AP Calculus AB

Q1 Interim Assessment

Test Booklet 3

Free Response Questions

October 2016

School: ______________________________

Student Name: ______________________________

Teacher: ______________________________

Period: ______________________________

AP® Calculus AB Exam

SECTION II: Free Response

DO NOT OPEN THIS BOOKLET OR BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ ☐☐ ☐☐☐☐
   Month Day Year
3. Six-digit school code
   ☐☐☐☐☐☐
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Instructions

The questions for Section II are printed in this booklet. Do not begin Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X², X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS AB

SECTION II, Part A

Time – 30 minutes

Number of problem – 2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE PROBLEMS

1. A particle moves along the x-axis with velocity, in meters per second, given by $v(t) = -1 + e^{1-t}$ for $t \geq 0$, where t is in seconds.

(a) Find the average rate of change of the velocity on the interval [2, 4]. Indicate units of measure.

(a) $\dfrac{v(4)-v(2)}{4-2} = \dfrac{-0.9502-(-0.6321)}{2}$ $= \dfrac{-0.3181}{2} = -0.159\, m/s^2$	$2: \begin{cases} 1: \dfrac{v(4)-v(2)}{4-2} \\ 1: \text{answer with units} \end{cases}$

(b) Find the value of $v'(3)$. Using the correct units, interpret the meaning of $v'(3)$ in terms of the motion of the particle.

(b) $v'(3) = -0.135$ $v'(3)$ **is the instantaneous acceleration (rate of change) in** m/s^2 **at** $t = 3$**.**	$3: \begin{cases} 1: v'(3) \\ 1: \text{intepretation} \\ 1: \text{units} \end{cases}$

(c) Is the speed of the particle increasing at time $t = 3$? Give a reason to your answer.

(c) $a(3) = v'(3) = -0.135 < 0$ $v(3) = -0.865 < 0$ **Speed is increasing at** $t = 3$ **since** $v(3) < 0$ **and** $a(3) < 0$**.**	$2: \begin{cases} 1: v(3) \\ 1: \text{answer with reason} \end{cases}$

(d) Find all values of t at which the particle changes direction. Justify your answer.

(d) $v(t) = 0$ **when** $1 = e^{1-t}$**, so** $t = 1$**.** $v(t) > 0$ **for** $t < 1$ **and** $v(t) < 0$ **for** $t > 1$**.** **Therefore, the particle changes direction at** $t = 1$**.**	$2: \begin{cases} 1: \text{solves } v(t) = 0 \text{ to get } t = 1 \\ 1: \text{justifies change in direction at } t = 1 \end{cases}$

2. Consider the curve given by $y^3 - xy = 2$.

(a) Show that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) $3y^2 \frac{dy}{dx} - \left(1 \cdot y + x \cdot \frac{dy}{dx}\right) = 0$ $3y^2 \frac{dy}{dx} - y - x\frac{dy}{dx} = 0$ $3y^2 \frac{dy}{dx} - x\frac{dy}{dx} = y$ $\frac{dy}{dx}\left(3y^2 - x\right) = y$ $\frac{dy}{dx} = \frac{y}{3y^2 - x}$	$3: \begin{cases} 2: \text{implicit differentiation} \\ 1: \text{verification} \end{cases}$

(b) Write an equation for the tangent line to the curve at the point $(-1,1)$.

(b) $\left.\frac{dy}{dx}\right\|_{(-1,1)} = \frac{1}{3 \cdot 1^2 - (-1)} = \frac{1}{4}$ **Tangent Line:** $y - 1 = \frac{1}{4}(x+1)$	$2: \begin{cases} 1: \text{slope} \\ 1: \text{tangent line equation} \end{cases}$

(c) Find the coordinates of all points at which the tangent line to the curve is vertical.

(c) $\frac{dy}{dx} = \frac{y}{3y^2 - x}$ **Vertical tangent line(s) occur at point(s) on the curve where** $3y^2 - x = 0$ **and** $y \neq 0$. $3y^2 - x = 0$ $x = 3y^2$ $y^3 - 3y^2 \cdot y = 2$ $y^3 - 3y^3 = 2$ $-2y^3 = 2$ $y^3 = -1$ $y = -1$ **and** $x = 3$ → $(3, -1)$	$4: \begin{cases} 1: \text{set } 3y^2 - x \text{ equal to zero} \\ 1: \text{substitute into original curve} \\ 1: y - \text{value} \\ 1: x - \text{value} \end{cases}$

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB
SECTION II, Part B
Time – 60 minutes
Number of problems – 4

NO CALCULATOR FOR THESE PROBLEMS.

DO NOT BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.

3. The twice–differentiable function f is defined for all real numbers and satisfies the conditions: $f(0) = 2$, $f'(0) = -4$, $f''(0) = 3$.

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ and in terms of a. Show the work that leads to your answers.

(a) $g'(x) = ae^{ax} + f'(x)$

$g'(0) = a - 4$

$g''(x) = a^2e^{ax} + f''(x)$

$g''(0) = a^2 + 3$

$$4 : \begin{cases} 1 : g'(x) \\ 1 : g'(0) \\ 1 : g''(x) \\ 1 : g''(0) \end{cases}$$

(b) The function h is given by $h(x) = \cos(kx)\, f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

(b) $h'(x) = f'(x)\cos(kx) - k\sin(kx)f(x)$
$h'(0) = f'(0)\cos(0) - k\sin(0)f(0) = f'(0) = -4$
$h(0) = \cos(0)f(0) = 2$
The equation of the tangent line is $y = -4x + 2$.

$$5: \begin{cases} 2: h'(x) \\ 3: \begin{cases} 1: h'(0) \\ 1: h(0) \\ 1: \text{equation of tangent line} \end{cases} \end{cases}$$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

4. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x.

(a) Let $h(x) = \frac{f(x)}{g(x)}$. Find $h'(2)$.

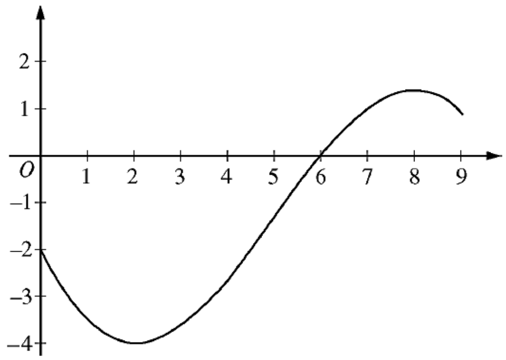
(a) $h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ $h'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$ $h'(2) = \dfrac{3\cdot 2 - 9\cdot 1}{[3]^2} = \dfrac{-3}{9} = \dfrac{-1}{3}$	$3: \begin{cases} 2: h'(x) \\ 1: h'(2) \end{cases}$

(b) Let $r(x) = f(g(x))$. Find $r'(1)$.

(b) $r'(x) = f'(g(x))\cdot g'(x)$ $r'(1) = f'(g(1))\cdot g'(1)$ $= f'(2)\cdot g'(1) = 2\cdot 5 = 10$	$2: \begin{cases} 1: r'(x) \\ 1: r'(1) \end{cases}$

(c) If $p(x) = f(x)(1 - x^2)$, determine whether $p(x)$ is increasing or decreasing at $x = 3$. Justify your answer.

(c) $p'(x) = f'(x) \cdot (1 - x^2) + f(x) \cdot (-2x)$ $p'(3) = f'(3) \cdot (1 - 3^2) + f(3) \cdot (-2 \cdot 3)$ $p'(3) = (-4) \cdot (-8) + (10) \cdot (-6) = -28$ **The function is decreasing at *x* = 3, because the** $p'(3) < 0$.	$4: \begin{cases} 2: p'(x) \\ 1: p'(3) \\ 1: \text{justification} \end{cases}$



Graph of $f'(x)$

5. Let f be a function defined on the closed interval $0 \le x \le 9$ with $f(2) = 8$. The graph of $f'(x)$, the derivative of f, is shown above.

(a) For $0 \le x \le 9$, find all values of x at which f has a relative minimum. Justify your answer.

(a) $f'(x)=0$ **at** $x=6$ $f'(x)$ **changes from negative to positive at** $x=6$. **Thus, *f* has a relative minimum at** $x=6$.	$2:\begin{cases} 1: x-\text{value} \\ 1: \text{justification} \end{cases}$

(b) For $0 \le x \le 9$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

(b) $f'(x)$ **changes from decreasing to increasing, or vice versa, at** $x=2$ **and** $x=8$. **Thus, the graph of *f* has points of inflection when** $x=2$ **and** $x=8$.	$2:\begin{cases} 1: x-\text{values} \\ 1: \text{justification} \end{cases}$

NO CALCULATOR ALLOWED

(c) Find all intervals on which the graph of *f* is both decreasing and concave up. Justify your answer.

(c) The graph of *f′(x)* is negative and increasing, therefore $f(x)$ is decreasing and concave up on $(2,6)$	$2:\begin{cases} 1: \text{interval} \\ 1: \text{explanation} \end{cases}$

(d) Let h be defined as $h(x) = xf(x)$. Find the equation of the line tangent to $h(x)$ at $x = 2$.

(d) $h(x) = xf(x) = 2(f(2)) = 2(8) = 16$ $h'(x) = f(x) + xf'(x)$ $h'(2) = f(2) + 2f'(2) = 8 + 2(-4) = 0$ **Tangent Line:** $y - 8 = 0(x - 2)$ **or** $y = 8$	$3:\begin{cases} 1: h(2) \\ 1: h'(2) \\ 1: \text{tangent line equation} \end{cases}$

6. Let f be a differentiable function for which $f(1) = 10$ and whose derivative is given by the equation $f'(x) = e^x(8 - x)$ for all $x \geq 0$.

(a) Find the x–coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f.

(a) $f'(x) = 0$ **at** $x = 8$ $f'(x)$ **changes from positive to negative at** $x = 8$. **Thus, *f* has a relative maximum at** $x = 8$.	$2: \begin{cases} 1: x - \text{value} \\ 1: \text{justification} \end{cases}$

(b) The graph of the function f has exactly one point of inflection. Find the x–coordinate of this point.

(b) $f''(x) = e^x(8 - x) + e^x(-1) = e^x(8 - x) - e^x$ $f''(x) = e^x(8 - x - 1) = e^x(7 - x)$ $f''(x) = 0$ **at** $x = 7$ $f''(x)$ **changes from positive to negative at** $x = 7$. **Thus, the graph of *f* has a point of inflection when** $x = 7$.	$3: \begin{cases} 1: f''(x) \\ 1: x - \text{value} \\ 1: \text{justification} \end{cases}$

(c) Find all intervals for which the graph of f is both increasing and concave up. Justify your answer.

(c) The graph of *f* is both increasing and concave up where $f'(x)$ and $f''(x)$ are positive: $(\mathbf{0}, \mathbf{7})$)	$2: \begin{cases} 1: \text{interval} \\ 1: \text{explanation} \end{cases}$

(d) Find the equation of the line tangent to f at $x = 1$.

(d) $f'(1) = e^1(8-1) = 7e$ **Tangent Line:** $y - 10 = 7e(x-1)$	$2: \begin{cases} 1: f'(1) \\ 1: \text{tangent line equation} \end{cases}$

STOP

END OF EXAM