21 February 2015

To: WINTER project file
From: Al Cooper
Subject: Analyzing circle maneuvers

1 The circle maneuver

The circle maneuver consists of at least two circles, one each in a left turn and a right turn, with a one-minute straight leg between left-turn circle(s) and right-turn circle(s) and with a one-minute straight leg before and after the maneuver. The preferred orientation is for the straight legs to be against or along the wind direction. The circles are best flown under control of the flight management system, with heading bug turned so as to keep ahead of the heading and give a constant-rate turn (with a rate that may change with altitude). The turn rate is often <3 min for a 360° turn at intermediate flight altitudes for the aircraft. It is important to have a uniform and non-turbulent wind field and to maintain the roll angle constant. [1]

2 Running the 'CircleManeuver.R' program

CircleManeuver.R is an R script for fitting a sinusoidal variation to a circle maneuver. It is located on tikal.eol.ucar.edu in ~cooperw/RStudio/WINTER and also in this github directory (linked here). Anyone can download this from github, but the program makes extensive use of 'Ranadu' functions so this should be used with that package, also available on tikal and on github. The program that produced this memo, CircleManeuver.Rnw, contains the same R code; CircleManeuver.R was extracted from CircleManeuver.Rnw using 'purl'. This memo therefore provides documentation to accompany use of CircleManeuver.R. Running that script will make plots and print text messages similar to those in this memo, so users can use those to construct a new report for future maneuvers.

Here are steps for running the program:

1. Step 1 is to find the maneuvers. The current version of Review.R, being used for WINTER, includes a section that searches for candidates for maneuvers, and that may be helpful when finding appropriate periods to analyze. For example, on WINTER tf01, processing with Review.R printed this message:
   ```
   ## [1] "possible circle:  195221--195749, 703-deg turn"
   ## [1] "possible circle:  195919--200437, 700-deg turn"
   ```

   The flight track for the period from 19:52:00 to 20:04:55 is shown in Fig. 1, which shows that this was indeed a circle maneuver. The first circle was not flown very well, so the

[1] The time for a 360° turn (T) at airspeed (V) and roll angle (ϕ) is given by $T = 2\pi V/(g \tan\phi)$. For the particular example used in this memo, the bank angle was about 26.7°and $T = 159$ s for flight at 125 m/s at an altitude of about 12,500 ft.

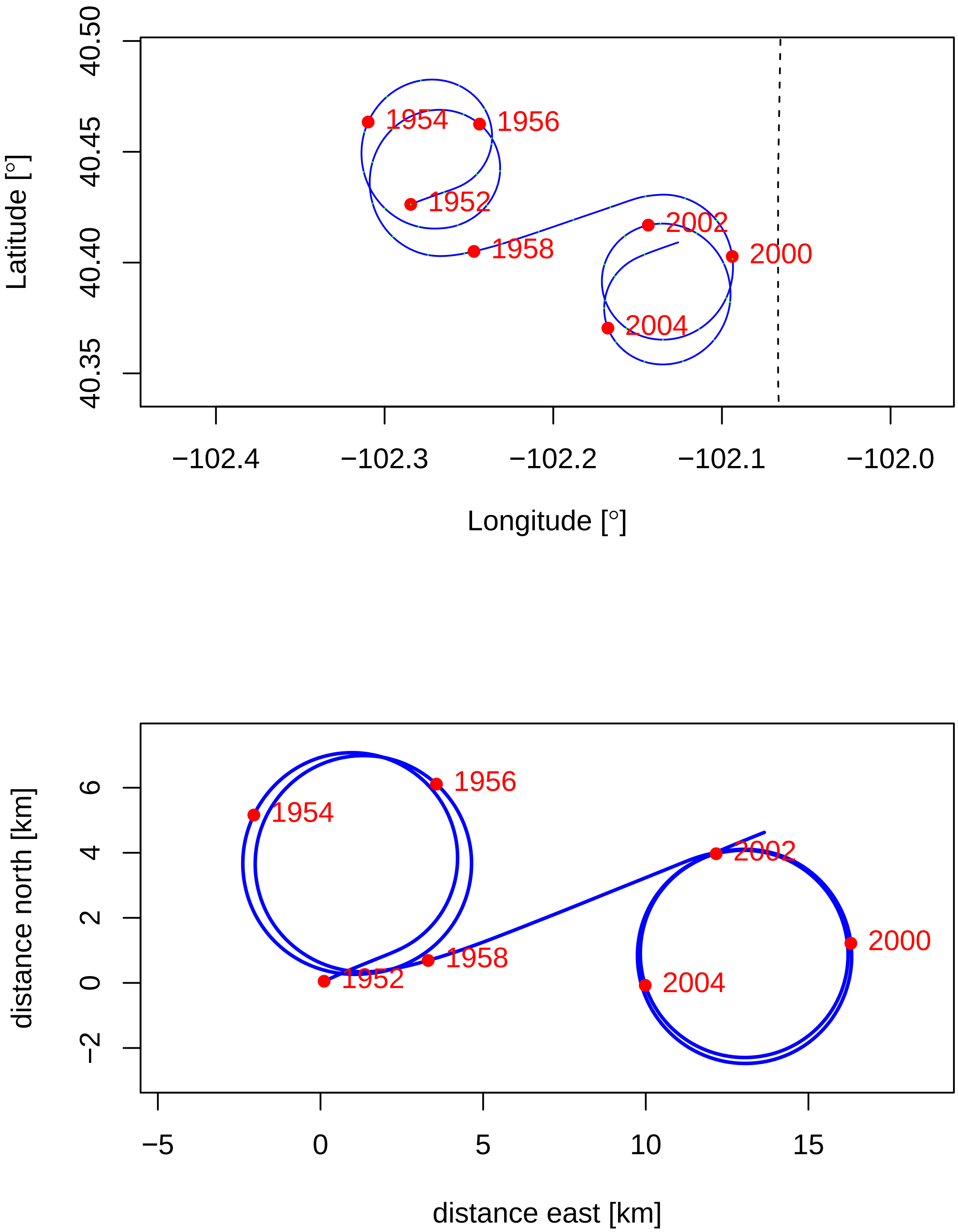

Figure 1: Portion of the flight track from WINTER tf01, 19:52:00 to 20:04:55, showing the circle maneuver flown at that time. The top panel shows the geographic position of the track, and the lower panel shows the track in a reference frame drifting with the wind.

period before 19:55 will be excluded from this example analysis. Deviations from normal attitude angles also are present for this first circle, and the sideslip shows a particularly large variation. Otherwise, this is is a good circle maneuver and a good example for analysis.

2. To use this routine, you have several options for configuration. The first is to use run-time arguments as follows, for example from a terminal window on tikal:
```
cd ~/RStudio/WINTER       ## cd to location of the routine
Rscript CircleManeuver.R WINTER tf01 195500 200410 0 0
```

   The second is to run using RStudio (or R directly) and execute the script CircleManeuver.R. In this case, the script will ask for the project name, flight number, and start and end times. In either method, if either time is -1 the script will search for the maneuver and if found will use the interval it finds. Also, if the data file is not found where it normally would reside, the script will ask for a complete path name for the file.

   A third option is to edit the default entries directly in the script and set 'UserInteraction <- FALSE' to prevent the program from stopping to ask for this information. The lines to set are in the first program 'chunk':
```
Flight <- tf01
Project <- ‘‘WINTER’’
startCircles <- 195200     # set to span the circle maneuver
endCircles <- 200455
UserInteraction <- FALSE
recalculateWind <- FALSE # option set by 5th runtime argument
findWindFromGPS <- FALSE # option set by 6th runtime argument
```

3. There are some optional sections in the 'CircleManeuver.R' script. The last two arguments in the Rscript call enable the first two, listed below.

   (a) There is an option enabled by 'recalculateWind' to recalculate the wind variables from original measurements. This can be used as a substitute for reprocessing the netCDF file with different options. For example, you can enter different sideslip sensitivity coefficients, heading offset, use different sets of measurements for calculating the wind, etc. [2] Another use for this option is to check suggested changes to see if applying corrections to, e.g., heading indeed results in reduced offsets as determined by the script.

   (b) An option enabled by 'findWindFromGPS' determines the wind only from the drift measured by GPS during the maneuver. Enabling this produces some additional text and plots showing results from supplemental fits that can complement the primary fit used in CircleManeuvers.R. When this option is set FALSE, the mean measured wind

[2] This was used in this example to recalculate the wind using different formulas for AKRD and SSRD, using the recommendations from FRAPPEprocessing.pdf.

direction is used instead of the wind produced by this supplemental fit. This option can be useful if there is suspicion that the mean measured wind direction is in error by more than a few degrees.

(c) The chunk 'search-for-circles' will look for circle maneuvers and use them if you provide start and/or end times of -1. This option is activated by supplying those flags in place of the times, either interactively or as the run-time arguments.

(d) There is an option 'supplemental-math' that applies only to this documentation program and causes it to print a long derivation that is duplicated elsewhere, notably in the wind uncertainty document. By default it is suppressed here but embedded for reference.

There were some special problems with the flight used for this example, so the 'recalculate-wind' option is used here although normally it should not be needed. [3]

3 Information that the circle maneuver provides

Circle maneuvers can provide this information on wind measurements:

1. An estimate of the mean wind, from GPS and heading only, that can be compared to the mean measured wind. The difference in magnitude should usually be within about 0.5 m/s for good wind measurements and steady conditions during the circle maneuver.

2. A test of the validity of TASX. For good measurements, the indicated error should be less than about 0.3 m/s.

3. An estimate of the offset error in sideslip (the first sensitivity coefficient).

4. An estimate of the offset in heading. The uncertainty in heading should be less than about 0.3° for normal operation, but this may vary from flight to flight and in the circle maneuver because of the horizontal accelerations imposed on the IRU by the maneuver.

3.1 The sinusoidal response expected if errors are present

The document on wind uncertainty contains a more extensive discussion of the basis for analysis of the circle maneuver. The key equation, with variables v and λ representing the true wind speed and direction, v_m the measured wind speed with error δv_m, V and δV the true airspeed and offset

[3] QCF failed and produced erroneously high values on this flight, so in processing PSFRD and QCFR were set as primary sensors for calculations including TASX. However, calculations for ATTACK and SSLIP were still based on the erroneous QCF and so were in error, and PCORs based on QCF were applied to PSFD to obtain PSFDC which was therefore also affected by the bad QCF measurements. For these reasons, the wind measurements and calculations of angle-of-attack and sideslip were repeated here using the variable QCFR and appropriate calibrations as in the FRAPPEprocessing.pdf memo.

in true airspeed, ψ and $\delta\psi$ the heading and offset in heading, ϕ the roll angle, and β and $\delta\beta$ the sideslip angle and offset in sideslip, is:

$$\delta v_m = -\delta V \cos\xi + \delta\xi\, V \sin\xi \tag{1}$$

where ξ is the angle between the flight direction relative to the air and the wind direction, defined as $\xi = \psi + \beta\cos\phi - \lambda$ and having error $\delta\xi = \delta\psi + \delta\beta\,\cos\phi$. For steady wind with speed v, the difference between the true wind speed and that measured around the circles is then

$$\delta v_m = v_m + A\,\cos\xi + B\,\sin\xi - v \tag{2}$$

where $A = -\delta V$, $B = V\delta\xi$, and v are three parameters that characterize the measurement errors. Best-fit values for these parameters can then be found by minimizing the error measure $\chi^2 = \sum(\delta v_m)^2$. Comparing wind measured upwind to that measured downwind results in a difference that is twice the error in true airspeed, while comparing wind measured while flying crosswind with wind from starboard to that measured while flying with wind from port results in a difference that is twice the heading error (corrected for sideslip error) multiplied by the true airspeed. However, fitting to the full circle makes it possible to use all the measurements to estimate these errors.

A problem with this approach is that the true wind direction λ is needed to provide the reference direction for ξ, yet an error in wind measurement will introduce an error in this reference direction. The circle pattern largely corrects for this by including equal weighting of errors from all directions of flight, but it still may be better to determine the wind direction solely from the drift of the pattern as measured by GPS. That largely removes the wind-measuring system from the process and so should provide a better reference direction for use in (2). This can be done by fitting the dead-reckoning flight track adjusted by the wind vector to the ground-speed track, or equivalently the dead-reckoning flight speed adjusted by the wind vector to the measured ground speed from GPS. This fit can include an unknown offset in heading, so that such an error will be corrected in the fit and should not affect the deduced wind direction. This option is normally suppressed but is included and discussed here to illustrate its use.

The horizontal components of the motion of the aircraft, as measured from the aircraft, are these:

$$\begin{aligned} v_{p,x} &= (V+\delta V)\sin(\xi' + \delta\xi') - v_x \\ v_{p,y} &= (V+\delta V)\cos(\xi' + \delta\xi') - v_y \end{aligned}$$

where $v_{p,x}$ and $v_{p,y}$ are the east and north components of the aircraft motion, v_x and v_y are the corresponding components of the wind, V is the true airspeed with possible error δV, ξ represents the direction of flight and so, for β=sideslip, α =angle-of-attack, and ϕ =roll, is $\xi' = \psi + \beta\cos\phi - \alpha\sin\phi$. [4] For fit parameters v_x, v_y, δV, and $\delta\xi$, best-fit values are found by minimizing the following error measure:

[4] The angle of attack enters here, while it did not in (2), because there the angle of attack and sideslip have already entered the calculation of wind so only the error in sideslip is relevant, while any error in angle of attack is neglected. That is the reason that the variable here has a prime accent.

$$\chi_2^2 = \sum \left((v_{p,x} - v_{g,x})^2 + (v_{p,y} - v_{g,y})^2 \right)$$

where $v_{g,x}$ and $v_{g,y}$ are the ground-speed components measured by GPS.

```
## [1] "wind determined from drift: 359.8 / 8.6"
```

The result is that the estimated wind direction is 359.8 with magnitude 8.6 m/s. This direction will then be used for the direction λ entering the relative angle ξ in (2). For comparison, the mean wind determined from the measurements was 1.0 / 8.5.

3.2 Results from fitting to the sinusoidal variation

```
## [1] "rms before and after fit adjustment: 1.08 0.39 m/s"
```

Figure 2 shows the measured wind speed as a function of the angle between the heading and the mean wind direction determined by the preceding fit. There is a clear sinusoidal variation, so it is useful to fit to these measurements to determine the magnitude of the errors. The best-fit results were a mean wind speed of 8.74 m/s, true airspeed correction of 0.14 m/s, and an offset in ξ of $-0.66°$. There is a significant reduction in the standard deviation of the measurement of wind speed, from 1.08 m/s in the original measurements to 0.39 after application of the corrections from the fit. Most of the standard deviation in the original measurements arises from the variation with ξ (adjusted heading). The fit provides confirmation of the validity of the measured true airspeed, after application of the corrections deduced from LAMS. (Cf. Cooper et al., 2014.) Note that, for this test case, PSFRD and QCFR are being used as primary pressure sensors and the corrections developed for that pair of pressure measurements are being used. The wind speed determined from the fit is also a very good match to the mean wind-speed measurement. However, the angular error $\delta\xi$ is large, significantly larger than the expected heading or sideslip uncertainty, and this indicates some problem needing further investigation.

The indicated error is $\delta\xi = \delta\psi + \delta\beta \cos\phi$, and it is not possible to separate the error in heading from the error in sideslip because $\cos\phi$ is constant throughout the left-turn and right-turn circles. Therefore, another approach to checking the sideslip angle is needed, as discussed in the next section.

4 Determining the sideslip offset

```
## Fit results: wind speed 8.74 m/s, TASX correction 0.14
## SS correction -0.57 deg., heading correction -0.08 deg.
```

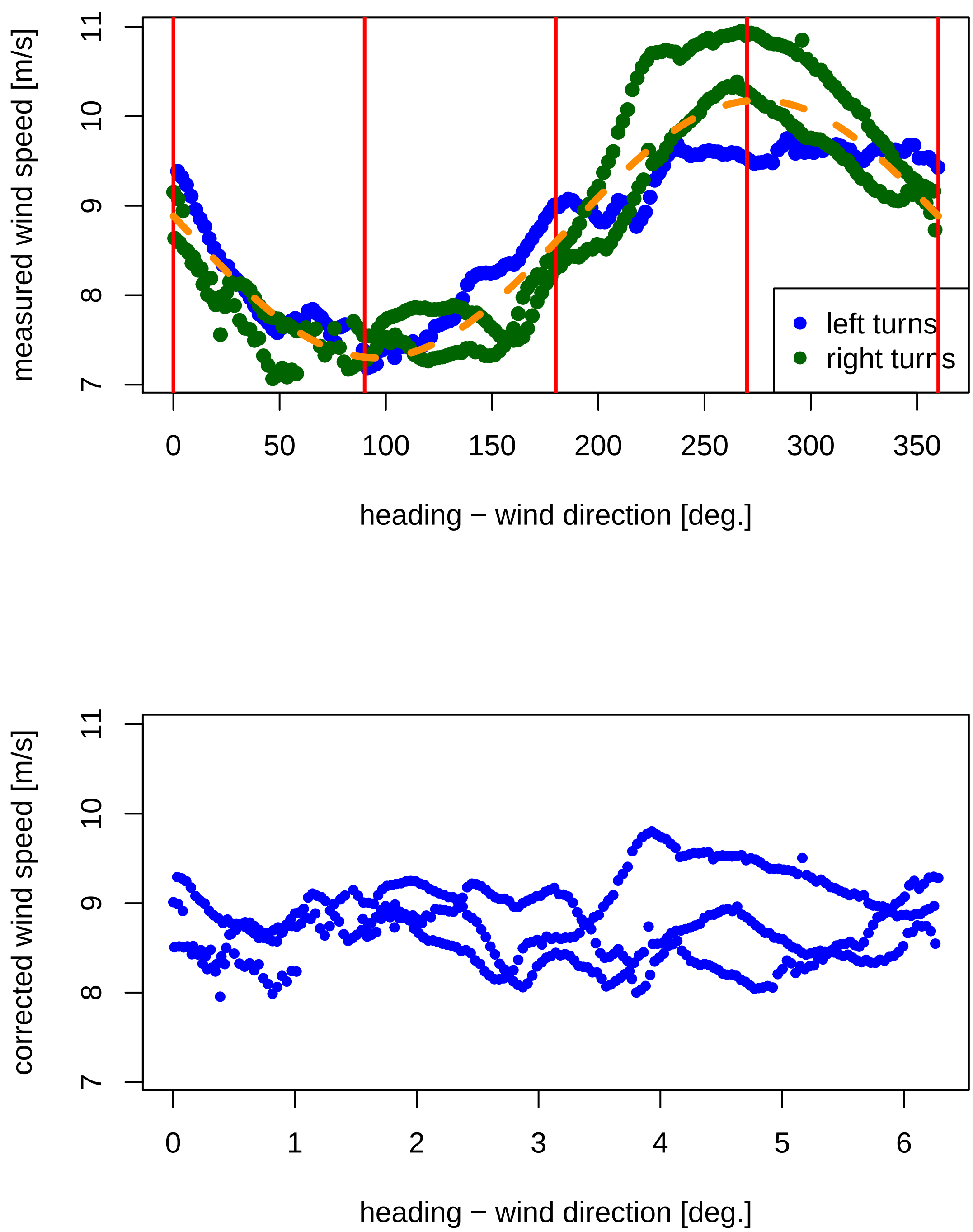

Figure 2: The measured wind speed as a function of the difference between heading and mean wind direction, for the measurements from the flight segment shown in the preceding figure. Top: before correction (with dashed orange line showing the correction function); bottom: after correction.

For level flight with zero vertical wind, the transformation equations from RAF Bulletin 23 indicate that, using some small-angle approximations, $-\sin\theta + \sin\phi\tan\beta + \cos\phi\tan\alpha = 0$ where θ is the pitch, ϕ the roll, α the angle of attack, and β the sideslip angle. Pitch and angle of attack can be assumed accurate because the former is checked, as part of the recalculation procedure, via determining the correction for Schuler oscillation and the latter is calibrated to low uncertainty via speed runs. Thus this equation can be used to deduce the offset in pitch measurement, while other maneuvers such as yaw maneuvers or reverse-heading maneuvers cannot separate a pitch offset from a heading offset. For small angles such that $\tan\beta \approx \beta$, $\tan\alpha \approx \alpha$ and $\sin\theta \approx \theta$, $\beta\sin\phi \approx \theta$-$\alpha\cos\phi$. With similar angle of attack and pitch in the left-turn and right-turn circles, but roll changing sign, it is expected that β will also change sign symmetrically from right-turn to left-turn circles. This can be checked via plots of sideslip vs time, as shown in the wind-uncertainty document, but here instead the sideslip will be checked by plotting the correction required for β to satisfy the preceding equation:

$$\delta\beta = (\theta - \alpha\cos\phi)/\sin\phi - \beta_m$$

where β_m is the measured sideslip and $\delta\beta$ is an assumed correction to that sideslip. For measurements from the circle maneuvers, the mean value of $\delta\beta$ was $-0.57^\circ \pm 0.02^\circ$, so this indicates that the error in ξ mostly arises from the error in sideslip. The residual error in heading is only -0.08°.

5 Conclusions

The circle maneuver from WINTER tf01 supports the validity of the wind measurements except for requiring a significant change in the sideslip offset. The indicated error in true airspeed and hence in the longitudinal component of the wind is only about 0.1 m/s, small compared to the expected uncertainty of about 0.3 m/s. The analysis indicates that a large correction to sideslip, of magnitude 0.57° should be made to the sensitivity coefficients previously recommended for FRAPPE.[5] With this correction to sideslip, the indicated heading correction is -0.08°. This is small enough to be within or close to likely uncertainty, so it may be preferable to impose no correction to heading. The corrections were iterated by making these changes and recalculating, leading to best results for 0.14 m/s subtracted from TASX, 0.57° subtracted from SSRD, and 0.14° subtracted from THDG.

6 Math details

– a long section of equations is suppressed here; uncomment %\mathtrue near line 64 to see it –

– End of Memo –

[5] The reason for the change in sideslip sensitivity coefficients vs those determined previously (cf. this document) is that the present analysis considers the effect of pitch on the deduced sideslip offset while it was assumed in the previous study that pitch would remain the same in right vs left turns. Surprisingly, that turns out not to be the case, so inclusion of pitch is important when determining the sideslip offset.

Reproducibility:

PROJECT:	CircleManeuver
ARCHIVE PACKAGE:	CircleManeuver.zip
CONTAINS:	attachment list below
PROGRAM:	CircleManeuver.Rnw
ORIGINAL DATA:	/scr/raf_data/WINTER/WINTERtf01.nc
GIT:	https://github.com/WilliamCooper/CircleManeuver.git

Attachments: CircleManeuver.Rnw
CircleManeuver.R
CircleManeuver.pdf
chunks/CircleSearch.R
SessionInfo