MAIN.

EXAMINATION SOLUTION FORM, Semester 1, 2014-15

Examiner (initials)	JMA	**Module Code**	PHY 2063.
Question Number	1.	**Page** (please number each page)	1 **of**

- Use **BLACK** ink only
- Legible hand-written model answers are preferred
- Indicate whether "bookwork" or not

MARKS

a) i) Heat energy required to raise temperature of (a specified e.g. 1mol or 1kg) of substance by 1K, under specified conditions (e.g. fixed volume) — 1

$$C_V = \left.\frac{dQ}{dT}\right|_V = T\left.\frac{\partial S}{\partial T}\right|_V$$ — 1

$\therefore$ Integrating gives $\Delta S = \int_{T_1}^{T_2} \frac{C_V}{T}\, dT$ — 1

[BW]

ii) Thermodynamic potentials are used to determine equilibriums in open systems (systems in contact with an external reservoir). — 1

For example Helmholtz free energy $F = U - TS$ is minimized for systems in contact with a heat bath at fixed temperature T. — 1

Equilibrium occurs when: $dF = 0$ (F is a minimum). — 1

[BW]

b) i) $$W = \frac{N!}{(N-n)!\, n!}$$ — 2.

N positions for a domain wall. n domain walls

ii) $F = En - T.S$ — 1

$S = k_B \ln W \simeq k_B (N\ln N - n\ln n - (N-n)\ln(N-n))$ — 2

using Stirling's approx. $\ln N! \simeq N\ln N - N$ — 1.

Combining these gives stated result

EXAMINATION SOLUTION FORM, Semester 1, 2014-15

Examiner (initials)		**Module Code**	PHY		
Question Number	1	**Page** (please number each page)	2	**of**	

- Use **BLACK** ink only
- Legible hand-written model answers are preferred
- Indicate whether "bookwork" or not

MARKS

iii) $\frac{dF}{dn} = 0$ in equilibrium — 1

$\Rightarrow 0 = \epsilon - k_B T\left(-\ln n + \ln(N-n)\right)$ — 2

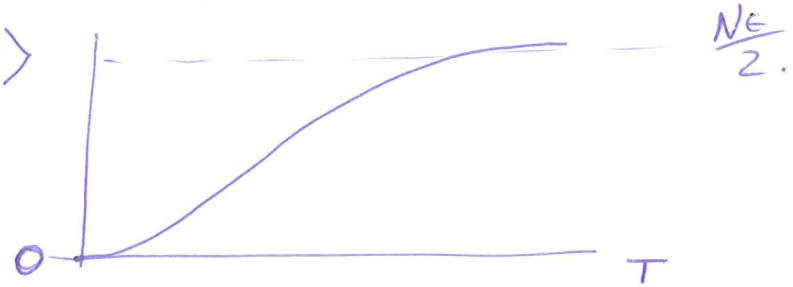
$\Rightarrow \frac{\epsilon}{k_B T} = \ln\left(\frac{N-n}{n}\right)$

$\Rightarrow \frac{N}{n} - 1 = e^{\epsilon/k_B T} \quad \therefore n = \frac{N}{1+e^{\epsilon/k_B T}}$

$\simeq N e^{-\epsilon/k_B T}$

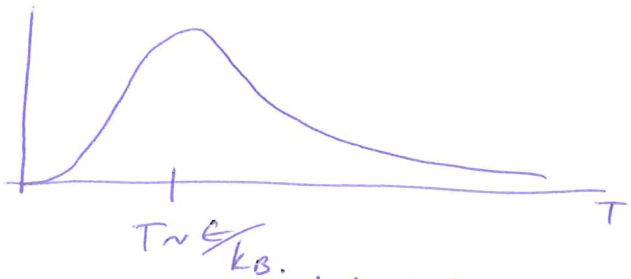
$\therefore \langle E \rangle = N\epsilon\left(e^{+\epsilon/k_B T}+1\right) \simeq N\epsilon e^{-\epsilon/k_B T}$ — 1

iv) $\langle E \rangle$



— 1

C_v



$T \sim \epsilon/k_B$. — 2

High T — equally divided between wall/no wall states $\Rightarrow \langle E \rangle = N\epsilon/2$.

Low T — very few walls & no thermal energy to create more $\therefore C_v$ Small. — 1

EXAMINATION SOLUTION FORM, Semester 1, 2014-15

Examiner (initials)		**Module Code**	PHY
Question Number	2.	**Page** (please number each page)	3 **of**

- Use **BLACK** ink only
- Legible hand-written model answers are preferred
- Indicate whether "bookwork" or not

MARKS

a) i) Each independent quadratic degree of freedom has thermal energy $\langle E \rangle = \frac{1}{2} k_B T$ provided the energy level spacing $\hbar\omega \ll k_B T$. (i.e. T is high compared to $\frac{\hbar\omega}{k_B}$). — 2. [BW]

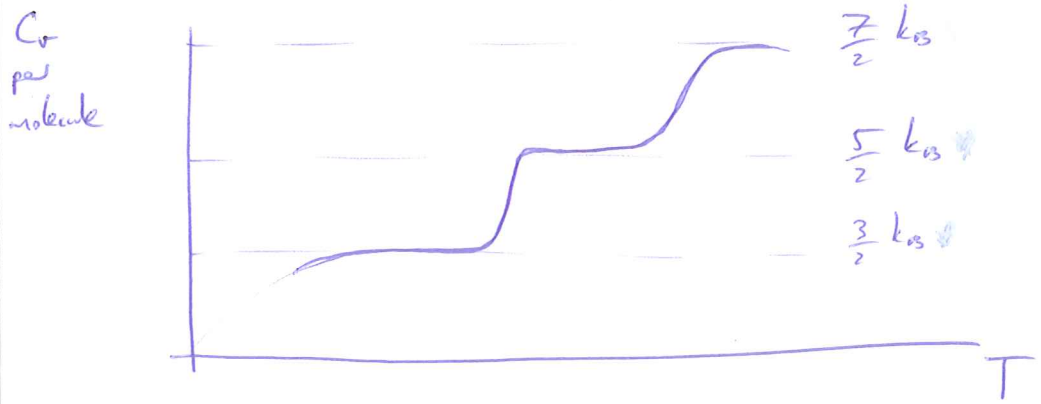
ii) A diatomic gas can store energy as follows:

translational kinetic energy — 3 components (x, y, z) $\Rightarrow \frac{3}{2} k_B T$. — 1

rotational kinetic energy — 2 components $\Rightarrow k_B T$. — 1

vibrational kinetic & potential energy $\Rightarrow k_B T$. — 1

These reach their equipartition values provided temperature is high enough. Typically vibrational modes require the highest T, then rotational, then translation for equipartition to apply. — 1



2 [BW]

EXAMINATION SOLUTION FORM, Semester 1, 2014-15

Examiner (initials)		**Module Code**	PHY
Question Number	2.	**Page** (please number each page)	4 **of**

- Use **BLACK** ink only
- Legible hand-written model answers are preferred
- Indicate whether "bookwork" or not

MARKS

b) i) $Z = \sum_{\text{states, } i} e^{-\epsilon_i / k_B T}$ — 1

here $\epsilon = +mB$ & $-mB$ — 1

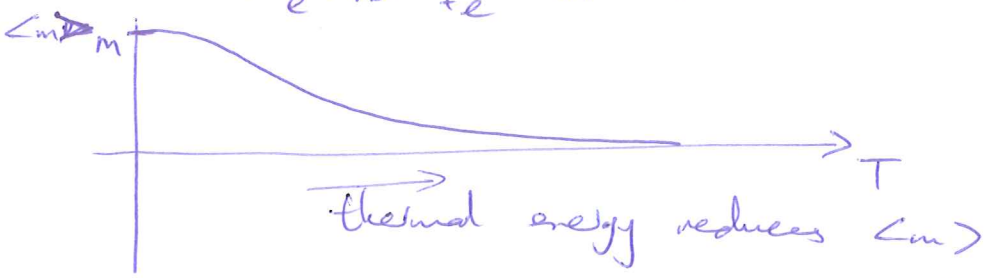
$\therefore Z = e^{mB/k_BT} + e^{-mB/k_BT}$. — 1

ii) $p_p = \dfrac{e^{mB/k_BT}}{e^{mB/k_BT} + e^{-mB/k_BT}} = \dfrac{1}{1 + e^{-2mB/k_BT}}$ — 1

$p_{ap} = \dfrac{1}{e^{2mB/k_BT} + 1}$ [or equivalent] — 1

iii) $\langle m \rangle = +m\, p_p + -m\, p_{ap}$

$= m\left(\dfrac{e^{mB/k_BT} - e^{-mB/k_BT}}{e^{mB/k_BT} + e^{-mB/k_BT}} \right) = m \tanh \dfrac{mB}{k_BT}$ — 2, 1



2

iv) $B \longrightarrow \frac{B}{2}$ effective level spacing halved.

$\Rightarrow p_{ap} = \dfrac{1}{e^{\frac{2mB}{k_B(\frac{T_{old}}{2})}} + 1} = \dfrac{1}{e^{\frac{2mB}{k_B T}} + 1}$.

$\therefore T_{old} = 2T$ i.e. temperature half of old value. — 2.

EXAMINATION SOLUTION FORM, Semester 1, 2014-15

Examiner (initials)		**Module Code**	PHY
Question Number	3.	**Page** (please number each page)	5 **of**

- Use **BLACK** ink only
- Legible hand-written model answers are preferred
- Indicate whether "bookwork" or not

MARKS

a) i) Order parameter — A quantity that is zero in the disordered (high T) phase & non-zero in the ordered (low T) phase. that describes the phase transition — 2. [BW]

ii) Paramagnet → ferromagnet — magnetisation — 2

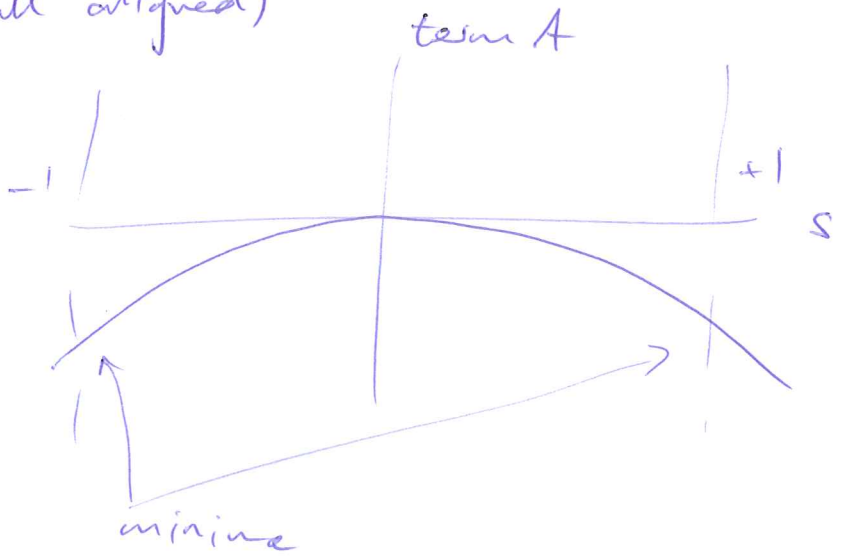
liquid → vapour — density difference — 2

[BW]

⋮

b) i) First term $\frac{zJ}{2}(1+s)(1-s)$

Interaction energy of average spins with neighbours. $s = \pm 1$ favored (all aligned) — 1 [BW]



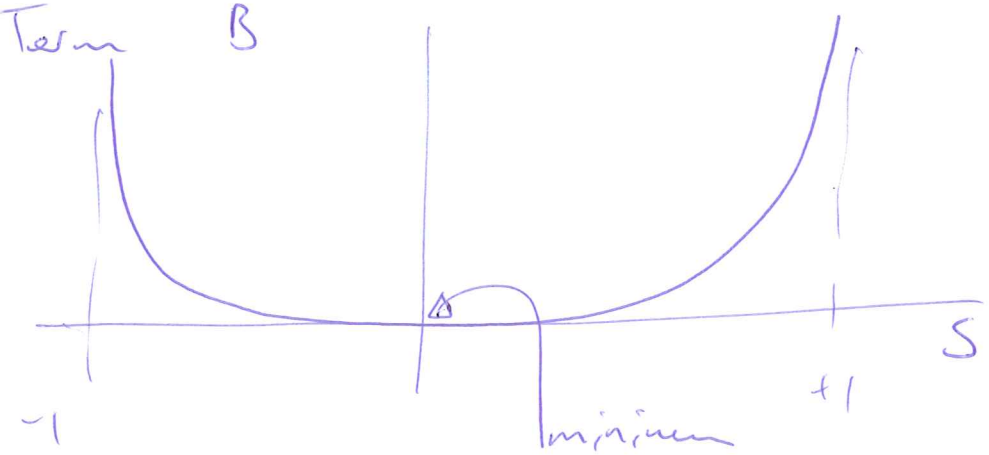
2

EXAMINATION SOLUTION FORM, Semester 1, 2014-15

Examiner (initials)		**Module Code**	PHY		
Question Number	3.	**Page** (please number each page)	6	**of**	

	MARKS
• Use **BLACK** ink only • Legible hand-written model answers are preferred • Indicate whether "bookwork" or not	

Term B



2

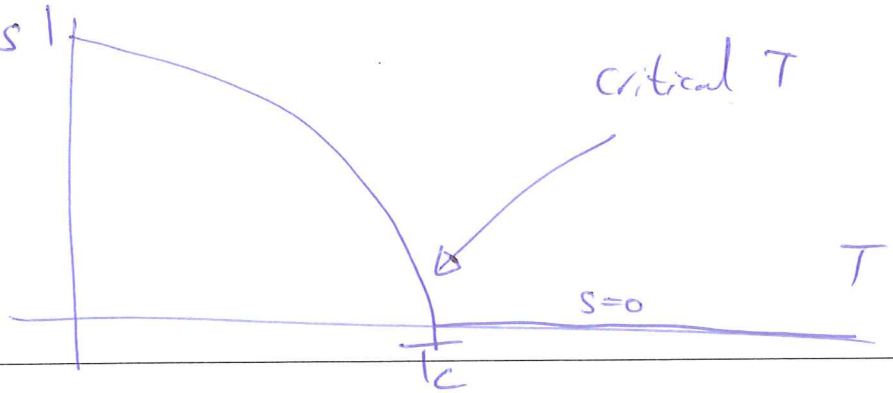
$\Rightarrow$ favors $s \neq 0$. (entropic term) 1 [BW]

ii) At equilibrium $\frac{\partial F}{\partial s} = 0$ 2

$$\Rightarrow 0 = \frac{zJ}{2}(-2s) - \frac{k_B T}{2}\left(-\ln(1+s) + \ln(1-s)\right)$$ 2

$$\Rightarrow \frac{1}{k_B T} = \frac{1}{2zJ} \ln \frac{1+s}{1-s}$$ 2

iii)



2.