Uncommon Schools | Change History.

AP Calculus BC

Q2 Interim Assessment

Test Booklet 1

Multiple Choice - Non-Calculator

January 2018

Student Name: _____

Period: _____

Teacher: _____

School: _____

AP® Calculus BC Exam

SECTION I: Multiple Choice

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
30
Time
60 minutes
Electronic Device
None allowed

Part B

Number of Questions
15
Time
45 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions. For Part A, fill in only the boxes for numbers 1 through 30 on the answer sheet. For Part B, fill in only the boxes for numbers 76 through 90 on the answer sheet.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, place the letter of your choice in the corresponding box on the answer sheet. Give only one answer to each question.

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

CALCULUS BC

SECTION I, Part A

Time – 60 minutes

Number of questions – 30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form for the choices, decide which of the choices given and place the letter of your choice in the corresponding box on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

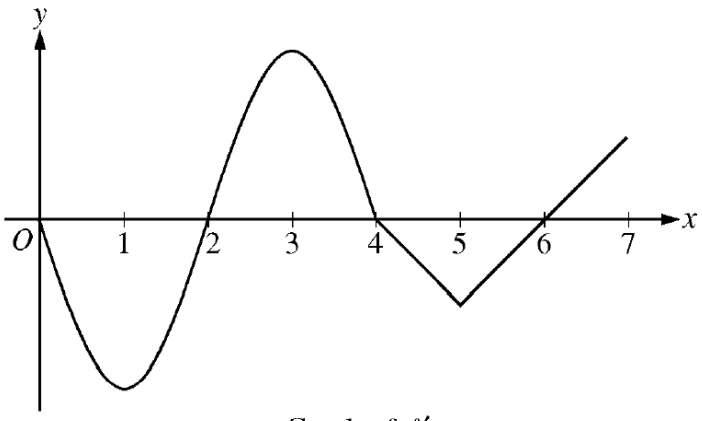
1. If $f(x) = \frac{1}{\sqrt{x}}$, then $f''(4) =$

(A) -4
(B) $-\frac{1}{16}$
(C) $\frac{3}{128}$
(D) 24

---

2. $\lim_{x \to 0} \frac{6x^3+12x^2}{5x^2-10x^3} =$

(A) $-\frac{3}{5}$
(B) 0
(C) $\frac{12}{5}$
(D) The limit does not exist.



Graph of f'

3. The graph of $f'(x)$, the derivative of the function f, is shown above. On which of the following intervals is f both concave up and decreasing?

(A) [0, 1] only
(B) [1, 2] only
(C) [2,3] only
(D) [1, 2] and [5, 6]

4. If $\int_{10}^{14} f(x)\,dx = -4$, then $\int_{14}^{10} (3f(x) - 10)dx =$

(A) -28

(B) -22

(C) 22

(D) 52

---

5. If $\frac{dy}{dt} = -10e^{-t/2}$ and $y(0) = 20$, what is the value of $y(6)$?

(A) $20e^{-6}$ (B) $20e^{-3}$ (C) $10e^{-3}$ (D) $5e^{-3}$

---

6. If $y = \tan(10x)$, $\frac{dy}{dx} =$

(A) $-10\sec^2(10x)$

(B) $10\sec^2(10x)$

(C) $-10\csc^2(10x)$

(D) $10\tan(10x)\sec(10x)$

7. The function f is continuous on the closed interval $[6, 8]$ and twice differentiable on the open interval $(6, 8)$. If $f'(x) < 0$ and $f''(x) > 0$ for all x on the open interval $(6, 8)$, which of the following could be a table of values for f ?

(A)

x	$f(x)$
6	12
7	10
8	8

(B)

x	$f(x)$
6	15
7	9.5
8	6.5

(C)

x	$f(x)$
6	4
7	7
8	14

(D)

x	$f(x)$
6	7
7	4
8	0

---

8. For values of h very close to 0, which of the following functions best approximates $f(x) = \frac{\csc(x+h) - \csc(x)}{h}$?

(A) $-\csc x \cot x$

(B) $-\csc^2 x$

(C) $\csc x \cot x$

(D) $\frac{\csc x}{x}$

---

9. The area of the region enclosed between the curves $f(x) = 9 - x^2$ and $g(x) = -3 + 2x^2$ is exactly:

(A) 8

(B) 16

(C) 32

(D) 64

10. If the substitution $x = 2 + 5t$ is used, which of the following is equivalent to $\int_0^2 \sin(2 + 5t)\, dt$?

(A) $\int_0^2 \sin x\, dx$

(B) $\frac{1}{5}\int_2^{12} \sin x\, dx$

(C) $\int_2^{12} \sin x\, dx$

(D) $5\int_2^{12} \sin x\, dx$

---

11. Let f be the function given by $f(x) = 2x^3 - 5$. What is the value of c that satisfies the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0, 2]$?

(A) $\frac{\sqrt{2}}{2}$

(B) $\frac{2\sqrt{3}}{3}$

(C) $\frac{4}{3}$

(D) 8

---

x	0	1	2	3
$f'(x)$	2	0	-3	8

12. The polynomial function f has selected values of its first derivative f' given in the table above. Which of the following statements must be true?

(A) f is decreasing on the interval $(0, 2)$.

(B) f has a local maximum at $x = 1$.

(C) f has a local minimum on the interval $(2, 3)$.

(D) f has an inflection point on the interval $(0, 2)$.

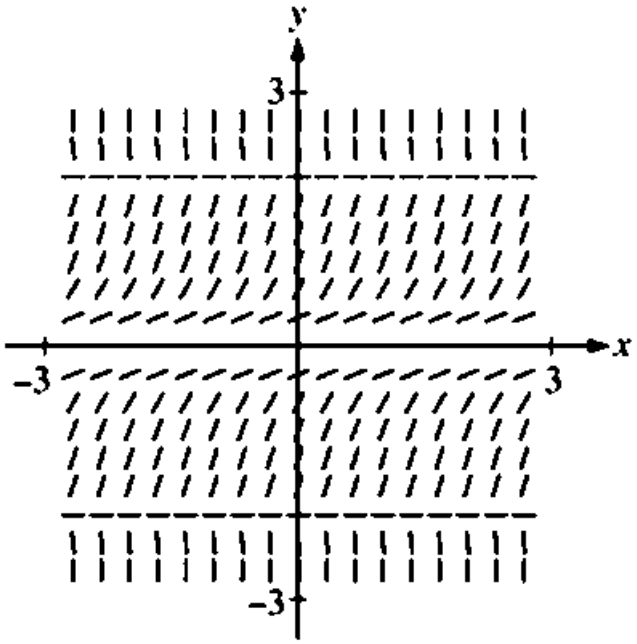
13. The function $f(x) = 80x^3 + 15x^4 - 6x^5$ has a relative minimum at $x =$

(A) -2

(B) 0

(C) 2

(D) 4



14. Shown above is a slope field for the differential equation $\frac{dy}{dx} = y^2(4 - y^2)$. If $y = g(x)$ is the solution to the differential equation with the initial condition $g(-2) = -1$, then $\lim_{x \to \infty} g(x)$ is

(A) $-\infty$ (B) -2 (C) 0 (D) 3

15. The region enclosed by the graph of $y = x^2$, the line $y = 4$, and the line $x = 0$ in the first quadrant is rotated about the x-axis. Which of the following integrals represents the volume of the resulting solid?

(A) $\pi \int_0^2 (4 - x^2) dx$

(B) $\pi \int_0^4 (16 - x^4) dx$

(C) $\pi \int_0^2 (16 - x^4) dx$

(D) $\pi \int_0^2 x^4 dx$

---

16. The function f is defined by $f(x) = \frac{x}{x-1}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope -1?

(A) $(0, 0)$ only

(B) $\left(-1, \frac{1}{2}\right)$ only

(C) $(2, 2)$ only

(D) $(0, 0)$ and $(2, 2)$

---

17. $\int x \cos x \, dx =$

(A) $x \sin x - \cos x + C$

(B) $x \sin x + \cos x + C$

(C) $x \sin x + C$

(D) $\frac{1}{2} x^2 \sin x + C$

x	$f(x)$	$f'(x)$
0	2	7
4	-10	3
7	4	-10

18. The table above gives selected values for a differentiable and increasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(4)$?

(A) -10

(B) $-\frac{1}{10}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

---

19. For time $t \geq 0$, the position of a particle traveling along a line is given by a differentiable function s. If s is increasing for $0 \leq t < 2$ and s is decreasing for $t > 2$, which of the following is the total distance the particle travels for $0 \leq t \leq 5$?

(A) $s(0) + \int_0^2 s'(t)\,dt - \int_2^5 s'(t)\,dt$

(B) $\int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$

(C) $\left|\int_0^5 s'(t)\,dt\right|$

(D) $\int_0^5 |s'(t)|\,dt$

---

20. $\int \frac{9}{x^2-7x+10}\,dx =$

(A) $27 \ln\left|\frac{x-5}{x-2}\right| + C$

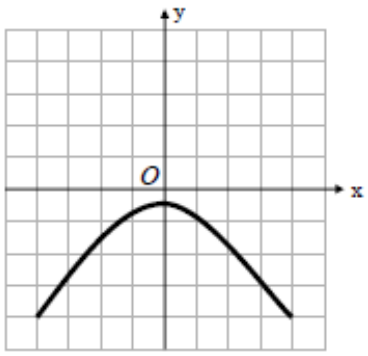
(B) $\frac{1}{2}\ln|x^2 - 7x + 10| + C$

(C) $3 \ln\left|\frac{x-5}{x-2}\right| + C$

(D) $3 \ln\left|\frac{x-2}{x-5}\right| + C$

21. The function f has the property that $f(x) < 0$, $f'(x) < 0$, and $f''(x) < 0$ are for all real values x. Which of the following could be the graph of f?

A)



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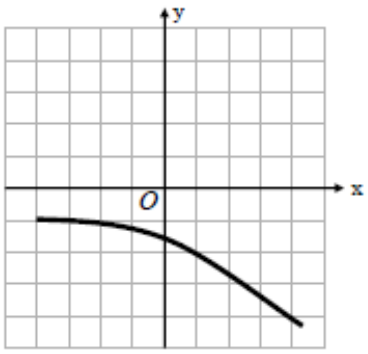

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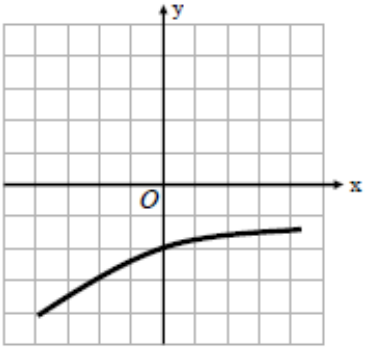
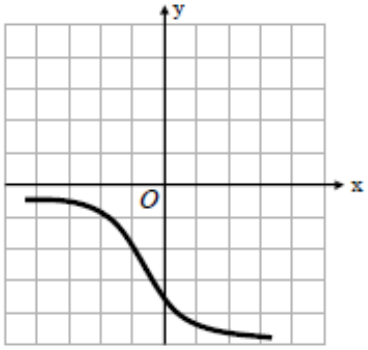


D)



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22. Let S be the region enclosed by the curve $y = \frac{1}{\sqrt{x}}$ and the lines $x = 1$ and $x = 3$. Find the volume of the solid whose base is the region S and whose cross sections perpendicular to the x-axis are semi-circles.

(A) $\frac{\pi}{12}$ units^2

(B) $\frac{\pi}{6}$ units^2

(C) $\frac{\pi \ln(3)}{4}$ units^2

(D) $\frac{\pi \ln(3)}{8}$ units^2

23. If $y = 6 - 2x$, what is the maximum value of the product $3xy$?

(A) $\frac{3}{2}$

(B) 3

(C) $\frac{9}{2}$

(D) $\frac{27}{2}$

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24. $\int_0^1 xe^{4x^2}\,dx =$

(A) e^4

(B) $\frac{1}{8}e^4$

(C) $8(e^4 - 1)$

(D) $\frac{1}{8}(e^4 - 1)$

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25. The region enclosed by the graph of $y = \sqrt{x-4}$ and the lines $y = 0$ and $x = 10$ is rotated about the x-axis. Which of the following gives the volume of the generated solid?

(A) $\int_4^{10} \sqrt{x-4}\,dx$

(B) $\pi\int_0^{10} (x-4)\,dx$

(C) $\pi\int_4^{10} \sqrt{x-4}\,dx$

(D) $\pi\int_4^{10} (x-4)\,dx$

26. The radius of a sphere is decreasing at a rate of 4 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere?

(A) -144π

(B) -96π

(C) -32π

(D) -24π

---

27. $\int_0^1 \frac{e^x-1}{e^x}\,dx =$

(A) $-e$

(B) $-\frac{1}{e}$

(C) $\frac{1}{e}$

(D) e

---

28. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1-\frac{M}{200}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t\to\infty} M(t)$?

(A) 50 (B) 200 (C) 500 (D) 1000

29. The function $y = c_1e^x + c_1xe^x$ is solution to which of the following differential equations?

(A) $y'' + 2y' + y = 0$

(B) $3y'' - 2y' + y = 0$

(C) $y'' - 2y' + y = 0$

(D) $y'' - y' + y = 0$

---

30. Which of the following is equivalent to $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)^3 \left(\frac{2}{n}\right)$?

(A) 0

(B) $\frac{13}{2}$

(C) 20

(D) 26

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO