Notes: A shearing matrix has the form

$$\begin{pmatrix} 1 & g \\ h & 1 \end{pmatrix}$$

where g and h are the shearing factors. For a shearing parallel to the x-axis, ie in the x-dir. only the transformation has this form

$$q_x = p_x + g\,p_y$$
$$q_y = p_y$$

In matrix form this is

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = \begin{pmatrix} 1 & g \\ 0 & 1 \end{pmatrix}\begin{pmatrix} p_x \\ p_y \end{pmatrix}$$

Similarly for a shearing in the y-dir. only

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ h & 1 \end{pmatrix}\begin{pmatrix} p_x \\ p_y \end{pmatrix}.$$

<u>Combinations</u>: You can apply multiple transformations but the order is important. You can combine these into a single matrix whose effect is the same as the individual transformations.

Eg. A rotation through an angle $\pi/6$ ccw. (ie $30°$) is followed by uniform scaling by a factor of 2. The rotation matrix is

$$R_{\pi/6} = \begin{pmatrix} \cos(\pi/6) & -\sin(\pi/6) \\ \sin(\pi/6) & \cos(\pi/6) \end{pmatrix}$$
$$= \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$$

The scaling matrix is
$$S = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

The combined result is their product, ie

$$SR_{\pi/6} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}$$

This matrix has the same effect as the two individual matrices.

Inverses and Transformations:

The transformations, rotation, reflection etc. are reversible and therefore we expect that their matrices have inverses and that the inverse matrix corresponds to the inverse transformation. For eg. a rotation ccw. through an angle θ about O is
$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

The inverse transformation is a clockwise rotation by an angle θ about O ie a rotation ccw through an angle $-\theta$. That is

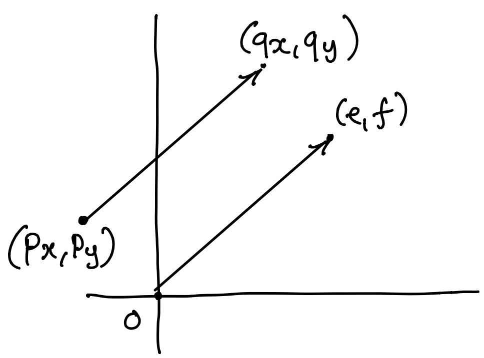
$$R_\theta^{-1} = R_{-\theta} = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

because $\cos(-\theta) = \cos\theta$ and $\sin(-\theta) = -\sin\theta$. As an exercise verify that $R_\theta R_{-\theta} = \mathbb{I}$. You'll need the trig. identity $\cos^2\theta + \sin^2\theta = 1$.

<u>Note on Translation</u>: To translate a pt. in the plane we do the following

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$$

We view a col. matrix like $\begin{pmatrix} e \\ f \end{pmatrix}$ as representing the coords. of a pt., in this case the pt. (e, f).



We also think of this as representing a displacement vector from the origin to the pt. (e, f).

Then translation using (e, f) simply moves any pt. by the same distance and in the same dir. We call $\begin{pmatrix} e \\ f \end{pmatrix}$

the translation vec.

Eg. The unit square at the origin has vertices with coords $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$. It is rotated clockwise about O through an angle of $\pi/6$. It is then reflected in y-axis. It is then scaled uniformly by a factor of 3 and finally it is translated using the vec. $(-1,2)$. Using homogeneous coords find and sketch the image of the unit square.

$$R_{-\pi/6} = \begin{pmatrix} \cos(\pi/6) & \sin(\pi/6) \\ -\sin(\pi/6) & \cos(\pi/6) \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix}$$

$$H_y = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

The combined transformation is

$$U H_y R_{-\pi/6} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix}$$

$$= \begin{pmatrix} -3\sqrt{3}/2 & -3/2 \\ -3/2 & 3\sqrt{3}/2 \end{pmatrix}$$

The homogeneous transformation matrix is

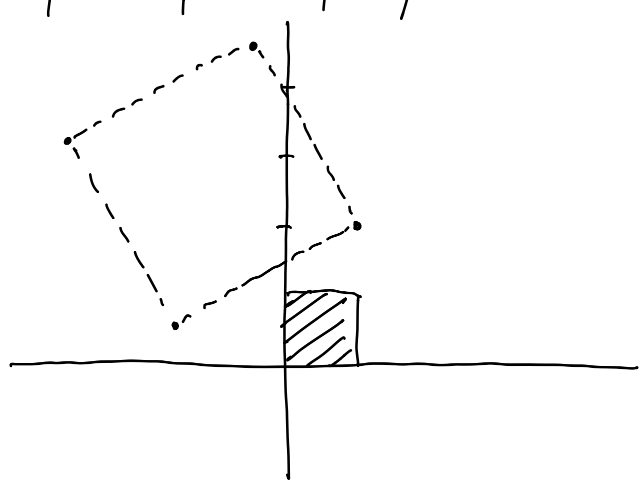
$$\begin{pmatrix} -3\sqrt{3}/2 & -3/2 & -1 \\ -3/2 & 3\sqrt{3}/2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

Transforming

$$\begin{pmatrix} -3\sqrt{3}/2 & -3/2 & 1 \\ -3/2 & 3\sqrt{3}/2 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & -3\sqrt{3}/2+1 & -3\sqrt{3}/2-3/2+1 & -3/2+1 \\ 2 & -3/2+2 & -3/2+3\sqrt{3}/2+2 & 3\sqrt{3}/2+2 \\ 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & -1.6 & -3.1 & -0.5 \\ 2 & 0.5 & 3.1 & 4.6 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$



Set Theory: A set is a well-defined collection of objects called it's members or elements. By well-defined we mean that given any object we can unambiguously say whether that object is in the set or not.

If A is a set and x is an element of the set x we write $x \in A$, otherwise we write $x \notin A$.

We can specify a set by
(1) listing all it's elements in braces, ie $\{ \}$
(2) specifying a rule that allows us to test whether any particular item is in the set or not.

Eg. $A = \{1, 2, 3\}$ is the set with members 1, 2 and 3.
Thus $1 \in A$ but $4 \notin A$ and Fido $\notin A$.