

CMPU 2012 Mathematics 2  
Problem Sheet 6: Predicate Logic

Q1. Let  $Q(x, y)$  denote the statement “ $x$  is greater than  $y$ ”. What are the truth values of the following?

- (a)  $Q(3, 1)$
- (b)  $Q(5, 5)$
- (c)  $Q(6, -6)$
- (d)  $Q(2^8, 256)$ .

Q2. Given the predicate  $P(x) : x > x^2$ , and the universe of discourse is the set of real numbers. What are the truth values of the following?

- (a)  $P(0)$
- (b)  $P(\frac{1}{2})$
- (c)  $P(2)$
- (d)  $P(-1)$
- (e)  $\exists x P(x)$
- (f)  $\forall x P(x)$ .

Q3. Let  $P(x)$  be the statement “ $x$  is happy”, where the universe of discourse for  $x$  is the set of students. Express each of the following quantifications in English.

- (a)  $\exists x P(x)$
- (b)  $\forall x \neg P(x)$
- (c)  $\exists x \neg P(x)$
- (d)  $\neg \forall x \neg P(x)$

Q4. Given the domain of discourse is the set of natural numbers  $\mathbb{N}$  (all positive integers) and the predicates:  $E(x) : x$  is even,  $O(x) : x$  is odd,  $H(x, y) : x < y$ ,  $P(x) : x$  is prime,  $M(x, y, n) : x \equiv y \pmod{n}$ . Write the following quantifications as English sentences and state whether they are true or false:

- (a)  $\exists x (E(x) \wedge O(x))$
- (b)  $\exists x (P(x) \wedge O(x))$
- (c)  $\forall x \forall y (E(x) \wedge O(y) \rightarrow M(x, y + 1, 2))$

Q5. Let  $L(x, y)$  be the predicate “ $x$  likes  $y$ ” and let the universe of discourse be the set of all people. Write the following quantifications as English sentences.

- (a)  $\forall x \forall y L(x, y)$ .
- (b)  $\forall x \exists y L(x, y)$ .
- (c)  $\forall x L(x, \text{George})$ .
- (d)  $\exists y \forall x L(x, y)$ .
- (e)  $\neg \exists y \forall x L(x, y)$ .

Q6. Given the domain of discourse is the set of natural numbers  $\mathbb{N}$  and the predicates:

$G(x) : x < 10$        $L(x) : x \geq 10$        $H(x, y) : x < y$

Use quantifiers to express the following statements.

- (a) There is no number that is both less than 10 and greater than or equal to 10.
- (b) For every number there is a number greater than it.
- (c) Any number less than 10 is less than any number greater than or equal to 10.

Q7. Given the following predicates:

$S(x) : x$  is a student,  $B(x) : x$  is a book,  $H(x, y) : x$  has  $y$ , where the universe of discourse is the universe, that is the set of all objects.

Use quantifiers to express the following statements.

- (a) Every student has a book.
- (b) Some student does not have any book.
- (c) Some student has all the books.
- (d) Not every student has a book.
- (e) There is a book which every student has.

**Answers:**

Q1.

(a) True, (b) False, (c) True, (d) False.

Q2.

(a) False, (b) True, (c) False, (d) False, (e) True, (f) False.

Q3.

(a) Some student is happy. (b) Every student is unhappy. (c) Some student is unhappy.  
(d) Not every student is unhappy.

Q4.

(a) There is a positive whole number that is both even and odd.  
(b) There is a positive whole number that is both prime and odd.  
(c) Every even positive integer is congruent to every odd positive integer plus one, mod 2.  
OR... Every odd positive integer plus one is even.

Q5.

(a) Everyone likes everyone. (b) Everyone likes someone. (c) Everyone likes George. (d)  
There is someone who everyone likes. (e) There is no one who everyone likes.

Q6.

(a)  $\exists x (G(x) \wedge L(x))$  (b)  $\forall x \exists y H(x, y)$  (c)  $\forall x \forall y (G(x) \wedge L(y) \rightarrow H(x, y))$ .

Q7.

(a)  $\forall x S(x) \rightarrow (\exists y B(y) \wedge H(x, y))$ .  
(b)  $\exists x S(x) \wedge (\forall y B(y) \rightarrow \neg H(x, y))$ .  
(c)  $\exists x S(x) \wedge (\forall y B(y) \rightarrow H(x, y))$ .  
(d)  $\neg(\forall x S(x) \rightarrow \exists y B(y) \wedge H(x, y))$ .  
(e)  $\exists y B(y) \wedge (\forall x S(x) \rightarrow H(x, y))$ .