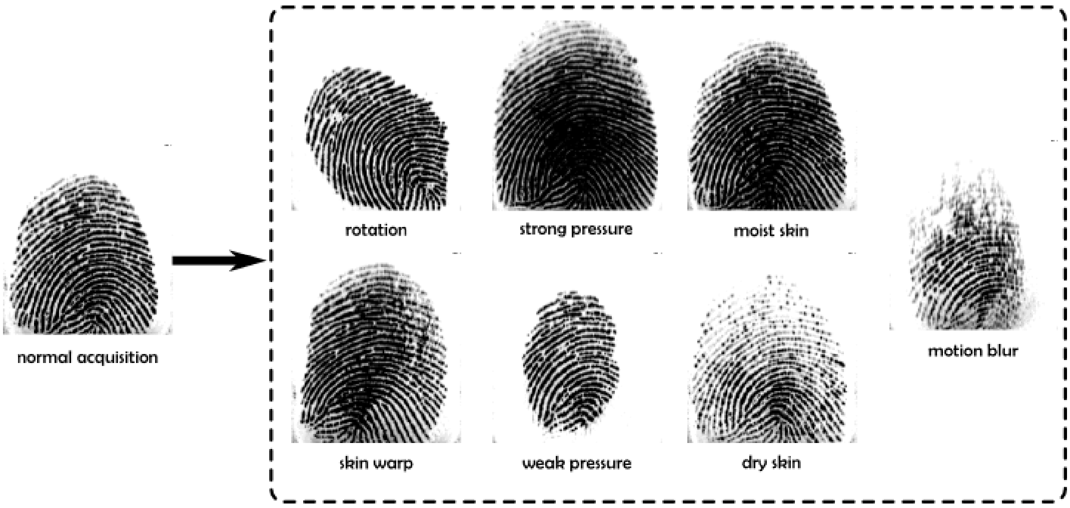
Digital analysis of fingerprints

1 – Presentation

1.1 Goal

Given a set of optical fingerprint images, find mathematical filters and models which best simulate artefacts that could occur during the fingerprint acquisition. An example is to try to answer the following question : given two real images of the same finger, how one goes from the image on the left to the right one ?

In the approach proposed in this project, seven algorithms need to be built to cover most common artefacts (motion blur, dry/moist finger, strong/weak finger pressure, rotations).



The goal of this project is to write a scientific application that will make use of the different mathematical filters and models suggested to solve fingerprint matching.

1.2 Library development

You will be working in a team. The library needs to be written in C++ and will contains a set of mathematical functions. You may use any software tools you have learn or heard of.

1.3 Work Plan

This document will guide you to the successful completion of your project. It contains multiple exercises, some of which you will be asked to answer in a final report ; those questions are labeled with a triangle at the beginning. It does not mean that you should skip the other ones because those extra questions may help you to debug/understand your work.

The assignment covers 4 topics, each having a “starter” and a “main course”. You will be asked to take at least three starters and two main courses, starter 1 being mandatory.

1.4 Grading

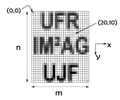
At the end of the project, you will turn a report and make a presentation of your work. The final grade will be based on - the technical writing of your report. - the presentation of your work. - the quality of the library you have develop. - your involvement in the project. - an evaluation by your pairs.

Each grading element will be detailed during the course of this project.

Read all the document at the beginning to make proper choice for libraries and class definitions.

2 – Image Loading, Saving and Pixels Manipulation

An image may be defined as a two-dimensional function $f(x, y)$ where x and y are spatial (plane) coordinates and the amplitude f at any pair of coordinates (x, y) is called the intensity or gray level of the image at that point (**Digital Image Processing**, *RC Gonzales, RE Woods*, 3rd Edition). Nowadays most of the images we encounter are digital such that x, y and the intensity values of f are all finite, discrete quantities. A digital image is composed of a finite number of elements named "pixels" (picture elements), each has one coordinate (x, y) and an intensity value in the integer range of $[0, 255]$. The size of an image (n, m) is usually defined as the number of pixels along its height n versus its width m.



The choice of a basis for pixel coordinates is arbitrary ; the convention is to choose the top left pixel as the origin $(0, 0)$ but one may define it in another way. Yet it needs to be defined properly from the beginning because any geometrical operation on the image will use pixel coordinates on this basis.

A common practice in image processing is to map intensity value originally in $[0, 255]$ to $[0, 1]$ when converted to a float ; again it is a convention which use is left to the project team. Note that it is important to map back the values to integers in $[0, 255]$ before saving any image.

2.1 Starter 1

The first round of questions is related to image loading, pixel manipulation and image saving.

- Describe your image frame for the project and the intensity range you will consider for the images.
- Look for an image processing library in C/C++ and load one image of the project. Some good options are openCV or Cimg libraries, but any other relevant solutions are welcomed. Support your choice.

At this point, you should build a class with appropriate members and methods which will help you thereafter in the project. You can assume that all images we will process are greyscale (and not RGB).

- Search for the minimum and the maximum in pixel intensity values, identify their values and data type, understand which one corresponds the black and the white. Create a method to cast all pixels values into floating numbers (most of the operations we will perform needs floating point arithmetic).
- By changing some pixel values, create white and black squares in the image as below :

- Save the result in a new image as a png file and check the integrity of the result.

- Let $s(x, y)$ be the image after performing a symmetry transform of the image $f(x, y)$ along the y axis. Express the mathematical relationship between those two images.
- Implement a method which does this operation, and save the resulting image in a new file. Do the same for a symmetry transform along the x and y diagonal axis.
- According to you, is this pixel swapping operation a rotation ? Justify your answer ; you can think of expressing the matrix which transforms the pixels coordinates during the symmetry operation and compute its determinant.

2.2 Main course 1

As a first order approximation and because of the spherical shape of the finger, one can simulate the pressure variation of the finger onto the sensor surface as a local spot on the image where pixels intensity is high, and decreasing as the distance to the center of the spot increases. The two images below give an example of what is observed in practice, the red square on the right image with a weak pressure of the finger is the location of the spot center, according to our definition :

The pixel operation to perform on the original image f is :

$$g(x, y) = c(x, y)f(x, y)$$

where $c(x, y) \in [0, 1]$ is a scalar coefficient - also known as a weight - and g the resulting image. The coefficient function will tend to decrease the pixel intensity of the original image as it goes towards zero, or keep it the same when close to one.

- Assuming first that the coefficient function is isotropic and defined as $c(r)$ with r being the euclidean distance between the center of the spot in the image and the pixel where the coefficient needs to be computed (with have $c : \mathcal{R} \to [0, 1]$), suggest few mathematical functions which monotonically decrease as $r \to \infty$, with $c(0) = 1$ and $\lim\_r \to \infty c(r) = 0$.
- Implement a function which takes as input the pixel coordinates of the spot, a set of pixels coordinates to which the coefficients should be computed, maybe some extra parameters which tune the function c, and outputs the coefficients values. You may want to test several functions.
- Test your function(s) on the image `clean_finger.png` and comment your results given what the image `weak_finger.png` give in practice.
- Looking at the images in the assignment, you may have noticed that the pixel intensity transform seems anisotropic. Recall the definition of anisotropy and explain why you can make this observation.
- Choose the function $c(r)$ which satisfied you the most in the previous question and adapt it to deal with anisotropic pixel operations (now you should consider $c(x, y)$ in its original definition).
- Implement a function which computes the coefficients according to your new function c, it would be nice to be able to choose the direction of the pixel intensity transform.
- Test it on the image `clean_finger.png` and comment your results compared to the first you got.

3 – Geometrical Warps

Image warping is the process through which the pixels coordinates (x, y) of an image are transformed according to a motion model. There exists many of those motion models : translation, rotation, similarity, affine or projective are amongst the most well known. It is of great use in many computer vision application, for example to create a seamless panorama from several images of a scene. The interested reader can take a look at section 2 of the article **Image Alignment and Stitching : A Tutorial** (*R. Szeliski*, 2006, foundations and Trends in Computer Graphics and Vision) to get a better understanding of the image warping operation.

3.1 Starter 2

This set of questions will explain you how to perform geometrical operations on images.

- Describe the motion model which relates pixels on the left image to the one on the right :

- Find a mathematical function $w : \mathcal{R}^2 \times \mathcal{P} \to \mathcal{R}^2$ which changes a pixel coordinates (x, y) given some parameters p in $\mathcal{P}$ accordingly to this motion model :
$$(x', y') = w(x, y; p)$$
- What are the parameters of this motion model ?
- Implement a method which, given a set of pixels coordinates and model parameters, outputs the new pixels coordinates.
- We now would like to visualize the transformed image ; in other words compute the new pixels intensity values given the pixels of the original image. What is the mathematical operation dedicated to this task ?
- Provide at least 2 examples of such methods with some explanation of how it could be applied in a two dimensional case. Give their algorithm complexity for an (n, m) image.
- Implement one of those methods, it should take as input : the original pixels coordinates, their values, and the transformed pixels coordinates. The output should be the new pixels values. The difficulty of the chosen method will be accounted for the final mark.
- How can you double check your algorithm using mathematical properties of the motion model you previously identified ? Think about w^{-1}. Give some test examples on your report which validate your algorithm, and comment on any disparity you would observe.
- ▷ Test your algorithm on the image `clean_finger.png` and try to parameterize at best your function $w(x, y; p)$ so that your output image is as close as possible to `warp1_finger.png`

3.2 Main Course 2

The finger skin elasticity, though useful for precise dexterous manipulation, is the consequence of many fails of fingerprint authentication systems. On the two following acquisitions of the same finger, the right acquisition is the result of a small rotation while the skin was under pressure during the acquisition :

Looking at the red square, one should notice that the black ridges of the fingerprint look squeezed in some part of the region and spaced out in the other part. The rest of the fingerprint image "almost" looks the same, such that this warp can be considered as local. Therefore, the warp function may look something like this :

$$w(x,y) = \big(x + \delta\_x(x,y), y + \delta\_y(x,y)\big)$$

with $\delta\_x : \mathcal{R}^2 \to \mathcal{R}$ and $\delta\_y : \mathcal{R}^2 \to \mathcal{R}$ being two functions which influence decreases - they should go towards zero - as the pixel coordinate (x, y) is away form the defined center of the warp. Note that piecewise warp function could be a solution but may create a discontinuity on the boundaries.

- Define a warp function which transform pixels coordinates according to this skin elasticity property. It should mimic this "squeezed and spaced out" observation made on real images. At least two parameters need to be considered for this warp : its location on the image (as a local warp, not all the image should be affected) and its strength. Other parameters could be of interest, like its orientation ; think that once the finger is under pressure, you can still translate or rotate it a bit.
- Implement this function and test it with different values of parameters on the image `clean_finger.png` to see how close to the reality (the image `warp2_finger.png`) your warp model can be.

4 – Linear Filtering

In image processing, linear filters are of great use for image denoising, resampling or features extraction. The convolution operation usually assumes spatially invariant filters, and is equivalent to linear filtering the image. The interested reader could look at **The design and use of steerable filters** (*WT Freeman, EH Adelson*, 1991, transactions on Pattern Analysis and Machine Intelligence) for an enlightening use of linear filters in the context of computer vision.

4.1 Starter 3

- Give a mathematical definition of the discrete convolution operation in 2 dimension. Explain with an illustration how it should work on a 2D grid.
- Implement a method which, given two matrices of arbitrary size, outputs the result of the convolution process between those two matrices ; the resulting matrix should be the same size as the first one in the function call. Find a suitable strategy to deal with image boundaries and explain it.
- What is the algorithm complexity of the convolution operation with respect to the number of pixels in both matrices ? Consider a naive implementation of the convolution.
- Explain the relationship between the Fourier transform and the convolution.
- Implement the convolution operation using the Fast Fourier Transform (use a library to compute 2D Fourier transforms like fftw if the image library you chose does not implement it), and double check your results comparing both convolution algorithms on an image of the project. Display the comparison in your report and try to explain any disparity.
- What is the algorithm complexity using the FFT to do the convolution operation ? Again consider a naive implementation of the FFT.
- Say we have a kernel of fixed size $(15, 15)$ pixels with which we want to convolve to an (n, m) image, when is it more efficient to use one method instead of the other with respect to the number of pixels in the image ?
- As a first order approximation, a motion blurred image f_b can be represented as the result of the convolution of the original (sharp) image f with a blurring kernel k :

  $$f\_b(x,y) = k * f(x,y)$$

  Can you suggest a blurring kernel $k(x, y)$ of maximum size $(15, 15)$ to which the image `clean_finger.png` is convolved to obtain the blurred right image `blurred_finger.png` ?

The kernel elements should sum up to 1 so that image energy is preserved. Don't expect to get good results against the real blurred image, but you should be able to simulate blur caused by a translation of the finger. You can find some hints about blur kernels in : **Removing camera shake from a single photograph** (*R. Fergus, B. Singh, A. Hertzmann, ST Roweis, WT Freeman*, 2006, SIGGRAPH).

4.2 Main Course 3

You probably end up with poor results while trying to simulate the motion blur for the last question. If you take a closer look at the image `blurred_finger.png`, you may notice that only the fingerprint surroundings look blurry and faded, while its center is still sharp. This seems consistent with the spherical shape of the finger and the properties of the skin

which make the finger stick to the surface. Thus the kernel blur should not be spatially invariant nor preserve the energy in the image :

$$f\_b(x, y) = k\_x, y * f(x, y)$$

where the subindex (x, y) for k_x, y expresses the dependency of the kernel with respect to its location on the image.

- ▷ Do you think you can use the FFT to implement this operation ?
- ▷ Given the center (x_c, y_c) of the fingerprint (we will assume that it is the point where the pressure is the highest), define a kernel function k_x, y whose energy decreases with its distance from the center point (x_c, y_c). The energy could decrease linearly or quadratically, but the sum of the kernel's elements has to remain in $[0, 1]$.
- ▷ Implement this function and test it on the image `clean_finger.png` ; you can consider first the identity kernel.
- ▷ Given the same center (x_c, y_c), define a kernel function which increases the blur on the image as its distance to the center point increases. You could first define a kernel for the fingerprint boundaries, and think of a way to make it evolve to the identity kernel as it distance to the center decreases.
- ▷ Implement this function and test it on the image `clean_finger.png` ;
  try to get closer to the image `blurred_finger.png` in tuning your algorithm.
- ▷ Given the kernel and the blurred image, do you think that the convolution operation could be reversed to deblur the image ? Give some explanation to support your answer, feel free to google the problem using appropriate keywords and give your sources (if any).

Note that in real life, the kernel is usually unknown and need to be estimated ; a really nice solution is suggested in **Removing camera shake from a single photograph** (*R. Fergus, B. Singh, A. Hertzmann, ST Roweis, WT Freeman*, 2006, SIGGRAPH).

5 – Morphological Filtering

Morphological filtering is a theory developed in the 1960s for the analysis and processing of discrete images (**An overview of morphological filtering**, *J. Serra, L. Vincent*,1992, Circuits, Systems and Signal Processing). It is a collection of non-linear operations related to the morphology of features in an image, such as boundaries, skeletons, . . . Similarly to the convolution operation, the image is scanned with a mask (called kernel previously) of predefined shape, this shape defines the region of interest or neighbourhood around a pixel ; it can be a square, circle, or diamond with the origin at the center is used. Morphological operations rely only on the relative ordering of pixel values, not on their numerical values, and therefore are especially suited to the processing of binary images, but those techniques can be extended to greyscale images as well. Their main goal is to remove imperfections on the images which could not be easily treated with linear or median filters, but other applications exists as we will see.

5.1 Starter 4

- Perform a non exhaustive bibliography on morphological filters to understand their action on images. Explain with an illustration how the dilatation and erosion operations work a 2D grid given a mask (aka structuring element). Cite your sources (articles, web links, . . .).
- According to you, what morphological operation would you perform to get the image on the right (`moist_finger.png`) given the image `clean_finger.png` ? Justify.

- Same question with the image on the right `dry_finger.png`.

- Because morphological filters are more easily applied on binary images, provide a mathematical explanation of the binarization operation of an image and implement such a method.
- explain how to properly choose the threshold parameter for the binarization operation. If you cannot find a clever way to do this, we recommend you to read **A threshold selection method from gray-level histograms**, *N. Otsu*, 1975, Automatica.
- Implement erosion and dilatation operations for binary images. Test your algorithm on the image `clean_finger.png` with several structural elements of different sizes till you approximate `moist_finger.png` and `dry_finger.png`. Detail your results on the report.
- Does the operation of successive erosion and dilation act the same as dilation and erosion ? Explain your answer with examples.

5.2 Main Course 4

The goal here is to extend the morphological operations to greyscale images, and adapt it to the fingerprint case. The input/output of your algorithm will now be greyscale images, however the structuring element will still be binary.

- Search documentation to understand how morphological filters can be adapt to greyscale images. Explain on your report and provide your sources.
- Implement the method to process greyscale images ; check on the image `clean_finger.png` if your result is correct.
- As noticed before, the spherical shape of the finger makes the processing relevant only locally. Suggest a way to adapt your algorithm to this observation either for the `moist_finger.png` and `dry_finger.png`.$; ideally, the transition should be seamless so that the processing does not create new non natural artefacts on the image. Implement your solution and show the improvement on the images.