CMPU 2012 Mathematics 2
Problem Sheet 8: Probability 1

Q1. Determine the probability of the following events

  (a) An even number appears on a fair die.

  (b) A king appears when a single card is drawn from an ordinary deck of 52 cards.

  (c) At least one tail appears in the toss of three fair coins.

  (d) A white marble appears in drawing a single marble from a basket containing 4 white, 3 red and 5 blue marbles.

Q2. Manchester United are playing Huddersfield and an online betting company suggest that Manchester United are twice as likely to win than Huddersfield. What is the probability that Manchester United win?

Q3. Celine, Kate, Sophia and Niamh are in a singing competition. If Celine is twice as likely to win than Sophia and Celine is three times more likely than Niamh, and Kate is 3 times more likely to win than Niamh. Calculate each of their probabilities of winning.

Q4. Rafael Nadal is playing 4 tennis matches in a row. Based on his recent record the probability that he will win any one of these matches is 0.9.

  (a) Find the probability that he loses his first match.

  (b) Find the probability that he loses at least one match.

Q5. A card is drawn from an ordinary deck of 52 cards. Determine which of the following events are mutually exclusive and explain why mathematically.

(a) A red card is drawn.

(b) A black card is drawn.

(c) A Jack is drawn.

(d) A diamond is drawn.

(e) A ten of clubs is drawn.

(f) An even number is drawn (ie. 2,4,6,8,10).

Q6. Determine which of the following pairs of events are independent and explain why mathematically.

(a) A coin is tossed twice, the first toss is heads and the second toss is tails.

(b) A King of clubs is drawn from an ordinary deck of 52 cards, it is not replaced and then a red card is drawn.

Q7. A card is drawn from an ordinary deck of 52 cards, events A, B and C are defined below:
A: a black card is drawn.,
B: a black Jack is drawn,
C: a clubs is drawn.
Are any of these events independent? Say why mathematically.

Q8. Jack is looking to buy a new phone on ebay. There are 9 that he is considering. 5 of the phones are Samsung, 3 are black and 2 are both black and Samsung. If he selects a phone at random find the following:

(a) What is the probability that the phone is a Samsung?

(b) What is the probability that the phone is not black?

(c) What is the probability the phone is black but not a Samsung?

(d) What is the probability that the phone is a Samsung or black?

(e) If the phone he picks is a Samsung what is the probability that it is also black?

(f) If the phone he picks is black what is the probability that it is also a Samsung?

Q9. A card player chooses a card at random from an ordinary deck of 52 cards.

  (a) If a club is chosen, what is the probability that it is also an Ace?

  (b) If a red card is chosen, what is the probability that it is a heart?

  (c) If a red card is chosen, what is the probability that it is a spade?

Answers:

Q1. (a) $\dfrac{1}{2}$,   (b) $\dfrac{1}{13}$,   (c) $\dfrac{7}{8}$,   (d) $\dfrac{1}{3}$

Q2. $\dfrac{2}{3}$

Q3. P(Celine Wins) = P(Kate Wins) = $\dfrac{6}{17}$, P(Niamh Wins) = $\dfrac{2}{17}$, P(Sophia Wins) = $\dfrac{3}{17}$.

Q4. (a) 0.1,   (b) 0.3439.

Q5. Events whose intersection is an empty set are independent, ie. $A \cap B = \{\}$. These are: (a) and (b),   (a) and (e),   (b) and (d),   (c) and (e),   (c) and (f),   (d) and (e).

Q6. (a) Independent since $P(A \cap B) = P(A)P(B) = \dfrac{1}{4}$ (where A is heads first and B is tails second).

(b) Not Independent since $P(B|A) = \dfrac{13}{51}$ and $P(B) = \dfrac{13}{52}$, so $P(B|A) \neq P(B)$ (where A is drawing a King of clubs first, B is drawing red card second.)

Q7. (a) Not Independent since $P(black\,Jack|black) = \dfrac{1}{13}$ and $P(black\,Jack) = \dfrac{1}{26}$ so $P(black\,Jack|black) \neq P(black\,Jack)$.

(b) Not Independent since $P(clubs|black) = \dfrac{1}{2}$ and $P(clubs) = \dfrac{1}{4}$ so $P(clubs|black) \neq P(clubs)$.

Q8. (a) $\dfrac{5}{9}$ (b) $\dfrac{2}{3}$ (c) $\dfrac{1}{9}$ (d) $\dfrac{2}{3}$ (e) $\dfrac{2}{5}$ (f) $\dfrac{2}{3}$.

Q9. (a) $\dfrac{1}{13}$ (b) $\dfrac{1}{2}$ (c) 0.