Uncommon Schools | Change History.

AP Calculus BC

Q1 Interim Assessment

Test Booklet 3

Free Response Questions

October 2016

School: _____

Student Name: _____

Teacher: _____

Period: _____

AP® Calculus BC Exam

SECTION II: Free Response

DO NOT OPEN THIS BOOKLET OR BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A
Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B
Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not begin Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS BC

SECTION II, Part A

Time – 30 minutes

Number of problem – 2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE PROBLEMS

Name: _____

1 1 1 1 1 1 1 1 1 1

1. Consider the curve known as the Folium of Descartes, given by the equation $x^3 + y^3 = 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{y-x^2}{y^2-x}$.

(b) Find the coordinate(s) of all points on the curve for which the tangent line is vertical.

Name: _____

1 1 1 1 1 1 1 1 1 1

1. Consider the curve known as the Folium of Descartes, given by the equation $x^3 + y^3 = 3xy$.

(c) Find the equation of the line tangent to the curve at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$.

(d) Let x and y be functions of time t that are related by the equation $x^3 + y^3 = 3xy$. At time $t = 2$, the values of $x = y = \frac{3}{2}$ and $\frac{dx}{dt} = 10$. Find the value of $\frac{dy}{dt}$ at time $t = 2$.

Name: _____

2 2 2 2 2 2 2 2 2 2

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

2. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$.

(a) Approximate the value of $r''(6)$. Indicate units of measure.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

Name: _____

2 2 2 2 2 2 2 2 2 2

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

2. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)dt$. Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)dt$? Give a reason for your answer.

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION II, Part B
Time – 60 minutes
Number of problems – 4

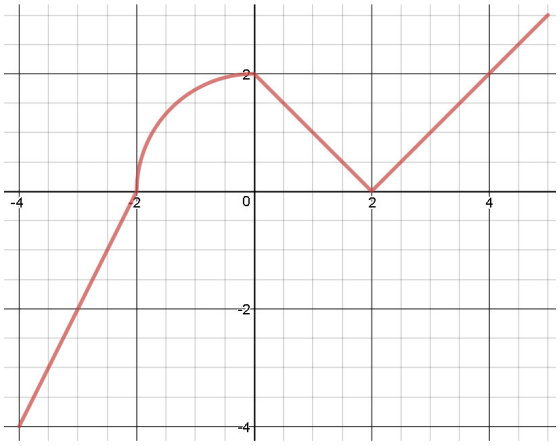
NO CALCULATOR FOR THESE PROBLEMS.

DO NOT BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.

Name: _____

3 3 3 3 3 3 3 3 3 3

NO CALCULATOR ALLOWED



Graph of $f(x)$

3. Let f be the continuous function defined on $[-4, 5]$, whose graph, consisting of a quarter circle centered at $(0, 0)$ and three line segments, is given above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.

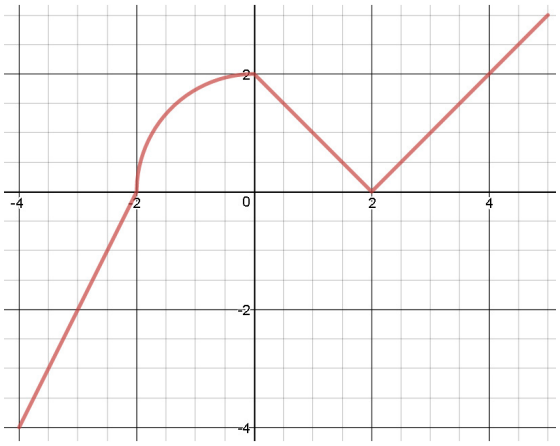
(a) Find $g(2)$, $g'(2)$, and $g''(2)$, or state that the value does not exist.

(b) For $-4 < x < 5$, find all values of x for which the graph of $g(x)$ has a point of inflection. Explain your reasoning.

Name: _____

3 3 3 3 3 3 3 3 3 3

NO CALCULATOR ALLOWED



Graph of $f(x)$

3. Let f be the continuous function defined on $[-4, 5]$, whose graph, consisting of a quarter circle centered at $(0, 0)$ and three line segments, is given above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.

(c) Find the interval(s) on which $g(x)$ is both increasing and concave down.

(d) Find the absolute maximum value of $g(x)$ on the closed interval $[-4, 5]$.

Name: _____

4 4 4 4 4 4 4 4 4 4

NO CALCULATOR ALLOWED

4. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the average acceleration of the particle on the interval $(0, 2)$.

(b) Find the instantaneous acceleration of the particle at time $t = 3$.

Name: _____

4 4 4 4 4 4 4 4 4 4

NO CALCULATOR ALLOWED

4. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

(c) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(d) Find all values of t at which the particle changes direction. Justify your answer.

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Name: _____

5 5 5 5 5 5 5 5 5 5

NO CALCULATOR ALLOWED

5. The function $f(x)$ is given by $f(x) = \frac{-9x+6x^2}{-3x^2}$. The twice-differentiable function $g(x)$ is defined for all real numbers and has the following properties:

$$g(3) = -2, \quad g'(3) = 7, \quad g''(3) = 13$$

(a) Identify the equation of the horizontal asymptote(s) of $f(x)$. Explain your reasoning.

(b) Consider the function $h(x) = f(x)g(x)$. Find the equation of the line tangent to the graph of $h(x)$ at $x = 3$.

Name: _____

5 5 5 5 5 5 5 5 5 5

NO CALCULATOR ALLOWED

5. The function $f(x)$ is defined by $f(x) = \frac{-9x+6x^2}{-3x^2}$ and the twice differentiable function $g(x)$ is defined for all real numbers and has the following conditions.

$$g(3) = -2, \quad g'(3) = 7, \quad g''(3) = 13$$

(c) Evaluate $\int_1^4 f(x)\,dx$. Show the work that leads to your answer.

Name: _____

6 6 6 6 6 6 6 6 6 6

NO CALCULATOR ALLOWED

6. Let f be a differentiable function for which $f(0) = 8$ and whose derivative is given by the equation $f'(x) = -2e^x(x - 8)$ for all $x \geq 0$.

(a) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(b) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

Name: _____

6 6 6 6 6 6 6 6 6 6

NO CALCULATOR ALLOWED

6. Let f be a differentiable function for which $f(0) = 8$ and whose derivative is given by the equation $f'(x) = -2e^x(x - 8)$ for all $x \geq 0$.

(c) Find all intervals for which the graph of f is both increasing and concave up. Justify your answer.

(d) Evaluate $\lim_{x \to 0} \frac{f(x)-8}{6x}$

STOP

END OF EXAM