CMPU 2012 Mathematics 2
Problem Sheet 10: Graph Theory

Q1. For the graphs $G_1, G_2, G_3, G_4, G_5, G_6$ shown below, state which of the following is:

(i) A connected graph.

(ii) A disconnected graph.

(iii) A complete graph.

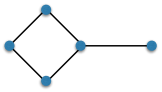
(iv) A cycle.

(v) A tree.

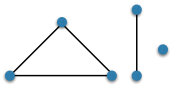
(vi) $\overline{G_4}$, the complement of G_4.

Note: there can be multiple answers for each part.

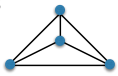
G_1



G_2

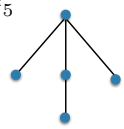


G_3



G_4

G_5



G_6

Q2. For the graphs G_1, G_2, G_3 and G_4 shown below state which of the following are:

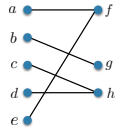
(i) A complete graph.

(ii) A bipartite graph.

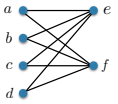
(iii) A complete bipartite graph.

For those that are bipartite or complete bipartite, illustrate the partition of the graph.

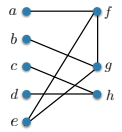
G_1



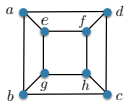
G_2



G_3



G_4



Q3. Determine how many edges each of the following graphs have, giving reasons for your answers:

(i) C_4

(ii) K_3

(iii) $K_{2,3}$

(iv) C_7

(v) K_{20}

(vi) $K_{12,13}$

Q4. For each of the following graphs answer the questions below:

(i) Is it bipartite?

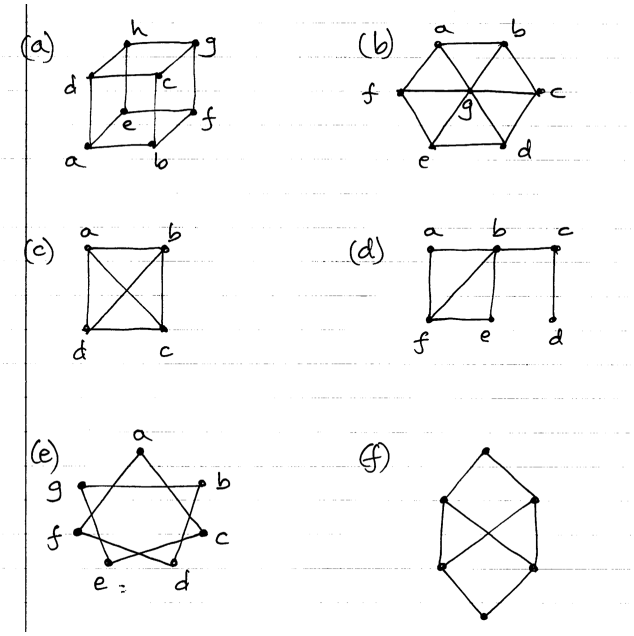
(ii) If it is bipartite, illustrate the partition of the graph.

(iii) Is it complete?

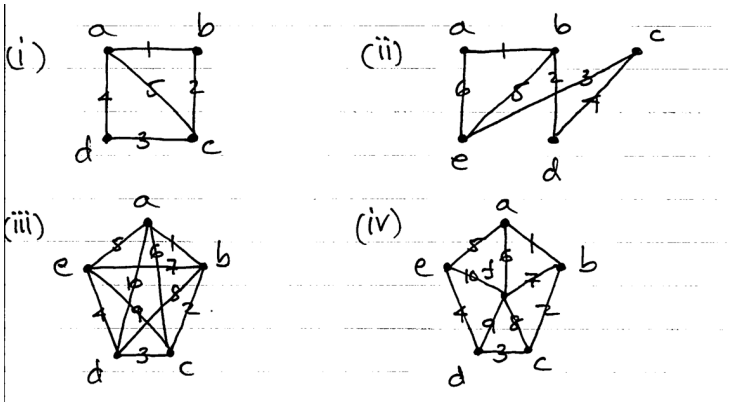
(iv) Is it complete bipartite?

(v) Does it have an Euler cycle?

(vi) Does it have an Euler path?



Q5. For each of the following graphs construct the adjacency and incidence matrices. Ignore the numbers (weights) on the edges of these graphs.



Q6. For each of the weighted graphs shown below use Kruskal's algorithm to find its minimal spanning tree, being sure to clearly show the steps involved. Illustrate the resulting spanning tree and calculate its weight.