finc584_ps4

30 October, 2022

Problem 1a

Multiple the density by the payoff amount to get the expected payoff. We can assume $k < \tau$ and drop all observations that don't fit this assumption. That leaves us with n = 50.

$$\frac{P_0(k)}{X_0} = \int_{-\infty}^{k} (e^k - e^r) a e^{\lambda r} dr = a\left(\frac{e^{k(1+\lambda)}}{\lambda} - \frac{e^{k(1+\lambda)}}{1+\lambda}\right) = a\frac{e^{k(1+\lambda)}}{\lambda(1+\lambda)}$$

Problem 1b

```
stock_price <- 2838.3

options <- readxl::read_xlsx(file.path(proj, 'options_07_26_2018.xlsx')) %>%
  select(c(1,2)) %>%
  rename(put_price = `put price`) %>%
  mutate(y = put_price/stock_price,
         k = log(strike) - log(stock_price))
```

```
## New names:
## * ‘‘ -> ‘...3‘
```

```
expected_payoff <- function(k, a, lambda){
  return( a*(exp(k + lambda*k)/(lambda*(1 + lambda))))
}


min.rss <- function(data, param){
  lapply(data$k, expected_payoff, a = param[1], lambda = param[2]) %>%
  unlist() %>%
  as.data.frame(.) %>% rename(exp_pay = 1) %>%
  summarize(rss = sum((exp_pay - data$y)^2)) %>%
    pull()
}


output <- optim(par = c(1,6), fn = min.rss, data = options %>% filter(k <= -0.15),
      hessian = T)

knitr::kable(cbind(c('a', 'lambda'),output$par), digits = 3)
```

a	0.0828670177608727
lambda	6.66323103737086

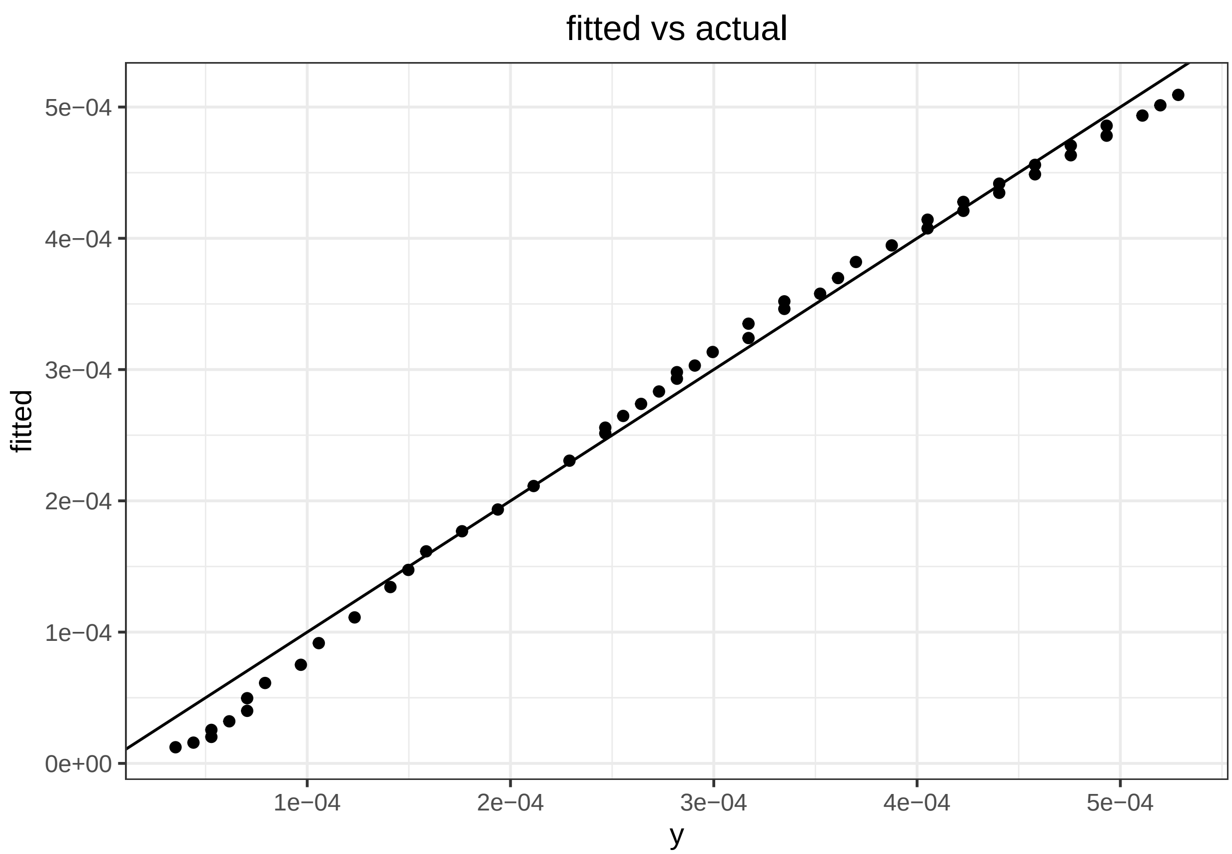
Problem 1c

```
nls_reg <- nls(y ~ a*(exp(k + lambda*k)/(lambda*(1 + lambda))) , data = options %>% filter(k < -0.15))

resid <- resid(nls_reg)

bread <- solve(t(nls_reg$m$gradient()) %*% nls_reg$m$gradient())
meat <-  t(nls_reg$m$gradient()) %*% diag(resid^2)  %*% nls_reg$m$gradient()

avar <- sqrt(diag(bread %*% meat %*% bread))

lower.ci <- coef(nls_reg) - qnorm(.975)%*%avar
upper.ci <- coef(nls_reg) + qnorm(.975)%*%avar
```

The confidence interval is 0.0701718, 0.0946858 for a and 6.3367481, 6.9664914 for lambda.

Problem 1d

```
options %>% filter(k < -0.15) %>%
  mutate(fitted = fitted(nls_reg)) %>%
  ggplot(aes(x = y, y = fitted)) +
  geom_point() +
  labs(title = "fitted vs actual") +
  geom_abline(intercept = 0, slope = 1) +
  theme_bw() +
  theme(plot.title = element_text(hjust =0.5))
```



The plot above shows that the model has decent fit towards the middle but fails to accurately capture the far left tail.

Problem 1e

We will take the expression given to us in the problem, integrate it, plug in the values estimated from the last section, and then compute the variance using the delta method.

$$\int_{-\infty}^{\tau} r^2 f(r)dr = a\left(\frac{r^2 e^{\lambda r}}{\lambda} + \frac{2re^{\lambda r}}{\lambda^2} + \frac{2e^{\lambda r}}{\lambda^3}\right)|_{-\infty}^{\tau}$$

which means that the annualized VIX is

$$VIX = \sqrt{\frac{252}{22} a e^{\lambda\tau}\left(\frac{\lambda^2\tau^2 - 2\lambda\tau + 2}{\lambda^3}\right)}$$

Thus,

$$\sqrt{n}(VIX(\hat{a}, \hat{\lambda}) - VIX) \sim N\left(0, \nabla VIX(a, \lambda)^T * \Sigma * \nabla VIX(a, \lambda)\right)$$

where

$$\nabla VIX = \left[\frac{252}{44 * VIX} * e^{\lambda\tau}\left(\frac{\lambda^2\tau^2 - 2\lambda\tau + 2}{\lambda^3}\right), \frac{a252e^{\lambda\tau}}{44\lambda^3 VIX} * \left((\tau - 3/\lambda)(\lambda^2\tau^2 - 2\lambda\tau + 2) + 2\tau(\lambda\tau - 1)\right)\right]$$

```
vix <- function(a, lambda, tau){
  sqrt((252/22)*a*exp(lambda * tau)*((lambda^2 * tau^2) - 2*lambda*tau + 2)/(lambda^3)) %>% as.vector()
}

vix_hat <- vix(coef(nls_reg)[1], coef(nls_reg)[2], -0.15)

grad_vix <- function(a, lambda, tau){
  vix_hat <- vix(coef(nls_reg)[1], coef(nls_reg)[2], -0.15)

  return(c((252/44*vix_hat)*exp(lambda * tau)*((lambda^2 * tau^2) - 2*lambda*tau + 2)/(lambda^3), #firs
    (252*a/(44* lambda^3 *vix_hat))*exp(lambda * tau)*((tau - 3/lambda)*(lambda^2 * tau^2 - 2*lambda*tau
    %>% as.vector())
}

g <- grad_vix(coef(nls_reg)[1], coef(nls_reg)[2], -0.15)

vix_sd <- sqrt(t(g) %*% bread %*% meat %*% bread %*% g)

vix.ci <- c(vix_hat - qnorm(.975)*vix_sd/sqrt(50), vix_hat + qnorm(0.975)*vix_sd/sqrt(50))
```

The point estimate is 0.0768375 and the confidence interval is (0.0760194, 0.0776555). These numbers are not very close to the actual VIX at all. I assume that is because we restricted our attention to the extreme left tail of the distribution while VIX puts equal weight on all outcomes.