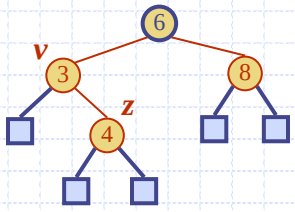
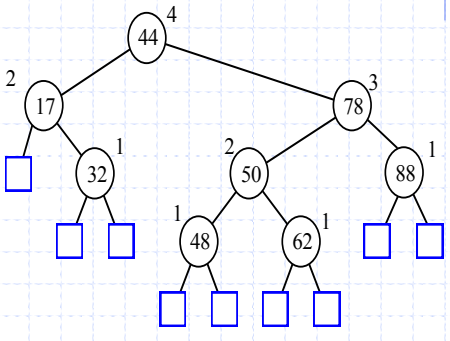
AVL Trees



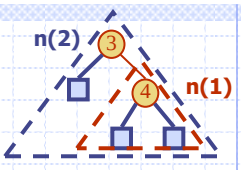
AVL Tree Definition

- **AVL trees are balanced.**
- An AVL Tree is a ***binary search tree*** such that for every internal node v of T, the *heights of the children of v can differ by at most 1.*



An example of an AVL tree where the heights are shown next to the nodes:

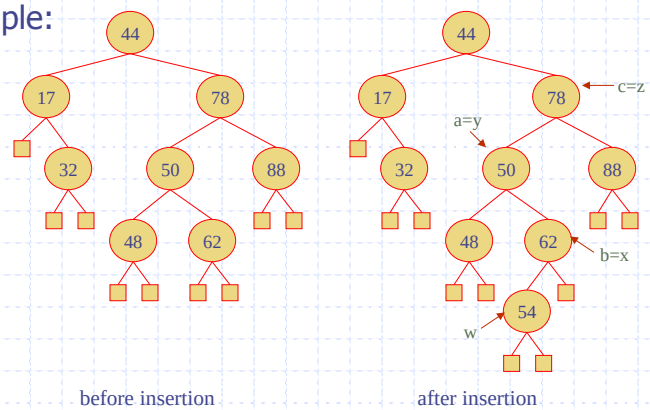
Height of an AVL Tree



- **Fact**: The ***height*** of an AVL tree storing n keys is O(log n).
- **Proof**: We won't go into it.

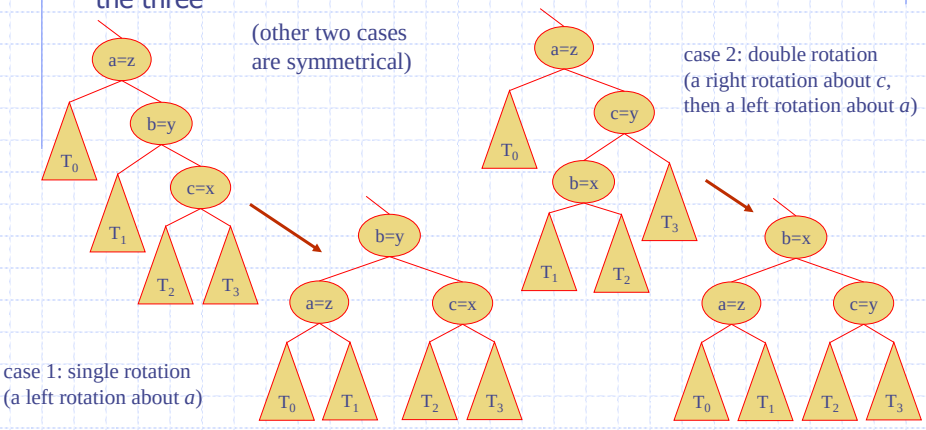
Insertion in an AVL Tree

- Insertion is as in a binary search tree
- Always done by expanding an external node.
- Example:

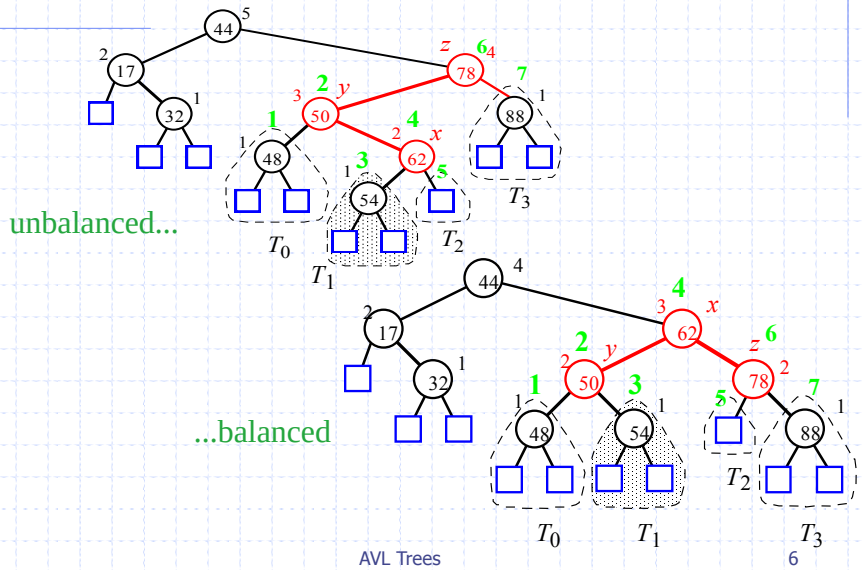


Trinode Restructuring

- let (*a*,*b*,*c*) be an inorder listing of *x*, *y*, *z*
- perform the rotations needed to make *b* the topmost node of the three

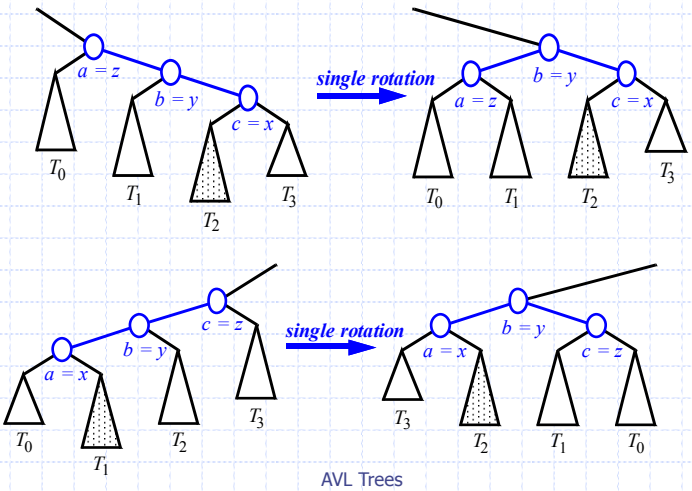


Insertion Example, continued



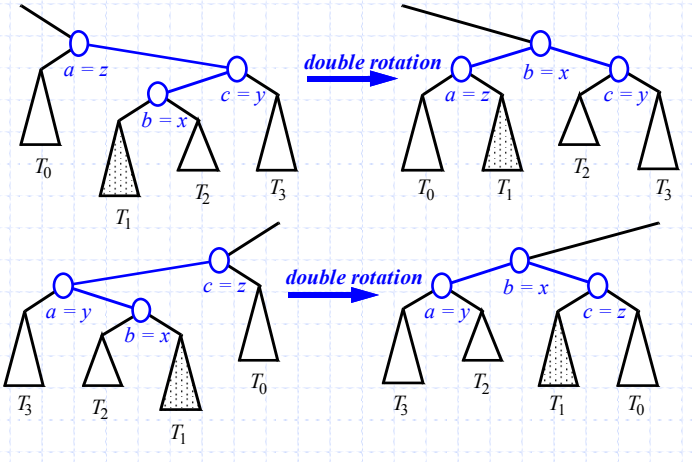
Restructuring (as Single Rotations)

- Single Rotations:



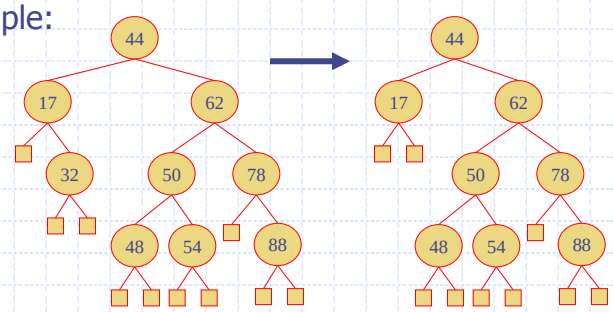
Restructuring (as Double Rotations)

- double rotations:



Removal in an AVL Tree

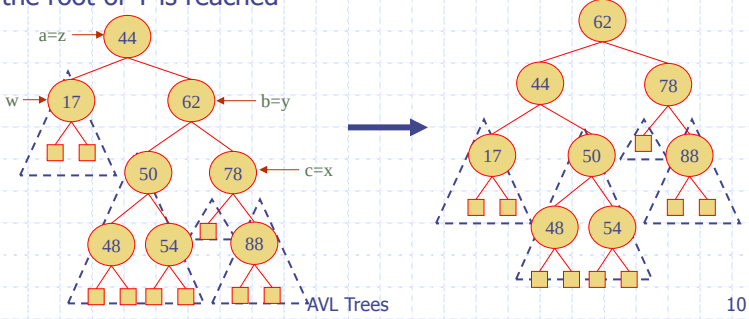
- Removal begins as in a binary search tree, which means the node removed will become an empty external node. Its parent, w, may cause an imbalance.
- Example:



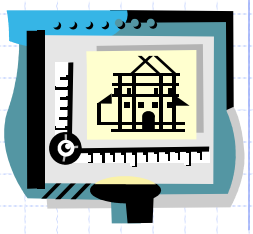
before deletion of 32 after deletion

Rebalancing after a Removal

- Let *z* be the first unbalanced node encountered while travelling up the tree from w. Also, let y be the child of z with the larger height, and let x be the child of y with the larger height.
- We perform restructure(x) to restore balance at z.
- As this restructuring may upset the balance of another node higher in the tree, we must continue checking for balance until the root of T is reached



Running Times for AVL Trees



- a single restructure is O(1)
  - using a linked-structure binary tree
- find is O(log n)
  - height of tree is O(log n), no restructures needed
- insert is O(log n)
  - initial find is O(log n)
  - Restructuring up the tree, maintaining heights is O(log n)
- remove is O(log n)
  - initial find is O(log n)
  - Restructuring up the tree, maintaining heights is O(log n)