UNIVERSITY OF SURREY©

Faculty of Engineering and Physical Sciences
Department Of Physics
Undergraduate Programmes in Physics

Module PHY2063; 15 credits

Energy and Entropy

FHEQ Level 5 (Year 2) Examination

Time allowed: 1.5 hours Semester 1 2013/14

Answer **TWO** questions only

Each question carries 20 marks.

Where appropriate the mark carried by an individual part of a question is indicated in square brackets [ ].

Additional Materials:

Department of Physics Formulae Booklet

Candidates may use only calculators which are non-programmable and with no alphanumeric memory.

1.

(a) (i) State the principle of equipartition of energy, and when it applies.

[2 marks]

(ii) Using your answer to (a)(i) calculate the heat capacity of a monoatomic crystalline solid in the high temperature limit.

[2 marks]

(b) (i) Show that the partition function of a two level system with non-degenerate energy levels of energy 0 and ϵ is

$$Z = 1 + e^{-\epsilon/(k_B T)}.$$

[4 marks]

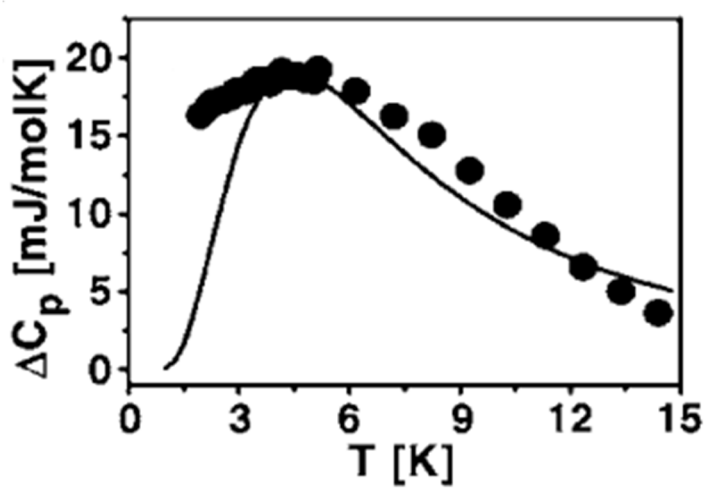
(ii) Calculate the average energy of the two level system.

[4 marks]

(iii) Sketch the resulting heat capacity and explain its main features.

[4 marks]

(c) Ti_2SC is a glassy solid. The difference between its heat capacity and that expected from the Debye model, ΔC_p, is shown below. The black circles are the experimental measurements of Drulis et al. (2008).



Explain the shape of the data, and calculate the spacing of the energy levels.

[4 marks]

2.

(a) (i) State the first law of thermodynamics in differential form, defining symbols used.

[3 marks]

(ii) The Helmholtz free energy, $F = U - TS$ is used to find equilibria in open systems at fixed temperature, T. Using your answer to (a)(i) show that the pressure, p, is given by

$$p = -\left(\frac{\partial F}{\partial V}\right)_T .$$

[3 marks]

(b) (i) Consider a simple discrete model of a gas consisting of V lattice sites with N particles occupying the sites. Calculate the total number of microstates of the system.

[3 marks]

(ii) Using Stirling's approximation show that the free energy is given by

$$F = U(T) - k_B T[V \ln V - N \ln N - (V - N) \ln(V - N)]$$

where $U(T)$ is the internal energy of the gas.

[2 marks]

(iii) Using your answer to (a)(ii) and (b)(ii) find the pressure in the system, and show that for $\frac{N}{V} \ll 1$ it results in the ideal gas law.

[4 marks]

(c) For gases at very high density or low temperature, it is expected that quantum effects will become important. Using the uncertainty principle and the thermal energy of a particle of mass m, show that this occurs at a number density n of

$$n \sim \left(\frac{k_B T m}{\hbar^2}\right)^{3/2}.$$

[5 marks]

3.

(a) Boltzmann's expression for entropy is given by

$$S = k_B \ln W .$$

(i) Define the symbols used here, and show that this expression for entropy is extensive.

[5 marks]

(ii) By considering a system in contact with a heat reservoir of temperature T derive the expression for the Boltzmann factor

$$p(\epsilon) \propto e^{-\epsilon/(k_B T)}.$$

[5 marks]

(iii) Give two examples of the use of the Boltzmann distribution.

[2 marks]

(b) It is proposed to measure Boltzmann's constant by observing the distribution of spherical polystyrene particles $1\mu m$ in diameter (colloids) suspended under gravity in glycerol.

(i) Calculate an expression for the number of particles as a function of height $n(h)$.

[4 marks]

(ii) It is found that at a temperature of $300\,K$, across a vertical distance of $15\,\mu m$ the number of particles has fallen by a factor e. Given that the density of glycerol is $1.025 g/cm^3$, and polystyrene is $1.053 g/cm^3$, calculate an estimate for k_B.

[4 marks]

Internal Examiner: Dr J M Adams

External Examiner: Prof AJ Horsewill

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