- QR factorization:
  - Normal equation: $A^T A\hat{x} = A^T b$.
    Plug in A and b to find $\hat{x} =$ best approximate solution (linear least-squares)
  - Compute projection of b onto A: $A(A^T A)^{-1}A^T b = \hat{x}$
  - Orthonormal vectors:
    $$q_n = \frac{a_n^{\perp}}{\parallel a_n^{\perp} \parallel_2}$$
    where
    $$a_n^{\perp} = a_n - e_{0,n}q_0 - \ldots - e_{n-1,n}q_{n-1}$$
    $$a_n^{\perp} = a_n - q_0^T a_n q_0 - \ldots - q_{n-1}^T a_n q_{n-1}$$
  - $A = QR$ where $Q = (q_0|\ldots|q_{n-1})$
    and $R = \begin{pmatrix} \parallel a_0 \parallel_2 & e_(0,1) & \ldots & e_{0,n-1} \\ & \parallel a_1^{\perp} \parallel_2 & \ddots & \vdots \\ & & \ddots & e_{n-2,n-1} \\ 0 & & & \parallel a_{n-1}^{\perp} \parallel_2 \end{pmatrix}$
- Row echelon form is the result of Gaussian elimination
- Space spanned by vectors: $A = \{a_0|...|a_{n-1}\}$ where A is the space
- $Ax = b$ for multiple solutions:
  $$A(x_s + \beta x_n) = b$$
  where x_s is s.t. $Ax_s = b$ and x_n s.t. $Ax_n = 0$
- Eigenvalues are scalars λ. λ is an eigenvalue of $A \iff Ax = \lambda x$ for some non-zero vector x. So:
  For 2x2 $M = \begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix}$, and for 3x3 $M = \begin{pmatrix} a-\lambda & b & c \\ d & e-\lambda & f \\ g & h & i-\lambda \end{pmatrix}$,
  and $\det(A - \lambda I) = 0$.
- Eigenvectors
- Determinants:
  - For 2x2 matrices $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\det(M) = ad - bc$
  - For 3x3 matrices $M = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$, $\det(M) = a(ei - hf) - b(di - fg) + c(dh - eg)$, or
    $\det(M) = aei + bfg + cdh - (afh + bdi + ceg)$
- Equivalent to "A is nonsingular"
  - A is invertible.
  - A^{-1} exists.
  - $AA^{-1} = A^{-1}A = I$.
  - A represents a linear transformation that is a bijection.
  - $Ax = b$ has a unique solution for all $b \in \mathbb{R}^n$.
  - $Ax = 0$ implies that x=0.
  - $Ax = e_j$ has a solution for all $j \in \{0, \ldots, n-1\}$
  - The determinant of A is nonzero: $det(A) \neq 0$.
  - LU with partial pivoting does not break down.
  - $\mathcal{N}(A) = 0$.
  - $\mathcal{C}(A) = \mathbb{R}^n$.
  - $\mathcal{R}(A) = \mathbb{R}^n$.
  - A has linearly independent columns.
  - A has linearly independent rows.