UNIVERSITY OF SURREY©

Faculty of Engineering & Physical Sciences

Department of Physics

Undergraduate Programmes in Physics

Module PHY2063; 15 Credits

Energy and Entropy

FHEQ Level 5 Examination

Time allowed: 1.5 hours Semester 1 2014/5

Answer TWO questions only

Each question carries 20 marks.

Where appropriate the mark carried by an individual part of a question is indicated in square brackets [ ].

Additional materials:
Department of Physics Formulae Booklet

Candidates may use only calculators which are non-programmable and with no alphanumeric memory.

Examiner: ________

Note: full credit will only be given where the solution shows appropriate working.

1.

(a)

(i) Define specific heat capacity and explain how its measurement can be used to determine changes in entropy. Use equations as appropriate.

[3 marks]

(ii) Explain briefly what is meant by a thermodynamic potential, using the Helmholtz free energy as an example. Include in your answer a statement of how it is used to find equilibrium.

[3 marks]

(b)

A paramagnet consists of $N+1$ spins in a linear array. In the figure below the up spins are represented by ↑, and the down spins by ↓. The colons (:) indicate a domain wall where spin reversal has occurred.

$$\uparrow\uparrow\uparrow:\downarrow\downarrow:\uparrow:\downarrow\downarrow\downarrow\downarrow:\uparrow:\downarrow:\uparrow\uparrow:\downarrow:\uparrow:\downarrow\downarrow:\uparrow:\downarrow:\uparrow\uparrow\uparrow:\downarrow:\uparrow:\downarrow\downarrow\downarrow:\uparrow:\downarrow:\uparrow$$

If we have n domain walls, each of energy ε, then the total energy of the system is

$$E = n\,\varepsilon.$$

(i) Write an expression for the number of ways n domain walls can be arranged between the $N+1$ spins.

[2 marks]

(ii) By using a suitable approximation show that the free energy of the system can be expressed as

$$F \approx \varepsilon\,n - k_B\,T\,[N\ln N - n\ln n - (N-n)\ln(N-n)].$$

[4 marks]

(iii) Calculate the equilibrium value of n, and hence determine an expression for the energy of the system.

[4 marks]

(iv) Sketch both the energy and hence the heat capacity of the system as a function of temperature. Discuss the microscopic origin of the high and low temperature behaviour.

[4 marks]

2.

(a) (i) State the principle of equipartition and when it applies.

[2 marks]

(ii) Explain how the principle of equipartition can be used to calculate the dependence of the heat capacity of a diatomic gas on temperature. Include in your answer a sketch of the heat capacity as a function of temperature, indicating when each degree of freedom achieves its equipartition value.

[6 marks]

(b) A system of non-interacting spin-$\frac{1}{2}$ particles each with magnetic moment m is in a magnetic field of flux density B. The energy of the system in the parallel and antiparallel states is

$$U = \pm m\, B.$$

(i) Calculate the partition function of the system.

[3 marks]

(ii) Write down the probabilities of being in the parallel and antiparallel states.

[2 marks]

(iii) Calculate the average magnetic moment of the system, and then sketch it as a function of temperature.

[5 marks]

(iv) Imagine that the magnetic field strength is halved without altering the distribution of the particles in the energy levels. Calculate the new temperature of the system in relation to the original temperature.

[2 marks]

3.

(a) (i) State what is meant by an *order parameter* of a phase transition.

[2 marks]

(ii) Give two examples of a phase transition for each example state an order parameter for the transition.

[4 marks]

(b) A mean field theory of a magnetic phase transition in a crystalline solid is written in terms of the average magnetisation per site s, which can take values between -1 and $+1$. The Helmholtz free energy, F, is

$$F = \underbrace{\frac{zJ}{2}(1+s)(1-s)}_{A} - \underbrace{\frac{k_B T}{2}[2\ln 2 - (1+s)\ln(1+s) - (1-s)\ln(1-s)]}_{B}$$

where z is the number of nearest neighbours, T is the temperature, and J is the interaction strength between the sites.

(i) State the physical meaning of the two terms labelled A and B, and by sketching them, or otherwise, show that one favours $s \sim 0$, whilst the other favours $s = \pm 1$.

[6 marks]

(ii) Show that the equilibrium value for s is given by

$$\frac{1}{k_B T} = \frac{1}{2\,zJ}\ln\frac{(1+s)}{(1-s)}$$

[6 marks]

(iii) Make a rough sketch of the temperature dependence of s, labelling the important features.

[2 marks]

Examiner: Dr J M Adams
External Examiner: Prof AJ Horsewill

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