Problem Sheet 6.

1. Below the transition temperature T_c (called the Curie temperature) ferromagnetic materials become spontaneously magnetized.

Material	T_c
Fe	1043 K
Co	1388 K
Ni	627 K.

If the magnetic coupling energy

$$E \sim \frac{\mu_0 \mu^2}{4\pi r^3}$$

& $\mu = \mu_B \approx 10^{-23}$ Am

$r \simeq 0.2$ nm

$$\Rightarrow \quad E \sim 1.25 \times 10^{-24} \text{ J} = 7.8 \, \mu\text{eV}.$$

If $E \simeq k_B T$

$$\Rightarrow \quad T \simeq 0.09 \text{ K}.$$

This is much lower than the transition temperature of e.g. Fe.
The quantum nature of the interaction between the ions is crucial & produces a stronger interaction between the atoms.

2. $$\langle m \rangle = m_0 \tanh \frac{m_0 B}{k_B T}$$

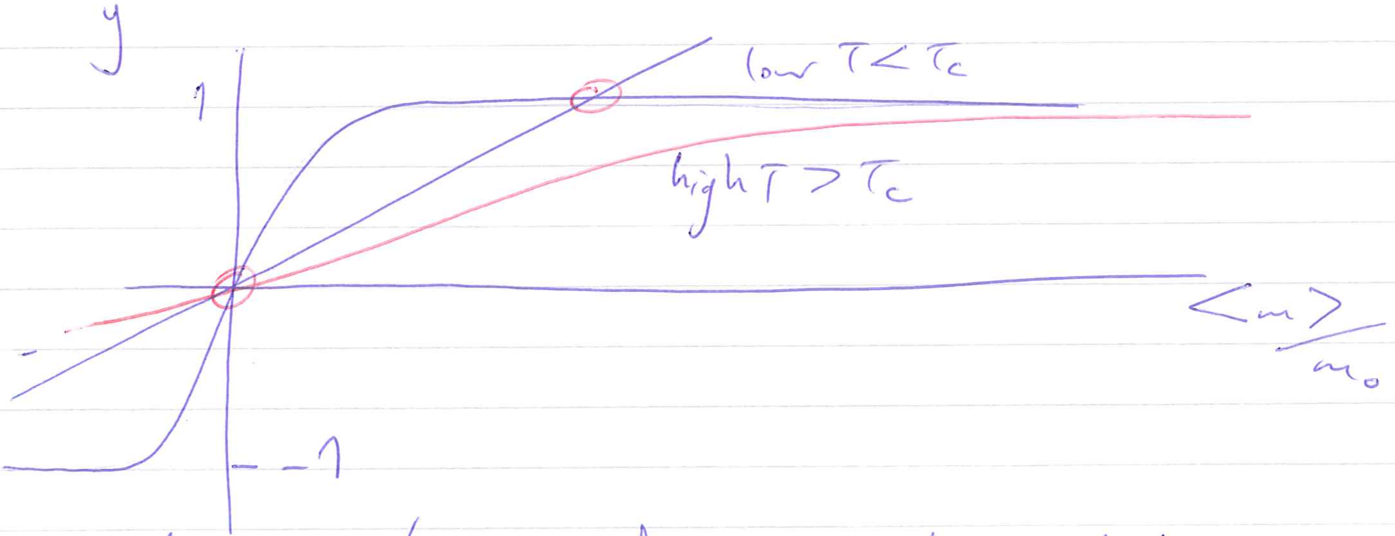
where $B = B_{int} + B_{ext} = \lambda \langle m \rangle + B_{ext}$.

If $B_{ext} = 0$

$$\langle m \rangle = m_0 \tanh \left(\frac{m_0 \lambda \langle m \rangle}{k_B T} \right).$$

Plot $y = \langle m \rangle / m_0$

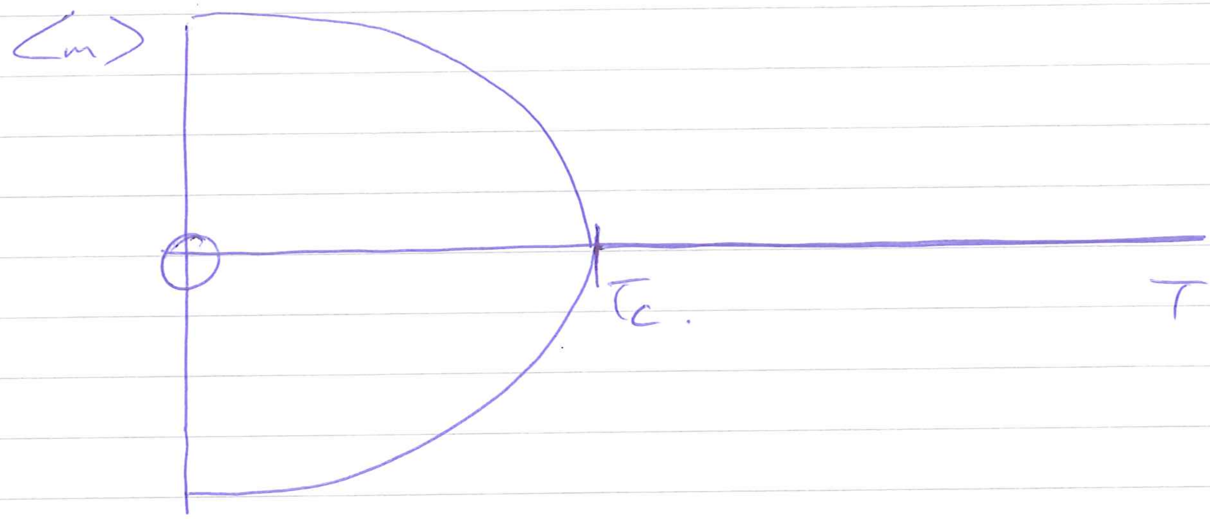
& $y = \tanh \dfrac{m_0 \lambda \langle m \rangle}{k_B T}$.



Solution if $\dfrac{d}{d\langle m \rangle} \tanh \dfrac{m_0 \lambda \langle m \rangle}{k_B T} = \dfrac{1}{m_0}$.

(Then we have additional crossings other than $\langle m \rangle = 0$).

$$\Rightarrow \quad \frac{m_0 \lambda}{k_B T} = \frac{1}{m_0} \quad \& \quad T_c = \frac{m_0^2 \lambda}{k_B}.$$



If $\quad \tanh x \simeq x - \frac{x^3}{3}$

$$\Rightarrow \quad \frac{\langle m \rangle}{m_0} \simeq \frac{m_0 \lambda \langle m \rangle}{k_B T} - \left(\frac{m_0 \lambda \langle m \rangle}{k_B T}\right)^3 \frac{1}{3}.$$

$$= \frac{\langle m \rangle}{m_0} \frac{T_c}{T} - \left(\frac{\langle m \rangle}{m_0} \frac{T_c}{T}\right)^3 \frac{1}{3}$$

$$\Rightarrow \quad 1 = \frac{T_c}{T} - \left(\frac{\langle m \rangle}{m_0}\right)^2 \frac{T_c^3}{3 T^3}.$$

$$\therefore \frac{\langle m \rangle}{m_0} = \pm \sqrt{\frac{T_c - T}{T} \cdot \frac{3 T^3}{T_c^3}}$$

$$= \pm \sqrt{(T_c - T)} \cdot \frac{T \sqrt{3}}{T_c^{3/2}}.$$

3.
$$F = \frac{1}{2}\alpha(T - T_c)P^2 + \frac{1}{4}bP^4 + \frac{1}{6}cP^6.$$

$$\frac{dF}{dP} = 0$$

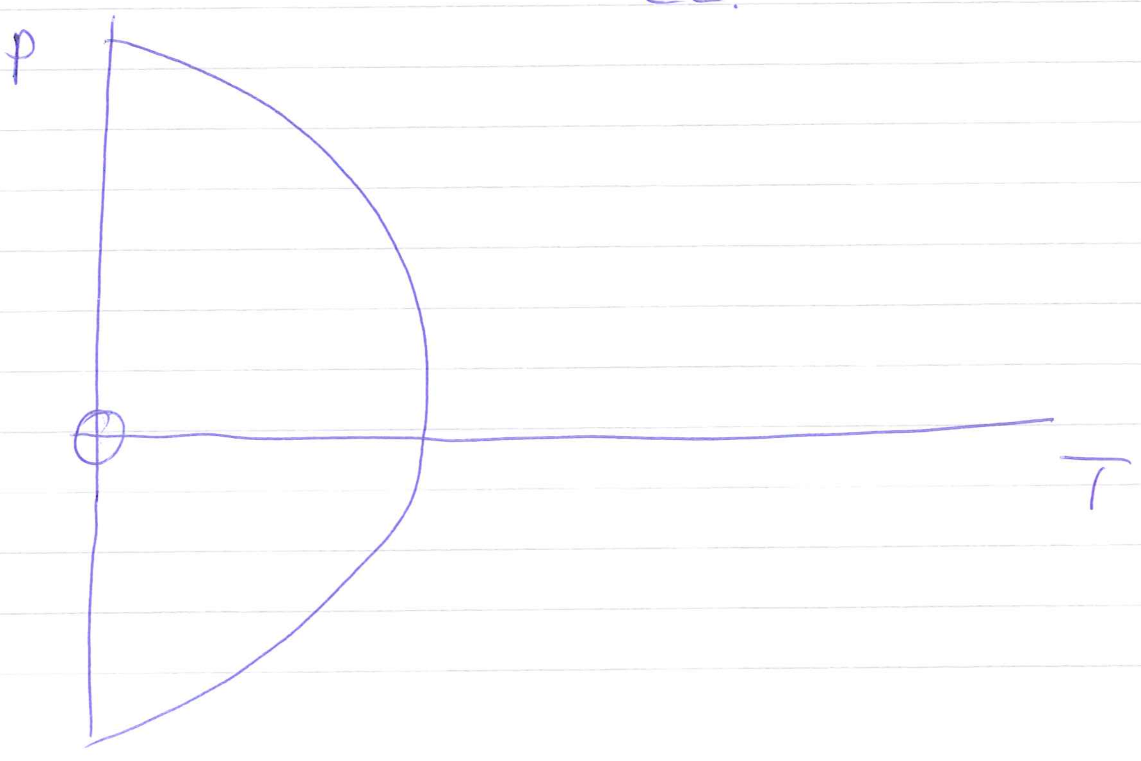
$$\Rightarrow 0 = \alpha(T - T_c)P + bP^3 + cP^5.$$

$$\therefore 0 = P(\alpha(T - T_c) + bP^2 + cP^4).$$

$$\Rightarrow P = 0 \quad \text{or}$$

$$cP^4 + bP^2 + \alpha(T - T_c) = 0.$$

$$\Rightarrow P^2 = \frac{-b \pm \sqrt{b^2 - 4.c.\alpha(T - T_c)}}{2c}.$$

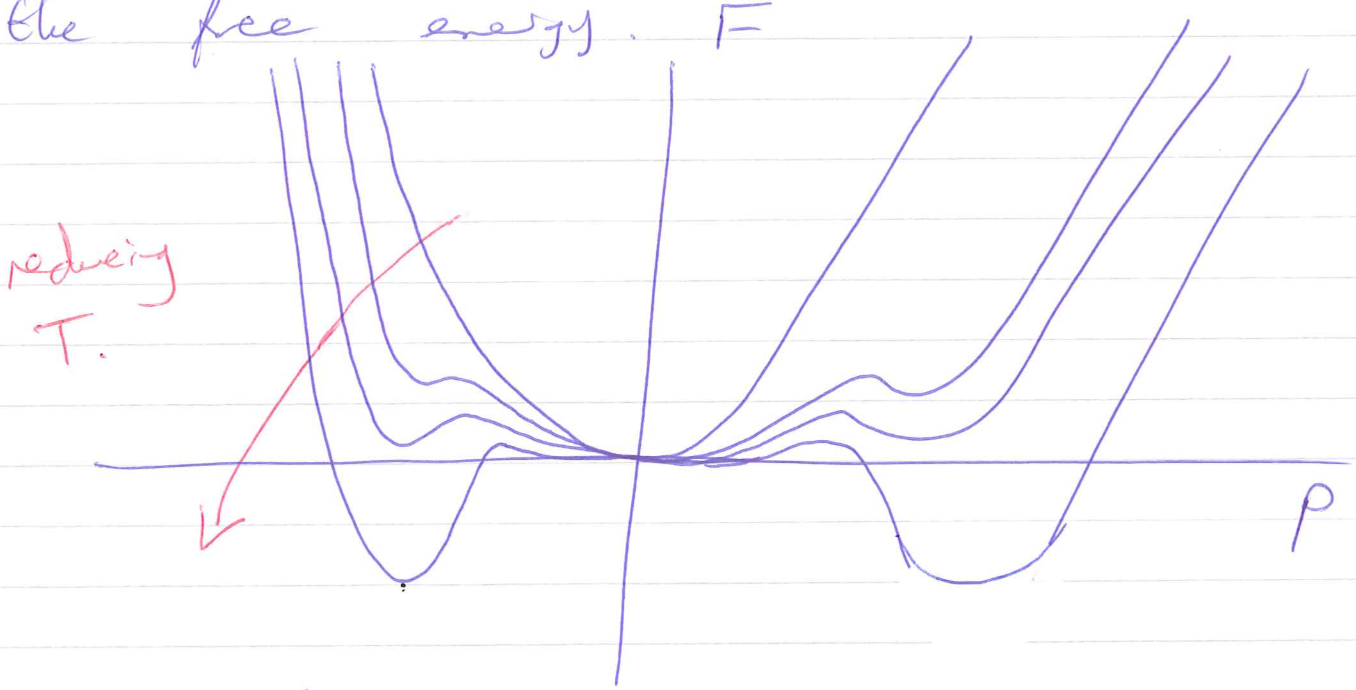


NB the shape is the same as $c = 0$ where

$$p^2 = -\frac{\alpha(T - T_c)}{b}.$$

If $b < 0$ & $c > 0$ then the type of transition changes.

This is clearest from a sketch of the free energy.



Instead of a continuous change in P from 0 there is a jump & hysteresis