UNIVERSITY OF SURREY©

Faculty of Engineering and Physical Sciences

Department Of Physics

BSc and MPhys Undergraduate Programmes in Physics

Module PHY2063; 15 credits

Energy and Entropy

FHEQ Level 5 (Year 2) Examination

Time allowed: 1.5 hours Semester 1 2012/13

Answer **TWO** questions only

Each question carries 20 marks.

Where appropriate the mark carried by an individual part of a question is indicated in square brackets [ ].

Additional Materials:
Department of Physics Formulae Booklet

Candidates may use only calculators which are non-programmable and with no alphanumeric memory. Calculators approved for use in the Physics Department are:

Casio: FX-82 Series, FX-83 Series, FX-85 ES, FX-115MS, 115W, 115S and FX-570W
Sharp: EL-531 LH
Texas Instruments: TI-30X
Tandy: EC-4031, EC-4032

1.

(a)

(i) State the second law of thermodynamics in terms of the entropy.

[3 marks]

(ii) Boltzmann's expression for entropy is given by $S = k_B \ln W$. Explain what is meant by W.

[3 marks]

(b) (i) A crystal has N lattice sites filled with N atoms of type A. If m of these atoms are replaced by atoms of type B find an expression for W. Hence, by using Stirling's approximation, an expression for the entropy of the crystal.

[Note: Stirling's approximation is given by $ln\, N! = N\, ln\, N - N$]

[6 marks]

(ii) If we denote the fraction of atoms of type B as x, then the Helmholtz free energy per site (up to an additive constant) can be written as

$$f = \frac{F}{N} = \epsilon\, x + \; k_B\, T\, [x \ln x + (1 - x) \ln(1 - x)],$$

where ϵ is the energy required to replace an atom of type A with type B.

Show that the equilibrium value for x obeys the expression

$$\frac{x}{1 - x} = e^{-\epsilon/(k_B T)}$$

[4 marks]

(c) If lead is placed in contact with tin, then some of the tin atoms are dissolved in the lead by replacing them on the lattice. At $300K$, 3% of the lead atoms are replaced by tin. Find the energy required to replace a lead atom with a tin atom, and hence estimate the solubility of tin in lead at $100K$.

[4 marks]

2.

(a) Give an equation for the partition function, Z, defining the symbols used. Statehow Z is used in the calculation of probabilities of the occupation of an energy state, and the Helmholtz free energy, F, in statistical physics.

[4 marks]

(b) (i) A quantum simple harmonic oscillator has energy levels $E_n = (n + \frac{1}{2})\, \hbar\omega$. Show that its partition function is given by

$$Z = \frac{e^{-\beta\, \hbar\omega/2}}{1 - e^{-\beta\, \hbar\omega}}$$

where $\beta = \frac{1}{k_B T}$.

Hence or otherwise show that the average energy of the system is

$$\langle E \rangle = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\,\beta\, \hbar\omega} - 1}$$

[8 marks]

(c) (i) State the principle of equipartition of energy.

[2 marks]

(ii) For dilute hydrogen bromide gas (HBr) the characteristic temperatures associated with the rotational and vibrational degrees of freedom are

$$T_{rot} = \frac{\hbar^2}{2\, I\, k_B} \sim 12.1K \text{ and } T_{vib} = \frac{\hbar\omega}{k_B} \sim 3000K.$$

Calculate its heat capacity, C_V, at $1000K$, stating the contributions of the translational, rotational and vibrational degrees of freedom. In each case give a justification for your answer.

[6 marks]

3.

(a)

(i) State the first law of thermodynamics. If you use an equation then define all terms used.

[2 marks]

(ii) From the differential expression for entropy

$$dS = \frac{dU + p\,dV}{T}$$

show that

$$\left.\frac{\partial S}{\partial U}\right|_V = \frac{1}{T}$$

[2 marks]

(iii) The probability, p, that a system occupies a state with energy U can be written as

$$p \propto e^{S/k_B}$$

By making a Taylor expansion of the entropy and inserting it into this expression show that the fluctuations in the energy obey

$$\langle \Delta U^2 \rangle = k_B T^2 C_V$$

where $C_V = \left.\frac{\partial U}{\partial T}\right|_V$ is the heat capacity at constant volume.

[Note: Normal distribution with standard deviation σ and mean μ is given by $p(x) \propto e^{-(x-\mu)^2/(2\,\sigma^2)}$*]*

[6 marks]

(b) (i) Give an equation for the grand partition function Ξ, defining all the symbols used.

[2 marks]

(ii) Show, for a reservoir of Fermi particles in contact with single quantum state of energy ϵ, that the average number of Fermi particles in this state is given by the Fermi-Dirac distribution

$$\langle n \rangle = \frac{1}{1 + e^{(\epsilon-\mu)/(k_B T)}}$$

[6 marks]

(iv) Sketch the Fermi-Dirac distribution, and comment on its limits at high and low temperatures.

[2 marks]

Internal Examiner: Dr J M Adams
External Examiner: Prof AJ Horsewill

[FINAL PAGE]