Important Note:

Matrix mult. isn't commutative, ie in general

$$AB \neq BA$$

Eg. $A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1(-1)+2(0) & 1(1)+2(2) \\ -1(-1)+0(0) & -1(1)+0(2) \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 5 \\ 1 & -1 \end{pmatrix}$$

$$BA = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1(1)+1(-1) & -1(2)+1(0) \\ 0(1)+2(-1) & 0(2)+2(0) \end{pmatrix}$$

$$= \begin{pmatrix} -2 & -2 \\ -2 & 0 \end{pmatrix}$$

ie $AB \neq BA$.

Note: For B to be an inverse of A we require that both

$$BA = \mathbb{I} \quad \text{and} \quad AB = \mathbb{I}$$

are true. Since $\mathbb{I}$ is a square matrix say $m \times m$,

$$BA = \mathbb{I}$$

then B must be $m \times n$ and A must be $n \times m$. But if $AB = \mathbb{I}$ then since A is $n \times m$ and B is $m \times n$ in order to get $\mathbb{I}$, an $m \times m$ matrix, we must have $m = n$.

Hence, only square matrices have inverses.

Some Properties of Matrix Algebra

A, B are matrices, c a scalar

1. $A+B = B+A$
2. $A+0 = A = 0+A$
3. $A-A = 0$
4. $A+(B+C) = (A+B)+C$
5. $A(B+C) = AB+AC$
6. $(A+B)C = AC+BC$
7. $c(A+B) = cA+cB$
8. $-A = (-1)A$
9. $A0 = 0A = 0$
10. $A(BC) = (AB)C$
11. $(AB)^{-1} = B^{-1}A^{-1}$
12. $(AB)^T = B^TA^T$

Transpose of a Matrix:

The transpose of a matrix A, denoted A^T is the matrix obtained by interchanging the rows and cols. of A. That is write the rows as cols. or vice-versa.

Eg. If $A = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$ then $A^T = \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$

Eg. $A = \begin{pmatrix} 1 & 3 & 6 \\ -1 & 4 & 5 \end{pmatrix}$ then $A^T = \begin{pmatrix} 1 & -1 \\ 3 & 4 \\ 6 & 5 \end{pmatrix}$

Note: If A is $m \times n$ then A^T is $n \times m$.

<u>Solving a pair of simultaneous linear eqns:</u>

We saw, if

$$ax + by = e$$
$$cx + dy = f$$

is such a pair of eqns. then in matrix form we get

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e \\ f \end{pmatrix}$$
$$A \quad \underset{\sim}{X} = C$$

ie

$$A\underset{\sim}{X} = C$$

If A^{-1} exists and we mult. across by it

$$A^{-1}A\underset{\sim}{X} = A^{-1}C$$
$$\mathbb{1}\underset{\sim}{X} = A^{-1}C$$
$$\underset{\sim}{X} = A^{-1}C$$

To solve, find A^{-1}, mult. by C and read off the soln.

For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A^{-1} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Eg. Solve

$$3x - 2y = 1$$
$$x + y = 6$$

In matrix form

$$\underset{A}{\begin{pmatrix} 3_{a} & -2_{b} \\ 1_{c} & 1_{d} \end{pmatrix}} \underset{X}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{C}{\begin{pmatrix} 1 \\ 6 \end{pmatrix}}$$

$\det(A) = ad - bc = 3(1) - (-2)(1) = 3 + 2 = 5$

Not zero so A^{-1} exists. and so does a soln.

$$A^{-1} = \frac{1}{5}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$$

Now

$$X = A^{-1}C$$
$$= \frac{1}{5}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 6 \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 1(1)+2(6) \\ -1(1)+3(6) \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 13 \\ 17 \end{pmatrix}$$

so $x = 13/5$, $y = 17/5$. Check by substitution in the original eqns.

Transformations:

We'll only deal with the 2-d case. Every point in the plane can be represented by a pair of nos., ie it's coordinates, (x,y). We can represent each such point by a col. matrix or vector ie

$$X = \begin{pmatrix} x \\ y \end{pmatrix}$$

For eg. $(2,3)$ would be $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

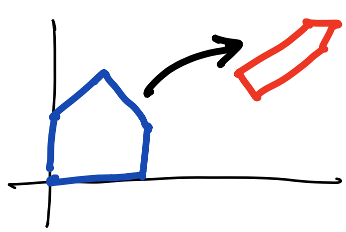
If we have a matrix $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then we can form

$$MX = \underset{2\times 2}{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} x \\ y \end{pmatrix}} = \underset{2\times 1}{\begin{pmatrix} ax+by \\ cx+dy \end{pmatrix}}$$

ie MX produces a col. matrix which can be thought of as another point in the plane. We think of M as acting on one pt. to produce another, ie a <u>transformation</u>.

I'll use the following notation, P and Q will be points with coords. (coordinates) (p_x, p_y) and (q_x, q_y). A matrix trans. (transformation) will send the pt. P to the pt. Q.

A transformation can be applied to a set of points.



trans. of a set of points.