Deutsch-Jozsa in a simulator

The homework called *Deutsch-Jozsa and Bernstein-Vazirani, classically* defined the Deutsch-Jozsa problem.

Implement four cases of the Deutsch-Jozsa algorithm and run the implementations on the quantum circuit simulator called Quirk, linked here: `https://algassert.com/quirk`. Use only gates in the Toolbox shown above the circuit in the grey area; avoid using the Make Gate facility.

The first two implementations should be for $n = 1$, with one implementation using an oracle for a constant function and the other implementation using an oracle for a balanced function. The next two implementations should be for $n = 2$, with one implementation using an oracle for a constant function and the other implementation using an oracle for a balanced function.

Submit screenshots that show your implementations and illustrate that they work, and submit a file with an explanation of how it works.

For the Deutsch-Jozsa algorithm, $n=1$

① Construct a circuit that has the main qubit at the top and the helper qubit at the bottom.

② Put each of two qubits in superposition by creating two H gates on the left

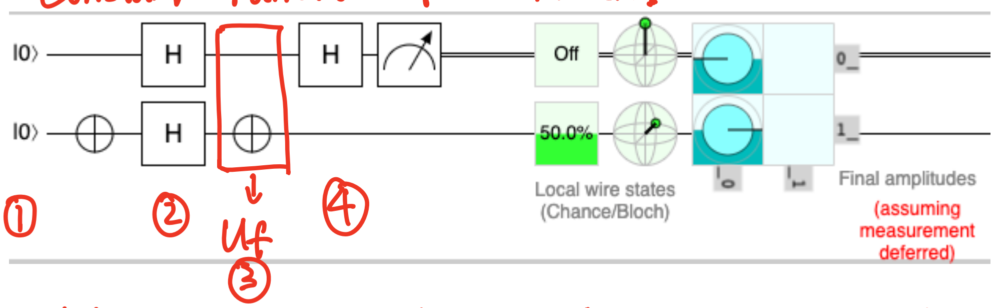
③ U_f creates the expression $f(0) \oplus f(1)$.

④ The H gate on the right will move $f(0) \oplus f(1)$ back down such that we can measure it.

f is constant if $f(0) \oplus f(1) = 0$

otherwise, f is balanced.

Constant function for $n=1$:

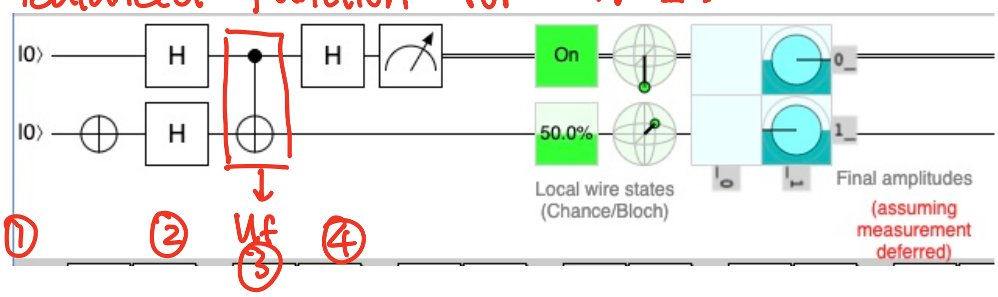


$U_f = I \otimes X$.

We measure the main qubit and we can see that the chance of being ON is 0. It means:

$f(0) \oplus f(1) = 0$ $\quad\therefore$ The function is constant.

Balanced function for $n=1$.



$U_f = CNOT$.

We measure the main qubit and we get 1. It means

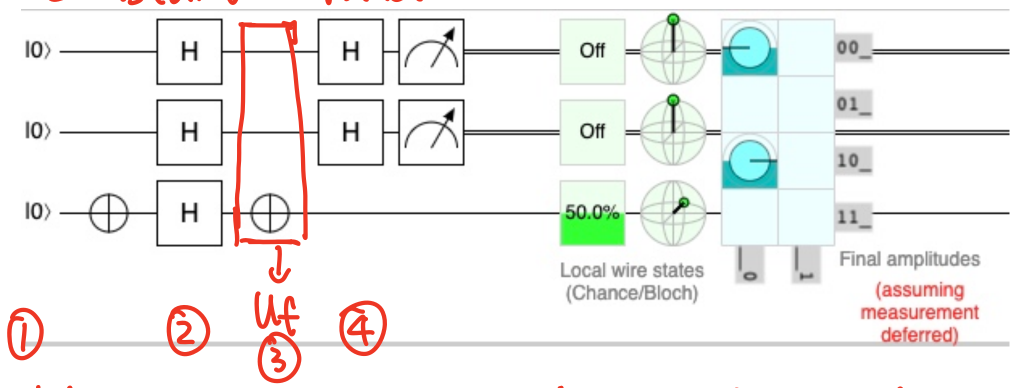
$f(0) \oplus f(1) = 1$ $\quad\therefore$ The function is balanced.

For $n=2$,

The circuit will have ① two main qubits and one helper qubit. ② Create one H gate for each qubit ③ Like in the case of $n=1$, we use a single call to U_f and ④ use H gates such that we can measure every main qubit

The idea is that we get 2 bits from the measurements. If all those bits are 0, we conclude that the function is constant, and otherwise it is balanced

Constant function for $n=2$.

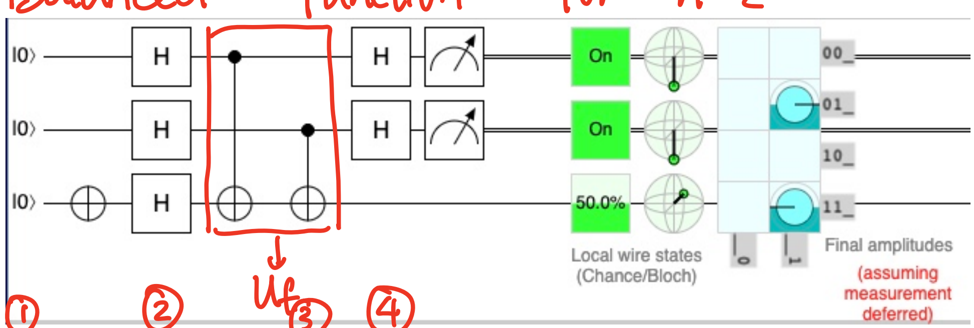


$U_f = I \otimes I \otimes X$

We measure two main qubits and we can see that the chance of being ON for both of them is 0.

$\therefore$ The function is constant.

Balanced function for $n=2$



$U_f = \text{CNOT}$

We measure two main qubits and we get $11 \neq 00$

$\therefore$ The function is balanced.