AP Calculus BC

Q2 Interim Assessment

Test Booklet 3

Free Response Questions

January 2018

Student Name: ______________________________

Period: ______________________________

Teacher: ______________________________

School: ______________________________

AP® Calculus BC Exam

SECTION II: Free Response

DO NOT OPEN THIS BOOKLET OR BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth ☐☐ ☐☐ ☐☐☐☐
   Month Day Year
3. Six-digit school code ☐☐☐☐☐☐
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not begin Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X², X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS BC

SECTION II, Part A

Time – 30 minutes

Number of problem – 2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE PROBLEMS

Name: ______________________

1 1 1 1 1 1 1 1 1 1

1. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

(b) Find the values of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

Name: ______________________

1 1 1 1 1 1 1 1 1 1

1. Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

   (c) Find the time t for which the amount of gradss clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.

   (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict at the time at which there will be 0.5 pounds of grass clippings remaining in the bin. Show the work that leads to your answer.

Name: ____________________________

2. A particle moves alogn a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t < 4$ for which the speed of the particle is 2.

(b) Write an expression involving an intergral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

Name: ____________________________

2 2 2 2 2 2 2 2 2 2

2. A particle moves alogn a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(c) Find all time t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

END OF PART A

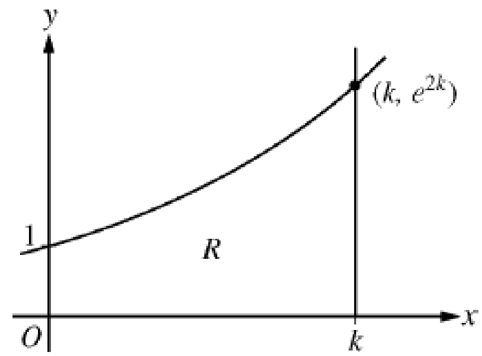
IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION II, Part B
Time – 60 minutes
Number of problems – 4

NO CALCULATOR FOR THESE PROBLEMS.

DO NOT BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.



3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.

(a) Find the area of region R in terms of k.

(b) The region R is rotated about the x-axis to form a solid. Find the volume, V, of the solid in terms of k.

Name: ____________________________

3 3 3 3 3 3 3 3 3 3

NO CALCULATOR ALLOWED

(c) The volume V, found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

Name: ______________________________

4 4 4 4 4 4 4 4 4 4

NO CALCULATOR ALLOWED

4. Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to this differential equation with initial condition $f(1) = 0$. For this particular solution $f(x) < 1$ for all values of x.

(a) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

(b) Find $\lim_{x \to 1} \frac{f(x)}{x^3 - 1}$. Show the work that leads to your answer.

Name: ____________________________

4 4 4 4 4 4 4 4 4 4

NO CALCULATOR ALLOWED

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 1 - y$ with initial condition $f(1) = 0$.

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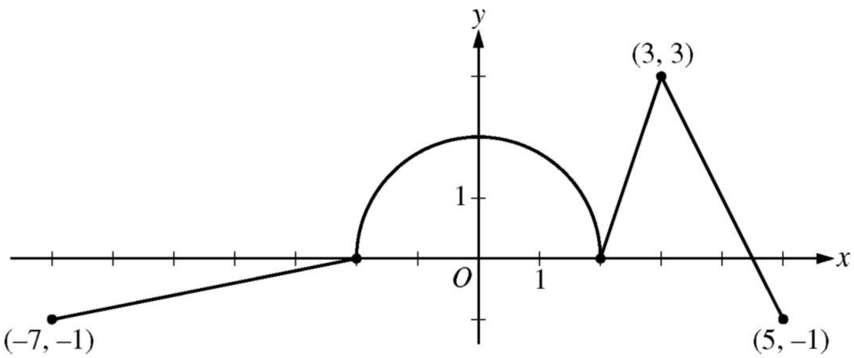
5 5 5 5 5 5 5 5 5 5

NO CALCULATOR ALLOWED



Graph of g'

5. The function g is defined and differentiable on the closed interval $[-7,5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

Name: ____________________________

6 6 6 6 6 6 6 6 6 6

NO CALCULATOR ALLOWED

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

6. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

   (a) Estimate $W''(5)$. Indicate units of measure.

   (b) Use the tangent line approximation to W at time $t = 30$ to predict the volume of the water, $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

Name: ____________________________

6 6 6 6 6 6 6 6 6 6

NO CALCULATOR ALLOWED

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

6. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the interval $0 \leq t \leq 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

   (c) Use a trapezoidal sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of the water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

   (d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

STOP

END OF EXAM