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AP Calculus AB

Q3 Interim Assessment

MC Test Booklet #1

April 2017

School: ______________________________

Student Name: ______________________________

Teacher: ______________________________

Period: ______________________________

AP® Calculus AB Exam

SECTION I: Multiple Choice

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
30
Time
60 minutes
Electronic Device
None allowed

Part B

Number of Questions
15
Time
45 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions. For Part A, fill in only the boxes for numbers 1 through 30 on the answer sheet. For Part B, fill in only the boxes for numbers 76 through 90 on the answer sheet.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, place the letter of your choice in the corresponding box on the answer sheet. Give only one answer to each question.

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

CALCULUS AB

SECTION I, Part A

Time—60 minutes

Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. $\int_{-1}^{3} (x^2 - 2x)\, dx =$

(A) $-\frac{2}{3}$ (B) $\frac{4}{3}$ (C) 8 (D) 12

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2. If $f(x) = e^{2x}(x^3 + 1)$, then $f'(2) =$

(A) $6e^4$ (B) $21e^4$ (C) $24e^4$ (D) $30e^4$

GO ON TO THE NEXT PAGE.

3. Let f be a differentiable function such that $f(2) = 4$ and $f'(2) = -\frac{1}{2}$. What is the approximation for $f(2.1)$ found by using the line tangent to the graph of f at $x = 2$?

(A) 2.95 (B) 3.95 (C) 4.05 (D) 4.1

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4. Let g be the function defined by $g(x) = x^4 + 4x^3$. How many relative extrema does g have?

(A) Zero (B) One (C) Two (D) Three

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5. The velocity of a particle moving along the x-axis is given by $v(t) = 2 - t^2$ for time $t > 0$. What is the average velocity of the particle from time $t = 1$ to time $t = 3$?

(A) -4 (B) -3 (C) $-\frac{7}{3}$ (D) $\frac{7}{3}$

t (hours)	0	2	7	9
$R(t)$ (tons per hour)	15	9	5	4

6. On a certain day, the rate at which material is deposited at a recycling center is modeled by the function R, where $R(t)$ is measured in tons per hour and t is the number of hours since the center opened. Using a trapezoidal sum with the three subintervals indicated by the data in the table, what is the approximate number of tons of material deposited in the first 9 hours since the center opened?

(A) 68 (B) 70.5 (C) 85 (D) 136

GO ON TO THE NEXT PAGE.

7. What is the total area of the regions between the curves $y = 6x^2 - 18x$ and $y = -6x$ from $x = 1$ to $x = 3$?

(A) 4 (B) 12 (C) 16 (D) 20

8. The function g is defined by $g(x) = x^2 + bx$, where b is a constant. If the line tangent to the graph of g at $x = -1$ is parallel to the line that contains the points $(0, -2)$ and $(3, 4)$, what is the value of b ?

(A) -1 (B) 2 (C) $\frac{5}{2}$ (D) 4

GO ON TO THE NEXT PAGE.

$$f(x) = \begin{cases} \dfrac{|x|}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

9. The function f is defined above. The value of $\int_{-5}^{3} f(x)\,dx$ is

(A) -2 (B) 2 (C) 8 (D) nonexistent

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10. Let g be a continuous function. Using the substitution $u = 2x - 1$, the integral $\int_{2}^{3} g(2x-1)\,dx$ is equal to which of the following?

(A) $\int_{2}^{3} g(u)\,du$ (B) $\frac{1}{2}\int_{2}^{3} g(u)\,du$ (C) $\int_{3}^{5} g(u)\,du$ (D) $\frac{1}{2}\int_{3}^{5} g(u)\,du$

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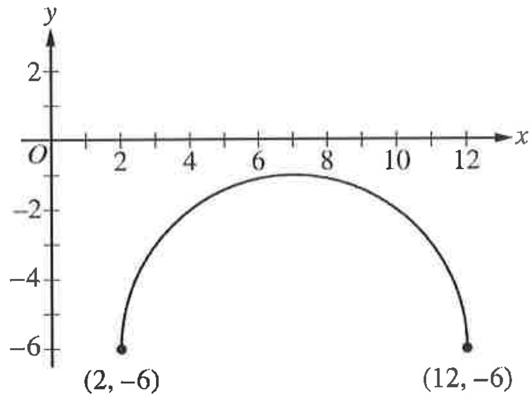

Graph of f

11. The graph of $y = f(x)$ consists of a semicircle with endpoints at $(2, -6)$ and $(12, -6)$, as shown in the figure above. What is the value of $\int_2^{12} f(x)\, dx$?

(A) $-\frac{25\pi}{2}$ (B) $\frac{25\pi}{2}$ (C) $-60 + \frac{25\pi}{2}$ (D) $60 - \frac{25\pi}{2}$

GO ON TO THE NEXT PAGE.

12. An object moves along a straight line so that at any time t its acceleration is given by $a(t) = 6t$. At time $t = 0$, the object's velocity is 10 and the object's position is 7. What is the object's position at time $t = 2$?

(A) 22 (B) 27 (C) 28 (D) 35

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13. If $y = \cos x - \ln(2x)$, then $\dfrac{d^3y}{dx^3} =$

(A) $\sin x - \dfrac{2}{x^3}$

(B) $-\sin x - \dfrac{2}{x^3}$

(C) $\sin x - \dfrac{1}{x^3}$

(D) $-\sin x - \dfrac{1}{x^3}$

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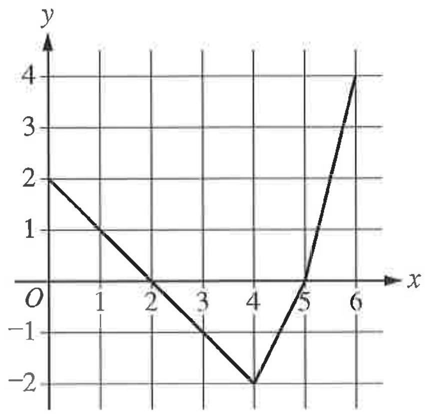

Graph of f

14. The graph of the function f, shown above, consists of three line segments. If the function g is an antiderivative of f such that $g(2) = 5$, for how many values of c, where $0 \le c \le 6$, does $g(c) = 3$?

(A) Zero (B) One (C) Two (D) Three

$$f(x) = \begin{cases} 6 + cx & \text{for } x < 1 \\ 9 + 2\ln x & \text{for } x \ge 1 \end{cases}$$

15. Let f be the function defined above, where c is a constant. If f is continuous at $x = 1$, what is the value of c ?

(A) 2 (B) 3 (C) 5 (D) 9

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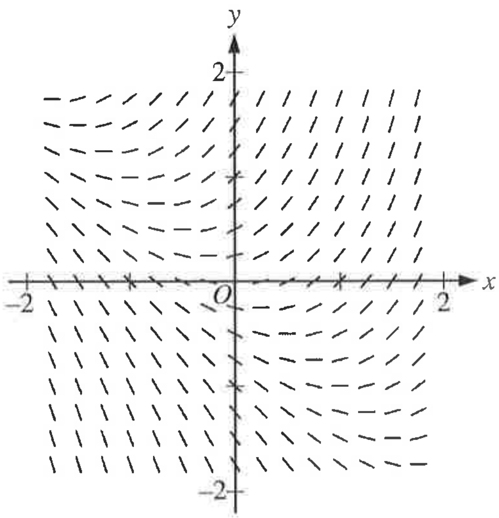
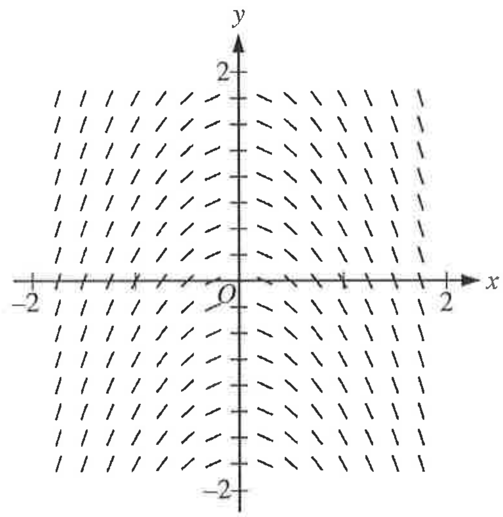
16. Which of the following could be a slope field for the differential equation $\frac{dy}{dx} = x^2 + y$?

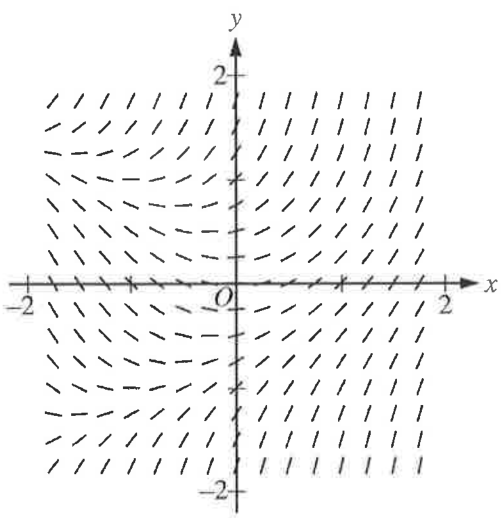
(A)



(B)



(C)



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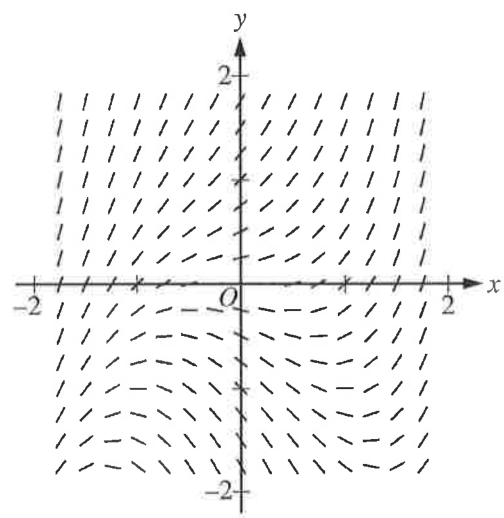

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17. The function $y = e^{3x} - 5x + 7$ is a solution to which of the following differential equations?

(A) $y'' - 3y' - 15 = 0$

(B) $y'' - 3y' + 15 = 0$

(C) $y'' - y' - 5 = 0$

(D) $y'' - y' + 5 = 0$

18. If $f(x) = \sin^{-1} x$, then $f'\left(\frac{\sqrt{3}}{2}\right) =$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{4}{7}$ (D) 2

GO ON TO THE NEXT PAGE.

19. $\lim_{x \to e} \dfrac{(x^{20} - 3x) - (e^{20} - 3e)}{x - e}$ is

(A) 0 (B) $20e^{19} - 3$ (C) $e^{20} - 3e$ (D) nonexistent

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20. Let $y = f(x)$ be a twice-differentiable function such that $f(1) = 2$ and $\dfrac{dy}{dx} = y^3 + 3$. What is the value of $\dfrac{d^2y}{dx^2}$ at $x = 1$?

(A) 12 (B) 66 (C) 132 (D) 165

GO ON TO THE NEXT PAGE.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	-6	9	-10	16
1	5	-3	3	-2
3	0	7	8	3

21. The table above gives values of f, f', g, and g' for selected values of x. If $h(x) = f(g(x))$, what is the value of $h'(1)$?

(A) -19 (B) -14 (C) 7 (D) 9

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22. Let $y = f(x)$ be the particular solution to the differential equation $\dfrac{dy}{dx} = \dfrac{x+1}{y}$ with the initial condition $f(0) = -2$. Which of the following is an expression for $f(x)$?

(A) $-2 - \sqrt{x^2 + 2x}$

(B) $-2 + \sqrt{x^2 + 2x}$

(C) $-\sqrt{x^2 + 2x + 4}$

(D) $\sqrt{x^2 + 2x + 4}$

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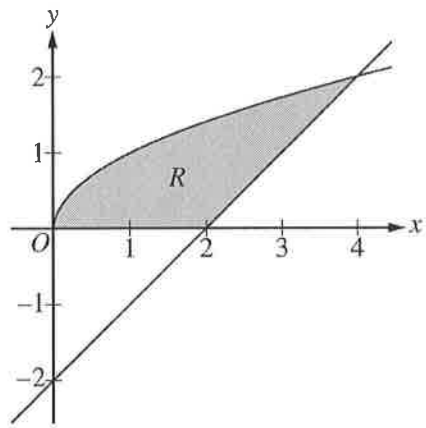

23. Let R be the shaded region bounded by the graph of $y = \sqrt{x}$, the graph of $y = x - 2$, and the x-axis, as shown in the figure above. Which of the following gives the volume of the solid generated when R is revolved about the x-axis?

(A) $\pi \int_0^4 \left(x - (x - 2)^2\right) dx$

(B) $\pi \int_0^4 (\sqrt{x} - (x - 2))^2 \, dx$

(C) $\pi \int_0^2 x \, dx + \pi \int_2^4 \left(x - (x - 2)^2\right) dx$

(D) $\pi \int_0^2 x \, dx + \pi \int_2^4 (\sqrt{x} - (x - 2))^2 \, dx$

GO ON TO THE NEXT PAGE.

24. $\lim_{x \to 3} \dfrac{\tan(x-3)}{3e^{x-3} - x}$ is

(A) 0 (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) nonexistent

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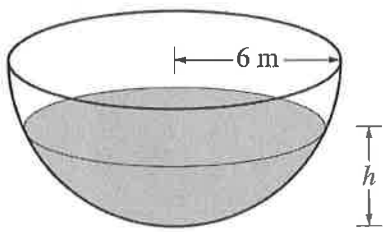
25. Let f be a function with first derivative defined by $f'(x) = \dfrac{3x^2 - 6}{x^2}$ for $x > 0$. It is known that $f(1) = 9$ and $f(3) = 11$. What value of x in the open interval $(1, 3)$ satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1, 3]$?

(A) $\sqrt{6}$ (B) $\sqrt{3}$ (C) $\sqrt{2}$ (D) 1

GO ON TO THE NEXT PAGE.

26. $\displaystyle\int_1^2 \frac{x^2 - x - 5}{x + 2}\, dx =$

(A) $-\dfrac{3}{2} + \ln\dfrac{4}{3}$ (B) $-\dfrac{25}{21}$ (C) $\dfrac{5}{2} + 3\ln\dfrac{3}{4}$ (D) $\dfrac{23}{45}$



27. A hemispherical water tank, shown above, has a radius of 6 meters and is losing water. The area of the surface of the water is $A = 12\pi h - \pi h^2$ square meters, where h is the depth, in meters, of the water in the tank. When $h = 3$ meters, the depth of the water is decreasing at a rate of $\frac{1}{2}$ meter per minute. At that instant, what is the rate at which the area of the water's surface is decreasing with respect to time?

(A) 3π square meters per minute

(B) 6π square meters per minute

(C) 9π square meters per minute

(D) 27π square meters per minute

GO ON TO THE NEXT PAGE.

28. Consider a triangle in the xy-plane. Two vertices of the triangle are on the x-axis at $(1, 0)$ and $(5, 0)$, and a third vertex is on the graph of $y = \ln(2x) - \frac{1}{2}x + 5$ for $\frac{1}{2} \le x \le 8$. What is the maximum area of such a triangle?

(A) $\frac{19}{2}$

(B) $2 \ln 2 + 9$

(C) $2 \ln 4 + 8$

(D) $2 \ln 16 + 2$

29. The function f is defined by $f(x) = x^3 + 4x + 2$. If g is the inverse function of f and $g(2) = 0$, what is the value of $g'(2)$?

(A) $-\frac{1}{16}$ (B) $-\frac{4}{81}$ (C) $\frac{1}{4}$ (D) 4

GO ON TO THE NEXT PAGE.

30. Which of the following limits is equal to $\int_2^5 x^2\, dx$?

(A) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{k}{n}\right)^2 \frac{1}{n}$

(B) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{k}{n}\right)^2 \frac{3}{n}$

(C) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{3k}{n}\right)^2 \frac{1}{n}$

(D) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + \frac{3k}{n}\right)^2 \frac{3}{n}$

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.