Uncommon Schools | Change History.

AP Calculus AB

Q3 Interim Assessment

April 2016

Section I – Part B (50 Minutes)

Calculators Allowed

Student Name: _____

School: _____

Teacher: _____

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

76. The function $P(t)$ models the population of the world, in billions of people, where t is the number of years since January 1, 2010. Which of the following is the best interpretation of the statement $P'(1) = 0.076$?

(A) On February 1, 2010, the population of the world was increasing at a rate of 0.076 billion people per year.

(B) On January 1, 2011, the population of the world was increasing at a rate of 0.076 billion people per year.

(C) On January 1, 2011, the population of the world was 0.076 billion people.

(D) From January 1, 2010 to January 1, 2011, the population of the world was increasing at an average rate of 0.076 billion people per year.

(E) When the population of the world was 1 billion people, the population of the world was increasing at a rate of 0.076 billion people per year.

x	0	2	4	6	8	10
$f(x)$	5	7	8	0	–15	–20

77. Let f be a differentiable function with selected values given in the table above. What is the average rate of change of f over the closed interval $0 \le x \le 10$?

(A) -6 (B) $-\frac{5}{2}$ (C) -2 (D) $-\frac{2}{5}$ (E) $\frac{2}{5}$

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78. The rate at which motor oil is leaking from an automobile is modeled by the function L defined by $L(t) = 1 + \sin\left(t^2\right)$ for time $t \ge 0$. $L(t)$ is measured in liters per hour, and t is measured in hours. How much oil leaks out of the automobile during the first half hour?

(A) 1.998 liters

(B) 1.247 liters

(C) 0.969 liters

(D) 0.541 liters

(E) 0.531 liters

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	4	2	π

79. The table above gives values of the differentiable functions f and g and their derivatives at $x = 0$.

If $h(x) = \dfrac{f(x)}{g(x)}$, what is the value of $h'(0)$?

(A) $\dfrac{8-3\pi}{4}$ (B) $\dfrac{3\pi-8}{4}$ (C) $\dfrac{4}{\pi}$ (D) $\dfrac{2-3\pi}{2}$ (E) $\dfrac{8+3\pi}{4}$

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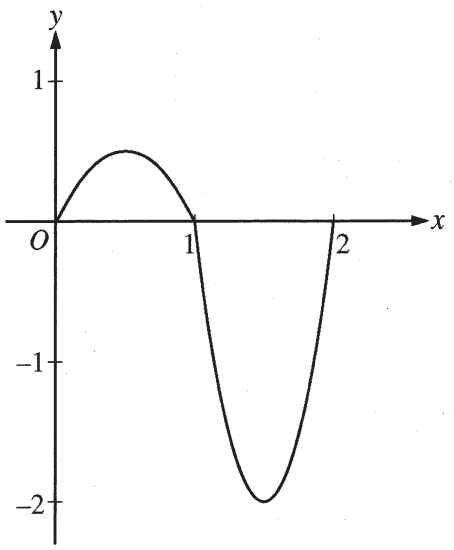

Graph of f'

80. The figure above shows the graph of f', the derivative of a function f, for $0 \le x \le 2$. What is the value of x at which the absolute minimum of f occurs?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) 2

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81. What is the area of the region enclosed by the graphs of $y = \sqrt{4x - x^2}$ and $y = \frac{x}{2}$?

(A) 1.707 (B) 2.829 (C) 5.389 (D) 8.886 (E) 21.447

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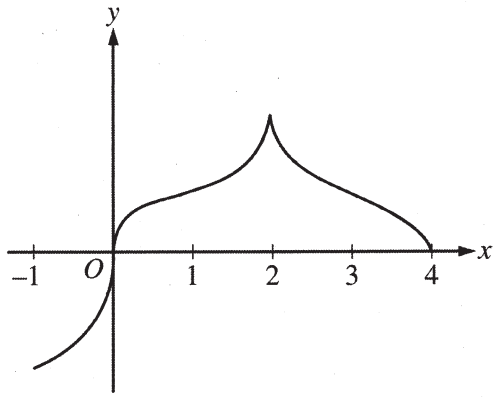

Graph of f'

82. The graph of f', the derivative of f, is shown above. The line tangent to the graph of f' at $x = 0$ is vertical, and f' is not differentiable at $x = 2$. Which of the following statements is true?

(A) f' does not exist at $x = 2$.

(B) f is decreasing on the interval $(2, 4)$.

(C) The graph of f has a point of inflection at $x = 2$.

(D) The graph of f has a point of inflection at $x = 0$.

(E) f has a local maximum at $x = 0$.

83. A particle moves along the x-axis so that its position at time $t > 0$ is given by $x(t)$ and $\frac{dx}{dt} = -10t^4 + 9t^2 + 8t$. The acceleration of the particle is zero when $t =$

(A) 0.387 (B) 0.831 (C) 1.243 (D) 1.647 (E) 8.094

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84. The function f is continuous on the closed interval $[1, 7]$. If $\int_1^7 f(x)\,dx = 42$ and $\int_7^3 f(x)\,dx = -32$, then $\int_1^3 2f(x)\,dx =$

(A) -148 (B) 10 (C) 12 (D) 20 (E) 148

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85. Let $y = f(x)$ define a twice-differentiable function and let $y = t(x)$ be the line tangent to the graph of f at $x = 2$. If $t(x) \geq f(x)$ for all real x, which of the following must be true?

(A) $f(2) \geq 0$

(B) $f'(2) \geq 0$

(C) $f'(2) \leq 0$

(D) $f''(2) \geq 0$

(E) $f''(2) \leq 0$

86. The vertical line $x = 2$ is an asymptote for the graph of the function f. Which of the following statements must be false?

(A) $\lim_{x \to 2} f(x) = 0$

(B) $\lim_{x \to 2} f(x) = -\infty$

(C) $\lim_{x \to 2} f(x) = \infty$

(D) $\lim_{x \to \infty} f(x) = 2$

(E) $\lim_{x \to \infty} f(x) = \infty$

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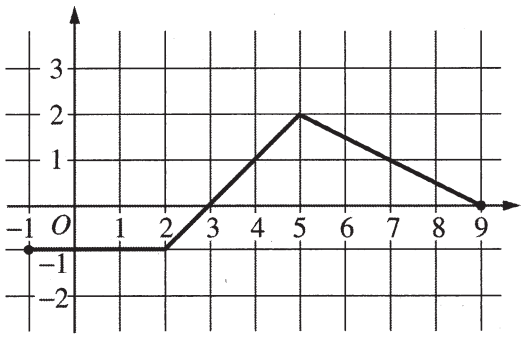

Graph of f

87. The graph of the piecewise linear function f is shown above. Let h be the function given by $h(x) = \int_{-1}^{x} f(t)\,dt$. On which of the following intervals is h increasing?

(A) $[-1, 3]$

(B) $[0, 5]$

(C) $[2, 5]$ only

(D) $[2, 9]$

(E) $[3, 9]$ only

88. The first derivative of the function f is given by $f'(x) = \sin\left(x^2\right)$. At which of the following values of x does f have a local minimum?

(A) 2.507 (B) 2.171 (C) 1.772 (D) 1.253 (E) 0

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89. If $\lim_{x \to a} f(x) = f(a)$, then which of the following statements about f must be true?

(A) f is continuous at $x = a$.

(B) f is differentiable at $x = a$.

(C) For all values of x, $f(x) = f(a)$.

(D) The line $y = f(a)$ is tangent to the graph of f at $x = a$.

(E) The line $x = a$ is a vertical asymptote of the graph of f.

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90. The temperature F, in degrees Fahrenheit (°F), of a cup of coffee t minutes after it is poured is given by $F(t) = 72 + 118e^{-0.093t}$. To the nearest degree, what is the average temperature of the coffee between $t = 0$ and $t = 10$ minutes?

(A) 93°F

(B) 119°F

(C) 146°F

(D) 149°F

(E) 154°F

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91. If $f'(x) = \cos\left(x^2\right)$ and $f(3) = 7$, then $f(2) =$

(A) 0.241 (B) 5.831 (C) 6.416 (D) 6.759 (E) 7.241

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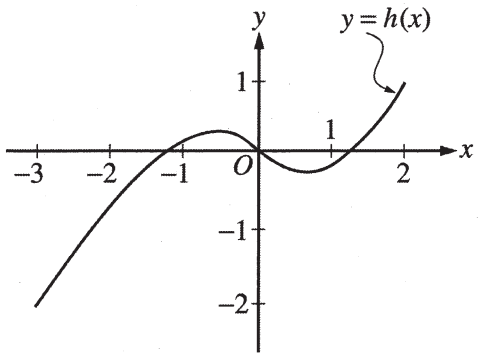

92. The graph of the function h is shown in the figure above. Of the following, which has the greatest value?

(A) Average value of h over $[-3,2]$

(B) Average rate of change of h over $[-3,2]$

(C) $\int_{-3}^{2} h(x)\,dx$

(D) $\int_{-3}^{0} h(x)\,dx$

(E) $h'(0)$