

Q1. Use the following matrices to answer parts (i), (ii) and (iii) below.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & -5 \\ -1 & 7 & 9 \\ 3 & 1 & 6 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 9 \\ 1 & 4 & 3 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 0 & 3 & 1 \\ 5 & 4 & -3 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} 3 & 3 \\ -7 & -9 \\ 8 & 1 \end{pmatrix}$$

$$\mathbf{E} = \begin{pmatrix} 0 & 1 \\ 1 & 5 \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} 4 & -2 \\ 7 & 17 \end{pmatrix}$$

$$\mathbf{G} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{H} = (12)$$

- (i). List all pairs of matrices from the above which are compatible for multiplication.
(ii). Calculate the products you listed in (i).
(iii). Calculate

(a). $\mathbf{AB} - 2\mathbf{DC}$

(b). \mathbf{AG}

(c). \mathbf{BAG}

(d). $(\mathbf{CD})^2 - 4\mathbf{EF}^2$

Q2. Calculate the determinant of the following 2×2 matrices:

(i).

$$\begin{pmatrix} -1 & 3 \\ 19 & 2 \end{pmatrix}$$

(ii).

$$\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$$

(iii).

$$\begin{pmatrix} -2 & 4 \\ -8 & 16 \end{pmatrix}$$

Q3. Find the inverse of the following matrices:

(i).

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

(ii).

$$\begin{pmatrix} 2 & -7 \\ 3 & -1 \end{pmatrix}$$

(iii).

$$\begin{pmatrix} 8 & 4 \\ -4 & 2 \end{pmatrix}$$

(iv).

$$\begin{pmatrix} 9 & 27 \\ 0 & 12 \end{pmatrix}$$

(v).

$$\begin{pmatrix} 12 & -9 \\ 8 & 2 \end{pmatrix}$$

Q4. Write each of the following systems of equations as a matrix equation and solve by finding the inverse of a matrix:

(i).

$$\begin{aligned} 3x + 4y &= -1 \\ 2x + 9y &= -6y \end{aligned}$$

(ii)

$$\begin{aligned} 3x + 4y &= 5 \\ 2x - 5y &= -12 \end{aligned}$$

(iii).

$$\begin{aligned} 7x - 2y &= 26 \\ 6x + 5y &= 29 \end{aligned}$$

(iv).

$$\begin{aligned} 3s + 2t &= 12 \\ 4s - t &= 5 \end{aligned}$$

(v).

$$\begin{aligned} 5c &= 1 - 3d \\ 2d + c + 4 &= 0 \end{aligned}$$

(vi).

$$\begin{aligned} 3p &= 2q \\ 4p + q + 11 &= 0 \end{aligned}$$

Q5. Write down the transpose of the following matrices:

(i).

$$\begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}$$

(ii).

$$\begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 1 & 2 & 3 \end{pmatrix}$$

(iii).

$$\begin{pmatrix} -1 & 0 & -1 \\ 1 & 2 & 3 \\ 1 & -8 & -14 \end{pmatrix}$$

(iv).

$$\begin{pmatrix} 1 & 3 & -1 & 0 \\ 2 & -1 & 0 & 4 \\ 0 & 1 & 6 & 1 \end{pmatrix}$$

Answers:

Q1:

(i). The allowed products are:

AA, AB, AD, AG,
BA, BB, BD, BG,
CA, CB, CD, CG,
DC, DE, DF,
EC, EE, EF,
FC, FE, FF,
GH, HH.

(ii).

$$\mathbf{AA} = \begin{pmatrix} -16 & 11 & -17 \\ 19 & 56 & 122 \\ 20 & 19 & 30 \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} -4 & -16 & 4 \\ 8 & 41 & 89 \\ 9 & 31 & 30 \end{pmatrix}$$

$$\mathbf{AD} = \begin{pmatrix} -51 & -20 \\ 20 & -57 \\ 50 & 6 \end{pmatrix}$$

$$\mathbf{AG} = \begin{pmatrix} -6 \\ 10 \\ 3 \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} 2 & 17 & 19 \\ 26 & 16 & 63 \\ 6 & 33 & 49 \end{pmatrix}$$

$$\mathbf{BB} = \begin{pmatrix} 2 & 8 & 22 \\ 9 & 37 & 36 \\ 4 & 18 & 46 \end{pmatrix}$$

$$\mathbf{BD} = \begin{pmatrix} -3 & -14 \\ 65 & 0 \\ -1 & -30 \end{pmatrix}$$

$$\mathbf{BG} = \begin{pmatrix} 0 \\ 9 \\ 2 \end{pmatrix}$$

$$\mathbf{CA} = \begin{pmatrix} 0 & 22 & 33 \\ -8 & 35 & -7 \end{pmatrix}$$

$$\mathbf{CB} = \begin{pmatrix} 1 & 7 & 30 \\ 2 & 2 & 32 \end{pmatrix}$$

$$\mathbf{CD} = \begin{pmatrix} -13 & -26 \\ -37 & -24 \end{pmatrix}$$

$$\mathbf{CG} = \begin{pmatrix} 1 \\ -8 \end{pmatrix}$$

$$\mathbf{DC} = \begin{pmatrix} 15 & 21 & -6 \\ -45 & -57 & 20 \\ 5 & 28 & 5 \end{pmatrix}$$

$$\mathbf{DE} = \begin{pmatrix} 3 & 18 \\ -9 & -52 \\ 1 & 13 \end{pmatrix}$$

$$\mathbf{DF} = \begin{pmatrix} 33 & 45 \\ -91 & -139 \\ 39 & 1 \end{pmatrix}$$

$$\mathbf{EC} = \begin{pmatrix} 5 & 4 & -3 \\ 25 & 23 & -14 \end{pmatrix}$$

$$\mathbf{EE} = \begin{pmatrix} 1 & 5 \\ 5 & 26 \end{pmatrix}$$

$$\mathbf{EF} = \begin{pmatrix} 7 & 17 \\ 39 & 83 \end{pmatrix}$$

$$\mathbf{FC} = \begin{pmatrix} -10 & 4 & 10 \\ 85 & 89 & -44 \end{pmatrix}$$

$$\mathbf{FE} = \begin{pmatrix} -2 & -6 \\ 17 & 92 \end{pmatrix}$$

$$\mathbf{FF} = \begin{pmatrix} 2 & -42 \\ 147 & 275 \end{pmatrix}$$

$$\mathbf{GH} = \begin{pmatrix} -12 \\ 0 \\ 12 \end{pmatrix}$$

$$\mathbf{HH} = (144)$$

(iii). (a)

$$\begin{pmatrix} -34 & -58 & 16 \\ 101 & 161 & 52 \\ -1 & -25 & 20 \end{pmatrix}$$

(iii). (b)

$$\begin{pmatrix} -6 \\ 10 \\ 3 \end{pmatrix}$$

(iii). (c)

$$\begin{pmatrix} 11 \\ 34 \\ 31 \end{pmatrix}$$

(iii). (d)

$$\begin{pmatrix} 543 & -138 \\ -1579 & -3794 \end{pmatrix}$$

Q2:

(i). -59

(ii). 2

(iii). 0

Q3:

(i). $\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$

(ii). $\frac{1}{19} \begin{pmatrix} -1 & 7 \\ -3 & 2 \end{pmatrix}$

(iii). $\frac{1}{16} \begin{pmatrix} 1 & -2 \\ 2 & 4 \end{pmatrix}$

(iv). $\frac{1}{36} \begin{pmatrix} 4 & -9 \\ 0 & 3 \end{pmatrix}$

(v). $\frac{1}{96} \begin{pmatrix} 2 & 9 \\ -8 & 12 \end{pmatrix}$

Q4:

(i). $x = -\frac{15}{37}, y = \frac{2}{37}$

(ii). $x = -1, y = 2$

(iii). $x = 4, y = 1$

(iv). $s = 2, t = 3$

(v). $c = 2, d = -3$

(vi). $p = -2, q = -3$

Q5:

(i). $\begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}$

(ii). $\begin{pmatrix} 2 & 5 & 1 \\ 3 & 6 & 2 \\ 4 & 7 & 3 \end{pmatrix}$

(iii). $\begin{pmatrix} -1 & 1 & 1 \\ 0 & 2 & -8 \\ -1 & 3 & -14 \end{pmatrix}$

(iv). $\begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ -1 & 0 & 6 \\ 0 & 4 & 1 \end{pmatrix}$