Eg. If $A = \{1, 2, 3, 4\}$ and $R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$ then R is a relation ie $R : A \to A$ and $R \subseteq A \times A$.

This relation is symmetric as can be seen by considering all possible cases. For eg $1R2$ and $2R1$ so that fine and the rest are trivial.

Eg. If $A = \{1, 2, 3, 4\}$, and $R = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$. This isn't symmetric since $1R2$ but $2\not R1$. It's also not reflexive because $2 \not R 2$ and $3 \not R 3$. (Instead of saying $2 \not R 2$ you can also say $(2,2) \notin R$.)

Eg. If $A = \{$all straight lines in the plane$\}$ and define $\perp$ on A by $a \in A, b \in A$ then $a \perp b$ if the lines a and b are at right angles. This is a relation on A.

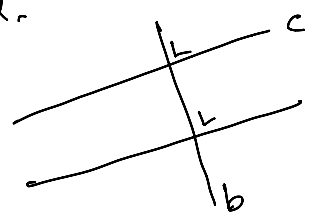
This is symmetric since if a is a line at right angles to b then b is also a line at right angles to a ie $a \perp b \Rightarrow b \perp a$. It's not reflexive since a line isn't perpendicular to itself.

<u>Transitivity</u>: A relation $R : A \to A$ is <u>transitive</u> if and only if whenever aRb and bRc we also have aRc.

Eg. $A = \{1,2,3,4\}$, $R = \{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$
If you consider the possible cases it's clear R is transitive.

Eg. $A = \{1,2,3,4\}$, $R = \{(1,3),(2,1)\}$
This isn't transitive since $2R1$ and $1R3$ but $2\not R3$ ie $(2,3) \notin R$.

Eg. $A = \{$all straight lines in the plane$\}$, $R = \perp$ as before.
This isn't transitive since if $a \perp b$ and $b \perp c$ then a and c are parallel.



To show a relation isn't reflexive, symmetric or transitive it's sufficient to find a single counter-example, ie a single case for which the property doesn't hold.

<u>Antisymmetric</u>: A relation $R: A \to A$ is antisymmetric if and only if given aRb and bRa then $a=b$.

Eg. Let $\mathbb{Z}$ be the set of all integers and define $R: \mathbb{Z} \to \mathbb{Z}$ by aRb if and only if $a \leq b$. Then R contains pairs like $(1,1)$, $(1,2)$, $(-1,2)$, $(0,10)$ etc. but doesn't contain $(0,-1)$

$(3,2)$ etc.

This relation is antisymmetric because $a \leq b$ and $b \leq a$ implies by the basic rules for arithmetic that $a=b$. R is reflexive because $a \leq a$ for every $a \in \mathbb{Z}$. It isn't symmetric, for eg $2 \leq 3$ but $3 \nleq 2$. It is transitive since $a \leq b$ and $b \leq c$ implies $a \leq c$.

Eg. $A = \{$all straight lines in the plane$\}$, $R = \perp$ as before. This isn't antisymmetric. As a counter-ex. take any two $\perp$ lines a, b. Then $a \perp b$, $b \perp a$ but $a \neq b$.

Eg. Define a relation $\mathcal{P}(A)$, $A \neq \emptyset$ by aRb if and only if $a \subseteq b$, where $a, b \in \mathcal{P}(A)$ ie a and b are subsets of A.

This is reflexive since $a \subseteq a$ for every a. It isn't symmetric It isn't symmetric since $\emptyset \subseteq A$ ie $\emptyset R A$ but $A \nsubseteq \emptyset$ ie $A \not R \emptyset$. It is transitive since $a \subseteq b$ and $b \subseteq c \Rightarrow a \subseteq c$. R is anti-symmetric since $a \subseteq b$ and $b \subseteq a$ implies that $a=b$.

<u>Equivalence Relations</u>:

A relation $R: A \longrightarrow A$, $A \neq \emptyset$, which is reflexive, symmetric and transitive is called an <u>equivalence relation</u>.

Eg. Let $T = \{$all plane triangles$\}$ and define a relation R on T by aRb if the triangles a and b are congruent, ie a and b have the same sides and angles or a and the reflection of b have the same sides and angles.