Power Set of a Set: The power set of a set A, is the set whose elements are the subsets of A. It's denoted
$$\mathcal{P}(A)$$

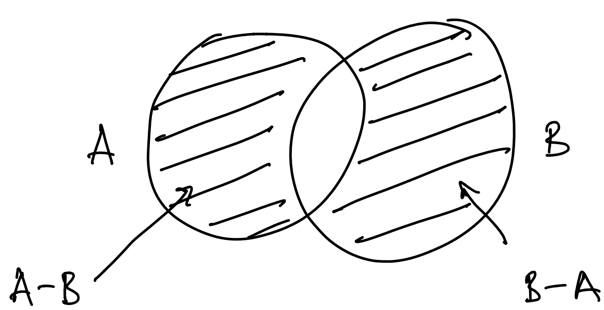
Eg. $A = \{1, 2, 3\}$
$$\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

Be methodical and remember to include $\emptyset$!

Another notation for the power set of A is 2^A.

If $x \in \mathcal{P}(A)$ then $x \subseteq A$.

Symmetric Difference: If A, B are sets, their symmetric difference is
$$(A-B) \cup (B-A).$$



The symmetric diff. is denoted $A \Delta B$. It's every in A and B which isn't common to both.

Eg. $A = \{1, 2, 3\}$, $C = \{2, 3, 4\}$
$A \Delta C = \{1, 4\}$.

<u>Cartesian Product</u>: If A, B are sets their <u>Cartesian product</u> denoted $A \times B$ is set of <u>ordered</u> pairs (a, b) where $a \in A$ and $b \in B$, ie

$$A \times B = \{(a, b) : a \in A, b \in B\}.$$

Eg. If $A = \{1, 2, 3\}$, $B = \{r, s\}$ then

$$\begin{aligned} A \times B = \{&(1, r), (1, s) \\ &(2, r), (2, s) \\ &(3, r), (3, s)\} \end{aligned}$$

Note that in general $A \times B \neq B \times A$. For the sets given

$$\begin{aligned} B \times A = \{&(r, 1), (r, 2), (r, 3), \\ &(s, 1), (s, 2), (s, 3)\} \end{aligned}$$

and $A \times B \neq B \times A$.

<u>Cardinality</u>: The <u>cardinality</u> of a set A, denoted by $\#(A)$ or $\|A\|$ is simply the no. of elements in A. For finite sets this straightforward, simply count the

elements. Infinite sets are more difficult and we can't worry about their size.

<u>Binary Relations</u>: If A, B are sets, a <u>binary relation</u> R from A to B is any subset of $A \times B$.

Eg. $A = \{1, 2, 3\}$, $B = \{r, s\}$

$A \times B = \{(1,r), (1,s), (2,r), (2,s), (3,r), (3,s)\}$

$R = \{(1,r), (1,s), (3,s)\}$ is a binary relation.

If $(a,b) \in R$ then we say a is related to b, denoted by aRb.

Eg. $A = \{$one, two, three, four, five$\}$, $B = \{1, 2, 3, 4, 5\}$ then we can define a relation as the set of pairs (word, no. of letters in that word) ie

$$R = \{(\text{one}, 3), (\text{two}, 3), (\text{three}, 5), (\text{four}, 4), (\text{five}, 4)\}.$$

Another relation, say S is the set of pairs (word, equivalent no.) ie

$$S = \{(\text{one}, 1), (\text{two}, 2), \ldots, (\text{five}, 5)\}.$$

The <u>domain</u> of a relation is the set of all the x-values where each pair in the relation has the form (x,y). The domain is a subset of A. The <u>range</u> is the set of y values.

Eg. $A = \{2,5,6,7\}$, $B = \{6,9,10,11,12\}$.
Define the relation R by aRb if and only if $2a<b$.

$$A \times B = \{(2,6),(2,9),(2,10),(2,11),(2,12)\\ (5,6),(5,9),(5,10),(5,11),(5,12)\\ (6,6),(6,9),(6,10),(6,11),(6,12)\\ (7,6),(7,9),(7,10),(7,11),(7,12)\}$$

$$R = \{(2,6),(2,9),(2,10),(2,11),(2,12),(5,11),(5,12)\}$$

Eg. $A = \{2,5,6,7\}$, $B = \{6,9,10,11,12\}$
Define the relation S by aSb if and only if $b = a+5$

$$S = \{(5,10),(6,11),(7,12)\}$$

<u>Properties of Relations:</u>

<u>Reflexive</u>: A relation $R: A \to A$ (this is functional notation, R is the name of the relation, A is the domain and B the range. Note that a function is a type of

relation ie every function is a relation but not vice-versa).
is reflexive if for every $a \in A$, aRa.

Eg. Define $R: \mathbb{N} \longrightarrow \mathbb{N}$ by xRy if $x \geq y$.

$$\mathbb{N} \times \mathbb{N} = \{ (1,1), (1,2), (1,3), \cdots \\ (2,1), (2,2), (2,3), \cdots \\ \vdots \quad \vdots \quad \vdots \quad \}$$

xRy if and only if $x \geq y$ so

$$R = \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3), \cdots \}$$

This relation is reflexive since xRx because $x \geq x$ for every $x \in \mathbb{N}$.

Symmetric: A relation $R: A \longrightarrow A$ is symmetric if aRb implies bRa.

Eg. The previous example isn't symmetric because for eg $2R1$ but $1 \not{R} 2$. ($\not{R}$ means "not related").