbf{w} - \mathbf{t}||_2^2 + C||\mathbf{w}||_2^2$;
4. Huge number of samples.

where $|| \cdot ||_1$ and $|| \cdot ||_2$ denote the 1-norm and the 2-norm of a vector, respectively, and C is a positive parameter.

1. NO: this is the Lasso loss function which does not have closed form solution.
2. NO: it would require to invert a matrix with the dimension of the parameter vector.
3. YES: this is Ridge regression which can be solved with LS.
4. YES: LS scales only linearly with the number of samples.

Exercise 9 (3 marks)

Consider the Thompson Sampling algorithm. Assume to have the following posterior distributions $Beta_i(\alpha_t, \beta_t)$ for arms $\mathcal{A} = \{a_1, \dots, a_5\}$ rewards, which are distributed as Bernoulli r.v.:

$$\begin{array}{llll} a_1: & \alpha_t = 1 & \beta_t = 6 & \hat{r}(a_1) = 0.13 \\ a_2: & \alpha_t = 4 & \beta_t = 6 & \hat{r}(a_2) = 0.55 \\ a_3: & \alpha_t = 31 & \beta_t = 31 & \hat{r}(a_3) = 0.4 \\ a_4: & \alpha_t = 8 & \beta_t = 1 & \hat{r}(a_4) = 0.90 \\ a_5: & \alpha_t = 5 & \beta_t = 7 & \hat{r}(a_5) = 0.95 \end{array}$$

where $\hat{r}(a_i)$ are random samples extracted from the posterior distributions.

1. Which arm would play the TS algorithm in the next round?
2. Do you think that there is an arm that is more promising to be the best one?
3. What is the UCB1 bound for arms a_3 and a_5? Assume that the Bayesian setting started from uniform $Beta(1, 1)$ priors.

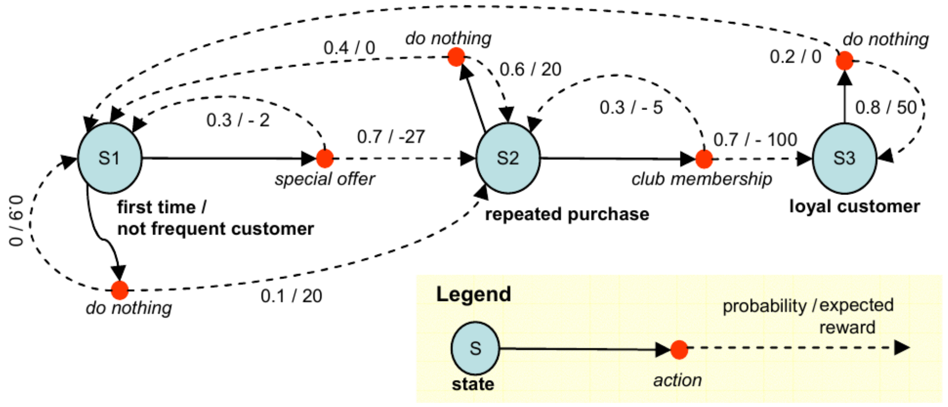
1. Arm a_5 since its sample is the highest one.
2. Arm a_4 only provided positive samples, thus, on average, it is the most promising one.
3. Since we have an overall number of pulls of $t = 5 + 8 + 60 + 7 + 10 = 90$ we have:

$$u(a_3) = \frac{30}{60} + \sqrt{\frac{2\log(90)}{60}}$$

$$u(a_5) = \frac{4}{10} + \sqrt{\frac{2\log(90)}{10}}$$

Exercise 10 (3 marks)

Consider the following MDP modeling an advertising problem:



where on the transition probabilities and the rewards are specified on the edges.

1. Provide the formulation of the Bellman expectation for V equations for the MDP in the figure in the case we consider the policy: $\pi(s_1; dn) = 1$ and $\pi(s_2; dn) = 1$ and with discount factor $\gamma = 0.5$.

2. Compute the value of state 2, i.e., $V(s_2)$ (justify your computations).

1. The Bellman equation for a state is of the form:

$$V^\pi(s) = \sum_a \pi(s,a) \left(R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^\pi(s') \right)$$

where $R(s,a)$ is the expected immediate reward. In its matricial form the equation (system of equations) is:

$$V = \begin{bmatrix} 2 \\ 12 \\ 20 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.4 & 0.6 & 0 \\ 0.2 & 0 & 0.8 \end{bmatrix}$$

2. Considering the first two equations and multiplying both sides by 20 we have:

$$20V(s_1) = 40 + 9V(s_1) + V(s_2)$$

$$20V(s_2) = 240 + 4V(s_1) + 6V(s_1)$$

leading to $V(s_1) = \frac{16}{3}$ and $V(s_2) = \frac{56}{3}$.