

CMPU 2012 Mathematics 2

Problem Sheet 9: Probability 2

Q1. Claire entered a competition and must choose one of 5 envelopes to receive a prize. Each envelope contains a sum of money according to the table below. If it is equally likely that Claire will pick any one of these envelopes, then calculate the expected value of her prize.

	Envelope 1	Envelope 2	Envelope 3	Envelope 4	Envelope 5
Cash prize	€10	€20	€50	€500	€5000

Q2. 8 horses are in a race and Jack has put a bet down. If horses 1,3,5, or 7 come in first place then he wins €100, if horses 2 or 4 come in first place then he wins €20, otherwise he wins nothing. The probability of horses 1 - 6 coming in first place are given below.

Horse	1	2	3	4	5	6	7	8
Probability of first place	0.1	0.05	0.15	0.15	0.2	0.1		

- (a) If horse 7 is twice as likely to win than horse 2, determine the probabilities of horse 7 and horse 8 winning the race.
- (b) Calculate the expected value of Jack's winnings.

Q3. For a Brown Thomas gift card a 6 digit identification code is created from the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, where all the digits are unique (ie. each digit can only appear once at most).

- (a) How many possible codes can be formed?
- (b) How many possible codes can be formed if the code must begin with a number less than 4 (ie. 0, 1, 2 or 3)?

- (c) What is the probability of getting a code that begins with a number less than 4 (ie. 0, 1, 2 or 3)?

Q4. A licence plate number is made up of 3 letters followed by 3 digits (0,1,2,3,...,9), where no letter or number can be repeated (e.g. WPG 273). Sam is buying a new car, what is the probability that she will get a licence plate that begins with her name (i.e. SAM ***)?

Q5. A mixed soccer team of 11 players must be chosen from 8 women and 14 men.

- (a) How many possible combinations of the team can be formed?
- (b) How many possible combinations of the are there with exactly 5 women?
- (c) How many possible combinations of the are there with at least 9 men?
- (d) What is the probability of having at least 9 men on the team?

Q6. In a computer game simulation, 3 characters are randomly chosen from 10 to appear in a fight scene. If there are 3 fairies, 4 dragons and 2 warriors and 1 goblin, what is the probability that a fairy, a dragon and a goblin appear in the scene?

Q7. Sarah works for a charity and stops people on Grafton street to try to convince them to donate €50. Each conversation is assumed to be an independent event and she has a success rate of 1 in 5. If Sarah talks to 12 people then calculate the probability that she:

- (a) convinces 5 people to donate,
- (b) convinces 10 people to donate.

Q8. Based on his recent performance, Roger Federer's fans believe that his probability of winning any given match is $\frac{9}{10}$ in an upcoming tournament. He will play 7 matches in the first stage and must win all 7 matches in order to reach the final. Assuming each match is an independent event, calculate the that probability that he:

- (a) Makes it to the final.
- (b) Loses at least one match.
- (c) Wins exactly 6 matches.

Q9. Grace's credit card details have just been hacked and now there is a probability of $\frac{1}{3}$ that the hackers will steal €100 from her every time she uses her card. Assuming that Grace uses her card 5 times and that each transaction is an independent event, calculate the following:

- (a) The probability that the hackers steal exactly €300 of Grace's money
(i.e. they successfully steal from Grace exactly three times).
- (b) The probability that the hackers steal at least €100 of Grace's money
(i.e. they successfully steal from Grace at least once).
- (c) If X is the amount of money that the hackers steal from Grace, then calculate the expected value of X , $E(X)$.