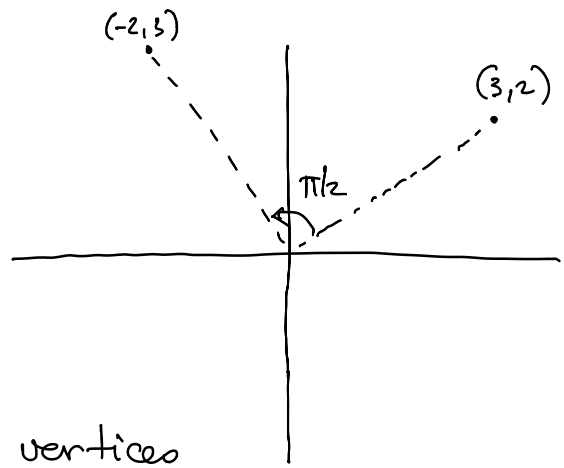
Eg. Find the image of the point with coords. $(3,2)$ under a ccw. rotation by $90°$ ie $\theta = \pi/2$.

The rotation matrix is

$$R_{\pi/2} = \begin{pmatrix} \cos(\pi/2) & -\sin(\pi/2) \\ \sin(\pi/2) & \cos(\pi/2) \end{pmatrix}$$
$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

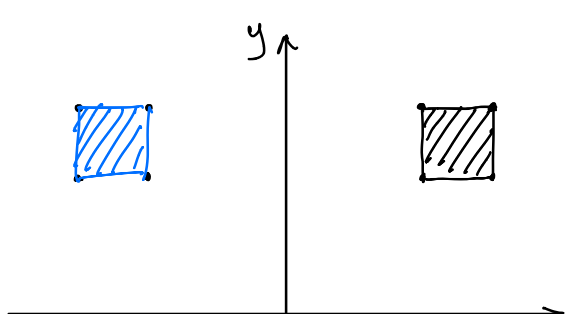
Acting on the pt. $(3,2)$ we get

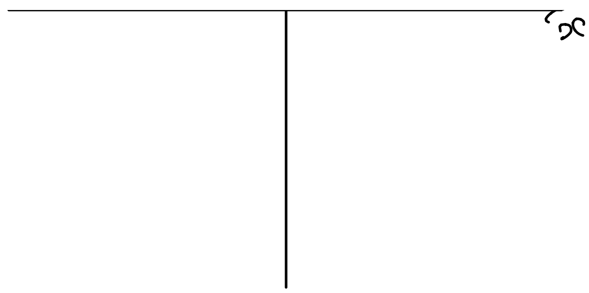
$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$




Exercise: Rotate the rectange with vertices $(2,2)$, $(3,2)$, $(3,3)$ and $(2,3)$ $90°$ ccw about the origin and sketch the original shape and it's image.

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 3 & 3 & 2 \\ 2 & 2 & 3 & 3 \end{pmatrix} = \begin{pmatrix} -2 & -2 & -3 & -3 \\ 2 & 3 & 3 & 2 \end{pmatrix}$$





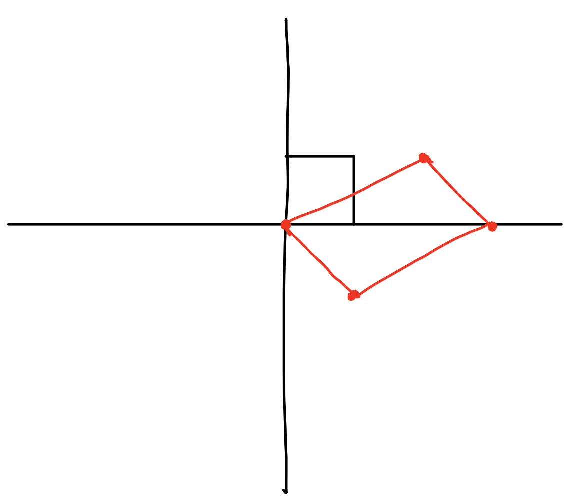

<u>Shearing</u>: A shearing transformation can be achieved using a matrix of the form

$$\begin{pmatrix} 1 & s \\ t & 1 \end{pmatrix}$$

ie
$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = \begin{pmatrix} 1 & s \\ t & 1 \end{pmatrix}\begin{pmatrix} p_x \\ p_y \end{pmatrix}$$

Eg. Apply the shear $\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$ to the unit square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 3 & 2 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$



<u>Translation:</u> A translation occurs when every point moves the same distance in the same direction. To achieve this we add a col. matrix representing the displacement ie the dist. and dir. of the translation.

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$$

Eg. Translate the unit square with vertices $(0,0)$, $(1,0)$ $(1,1)$ and $(0,1)$ using the matrix $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

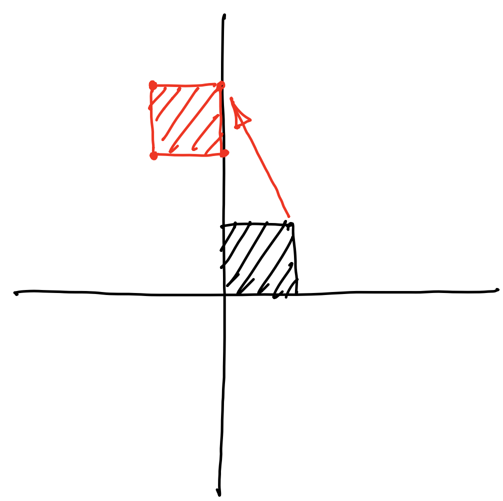
We get

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$



Exercise: A square has vertices $(1,1)$, $(2,1)$, $(2,2)$ and $(1,2)$. Find and sketch the image of the square if it is translated by $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and then rotated about the origin ccw. by an angle of $\pi/2$.

We can apply multiple transformations consequtively but the results will depend on the order of the operations. The transformations of scaling, rotation reflection and shearing are called linear transformation and are applied by matrix multiplication. If we include translation we get a type of transformation of the form

$$Q = LP + T$$

where $L$ is a linear trans. (transformation), $T$ is a translation and $Q$ and $P$ are col. matrices for the original pt. and it's image.

<u>Homogeneous Coordinates:</u>

An affine trans. involves matrix mult. and addition. Homogeneous coords. allow this to be combined in a single mult. A linear trans. has the form

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} p_x \\ p_y \end{pmatrix}$$

while for a trans.

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$$

In homogeneous coords the matrices for the points become
$$\begin{pmatrix} p_x \\ p_y \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} q_x \\ q_y \\ 1 \end{pmatrix}$$

and the affine trans. consisting of the Linear trans. followed by the translation becomes

$$\begin{pmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{pmatrix}$$

ie
$$\begin{pmatrix} q_x \\ q_y \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} ap_x + bp_y + e \\ cp_x + dp_y + f \\ 0 + 0 + 1 \end{pmatrix}$$

Without homogeneous coords.
$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} p_x \\ p_y \end{pmatrix} = \begin{pmatrix} ap_x + bp_y \\ cp_x + dp_y \end{pmatrix}$$
$$\begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} ap_x + bp_y \\ cp_x + dp_y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$$
$$= \begin{pmatrix} ap_x + bp_y + e \\ cp_x + dp_y + f \end{pmatrix}$$