EXAMINATION SOLUTION FORM, Semester 1, 2012-13

Examiner (initials)	JMA	**Module Code**	MAIN PHY2063
Question Number	1.	**Page** (please number each page)	1 of

- Use BLACK ink only
- Writing must be LEGIBLE
- Indicate whether "bookwork" or not

MARKS (to total 20 for each question)

a) i) For a thermally isolated system the change in entropy for any process $dS \geq 0$. — 3

ii) W — the number of microstates associated with a particular set of constraints (e.g. volume, energy, pressure etc.) — 3

b) i) For N objects $(N-m)$ type A, m type B

$$\Rightarrow W = \frac{N!}{m!(N-m)!}$$ — 2

$$\Rightarrow \ln W = N\ln N - N - m\ln m + m - (N-m)\ln(N-m) + N - m$$

$$= N\ln N - m\ln m - (N-m)\ln(N-m)$$ — 3

$$\therefore S = k_B \ln W$$

$$= k_B [N\ln N - m\ln m - (N-m)\ln(N-m)]$$ — 1

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Examiner (initials): JMA

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Question Number: 1.

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MARKS (to total 20 for each question)

ii) In equilibrim $\frac{\partial f}{\partial x} = 0$ — 2

$\Rightarrow 0 = \epsilon + k_B T\left(\ln x \cancel{+1} - \ln(1-x) \cancel{-1} \right)$ — 1

$\therefore \frac{x}{1-x} = e^{-\epsilon/k_B T}$ — 1

c) Set $x = 0.03$

$\Rightarrow \epsilon = -k_B T \ln\left(\frac{x}{1-x}\right)$

$\Rightarrow \epsilon = -k_B T(-3.476) = +1.44 \times 10^{-20}\,\mathrm{J}$

$\cong 90\ \mathrm{meV}$ — 2

$\Rightarrow$ at $100\mathrm{K}$

$\frac{x}{1-x} = 2.92 \times 10^{-5} \Rightarrow x \simeq \frac{2.92 \times 10^{-5}}{1 + 2.92 \times 10^{-5}}$ — 2

$\simeq 2.9 \times 10^{-5}$

$\simeq 3 \times 10^{-3}\ \%$.

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MARKS (to total 20 for each question)

a) $Z = \sum_i e^{-\epsilon_i/k_B T}$ — 1

Sum over all states

The probability of state with energy ϵ_i is — 2

$p_i = \frac{e^{-\epsilon_i/k_B T}}{Z}$

The Helmholtz free energy is given — 1

$F = -k_B T \ln Z$

b) i) $Z = \sum_n e^{-E_n/k_B T} = \sum_n e^{-\frac{\hbar\omega}{k_B T}(n+\frac{1}{2})}$ — 2

$= e^{-\hbar\omega/2k_B T} \sum_{n=0}^{\infty} e^{-\frac{\hbar\omega}{k_B T} n}$ — 2

$= \frac{e^{-\hbar\omega\beta/2}}{1 - e^{-\beta\hbar\omega}}$

$\langle E \rangle = -\frac{\partial}{\partial\beta} \ln Z$ — 1

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Question Number 2 | **Page** (please number each page) 4 **of**

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MARKS (to total 20 for each question)

$$\langle E \rangle = -\frac{\partial}{\partial \beta}\left(-\frac{\hbar\omega\beta}{2} - \ln\left(1 - e^{-\beta\hbar\omega}\right)\right)$$

$$= \frac{\hbar\omega}{2} + \frac{1}{1-e^{-\beta\hbar\omega}}\left(+\hbar\omega e^{-\beta\hbar\omega}\right) \qquad 2$$

$$= \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{+\beta\hbar\omega}-1} \qquad 1$$

c) i) Each degree of freedom associated with a quadratic energy has $\frac{1}{2}k_BT$ of energy associated with it. 2

ii) As $T = 1000K \gg T_{rot}$ the rotational degree of freedom has $\frac{1}{2}k_BT \cdot \times 2$ 2

$T \ll T_{vib} \Rightarrow$ hence this degree of freedom is quenched & doesn't contribute to heat capacity. 2

$T \gg T_{tr} \Rightarrow \frac{3}{2}k_BT$ for translational component 1

$\Rightarrow \langle E \rangle = \frac{5}{2}k_BT \Rightarrow C_v = \frac{5}{2}k_B$ 1

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Question Number: 3. | Page (please number each page): 5 | of

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MARKS (to total 20 for each question)

a) i) Energy is conserved if heat is taken into account

$$\underbrace{dU}_{\text{internal energy change}} = \underbrace{dW}_{\text{work done on system}} + \underbrace{dQ}_{\text{heat supplied}}$$

2

ii)
$$dS = \frac{dU}{T} + \frac{p}{T}\,dV$$

S is a function of state hence

$$dS = \frac{\partial S}{\partial U}\,dU + \frac{\partial S}{\partial V}\,dV$$

$$\Rightarrow \left.\frac{\partial S}{\partial U}\right|_V = \frac{1}{T} \quad \& \quad \left.\frac{\partial S}{\partial V}\right|_U = \frac{p}{T}.$$

2

iii) $p \propto \exp\left(S/k_B\right)$

$$S(U+\Delta U) \cong S(U) + \frac{\partial S}{\partial U}\Delta U + \frac{\partial^2 S}{\partial U^2}\frac{\Delta U^2}{2}$$

zero in equilibrium.

2

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MARKS (to total 20 for each question)

$$\therefore \quad p \propto \exp\left\{ \frac{S(U)}{k_B} + \frac{\partial^2 S}{\partial U^2} \frac{\Delta U^2}{2k_B} \right\}$$

Since $\quad \dfrac{\partial S}{\partial U} = \dfrac{1}{T} \Rightarrow \dfrac{\partial^2 S}{\partial U^2} = -\dfrac{1}{T^2}\dfrac{\partial T}{\partial U}$

$$= -\frac{1}{T^2}\frac{1}{C_V}$$

2

$$\Rightarrow \quad p \propto \exp\left\{ -\frac{\Delta U^2}{2C_V k_B T^2} \right\}$$

2

Comparing with $p(x)$

$$\Rightarrow \quad \langle \Delta U^2 \rangle = C_V k_B T^2$$

b) i)

$$\Xi = \sum_i e^{-(\epsilon_i - \mu N_i)/k_B T}$$

Sum over all states

particles

energy of state i

2

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MARKS (to total 20 for each question)

ii) Fermions obey the Pauli exclusion principle $\Rightarrow$ only 1 particle can occupy any ~~state~~ particular quantum state.

$$\Xi = \underbrace{1}_{N_i = 0,\ \epsilon_i = 0} + \underbrace{e^{-(\epsilon-\mu)/k_BT}}_{N_i = 1,\ \epsilon_i = \epsilon}$$

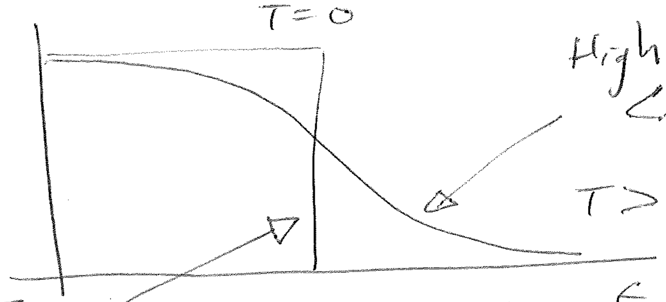
3

$$\langle n \rangle = \frac{1}{\Xi}\cdot 0 + \frac{e^{-(\epsilon-\mu)/k_BT}}{\Xi}$$

3

$$= \frac{1}{1+e^{(\epsilon-\mu)/k_BT}}$$

iv) $\langle n \rangle$



High $T \Rightarrow \langle n \rangle \simeq e^{-(\epsilon-\mu)/k_BT}$ (Classical/Maxwell distribution)

$T > 0$

2

Low $T \Rightarrow$ Only $\epsilon < \mu$ occupied. (Quantum $\Rightarrow$ Pauli Exclusion)

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