Uncommon Schools | Change History.

AP Calculus BC

Q3 Interim Assessment

Test Booklet 3

Free Response Questions

April 2017

Student Name: _____

Period: _____

Teacher: _____

School: _____

AP® Calculus BC Exam

SECTION II: Free Response

DO NOT OPEN THIS BOOKLET OR BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not begin Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE PROBLEMS.

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t (minutes)	0	2	5	7	10
$h(t)$ (inches)	3.5	10.0	15.5	18.5	20.0

1. The depth of water in tank A, in inches, is modeled by a differentiable and increasing function h for $0 \le t \le 10$, where t is measured in minutes. Values of $h(t)$ for selected values of t are given in the table above.

   (a) Use the data in the table to find an approximation for $h'(6)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Approximate the value of $\int_0^{10} h(t)\,dt$ using a right Riemann sum with the four subintervals indicated by the data in the table. Is this approximation greater than or less than $\int_0^{10} h(t)\,dt$? Give a reason for your answer.

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(c) The depth of water in tank B, in inches, is modeled by the function $g(t) = 3.2 + 17.5\sqrt{\sin(0.16t)}$ for $0 \le t \le 10$, where t is measured in minutes. Find the average depth of the water in tank B over the interval $0 \le t \le 10$. Is this value greater than or less than the average depth of the water in tank A over the interval $0 \le t \le 10$? Give a reason for your answer.

(d) According to the model given in part (c), is the depth of the water in tank B increasing or decreasing at time $t = 6$? Give a reason for your answer.

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2. For $t \geq 0$, a particle moving in the xy-plane has the position vector $\langle x(t), y(t) \rangle$ at time t, where $\frac{dx}{dt} = -1 + e^{\sin t}$ and $\frac{dy}{dt} = \cos(t^2)$. At time $t = 2$, the position of the particle is $(5, 7)$.

(a) Find the acceleration vector of the particle at time $t = 2$.

(b) Find the total distance traveled by the particle over the time interval $1.8 \leq t \leq 2$.

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(c) Find the x-coordinate of the position of the particle at time $t = 1$.

(d) At time $t = \sqrt{\frac{7\pi}{2}}$, the line tangent to the path of the particle is horizontal. Find the particle's speed at time $t = \sqrt{\frac{7\pi}{2}}$. Determine whether the particle is moving to the left or to the right at that time. Give a reason for your answer.

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END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS.

DO NOT BEGIN PART B UNTIL YOU ARE TOLD TO DO SO.

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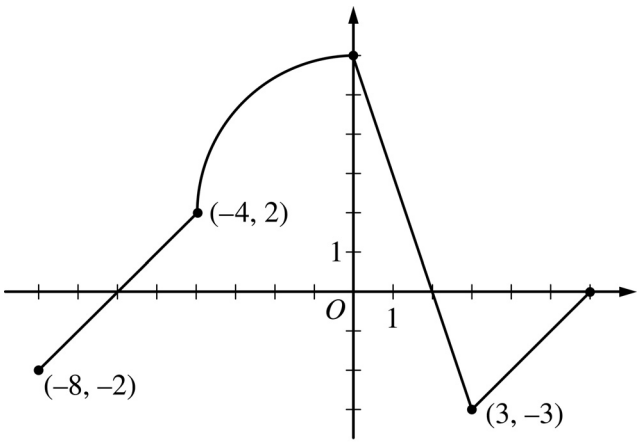
NO CALCULATOR ALLOWED



Graph of g

3. A continuous function g is defined on the closed interval $-8 \le x \le 6$. The graph of g, shown above, consists of three line segments and a quarter of a circle centered at the point $(0, 2)$. Let f be the function given by $f(x) = \int_{-8}^{x} g(t)\,dt$.

   (a) Find all values of x in the interval $-8 < x < 6$ at which f has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

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NO CALCULATOR ALLOWED

(b) Find $f(0)$.

(c) Find $\lim_{x \to -4} \dfrac{f(x)}{x^2 + 4x}$.

(d) Let h be the function defined by $h(x) = \dfrac{g(x)}{x^2 + 1}$. Find $h'(1)$.

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NO CALCULATOR ALLOWED

4. Consider the differential equation $\frac{dy}{dx} = (y - 2)(x^2 + 1)$.

(a) Find $y = g(x)$, the particular solution to the given differential equation with initial condition $g(0) = 5$.

(b) For the particular solution $y = g(x)$ found in part (a), find $\lim_{x \to -\infty} g(x)$.

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NO CALCULATOR ALLOWED

(c) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 3$. Find the value of $\frac{d^2y}{dx^2}$ at the point (1, 3). Is the graph of $y = f(x)$ concave up or concave down at the point (1, 3) ? Give a reason for your answer.

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NO CALCULATOR ALLOWED

x	1	2	3	4	5
$f(x)$	4	8	7	3	–1
$f'(x)$	5	–2	–1	6	2
$f''(x)$	2	–3	1	–1	3

5. The table above gives values of a function f, its first derivative f', and its second derivative f'' for selected values of x. The function f'' is continuous for all real numbers.

   (a) Let g be the function given by $g(x) = (2x - 1)f(x)$. Find $g'(3)$.

   (b) Let h be the function given by $h(x) = f(f(x))$. Find $h'(4)$.

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NO CALCULATOR ALLOWED

(c) Find the value of $\int_1^5 x f''(x)\, dx$.

(d) Is there a value c, for $3 < c < 4$, such that $f''(c) = 7$? Justify your answer.

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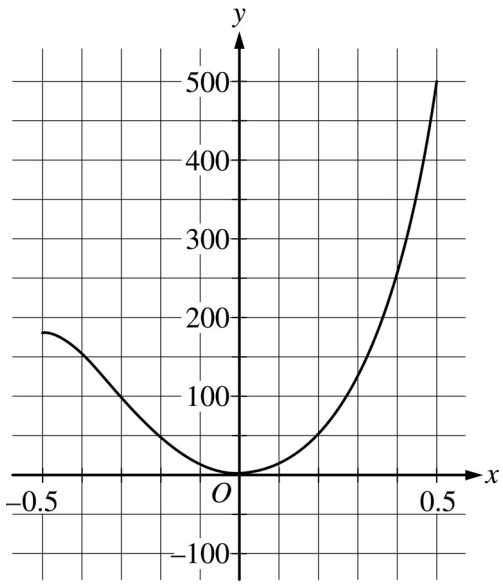
NO CALCULATOR ALLOWED



Graph of $f^{(5)}$

6. Let f and g be the functions given by $f(x) = xe^{x^3}$ and $g(x) = \int_0^x f(t)\,dt$. The graph of $f^{(5)}$, the fifth derivative of f, is shown above for $-\frac{1}{2} \le x \le \frac{1}{2}$.

   (a) Write the first four nonzero terms and the general term of the Taylor series for e^x about $x = 0$.
   Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

NO CALCULATOR ALLOWED

(b) Write the first four nonzero terms of the Taylor series for g about $x = 0$.

(c) Find the value of $g^{(5)}(0)$.

(d) Let $P_5(x)$ be the fifth-degree Taylor polynomial for g about $x = 0$. Use the Lagrange error bound along with information from the given graph to find an upper bound on $\left| P_5\left(\frac{1}{2}\right) - g\left(\frac{1}{2}\right) \right|$.

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END OF EXAM