Eg. Suppose A is the set of all students in this class, then Paul $\notin A$ but Darren $\in A$.

There doesn't have to be any relationship between the elements of a set. In fact the set itself creates a relationship between them - that of being in the same set.

Eg. $A = \{\text{cow}, 9, \sin x\}$ is a perfectly valid, if slightly odd set.

Notes: We don't allow duplicates in a set - all the elements must be distinguishable in some way. The order of the elements doesn't matter. Thus $\{9, \text{cow}, \sin x\}$ is the same set as above.

In maths we generally consider sets of items which are of the same type. For eg. nos., matrices, funcs. etc.

<u>Set Builder Notation</u>:

The 2nd. way to specify a set is to give a rule that unambiguously identifies every member of the set. This takes the form

$$\{x \mid P(x)\}$$

The braces indicate a set definition. The var. x stands for any object, $|$ means "such that" (sometimes $:$ is used instead of $|$), and $P(x)$ is a logical statement called a predicate which depends on the variable, and which has either a true or false value. Thus

$$A = \{ x \mid P(x) \}$$

means A is the set of all things for which $P(x)$ is true.

Eg. $A = \{ x \mid x \text{ is a positive integer} \}$

$P(x)$ in this case is "x is a positive integer". We can test any item using $P(x)$ to see if it's A.

$P(\text{Blue})$ = false since the colour blue isn't an integer.
$P(\text{Paul})$ = false therefore Paul $\notin A$.
$P(3)$ = true " $3 \in A$
$P(-1)$ = false " $-1 \notin A$
$P(\pi)$ = false " $\pi \notin A$

Sometimes we'll define sets in a rather loose manner, hopefully the interpretation is clear.

Eg. $A = \{1, 3, 5, \cdots\}$ where $\cdots$ (an ellipsis) means and so on. This set could also be specified as

$$A = \{x \mid x \text{ is an odd positive integer}\}$$

or

$$A = \{x \mid x > 0,\ x = 2n+1 \text{ for some whole no. } n\}$$

Eg. $\mathbb{Z} = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$ is just the set of integers.

<u>Equality</u>: Two sets A and B are <u>equal</u> if and only if they contain exactly the same elements. If they are equal we write $A = B$.

Another way to specify $A = B$ is that if $a \in A \Rightarrow a \in B$ and vice-versa.

<u>Subsets</u>: If a set A contains every element of a set B then we say <u>B is a subset of A</u> and write

$$B \subseteq A$$

That is $B \subseteq A$ if and only if $b \in B \Rightarrow b \in A$. If B is not a subset of A we write $B \not\subseteq A$. For this to be true B must contain at least one element not in A. If $B \subseteq A$ but $B \neq A$ then we write $B \subset A$, and we say B is a proper subset of A.

Equality can be recast in terms of subsets:

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Properties:

(1) $A \subseteq A$

(2) $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

(3) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

Note: It's not correct to write $A \in B$ if what we mean is $A \subseteq B$. For eg. if $A = \{a, b, c\}$ then we say $a \in A$ or $\{a\} \subset A$ and if $B = \{b\}$ then it's correct to say $B \subset A$ but not to say $B \in A$. It's also not correct to say $b \subset A$.

For the moment A, B and C will be sets.

Set Union: The union of A and B, denoted by $A \cup B$ is the set containing every element in A and every element in B.

Eg. $A = \{1, 2, 3\}$, $B = \{\text{blue}, 2, \pi\}$ then
$A \cup B = \{1, 2, 3, \text{blue}, \pi\}$

Properties:

(1) $A \cup A = A$

(2) $A \cup B = B \cup A$ (commutivity)

(3) $A \cup (B \cup C) = (A \cup B) \cup C$ (associativity)

(4) $A \subseteq A \cup B$, $B \subseteq A \cup B$

(5) $A \subseteq B$ if and only if $A \cup B = B$

Intersection: The intersection of A and B, denoted $A \cap B$ is the set of all element in both A and B. That is, the common elements of A and B.

Eg. $A = \{1, 2, 3\}$, $B = \{\text{blue}, 2, \pi\}$

$A \cap B = \{2\}$

Properties:

(1) $A \cap A = A$

(2) $A \cap B = B \cap A$

(3) $(A \cap B) \cap C = A \cap (B \cap C)$

Properties of unions & intersections

(1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ distributivity

(2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ "