

# Empirical Representation of AKRD

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RAF Presentation 14 November 2016

# OVERVIEW

The topic addressed here:

AKRD is calculated using a standard empirical representation.  
That representation seems to be project-dependent.

- It is hard to understand why it should vary.
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## The standard representation:

$$\alpha = c_0 + \frac{\Delta p_\alpha}{q} (c_1 + c_2 M)$$

Fit to a reference that assumes zero vertical wind:

$$\alpha^* = \theta - \frac{w_p}{V}$$

# SOME WORRIES

Adjustment of the mean value:

$$\langle \alpha \rangle = c_0 + c_1 \left\langle \frac{\Delta p_\alpha}{q} \right\rangle + c_2 \left\langle \frac{\Delta p_\alpha}{q} M \right\rangle$$

- For given  $\alpha$ , expect  $\Delta p_\alpha$  and  $q$  to be approx. proportional
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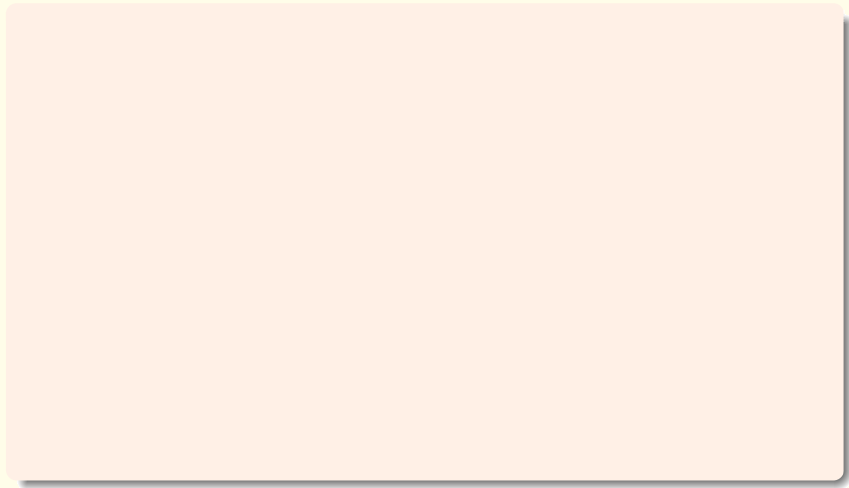
This affects sensitivity to  $\Delta p_\alpha / q$

The sensitivity to radome-pressure fluctuations is now:

$$\alpha' = (c_1 + c_2 M) \left( \frac{\Delta p_\alpha}{q} \right)'$$

- Response to radome fluctuations is then altitude dependence.
- There does not appear to be support for this.
- $c_2$  is project-dependent, depending on flight patterns.

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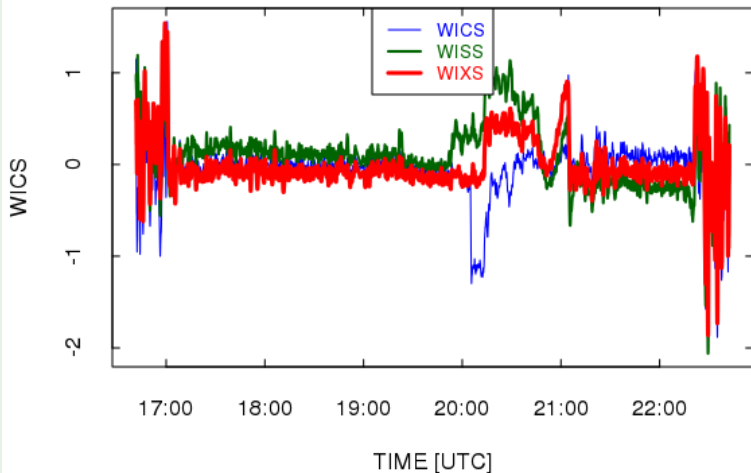
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The fluctuating component may be universal, while the slowly varying component is likely project-dependent.



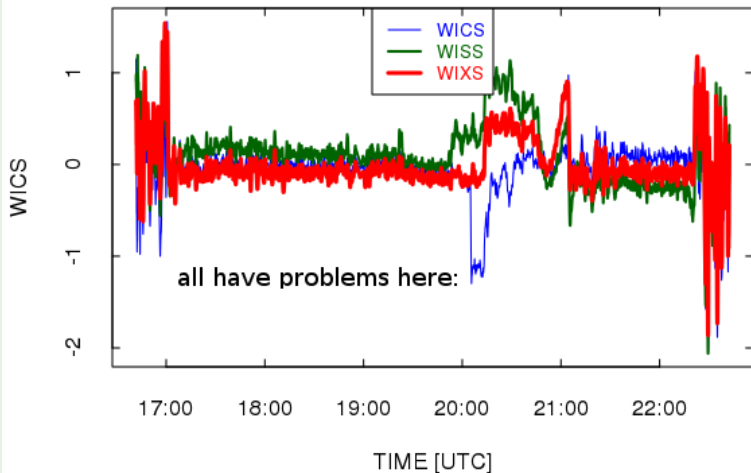
## EXAMPLE TO USE: ORCAS Flight 6

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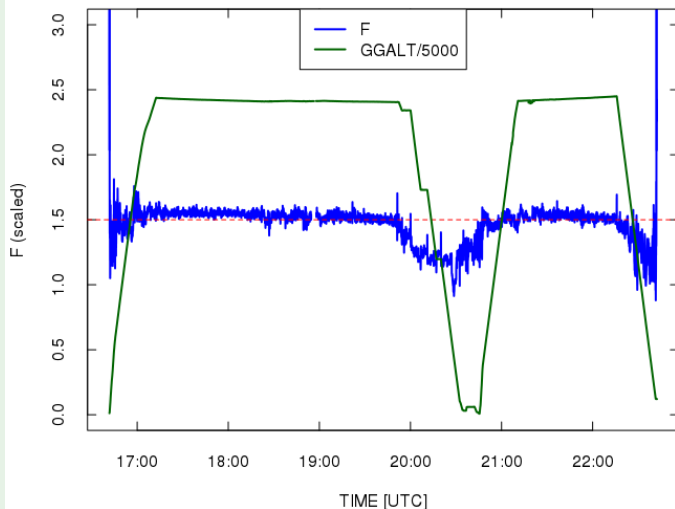
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Variations in  $F$  then can be used to identify unusual flight conditions like use of flaps, etc.

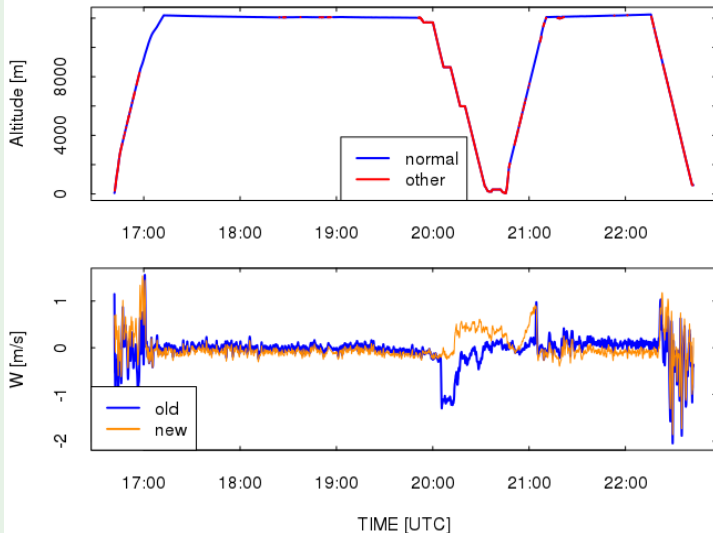
# PLOT $F$ FOR A REPRESENTATIVE FLIGHT:

## ORCAS Flight 6: Vertical Wind Errors Near 20:00



# FLAG ERRORS USING $F < 1.45$ :

Good vertical wind for normal flight:





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## COMPLEMENTARY FILTER:

- ① Split variables representing  $\alpha$  (like  $\Delta p_\alpha/q$ ) into two components:
  - (a) high-frequency component, 10-min cutoff
  - (b) low-frequency component, variable minus high-frequency component
- ② Fit these components separately so as to avoid mixing primary sensitivity and representation of the long-term mean. Fit to similarly filtered reference values.
- ③ Add the two contributions to obtain a final  $\alpha$ .

# THE COMPLEMENTARY FILTER

## Advantages:

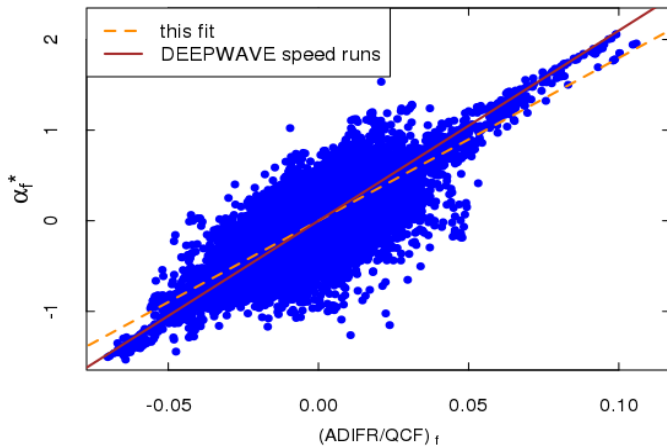
- This provides separate representation of the fast and slow components.
  - *This avoids having to use a single formula for both.*
  - *The most important fast component will not be distorted.*
- The result can be a reference to which to compare other approaches that might be easier to implement.
- The slow component is not removed, so valid larger regions of updraft can be measured.

## Disadvantage:

- This implementation requires a second-pass calculation.
- It will require further exploration to see if this can be implemented in nimbus instead.

# THE HIGH-FREQUENCY COMPONENT:

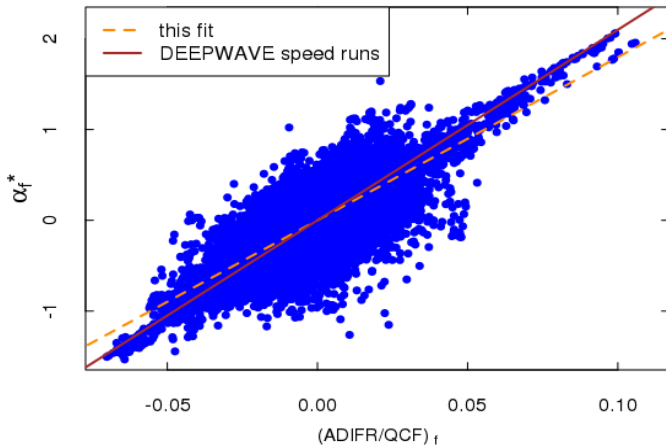
Combined ORCAS and CSET flights



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Combined ORCAS and CSET flights

what is wrong with “this fit”? (next slide)



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## Speed runs are best:

- Usually, little vertical wind, so no error in  $\alpha^*$
- DEEPWAVE value of  $c_1$  was 20.986. Result for many projects is similar when speed runs are used.
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## Regression fitting: A problem can arise from measurement errors.

- With spread-out scatterplots, there is often an apparent bias toward a more level result.
- Because there is a limited range represented in these composite fluctuations, the correlation coefficient is small even though the residual error is  $<0.1^\circ$ .
- If we are to assume a universal value for all projects, it should be based on speed runs. The DEEPWAVE value is perhaps best.



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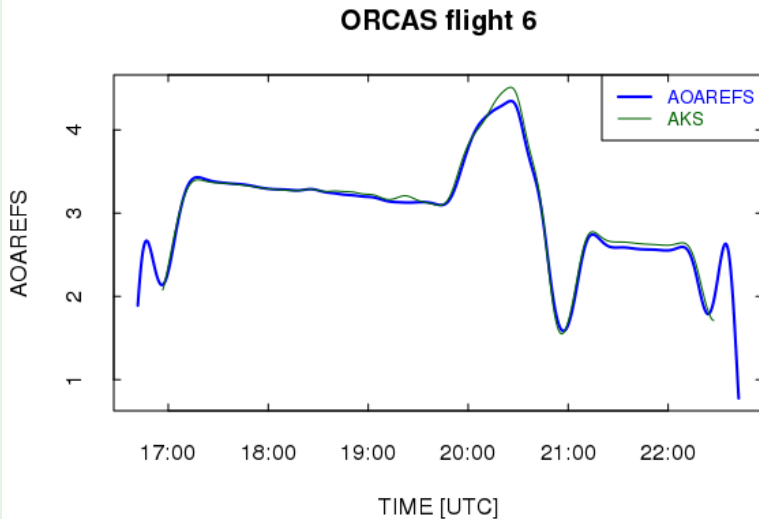
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A simple fit works well:

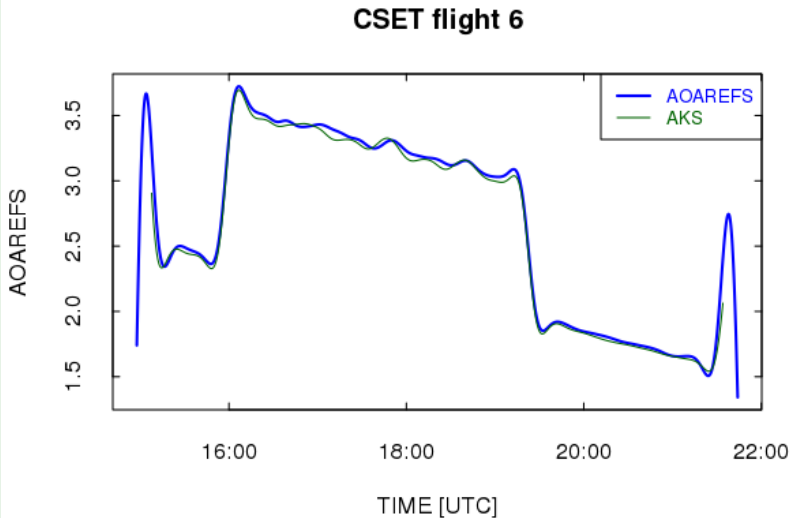
$$\alpha_s^* \sim d_0 + d_1 \left( \frac{\{\text{AKRD}\}}{\{\text{QCF}\}} \right)_s + d_2 \{\text{QCF}\}_s$$

Two-project residual:  $0.061^\circ$ ,  $R^2 = 0.99$

# THE LOW-FREQUENCY COMPONENT - ORCAS FLIGHT



# THE LOW-FREQUENCY COMPONENT - CSET FLIGHT





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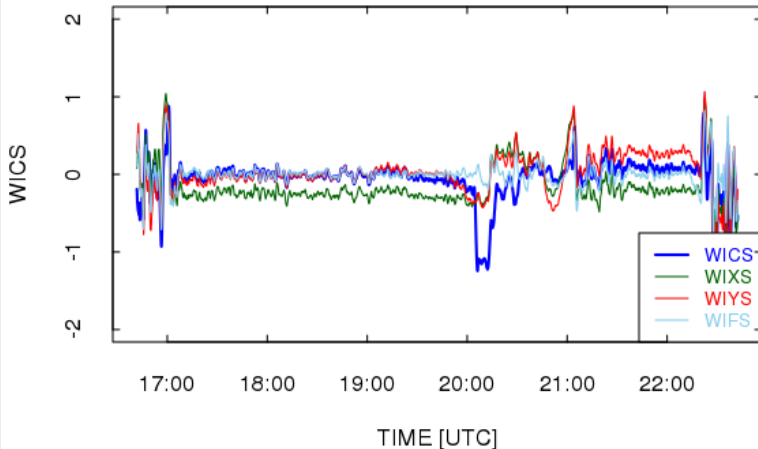
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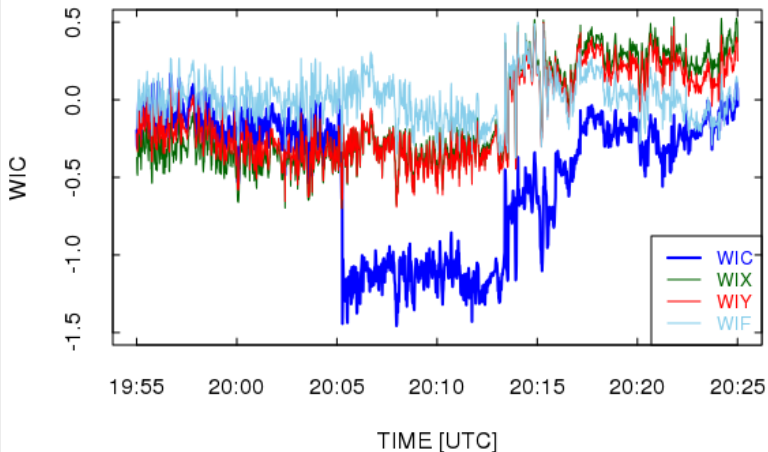
# ALL VERTICAL WIND VARIABLES

**ORCAS flight 6: mean WIY is 0.03, WIF is -0.00**



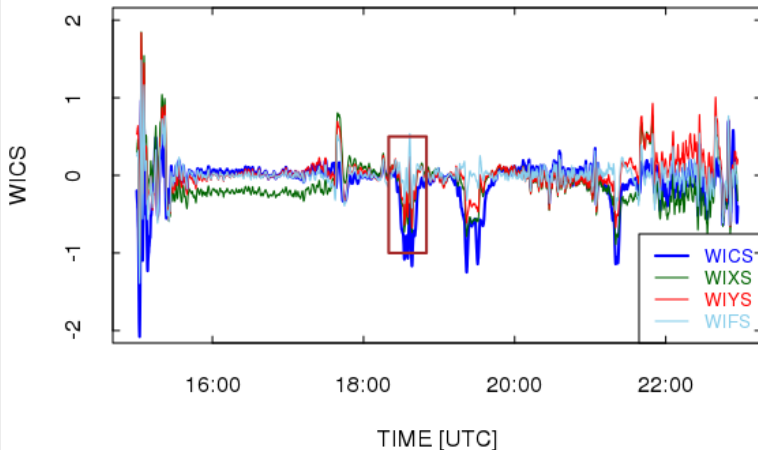
(SHORTER PERIOD)

### ORCAS flight 6

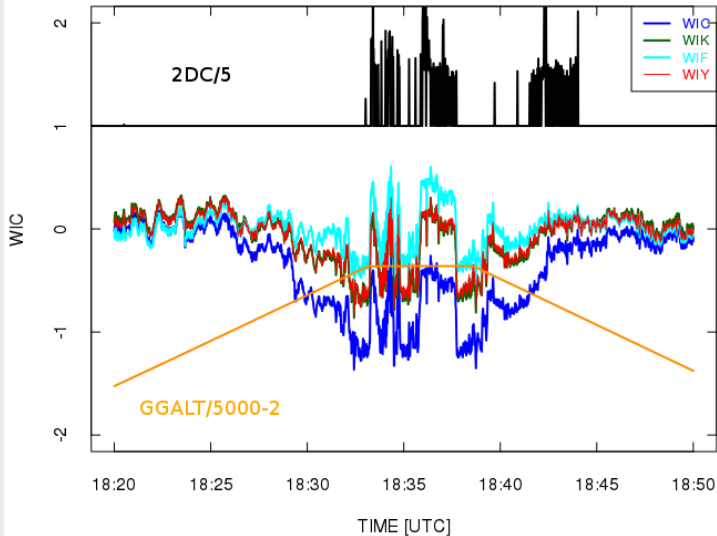


# A FLIGHT WITH DOWNDRAFT PERIODS

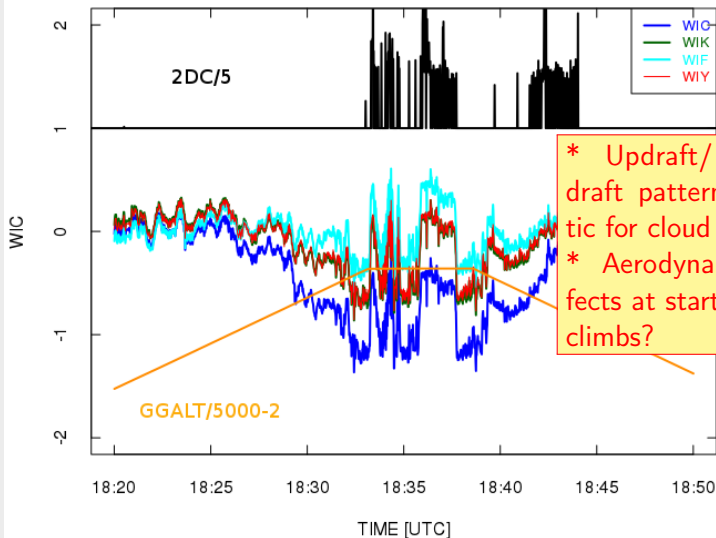
ORCAS flight 11: mean WIY is 0.03, WIF is 0.00



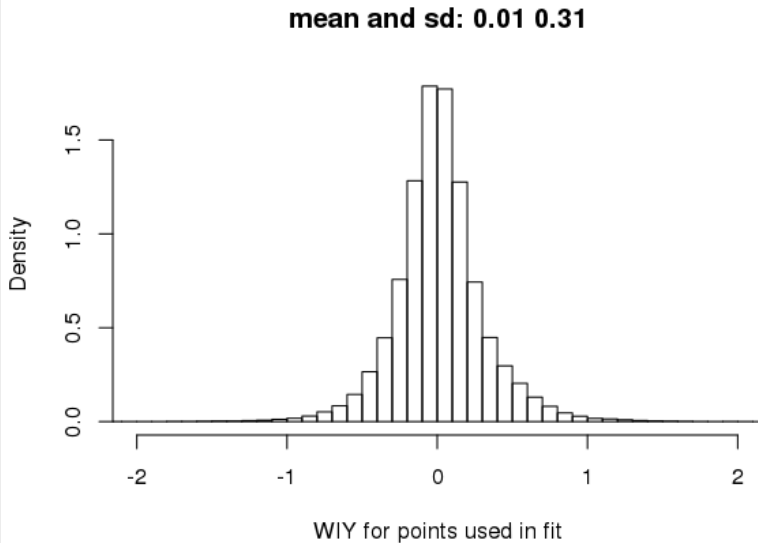
# EXPANSION OF BOX ON PREVIOUS SLIDE



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# RETURN TO QUESTION OF BIAS IN DATA FOR FIT





# RECOMMENDATIONS (BASED ON CSET/ORCAS)

## Consider the complementary-filter solution:

- ① Best representation of component with periods  $< 10$  min.
- ② “Fast” representation is consistent, all projects.
- ③ “Slow” representation is consistent for these two projects, maybe others.
- ④ The combination continues to show suggestions of aerodynamic effects
  - (a) lasting a few minutes at start and end of climbs
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## Alternatives:

- 1 Conventional all-two-project fit works fairly well.
- 2 Even better if QCF dependence is added.
- 3 Disadvantage: Leads to variable high-frequency sensitivity.

# IMPLEMENTATION STRATEGIES:

## Second-pass processor:

**Advantages:**

- \* Can do forward/backward filtering to avoid transient effects;
- \* Could be combined with other second-pass tasks like height-above-terrain and pitch/heading corrections, maybe someday Kalman filtering.

**Disadvantages:** Requires extra processing and attention.

## Incorporate in nimbus:

**Feasibility?** Not sure, but probably reasonably straightforward.

**Single-pass filtering:** Will have a disadvantage; probably very little difference vs forward/backward. **Need to test this.**