



AP Calculus AB
Q1 Interim Assessment
Test Booklet 3
Free Response Questions
October 2016

School: _____

Student Name: _____

Teacher: _____

Period: _____

AP[®] Calculus AB Exam

SECTION II: Free Response

DO NOT OPEN THIS BOOKLET OR BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time

1 hour, 30 minutes

Number of Questions

6

Percent of Total Score

50%

Writing Instrument

Either pencil or pen with black or dark blue ink

Weight

The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions

2

Time

30 minutes

Electronic Device

Graphing calculator required

Percent of Section II Score

33.3%

Part B

Number of Questions

4

Time

60 minutes

Electronic Device

None allowed

Percent of Section II Score

66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name

First letter of your first name

2. Date of birth

Month Day Year

3. Six-digit school code

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not begin Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as `fnInt(X^2, X, 1, 5)`.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS AB

SECTION II, Part A

Time – 30 minutes

Number of problem – 2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE PROBLEMS

1**1****1****1****1****1****1****1****1****1**

1. A particle moves along the x -axis with velocity, in meters per second, given by $v(t) = -1 + e^{1-t}$ for $t \geq 0$, where t is in seconds.

(a) Find the average rate of change of the velocity on the interval $[2, 4]$. Indicate units of measure.

(b) Find the value of $v'(3)$. Using the correct units, interpret the meaning of $v'(3)$ in terms of the motion of the particle.

1**1****1****1****1****1****1****1****1****1**

(c) Is the speed of the particle increasing at time $t = 3$? Give a reason to your answer.

(d) Find all values of t at which the particle changes direction. Justify your answer.

2**2****2****2****2****2****2****2****2****2**

2. Consider the curve given by $y^3 - xy = 2$.

(a) Show that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(b) Write an equation for the tangent line to the curve at the point $(-1, 1)$.

2**2****2****2****2****2****2****2****2****2**

(c) Find the coordinates of all points at which the tangent line to the curve is vertical.

END OF PART A

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB
SECTION II, Part B
Time – 60 minutes
Number of problems – 4

NO CALCULATOR FOR THESE PROBLEMS.

DO NOT BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

3. The twice-differentiable function f is defined for all real numbers and satisfies the conditions: $f(0) = 2$, $f'(0) = -4$, $f''(0) = 3$.
- (a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show the work that leads to your answers.

3**3****3****3****3****3****3****3****3****3****NO CALCULATOR ALLOWED**

- (b) The function h is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

4**4****4****4****4****4****4****4****4****4****NO CALCULATOR ALLOWED**

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

4. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x .

(a) Let $h(x) = \frac{f(x)}{g(x)}$. Find $h'(2)$.

(b) Let $r(x) = f(g(x))$. Find $r'(1)$.

4**4****4****4****4****4****4****4****4****4****NO CALCULATOR ALLOWED**

- (c) If $p(x) = f(x)(1 - x^2)$, determine whether $p(x)$ is increasing or decreasing at $x = 3$. Justify your answer.

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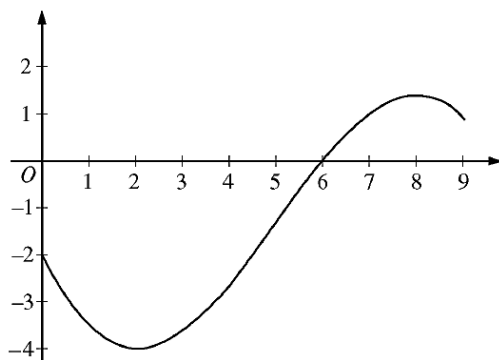
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NO CALCULATOR ALLOWED

Graph of $f'(x)$

5. Let f be a function defined on the closed interval $0 \leq x \leq 9$ with $f(2) = 8$. The graph of $f'(x)$, the derivative of f , is shown above.

(a) For $0 \leq x \leq 9$, find all values of x at which f has a relative minimum. Justify your answer.

(b) For $0 \leq x \leq 9$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

5**5****5****5****5****5****5****5****5****5****NO CALCULATOR ALLOWED**

(c) Find all intervals on which the graph of f is both decreasing and concave up. Justify your answer.

(d) Let h be defined as $h(x) = xf(x)$. Find the equation of the line tangent to $h(x)$ at $x = 2$.

6**6****6****6****6****6****6****6****6****6****NO CALCULATOR ALLOWED**

6. Let f be a differentiable function for which $f(1) = 10$ and whose derivative is given by the equation $f'(x) = e^x(8 - x)$ for all $x \geq 0$.
- (a) Find the x -coordinate of the critical point of f . Determine whether this point is a relative minimum, a relative maximum, or neither for the function f .
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- (b) The graph of the function f has exactly one point of inflection. Find the x -coordinate of this point.

6**6****6****6****6****6****6****6****6****6****NO CALCULATOR ALLOWED**

(c) Find all intervals for which the graph of f is both increasing and concave up. Justify your answer.

(d) Find the equation of the line tangent to f at $x = 1$.

STOP

END OF EXAM