Uncommon Schools | Change History.

AP Calculus AB

Q2 Interim Assessment

January 2016

Section II – Part A (30 Minutes)

Calculators Allowed

Student Name: ______________________________

School: ______________________________

Teacher: ______________________________

SECTION II – PART A DIRECTIONS
30 Minutes: 2 Open Response *(9 points each)*

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

1. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Estimate $W''(5)$. Indicate units of measure.

(b) Use the tangent line approximation to W at time $t = 30$ to predict the volume of the water, $W(t)$, in GL, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

(c) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

(a) $\frac{W'(10)-W'(0)}{10-0} = \frac{0.7-0.6}{10}$ $= \frac{0.1}{10} = 0.01 GL/day^2$	$3: \begin{cases} 1: \frac{W'(10)-W'(0)}{10-0} \\ 1: \text{answer} \\ 1: \text{units} \end{cases}$
(b) An equation of the tangent line is $y = 0.5(t-30)+125$ **or** $y = 0.5t + 110$. $W(32) \approx 0.5(32-30)+125 = 126$	$3: \begin{cases} 2: \text{tangent line} \\ 1: \text{answer} \end{cases}$
(c) $\frac{dA}{dt} = (0.3)\frac{2}{3}W^{-1/3} \cdot \frac{dW}{dt} = \frac{0.2}{\sqrt[3]{W}} \cdot \frac{dW}{dt}$ $\left.\frac{dA}{dt}\right\|_{t=30} = \frac{0.2}{\sqrt[3]{125}} \cdot 0.5 = 0.02$	$3: \begin{cases} 2: dA/dt \\ 1: \text{answer} \end{cases}$

2. The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and

$$g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right).$$

(a) Find all values of x in the interval [0.12, 1] at which the graph of g has a horizontal tangent, and determine whether g has a local maximum, a local minimum, or neither at each of these values of x. Justify your answers.

(b) On what subintervals of (0.12, 1), if any, is the graph of g concave down? Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 1$.

(d) Does the line tangent to the graph of g at $x = 1$ lie above or below the graph on (0.3, 1)? Why?

(a) $g'(x) = 0$ **at x = 0.163 and 0.359** $g(x)$ **has a local maximum at x = 0.163 because** $g'(x)$ **changes from (+) to (–).** $g(x)$ **has a local minimum at x = 0.359 because** $g'(x)$ **changes from (–) to (+).**	$3: \begin{cases} 1: \text{x — values} \\ 1: \text{justification for local max} \\ 1: \text{justification for local min} \end{cases}$
(b) $g(x)$ **is concave down on (0.129, 0.223), because** $g''(x) < 0$.	$2: \begin{cases} 1: \text{interval} \\ 1: \text{justification} \end{cases}$
(c) $g'(1) = 0.909$ $y - 2 = 0.909(x - 1)$	$2: \begin{cases} 1: g'(1) \\ 1: \text{tangent line} \end{cases}$
(d) The tangent line is below the graph, because the graph of g is concave up $(g''(x) > 0)$ **for (0.3, 1).**	$2: \begin{cases} 1: \text{answer} \\ 1: \text{reason} \end{cases}$

END OF SECTION II – PART A

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS PART ONLY.
DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.