EXAMINATION SOLUTION FORM, Semester 1, 2013-14

Examiner (initials)	JMA	Module Code	PHY 2063
Question Number	1	Page (please number each page)	1 of

- Use **BLACK** ink only
- Model answers must be **hand-written**
- Ensure writing is **legible**
- Indicate whether "bookwork" or not

MARKS (to total 20 for each question)

a) i) Each quadratic degree of freedom has $\frac{1}{2}k_BT$ of thermal energy. — 1

This applies in the high temperature limit when $k_BT \gg E$ (energy level spacing) — 1 BW

ii) Each atom in a crystalline solid has 3 translational & 3 positional degrees of freedom — 1

$$\Rightarrow \quad u = (3+3)\cdot\frac{1}{2}k_BT = 3k_BT \text{ per atom}$$

$$\therefore C_v \simeq C_p = \left.\frac{dQ}{dT}\right|_v = \left.\frac{\partial u}{\partial T}\right|_v = 3k_B \text{ per atom}$$ — 1 BW

b) i)

$$Z = \sum_i e^{-\epsilon_i/k_BT}$$

$$= 1 + e^{-\epsilon/k_BT} \quad \text{as } \epsilon_i = 0, 1$$ — 4 BW

ii) $$\langle\epsilon\rangle = 0\cdot p_0 + \epsilon\cdot p_\epsilon$$

$$p_0 = \frac{1}{Z} \qquad p_\epsilon = \frac{e^{-\epsilon/k_BT}}{Z}.$$

$$\therefore \langle\epsilon\rangle = \frac{\epsilon e^{-\epsilon/k_BT}}{1+e^{-\epsilon/k_BT}} = \frac{\epsilon}{1+e^{\epsilon/k_BT}}.$$ — 4 BW

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Examiner (initials): JMA

Module Code:

Question Number: 1

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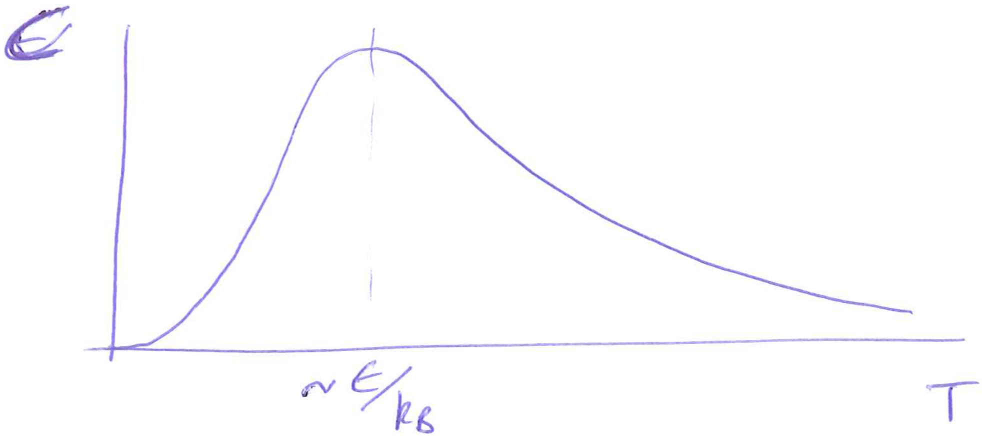
iii) $C = \dfrac{d\langle \epsilon \rangle}{dT} = k_B \dfrac{\epsilon^2}{(k_B T)^2} \dfrac{1}{(1+e^{\epsilon/k_B T})^2} e^{\epsilon/k_B T}$



2

$T \to 0 \Rightarrow$ not enough thermal energy to excite system $\therefore C \to 0$ 1

$T \to \infty \Rightarrow$ equal occupation of two levels so no way to absorb heat $C \to 0$ 1

BW

c) The data looks similar to a two level system. We expect that the solid has an additional two level system structure. Peak is at $T \sim 4.5K$ 2.

$\therefore \dfrac{\epsilon}{k_B} \sim 4.5K$ or $\epsilon \sim 388 \mu eV$ 2

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Examiner (initials): JMA.

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Question Number: 2

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MARKS (to total 20 for each question)

a) i) $dU = dQ + dW$ — 3 BW

internal energy; heat supplied to ~~gas~~ system; work done on system

ii) $F = U - TS$

$dF = dU - TdS - SdT$

$= \cancel{TdS} - pdV - \cancel{TdS} - SdT$ — 2

$\therefore p = -\left.\frac{\partial F}{\partial V}\right|_T$ — 1 BW

b) i) $W = \frac{V!}{N!\,(V-N)!}$ — 3

ii) $\ln N! \simeq N\ln N - N$

$\therefore S = k_B \ln W$

$\Rightarrow F = U - TS$

$= U(T) - k_B T(V\ln V - N\ln N - (V-N)\ln(V-N))$ — 2.

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b) iii) $P = -\frac{\partial F}{\partial V}\Big\vert_T$	
$\therefore p = -\left(-k_BT\left(1 + \ln V - 1 - \ln(V-N)\right)\right)$	
$= -k_BT \ln\left(1 - \frac{N}{V}\right)$	1
$\simeq k_BT\frac{N}{V}$	2
or $pV = Nk_BT$ (ideal gas law)	1
c) $E = \frac{p^2}{2m} \simeq k_BT$	1
$\Delta x\, \Delta p \sim \hbar$	1
$\Delta x \sim \left(\frac{1}{n}\right)^{\frac{1}{3}} \quad \therefore \quad n^{\frac{1}{3}} \sim \frac{\Delta p}{\hbar} = \sqrt{\left(\frac{2mk_BT}{\hbar^2}\right)}$	2
$\therefore n \sim \left(\frac{mk_BT}{\hbar^2}\right)^{3/2}$	1

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Examiner (initials) | Module Code

Question Number: 3.

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a) i) $S = k_B \ln W$

entropy — S; Boltzmann's const. — k_B; # microstates (realisations of system) — W

3. BW

Extensive variables are proportional to system size. If we have two systems

$\Rightarrow W_T = W_1 \cdot W_2$

& $S_T = k_B \ln W_T = k_B(\ln W_1 + \ln W_2)$

$= S_1 + S_2$

2 BW

ii) Consider a system in contact with a heat reservoir. The system has energy ϵ & the reservoir $U_T - \epsilon$.

$\Rightarrow W_T = 1 \cdot W_{res}(U_T - \epsilon)$

1 μ state of system — 1; # μ states of reservoir — $W_{res}(U_T - \epsilon)$

1

$P_\epsilon \propto W_{res}(U_T - \epsilon)$

1

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$S = k_B \ln W$

$\ln p_\epsilon \propto \frac{1}{k_B} S_{res}(U_T - \epsilon)$. 1

$\simeq \frac{1}{k_B} S_{res}(U_T) - \frac{\epsilon}{k_B}\frac{\partial S_{res}}{\partial U} + \ldots$ 1

$= C - \frac{\epsilon}{k_B T}$.

$\therefore p_\epsilon \propto e^{-\epsilon/k_B T}$. 1 BW

iii) • Maxwell Boltzmann distribution. 2
• Isothermal atmosphere model BW

b) i) $P = V(\rho_s - \rho_g)gh$ 2

$\uparrow$ potential energy.

$\therefore n(h) = n(0)\, e^{-P/k_B T}$. 2

ii) $\ln \frac{n(h)}{n(0)} = -\frac{P}{k_B T}$ $\therefore$ $k_B = \frac{V(\rho_s - \rho_g)g\cdot h}{T\left(\ln \frac{n(0)}{n(h)}\right)}$, $\ln \frac{n(0)}{n(h)} = 1$

$k_B \simeq 5.75 \times 10^{-23}\ JK^{-1}$ 4.

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