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```

1 Graph Theory

1.1 Adjacency List

```
1 vector < int > list[5];
  void Adjacency_List(){
       // initial
      for (int i = 0; i < 5; i++)
6
7
           list[i].clear();
8
      int a, b; // start & end of an edge
10
11
      while (cin >> a >> b)
12
           list[a].push_back(b);
13
           // list[b].push_back(a);
14 }
```

1.2 DFS

```
1  vector < int > G[N];
2  bitset < N > vis;
3  void dfs(int s) {
4     vis[s] = 1;
5     for (int t : G[s]) {
6         if (!vis[i])
7         dfs(i);
8     }
9 }
```

1.3 BFS

```
1 | vector < int > G[N];
2 bitset<N> vis;
  void bfs(int s) {
       queue<int> q;
       q.push(s);
6
       vis[s] = 1;
       while (!q.empty()) {
7
8
           int v = q.front();
           q.pop();
9
10
           for (int t : G[v]) {
11
                if (!vis[t]) {
12
                    q.push(t);
13
                    vis[t] = 1;
14
                }
15
           }
16
       }
17 }
```

1.4 Disjoint Set and Kruskal

```
1 struct Edge{
       int u, v, w;
2
       // bool operator < (const Edge &rhs) const {
3
            return w < rhs.w; }</pre>
4 };
5
 6
  vector<int> parent;
  vector<Edge> E;
  bool cmp(Edge edge1, Edge edge2){
9
10
       return edge2.w > edge1.w;
11
12
  int find(int x){
14
       if(parent[x] < 0){
15
           return x;
16
17
       return parent[x] = find(parent[x]);
18 }
19
  bool Uni(int a, int b){
20
21
       a = find(a);
       b = find(b);
22
23
       if(a == b){
24
           return false;
25
       if(parent[a] > parent[b]){
26
27
           swap(a, b);
28
29
       parent[a] = parent[a] + parent[b];
30
       parent[b] = a;
31
       return true;
32 }
33
34
  void Kruskal() {
35
36
       int cost = 0:
37
       sort(E.begin(), E.end()); // sort by w
38
39
       // sort(E.begin(), E.end(), cmp);
40
41
       // two edge in the same tree or not
       for (auto it: E){
42
43
           it.s = Find(it.s);
44
           it.t = Find(it.t);
45
           if (Uni(it.s, it.t)){
46
                cost = cost + it.w;;
47
48
       }
49
50
  int main(){
52
53
       // create N space and initial -1
       parent = vector<int> (N, -1);
54
55
56
       for(i = 0; i < M; i++){
57
           cin >> u >> v >> w;
58
           E.push_back({u, v, w});
59
60
       Kruskal();
61
62
63
       return 0;
64 }
```

1.5 Floyd-Warshall

1.6 Dijkstra

```
1 struct edge {
2
    int s, t;
     LL d;
3
4
     edge(){};
     edge(int s, int t, LL d) : s(s), t(t), d(d) {}
6 };
7
8 struct heap {
9
    LL d;
10
     int p; // point
11
     heap(){};
     heap(LL d, int p) : d(d), p(p) {}
12
     bool operator<(const heap &b) const { return d >
13
         b.d; }
14 };
15
16 int d[N], p[N];
17 vector<edge> edges;
18 vector<int> G[N];
19 bitset < N > vis;
20
21
  void Dijkstra(int ss){
22
       priority_queue < heap > Q;
23
24
       for (int i = 0; i < V; i++){</pre>
25
26
            d[i] = INF;
27
28
       d[ss] = 0;
29
       p[ss] = -1:
30
       vis.reset() : Q.push(heap(0, ss));
31
32
       heap x;
33
34
       while (!Q.empty()){
35
36
            x = Q.top();
37
            Q.pop();
38
            int p = x.p;
39
40
            if (vis[p])
41
                continue:
            vis[p] = 1;
42
44
            for (int i = 0; i < G[p].size(); i++){</pre>
                edge &e = edges[G[p][i]];
45
46
                if (d[e.t] > d[p] + e.d){
                     d[e.t] = d[p] + e.d;
47
48
                     p[e.t] = G[p][i];
49
                     Q.push(heap(d[e.t], e.t));
50
51
            }
       }
52
53 }
```

1.7 Fenwick Tree

```
1 // Binary Indexed Tree
2 // Build: O(NlogN)
3 // Space: O(N)
4 // update: O(logN)
5 // Cal Interval Sum: O(logN)
6 const int N = 10000000;
7 \mid int \ t[N + 1]; // 第零格無作用,數列從第一項到第 N 項
8 // 快速求出最低位的 bit (1)
9 int lower_bit(int n){
10
      return n & -n;
11 }
12 // value[1] + value[2] + ... + value[n]
13 int sum(int n){
14
      int s = 0;
      while (n > 0){
15
          s = s + t[n];
16
```

```
17
           n = n - lower_bit(n);
      }
18
19
       return s;
20 }
21
  // value[n] = value[n] + d
  void add(int n, int d){
22
       while (n \le N)
23
24
           t[n] = t[n] + d;
25
           n = n + lower_bit(n);
26
27 }
  // value[a] + value[a+1] + ... + value[b]
28
29 int query(int a, int b){
       if (a > b){
30
31
           swap(a, b);
32
33
       return sum(b) - sum(a - 1);
34 }
```

2 Dynamic Programming

2.1 Fibonacci

```
1 / f(n) = f(n - 1) + f(n - 2)
2 // f(0) = 0, f(1) = 1
3 int dp[30];
4 int f(int n){
      if (dp[n] != -1){
5
6
           return dp[n];
7
8
       return dp[n] = f(n - 1) + f(n - 2);
9 }
10
11
  int main(){
       memset(dp, -1, sizeof(dp));
12
13
       dp[0] = 0;
14
       dp[1] = 1;
15
       cout << f(25) << '\n';
16 }
```

2.2 Pascal Triangle

```
1 | // init : f(i, 0) = f(i, i) = 1
2 // tren: f(i, j) = f(i - 1, j) + f(i - 1, j - 1)
3
  #define N 30
  int dp[N][N];
5
  void Pascal_Traingle(void){
       for(int i = 0; i < N; i++){
7
           dp[i][0] = dp[i][i] = 1;
           for(int j = 1; j < i; j++){</pre>
8
9
               dp[i][j] = dp[i - 1][j] + dp[i - 1][j -
                    17:
10
           }
11
      }
12 }
```

2.3 Robot

```
1  // f(1, j) = f(i, 1) = 1
2  // f(i, j) = f(i - 1, j) + f(i, j - 1)
3  int dp[105][105];
4  dp[1][1] = 1;
5  for(int i = 1; i <= 100: ++i){
6    for(int j = 1; j <= 100; ++j){
7        if(i + 1 <= 100) dp[i + 1][j] += dp[i][j];
8        if(j + 1 <= 100) dp[i][j + 1] += dp[i][j];
9  }
10 }</pre>
```

2.4 Max Interval Sum

```
1 // No Limit
2 int ans = A[1];
3 sum[1] = dp[1] = A[1];
5 | for(int i = 2; i \le n; ++i) \{
       sum[i] = A[i] + sum[i - 1];
7
       dp[i] = min(dp[i - 1], sum[i]);
8
       ans = max(ans, sum[i] - dp[i - 1]);
9 }
10
11 // length <= L
12 int a[15] = {0, 6, -8, 4, -10, 7, 9, -6, 4, 5, -1};
13 int sum[15];
14
15 int main(){
       int L = 3, ans = 0;
16
       for (int i = 1; i <= 10; ++i)
17
18
19
             sum[i] = a[i] + sum[i - 1];
20
       }
21
       deque<int> dq;
22
       dq.push_back(0);
        for (int i = 1; i <= 10; ++i){</pre>
23
            \quad \textbf{if} \ (\texttt{i} \ \text{-} \ \mathsf{dq.front()} \ \texttt{>} \ \mathsf{L}) \{
24
25
                 dq.pop_front();
26
            }
            ans = max(ans, sum[i] - sum[dq.front()]);
27
            while(!dq.empty() && sum[i] < sum[dq.back()]){</pre>
28
29
                 dq.pop_back();
30
31
            dq.push_back(i);
32
33
        cout << ans << '\n';
34 }
```

2.5 Max Area

```
1 const int N = 25;
2
  int main(){
4
       int n;
5
       cin >> n;
       vector < int > H(n + 5), L(n + 5), R(n + 5);
       for (int i = 0; i < n; ++i){
           cin >> H[i];
9
10
       stack<int> st;
11
       // calculate R[]
       for (int i = 0; i < n; ++i){
12
13
           while (!st.empty() && H[st.top()] > H[i]){
               R[st.top()] = i - 1;
14
15
                st.pop();
           }
16
17
           st.push(i);
18
19
       while (!st.empty()){
20
           R[st.top()] = n - 1;
21
           st.pop();
22
23
       // calculate L[]
       for (int i = n - 1; i \ge 0; --i){
24
25
           while (!st.empty() && H[st.top()] > H[i]){
26
               L[st.top()] = i + 1;
27
                st.pop();
           }
28
29
           st.push(i);
30
       while (!st.empty()){
31
32
           L[st.top()] = 0;
33
           st.pop();
34
35
       int ans = 0;
       for (int i = 0; i < n; ++i){
36
```

2.6 LCS

```
1 // init : dp[i][0] = dp[0][i] = 0
  // tren : dp[i][j] =
      // if a[i] = b[j]
          // dp[i - 1][j - 1] + 1
4
5
       // else
6
          // max(dp[i - 1][j], dp[i][j - 1])
7
  // LIS
8
       // init : dp[0] = 0
      // tren : dp[i] = max\{dp[j] \mid j < i \text{ and } A[j] <
9
           A[i] + 1
  // LIS → LCS (嚴格遞增)
       // A 為原序列, B = sort(A)
11
12
       // 對 A, B 做 LCS
  // LCS → LIS (數字重複、有數字在 B 裡面不在 A 裡面)
       // A, B 為原本的兩序列
       // 對 A 序列作編號轉換,將轉換規則套用在 B
15
       // 對 B 做 LIS
17
  int dp[a.size() + 1][b.size() + 1];
  for(int i = 0; i <= a.size(); i++){</pre>
19
      dp[i][0]= 0;
20
  }
  for(int i = 0; i <= b.size(); i++){</pre>
21
22
      dp[0][i] = 0;
23 }
24
25
  for(int i = 1; i <= a.size(); i++){</pre>
26
       for(int j = 1; j <= b.size(); j++){</pre>
          if(a[i - 1] == b[j - 1]){
27
28
               dp[i][j] = dp[i - 1][j - 1] + 1;
29
30
           else{
               dp[i][j] = max(dp[i - 1][j], dp[i][j -
31
                   1]);
32
          }
33
      }
  }
34
35
36 return 0;
```

2.7 0-1 Bag

```
1 // 不放:重量和價值不變
2
      // to f(i, j) = f(i - 1, j)
  // 放:重量 + w_i,價值 + v_i
3
       // to f(i, j) = f(i - 1, j - w_i) + v_i
  // tren: f(i, j) = max(f(i - 1, j), f(i - 1, j - w_i))
       + v i)
  int dp[MXN + 1][MXW + 1];
  memset(dp, 0, sizeof(dp));
  for (int i = 1; i <= MXN; ++i){</pre>
8
      for (int j = 0; j < w[i]; ++j){
9
           dp[i][j] = dp[i - 1][j];
10
11
       for (int j = w[i]; j <= MXW; ++j){</pre>
12
13
           dp[i][j] = max(dp[i - 1][j - w[i]] + v[i],
               dp[i - 1][j]);
14
15 }
16 cout << dp[MXN][MXW] << '\n';</pre>
```

2.8 Infinite Bag

```
1 / f(i, j) = max(f(i - 1, j), f(i - 1, j - wi) + vi,
       f(i, j - wi) + vi)
2
       // coin chage
          // 最少幾枚能湊成 M 元
3
4
                   f(i,j)=min(f(i-1,j),f(i-1,j-ci)+1,f(i,j-ci)+1)
5
           // 多少種能湊成 M 元
6
              // f(i, j) = f(i - 1, j) + f(i, j - ci)
7 int dp[MXW];
8 memset(dp, -INF, sizeof(dp));
9|dp[0] = 0;
10 for (int i = 0; i < N; ++i){
      for (int j = w[i]; j <= MXW; ++j){</pre>
11
12
          dp[j] = max(dp[j - w[i]] + v[i], dp[j]);
13
14 }
```

2.9 Tree

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 \mid const \mid int \mid MXV = 15;
 4 vector<int> G[MXV];
5 int high[MXV][2];
6 int ans[MXV], height[MXV];
8 void dfs(int u){
       height[u] = 1;
       for (int v : G[u]){
10
11
           dfs(v);
12
           height[u] = max(height[u], height[v] + 1);
           if (high[u][0] == 0 || height[high[u][0]] <</pre>
13
                height[v]){
                high[u][1] = high[u][0];
14
                high[u][0] = v;
15
16
           }
17
           else if (high[u][1] == 0 ||
                height[high[u][1]] < height[v]){
                high[u][1] = v;
18
19
           }
       }
20
21 }
22
23 void dfs2(int u, int legnth){
       ans[u] = height[high[u][0]] +
24
            max(height[high[u][1]], legnth) + 1;
25
       for (int v : G[u]){
26
           if (v == high[u][0]){
                dfs2(v, max(height[high[u][1]], legnth) +
27
                     1);
28
           }
           else{
29
30
                dfs2(v, max(height[high[u][0]], legnth) +
                    1);
31
           }
       }
32
33 }
34
35 int main(){
36
       int n;
37
       cin >> n;
38
       for (int i = 1; i < n; ++i){
           int x, y;
39
40
           cin >> x >> y;
41
           G[x].emplace_back(y);
       }
42
43
       dfs(1);
       dfs2(1, 0);
44
45
       for (int i = 1; i <= n; ++i){</pre>
           cout << ans[i] << '\n';</pre>
46
47
       }
48 }
```