DUBLIN INSTITUTE OF TECHNOLOGY

DT228 BSc. (Honours) Degree in Computer Science

Year 2

DT282 BSc. (Honours) Degree in Computer Science (International)

Year 2

WINTER EXAMINATIONS 2016/2017

MATHEMATICS 2 [CMPU2012]

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THURSDAY 12TH JANUARY 09.30 A.M. – 11.30 A.M.

TWO HOURS

INSTRUCTIONS
QUESTION 1 IS COMPULSORY
ANSWER QUESTION 1 AND TWO OF THE OTHER THREE QUESTIONS
ALL YOUR WORK MUST BE SHOWN

MATHEMATICAL TABLES ARE AVAILABLE

1. **a)** Find the residue of the following:

(i) $2^{50} \bmod 9$
(ii) $3^{401} \bmod 8$
(iii) $4^{79} \bmod 7$
(iv) $4^{3031} \bmod 5$

[8]

b) Assume the domain of discourse is the set of integers $\mathbb{Z}$. Given the predicates:

$$E(x) : x \text{ is even}$$
$$O(x) : x \text{ is odd}$$
$$C(x, y, n) : x \equiv y \bmod n$$

then express the following statements symbolically using logical notation.

(i) Not every integer is even
(ii) No integer is both even and odd
(iii) All even integers are congruent mod 2
(iv) For every integer, there is an integer to which it is not congruent mod 5

[8]

c) Consider the experiment of flipping a coin twice, where the coin is **biased** such that $p(\text{Heads}) = 0.6$. The number of heads that result is recorded as a discrete random variable, X.

(i) What is meant by the term *random variable*?
(ii) Summarise the probability distribution of X in an appropriate table.
(iii) What is the expected value of X, $E(X)$?
(iv) What is the variance of X, $Var(X)$?

[8]

d) Using the Chinese Remainder Theorem, solve the simultaneous congruence equations:

$$\left\{ \begin{array}{l} x \equiv 3 \bmod 5 \\ x \equiv 4 \bmod 6 \\ x \equiv 2 \bmod 7 \end{array} \right\}$$

[8]

e) Using *proof by induction* prove the following $\forall n \in \mathbb{N}$:

$$\sum_{i=1}^{n} \frac{1}{2^i} = \frac{2^n - 1}{2^n}$$

Be sure to label all steps clearly.

[8]

2. a) Define *mathematically* (using formulae) what is meant by the following terms:

(i) Mutually exclusive events
(ii) Independent events

[6]

b) In a game of bridge, you receive 13 cards (called a hand) from a normal pack of 52 playing cards. In your hand, what is the probability that:

(i) Every card is a diamond
(ii) Every card is of the same suit
(iii) You receive at least two hearts

[12]

c) Bits are sent over a communication channel in packets of 8. Assume that the probability of a bit being corrupted over this channel is 0.2, and that such errors are independent.

(i) What distribution can we use to model the number of corrupted bits in any packet of 8 bits? Use this distribution to answer the questions below.
(ii) What is the probability that none of the 8 bits will be corrupted?
(iii) What is the probability that at least 2 of the 8 bits will be corrupted?
(iv) For the transmission of a single packet, find the **expected value** and the **variance** of the number of corrupted bits.

[12]

3. a) Use *Unique Factorisation* to find the least common multiple (LCM) of the integers below. That is, find:

$$< 342, 1197 >$$

[5]

b) Using the extended Euclidean algorithm, find the general solution of the linear Diophantine equation:

$$61x + 19y = 5$$

[10]

c) For the given encryption matrix E, find the decryption matrix D (its inverse) mod 7, and using this, **decode** the message below: (use * for $0, A = 1, B = 2$, etc.):

$$\left\{ E = \begin{pmatrix} 3 & 1 \\ 3 & 4 \end{pmatrix} \bmod 7,\ \text{Cyphertext} = EACFCD \right\}$$

[15]

4. a) Define the following terms in relation to graph theory:

(i) connected graph
(ii) simple cycle
(iii) Euler cycle

[6]

b) Construct the adjacency matrix for the graph G shown in Figure 1.

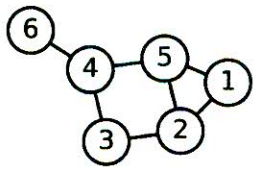

Figure 1: Graph G

[4]

c) For the graph G shown in Figure 1, answer each of the following questions. You must justify your answer in each case.

(i) Is G connected?
(ii) Is G bipartite?
(iii) Is G complete?
(iv) Does G have an Euler path?

[8]

d) Define a minimal weight spanning tree for a graph.
Use Kruskel's algorithm on the weighted graph H shown in Figure 2 in order to find its minimal spanning tree, being sure to clearly show the steps involved. What is the weight of the resulting tree?

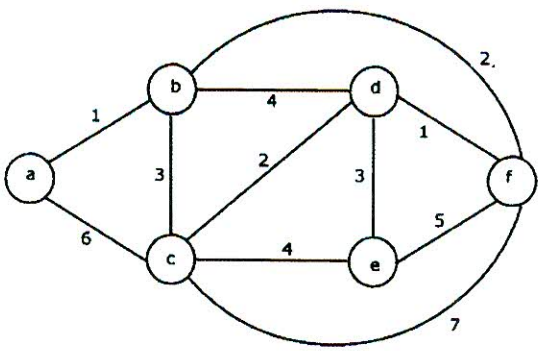

Figure 2: Weighted graph H

[12]