Part A

For these questions assume we are using numerical values 0-25 for the letters A-Z

Q1. For each of the following matrices, working modulo 26, encrypt the message, 'GREEN EGGS AND HAM':

i. $\begin{pmatrix} 3 & 10 \\ 9 & 7 \end{pmatrix}$

ii. $\begin{pmatrix} 8 & 9 \\ 3 & 11 \end{pmatrix}$

Q2. For each of the following matrices find it's inverse modulo 26 and hence decrypt the associated message:

i. $\begin{pmatrix} 13 & 4 \\ 9 & 1 \end{pmatrix}$, (RDSRQOVUQBCZANQWRDDSAKOB)

ii. $\begin{pmatrix} 3 & 10 \\ 9 & 7 \end{pmatrix}$, (RLOQNZOFXMCQKGQIVDAZ)

iii. $\begin{pmatrix} 3 & 5 \\ 6 & 3 \end{pmatrix}$, (XAYQNNJXBLYA)

Part B

Q1. For each of the following determine if they have solutions and find all the incongruent solutions:

i. $2x \equiv 5 \pmod{7}$

ii. $3x \equiv 2 \pmod{7}$

iii. $3x \equiv 6 \pmod{9}$

iv. $6x \equiv 3 \pmod{9}$

v. $17x \equiv 14 \pmod{21}$

vi. $19x \equiv 30 \pmod{40}$

vii. $15x \equiv 9 \pmod{25}$

viii. $9x \equiv 5 \pmod{25}$

ix. $128x \equiv 833 \pmod{1001}$

x. $103x \equiv 444 \pmod{999}$

xi. $987x \equiv 610 \pmod{1597}$

xii. $980x \equiv 1500 \pmod{1600}$

xiii. $6789783x \equiv 2474010 \pmod{28927591}$

Q2. Find the inverse modulo 17 of each of the following

$$(a) \quad 4 \qquad (b) \quad 5 \qquad (c) \quad 7 \qquad (d) \quad 16$$

and hence solve

i. $4x \equiv 9 \pmod{17}$

ii. $5x \equiv 11 \pmod{17}$

iii. $7x \equiv 2 \pmod{17}$

iv. $16x \equiv 12 \pmod{17}$

v. $16x \equiv 28 \pmod{17}$

Q3. Determine which integers a, with $1 \leq a \leq 11$ have an inverse modulo 12.

Answers start on next page. Please try the questions before looking at the answers!

Answers Part A

Q1.

i. GRAMBPASCGRIVLGJ

ii. TXQEKKYGOCBUEVRD

Q2.

i. TO SLEEP PERCHANCE TO DREAMX

ii. BEWARE OF THE MESSENGER

iii. TOO MANY DAVES

Answers Part B

Answers:

Q1.

i. $x \equiv 6 \pmod 7$

ii. $x \equiv 3 \pmod 7$

iii. $x \equiv 2, 5 \text{ or } 8 \pmod 9$

iv. $x \equiv 2, 5, \text{ or } 8 \pmod 9$

v. $x \equiv 7 \pmod{21}$

vi. $x \equiv 10 \pmod{40}$

vii. There are no solutions.

viii. $x \equiv 20 \pmod{25}$

ix. $x \equiv 812 \pmod{1001}$

x. $x \equiv 111 \pmod{999}$

xi. $x \equiv 1596 \pmod{1597}$

xii. $x_t \equiv (875 + 80t) \pmod{1600}, t = 0, 1, \cdots, 19$

xiii. $x_t \equiv (247320 + 3157t) \pmod{28927591}, t = 0, 1, \cdots 9162$

Q2. The inverses are:

$$(a) \quad 13 \qquad (b) \quad 7 \qquad (c) \quad 5 \qquad (d) \quad 16$$

The solutions to the equations are:

i. $x \equiv 15 \pmod{17}$

ii. $x \equiv 9 \pmod{17}$

iii. $x \equiv 10 \pmod{17}$

iv. $x \equiv 5 \pmod{17}$

v. $x \equiv 6 \pmod{17}$

Q3. Only $1, 5, 7, 11$ have inverses modulo 12.