Uncommon Schools | Change History.

AP Calculus AB

Q2 Interim Assessment

January 2016

Section II – Part B (60 Minutes)

No Calculators Allowed

Student Name: ________________________________

School: ________________________________

Teacher: ________________________________

SECTION II – PART B DIRECTIONS
60 Minutes: 4 Open Response *(9 points each)*

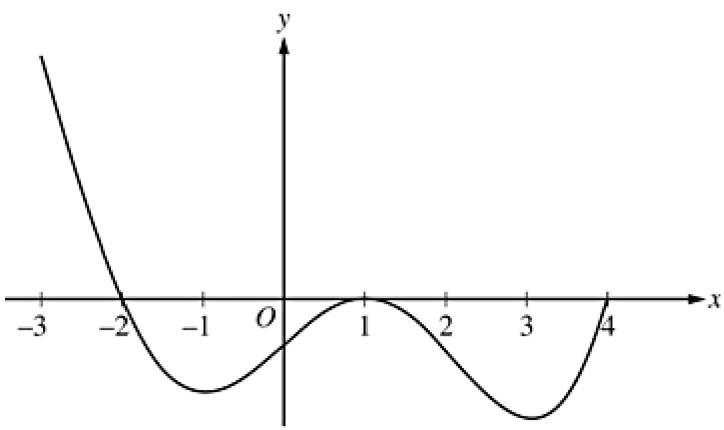
3. The figure below shows the graph of f', the derivaive of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $(-3, 4)$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection of $f(x)$. Give a reason for your answer.

(d) If $h(x) = f(\sin(\pi x))$, find $h'(x)$.



Graph of f'

(a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$. $f'(x)$ changes from positive to negative at $x = -2$. Therefore, f has a relative maximum at $x = -2$.	$2: \begin{cases} 1: \text{identifies } x = -2 \\ 1: \text{answer with reason} \end{cases}$
(b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.	$3: \begin{cases} 1: \text{intervals} \\ 1: f' \text{is decreasing} \\ 1: f'(x) < 0 \end{cases}$
(c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points. The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point. (Or any equivalent reason.)	$2: \begin{cases} 1: x = -1, 1, 3 \\ 1: \text{answer with reason} \end{cases}$
(d) $h'(x) = f'(\sin(\pi x))\cos(\pi x)\pi$	$2: h'(x)$

4. Let f be the function given by $f(x)=(x^2-2x-1)e^x$.

(a) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

(b) Find the interval(s) on which f is increasing. Justify your answer.

(c) Find the interval(s) on which f is concave down. Justify your answer.

(a) $\lim_{x\to\infty} f(x)=\infty$ **or DNE** $\lim_{x\to-\infty} f(x)=0$	$2:\begin{cases} 1: x\to\infty \\ 1: x\to-\infty \end{cases}$
(b) $f'(x)=(2x-2)e^x+(x^2-2x-1)e^x$ $f'(x)=(x^2-2x-1+2x-2)e^x=(x^2-3)e^x$ $x=\pm\sqrt{3}$ → **critical numbers** $f(x)$ **is increasing on** $(-\infty,-\sqrt{3})\cup(\sqrt{3},\infty)$ **because** $f'(x)>0$.	$4:\begin{cases} 1: f'(x) \\ 1: \text{critical numbers} \\ 1: \text{intervals} \\ 1: \text{justification} \end{cases}$
(c) $f''(x)=(2x)e^x+(x^2-3)e^x$ $f''(x)=(x^2+2x-3)e^x=(x+3)(x-1)e^x$ $x=-3, 1$ → **possible points of inflection** $f(x)$ **is concave down on** $(-3,1)$ **because** $f''(x)<0$.	$3:\begin{cases} 1: f''(\text{x}) \\ 1: \text{possible points of inflection} \\ 1: \text{answer with justification} \end{cases}$

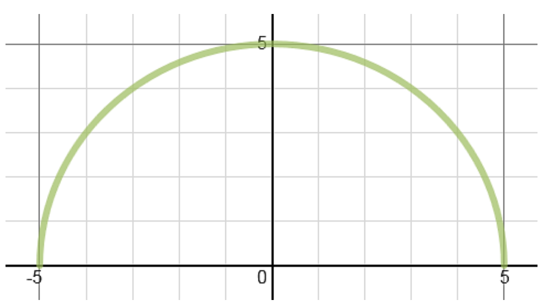
5. The function *f* is defined by $f(x)=\sqrt{25-x^2}$ **for [–5, 5].**

(a) Find $f'(x)$.

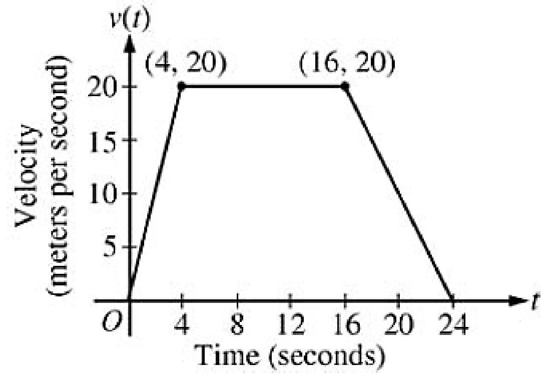
(b) Write an equation for the line normal to the graph of *f* at *x* = –3.

(c) Find the value of $\int_{-5}^{5} f(x)dx$. **Show the work that leads to your answer.**

(d) Let *g* be the function defined by $g(x)=\begin{cases} f(x) & -5\le x\le -3 \\ x+6 & -3<x\le 5\end{cases}$. **Is *g* continuous at *x* = –3? Write a concluding statement justifying your answer.**

(a) $f'(x)=\frac{1}{2}(25-x^2)^{-1/2}(-2x)=\frac{-x}{(25-x^2)^{1/2}}=\frac{-x}{\sqrt{25-x^2}}$	$2: f'(x)$
(b) $f'(-3)=\frac{1}{2}(25-9)^{-1/2}(6)=\frac{6}{2(4)}=\frac{3}{4}$ $y-4=\frac{-4}{3}(x+3)$	$2:\begin{cases}1: f'(-3)\\ 1: \text{tangent line}\end{cases}$
(c) $\int_{-5}^{5}\sqrt{25-x^2}\,dx$ → **area of semi-circle with radius = 5** $\int_{-5}^{5}\sqrt{25-x^2}\,dx=\frac{25\pi}{2}$ 	$2:\begin{cases}1: \text{appropriate work}\\ 1: \text{answer}\end{cases}$
(d) $\lim_{x\to -3^-} f(x)=4$ $\lim_{x\to -3^+} x+6=3$ **Since** $\lim_{x\to -3^-} g(x)\ne \lim_{x\to -3^+} g(x)$, **the function *g*(*x*) is not continuous at *x* = –3.**	$3:\begin{cases}1: \text{left}-\text{hand limit}\\ 1: \text{right}-\text{hand limit}\\ 1: \text{concluding statement}\end{cases}$

6. A car is traveling on a straight road. For $0 \leq t \leq 24$ secs, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph.



(a) Find $\int_0^{24} v(t)dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Indicate units of measure. Does the Mean Value Theorem guarantee a value for c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

(a) $\int_0^{24} v(t)\,dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$	1 : value
(b) $v'(4)$ does not exist because $\lim_{t\to 4^-}\left(\frac{v(t)-v(4)}{t-4}\right) = 5 \neq 0 = \lim_{t\to 4^+}\left(\frac{v(t)-v(4)}{t-4}\right)$. $v'(20) = \frac{20-0}{16-24} = -\frac{5}{2}$ m/sec^2	$3 : \begin{cases} 1 : v'(4) \text{ does not exist, with explanation} \\ 1 : v'(20) \\ 1 : \text{units} \end{cases}$
(c) $a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$ $a(t)$ does not exist at $t = 4$ and $t = 16$.	$2 : \begin{cases} 1 : \text{finds the values } 5, 0, -\frac{5}{2} \\ 1 : \text{identifies constants with correct intervals} \end{cases}$
(d) The average rate of change of v on $[8, 20]$ is $\frac{v(20)-v(8)}{20-8} = -\frac{5}{6}$ m/sec^2. No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.	$3: \begin{cases} 1: \text{average rate of change} \\ 1: \text{answer with units} \\ 1: \text{explanation} \end{cases}$

END OF EXAM