Problem Sheet 9 Solutions

Q1

$$E(X) = \frac{1}{5}(10) + \frac{1}{5}(20) + \frac{1}{5}(50) + \frac{1}{5}(500)$$

$$+\frac{1}{5}(5000) = €1116$$

Q. 2 (a)

$$\text{Horse } 7 = 2(0\cdot05) = 0\cdot1$$

$$\text{Horse } 8 = 1 - (0\cdot1 + 0\cdot05 + 0\cdot15 + 0\cdot15$$
$$+ 0\cdot2 + 0\cdot1 + 0\cdot1)$$

$$= 0\cdot15.$$

(b) $E(X) = 100(0\cdot1) + 20(0\cdot05) + 100(0\cdot15)$

$$+ 20(0\cdot15) + 100(0\cdot2) + 0(0\cdot1)$$

$$+100(0\cdot1) + 0(0\cdot15) = €59$$

Q. 3 (a) $10 \times 9 \times 8 \times 7 \times 6 \times 5 = 151200$

(b) $4 \times 9 \times 8 \times 7 \times 6 \times 5 = 60480$

(c) $P = \dfrac{60480}{151200} = \dfrac{2}{5}$

Q.4 Total number of possible number plates

$= 26 \times 25 \times 24 \times 10 \times 9 \times 8$

$= 11232000$

Number of plates beginning with "SAM"

$= 1 \times 1 \times 1 \times 10 \times 9 \times 8$

$= 720$

$\Rightarrow$ P(getting a plate beginning with "SAM") =

$$\frac{720}{11232000} = \frac{1}{15600}$$

Q.5. (a) $\binom{22}{11} = 705432$

(b) $\binom{8}{5} \times \binom{14}{6} = 168168$

(c) $\binom{14}{9}\binom{8}{2} + \binom{14}{10}\binom{8}{1} + \binom{14}{11} = 64428$

Q5 (d) P(at least 9 men) $= \frac{64428}{705432}$

Q6. Total number of combinations $= \binom{10}{3} = 120$

No. of combinations with a fairy, a dragon and a goblin $= \binom{3}{1}\binom{4}{1}\binom{1}{1}$

$= 12$

P(fairy, dragon, goblin) $= \frac{12}{120} = \frac{1}{10} = 0.1$

Q.7 (a) $\binom{12}{5}\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^7 = 0.05315$

(b) $\binom{12}{10}\left(\frac{1}{5}\right)^{10}\left(\frac{4}{5}\right)^2 = 4.325\times10^{-6}$

Q.8 (a) P(wins all matches)

$$= \binom{7}{7}(9/10)^7(1/10)^{7-7} = 0.48$$

(b) P(looses at least one match)

= 1 - P(wins all matches)

= 1 - 0.48 = 0.52

(c) $\binom{7}{6}\left(\frac{9}{10}\right)^6\left(\frac{1}{10}\right)^1 = 0.372$

Q.9 (a) $\binom{5}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 = \frac{40}{243}$

(b) P(steal at least once) =

1 - P(don't steal anything) =

$$1 - \binom{5}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{5-0} = \cancel{\frac{32}{243}}\ \frac{211}{243}$$

(c)

$$P(0) = \binom{5}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^5 = \frac{32}{243}$$

$$P(1) = \binom{5}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^4 = \frac{80}{243}$$

$$P(2) = \binom{5}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3 = \frac{80}{243}$$

$$P(3) = \binom{5}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 = \frac{40}{243}$$

$$P(4) = \binom{5}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 = \frac{10}{243}$$

$$P(5) = \binom{5}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^0 = \frac{1}{243}$$

$$E(X) = 0\,P(0) + 100\,P(1) + 200\,P(2) + 300\,P(3) + 400\,P(4) + 500\,P(5)$$

$$= \frac{40500}{243} = 166.67$$