

Curvas Elípticas

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soma

$$\begin{array}{c|c} P \neq Q & P=Q \\ L = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} & L = \frac{(3 \cdot (X_1)^2 + A)}{(2 \cdot Y_1)} \end{array}$$

$$X_3 = (L^2 - X_1 - X_2) \bmod p$$

$$Y_3 = [L \cdot (X_1 - X_3) - Y_1] \bmod p$$

Negação

$$\begin{aligned} \text{primo} &= 11 \\ P &= (0, 2) \\ -P &= (0, (11-2)) \\ -P &= (0, 9) \end{aligned}$$

Subtração

$$\begin{aligned} P-Q \\ P+(-Q) \end{aligned}$$

Multiplicação

$$3P = (P+P)+P$$

soma - lambda

$$L = \frac{P \neq Q}{(Y_2 - Y_1)} \mid \frac{P=Q}{(3 \cdot (X_1)^2 + A) \cdot (2 \cdot Y_1)}$$

Soma - ajuste lambda

$$L = \frac{P \neq Q}{(Y_2 - Y_1) \cdot ((X_2 - X_1) \cdot \text{modinverse } p)}$$

$$L = \frac{P=Q}{(3 \cdot ((X_1)^2 \bmod p) + A) \cdot ((2 \cdot Y_1) \cdot \text{modinverse } p)}$$

modinverse - inverso modular

$3 * 0 \equiv 0 \pmod{7}$
 $3 * 1 \equiv 3 \pmod{7}$
 $3 * 2 \equiv 6 \pmod{7}$
 $3 * 3 \equiv 9 \equiv 2 \pmod{7}$
 $3 * 4 \equiv 12 \equiv 5 \pmod{7}$
 $3 * 5 \equiv 15 \pmod{7} \equiv 1 \pmod{7}$ <----- RESTO 1, ENCONTROU O INVERSO!
 $3 * 6 \equiv 18 \pmod{7} \equiv 4 \pmod{7}$

5 é o inverso modular de 3 mod 7

$$Y^2 = X^3 + 4X + 4 \bmod 11$$

p=11

qtd_residuos_quadraticos = (11-1) / 2

qtd_residuos_quadraticos = 5

$Y' = (i)^2 \bmod p$	$Y' = (p-i)^2 \bmod p$	crivos
$1^2 \bmod 11$	$10^2 \bmod 11$	1
$2^2 \bmod 11$	$9^2 \bmod 11$	4
$3^2 \bmod 11$	$8^2 \bmod 11$	9
$4^2 \bmod 11$	$7^2 \bmod 11$	5
$5^2 \bmod 11$	$6^2 \bmod 11$	3

X	Crivo = $X^3 + 4X + 4 \bmod 11$	Y'	Y''	PONTOS
0	$(0^3 + 4 \cdot 0 + 4) \bmod 11 = 4$	2	9	(0,2) (0,9)
1	$(1^3 + 4 \cdot 1 + 4) \bmod 11 = 9$	3	8	(1,3) (1,8)
2	$(2^3 + 4 \cdot 2 + 4) \bmod 11 = 9$	3	8	(2,3) (2,8)
3	$(3^3 + 4 \cdot 3 + 4) \bmod 11 = 10$			
4	$(4^3 + 4 \cdot 4 + 4) \bmod 11 = 7$			
5	$(5^3 + 4 \cdot 5 + 4) \bmod 11 = 6$			
6	$(6^3 + 4 \cdot 6 + 4) \bmod 11 = 2$			
7	$(7^3 + 4 \cdot 7 + 4) \bmod 11 = 1$	1	10	(7,1) (7,10)
8	$(8^3 + 4 \cdot 8 + 4) \bmod 11 = 9$	3	8	(8,3) (8,8)
9	$(9^3 + 4 \cdot 9 + 4) \bmod 11 = 10$			
10	$(10^3 + 4 \cdot 10 + 4) \bmod 11 = 10$			

$$E_{11} = \{P_{\infty}, (0,2), (0,9), (1,3), (1,8), (2,3), (2,8), (7,1), (7,10), (8,3), (8,8)\}$$