

1. Let  $A = \{2, 4, 6, 7\}$ ,  $B = \{6, 9, 10, 11, 12\}$  and define the relations

- (i).  $R : A \rightarrow B$  with  $aRb$  if and only if  $2a < b$
- (ii).  $S : A \rightarrow B$  with  $aSb$  if and only if  $b = a + 5$

Answer the following:

- (a). List the element in  $R$
- (b). List the elements in  $S$

2. Let  $A = \{1, 2, 5, 7\}$ ,  $B = \{2, 6, 7, 9, 10, 12\}$  and define the relations

- (i).  $R : A \rightarrow B$  with  $aRb$  if and only if  $b$  is divisible by  $2a$
- (ii).  $S : A \rightarrow B$  with  $aSb$  if and only if  $a = b - 7$

Answer the following:

- (a). List the element in  $R$
- (b). List the elements in  $S$

3. Define the relation  $R : \mathbb{N} \rightarrow \mathbb{N}$  on  $\mathbb{N}$ , the set of all positive integers, by

$$R = \{(a, b) : ab \text{ is even}\}$$

Determine if  $R$  is

- (i). Reflexive
- (ii). Symmetric
- (iii). Transitive

Is  $R$  an equivalence?

4. Let  $A = \{1, 2, 3\}$  and define the relations  $R : A \rightarrow A$ ,  $S : A \rightarrow A$  and  $T : A \rightarrow A$  by

$$\begin{aligned} R &= \{(1, 1), (1, 2), (1, 3), (3, 3)\} \\ S &= \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\} \\ T &= \{(1, 1), (1, 2), (2, 2), (2, 3)\} \\ U &= A \times A \end{aligned}$$

Determine whether each of the relations is

- (a). Reflexive
- (b). Symmmetric
- (c). Transitive
- (d). Antisymmetric

For any relations that are equivalences write down the resulting partition of the set  $A$  into equivalence classes.

**5.** Give an example of a relation on  $A = \{1, 2, 3\}$  which is

- (a). Both symmetric and antisymmetric
- (b). Neither symmetric nor antisymmetric
- (c). Transitive

**6.** Let  $R$  be the relation on the set  $A = \{1, 2, 3, 4, 5, 6\}$  given by:

$$R = \{(1, 1), (1, 5), (2, 2), (2, 3), (2, 6), (3, 2), (3, 3), (3, 6), (4, 4), (5, 1), (5, 5), (6, 2), (6, 3), (6, 6)\}.$$

Determine whether  $R$  is

- (a). Reflexive
- (b). Symmetric
- (c). Transitive
- (d). Antisymmetric

If  $R$  is an equivalence then write down the equivalence classes of  $R$ .

**7.** Given the sets  $A = \{1, 2\}$  and  $B = \{a, b, c\}$  describe all possible functions from  $A$  to  $B$ .

**8.** Given the sets  $A = \{1, 2, 3, 4\}$  and  $B = \{4, 6, 9\}$  which of the following relations are functions? For those which are not functions state the reason.

- (a).  $\{(1, 4), (2, 6), (3, 6), (4, 4)\}$
- (b).  $\{(1, 4), (2, 6), (3, 4), (4, 6), (1, 6)\}$
- (c).  $\{(1, 6), (2, 6), (3, 4)\}$
- (d).  $\{(1, 6), (2, 4), (3, 4), (4, 4)\}$
- (e).  $\{(1, 4), (1, 6), (1, 9)\}$
- (f).  $\{(4, 6), (2, 4), (4, 9), (1, 4)\}$