DBA5109 Quantitative Risk Management

Group Project – Option B

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1. Data Exploration and Analysis

1.1 Datasets Overview

In this project, we utilised dataset derived from the Kenneth R. French Data Library, which includes following data from January 1986 to December 2015, providing a total of 30 years of data:

- Mkt-RF: The excess return on the market portfolio, i.e., the value-weight return of all applicable firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ;
- RF: The risk-free rate;
- Monthly returns of 43 U.S. industry portfolios.

1.2 Data Preprocessing

To prepare the dataset for portfolio analysis, we first computed the excess returns of 43 industry portfolios by subtracting the risk-free rate from their monthly returns. Since the excess market return (Mkt-RF) was provided, no further adjustment was needed for the market portfolio.

1.3 Basic Portfolio Construction and In-sample analysis

(a) Performance metrics (In-sampling)

Portfolio	Expected Return (%)	Standard Deviation (%)	Sharpe Ratio
MKT	7.636	15.535	0.492
EWP	8.620	16.132	0.325
TAN	23.002	18.321	1.071
GMV	6.746	9.922	0.340

Summary:

- **Market Portfolio (MKT):**
  The market portfolio delivers a 7.636% annual return with 15.535% volatility, offering a solid 0.492 Sharpe ratio as a benchmark, though it's outperformed by the tangency portfolio in both returns and risk-adjusted performance.
- **Equally Weighted Portfolio (EWP):**
  While the equally weighted portfolio generates a slightly higher 8.620% return than the market, its 16.132% volatility and low 0.325 Sharpe ratio demonstrate that simple equal weighting is less efficient than optimized portfolio strategies.
- **Tangency Portfolio (TAN):**
  The tangency portfolio delivers exceptional 23.002 returns with 18.321% risk, boasting a market-beating 1.071 Sharpe ratio that comes with the practical challenge of requiring significant short positions in some assets.
- **Global Minimum Variance Portfolio (GMV):**
  With the lowest 9.922% volatility but modest 6.746% returns, the GMV portfolio's 0.344 Sharpe ratio makes it ideal for risk-averse investors, offering stability without aggressive short-selling strategies.

In summary, each portfolio has its own strength and thus caters to different risk-level investors. The tangency portfolio (TAN) is the best choice for investors seeking maximum risk-adjusted returns due to its highest Sharpe ratio, while the global minimum variance portfolio (GMV) is ideal for those prioritizing low volatility over returns. The equally weighted portfolio (EWP) is simple to implement but underperforms optimized portfolios, and the market portfolio (MKT) serves as a solid benchmark but can be improved upon through optimization.

(b) The σ $vs.$ $E[r]$ diagram

The figure 1.3.1 presents the risk-return trade-off of portfolios constructed using in-sample data (1986–2010). The x-axis represents the annualized standard deviation (σ) as a measure of risk, and the y-axis shows the annualized expected return (E[r]). The blue curve represents the Efficient Frontier (EF), which plots the set of optimal portfolios offering the highest expected return for a given level of risk.

While the TAN and GMV lie on the efficient frontier as expected, EWP does not—it bears relatively high risk with moderate returns, reinforcing the value of optimization.

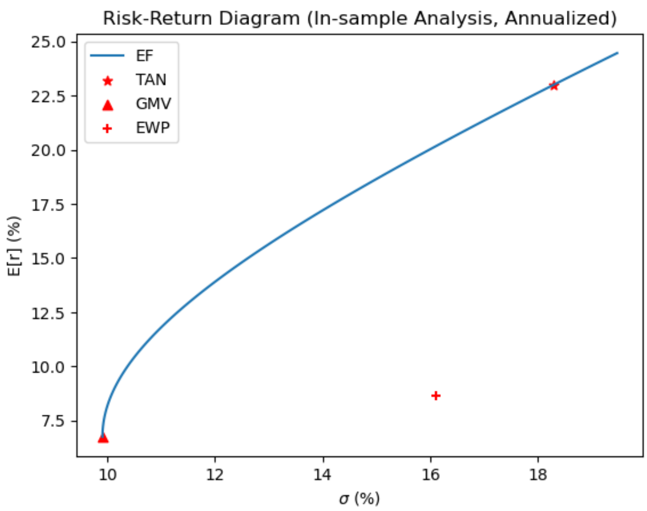

Figure 1.3.1- **σ $vs.$ $E[r]$diagram** (In-sample Analysis)

(c) The β $vs.$ $E[r]$diagram

The figure 1.3.2 plots each industry portfolio's beta (β) on the x-axis against its annualized expected return (E[r]) on the y-axis. The dashed line represents the Capital Asset Pricing Model (CAPM)'s Security Market Line (SML), which models the expected return as a linear function of beta.

As shown in Figure 1.3.2, some industries lie above the SML, suggesting they outperform CAPM predictions (positive alpha). Others fall below the SML, indicating underperformance relative to their beta. And one thing worth attentioning is that TAN lies significantly above the SML, suggesting it captures alpha and offers excess return beyond what CAPM predicts.

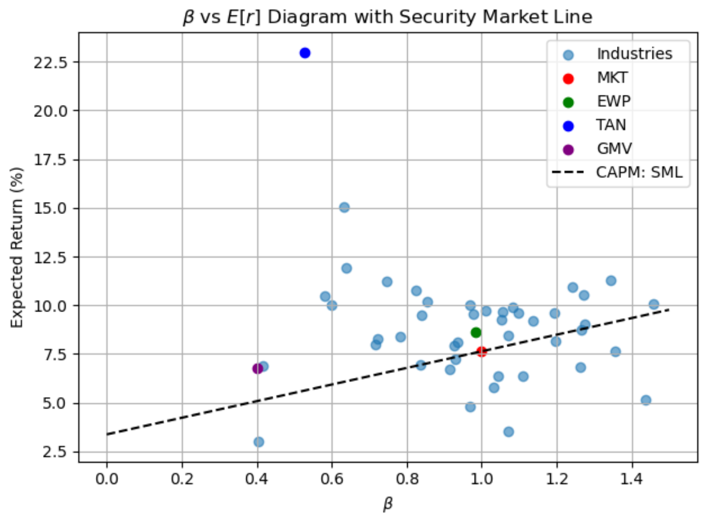

Figure 1.3.2- β vs. $E[r]$diagram

1.4 Robust Portfolio Construction and Out-of-sample Analysis

(a) The σ vs. $E[r]$diagram

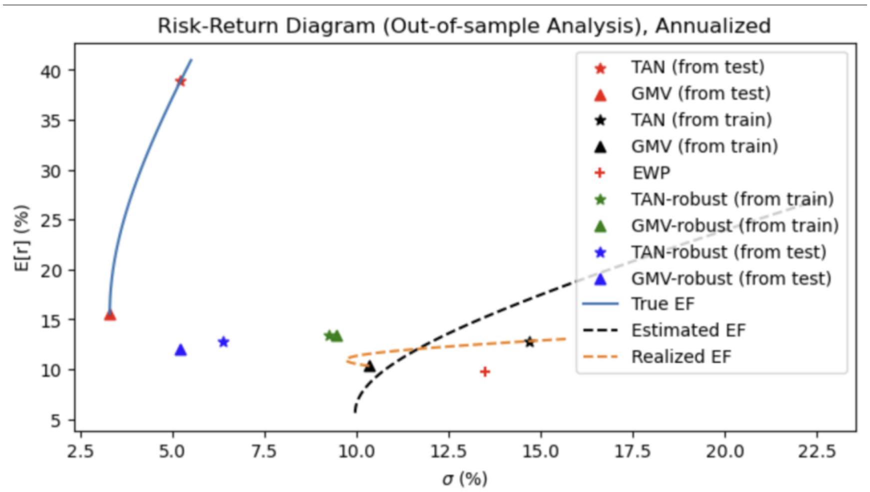

Figure 1.4.1- σ vs. $E[r]$diagram (Out-of-sample)

(b) Compare the out-of-sample performances of MKT, EWP, TAN, TAN-robust, GMV, and GMV-robust portfolios.

Performance metrics (Out-of-sampling)

Portfolio	Expected Return (%)	Standard Deviation (%)	Sharpe Ratio
MKT	6.795	16.110	0.422
EWP	9.775	13.592	0.718
TAN	12.817	14.806	0.864
GMV	10.332	0.864	0.987
TAN-robust	13.420	0.987	1.436

Portfolio	Expected Return (%)	Standard Deviation (%)	Sharpe Ratio
GMV-robust	13.384	13.420	1.401

The market (MKT) portfolio, which serves as a broad benchmark, exhibits an expected annual return of about 6.795% with a relatively high annualized standard deviation of 16.110%, resulting in the lowest Sharpe ratio of 0.422. In contrast, the equally weighted portfolio (EWP) achieves a higher expected return of roughly 9.775% and a standard deviation of 13.592%, yielding an improved Sharpe ratio of 0.718, which suggests that even a naïve equal weighting can outperform the market portfolio when estimation error is considered.

The standard tangency portfolio (TAN), constructed using the traditional sample estimates for expected returns and covariance, delivers an expected return of approximately 12.817% and a risk level of 14.806%, with a Sharpe ratio of 0.864, showing a better risk–reward trade-off compared to MKT and EWP. Meanwhile, the global minimum variance portfolio (GMV), which focuses solely on reducing risk through the minimization of volatility, realizes a return of about 10.332% with a lower standard deviation of 10.443% and a Sharpe ratio of 0.987, underscoring the benefits of risk minimization for enhanced out-of-sample performance.

More impressively, both robust portfolios significantly outperform their traditional counterparts. The TAN-robust portfolio, which incorporates a shrinkage estimator for the covariance matrix along with a CAPM-based expected return estimate, achieves an expected return of around 13.420% with a substantially reduced risk of 9.332% and an outstanding Sharpe ratio of 1.436. Similarly, the GMV-robust portfolio attains an expected return of approximately 13.384% with a standard deviation of 9.540% and a Sharpe ratio of 1.401. These results clearly demonstrate that employing robust techniques to mitigate estimation error can lead to both higher returns and lower volatility, thereby improving the overall risk-adjusted performance.

In summary, while the market portfolio underperforms on a risk-adjusted basis, the robust versions of the tangency and global minimum variance portfolios deliver superior out-of-sample performance by effectively reducing noise and estimation risk. The robust portfolios not only provide higher expected returns but also lower volatility, as reflected in the markedly higher Sharpe ratios compared to their traditional counterparts, confirming that robust estimation methods can be highly beneficial for portfolio optimization.

(c) Contrast the table for out-of-sample performance with that for in-sample performance.

The in-sample analysis shows that while the tangency portfolio appears to deliver exceptionally high returns with high risk (yielding a Sharpe ratio of 1.071) compared to its more modest out-of-sample performance (with a Sharpe ratio of 0.864), and both the global minimum variance and equally weighted portfolios display significant shifts in returns, risk, and Sharpe ratios between in-sample and out-of-sample periods—unlike the stable market portfolio—these discrepancies suggest that portfolios optimized on historical data may be overfitted and thus require out-of-sample testing to reliably assess their true performance, especially since the evaluation uses training data from 1986 to 2010, a period marked by major financial events such as Black Monday in 1987[1], the Asian Financial Crisis in 1997[2], the Russian Financial Crisis in 1998 [3], the Dot-Com Bubble in the early 2000s[4], and the Global Financial Crisis of 2007–2008[5] that substantially influenced portfolio management strategies, while the test period from 2010 to 2015 reflects a recovery phase still facing challenges like the European Sovereign Debt Crisis[6], the 2013 Taper Tantrum[7], and declining commodity prices [8].

2. Data Challenge

2.1 Data Construction

Based on explanations above, we selected data from 2005 to 2015 because this period spans a variety of market conditions, including the calm leading up to the 2007–2008 financial crisis, the crisis itself, and the subsequent recovery. Using this mixed period helps ensure that our constructed portfolio is robust and represents different economic environments, so we can meaningfully evaluate its performance when applied to the out-of-sample period (2016–2020).

2.2 Portfolio Exploration

2.2.1 **LLM**

The system employs a suite of optimization methods—Sharpe optimization, mean-variance optimization, Black-Litterman, and equal weighting—integrated with a large language model (specifically GPT-4o) to automate tool selection and decision-making. The goal is to allocate weights that optimize risk-adjusted returns, validated through rigorous backtesting with data from 2006 to 2015.

The system follows a structured pipeline, executed in five sequential steps, with the LLM enhancing automation and decision-making. Each step is detailed below, including its purpose, outputs, and the task description sent to the LLM to invoke it. All the task descriptions will be treated as part of this prompt, "You are an intelligent agent tasked with analyzing financial data. Your current task is: "{task}". Below are the available tools and their descriptions: {tool_descriptions} Choose the most appropriate tool for the task and return only the tool name (e.g., "stats_tool")", and sent to the LLM.

- Stats tool

  Purpose: Compute statistical inputs for optimization.

  Outputs: mean returns, standard deviations and covariance matrix over the 43 stocks

  Task: Calculate statistics of the data for the 43 stocks
- Split data tool

  Purpose: Divide data into training and testing sets for model training and validation.

Outputs: training data (240×43), testing data (120×43)
Prompt: Split data into training and testing sets

- Optimization methods: Sharpe optimization, Mean-variance optimization, Black-Litterman and Equal weight
  Purpose: Directly optimize the Sharpe ratio and weights using different methods.
  Outputs: weight vector from each method
  Prompt: Optimize weights using portfolio optimization
- Backtest tool
  Purpose: Evaluate each method's weights on the test set.
  Outputs: Sharpe ratio, expected return and volatility
  Prompt: Backtest weights on test data
- Reflection tool
  Purpose: Select the optimal method based on backtest results.
  Outputs: Selected methods Sharpe ratio and weights
  Prompt: Directly call the reflection tool

Role of LLM

The LLM (GPT-4o) is integral to the system, performing two critical functions:

- **Tool Selection**:
  - For each task (e.g., "calculate statistics," "optimize weights"), the LLM evaluates tool descriptions and selects the appropriate one (e.g., stats_tool, optimization_tool).
  - Uses a prompt-based approach, ensuring adaptability to varying task formulations.
  - Example: For "split data into training and testing sets," it reliably chooses split_data_tool.
- **Decision-Making (Reflection)**:
  - In the reflection step, the LLM analyzes backtest results, considering Sharpe ratios, returns, volatilities, and weight distributions.
  - Provides a human-like justification, improving interpretability (e.g., "Sharpe Optimization is chosen due to its superior risk-adjusted return").
  - Enhances robustness by cross-validating numerical results with contextual reasoning.

The LLM's integration reduces manual intervention, automates complex decisions, and ensures the system is scalable to new methods or datasets.

Results

Method	Expected Return (%)	Standard Deviation (%)	Sharpe Ratio
Sharpe Optimization	11.9	11.74	1.01

Mean-Variance	14.89	15.3	0.97
Black-Litterman	8.4	11.54	0.73
Equal Weight	8.56	17.52	0.49

Table 2.2.1.1-LLM Performance Metric

2.2.2 Conditional Value at Risk (CvaR) Optimization

Background

In out-sampling analysis, the portfolio tools we applied—such as Tangency Portfolio (TAN), Global Minimum Variance (GMV), and their robust counterparts—primarily focused on optimizing mean-variance trade-offs. However, these approaches do not explicitly account for extreme losses in the tails of the return distribution.

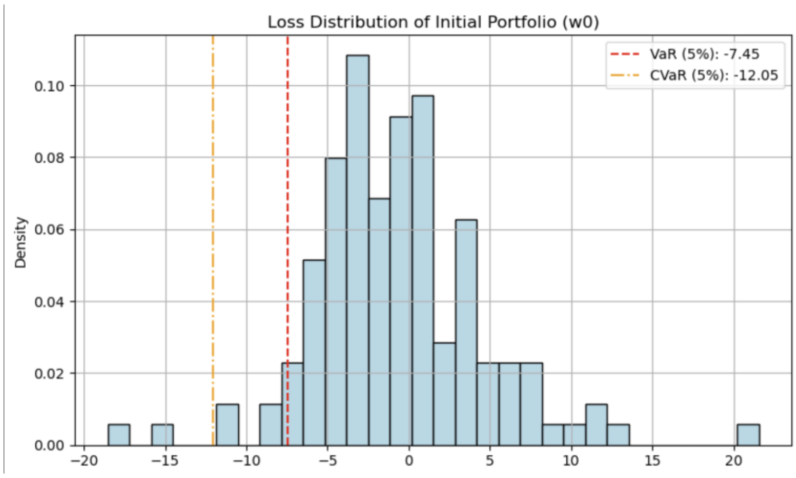

Figure 2.2.2.1-Loss Distribution (from 2005 to 2015)

Hence, we first examined the loss distribution of the initial portfolio using market weights (from 2005 to 2015). As shown in the figure above (see Figure 2.2.2.1), significant tail risk is observed. To better tackle this issue and manage such tail risk, we were motivated to introduce Expected Shortfall, also called CVaR. Unlike Value at Risk (VaR), CVaR estimates the expected loss beyond the loss is greater than VaR level, offering a more comprehensive and conservative measure for downside protection.

Objective

To minimize CVaR portfolio at 5% level under constraints and comparing its effectiveness with standard approaches.

Settings for CVaR Portfolio Optimization:

1. Initial guess: Market weights based on beta.

2. Constraints:
- Portfolio weights sum to 1.
- Expected return not less than market average.
- Monthly volatility limited to 3% (annualized ~12%).

Results

We plotted a risk-return diagram for all portfolios as figure 2.2.2.2 and summarized performance of all portfolios in performance metrics as Table 2.2.2.1. The diagram clearly shows that CVaR-Min lies to the right of robust portfolios, indicating higher return with a bit more volatility. Robust portfolios cluster closer to the minimum variance frontier. Interestingly, Tangency portfolio shows a strong return but with unrealistic levels; thus, we suggest it is likely due to overfitting issue on training data, since there is a 5 years (2011-2015) overlap in our training and testing data.

Portfolio	Expected Return (%)	Standard Deviation (%)	Sharpe Ratio
MKT	3.410	15.535	0.492
EWP	9.775	13.592	0.718
TAN	49.889	14.806	3.162
GMV	11.227	7.274	1.541
TAN-robust	14.452	7.701	1.874
GMV-robust	14.453	7.835	1.842
CvaR-Min	18.271	9.184	1.987

Table 2.2.2.1-Performance Metric (with CVaR)

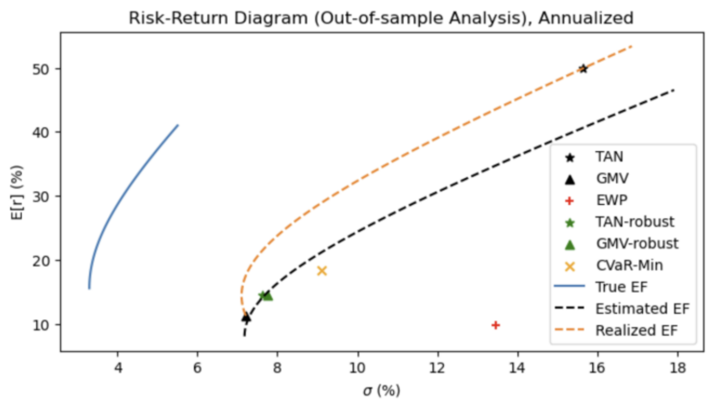

Figure 2.2.2.2- σ $vs.$ $E[r]$Diagram (with CVaR)

Key Insights

The CVaR-Min portfolio delivered both the highest expected return and Sharpe Ratio among all tested strategies while keeping risk within a reasonable range. We can see that CVaR-Min effectively controls tail risk, which is beneficial for investors with downside risk aversion. The CVaR-Minimizing portfolio proves to be a strong alternative to traditional and shrinkage-based methods, offering superior expected returns with controlled risk. This approach is well-suited for investors balancing performance and downside protection in uncertain market conditions.

2.3 Recommendations

The table shows that the CVaR-Min portfolio delivered an attractive expected return of 18.271% with a relatively low standard deviation of 9.184% and a Sharpe ratio of 1.987, indicating a favorable risk-adjusted performance. This portfolio is constructed using the Conditional Value at Risk (CVaR) minimization approach, which focuses on reducing the potential for extreme losses by limiting exposure to the worst-performing scenarios. In the period from 2010 to 2015, the global market experienced significant volatility, with recovery efforts still ongoing after the 2007–2008 Global Financial Crisis. During this time, several factors—including the European debt crisis, the 2013 Taper Tantrum, and a notable decline in commodity prices—had a pronounced impact, particularly on sectors such as oil, gas, and mining.

Because these sectors were especially susceptible to sharp declines during the 2010–2015 period, the CVaR-Min portfolio strategy naturally assigns lower or even zero weights to them, as shown in the image. By minimizing exposure to industries that were most affected by the downturn in commodity prices, the portfolio is better protected against the kind of sudden, severe market shocks that can erode returns. In contrast, industries that displayed more stable performance during this turbulent period received more favourable allocations. This careful rebalancing not only protects against extreme risk but also helps ensure that the portfolio maintains a competitive return level while offering downside protection. In summary, the CVaR-Min portfolio's allocation strategy—evident from the image—reflects a deliberate effort to mitigate risk in sectors vulnerable to market stress, making it an appealing choice in a post-crisis environment that is still characterized by uncertainty and lingering volatility.

REFERENCE

[1] Wikipedia contributors. (n.d.). *Black Monday (1987)*. In Wikipedia, The Free Encyclopedia. Retrieved March 29, 2024, from https://en.wikipedia.org/wiki/Black_Monday_(1987)

[2] Wikipedia contributors. (n.d.). *Asian financial crisis*. In Wikipedia, The Free Encyclopedia. Retrieved March 28, 2024, from https://en.wikipedia.org/wiki/Asian_financial_crisis

[3] Wikipedia contributors. (n.d.). *1998 Russian financial crisis*. In Wikipedia, The Free Encyclopedia. Retrieved March 28, 2024, from https://en.wikipedia.org/wiki/1998_Russian_financial_crisis

[4] Wikipedia contributors. (n.d.). *Dot-com bubble*. In Wikipedia, The Free Encyclopedia. Retrieved March 28, 2024, from https://en.wikipedia.org/wiki/Dot-com_bubble

[5] Wikipedia contributors. (n.d.). *Global financial crisis*. In Wikipedia, The Free Encyclopedia. Retrieved March 28, 2024, from https://en.wikipedia.org/wiki/Global_financial_crisis

[6] Wikipedia contributors. (n.d.). *European debt crisis*. In Wikipedia, The Free Encyclopedia. Retrieved March 28, 2024, from https://en.wikipedia.org/wiki/European_debt_crisis

[7] Wikipedia contributors. (n.d.). *Taper tantrum*. In Wikipedia, The Free Encyclopedia. Retrieved March 28, 2024, from https://en.wikipedia.org/wiki/Taper_tantrum

[8] Wikipedia contributors. (n.d.). *Oil price*. In *Wikipedia, The Free Encyclopedia*. Retrieved March 28, 2024, from https://en.wikipedia.org/wiki/Oil_price