Predictive Analytics Lecture 1

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Define: Prediction and Forecast

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"statement about an uncertain event", "informed guess or opinion"

predict (v.) 1620s (implied in predicted), "*foretell, prophesy*," a back formation from prediction or else from Latin praedicatus, past participle of praedicere "foretell, advise, give notice,"

forecast (n.) early 15c., "*forethought, prudence*," probably from forecast (v.). Meaning "conjectured estimate of a future course" is from 1670s.

I will be using predict and forecast interchangeably.

Examples

We make predictions all the time, saying things like:

- "Apple stock will go up tomorrow",
- "This condo will sell for $500K"

and sometimes unknowingly

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How do we make predictions? We use a *model*.

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Outputs? health, wealth and wisdom

Synonyms for Inputs and Outputs

Here, the inputs and outputs are

- *features*
- *attributes*
- *characteristics*
- *variables* / *variates*

of a person. A person features health, a person has the characteristic of going to bed early.

What are "observations"?

Here, we have features of a person. Generally, inputs and outputs are features of the

- *observation* or
- *unit* or
- *record* or
- *subject*.

Thus the model relates some *feature(s) of the observation* to other *feature(s) of the observation*. Here, we are relating specific people's bedtime schedule and waking schedule to their health, wealth and wisdom.

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The Model as a Functional Relationship

Thus the model relates some *measured feature(s) of the observation* to other *measured feature(s) of the observation*. The relationship is a function taking in inputs (within the parentheses) and "returning" the outputs (the equal sign). For any observation,

$$\begin{array}{c}\text{the measured}\\ \text{outputs of an}\\ \text{observation}\end{array} = \text{model}\left(\begin{array}{c}\text{the measured}\\ \text{inputs of an}\\ \text{observation}\end{array}\right)$$

It is traditional to put the outputs on the left hand side. This is assumed that the outputs were measured. This type of observation is called

- old or
- historical or
- known

and predictions here are not needed (obviously). In our aphorism model, for the observation being a known person named Joe:

$$\begin{bmatrix}\text{a measured quantity of Joe's health}\\ \text{a measured quantity of Joe's wealth}\\ \text{a measured quantity of Joe's wisdom}\end{bmatrix} = \text{model}\left(\begin{bmatrix}\text{a measured quantity of Joe's bedtime}\\ \text{a measured quantity of Joe's waketime}\\ \vdots\end{bmatrix}\right)$$

Updated Definition of Prediction

Now we can hone our definition of prediction. For a

- new or
- heretofore unseen or
- future

observation, where the inputs have been measured / assessed but the output has not been measured / assessed,

$$\underbrace{\begin{pmatrix}\text{the guessed}\\ \text{output}\\ \text{measurements}\end{pmatrix}}_{\text{prediction}} = \text{model}\left(\begin{pmatrix}\text{the measured}\\ \text{inputs of an}\\ \text{observation}\end{pmatrix}\right)$$

$$\begin{bmatrix}\text{a guessed quantity of Bob's health}\\ \text{a guessed quantity of Bob's wealth}\\ \text{a guessed quantity Bob's wisdom}\end{bmatrix} = \text{model}\left(\begin{bmatrix}\text{a measured quantity of Bob's bedtime}\\ \text{a measured quantity of Bob's waketime}\\ \vdots\end{bmatrix}\right)$$

Measurements as Variables

Instead of "a measured quantity ..." we can use algebraic *variables* to denote the numerical quantities. It is traditional to use x's to represent inputs and y's to represent outputs. Here would be the relationship for Joe:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \text{model}\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}\right)$$

and for Bob:

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} = \text{model}\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}\right)$$

We will use the "hat" symbol (ˆ) to indicate a prediction of the output $\hat{y}$ to distinguish it from a known value of the output y.

More Vocabulary

Even though measured inputs and outputs are features of an observation, they each go by special names that emphasize their roles.

Each output y is called a

- *response* (the model "responds" to inputs)
- *outcome* / *outcome metric* (the result of inputs)
- ~~*endpoint*~~ (only used in clinical trial context)

and they are the target of prediction — what we want to ultimately predict.

Inputs x's then can go by the following terms of art:

- *covariates* (because the vary with the response, co-vary)
- *predictors* (since they will be the inputs used to make predictions)

and they are what we make use of to predict. I will try to use "response" and "predictors" in this course.

Mathematical Model

Now that we have predictors and responses measured an numeric and an equal sign relating them. We have officially created a *mathematical model*. The word "model" now will be represented as a function, f. So for an old observation,

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = f\left(x_1, x_2, \ldots\right)$$

and a new observation,

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} = f\left(x_1, x_2, \ldots\right)$$

It is said that the "model explains the response". What does this mean?

Science is based on Mathematical Models

We have become quite successful at shrink-wrapping interesting variables in the world around us

$$F$$

Focus: models with univariate responses.

Although general models have any number of outputs, this semester we will only consider models with one output. Thus, we will be looking at models such as

Early to bed and early to rise makes a man healthy.

We picked the most interesting output. So, for an old observation,

$$y = f(x_1, x_2, \ldots)$$

and a new observation,

$$\hat{y} = f(x_1, x_2, \ldots)$$

Many possible models

Early to bed and early to rise makes a man healthy.

What is the response metric?

Many possible models

Early to bed and early to rise makes a man healthy.

What is the response metric? What does "healthy" mean?

- Healthy for his whole life? Unlikely the model means this...
- Healthy for ages 25-65? Since we can expect health in infanthood and adolescence but not in elderly years

One also gets a feeling from the wording, there is either "healthy" or "not healthy". Thus the response metric will be the *categorical* data type and the model would be called a *classification* model.

Categorical measurements consist of discrete, mutually exclusive *levels*. Here, {healthy, not healthy}. Generally, {a, b, c, ...}. Metrics with a large number of levels are difficult to model — keep it low.

I there are two levels, it is called *binary* and the model would be called a "binary response model" (or classification) with elements 0 and 1.

Define the response clearly

Response: Healthy for ages 25–65

We still need a clear definition. Ideas? How about: healthy means no incidence of a "major" disease between the ages of 25–65? This can be assessed with medical records.

Define the predictors clearly

x_1: bedtime schedule

Definition?

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Definition? Average bedtime.

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Definition? Average bedtime. How to measure / assess? Survey?

x_2: waketime schedule

Definition?

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x_2: waketime schedule

Definition? Average time to rise in the morning assessed via survey

Thus, x_1 and x_2 are a variant of a *timestamp* data type.

Dataframes

The historical *data frame* or *dataset* or even more colloquially, the "data" looks like:

Healthy? 1 = yes (y)	Average Bedtime (x_1)	Average Waketime (x_2)
1	9:32PM	6:42AM
0	11:55PM	7:53AM
0	10:33PM	7:02AM
⋮		

Dataframes have n observations and p predictors. Here $n =$

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Dataframes have n observations and p predictors. Here $n = 3$ (only those viewable above) and $p = 2$. Thus, it is a matrix with n rows and $p + 1$ columns. The "+1" is for the response which is not a predictor.
What would a new observation look like? Tony went to bed on average 9:53PM and awoke on average at 6:13AM. Did he have a healthy life or not?
We don't know the model yet...

Mathematical Models are Deterministic

Early to bed and early to rise makes a man healthy.

which, after measuring inputs and outputs and mathematizing, becomes $y = f(x_1, x_2)$.

From the wording, it seems the model is unequivocal and deterministic. This means

Mathematical Models are Deterministic

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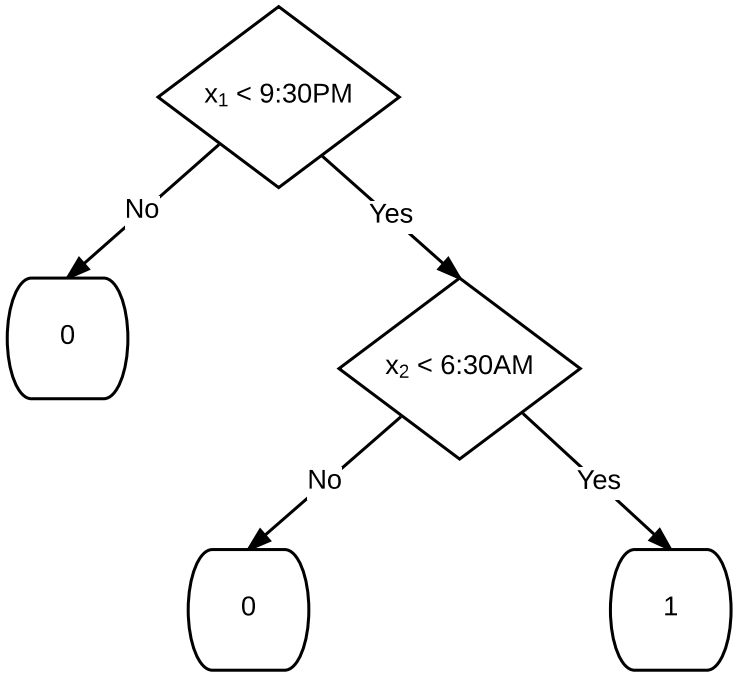
which, after measuring inputs and outputs and mathematizing, becomes $y = f(x_1, x_2)$.

From the wording, it seems the model is unequivocal and deterministic. This means that for any input values (the measured values of x_1 and x_2), the output (the response) will have only one unique value.

Thus the functional form likely looks like a *decision tree model*.



Is the Model *Really* Deterministic?

This is an ancient question... it touches on the free will vs. determinism debate. We will punt on the philosophy and ask: is this model deterministic?

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This is an ancient question... it touches on the free will vs. determinism debate. We will punt on the philosophy and ask: is this model deterministic? NO.

Thus, this model is wrong. Why? We can find at least one person who does not have a matching response when inputs are evaluated in f. Seems obvious but...

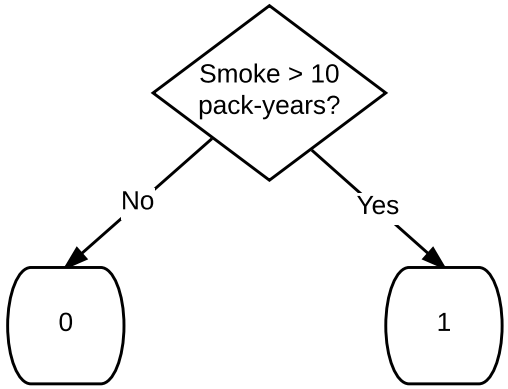
Smoking and Lung Cancer

Consider the model with the binary input

y: contract lung cancer at some point (1) or not (0)

x_1: smoke 10 pack years or more at some point in a lifetime (1) or not (0) and the response

Do you think the model should look like the below?



No... in fact "only" 16% of smokers get lung cancer compared to about 0.4% of non-smokers. Thus, the simpel model above is wrong because some responses (that is features of certain individuals) will not "fit" the model. Thus, should we throw out the whole enterprise of modeling?

Statistical Models

Mathematical models such as

$$y = f(x_1, x_2, \ldots)$$