**Faculty of Engineering and Physical Sciences**
**Department of Physics**

UNIVERSITY OF SURREY

## Year 2, Energy and Entropy
## Problem Set 6

### Phase Transitions

***1.*** The energy associated with the magnetic coupling between spins is roughly

$$\frac{\mu_0 \mu^2}{4\pi r^3}$$

The magnitude of the magnetic moments $\mu$ is of order a Bohr magneton, $\mu_B = e\hbar/2m \approx 10^{-23}$ A m. If the atoms are about 0.2nm apart, estimate the magnetic coupling and the corresponding transition temperature. Compare this temperature with the transition temperature of typical ferromagnets.

***2.*** The Weiss theory of ferromagnetism makes use of the equations

$$\langle m \rangle = m_0 \tanh\left(\frac{m_0 B}{k_B T}\right) \quad \text{and} \quad B = B_{\text{int}} + B_{\text{ext}} = \lambda \langle m \rangle + B_{\text{ext}}$$

By looking for graphical solutions of these equations when the externally applied magnetic field $B_{\text{ext}}$ is zero, show that the theory predicts a ferromagnetic phase transition at a temperature $T_c = \lambda m_0^2 / k_B$ and make a plot of $\langle m \rangle$ as a function of $T$. Making use of the result $\tanh x = x - x^3/3$, prove that just below $T_c$ $\langle m \rangle$ varies like $\sqrt{T_c - T}$.

***3.*** A ferroelectric crystal has a free energy of the form

$$F = \tfrac{1}{2}\alpha(T - T_c)P^2 + \tfrac{1}{4}bP^4 + \tfrac{1}{6}cP^6$$

where $P$ is the electric polarization of the system.

Calculate the minimum of the free energy for the case $b > 0$ and $c > 0$ and sketch the form of the polarization $P$ as a function of temperature.

What happens when $b < 0$ and $c > 0$?