1. Lead Controller

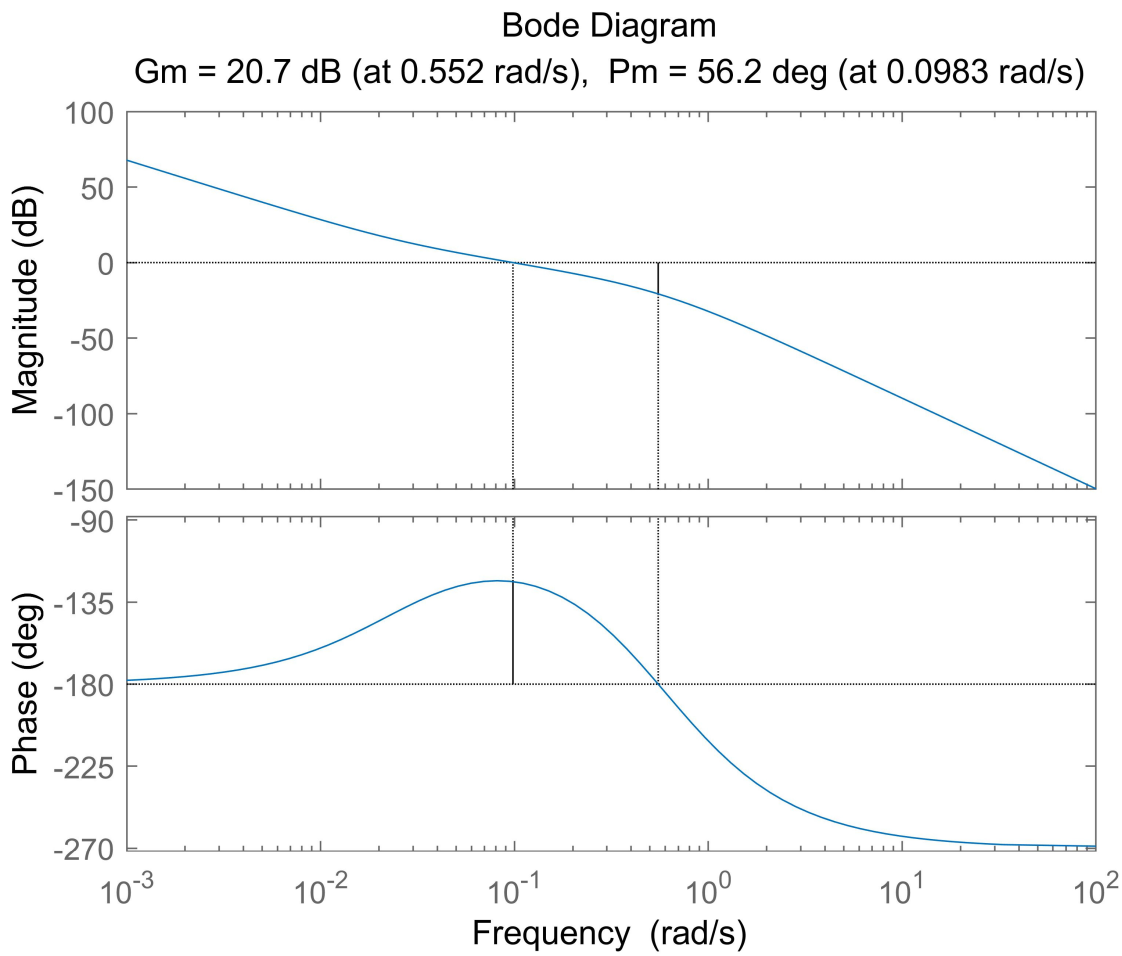
(a) The lead controller is chosen with parameters:

$$k = \frac{1}{6000}$$
$$T = 40$$
$$\alpha = 0.05$$

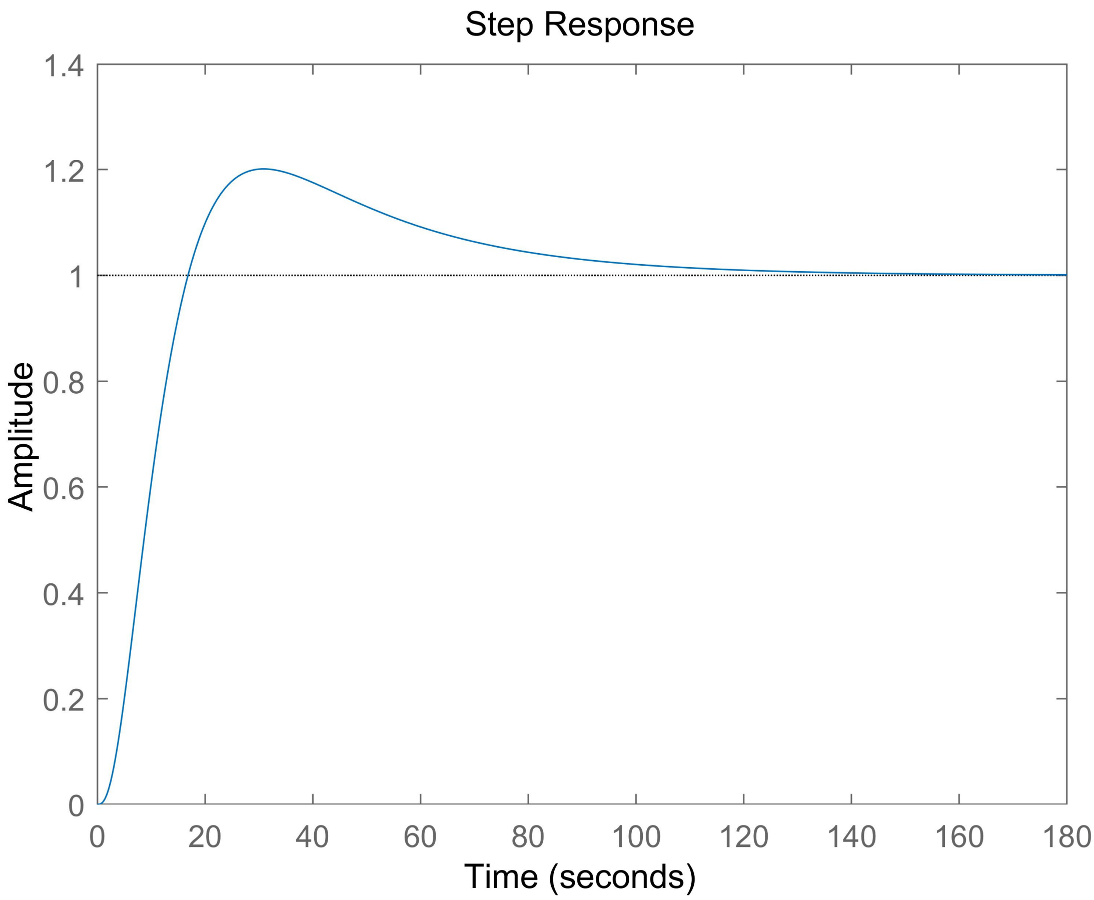
The resulting lead controller is:

$$K(s) = \frac{1}{6000}\frac{40s+1}{2s+1}$$

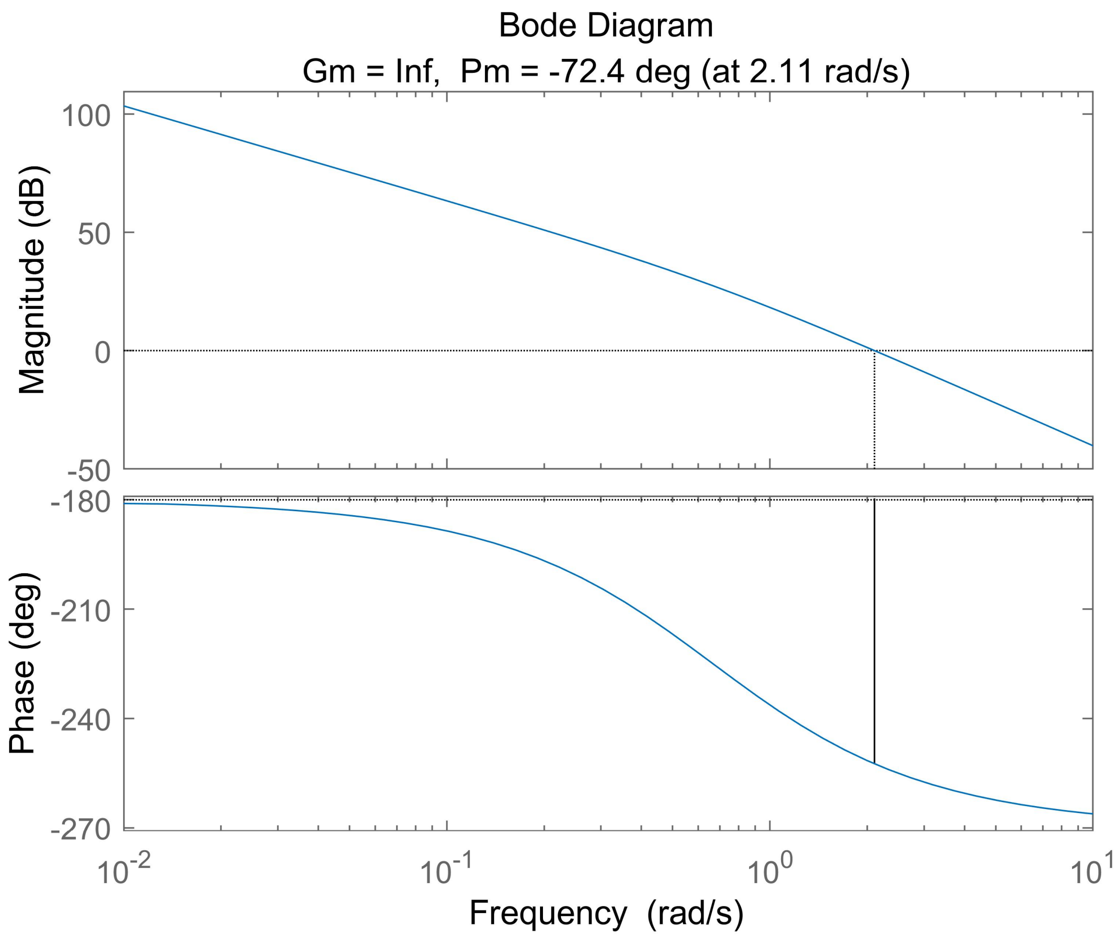
(b) The Bode plot is attached below.



(c) The closed-loop step response is attached below.



(d) It's not possible to achieve all four specifications. As shown in the open-loop Bode plot of the uncompensated transfer function (attached below), at frequency $4\,rad/s$, the phase is less than $-240°$. To achieve a phase margin of at least $40°$ while having the gain crossover frequency of at least $4\,rad/s$, a phase lead of more than $100°$ has to be introduced at the frequency of at least $4\,rad/s$, which is impossible for a single lead compensator. A single lead compensator can provide a phase lead of only $90°$ asymptotically. So, requirement III and requirement IV are contradicting and cannot be achieved simultaneously. The lead compensator designed above achieve requirements I, II, and IV.



2. Cascade Control

(a)

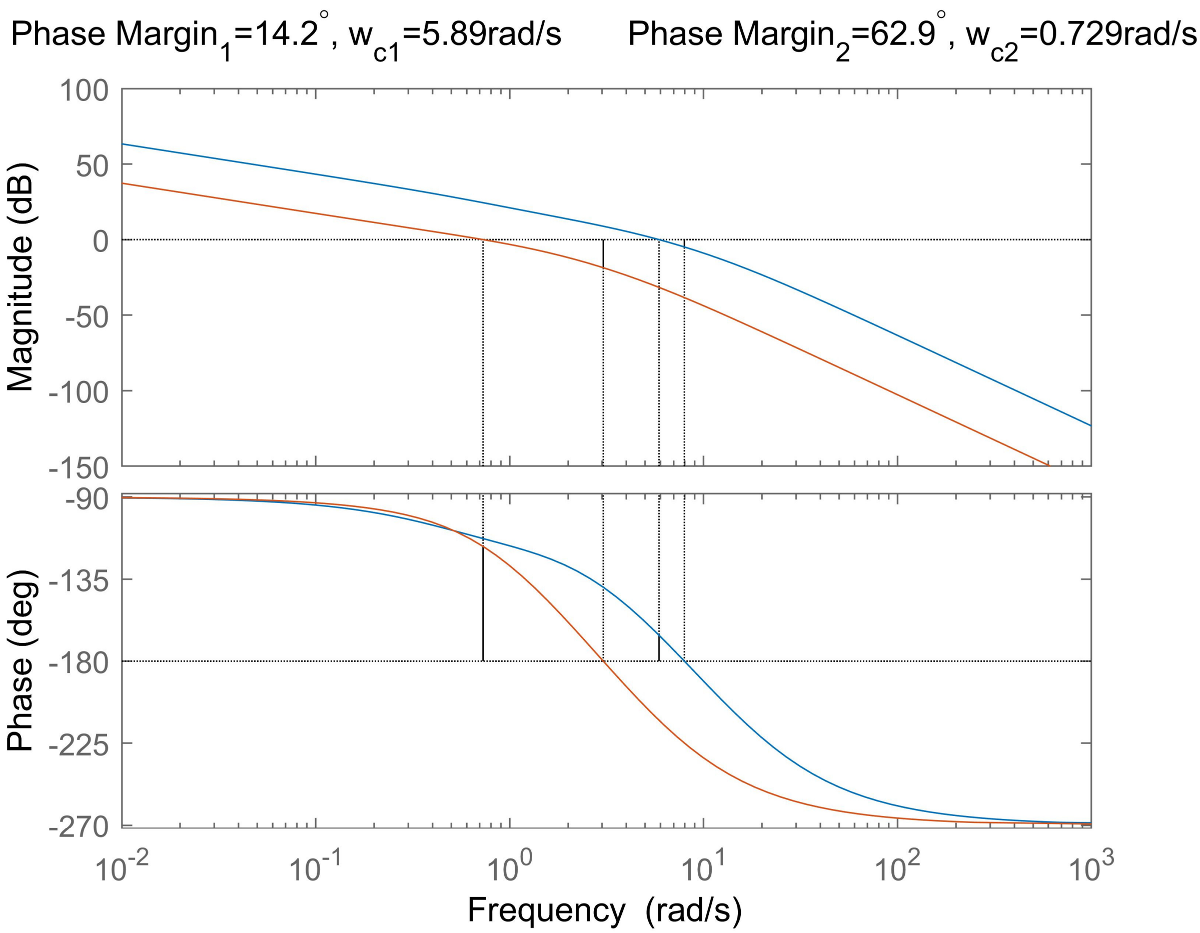
$$G_{out}(s) = \frac{49.05}{s(s+\frac{2}{3})(s+5)}$$

(b)

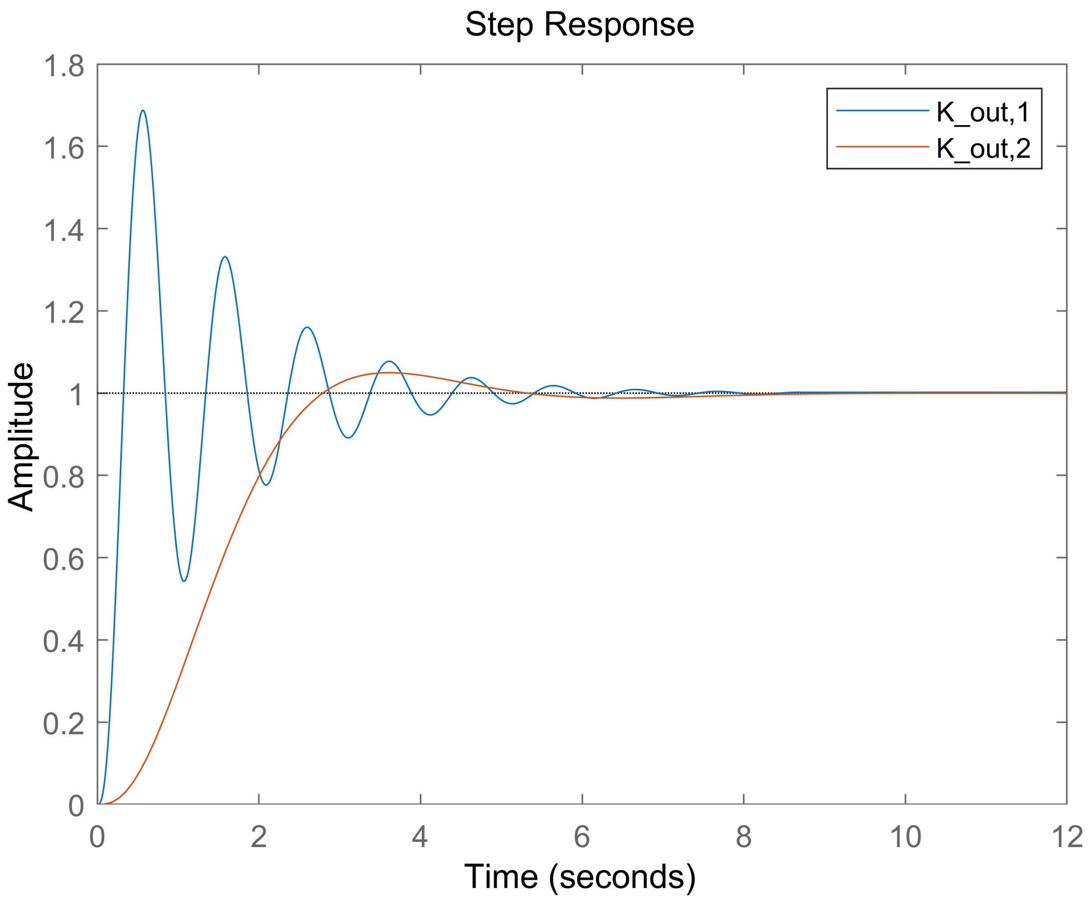
$$k_1 = 1, \alpha_1 = 7-4\sqrt{3}, T_1 = 1 \rightarrow K_{out,1}(s) = \frac{s+1}{\left(7-4\sqrt{3}\right)s+1}$$

$$k_2 = \frac{1}{20}, \alpha_2 = \frac{1}{3}, T_1 = \sqrt{3} \rightarrow K_{out,2}(s) = \frac{1}{20}\frac{\sqrt{3}s+1}{\frac{\sqrt{3}}{3}s+1}$$

(c) The overlaid Bode plots are attached below.



(d) The overlaid closed-loop step response are attached below.



(e) Using `stepinfo()`, the table is filled below.

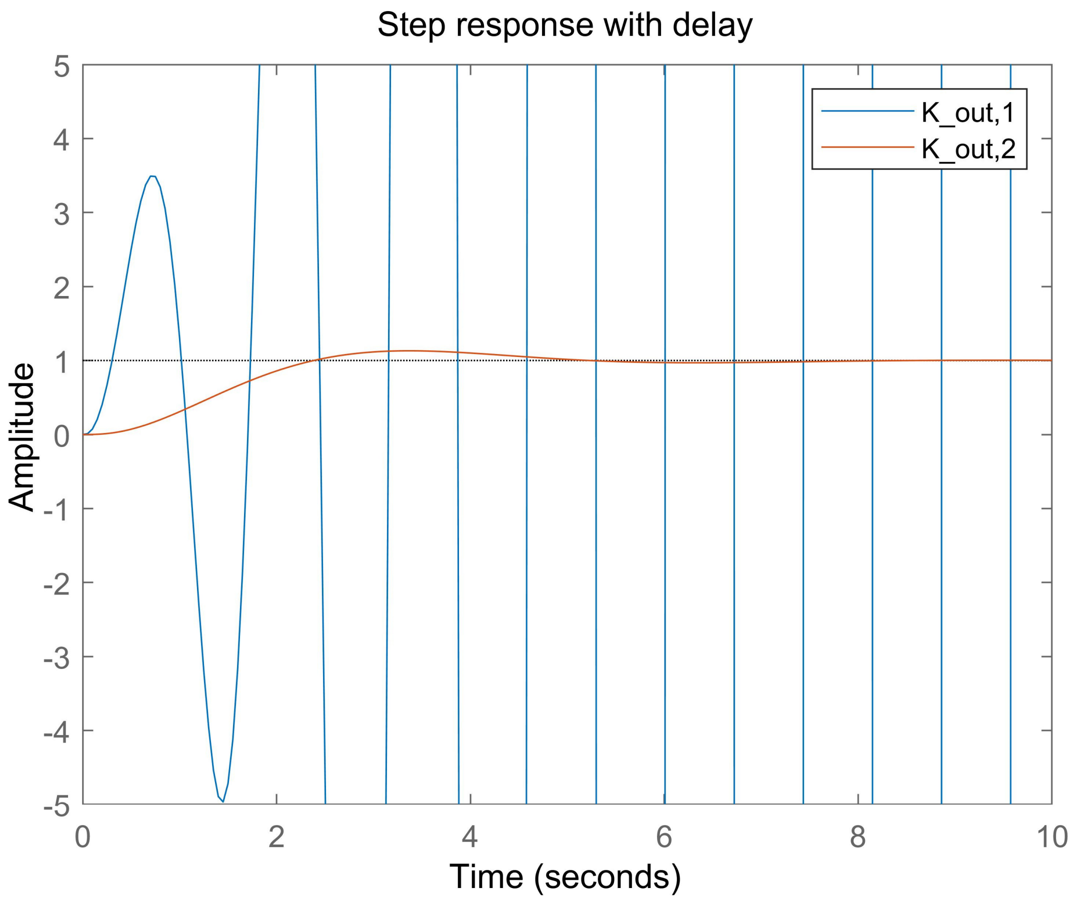
Controller	$K_{out,1}(s)$	$K_{out,1}(s)$	No Cascade Control
Rise Time	0.1929	1.7320	11.0126
% Overshoot	68.7712	4.9654	20.1149

3. Model Mismatch

(a)

Controller	$K_{out,1}(s)$	$K_{out,2}(s)$
Maximum delay T_d	$42.1ms$	$1505.8ms$

(b) The step responses with delay of $200ms$ is attached below. Since $200ms > 0.0421s$, which exceeds the maximum delay allowed for controller 1, the closed-loop system becomes unstable and the step response diverges. While for the controller 2, it's within the range of allowed delay. The closed-loop system is still stable and the step response converges. The results of step responses agree with the maximum delay analysis.



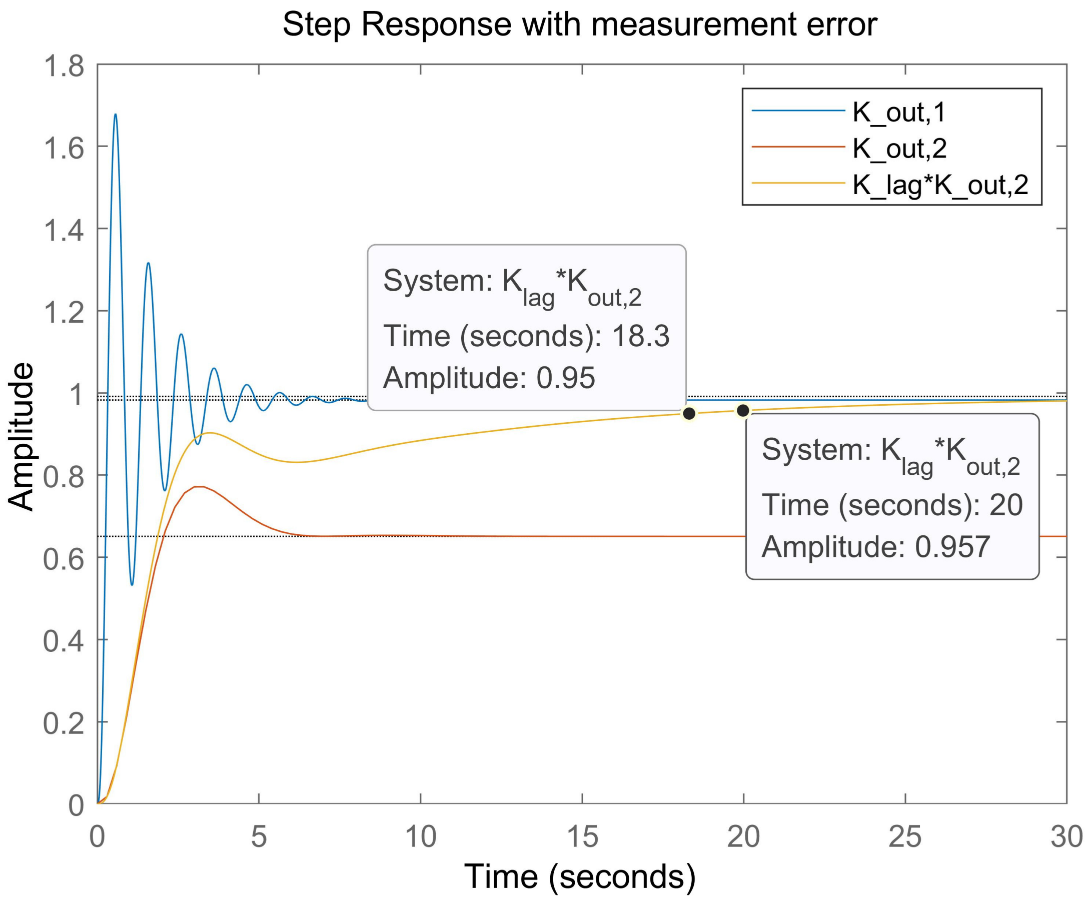
(c)

$$S_{err}(s) = \frac{G_{out}}{1 + K_{out}G_{out}} = \frac{49.05}{s\left(s + \frac{2}{3}\right)(s+5) + 49.05K_{out}}$$

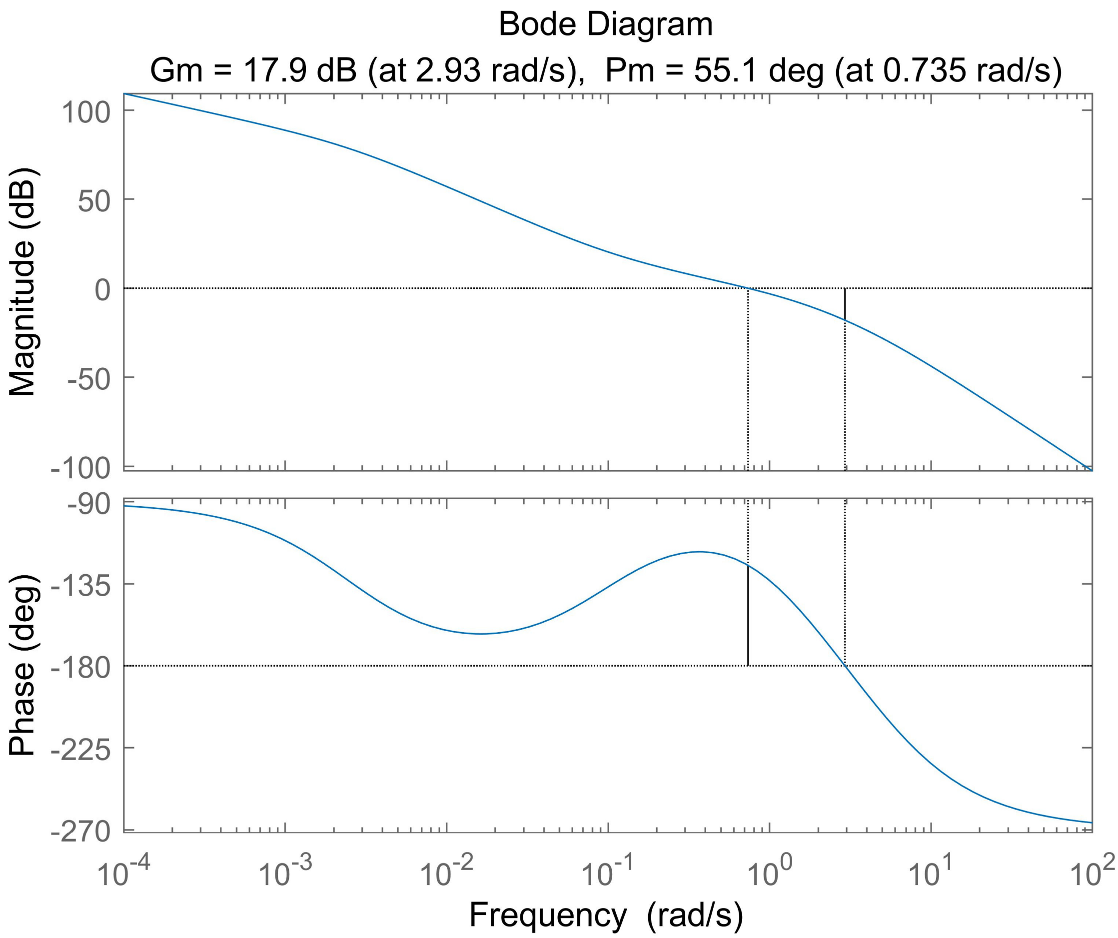
(d)

$$\alpha_{lag} = 40, T_{lag} = 10 \rightarrow K_{lag}(s) = 40\frac{10s+1}{400s+1}$$

(e) The overlaid step responses with measurement error for both uncompensated and compensated plants are attached below. As shown in the plot, the compensated controller 2 rises to within 5cm error at 18.3s. And its steady state error is smaller than using the uncompensated controller 1 with much less oscillation.



(f) The Bode plot of $K_{lag}(s) * K_{out,2}(s)$ is attached below. As shown in the plot, the phase margin is $55.1°$, which is above $55°$.



Full MATLAB code:

```
%% Lead Compensator
G_CF = tf(9.81, [1 2/3 0 0]);

K = tf([40/6000 1/6000], [2 1]);

margin(G_CF)

margin(K*G_CF)

step(feedback(K*G_CF,1))

%% Cascade Control
g_theta = tf(1,[1 0]);
tf_theta = feedback(5*g_theta,1);

G_no_theta = tf(9.81, [1 2/3 0]);

G_out = tf_theta * G_no_theta;

% Crossover @>=4rad/s
K1 = tf([1 1],[1*(7-4*sqrt(3)) 1]);
% bode(K1*G_out);
% step(feedback(K1*G_out,1));
% stepinfo(feedback(K1*G_out,1));



% Phase margin >= 60 degree
K2 = tf([sqrt(3)/20 1/20],[1/3*sqrt(3) 1]);
% bode(K2*G_out);
% step(feedback(K2*G_out,1));
% stepinfo(feedback(K1*G_out,1));
margin(K1*G_out)
hold on
margin(K2*G_out)
hold off
title("Phase Margin_1=14.2^{\circ}, w_{c1}=5.89rad/s      Phase Margin_2=62.9^{\circ}, w_{c2}=0.729rad/s")

step(feedback(K1*G_out,1));
hold on
step(feedback(K2*G_out,1));
legend('K_{out,1}','K_{out,2}')

stepinfo(feedback(K*G_CF,1))
```

```matlab
stepinfo(feedback(K1*G_out,1))
stepinfo(feedback(K2*G_out,1))



%% Model Mismatch
% Max delay
[Gm1,Pm1,Wcg1,Wcp1] = margin(K1*G_out);
max_time_delay1 = deg2rad(Pm1)/Wcp1;

[Gm2,Pm2,Wcg2,Wcp2] = margin(K2*G_out);
max_time_delay2 = deg2rad(Pm2)/Wcp2;

% Step response with delay
tf_delay = tf(1,1,'InputDelay',0.2);
step(feedback((K1*G_out),tf_delay),10)
ylim([-5,5])
hold on

step(feedback((K2*G_out),tf_delay),10)
ylim([-5,5])
hold off
title("Step response with delay")
legend('K_{out,1}','K_{out,2}')

%S error
syms x
S1_num = sym2poly(49.05*((7-4*sqrt(3))*x+1));
S1_den = sym2poly(x*((7-4*sqrt(3))*x+1)*(x+2/3)*(x+5) +49.05*(x+1));
S1 = tf(S1_num, S1_den);

S2_num = sym2poly(49.05*(sqrt(3)/3*x+1));
S2_den = sym2poly(x*(sqrt(3)/3*x+1)*(x+2/3)*(x+5)+(49.05/20)*(sqrt(3)*x+1));
S2 = tf(S2_num, S2_den);

%Step response with error
T1 = feedback(K1*G_out,1);
tf_error_1 = T1 - pi/180*S1;

T2 = feedback(K2*G_out,1);
tf_error_2 = T2 - pi/180*S2;

% Lag Compensator
K_lag = tf([400 40],[400 1]);
```

```
S_lag_num = sym2poly(49.05*(sqrt(3)/3*x+1)*(400*x+1));
S_lag_den = sym2poly((sqrt(3)/3*x+1)*(400*x+1)*x*(x+2/3)*(x+5)+(49.05/20)*(40)*(sqrt(3)*x+1)*(10*x+1));
S_lag = tf(S_lag_num, S_lag_den);

T_lag = feedback(K_lag*K2*G_out,1);
tf_error_lag = T_lag - pi/180*S_lag;

step(tf_error_1,30)
hold on
step(tf_error_2,30)
hold on
step(tf_error_lag,30)
title("Step Response with measurement error")
legend('K_{out,1}','K_{out,2}','K_{lag}*K_{out,2}')
hold off

margin(K_lag*K2*G_out)
```