Question 1

Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30,$ the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30.$ Indicate units of measure.

(b) Find the value of $A'(15).$ Using correct units, interpret the meaning of the value in the context of the problem.

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30.$

(d) For $t > 30,$ $L(t),$ the linear approximation to A at $t = 30,$ is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

(a) $\dfrac{A(30) - A(0)}{30 - 0} = -0.197$ (or -0.196) lbs/day	1 : answer with units
(b) $A'(15) = -0.164$ (or -0.163) The amount of grass clippings in the bin is decreasing at a rate of 0.164 (or 0.163) lbs/day at time $t = 15$ days.	$2 : \begin{cases} 1 : A'(15) \\ 1 : \text{interpretation} \end{cases}$
(c) $A(t) = \dfrac{1}{30}\displaystyle\int_0^{30} A(t)\,dt \Rightarrow t = 12.415$ (or 12.414)	$2 : \begin{cases} 1 : \dfrac{1}{30}\displaystyle\int_0^{30} A(t)\,dt \\ 1 : \text{answer} \end{cases}$
(d) $L(t) = A(30) + A'(30)\cdot(t - 30)$ $A'(30) = -0.055976$ $A(30) = 0.782928$ $L(t) = 0.5 \Rightarrow t = 35.054$	$4 : \begin{cases} 2 : \text{expression for } L(t) \\ 1 : L(t) = 0.5 \\ 1 : \text{answer} \end{cases}$

Question 2

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) Solve $|v(t)| = 2$ on $2 \le t \le 4$.
$t = 3.128$ (or 3.127) and $t = 3.473$

$2 : \begin{cases} 1 : \text{considers } |v(t)| = 2 \\ 1 : \text{answer} \end{cases}$

(b) $s(t) = 10 + \int_0^t v(x)\,dx$

$s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$

$2 : \begin{cases} 1 : s(t) \\ 1 : s(5) \end{cases}$

(c) $v(t) = 0$ when $t = 0.536033,\ 3.317756$
$v(t)$ changes sign from negative to positive at time $t = 0.536033$.
$v(t)$ changes sign from positive to negative at time $t = 3.317756$.

Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).

$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$

(d) $v(4) = -11.475758 < 0,\ a(4) = v'(4) = -22.295714 < 0$

The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.

2 : conclusion with reason

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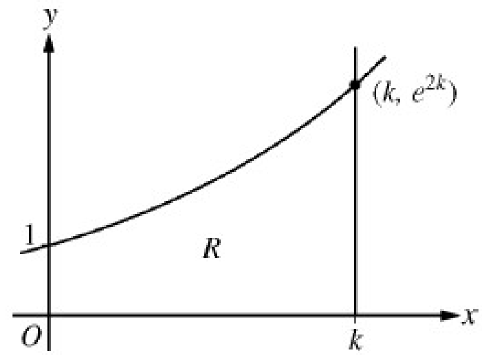
Question 3 (Modified)



1. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.

   (a) Find the area of region R in terms of k.
   (b) The region R is rotated about the x-axis to form a solid. Find the volume, V, of the solid in terms of k.
   (c) The volume V, found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

(a) Area = $\int_0^k e^{2x}\,dx = \frac{1}{2}e^{2x}\Big|_{x=0}^{x=k} = \frac{1}{2}e^{2k} - \frac{1}{2}$

$2: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(b) Volume = $\pi\int_0^k (e^{2x})^2 dx = \pi\int_0^k e^{4x}\,dx = \frac{\pi}{4}\,e^{4x}\Big|_{x=0}^{x=k} = \frac{\pi}{4}e^{4k} - \frac{\pi}{4}$

$4: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

(c) $\frac{dV}{dt} = \pi e^{4k}\frac{dk}{dt}$

When $k = \frac{1}{2}, \frac{dV}{dt} = \pi e^2 \cdot \frac{1}{3}$.

$3: \begin{cases} 1: \text{applies chain rule} \\ 1: \text{answer} \end{cases}$

Question 5

Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x.

(a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

(b) Find $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with the initial condition $f(1) = 0$.

(a) $$f\left(\frac{1}{2}\right) \approx f(1) + \left(\left.\frac{dy}{dx}\right|_{(1,\,0)}\right) \cdot \Delta x$$
$$= 0 + 1 \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2}$$
$$f(0) \approx f\left(\frac{1}{2}\right) + \left(\left.\frac{dy}{dx}\right|_{\left(\frac{1}{2},\,-\frac{1}{2}\right)}\right) \cdot \Delta x$$
$$\approx -\frac{1}{2} + \frac{3}{2} \cdot \left(-\frac{1}{2}\right) = -\frac{5}{4}$$

$2: \begin{cases} 1: \text{Euler's method with two steps} \\ 1: \text{answer} \end{cases}$

(b) Since f is differentiable at $x = 1$, f is continuous at $x = 1$. So, $\lim_{x \to 1} f(x) = 0 = \lim_{x \to 1}\left(x^3 - 1\right)$ and we may apply L'Hospital's Rule.

$$\lim_{x \to 1} \frac{f(x)}{x^3 - 1} = \lim_{x \to 1} \frac{f'(x)}{3x^2} = \frac{\lim_{x \to 1} f'(x)}{\lim_{x \to 1} 3x^2} = \frac{1}{3}$$

$2: \begin{cases} 1: \text{use of L'Hospital's Rule} \\ 1: \text{answer} \end{cases}$

(c) $$\frac{dy}{dx} = 1 - y$$
$$\int \frac{1}{1 - y}\, dy = \int 1\, dx$$
$$-\ln|1 - y| = x + C$$
$$-\ln 1 = 1 + C \Rightarrow C = -1$$
$$\ln|1 - y| = 1 - x$$
$$|1 - y| = e^{1-x}$$
$$f(x) = 1 - e^{1-x}$$

$5: \begin{cases} 1: \text{separation of variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } y \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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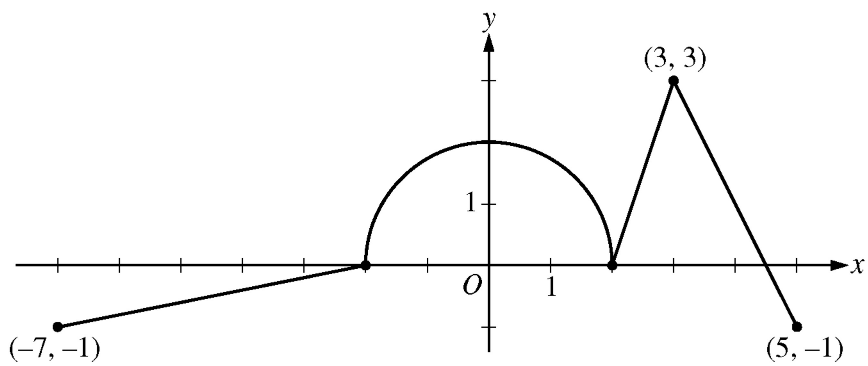
Question 5



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(a) $g(3) = 5 + \int_0^3 g'(x)\,dx = 5 + \frac{\pi \cdot 2^2}{4} + \frac{3}{2} = \frac{13}{2} + \pi$ $g(-2) = 5 + \int_0^{-2} g'(x)\,dx = 5 - \pi$	$3 : \begin{cases} 1 : \text{uses } g(0) = 5 \\ 1 : g(3) \\ 1 : g(-2) \end{cases}$
(b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.	$2 : \begin{cases} 1 : \text{identifies } x = 0, 2, 3 \\ 1 : \text{explanation} \end{cases}$
(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$ On the interval $-2 \le x \le 2$, $g'(x) = \sqrt{4 - x^2}$. On this interval, $g'(x) = x$ when $x = \sqrt{2}$. The only other solution to $g'(x) = x$ is $x = 3$. $h'(x) = g'(x) - x > 0$ for $0 \le x < \sqrt{2}$ $h'(x) = g'(x) - x \le 0$ for $\sqrt{2} < x \le 5$ Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.	$4 : \begin{cases} 1 : h'(x) \\ 1 : \text{identifies } x = \sqrt{2}, 3 \\ 1 : \text{answer for } \sqrt{2} \text{ with analysis} \\ 1 : \text{answer for 3 with analysis} \end{cases}$

AP® Calculus BC
2013 Scoring Guidelines

Question 3 (Modified)

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

1. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

   (a) Estimate $W''(5)$. Indicate units of measure.

   (b) Use the tangent line approximation to W at time $t = 30$ to predict the volume of the water, $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

   (c) Use a trapezoidal sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of the water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

   (d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t =$ 30 days.

(a)

$$W''(5) \approx \frac{0.7 - 0.6}{10 - 0} = 0.01\,\frac{GL}{day^2}$$

$2: \begin{cases} 1: \text{AROC} \\ 1: \text{Answer + Units} \end{cases}$

(b) An equation of the tangent line is $y = 0.5(t - 30) + 125$

$$W(32) \approx 0.5(32 - 30) + 125 = 126$$

1: answer

(c)

$$\int_0^{30} W'(t)\,dt \approx 10\left(\frac{0.7 + 0.6}{2}\right) + 12\left(\frac{1.0 + 0.7}{2}\right) + 8\left(\frac{1.0 + 0.5}{2}\right) = 22.7$$

$$W(0) = W(30) - \int_0^{30} W'(t)dt = 125 - 22.4 = 102.3$$

$3: \begin{cases} 1: \text{Trapezoidal Sum} \\ 1: \text{Approximation} \\ 1: \text{Answer} \end{cases}$

(d)

$$\frac{dA}{dt} = (0.3)\left(\frac{2}{3}\right)W^{-\frac{1}{3}} \cdot \frac{dW}{dt} = \frac{0.2}{\sqrt[3]{W}}\frac{dW}{dt}$$

$$\left.\frac{dA}{dt}\right|_{t=30} = \frac{0.2}{\sqrt[3]{125}} \cdot 0.5 = 0.02$$

$3: \begin{cases} 2: \frac{dA}{dt} \\ 1: \text{Answer + Units} \end{cases}$