1. Lead Controller

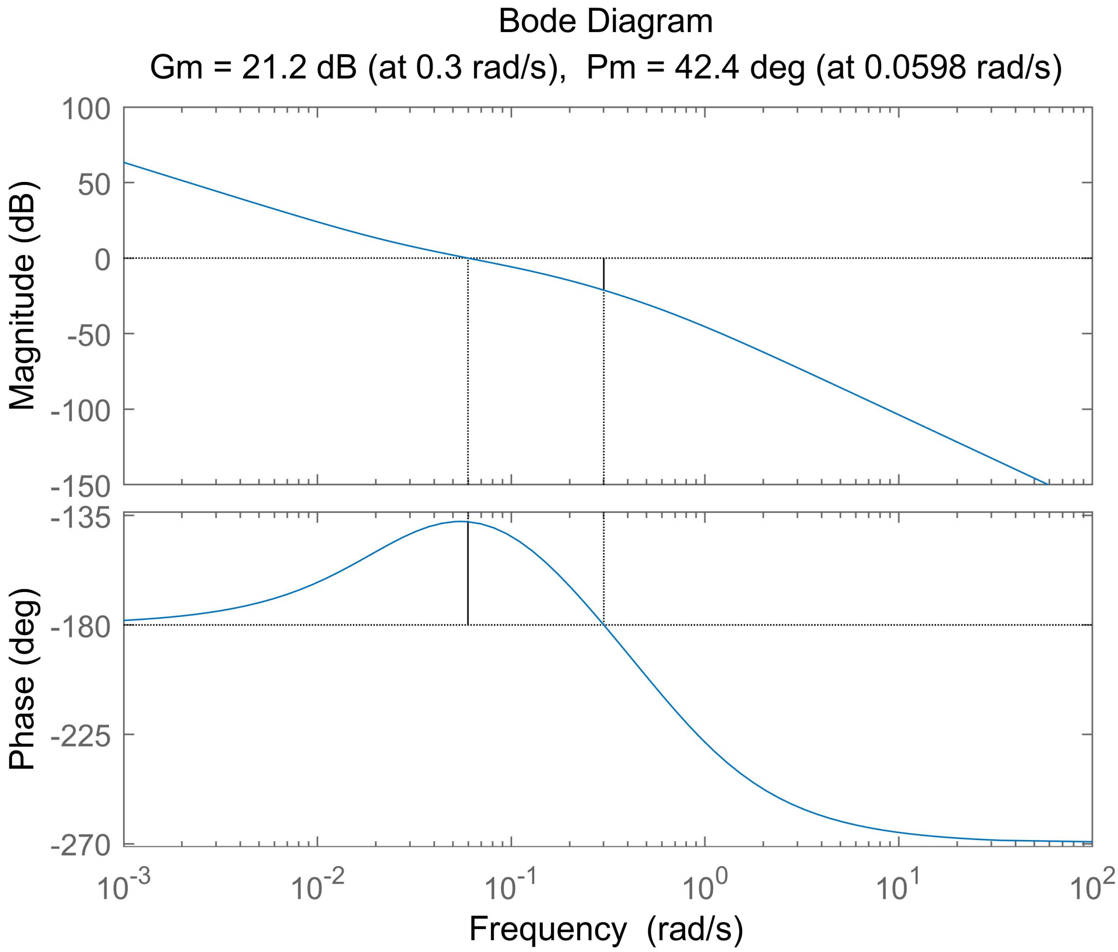
(a) The lead controller is chosen with parameters:

$$k = \frac{1}{10000}$$
$$T = 40$$
$$\alpha = 0.15$$

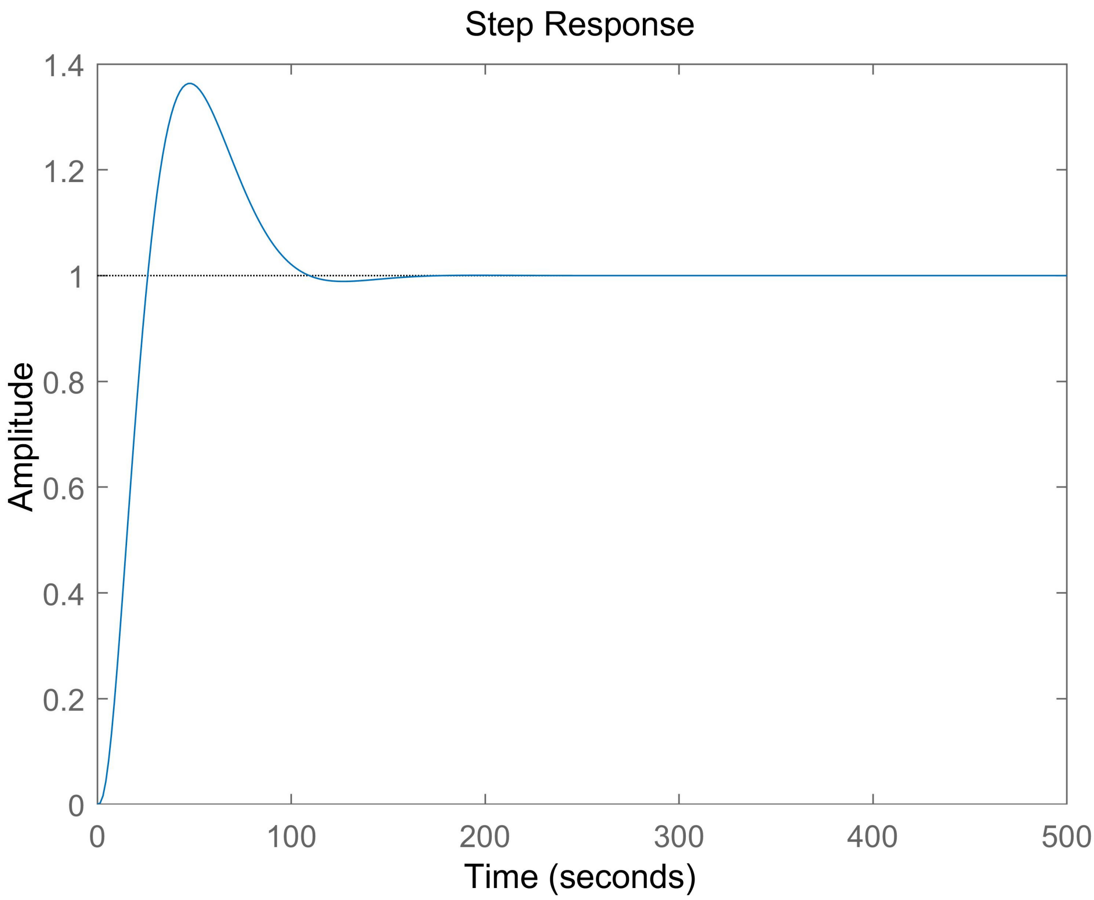
The resulting lead controller is:

$$K(s) = \frac{1}{10000}\frac{40s + 1}{6s + 1}$$

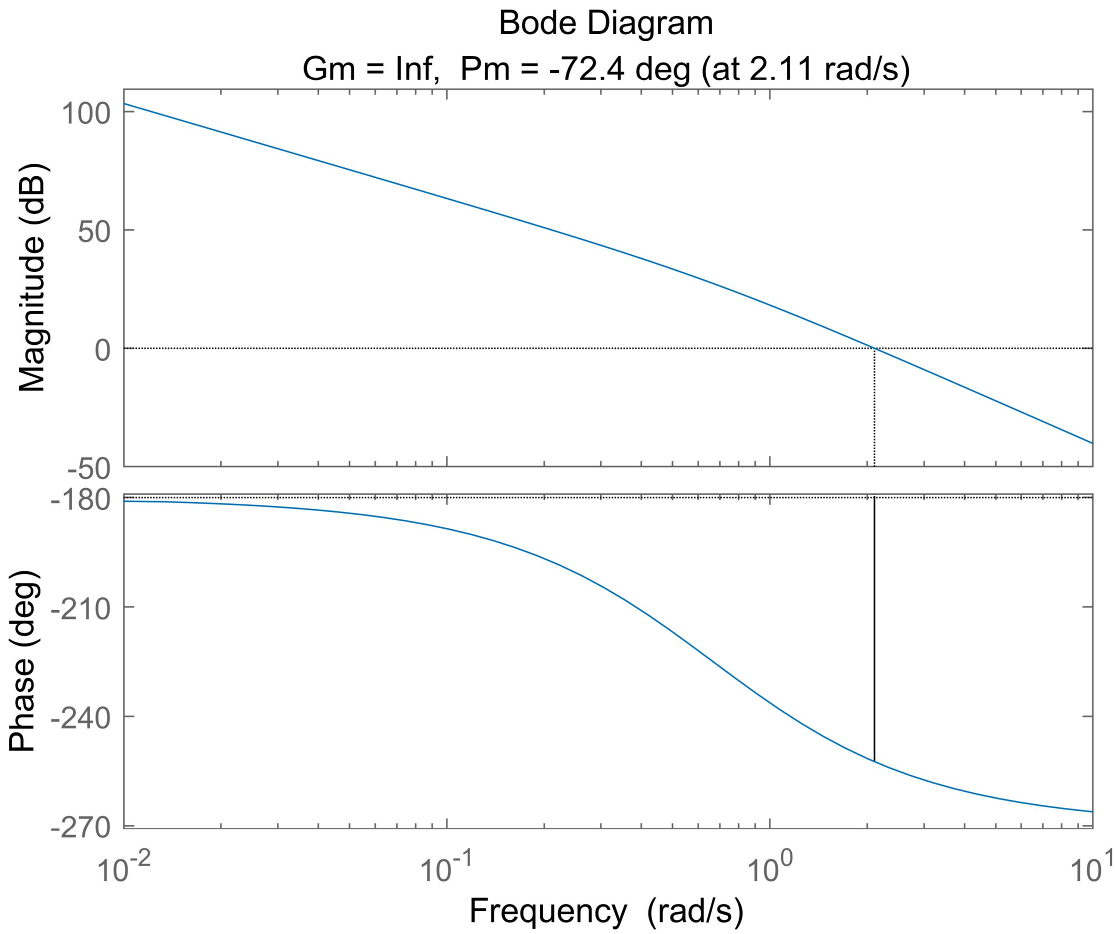
(b) The Bode plot is attached below.



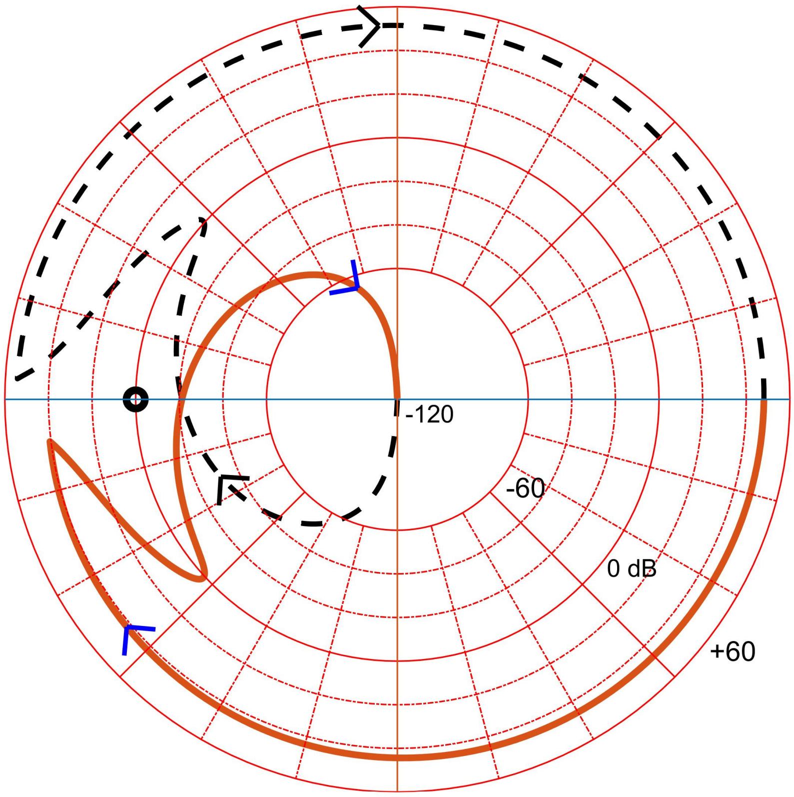
(c) The closed-loop step response is attached below.



(d) It's not possible to achieve all four specifications. As shown in the open-loop Bode plot of the uncompensated transfer function (attached below), at frequency $4\,rad/s$, the phase is less than $-240°$. To achieve a phase margin of at least $40°$ while having the gain crossover frequency of at least $4\,rad/s$, a phase lead of more than $100°$ has to be introduced at the frequency of at least $4\,rad/s$, which is impossible for a single lead compensator. A single lead compensator can provide a phase lead of only $90°$ asymptotically. So, requirement III and requirement IV are contradicting and cannot be achieved simultaneously.



The lead compensator designed above achieves requirements I, II, and IV. Since the original plant has an integrator, its steady state error for a step response is zero. The asymptotic stability can also be shown using the Nyquist plot, which is attached below. And the phase margin is 42.4°, as shown in the above Bode plot in part (b).



2. Cascade Control

(a)

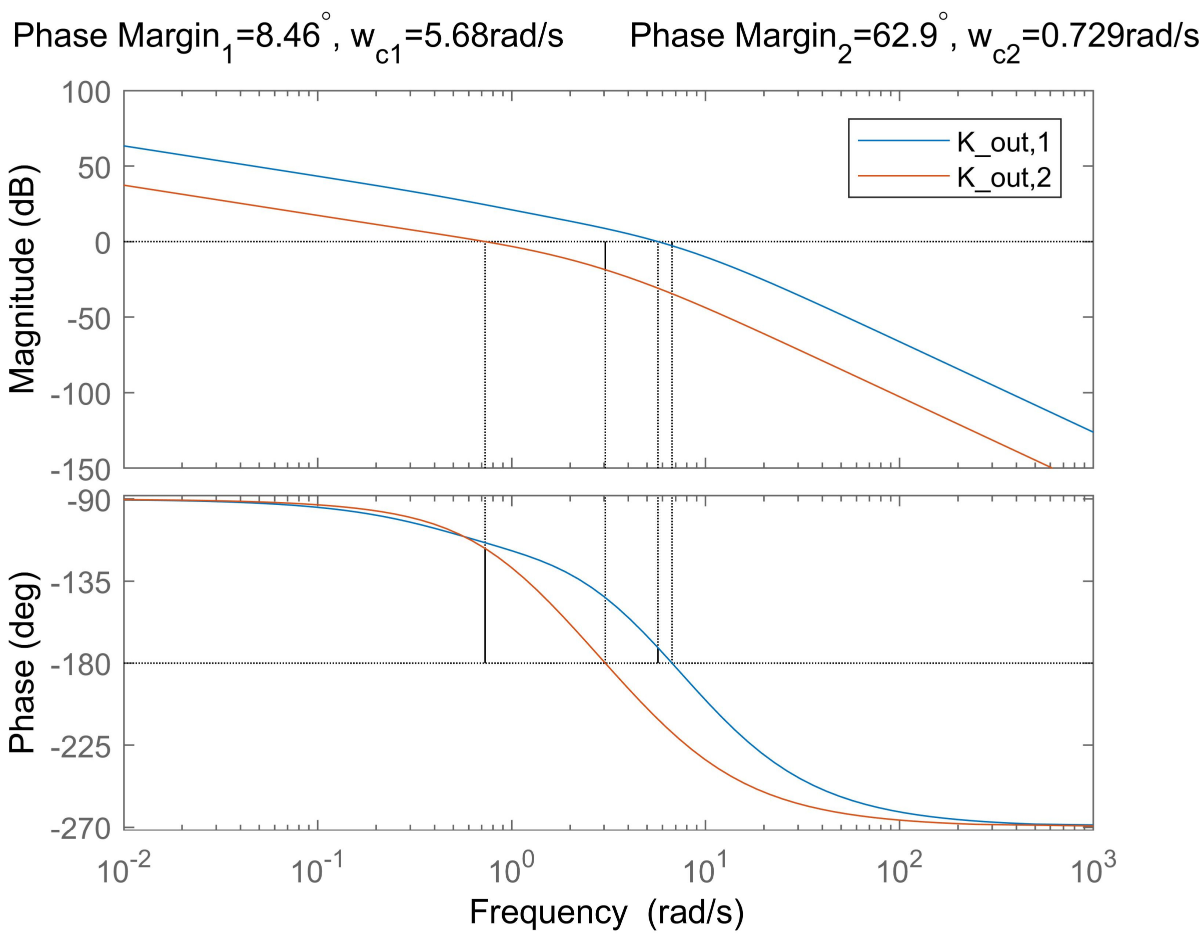
$$G_{out}(s) = \frac{49.05}{s(s+\frac{2}{3})(s+5)}$$

(b)

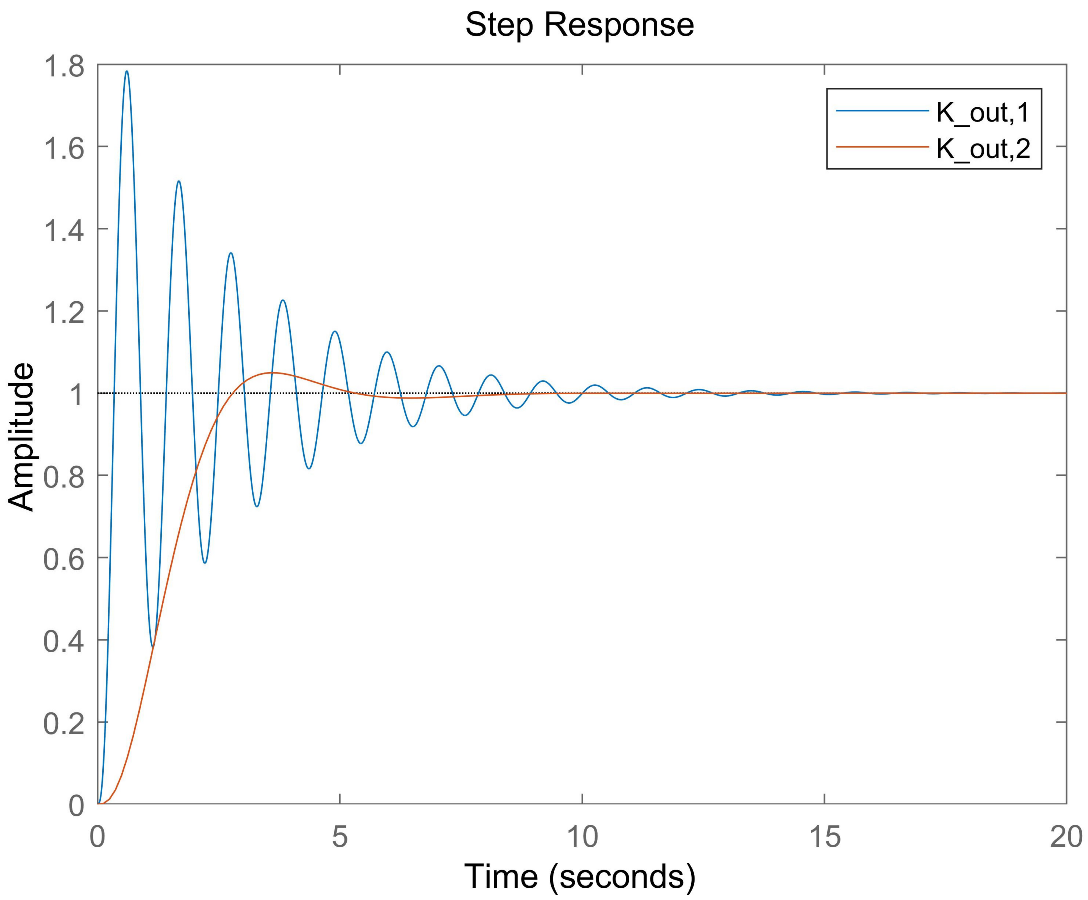
$$k_1 = 1, \alpha_1 = 0.1, T_1 = 1 \rightarrow K_{out,1}(s) = \frac{s+1}{0.1s+1}$$

$$k_2 = \frac{1}{20}, \alpha_2 = \frac{1}{3}, T_1 = \sqrt{3} \rightarrow K_{out,2}(s) = \frac{1}{20}\frac{\sqrt{3}s+1}{\frac{\sqrt{3}}{3}s+1}$$

(c) The overlaid Bode plots are attached below.



(d) The overlaid closed-loop step responses are attached below.



(e) Using `stepinfo()`, the table is filled below.

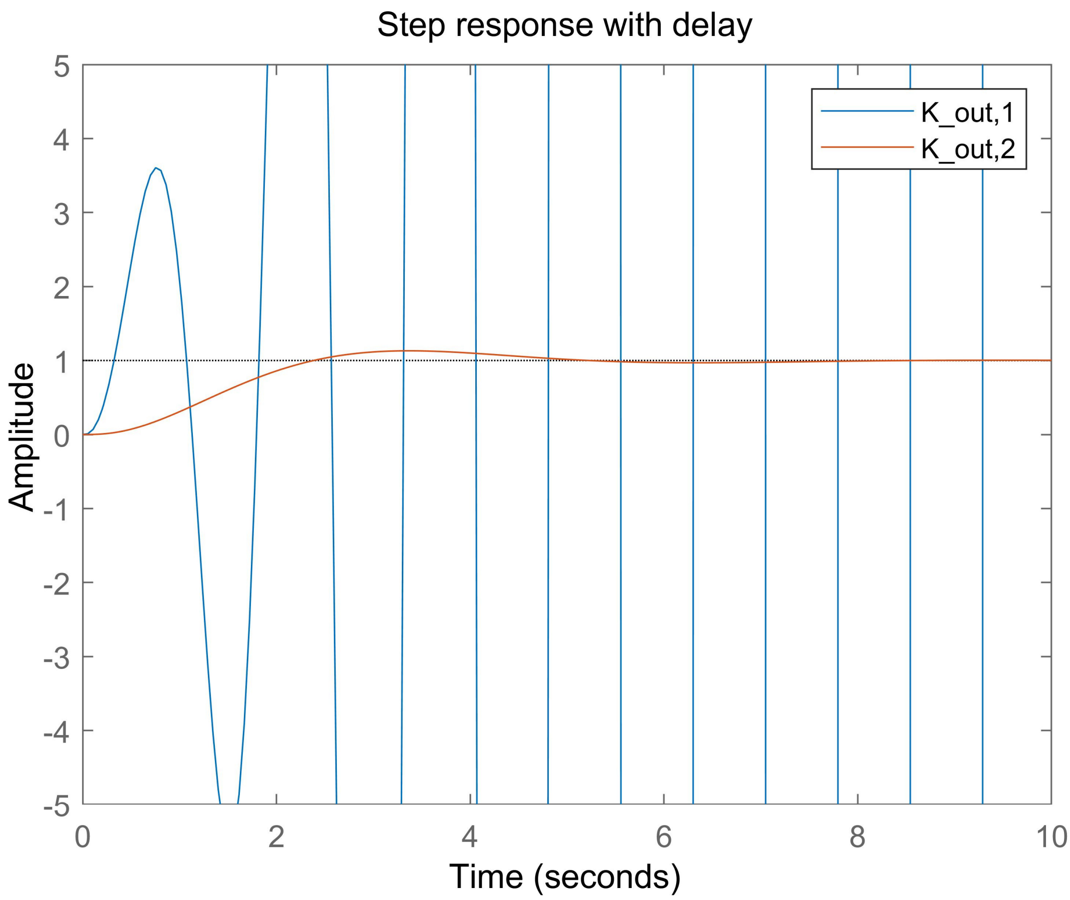
Controller	$K_{out,1}(s)$	$K_{out,2}(s)$	No Cascade Control
Rise Time(s)	0.2012	1.7320	17.1033
% Overshoot	78.4441	4.9654	36.3413

3. Model Mismatch

(a)

Controller	$K_{out,1}(s)$	$K_{out,2}(s)$
Maximum delay T_d(ms)	26.0	1505.8

(b) The step responses with delay of $200ms$ is attached below. Since $200ms > 26.0ms$, which exceeds the maximum delay allowed for controller 1, the closed-loop system becomes unstable and the step response diverges. While for the controller 2, it's within the range of allowed delay. The closed-loop system is still stable and the step response converges. The results of step responses agree with the maximum delay analysis.



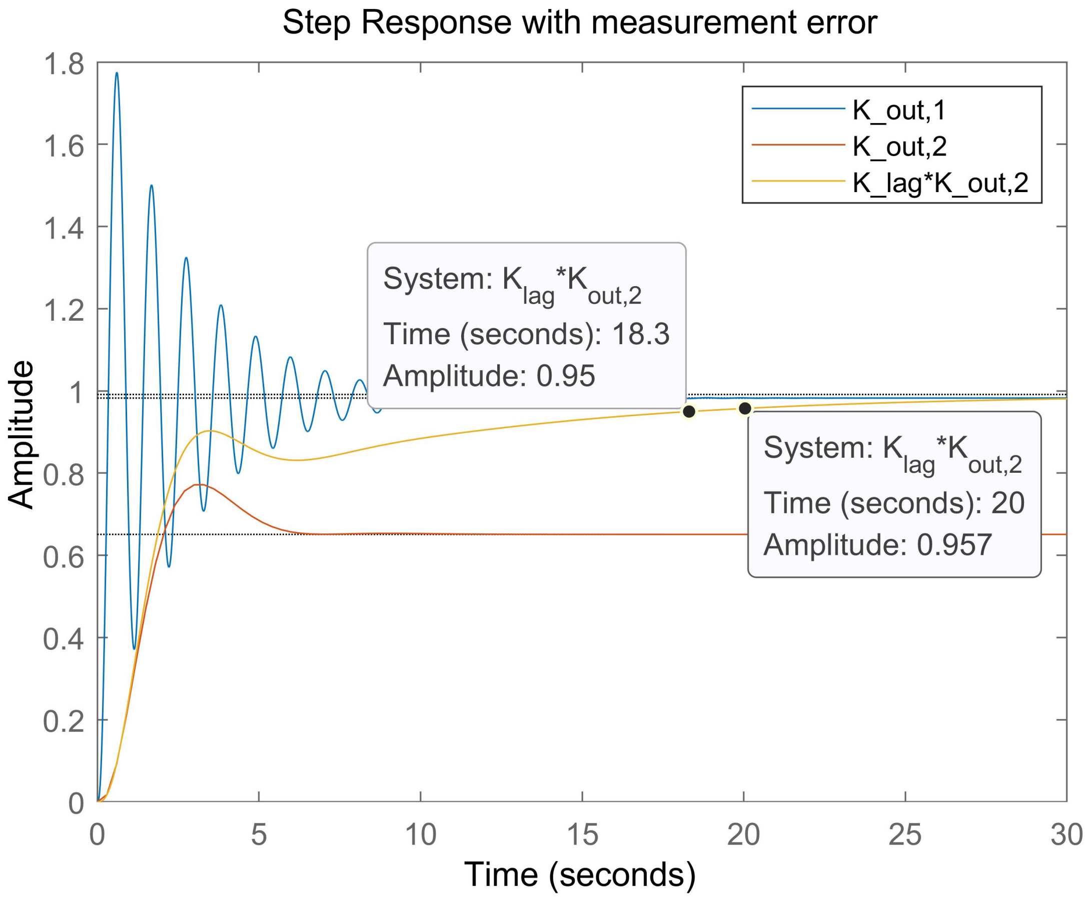
(c)

$$S_{err}(s) = \frac{G_{out}(s)}{1 + K_{out}(s)G_{out}(s)} = \frac{49.05}{s\left(s+\frac{2}{3}\right)(s+5) + 49.05K_{out}}$$

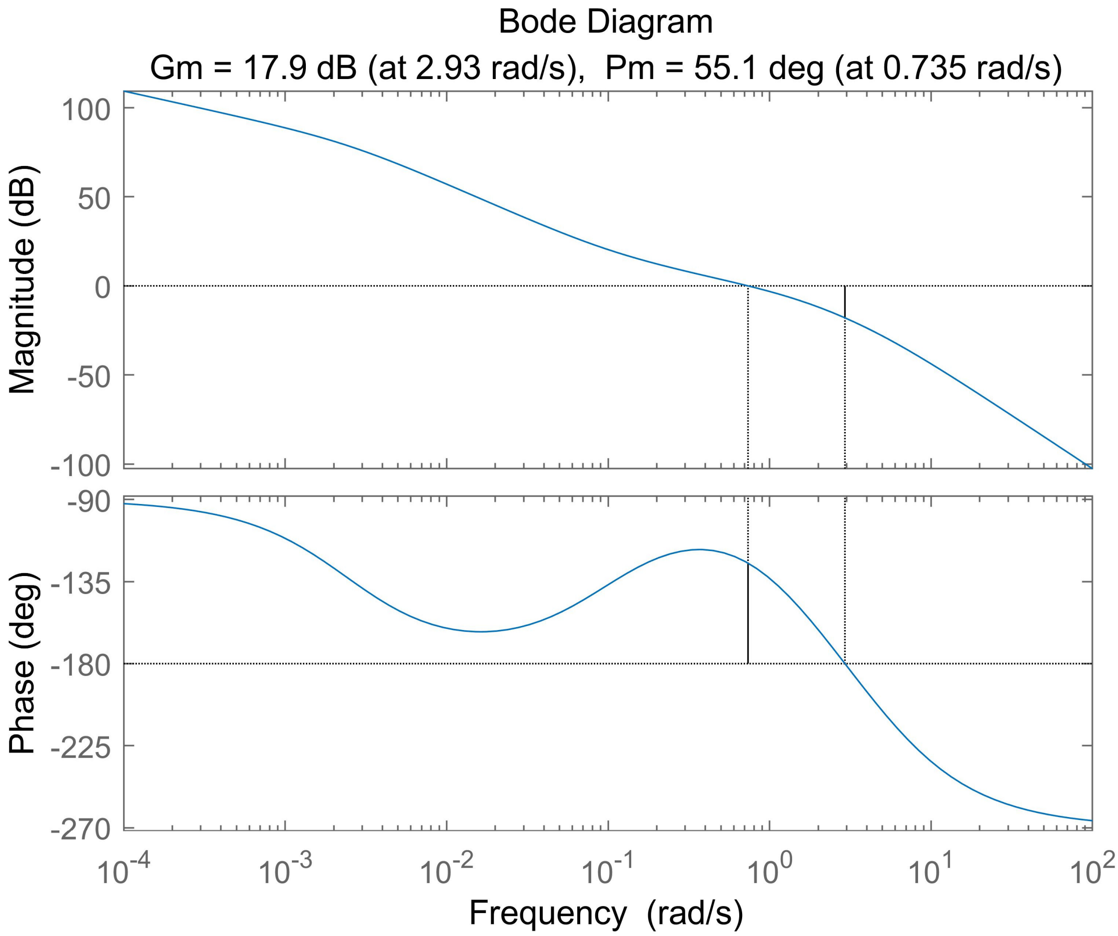
(d)

$$\alpha_{lag} = 40, T_{lag} = 10 \rightarrow K_{lag}(s) = 40\,\frac{10s+1}{400s+1}$$

(e) The overlaid step responses with measurement error for both uncompensated and compensated plants are attached below. As shown in the plot, the compensated controller 2 rises to within 5cm error at 18.3s. And its steady state error is much smaller than using the controller 2, and the steady state error is even smaller than using the uncompensated controller 1 with much less oscillation.



(f) The Bode plot of $K_{lag}(s) * K_{out,2}(s)$ is attached below. As shown in the plot, the phase margin is $55.1°$, which is above $55°$.



Full MATLAB code:

```
%% Lead Compensator
G_CF = tf(9.81, [1 2/3 0 0]);

K = tf([40/10000 1/10000], [40*0.15 1]);

margin(G_CF)

margin(K*G_CF)

nyqlog(K*G_CF)

step(feedback(K*G_CF,1),500)

%% Cascade Control
g_theta = tf(1,[1 0]);
tf_theta = feedback(5*g_theta,1);

G_no_theta = tf(9.81, [1 2/3 0]);

G_out = tf_theta * G_no_theta;

% Crossover @>=4rad/s
K1 = tf([1 1],[1*(0.1) 1]);
% bode(K1*G_out);
% step(feedback(K1*G_out,1));
% stepinfo(feedback(K1*G_out,1));


% Phase margin >= 60 degree
K2 = tf([sqrt(3)/20 1/20],[1/3*sqrt(3) 1]);
% bode(K2*G_out);
% step(feedback(K2*G_out,1));
% stepinfo(feedback(K1*G_out,1));

margin(K1*G_out)
hold on
margin(K2*G_out)
hold off
title("Phase Margin_1=8.46^{\circ}, w_{c1}=5.68rad/s      Phase Margin_2=62.9^{\circ}, w_{c2}=0.729rad/s")

step(feedback(K1*G_out,1),20);
hold on
step(feedback(K2*G_out,1),20);
```

```
legend('K_{out,1}','K_{out,2}')

stepinfo(feedback(K*G_CF,1))
stepinfo(feedback(K1*G_out,1))
stepinfo(feedback(K2*G_out,1))



%% Model Mismatch
% Max delay
[Gm1,Pm1,Wcg1,Wcp1] = margin(K1*G_out);
max_time_delay1 = deg2rad(Pm1)/Wcp1;

[Gm2,Pm2,Wcg2,Wcp2] = margin(K2*G_out);
max_time_delay2 = deg2rad(Pm2)/Wcp2;

% Step response with delay
tf_delay = tf(1,1,'InputDelay',0.2);
step(feedback((K1*G_out),tf_delay),10)
ylim([-5,5])
hold on

step(feedback((K2*G_out),tf_delay),10)
ylim([-5,5])
hold off
title("Step response with delay")
legend('K_{out,1}','K_{out,2}')

%S error
syms x
S1_num = sym2poly(49.05*((7-4*sqrt(3))*x+1));
S1_den = sym2poly(x*((7-4*sqrt(3))*x+1)*(x+2/3)*(x+5) +49.05*(x+1));
S1 = tf(S1_num, S1_den);

S2_num = sym2poly(49.05*(sqrt(3)/3*x+1));
S2_den = sym2poly(x*(sqrt(3)/3*x+1)*(x+2/3)*(x+5)+(49.05/20)*(sqrt(3)*x+1));
S2 = tf(S2_num, S2_den);

%Step response with error
T1 = feedback(K1*G_out,1);
tf_error_1 = T1 - pi/180*S1;

T2 = feedback(K2*G_out,1);
tf_error_2 = T2 - pi/180*S2;
```

```
% Lag Compensator
K_lag = tf([400 40],[400 1]);

S_lag_num = sym2poly(49.05*(sqrt(3)/3*x+1)*(400*x+1));
S_lag_den = sym2poly((sqrt(3)/3*x+1)*(400*x+1)*x*(x+2/3)*(x+5)+(49.05/20)*(40)*(sqrt(3)*x+1)*(10*x+1));
S_lag = tf(S_lag_num, S_lag_den);

T_lag = feedback(K_lag*K2*G_out,1);
tf_error_lag = T_lag - pi/180*S_lag;

step(tf_error_1,30)
hold on
step(tf_error_2,30)
hold on
step(tf_error_lag,30)
title("Step Response with measurement error")
legend('K_{out,1}','K_{out,2}','K_{lag}*K_{out,2}')
hold off

margin(K_lag*K2*G_out)
```