Note: $A \cap B \subseteq A$ and $A \cap B \subseteq B$. Also

$$A \subseteq B \text{ if and only if } A \cap B = A.$$

Null Set: The null or empty set, denoted $\emptyset$ is the set which contains no elements. Also denoted $\{\}$. We define $\emptyset$ to be a subset of every other set.

Properties:

1. $\emptyset \cup \emptyset = \emptyset$
2. $\emptyset \cap \emptyset = \emptyset$
3. $A \cup \emptyset = A$
4. $A \cap \emptyset = \emptyset$

For completeness I'll list some properties using operations we haven't covered yet.

5. $\emptyset - \emptyset = \emptyset$
6. $A - A = \emptyset$
7. $A - \emptyset = A$
8. $\emptyset - A = \emptyset$

Disjoint Sets: Two sets are disjoint if their intersection is empty, ie A, B are disjoint if $A \cap B = \emptyset$.

A collection of sets $A_1, A_2, \cdots, A_n$ is pairwise disjoint if and only if

$$A_i \cap A_j = \emptyset \quad \text{for all } i \text{ and } j.$$

This means that $A_1 \cap A_2 = \emptyset$, $A_1 \cap A_3 = \emptyset$, $\cdots$, $A_1 \cap A_n = \emptyset$, $A_2 \cap A_3 = \emptyset$, $\cdots$ etc.

It can happen that $A_1 \cap A_2 \cap \cdots \cap A_n = \emptyset$ without the sets being pairwise disjoint. For eg. $A_1 \cap A_2 \cap A_3 = \emptyset$ does not imply A_1, A_2 and A_3 are pairwise disjoint. For eg, if

$$A_1 = \{1, 2\}, \quad A_2 = \{2, 3\}, \quad A_3 = \{3, 1\}$$

then $A_1 \cap A_2 \cap A_3 = \emptyset$. but they are not pairwise disjoint since $A_1 \cap A_2 = \{2\}$, $A_1 \cap A_3 = \{1\}$ etc.

<u>Set Partition</u>: Sometimes it's useful to split a set into a collection of disjoint subsets. Such a subdivision of a set is called a partition of that set.

If A is a set then $A_1, \cdots, A_n$ is a partition of A if

(1) $A_1, \cdots, A_n$ is a disjoint collection of subsets of A

(2) $A_1 \cup A_2 \cup \cdots \cup A_n = A$.

That is $A_1, \cdots, A_n$ are pairwise disjoint and every element in A is in some A_i.

Eg. If $A = \{1, 2, 3, 4, 5, 6\}$, then
$A_1 = \{1\}$, $A_2 = \{2, 5\}$, $A_3 = \{3, 4, 6\}$ is a partition of
A.

<u>Set Difference</u>: If A, B are sets, the set difference $A - B$ (some books use $A \setminus B$) is the subset of A of all the elements which are in A but not in B. Since every element (if any) in $A-B$ is in A, it follows that

$$A - B \subseteq A.$$

If we form $A - A$ we get the subset of A containing all the elements of A which are not in A, ie $A - A = \emptyset$.

Eg. $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6\}$ then

$$A - B = \{1, 3, 5\}, \text{ and } B - A = \emptyset$$

Note that $A - B \neq B - A$ in general.

<u>Complement of a Set</u>: If A, B are sets, the complement of A in B is

$$B - A$$

ie it's the portion that remains when you take away A's elements. With complements you've usually fixed some largest set of interest and complements are taken with respect to that set.

In probability theory we are always dealing with sets which are subsets of some clearly specified larger set, usually denoted Ω, the sample space. In this case we use a special notation for complements of a set A in Ω, namely A^c called the complement of A. Thus

$$A^c = \Omega - A.$$

Some authors use $\hat{A}$ instead of A^c but I won't.

Properties (assuming relative to a largest set Ω):

(1) $\Omega^c = \emptyset$

(2) $\emptyset^c = \Omega$

(3) $(A^c)^c = A$

(4) $A \cup A^c = \Omega$

(5) $A \cap A^c = \emptyset$

(6) $A \subseteq B$ then $B^c \subseteq A^c$

(7) $A - B = A \cap B^c$

(8) $(A - B)^c = A^c \cup B$

(9) $A^c - B^c = B - A$

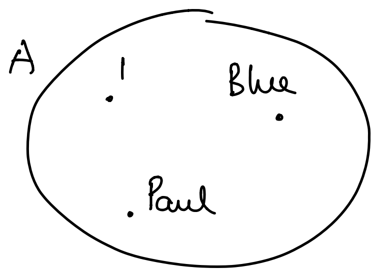
There are two important properties called De Morgan laws:

(10) $(A \cup B)^c = A^c \cap B^c$

(11) $(A \cap B)^c = A^c \cup B^c$

<u>Venn Diagrams</u>: Sets are represented by closed loops. Some or all of the elements can be shown in the loop.

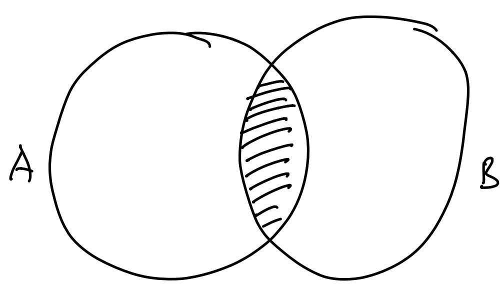
Eg. $A = \{1, \text{Paul}, \text{blue}\}$, then



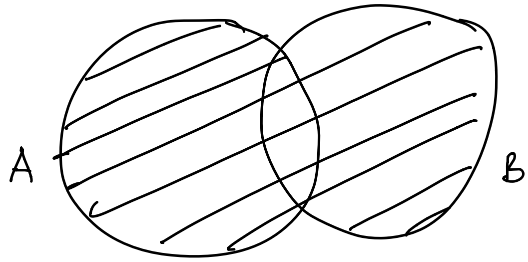
is a Venn diagram.

Most commonly, Venn diagrams are used to picture relationships between two or more sets, often by shading some portion of the diagram to illustrate a relationship of interest.

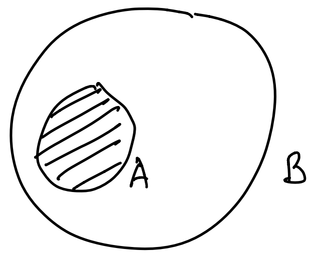
Eg. $A \cap B$



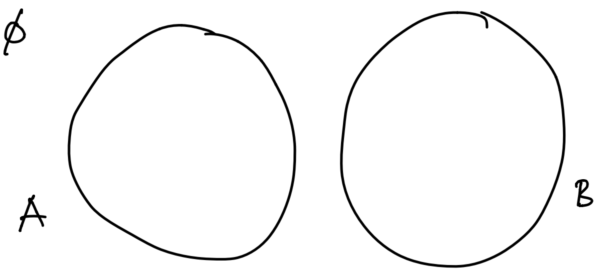
Eg. $A \cup B$



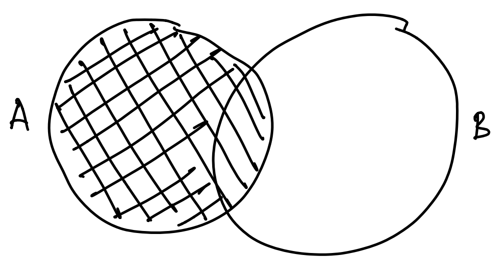
Eg. $A \subset B$



Eg. $A \cap B = \emptyset$

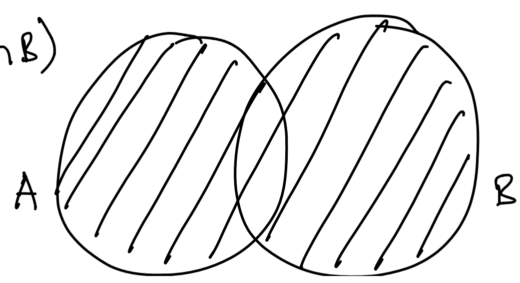


Eg. $A \cup (A - B) = A$

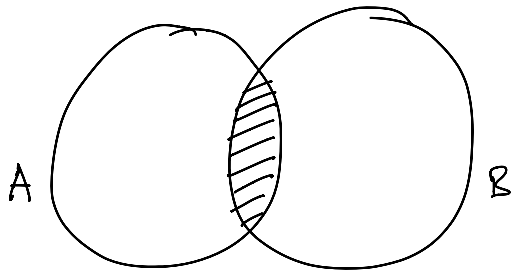


Eg. $(A \cup B) - (A \cap B)$

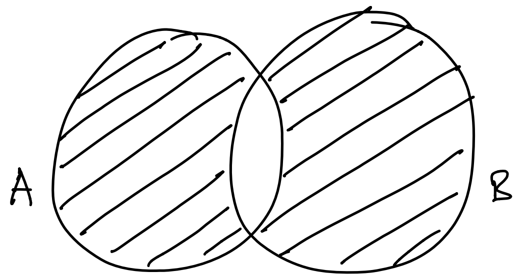
$A \cup B$:



$A \cap B$:



$(A \cup B) - (A \cap B)$:



More on complements – clarify.