



Parallel Parsing

The Implementation of a Parallel LL Parser Generator

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Introduction

- Parallel LL parser generator.
- $\text{LLP}(q, k) \subseteq \text{LL}(k)$.
- Pareas uses $\text{LLP}(1, 1)$.
- Two mistakes in the paper that describes LLP.
- Tested the parser generator thoroughly.

LL parsing

Here is a LL(1) grammar.

$$1) T \rightarrow R \quad 2) T \rightarrow aTc \quad 3) R \rightarrow \varepsilon \quad 4) R \rightarrow bR$$

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Accepted! the string “*abc*” can be parsed.

LLP Parsing

Augment the grammar, this grammar is LL(1) and LLP(1, 1).

$$0) T' \rightarrow \vdash T \dashv$$

Now we parse the string " $\vdash abc \dashv$ " instead and a LLP table is needed.

1. Initial pushdown store.
2. Final pushdown store.
3. Left parse.

	\vdash	a	b	c	\dashv
ε	$(T', T \dashv, 0)$				
\vdash		$(T, Tc, 2)$	$(T, R, (1, 4))$		$(T \dashv, \varepsilon, (1, 3))$
a		$(T, Tc, 2)$	$(T, R, (1, 4))$	$(Tc, \varepsilon, (1, 3))$	
b			$(R, R, 4)$	$(Rc, \varepsilon, 3)$	$(R \dashv, \varepsilon, 3)$
c				$(c, \varepsilon, ())$	$(\dashv, \varepsilon, ())$

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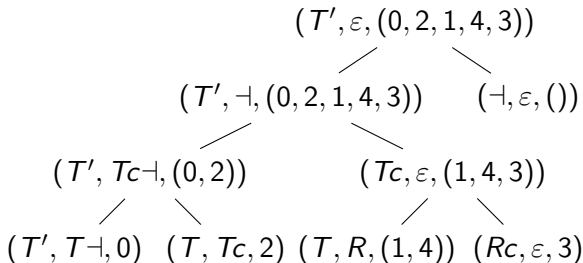
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3. Left parse.

(ε, \vdash)	(\vdash, a)	(a, b)	(b, c)	(c, \dashv)
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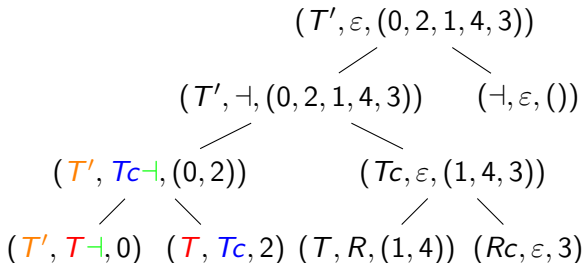
LLP Parsing using Parallel Reduce

Use the associative **glue** operation.



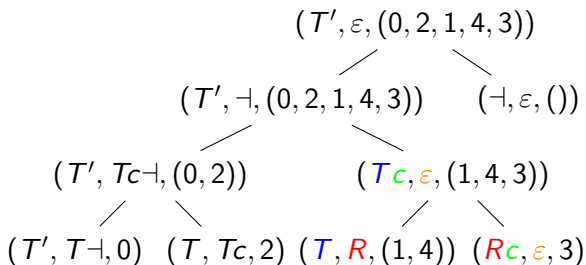
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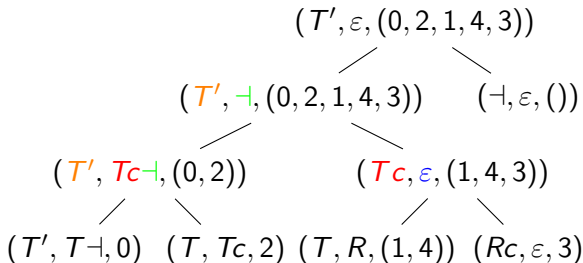
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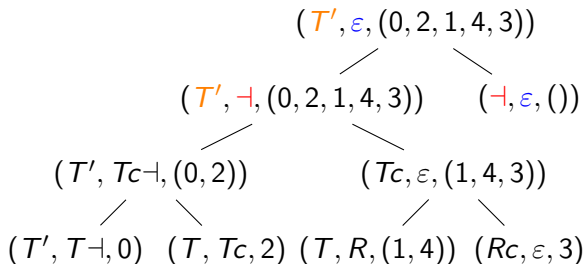
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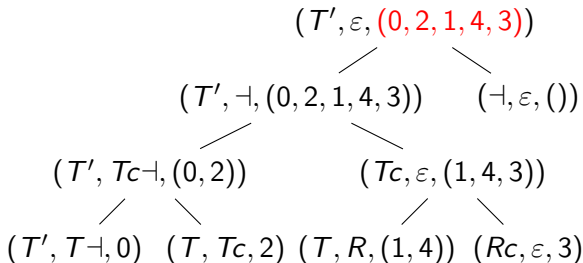
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LLP Parsing using Bracket Matching

(ε, \vdash)	(\vdash, a)	(a, b)	(b, c)	(c, \dashv)
$(T', T \dashv, 0)$	$(T, Tc, 2)$	$(T, R, (1, 4))$	$(Rc, \varepsilon, 3)$	$(\dashv, \varepsilon, ())$

LLP Parsing using Bracket Matching

(ε, \vdash)	(\vdash, a)	(a, b)	(b, c)	(c, \dashv)
$(T', T \dashv, 0)$	$(T, Tc, 2)$	$(T, R, (1, 4))$	$(Rc, \varepsilon, 3)$	$(\dashv, \varepsilon, ())$
$(T', T \dashv)$	(T, Tc)	(T, R)	(Rc, ε)	(\dashv, ε)

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$(T', T \dashv)$	(T, Tc)	(T, R)	(Rc, ε)	(\dashv, ε)
$(T', \dashv T)$	(T, cT)	(T, R)	(Rc, ε)	(\dashv, ε)

LLP Parsing using Bracket Matching

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$(T', \dashv T)$	(T, cT)	(T, R)	(Rc, ε)	(\dashv, ε)
$(\varepsilon, [\dashv]^T)$	$(\vdash]^T, [^c]^T)$	$(\vdash]^T, [^R)$	$(\vdash]^R]^c, \varepsilon)$	$(\vdash]^{\dashv}, \varepsilon)$

LLP Parsing using Bracket Matching

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$(T', T \dashv)$	(T, Tc)	(T, R)	(Rc, ε)	(\dashv, ε)
$(T', \dashv T)$	(T, cT)	(T, R)	(Rc, ε)	(\dashv, ε)
$(\varepsilon, [\dashv [^T)$	$(\lceil^T, [^c [^T)$	$(\lceil^T, [^R)$	$(\lceil^R]^c, \varepsilon)$	$(\lceil^{\dashv}, \varepsilon)$

Now perform bracket matching and assert their types match up.

$$[\dashv [^{\textcolor{blue}{T}}]^{\textcolor{blue}{T}} [^{\textcolor{green}{c}} [^{\textcolor{orange}{T}}]^{\textcolor{orange}{T}} [^{\textcolor{red}{R}}]^{\textcolor{red}{R}}]^{\textcolor{green}{c}}]^{\dashv}$$

LLP Parsing using Bracket Matching

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$(T', T \dashv)$	(T, Tc)	(T, R)	(Rc, ε)	(\dashv, ε)
$(T', \dashv T)$	(T, cT)	(T, R)	(Rc, ε)	(\dashv, ε)
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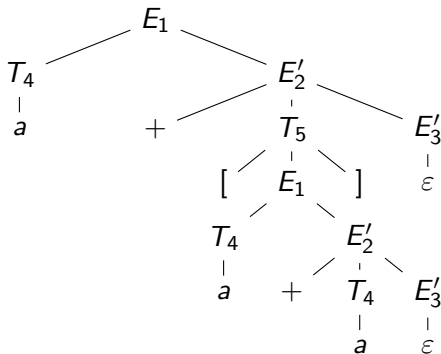
Construct the production sequence.

$$0, 2, 1, 4, 3$$

Syntax Tree

$$1) E \rightarrow TE' \quad 2) E' \rightarrow +TE' \quad 3) E' \rightarrow \varepsilon \quad 4) T \rightarrow a$$

$$(a + [a + a], E, ()) \vdash^* (\varepsilon, \varepsilon, (1, 4, 2, 5, 1, 4, 2, 4, 3, 3))$$



LLP Table

To perform LLP parsing a LLP table is needed.

1. Find the initial pushdown.
2. Parse the first symbol of the lookahead string to get the final pushdown store.
3. Construct LLP configuration.

The Problem

Consider the following augmented LL(2) grammar.

$$0) S' \rightarrow \vdash S \dashv \quad 1) A \rightarrow \varepsilon \quad 2) S \rightarrow aAa \quad 3) A \rightarrow a$$

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The PSLS (Prefix of a Suffix of a Leftmost Sentential) table. The grammar is LLP(2,2) since all the sets are singletons.

	\dashv	$a \dashv$	aa
\vdash			$\{S\}$
$\vdash a$		$\{A\}$	$\{A\}$
aa	$\{\dashv\}$	$\{a\}$	

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A small example of a PSLS value.

$$S \Rightarrow_{lm}^* \vdash S \dashv \Rightarrow \vdash aAa \dashv \Rightarrow^* \vdash aa \dashv$$

This corresponds to the entry $\text{PSLS}(\vdash a, a \dashv) = \{A\}$.

The Problem

Construct the LL(2) table.

	$\vdash a$	aa	$a \vdash$
S'	$S' \rightarrow \vdash S \vdash$		
S		$S \rightarrow aAa$	
A		$A \rightarrow a$	$A \rightarrow \varepsilon$

The Problem

Construct the LL(2) table.

	$\vdash a$	aa	$a \dashv$
S'	$S' \rightarrow \vdash S \dashv$		
S		$S \rightarrow aAa$	
A		$A \rightarrow a$	$A \rightarrow \varepsilon$

Try LL parsing the initial pushdown store $\text{PSLS}(\vdash a, a \dashv) = \{A\}$.

$$(a \dashv, A, ()) \vdash (a \dashv, \varepsilon, 1)$$

- The first symbol cannot be parsed so the algorithm fails.
- One of the authors says this is a mistake.
- This mistake is also found in the authors PhD thesis about LLP.

The Problem

The problem is due to the PSLS definition.

$$\begin{aligned} \text{PSLS}(x, y) = & \{ \alpha : \exists S \Rightarrow_{lm}^* wuA\beta \Rightarrow wxB\gamma \Rightarrow^* wxy\delta, \\ & w, u \in T^*, A, B \in N, \alpha, \beta, \gamma, \delta \in (N \cup T)^*, u \neq x, \\ & \alpha \text{ is the shortest prefix of } B\gamma \text{ such that } \text{FIRST}_1(y) \subseteq \text{FIRST}_1(\alpha) \} \\ \cup & \{ a : \exists S \Rightarrow^* wuA\beta \Rightarrow wxa\gamma \Rightarrow^* wxy\delta, \\ & a = \text{FIRST}_1(y), w, u \in T^*, \beta, \gamma, \delta \in (N \cup T)^*, u \neq x \} \end{aligned}$$

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$$\text{FIRST}_1(y) \subseteq \text{FIRST}_1(\alpha)$$

$$\not\Rightarrow$$

$$(y, \text{PSLS}(x, y), ()) \vdash^* (b, \omega, \pi) \text{ where } ab = y, a \in T \text{ and } b \in T^*$$

The Problem

$$\begin{aligned} \text{PSLS}_k(x, y) = & \{ \alpha : \exists S \Rightarrow_{lm}^* wuA\beta \Rightarrow wxB\gamma \Rightarrow^* wxy\delta, \\ & w, u \in T^*, A, B \in N, \alpha, \beta, \gamma, \delta \in (N \cup T)^*, u \neq x, \\ & \alpha \text{ is the shortest prefix of } B\gamma \text{ such that } y \in \text{FIRST}_k(\alpha) \} \\ \cup & \{ y : \exists S \Rightarrow^* wuA\beta \Rightarrow wxa\gamma \Rightarrow^* wxy\delta, \\ & a = \text{FIRST}_1(y), w, u \in T^*, \beta, \gamma, \delta \in (N \cup T)^*, u \neq x \} \end{aligned}$$

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- The time complexity is not impacted.
- The problem of deriving too many symbols.

“Since the length of both α_i and ω_i is limited by some constant z for the grammar, the time complexity is $O(z) = O(1)$.” (Vagner, 2007)

The Problem

Construct the new PSLS_2 table.

	\vdash	$a \vdash$	aa
\vdash			$\{S\}$
$\vdash a$		$\{Aa \vdash\}$	$\{Aa\}$
aa	$\{\vdash\}$	$\{a \vdash\}$	

The Problem

Construct the new PSLS_2 table.

	\neg	$a \neg$	aa
\vdash			$\{S\}$
$\vdash a$		$\{Aa \neg\}$	$\{Aa\}$
aa	$\{\neg\}$	$\{a \neg\}$	

Try LL parsing the initial pushdown store $\text{PSLS}_2(\vdash a, a \neg) = \{A\}$.

$$(a \neg, Aa \neg, ()) \vdash (a \neg, a \neg, 1) \vdash (\neg, \neg, 1)$$

Now the final pushdown store can be found. Resulting in the LLP configuration $(Aa \neg, \neg, 1)$.

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1. Not tested.
2. It was created by Vladislav Vagner.

The Problem

Using Vladislav Vagners newer PSLS definition.

	\neg	$a \neg$	aa
\vdash			$\{S\}$
$\vdash a$		$\{Aa\}$	$\{A\}$
aa	$\{\neg\}$	$\{a\}$	

Do LL parsing.

$$(a \neg, Aa, ()) \vdash (a \neg, a, 1) \vdash (\neg, \varepsilon, 1)$$

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- Algorithm 2.3 should use $v_j \in \text{FIRST}_k(\gamma)$ not $y \in \text{FIRST}_k(\gamma)$.
- When showing the infinite loop, FIRST_k should be FIRST_1 .

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- Parsing more complex grammars, an example is allowing “—” to mean both the binary and unary operation for numbers, while ignore parenthesis. When testing this such a grammar was found to be $LLP(2, 1)$.
- There exists a larger grammar class LLP^* which can parse all regular languages and more of LL.

A Little Example

A LISP Parser