W228/201

DUBLIN INSTITUTE OF TECHNOLOGY
KEVIN STREET, DUBLIN 8

CMPU 2012

B. Sc. (Honours) Degree in Computer Science Year 2

Semester 1 Examinations 2014/15

Mathematics 2

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9.30 - 11.30 am

Question 1 is **compulsory**
Answer Question 1 **and** two of the other three questions.
All your work must be shown.

Mathematical Tables are available.

1. (a) Find the residue, modulo 391, of 3^{31} and hence calculate the residue, modulo 391, of $501(3^{31})$.

(8 marks)

(b) Using the Chinese Remainder Theorem solve the simultaneous congruence equations:

$$x = 1(\text{mod } 5)$$
$$x = 2(\text{mod } 6)$$
$$x = 3(\text{mod } 7).$$

(8 marks)

(c) Assuming the domain of discourse is the set of integers $\mathbb{Z}$, define suitable predicates and write the following statements symbolically using logical notation:

i. No integer which is even is also odd.
ii. Some odd integers are divisible by 3.
iii. Every even integer is divisible by 2.
iv. Some integers which are divisible by 3 are even and some are odd.
v. Some integers are even and some integers are odd.
vi. Zero is even.
vii. Every integer is either even or odd.
viii. Nine is odd and divisible by 3.

(8 marks)

(d) In a class of 90 students, 50 are taking mathematics, 30 are taking computer science and 20 are taking both. If a student is selected at random from the the class then answer the following giving each answer as a fraction:

i. What is the probability the student is studying mathematics?
ii. What is the probability the student is not studying computer science?
iii. What is the probability the student is studying computer science but not mathematics?
iv. What is the probability the student is studying mathematics or computer science?
v. If the student is studying mathematics what is the probability that they are also studying computer science?

(8 marks)

(e) Using proof by induction prove the following results

i. $1+3+5+\cdots+(2n-1)=n^2, \forall n \in \mathbb{Z}_+$

ii. $3|(n^3-n), \forall n \in \mathbb{Z}_+$

(8 marks)

2. (a) Give the mathematical definition of the term **independent events**.

(2 marks)

(b) Two fair dice are thrown. Using the reduced sample space, find the probability that the sum of the two dice is 8 or greater if

i. The first die is a 4

ii. At least one of the dice is a 4

(4 marks)

(c) A random experiment consists of rolling two fair dice once. Define the events

A : the numbers rolled are not the same

B : the sum of the two dice is 9

C : the sum of the two dice is 6

D : both numbers rolled are even

E : one number rolled is even and the other is odd

and calculate $P(B|A), P(C|A), P(D|A)$ and $P(E|A)$ giving your answer as a fraction in each case. Is event B independent of A?

(14 marks)

(d) A competitor in the Olympic Rifle Shooting event takes five shots at a target. They have a one in four chance of scoring a bullseye on any one shot. Assuming that each shot is an independent Bernoulli trial with a bullseye denoting success, calculate the following:

i. The probability that the competitor scores exactly one bullseye.

ii. The probability that the competitor scores exactly two bullseyes.

iii. The probability that the competitor scores no bullseyes.

iv. The probability that the competitor scores at least three bullseyes.

v. The expected value for all five shots.

In each case give your answer as a fraction.

(10 marks)

3. (a) Using the extended Euclidean algorithm, find the general solution of the linear Diophantine equation

$$273x - 391y = 27.$$

and hence determine the inverse of 273 modulo 391.

(15 marks)

(b) The ciphertext

EHQFPS

was encrypted by means of a digraph cipher, using the matrix

$$A = \begin{pmatrix} 2 & 3 \\ 7 & 8 \end{pmatrix}$$

modulo 26, where $A = 0, B = 1, \ldots, Z = 25$. Find the inverse of A modulo 26 and hence retrieve the message plaintext.

(15 marks)

4. (a) Define the terms

i. Graph

ii. Connected graph

iii. Euler cycle

(6 marks)

(b) Construct the adjacency matrix for the graph G shown in Figure 1.

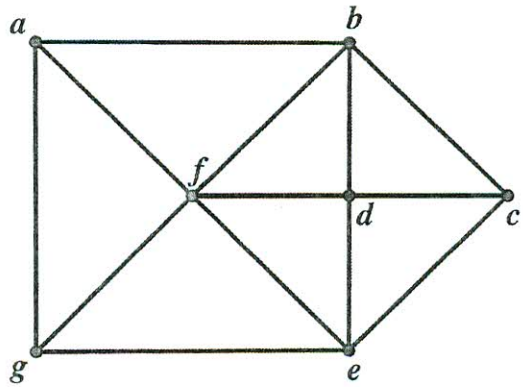

Figure 1: The graph G.

(4 marks)

(c) For the graph G shown in Figure 1 (see previous page), answer each of the following questions. Be careful to justify your answers.

i. Is G connected?

ii. Is G bipartite?

iii. Is G complete?

iv. Does G have an Euler cycle?

(10 marks)

(d) Write down the steps of Kruskal's algorithm for constructing a minimal weight spanning tree for a graph and use it to construct a minimal spanning tree for the weighted graph H shown in Figure 2. What is the weight of this minimal spanning tree?

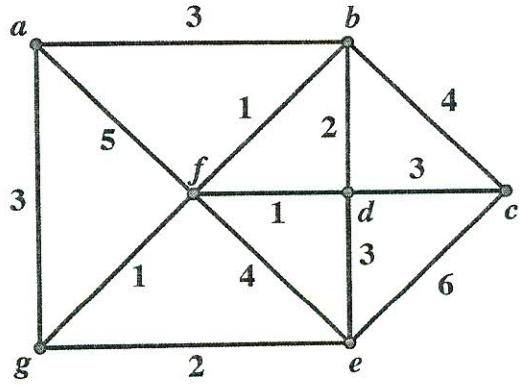

Figure 2: The weighted graph H.

(10 marks)