Solutions PS10

Q1 (i) Connected : G_1, G_3, G_4, G_5

(ii) disconnected: G_2

(iii) Complete: G_3

(iv) Cycle : G_4

(v) Tree : G_5, G_6.

(vi) $\overline{G_4}$: G_6.

Q. 2 (i) complete : none

(ii) bipartite : G_1, G_2, G_4

(iii) complete bipartite : G_2

Q. 3 (i) A cycle with 4 vertices has 4 edges.

(ii) A complete graph with 3 vertices has $\binom{3}{2}$ edges = 3 edges since every vertex is connected to every other vertex.

(iii) A complete bipartite graph partitioned into 2 and 3 sets of vertices V_1, V_2 has

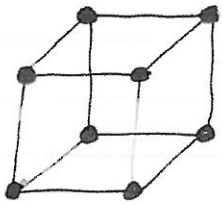
$2 \times 3 = 6$ edges since every vertex in set V_1 is connected to every vertex in V_2.

(iv) C_7 has 7 edges (see reasons in (a))

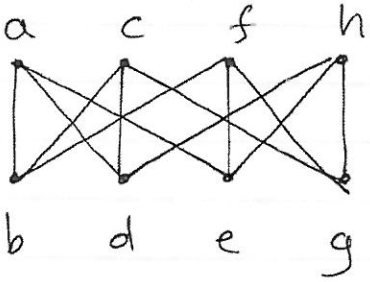
(v) K_{20} has $\binom{20}{2} = 190$ edges (see reasons in (b))

(vi) $K_{12,13}$ has $12 \times 13 = 156$ edges (see reasons in (c))

Q4.

(a)(i) 

Yes – adjacent vertices can be coloured differently.

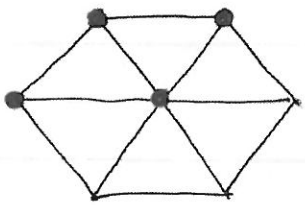
(ii) 

(iii) No – a and c are not adjacent.

(iv) No – a and g are not adjacent.

(v) No – it's vertices don't all have even degree.

(vi) No – more than two vertices have odd degree.

(b)(i) 

No – cannot colour adjacent vertices differently.

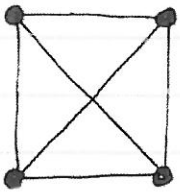
(ii) Not bipartite.

(iii) No – a and c are not adjacent.

(iv) No – not bipartite

(v) No – there are vertices with odd degree.

(vi) No – there are more than two vertices with odd degree.

(c)(i)  No – cannot colour adjacent vertices differently.

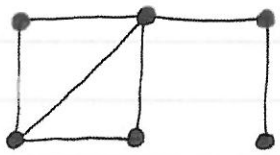
(ii) Not bipartite.

(iii) Yes – every vertex is adjacent to every other.

(iv) No – not bipartite.

(v) No – there are vertices with odd degree.

(vi) No – there are more than two vertices of odd degree

(d)(i)  No – cannot colour adjacent vertices differently.

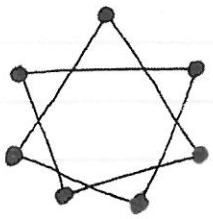
(ii) Not bipartite

(iii) No – a and e are not adjacent

(iv) No – not bipartite

(v) No – there are vertices with odd degree

(vi) Yes – there are exactly two vertices with odd degree, (deg(f) = 3, deg(d) = 1).

(e)(i)  No – cannot colour adjacent vertices differently.

(ii) Not bipartite.

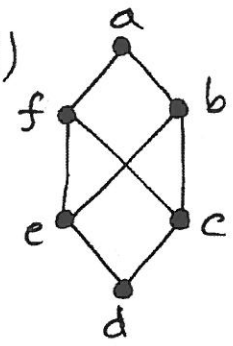
(iii) No – a and b are not adjacent.

(iv) No – not bipartite.

(v) Yes – every vertex has even degree.

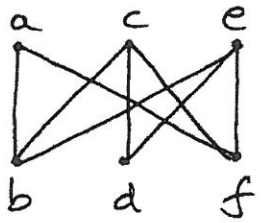
(vi) No – every vertex has even degree.

(f) (i)



Yes – adjacent vertices can be coloured differently.

(ii)



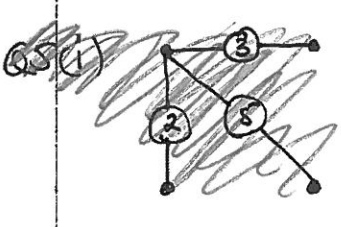
(iii) No – f and b are not adjacent.

(iv) No – a and d are not adjacent.

(v) No – there are vertices with odd degree.

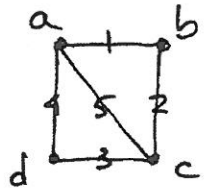
(vi) No – there are more than two vertices with odd degree.

Q5 (i)



Q.5

(i)



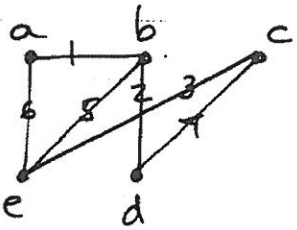
	a	b	c	d
a	0	1	1	1
b	1	0	1	0
c	1	1	0	1
d	1	0	1	0

Adjacency matrix

	1	2	3	4	5
a	1	0	0	1	1
b	1	1	0	0	1
c	0	1	1	0	1
d	0	0	1	1	0

incidence matrix.

(ii)



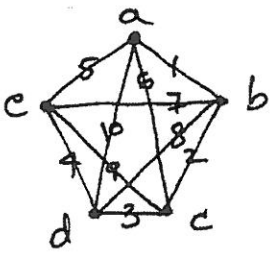
	a	b	c	d	e
a	0	1	0	0	1
b	1	0	0	1	1
c	0	0	0	1	1
d	0	1	1	0	0
e	1	1	1	0	0

Adjacency matrix

	1	2	3	4	5	6
a	1	0	0	0	0	1
b	1	1	0	0	1	0
c	0	0	1	1	0	0
d	0	1	0	1	0	0
e	0	0	1	0	1	1

Incidence matrix

(iii)



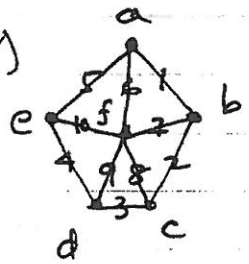
	a	b	c	d	e
a	0	1	1	1	1
b	1	0	1	1	1
c	1	1	0	1	1
d	1	1	1	0	1
e	1	1	1	1	0

Adjacency Matrix

	1	2	3	4	5	6	7	8	9	10
a	1	0	0	0	1	1	0	0	0	1
b	1	1	0	0	0	0	1	1	0	0
c	0	1	1	0	0	1	0	0	1	0
d	0	0	1	1	0	0	0	1	0	1
e	0	0	0	1	1	0	1	0	1	0

Incidence matrix.

(iv)

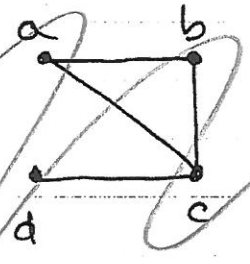


	a	b	c	d	e	f
a	0	1	0	0	1	1
b	1	0	1	0	0	1
c	0	1	0	1	0	1
d	0	0	1	0	1	1
e	1	0	0	1	0	1
f	1	1	1	1	1	0

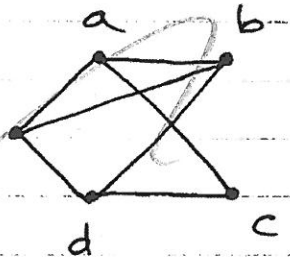
Adjacency Matrix

	1	2	3	4	5	6	7	8	9	10
a	1	0	0	0	1	1	0	0	0	0
b	1	1	0	0	0	0	1	0	0	0
c	0	1	1	0	0	0	0	1	0	0
d	0	0	1	1	0	0	0	0	1	0
e	0	0	0	1	1	0	0	0	0	1
f	0	0	0	0	0	1	1	1	1	1

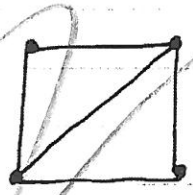
Q2 (i)



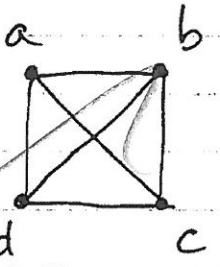
(ii)



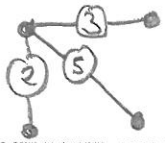
Q3 (i)



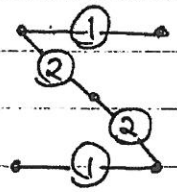
(ii)



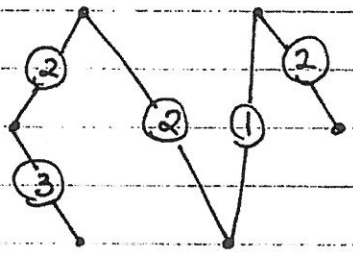
Q.6 (i)



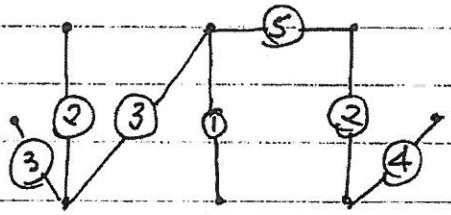
(ii)



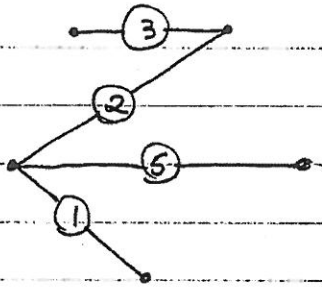
(iii)



(iv)



(v)



(vi)