

DUBLIN INSTITUTE OF TECHNOLOGY
KEVIN STREET, DUBLIN 8.

BSc. (Honours) Degree in Computer Science

Year 2

SEMESTER 1 EXAMINATIONS 2013/14

MATHEMATICS 2

Monday 6th January 2014
1.00pm – 3.00pm

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Mr. D. Tracey

Question 1 is **compulsory**.
Answer question 1 **and** two of the other three questions.
All your work must be shown.

Mathematical Tables are available.

1.

- (a) Consider the experiment of flipping a fair coin three times. The number of tails that appear is noted as a discrete random variable:

X : number of tails that appear in 3 flips of a fair coin.

- i. Summarise the probability distribution of X in a table, listing the possible values beside the probability of that value.
- ii. What is the expected value of the random variable X , $E[X]$?

[8 marks]

- (b) Given the following predicates: $B(x) = "x \text{ is a boy}"$, $G(x) = "x \text{ is a girl}"$ and $S(x) = "x \text{ is a student}"$. Write the following statements symbolically:

- i. Every student is either a boy or a girl
- ii. Some students are boys and some are girls
- iii. A student that is not a boy, is a girl

[8 marks]

- (c) Use Euclid's algorithm to find $\gcd(40902, 24140)$.

[8 marks]

- (d) Compute the following numbers without using a calculator. Show all your work:

- i. $9^{40} \bmod 10$
- ii. $4^{20} \bmod 9$
- iii. $2^{21} \bmod 7$

[8 marks]

- (e) Prove by mathematical *induction* that the sum of the first n terms in the series below is $\frac{5n(n+1)}{2}$. That is, show that $P(1)$ is true, and $P(n) \rightarrow P(n+1)$.

$$P(n) = "5 + 10 + 15 + \dots + (5n) = \frac{5n(n+1)}{2}"$$

[8 marks]

2.

- (a) Define *mathematically* (using formulas) what is meant by the following terms;

- i. Mutually exclusive events
- ii. Independent events

[4 marks]

- (b) Three horses A , B and C race. A is twice as likely to win as B , and B is twice as likely to win as C . Find their probabilities of winning denoted $P(A)$, $P(B)$ and $P(C)$.

[6 marks]

- (c) Suppose E is the event that a randomly generated bit string of length 4 begins with a 1, and F is the event that a randomly generated bit string contains an even number of 0's. Are E and F independent?

[7 marks]

- (d) Whenever horses A, B and C race together, their respective probabilities of winning are 0.25, 0.55 and 0.2. They race three times.

- i. Compute the probability that horse B wins all three races.
- ii. Compute the probability that horse C wins the first 2 races but loses the third race.
- iii. Compute the probability that A, B and C each win one race.

[13 marks]

3.

- (a) Find all integer solutions of the following Diophantine equation

$$35x + 49y = 427$$

[10 marks]

- (b) Show that the modular inverse mod 5 of

$$E = \begin{pmatrix} 2 & 1 \\ 3 & 3 \end{pmatrix} \text{ is } D = \begin{pmatrix} 1 & 3 \\ 4 & 4 \end{pmatrix}.$$

Hence show how to encrypt the string "CAAB" using E as the encryption matrix, and find the encrypted string. Assume letters A to Z are represented by 1 to 26, and '*' represents 0.

[12 marks]

- (c) Find the general solutions of the following congruence equations:

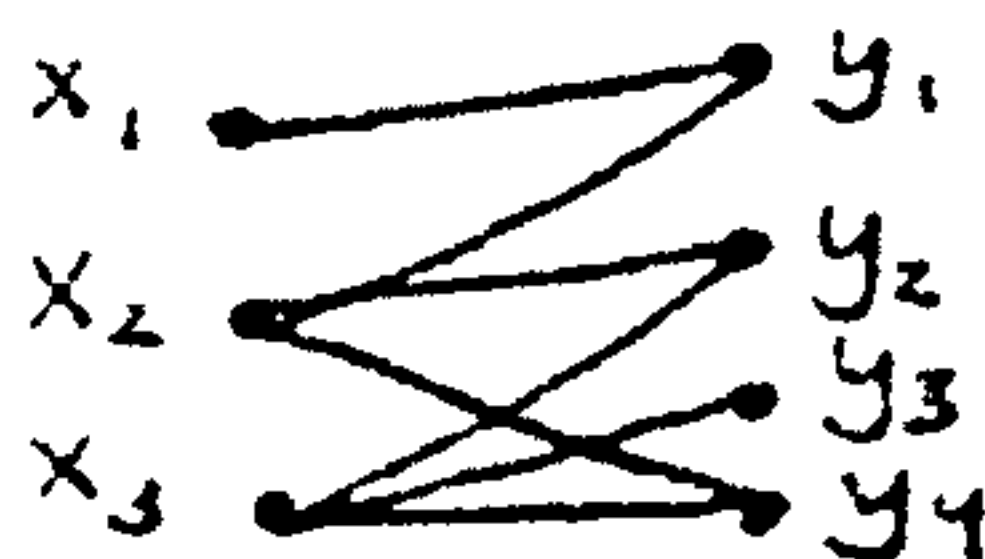
- i. $7x \equiv 4 \pmod{9}$
- ii. $x \equiv 4 \pmod{5}$ and $x \equiv 3 \pmod{7}$

[8 marks]

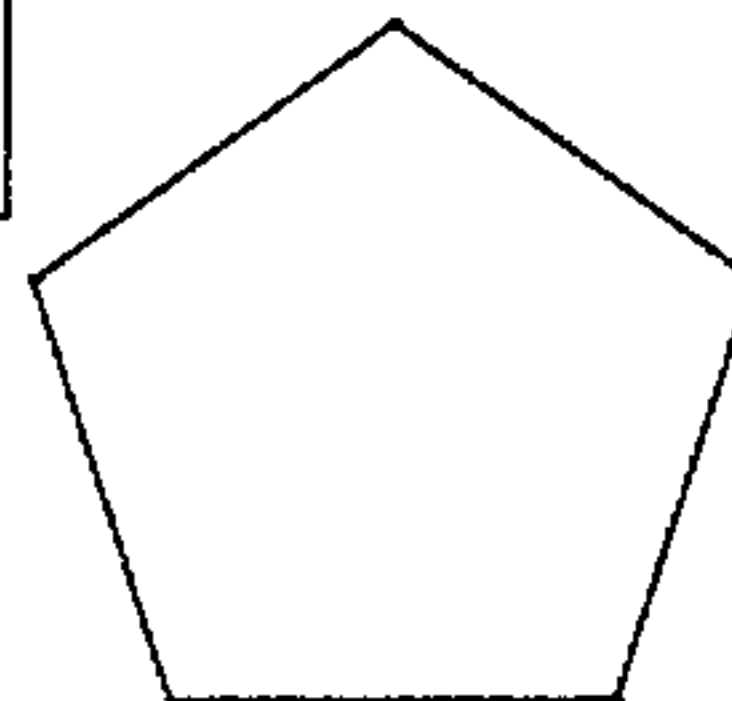
4.

- (a) Define clearly the term "bi-partite graph". For each of the graphs below, determine if it is bi-partite or not. If it is, then draw it in standard form or explain why it is not.

G_1



G_2



[8 marks]

- (b) Define the following terms in relation to graph theory:

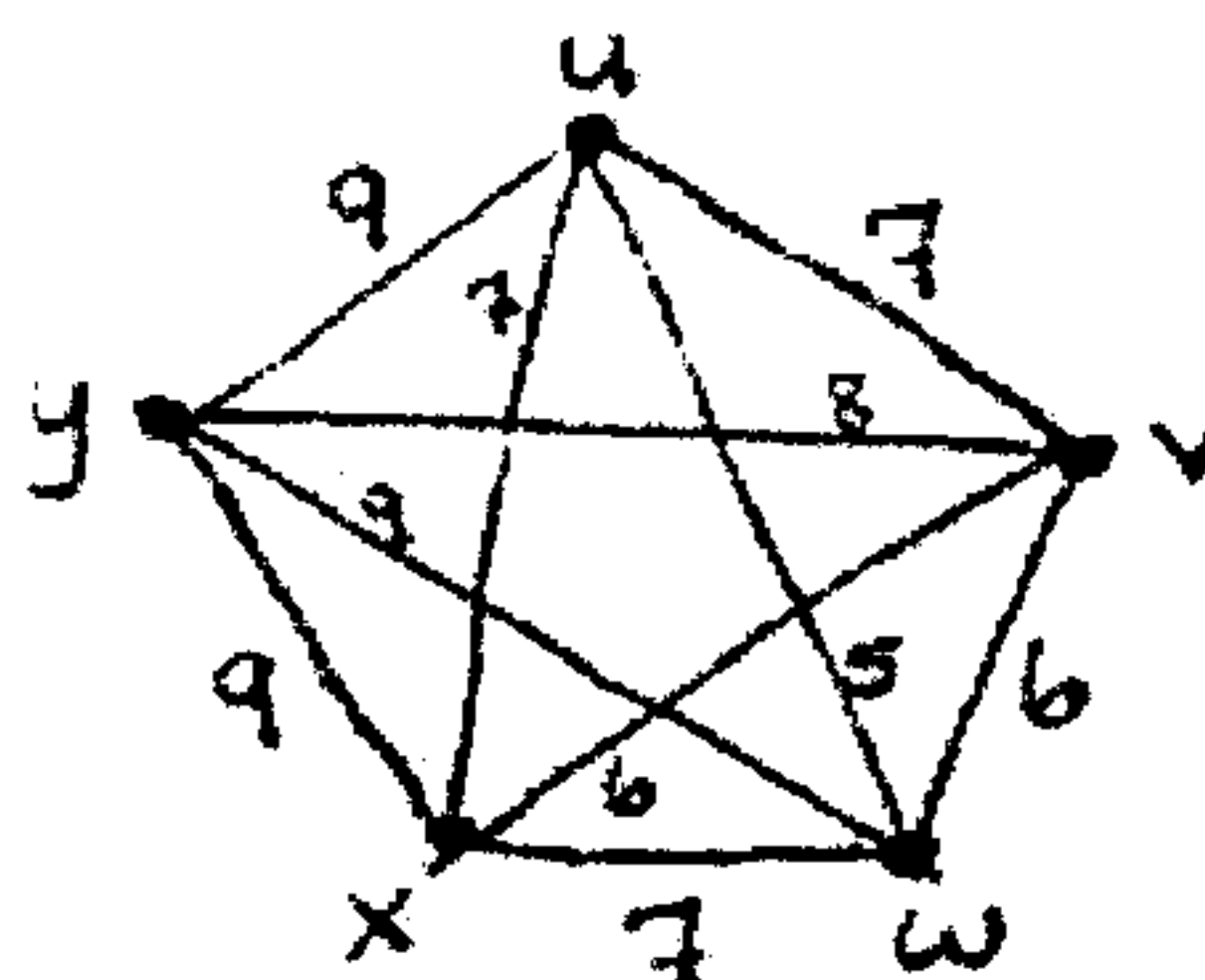
- i. The degree of a vertex
- ii. A closed walk
- iii. A minimum spanning tree

[6 marks]

- (c) Write down clearly the steps of Kruskal's Algorithm for constructing a minimum weight spanning tree for a graph G . Compute a *spanning tree* for the graph below, showing clearly each step.

[10 marks]

G_1



- (d) How many edges have the following graphs and give reasons for your answers
- K_8
 - C_{22}
 - $K_{8,12}$

[6 marks]