The problem being addressed

As originally developed from the LAMS observations, the fit for the PCOR Δp was expressed in terms of raw measurements QCF, PSF, Mach(QCF, PSF), ADIFR, and QCR. The advantage is that, by using only raw measurements, the function then does not depend on any other derived parameters. However, a problem arises if the sensitivity coefficients for the radome change, as is apparently the case. Because a likely dependence of Δp is on angle of attack, if the value of ADIFR changes for a given angle of attack then the original formula will lead to an incorrect value for Δp .

This suggests reformulating the equation for Δp in terms of angle-of-attack α rather than ADIFR and QCR directly. However, then a problem with implementation arises because QCXC, now used to find α, depends on Δp but a reformulated Δp will depend on α. This note suggests a complicated formula as a way around that circularity problem.

The math

The formula for Δp developed from fits to the LAMS data was:

$$\frac{\Delta p}{p} = a_0 + a_1 \frac{q_{\mathrm{m}}}{p_{\mathrm{m}}} + a_2 M^3 + a_3 \frac{\Delta p_\alpha}{\Delta q_{\mathrm{r}}} \tag{1}$$

with values of the coefficients $\{a_{i,i=0-3}\}$ respectively $\{-0.00076, 0.073, -0.0864, 0.0465\}$. The resulting formula for corrected q_c (QCFC) is then

$$q_c = q_m - \Delta p \tag{2}$$

The formula for α is:

$$\alpha = b_0 + \frac{\Delta p_\alpha}{q_c}(b_1 + b_2 M) \tag{3}$$

where M is evaluated from the corrected measurements q_c and p_c. However, this term is a minor contributor to the equation and M and q_c can be calculated from the uncorrected measurements q_m and p_m with negligible effect on the resulting value of Δp. The default coefficients from the Processing Algorithms document are b_i={4.604, 18.67, 6.49}; for CONTRAST the appropriate coefficients were found to be {4.34685, 20.10448, 1.36492}, and still other coefficients apply to DEEPWAVE.

Equation 3 solved for the pressure ratio needed in Eq. 1 is

$$\frac{\Delta p_\alpha}{q_r} = \frac{(\alpha - b'_0)}{(b'_1 + b'_2 M)} \frac{q_c}{q_r} \tag{4}$$

where primes on the coefficients $\{b'_i\}$ indicate that these should be the coefficients applicable at the time of the LAMS calibration that determined the coefficients in Eq. 1; i.e., for PREDICT where $\{b'_i\}=\{4.604, 18.67, 6.49\}$. However, α from subsequent flights when the radome sensitivity coefficients might have changed is given by Eq. 3 with new coefficients $\{b_i\}$. Substituting in Eq. 2 and using Eq. 1 results in the following expression for q_c:

$$q_c = q_m - p_m \left(a_0 + a_1 \frac{q_m}{p_m} + a_2 M^3(q_m, p_m) + a_3 \frac{(\alpha - b'_0)}{(b'_1 + b'_2 M)} \frac{q_c}{q_r} \right)$$

or

$$q_c = \frac{q_m - p_m \left(a_0 + a_1 \frac{q_m}{p_m} + a_2 M^3(q_m, p_m) \right)}{1 + p_m a_3 \frac{(\alpha - b'_0)}{(b'_1 + b'_2 M)} \frac{1}{q_r}} \tag{5}$$

where Eq. 3 should be used to evaluate α. For this purpose only, q_m could be substituted into Eq. 3 with negligible effect on the result obtained from Eq. 5.

The Circularity Problem

A problem with this approach, encountered as we tried to apply this to CONTRAST data, is that sensitivity coefficients are determined using Eq. 3, which depends on q_c, but q_c is evaluated using Eq. 5 which depends on α. For CONTRAST this was resolved by iteration, in which an initial estimate of $\{b_i\}$ was used to evaluate q_c, then flight data were used to fit for the coefficients in Eq. 3, and this new estimate of $\{b_i\}$ was used to repeat the calculation of q_c, etc., until $\{b_i\}$ no longer changed.

This is impractical for routine use, so a different approach is needed.

Suggested Solution

One change that avoids this circular dependence is to express Eq. 3 in terms of q_m (QCF) instead of q_c (QCXC). In the past, q_r (QCR) was used, but this was changed because invalid measurements from the radome, caused by the radome becoming blocked by ice accretion or by freezing of residual water in the lines from the pressure port to the transducer, were much more frequent than from the pitot tube and this caused problems with many derived measurements that depend on pressure. However, it is just as reliable to base the denominator on q_m (QCF):

$$\alpha = b_0^* + \frac{\Delta p \alpha}{q_m} (b_1^* + b_2^* M) \tag{6}$$

This makes it possible to determine the coefficients $\{b_i^*\}$ independent of the result for Δp (except through a minor residual dependence of M on Δp), which turned out to be barely detectable.

The procedure recommended for the future, and for reprocessing past projects other than CON-TRAST which has already been done as outlined above, is to use these sensitivity coefficients for angle-of-attack with Eq. 6:

pre-SAANGRIA-TEST $\{b_i\} = \{5.5156, 19.0686, 2.0840\}$

SAANGRIA-TEST and later: $\{b_i\} = \{4.6049, 18.4376, 6.7546\}$

Requirements for a PCOR function

Evaluation of $\Delta p = q_m - q_c$ then requires this input:

1. Raw measurements p_m=PCF, q_m=QCF, Δp_α=ADIFR, q_r=QCR.

2. PCOR calibration coefficients $\{a_i\}$ as listed above. These are fixed and can be coded into the PCOR function.

3. Current radome sensitivity coefficients $\{b_i\}$. These change and should be passed to the PCOR function.

4. The sensitivity coefficients $\{b_i'\}$ at the time the formula for Δp was originally determined using LAMS, as given above. These do not change and can be coded into the PCOR function.

Ad-Hoc Suppression of Flap-Deployment-or-Retraction Errors

Two additional steps are needed to avoid bad values at slow airspeed:

1. Evaluate the denominator in (5) separately. In normal operation, this should be close to 1, with only a small correction for the last term in the denominator. To avoid a region of singularity, when this denominator falls below 0.85 force it to 0.85. This only occurs for TAS<100, so it won't affect normal research data.

2. With this imposed limit on the denominator, the value of Δp determined from (5) using $\Delta p = q_m - q_c$ will still have undesirable effects at low airspeed. The problem region only occurs for TAS < 100, and QCF < 40 always leads to TAS < 100, so a way to taper the value of Δp for the problem region (to move Δp toward zero as the airspeed decreases) is to taper values of the correction DP=Δp for QCF<40 as follows:

```
if(QCF < 40.) {DP *= (QCF/40.)**3}
```

This avoids the creation of unrealistic perturbations during initial climb or final descent, where it appears that deployment or retraction of flaps causes a change in angle-of-attack through a region of singularity and also causes an airflow change not represented by the standard PCOR function. This approach is arbitrary but does keep the measurements from looking too bad during these transitions.

Documentation

This note is PcorCalculation.pdf, generated by PcorCalculation.lyx, both on cooperw@ucar.edu Google Drive, directory "Algorithms",available to anyone within UCAR. Appropriate modifications to Section 4.4 of the document re algorithms, ProcessingAlgorithms.pdf (same location), have also been made to reflect the recommended processing approach described above.

The fits leading to the recommended values of $\{b_i\}$ above were obtained using the R program AKRDcoef.R located in the EOL directory ~cooperw/RStudio/CONTRAST. For CONTRAST, the program used was TestReprocessing.R in the same directory. Associated processing for vertical wind for CONTRAST was studied and documented using the program CalibrationCONTRAST.Rnw in that same directory; this also generated the memo CalibrationCONTRAST.pdf in Google Drive for cooperw@ucar.edu, directory Algorithms.

For a possible implementation, see ~cooperw/RStudio/CONTRAST/CheckPcor.R or ~cooperw/RStudio/Ranadu/R/PcorFunction.R.

– End of This Note –