Uncommon Schools | Change History.

AP Calculus BC

Q3 Interim Assessment

Test Booklet 1

Multiple Choice (Non-Calc)

March 2018

School: ______________________________

Student Name: ______________________________

Teacher: ______________________________

Period: ______________________________

CALCULUS BC

SECTION I, Part A

Time—60 minutes

Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1} x = \arcsin x$).

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1. If $f(x) = (x^2 - 3)^4$, then $f'(1) =$

(A) -64 (B) -32 (C) -16 (D) 32

2. $\int_{-2}^{1} (8x^3 - 3x^2)\, dx =$

(A) -561 (B) -90 (C) -39 (D) 81

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3. An object moves in the xy-plane so that its position at any time t is given by the parametric equations $x(t) = t^3 - 3t^2 + 2$ and $y(t) = \sqrt{t^2 + 16}$. What is the rate of change of y with respect to x when $t = 3$?

(A) $\frac{1}{90}$ (B) $\frac{1}{15}$ (C) $\frac{3}{5}$ (D) $\frac{5}{2}$

4. Snow is falling at a rate of $r(t) = 2e^{-0.1t}$ inches per hour, where t is the time in hours since the beginning of the snowfall. Which of the following expressions gives the amount of snow, in inches, that falls from time $t = 0$ to time $t = 5$ hours?

(A) $2e^{-0.5} - 2$

(B) $0.2 - 0.2e^{-0.5}$

(C) $4 - 4e^{-0.5}$

(D) $20 - 20e^{-0.5}$

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5. If $e^x - y = xy^3 + e^2 - 18$, what is the value of $\frac{dy}{dx}$ at the point $(2, 2)$?

(A) $e^2 - 32$ (B) $\frac{e^2 - 9}{24}$ (C) $\frac{e^2 - 8}{25}$ (D) $\frac{e^2}{13}$

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t (hours)	0	2	7	9
$R(t)$ (tons per hour)	15	9	5	4

6. On a certain day, the rate at which material is deposited at a recycling center is modeled by the function R, where $R(t)$ is measured in tons per hour and t is the number of hours since the center opened. Using a trapezoidal sum with the three subintervals indicated by the data in the table, what is the approximate number of tons of material deposited in the first 9 hours since the center opened?

(A) 68 (B) 70.5 (C) 85 (D) 136

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7. At time $t \geq 0$, a particle moving in the xy-plane has a velocity vector given by $v(t) = \left\langle \cos(2t), e^{3t} \right\rangle$. What is the acceleration vector of the particle?

(A) $\left\langle -2\sin(2t), 3e^{3t} \right\rangle$

(B) $\left\langle -2\sin(2t), e^{3t} \right\rangle$

(C) $\left\langle \dfrac{1}{2}\sin(2t), \dfrac{1}{3}e^{3t} \right\rangle$

(D) $\left\langle 2\sin(2t), 3e^{3t} \right\rangle$

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8. Consider the geometric series $\sum_{n=1}^{\infty} a_n$, where $a_n > 0$ for all n. The first term of the series is $a_1 = 48$, and the third term is $a_3 = 12$. Which of the following statements about $\sum_{n=1}^{\infty} a_n$ is true?

(A) $\sum_{n=1}^{\infty} a_n = 64$

(B) $\sum_{n=1}^{\infty} a_n = 96$

(C) $\sum_{n=1}^{\infty} a_n$ converges, but the sum cannot be determined from the information given.

(D) $\sum_{n=1}^{\infty} a_n$ diverges.

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$$f(x) = \begin{cases} \dfrac{|x|}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

9. The function f is defined above. The value of $\int_{-5}^{3} f(x)\,dx$ is

(A) -2 (B) 2 (C) 8 (D) nonexistent

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10. Which of the following series can be used with the limit comparison test to determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ converges or diverges?

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$ (B) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ (C) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ (D) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

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11. $\int_1^e x^4 \ln x \, dx =$

(A) $\dfrac{6e^5 - 1}{25}$ (B) $\dfrac{4e^5 + 1}{25}$ (C) $\dfrac{1 - e^3}{3}$ (D) e^4

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12. An object moves along a straight line so that at any time t its acceleration is given by $a(t) = 6t$. At time $t = 0$, the object's velocity is 10 and the object's position is 7. What is the object's position at time $t = 2$?

(A) 22 (B) 27 (C) 28 (D) 35

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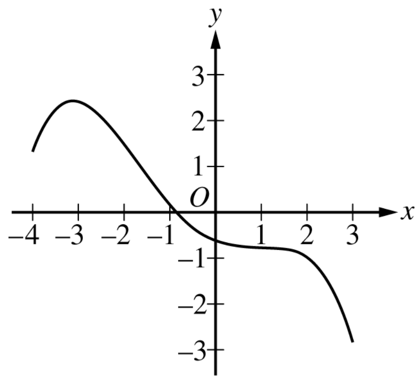

Graph of f

13. The graph of a differentiable function f is shown above on the closed interval $[-4, 3]$. How many values of x in the open interval $(-4, 3)$ satisfy the conclusion of the Mean Value Theorem for f on $[-4, 3]$?

(A) Zero (B) One (C) Two (D) Three

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14. $\int_0^x \sin(t^6)\, dt =$

(A) $\dfrac{x^2}{2} - \dfrac{x^4}{4} + \dfrac{x^6}{6} - \cdots + \dfrac{(-1)^{n+1}x^{2n}}{2n} + \cdots$

(B) $\dfrac{x^2}{2} - \dfrac{x^4}{4 \cdot 3!} + \dfrac{x^6}{6 \cdot 5!} - \cdots + \dfrac{(-1)^{n+1}x^{2n}}{2n \cdot (2n-1)!} + \cdots$

(C) $\dfrac{x^7}{7} - \dfrac{x^{19}}{19} + \dfrac{x^{31}}{31} - \cdots + \dfrac{(-1)^{n+1}x^{6(2n-1)+1}}{6(2n-1)+1} + \cdots$

(D) $\dfrac{x^7}{7} - \dfrac{x^{19}}{19 \cdot 3!} + \dfrac{x^{31}}{31 \cdot 5!} - \cdots + \dfrac{(-1)^{n+1}x^{6(2n-1)+1}}{(6(2n-1)+1)\cdot(2n-1)!} + \cdots$

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15. The speed of a runner, in miles per hour, on a straight trail is modeled by $f(m) = \dfrac{1}{10}(-2m^3 + 9m^2 - 12m) + 7$, where m is the runner's distance, in miles, from the start of the trail. What is the maximum speed of the runner for $0 \le m \le 3$?

(A) 6.5 (B) 6.6 (C) 7.0 (D) 7.5

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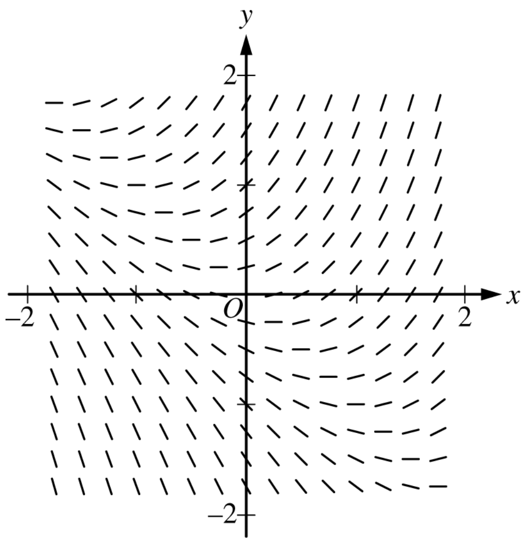
16. Which of the following could be a slope field for the differential equation $\frac{dy}{dx} = x^2 + y$?

(A)



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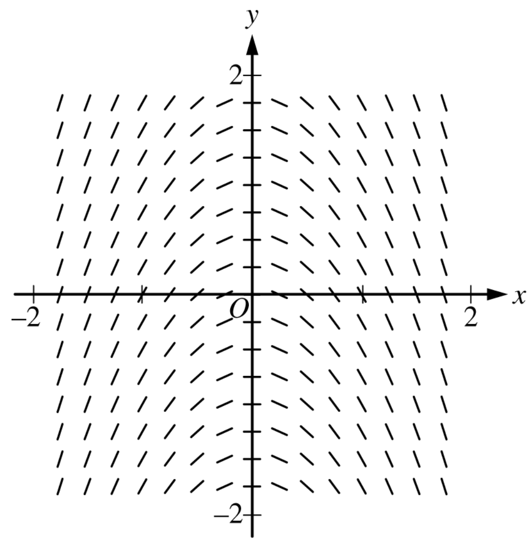

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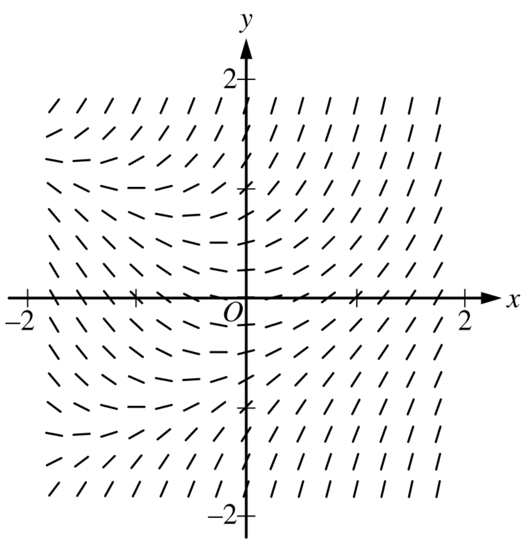
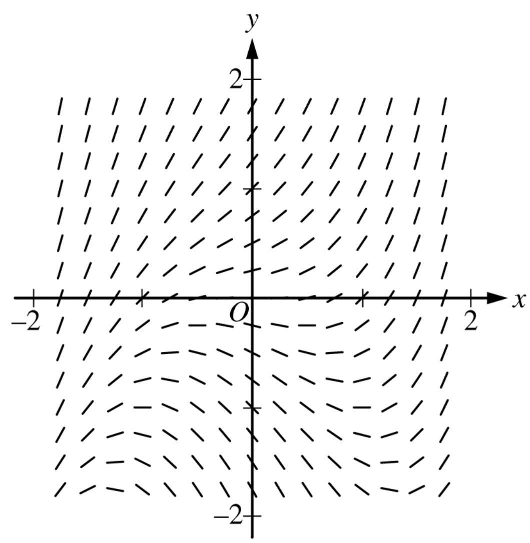


(D)



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17. $\displaystyle\int \frac{8x-10}{(2x-1)(x+1)}\,dx =$

(A) $-4\ln|2x-1| + 6\ln|x+1| + C$

(B) $-2\ln|2x-1| + 6\ln|x+1| + C$

(C) $3\ln|2x-1| - 4\ln|x+1| + C$

(D) $6\ln|2x-1| - 4\ln|x+1| + C$

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18. What is the slope of the line tangent to the polar curve $r = 2\cos\theta - 1$ at the point where $\theta = \pi$?

(A) -3 (B) 0 (C) 3 (D) The slope is undefined.

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19. $\lim\limits_{x \to e} \dfrac{(x^{20} - 3x) - (e^{20} - 3e)}{x - e}$ is

(A) 0 (B) $20e^{19} - 3$ (C) $e^{20} - 3e$ (D) nonexistent

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20. What is the radius of convergence for the series $\sum\limits_{n=0}^{\infty} \dfrac{(x - 5)^n}{2^n (2n + 3)^2}$?

(A) $\dfrac{1}{2}$ (B) 1 (C) 2 (D) 5

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21. Which of the following statements about the integral $\int_0^{\pi} \sec^2 x \, dx$ is true?

(A) The integral is equal to 0.

(B) The integral is equal to $\frac{2}{3}$.

(C) The integral diverges because $\lim_{x \to \frac{\pi}{2}^-} \sec^2 x$ does not exist.

(D) The integral diverges because $\lim_{x \to \frac{\pi}{2}^-} \tan x$ does not exist.

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22. Which of the following series are conditionally convergent?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

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23. $\displaystyle\int_0^{\ln 2} \frac{e^x}{1 + (e^x - 1)^2}\,dx =$

(A) $\arctan(\ln 2)$ (B) $\ln 2$ (C) $\dfrac{\pi}{4}$ (D) $\dfrac{\pi}{2}$

24. $\displaystyle\lim_{x \to 3} \frac{\tan(x - 3)}{3e^{x-3} - x}$ is

(A) 0 (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) nonexistent

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25. The volume of a sphere is increasing at a rate of 6π cubic centimeters per hour. At what rate, in centimeters per hour, is its diameter increasing with respect to time at the instant the radius of the sphere is 3 centimeters? (Note: The volume of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$.)

(A) $\frac{1}{3}$ (B) 1 (C) $\sqrt{6}$ (D) 6

26. The function $f(\theta) = \theta^3 - 6\theta^2 + 9\theta$ satisfies $f(\theta) \geq 0$ for $\theta \geq 0$. During the time interval $0 \leq t \leq 2\pi$ seconds, a particle moves along the polar curve $r = f(\theta)$ so that at time t seconds, $\theta = t$. On what intervals of time t is the distance between the particle and the origin increasing?

(A) $0 \leq t \leq 3$ only

(B) $0 \leq t \leq 2\pi$

(C) $1 \leq t \leq 3$ only

(D) $0 \leq t \leq 1$ and $3 \leq t \leq 2\pi$ only

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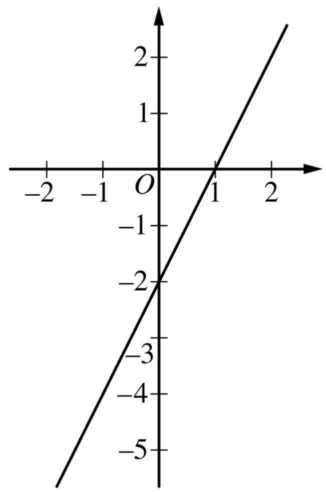

Graph of f

27. The graph of the function f is shown above for $-2 < x < 2$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. On what open interval is g negative and decreasing?

(A) $-2 < x < 0$ only

(B) $-2 < x < 1$

(C) $0 < x < 1$ only

(D) $0 < x < 2$

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28. The function N satisfies the logistic differential equation $\frac{dN}{dt} = \frac{N}{10}\left(1 - \frac{N}{850}\right)$, where $N(0) = 105$. Which of the following statements is false?

(A) $\lim_{t\to\infty} N(t) = 850$

(B) $\frac{dN}{dt}$ has a maximum value when $N = 105$.

(C) $\frac{d^2N}{dt^2} = 0$ when $N = 425$.

(D) When $N > 425$, $\frac{dN}{dt} > 0$ and $\frac{d^2N}{dt^2} < 0$.

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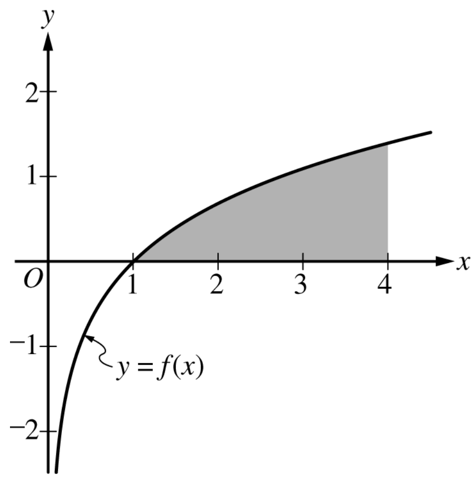

29. The function f is given by $f(x) = \ln x$. The graph of f is shown above. Which of the following limits is equal to the area of the shaded region?

(A) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \left(1 + \ln\left(\frac{3k}{n}\right)\right)\frac{3}{n}$

(B) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \ln\left(1 + \frac{3k}{n}\right)\frac{3}{n}$

(C) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \ln\left(\frac{4}{n}\right)\left(1 + \frac{4k}{n}\right)$

(D) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \ln\left(1 + \frac{4k}{n}\right)\frac{4}{n}$

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30. The Taylor series for a function f about $x = 0$ is given by $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!} x^{2n}$ and converges to f for all real numbers x. If the fourth-degree Taylor polynomial for f about $x = 0$ is used to approximate $f\left(\frac{1}{2}\right)$, what is the alternating series error bound?

(A) $\frac{1}{2^4 \cdot 5!}$

(B) $\frac{1}{2^5 \cdot 6!}$

(C) $\frac{1}{2^6 \cdot 7!}$

(D) $\frac{1}{2^{10} \cdot 11!}$

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.