Faculty of Engineering and Physical Sciences
Department of Physics

Year 2, Energy and Entropy
Problem Set 1

This problem sheet aims to help you recall the use of partial derivatives, and the thermodynamics you covered in Year 1 as part of Properties of Matter.

Partial differentiation

1. (a) For the function $f(x, y) = y + x^2 + e^{xy}$

(i) Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

(ii) Show that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

(b) Using the ideal gas equation $p(V, T) = \frac{RT}{V}$

(i) Show that the isothermal bulk modulus $\kappa_T = -V \left(\frac{\partial p}{\partial V}\right)_T = \frac{RT}{V}$

(ii) Compute the partial derivative of κ_T with respect to V and explain the significance of positive and negative values of this derivative.

2. The differential $A(x, y)dx + B(x, y)dy$ is exact if $\frac{\partial A}{\partial y} = \frac{\partial B}{\partial x}$.

(a) Determine whether the following are exact

(i) $dx = (10y + 6z)dy + 6ydz$

(ii) $dx = (3y^2 + 4yz)dy + (2yz + y^2)dz$

(b) An ideal gas is kept at a *constant pressure* (e.g. by a weight on top of a container). The internal energy of the system can be increased by doing mechanical work, or supplying heat.

(i) Show that the internal energy $dU = \frac{3}{2}RdT - pdV$ is an exact differential, and is hence independent of the way in which the work is done on the system (U is a function of state).

(ii) The entropy change is $dS = \frac{dU}{T} + \frac{p}{T}dV$. Show that S is also a function of state.

Thermodynamics Revision

3. A cubic metre of air (diatomic gas) at 0°C and 1 atm is compressed reversibly to 10atm.

(a) What is its final temperature if it is compressed adiabatically?

(b) How much heat must be removed it if is compressed isothermally?

4. Calculate the changes in entropy of the Universe as a result of the following operations:

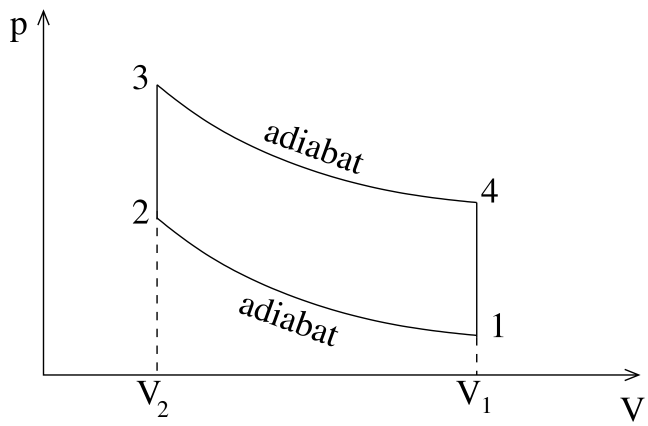
(a) A copper block of mass 0.4kg and thermal capacity 150 J K^{-1} at 100°C is placed in a lake at 10° C.

(b) The same block at 10°C is dropped from a height of 100m into the lake.

(c) Two such blocks, at 100°C and 0°C are joined together.

5. At 10K the molar heat capacity of gold is 0.43JK^{-1} mol^{-1}. Assume that C_P of gold varies as $C_P = aT$ over the range 0 to 10K. Determine the entropy at 10K assuming that $S(T = 0) = 0$.

6. A reversible cycle which idealizes that used in the petrol engine is the *air standard Otto cycle*; explain which part of the cycle represents which process in a real petrol engine.



(a) Show that its efficiency is $\eta = 1 - r^{1-\gamma}$ where $r = V_1/V_2$ is the *compression ratio* and γ is the ratio of heat capacities.

(b) Suppose that an advance in engine design were to allow an increase in r by 10% from its present value for cars of around 10. Estimate the reduciton in annual UK emmissions of carbon dioxide that would result.

7. The differential of the internal energy of a surface of a liquid with surface tension γ and area A may be written as

$$dU = TdS + \gamma dA.$$

Write down the corresponding form of the Helmholtz free energy, $F = U - TS$. Using the fact that these equations involve exact differentials derive the Maxwell relation

$$\left(\frac{\partial S}{\partial A}\right)_T = -\left(\frac{\partial \gamma}{\partial T}\right)_A .$$

The internal energy and the entropy are proportional to the area A. Show that the internal energy per unit area is

$$u(T) = \frac{U}{A} = \gamma - T\left(\frac{\partial \gamma}{\partial T}\right)_A .$$