Formulae to Estimate Peak Effluent Flows from Brasilian Dams Break in 21th Century

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Abstract

To predict the outcoming flood after a dam failure, many computational and empirical approaches have been developed throughout history. All those includes a certain number of parameters and are intended to help engineers in quick estimates and planning more detailed studies. In this article we aimed to evaluate the different empirical formulas proposed by researchers to estimate the outflow peak dambreak situations and compare the results in two different case studies, the overtopping and piping failure of Jurumirim and Chavantes dams. Also, HEC-RAS software was implemented and used as reference to calculate the discharge peaks. Considering the large amount of available formulas, a statistical analysis where employed to evaluate and classify the results. Finally, the pertinence and range of application of the formulas is discussed

Introduction

To minimize the losses associated with the failure of a dam, the Brazilian National Dam Safety Policy since 2010 requires the elaboration of a Risk Plan (ANA, 2010) that includes estimating the potential impacts of a dam break and the corresponding Emergency Action Plan (EAP). The EAP contains information such as the inundation maps and the procedures to be performed in cases of emergency (Lauriano, 2009). Its development is based on the forecast and calculation of maximum flooded levels, water velocities and maximum discharges resulting from the rupture scenarios aiming objectives such as reducing human, infrastructure and biodiversity losses. In order to guarantee the safety of hydraulic structures, the Brazilian regulations require a constant update of the Dam Safety Manual (ANA, 2016) which describes the dam safety inspection procedure. That is why it is more than justified the need to develop useful and practical tools to classify the hazard, plan emergency actions, map and assess the potential risks (Graham, 1998). One of these tools is the modeling of the effluent flow of a dam rupture, by empirical. mathematical and computational approaches. Computational modeling uses mathematical models and numerical and their use models has proved important for predicting hydraulic and hydrological phenomena.

A well know computational model is HEC-RAS software (**Brunner**, **1995**) which requires little input data and can presents a simplified view of one-dimensional constant flow studies, unidimensional and bidimensional unsteady flow calculations, sediment transport / moving bed calculations and modeling of temperature and water quality for a complete network of natural and/or artificial channels. Furthermore, it is free and works with small simplifications of the Saint

Venant equation, using an implicit method of finite difference, which provide a high degree of precision and reliability (Mbajiorgu, 2017).

Hence, this work uses the HEC-RAS software to evaluate and compare the performance of several empirical-practical formulas proposed by researchers in the area, to estimate the peak flow from a dam failure by piping and overtopping. The calculation processes for the study of emergency. Thus, this project aims, mainly, to identify practical formulas and compare their results to a more sophisticated approach using two case studies, the dams Jurumirim and Chavantes dams, two HPP located in São Paulo State, Brazil.

Material e Methods

Preliminary, various formulas were compiled to estimate the effluent peak flow from a rupture (**Table 1**) whose parameters area taken from the design plans of the studies structures (**Figure 1**) and (**Figure 2**) and reports of previous studies dam break studies (FCTH, 2003).

Author	Formula	Depends on
Pierce (2008) ¹	$Qp = 0.784 \times Hw^{2.668}$	Hw
Pierce (2008) ²	$Qp = 2.325 \times Ln(Hw)^{6.405}$	Hw
Pierce (2008) ³	$Qp = 0.00919 \times V^{0.745}$	v
Singh and Snorrason	$Qp = 1.776 \times V^{0.47}$	
(1984) ¹	QP = 1.778 × V	V
U. S. Bureau of Reclamation (1982)	$Qp = 19.1 \times hw^{1.85}$	
Enveloped Equation	QP = 17.1 × 11W	hw
Soil Conservation Service	$Qp = 16.6 \times hw^{1.85}$	
(1981) apud Wahl (2008)	QP = 10.0 × 11.1	hw
Singh and Snorrason (1984) ²	$Qp = 13.4 \times H^{1.89}$	н
Singh and Snorrason	$Qp = 1.776 \times V^{0.47}$	
(1984) ³		V
Costa (1985)¹	$Qp = 1.122 \times V^{0.57}$	V
Evans (1986)	$Qp = 0.72 \times V^{0.53}$	V
Lou (1981) apud Faria	$Qp = 7.683 \times H^{1.909}$	н
(2019) Froehlich (1995b)	$Qp = 0.607 \times V^{0.295} \times hw^{1.24}$	
MacDonald and	QP = 0.007 A V A NV	V, hw
Langridge - Monopolis	$Qp = 1.154 \times (V \times hw)^{0.412}$	
(1984)		V, hw
MacDonald and		
Langridge - Monopolis	$Qp = 3.85 \times (V \times hw)^{0.411}$	
(1984) Enveloped equation		V, hw
Hagen(1982) ¹	$Qp = 0.54 \times (V \times H)^{0.5}$	V. H
Hagen(1982) ²	$Qp = 1.205 \times (H \times V)^{0.48}$	
Vertedor de soleira	QP = 1:203 \(\times (H \(\times V))\)	V, H
espessa - Singh (1996)	$Qp = 1.7 \times b \times h^{\frac{3}{2}}$	
apud Faria (2019)	CP	b,h
Costa (1985)²	$Qp = 0.981 \times (S \times H)^{0.42}$	S,H
Costa (1985) Envolved	$Qp = 2.634 \times (S \times H)^{0.44}$	
Equation	ξP 2.00 (γ (1)	S,H
	$\left(\begin{array}{c} 1.94 \times \frac{As}{} \right)^3$	
Wetmore e Fread (1981)	$Qp = 1.7 \times Bt \times \left\{ \frac{1.94 \times \frac{As}{Bb}}{Tp + \left[\frac{1.94 \times As}{Bb \times \sqrt{H}} \right]} \right\}$	
apud Faria (2019)	$\left[\text{Tp} + \left \frac{1.94 \times \text{As}}{\text{Bb} \times \sqrt{\text{H}}} \right \right]$	
	([55 × 411])	As,H
Xu and Zhang (2009)	$\frac{Qp}{\sqrt{g \times V^{5/3}}} = 0.175 \times \left(\frac{H}{Hr}\right)^{0.199} \times \left(\frac{V^{1/3}}{hw}\right)^{-1.274} \times e^{B4}$	
, , , , , , , , , , , , , , , , , , , ,	$\sqrt{g \times V^{5/3}}$ (hw)	V,H,Hw
Mohamed (2001) ²	$Qp = 0.98 \times A \times \sqrt{2 \times g \times (Hw - Hp)} $ (Only Piping)	A,Hw,Hp
Apud NRCS (2005)	$Qp = 65 \times Hw^{1.85}$ $Qp = 1.1 \times Br^{1.35}$	
Saint Venant apud Faria	8 _ 3	Hw, V,Ab
(2019)	$Qp = \frac{8}{27} \times b \times \sqrt{g} \times Y_{\text{medio}}^{\frac{3}{2}}$	b,Hw,hmin-op
Mohamed (2001) ¹	$Op = 3 \times b \times (Hw - Hc)^{1.5}$	b,Hw,Hc
	(1)1/2 (4)5/2	
Macchione (2008) ¹	$Qp = \left(\frac{1}{2} \times g\right) \times \left(\frac{1}{5} \times (Hw - Y)\right) \times tan(\beta)$	Hw,B,m,H
	$\begin{aligned} & Q_{\rm P} = \left(\frac{1}{2} \times g\right)^{1/2} \times \left(\frac{4}{5} \times ({\rm Hw} - {\rm Y})\right)^{5/2} \times \tan(\beta) \\ & Q_{\rm P} = \left(\frac{1}{2} \times g\right)^{1/2} \times ({\rm Hw}({\rm Hw} - {\rm 2Y}))^{3/2} \times ({\rm Hw} - {\rm Y})^{-1/2} \times \tan(\beta) \end{aligned}$	nw,b,in,H
Macchione(2008) ²	$Qp = \left(\frac{1}{2} \times g\right) \times (Hw(Hw - 2Y))^{3/2} \times (Hw - Y)^{-1/2} \times \tan(\beta)$	Hw,B,m,H
	\/	1.147,10,111,11

Table 1. Formulas to estimate the peak effluent flow from a rupture **Source:** Author

The step consisted in implementing the HEC-RAS software (Figure 1) with the information for each simulated case (Table 2) and (Table 3). Each case varies its breach geometry of boundary conditions, initial piping elevation in the piping cases, Starting WS in some cases, the pool elevation at failure and breach formation time. It was necessary to adopt an equivalent breach since because the software only accepts breach with a symmetrical trapezoidal or triangular shapes.

Jurumirim Dam									
S	imulation	1	2	3	4	5			
Center Station of the breach		3035	3035	1110	3035	3035			
Final Botton	n Longitudinal Width	25	68	260	25	68			
Final B	ottom Elevation	533	539	554	533	539			
Lef	Left Side Slope			8	6.4	6.4			
Righ	nt Side Slope	2.7	2.7	8	2.7	2.7			
Breach	Weir Coefficient	1.44							
Breach Fo	rmation Time (hrs)			7.35					
Fa	ilure Mode			Piping					
Pipir	ng Coefficient			0.5					
Initial I	Piping Elevation	550	550	554	568	568			
Trig	ger Failure at		٧	NS Ele	V				
St	tarting WS	568							
Top of	Dam Elevation	570							
Breach Bottom Elevation			533						
Pool Ele	vation at Failure			568					
Pool Vo	olume at Failure	7100000							
Dam	Crest Width			6					
Slope of US	Dam Face Z1 (H:V)			6.4					
Slope of US	Dam Face Z2 (H:V)	2.7							
Ear	rth Fill Type	- 1	Fine H	omoge	eneous	6			
	Dam Type	Concrete - faced dam							
Dar	n Erodibility	High	High	High	High	High			
	05/jan	800							
	06/jan	800							
Boundary	07/jan	800							
conditions	08/jan			800					
	09/jan			800					
	10/jan	800							

Jurumirim Dam														
S	imulation	6	7	8	9	10	11	12	13	14	15	16	17	18
Center Station of the breach		3035	3035	3035	3035	3035	3035	3035	3035	3035	3035	1110	1110	1110
Final Bottom	n Longitudinal Width	25	25	25	25	25	25	25	68	25	68	260	260	260
Final Bo	ottom Elevation	533	533	533	533	533	533	533	539	533	539	554	554	554
Lef	t Side Slope	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	8	8	8
Righ	nt Side Slope	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	8	8	8
Breach '	Weir Coefficient							1.44						
Breach Fo	rmation Time (hrs)	7.17	7.27	7.17	7.27	7.27	7.27	7.17	7.17	7.17	7.17	7.17	7.17	7.17
Fai	ilure Mode				•		Ove	rtopp	ing		•			
Trigg	ger Failure at						٧	VS Ele	v					
Starting WS		568	568	568	568	568	568	568	568	568	568	568	568	568
Top of	Dam Elevation	570	570	570	570	570	570	570	570	570	570	570	570	570
Breach B	Bottom Elevation	539	539	539	539	539	539	539	539	539	539	539	539	539
Pool Elevation at Failure		568	570	568	570	570	570	568	568	568	568	568	568	568
Pool Vo	olume at Failure	7750000												
Dam	Crest Width	6												
Slope of US	Dam Face Z1 (H:V)	20.4												
Slope of US	Dam Face Z2 (H:V)	0												
Ear	th Fill Type	Fine Homogeneous												
	Dam Type	Concrete - faced dam												
Dan	n Erodibility	High												
	05/jan	800	800	1000	1000	1000	1000	1000	800	1000	1000	800	1000	1000
	06/jan	800	800	1200	1200	1500	1200	1500	800	1200	1200	800	1200	1200
Boundary	07/jan	800	800	1500	1500	2000	1500	2000	800	1500	1500	800	1500	1500
conditions	08/jan	800	800	1800	1800	2500	1800	2500	800	1800	1800	800	1800	1800
	09/jan	800	800	2000	2000	3000	2000	3000	800	2000	2000	800	2000	2000
	10/jan	800	800	2500	2500	3800	2500	3800	800	2500	2500	800	2500	2500
	10/jan	800	800	2500	2500	3800	2500	3800	800	2500	2500	800	2500	2500

Table 2. Input data of the Jurumirim Dam in the software HEC-RAS.

Source: Author

	Chavantes Dam					
S	imulation	1	2			
Center Sta	ation of the breach	1315				
Final Botton	n Longitudinal Width	50	00			
Final B	ottom Elevation	4:	10			
Lef	t Side Slope	- 2	2			
Righ	nt Side Slope	3				
Breach	Breach Weir Coefficient					
Breach Fo	rmation Time (hrs)	10	.86			
Fa	ilure Mode	Pip	ing			
Pipir	ng Coefficient	0	.5			
Initial I	Piping Elevation	421	473.5			
Trig	ger Failure at	WS	Elev			
St	tarting WS	47	74			
Top of	Top of Dam Elevation					
Breach I	Breach Bottom Elevation					
Pool Ele	Pool Elevation at Failure					
Pool Vo	Pool Volume at Failure					
Dam	Crest Width	11				
Slope of US	Dam Face Z1 (H:V)	0	.9			
Slope of US	Slope of US Dam Face Z2 (H:V)					
		No	n-			
Ea	rth Fill Type	homog	eneous			
		or Ro	ckfill			
[Dam Type	Dam	with			
Dar	n Erodibility	Hi	gh			
	05/jan	35	00			
	06/jan 35					
Boundary	07/jan	35	00			
conditions	08/jan	35	00			
	09/jan	3500				
	10/jan	3500				

	Chava	ntes Da	m						
S	imulation	3	4	5	6	7			
Center Sta	ation of the breach			1315					
Final Botton	n Longitudinal Width			500					
Final B	ottom Elevation			410					
Lef	t Side Slope			2					
Righ	nt Side Slope			3					
Breach	Weir Coefficient			1.44					
Breach Fo	rmation Time (hrs)	11.12	11.12	11.12	11.19	11.19			
Fa	ilure Mode		Ove	ertoppir	ng				
Trigg	ger Failure at		V	VS Elev					
St	tarting WS			474					
Top of	Dam Elevation	479.6							
Breach E	Bottom Elevation	391.8							
Pool Ele	evation at Failure	474	474.5	474	474	475.5			
Pool Vo	olume at Failure		9	500000					
Dam	Crest Width			11					
Slope of US	S Dam Face Z1 (H:V)	0.9							
Slope of US	S Dam Face Z2 (H:V)	0.9							
Ear	rth Fill Type	Non-homogeneous or Rockfill							
[Dam Type	Dam with corewall							
Dar	n Erodibility	High							
	05/jan	3500	3500	5000	5000	5000			
	06/jan	3500	3500	8000	8000	8000			
Boundary	07/jan	3500	3500	10000	10000	10000			
conditions	08/jan	3500	3500	12000	12000	12000			
	09/jan	3500	3500	20000	20000	20000			
	10/jan	3500	3500	35000	35000	35000			

Table 3. Input data of the Chavantes Dam in the software HEC-RAS Source: Author

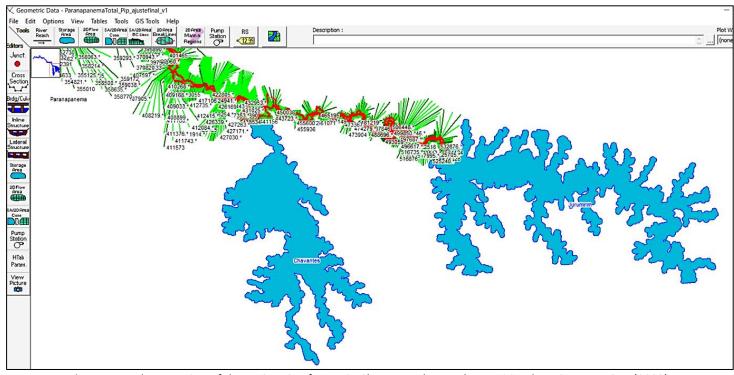


Figure 1. Implementation of the HEC-RAS software in Chavantes dam and Jurumirim dam. Source: FCTH (2003).

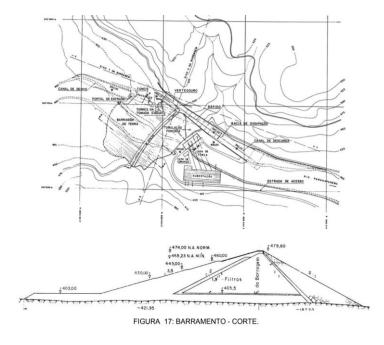


Figure 2. Elevation and section plan of the Chavantes dam. **Source:** FCTH (2003)

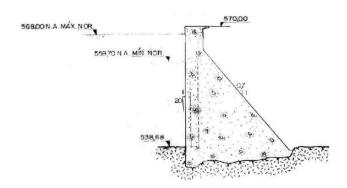


Figure 3. Section plan of the Jurumirim concrete dam. **Source:** FCTH (2003)

Results and Discussion

The software HEC-RAS gave us hydrographs that have information about the maximum flow and volume in 5 cases of piping failure and 13 cases of overtoppping failure simulated in Jurumirim Dam (Figure 4). Two cases in piping failure and 5 cases in overtopping failure in Chavantes Dam were analyzed as well (Figure 5).



Figure 4. Maximum Effluent Flow in the rupture of the Jurumirim dam. **Source:** Author

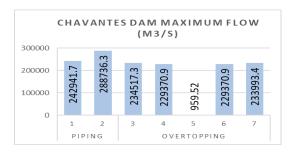


Figure 5. Maximum Effluent Flow in the rupture of the Chavantes dam **Source:** Author

These results were compared with the results generated from the formulas proposed by the mentioned authors. We can observe the following comparative table of case 7 with rupture by overtopping in the Jurumirim dam. In this case, the flow rate calculated by the HEC-RAS software equal to 39871.1 m3/s (Figure 6).

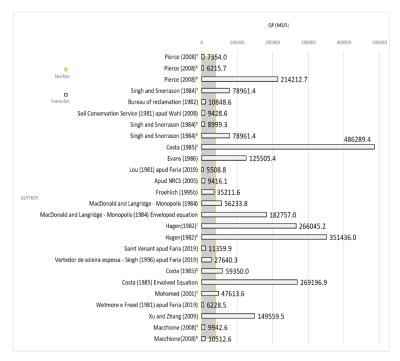


Figure 6. Comparative table of the peak effluent flows of a breach calculated by the HecRas software (dark gray) and by the proposed formulas (light gray) **Source:** Author

The degree of uncertainty between the results, provided in each case of Jurumirim dam and Chavantes dam, by the HEC-RAS software and those generated by the proposed formulas was calculated and classified (Figure 7) and (Figure 8). We can see that very few formulas are able to predict results close those that obtained HEC-RAS regardless of the amount of parameters considered what let us to postulate that the correct selection of information to use with the formulas is more important than detailed parametrization.

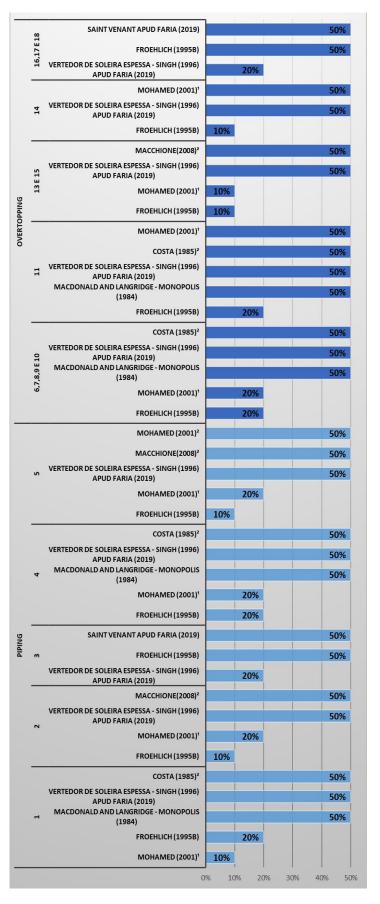


Figure 7. Degree of uncertainty between the results provided in each case by the HEC-RAS software and those generated by the proposed in formulas in Jurumirim dam. **Source:** Author

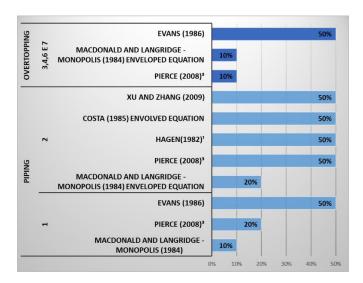


Figure 8. Degree of uncertainty between the results provided in each case by the HEC-RAS software and those generated by the proposed in formulas in Chavantes dam. **Source:** Author

The case 12 (rupture by overtopping in the Jurumirim dam) and case 5 (rupture by overtopping in the Chavantes dam) were not successful due to inconsistent results or not being able to be simulated by the HEC-RAS software. Therefore, these were not considered in the analysis. Taking into consideration that the number of remaining cases is valid and reliable.

We can select 3 to 5 formulas that are really closer to the results of the HEC-RAS software simulated in rupture by Piping and Overtopping.

According to **Wahl (2004)** the Predictions of peak flow have uncertainties of about 50% to 100% order of magnitude, except the Froehlich peak flow equation, which has an uncertainty of about 33% order of magnitude. Affirmation that was partially verified because the most accurate values resulting from this study have a range of 10% -50%. Also, Froehlich's most accurate and most generalized in this article.

Multiple linear regression analysis was used to develop an equation for predicting peak outflow from a breached embankment dam (Froehlich, 1995b). Even though initially the formula proposed by Froehlich and Macchione was analyze for embankment dams, they have a good performance whenh compared to the results of the simulations made in the Jurumirim dam on the Hec Ras software.

Also, Pierce expanded the embankment breach database by 44 case studies yielding a composite database of 87 cases. Linear, linear compound, curvilinear, and multivariable regression analyzes were performed on the composite database to develop best fit and envelope relationships correlating the height of water behind the dam H, the volume of water behind the dam V, the dam factor HV, and both the height and volume of water behind the dam H and V to the peak-breach discharge Qp (Pierce, 2010).

Regarding the Hydrogram with Parabolic Decay, this represents the emptying time more gradually than the triangular one, being more consistent with cases that have already occurred in earth dams, in which the descent stretches seek to represent an approximately exponential decay (Faria, 2019). Thus, for most cases analyzed, with

the exception of hydrographs generated by simulations 5 of the Chavantes dam and simulations 11 and 15 of the Jurumirim dam

Finally, based on the results generated, the number of cases analyzed by the authors in the creation of their formulae, and the revised literature recommends the use of the formulas proposed by **Froehlich** (1995b) and **Singh** (1996) in concrete dams and the formula de **Pierce** (2008) 3 on earth dams. In the second instance, the formulas mentioned in the work of **Mohamed** (2001) in concrete dams and those of **MacDonald Envelope equation** (1984) and **Evans** (1986) in earth dams can be used.

Also, were found formulae for the calculation of other variables of the breach (Table 4). In this way, there are a broad study panorama that could be analyzed in the future

V . II I . I .	Amount of	of "n" input variables								
Variable to calculate	formulas	n=1	n=2	n=3	n=4	n=5	n=6	n=7		
Peak Flow	26	11	7	4	1	2	1	0		
Breach formation time	8	3	4	0	0	0	1	0		
Volume eroded	2	0	2	0	0	0	0	0		
Breach area	4	0	0	3	1	0	0	0		
Average breach width	1	0	0	1	0	0	0	0		
Erosion rate	6	0	0	0	2	0	1	3		
Breach width as a function of time	4	0	0	0	1	1	2	0		
Breach height as a function of time	1	0	0	0	1	0	0	0		
Rate of water depletion	5	0	0	0	1	1	2	1		

Table 4. Classification of the formulae searched for the calculation of other variables of the breach. **Source:** Author

Conclusions

This article summarized the most important contributions to predict dam break flow from a historical review of the empirical and practical formulas and compared them to the results obtained from a computational. More than 50 different formulations were found in the literature, derived from different experiences and approach methods.

In general, in the Jurumirim dam, the most accurate formulas in all cases are the formulae proposed by Froehlich (1995B) and Singh (1996). Second, the formula proposed by Mohamed (2001) 1 is accurate in most of the cases analyzed. Third, the formulas proposed by the U.S. Army corps of Engineering, by Macchione, by Costa (1985) 2 and by MacDonald (1984) are met in some cases.

On the other hand, in the case of the Chavantes dam, the most precise formula in all cases is the one proposed by Pierce (2008)3. Second, the formula proposed by Evans (1986) and MacDonald Enveloped equation (1984) is fulfilled in most of the cases analyzed. Third, in some cases of Piping rupture there are the formulas of Xu and Zhang (2009), Costa (1985) enveloped equation, Hagen (1982) and MacDonald (1984).

Some limitations were found since some equations are homogeneous, have poor parameter description and are not always clear to the final user. The sue of the 1D HEC-RAS simulations to compare can also be criticized in terms of the accuracy when aspects as the symmetrical breach representation, high sediment concentration flows and so.

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