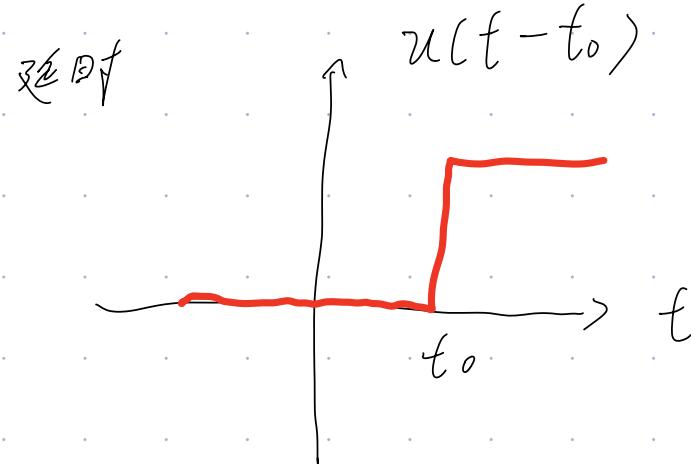
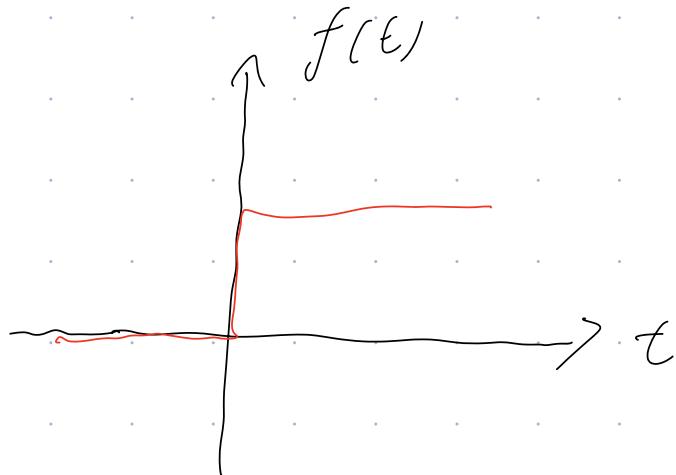


Signal

1. 單位階跃信号 $u(t)$

$$u(t) = \begin{cases} 0 & (t < 0) \\ 1 & (t > 0) \end{cases}$$



$$2. \text{ 单位冲激信号 } \delta(t) = \frac{d}{dt} u(t)$$

$$\left\{ \begin{array}{l} \int_{-\infty}^{+\infty} \delta(t) dt = 1 \quad \text{强度(面积)} = 1 \\ \delta(t) = 0 \quad (t \neq 0) \quad \text{强度无穷大} \end{array} \right.$$

property

$$1. \text{ 线性性质} \quad X(t) \delta(t) = X(0) \delta(t)$$

$$\int_{-\infty}^{+\infty} X(t) \delta(t) dt = X(0)$$

$$2. \text{ 尺度运算} \quad \delta(at) = \frac{1}{|a|} \delta(t)$$

$$3. \text{ even func.} \quad \delta(-t) = \delta(t)$$

3. 沖激偶信號 $\delta'(t) = \frac{d}{dt} \delta(t)$

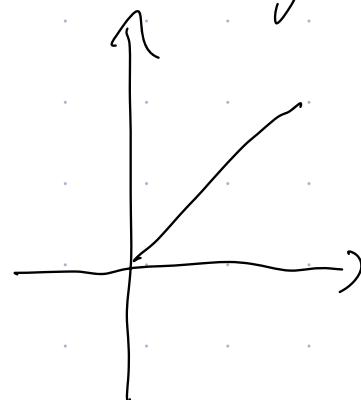
$$\int_{-\infty}^{+\infty} \delta'(t) f(t) dt = -f(0)$$

微近法

$$\int_{-\infty}^{+\infty} \delta'(t) dt = 0$$

$S=0$

$y = r(t)$ 電壓信號



$$V(\epsilon) \xrightarrow{\frac{d}{d\epsilon}} U(\epsilon) \xrightarrow{\frac{d}{dt}} S(\epsilon) \xrightarrow{\frac{d}{dt}} S'(t)$$

\approx

continuous

$$\left\{ \begin{array}{l} \text{周期} \quad E = \lim_{T \rightarrow \infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt \\ \text{非周期} \quad E = \int_{-\infty}^{+\infty} |f(t)|^2 dt \\ P = \lim_{T \rightarrow \infty} \frac{E}{T} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt \end{array} \right.$$

discrete

$$\left\{ \begin{array}{l} E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 \\ P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{K=-N}^N |x[k]|^2 \end{array} \right.$$

Fourier series

周期信号为 $f(t)$ ，周期 = T_1 ， $\omega_1 = \frac{2\pi}{T_1}$

$$a_0 = \frac{1}{T} \int_{-\infty}^{+\infty} f(t) dt \quad \text{面积} / -TT$$

$$a_n = \frac{2}{T} \int_T \cos(n\omega_0 t) f(t) dt$$

$$b_n = \frac{2}{T} \int_T \sin(n\omega_0 t) f(t) dt$$

FT

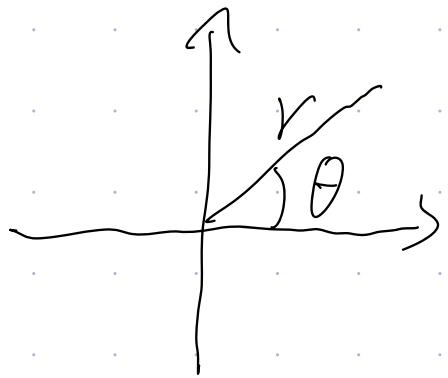
$$F(w) = \int_{-\infty}^{+\infty} f(t) e^{-jwt} dt$$

$$f(t) \longleftrightarrow F(w)$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(w) e^{jwt} dt$$

$$F(w) \text{ 可写为 } |F(w)| e^{j\varphi(w)}$$

$|F(w)| - w, \varphi(w) - w$ 曲线称为非周期信号
的幅度谱和相位谱



$$1 Hz = 1 s$$

$$\omega = 2\pi f$$

$$\sin \theta = \sin \omega t$$

冲激函数匹配法

e.g. $y''(t) + 2y'(t) + y(t) = f''(t) + 2f'(t)$

$y(0_-) = 1, y'(0_-) = -1, f(t) = u(t), \text{ find } y(0_+), y'(0_+)$

解：

$$f''(t) + 2f'(t) = \delta'(t) + 2u(t)$$

移项
积分直到出现 $u(t)$ 为止

1 $\left\{ \begin{array}{l} y''(t) = a\delta'(t) + b\delta(t) + cu(t) \end{array} \right.$

2 $\left\{ \begin{array}{l} y'(t) = a\delta(t) + bu(t) + \cancel{c\delta'(t)} \quad y'(0_+) = y'(0_-) + b \\ y(t) = au(t) \quad y(0_+) = y(0_-) + a \end{array} \right.$

(a = 1)

$$\begin{cases} b+2a=0 \\ c+2b+a=2 \end{cases}$$

1. $r''(t) + 3r'(t) + 2r(t) = 2e(t) + bu(t)$
 $\Rightarrow r(0_-) = 2, r'(0_-) = 0, e(t) = u(t)$

解:

$$r^2 + 3r + 2 = 0$$

$$r_1 = -1, r_2 = -2$$

$$r_h(t) = C_1 e^{-t} + C_2 e^{-2t}$$

$$\therefore r(t) = C_1 e^{-t} + C_2 e^{-2t} + 3$$

$$\begin{cases} r''(t) = a\delta(t) + bu(t) \\ r(t) = a u(t) \quad r(0_+) = r(0_-) + a \\ r(t) = a t u(t) \quad r(0_+) = r(0_-) \end{cases}$$

$$\therefore a = 2$$

无界扰动

$$r'(t) + bu(t) = 2\delta(t) + bu(t)$$

$\leftarrow \leftarrow \leftarrow$

$S(t)$ 元特解

\therefore 設 $V_p(t) = B$

$$0 + 0 + 2B = 6$$

$$\therefore B = 3$$

$$\therefore V'(0_+) = V(0_-) + 2$$

$$V(0_+) = V(0_-) = 0$$

$$\begin{cases} -C_1 - 2C_2 = 0 + 2 \\ C_1 + C_2 + 3 = 2 \end{cases} \quad \begin{cases} C_1 = 0 \\ C_2 = -1 \end{cases}$$

$$\therefore V(t) = (-e^{-2t} + 3) u(t)$$

最後 $+ u(t)$

$$2. \quad y''(t) + 4y'(t) + 3y(t) = f(t),$$

$$y(0_-) = y'(0_-) = 1, \quad f(t) = u(t)$$

$$\text{解得: } \lambda^2 + 4\lambda + 3 = 0$$

$$WT - 1 \quad 1 \quad 1 \quad 1 \quad -$$

$$V_1 = -1, V_2 = -3$$

$$y_h(t) = C_1 e^{-t} + C_2 e^{-3t}$$

$$f(t) = u(t), \text{ 设 } y_p(t) = B$$

$$0 + 0 + 3B = 1 \\ \therefore B = \frac{1}{3}$$

$$y = C_1 e^{-t} + C_2 e^{-3t} + \frac{1}{3}$$

$$f(t) = u(t) \neq \cancel{\sin \theta \cos \theta}$$

$$\therefore y(0+) = y(0-) = 1$$

$$y'_m = -y'_{m-1} - 1$$

$$J(v_f) = J(v-1-1)$$

def γ

零输入/零状态响应

y_{zs} : zero status, $y_{zs}^{(k)} = 0$

y_{zi} : zero input, $y_{zi}^{(k)}(0_-) = y_{zi}^{(k)}(0_+)$

$$r''(t) + 3r'(t) + 2r(t) = 2e(t) + 6e(t)$$

$$\text{已知 } r(0_+) = 2, r'(0_+) = 0, e(t) = u(t)$$

解：

$$y = y_{2i} + y_{2s}$$

$$\text{求 } y_{2i} \Rightarrow \text{齐次解}$$

$$r^2 + 3r + 2 = 0$$

$$r_1 = -1, r_2 = -2$$

$$V_{2i}(t) = C_1 e^{-t} + C_2 e^{-2t}$$

$$V_{2i}(0_+) = V_{2i}(0_-) = 2$$

$$V_{2i}'(0_+) = V_{2i}'(0_-) = 0$$

r - 11

$$\begin{cases} C_1 + C_2 = 2 \\ -C_1 - 2C_2 = 0 \end{cases} \quad \begin{cases} C_1 = 7 \\ C_2 = -2 \end{cases}$$

$$\therefore V_{Zj}(t) = (4e^{-t} - 2e^{-2t})n(t)$$

2.

$$v'(t) + 3v(t) + 2r(t) = 2e(t) + 6c(t)$$

$$\text{已知 } v(0_-) = 2, v'(0_-) = 0, e(t) = u(t)$$

设 $y_{2s} \Rightarrow \frac{x}{t} + \frac{t}{x}$

$$V^2 + 3V + 2 = 0$$

$$V_1 = -1, V_2 = -2$$

$$V_{2s}(t) = C_1 e^{-t} + C_2 e^{-2t}$$

$$\therefore 2e(t) + 6c(t) = 2\delta(t) + 6u(t)$$

设 $y_{2sp} = \beta$

$$0 + 0 + 2\beta = 6$$

$$\beta = 3$$

$$V_{2s}(t) = 3 + C_1 e^{-t} + C_2 e^{-2t}$$

$$\begin{matrix} 1 \\ 3 \\ 2 \end{matrix} \left\{ \begin{array}{l} V_{ZS}'' = a \delta(t) + b u(t) \\ V_{ZS}' = a u(t) \quad V_{ZS}'(0_+) = V_{ZS}(0_+) + a \\ V_{ZS} = a t u(t) \quad V_{ZS}(0_+) = V_{ZS}(0_-) \end{array} \right.$$

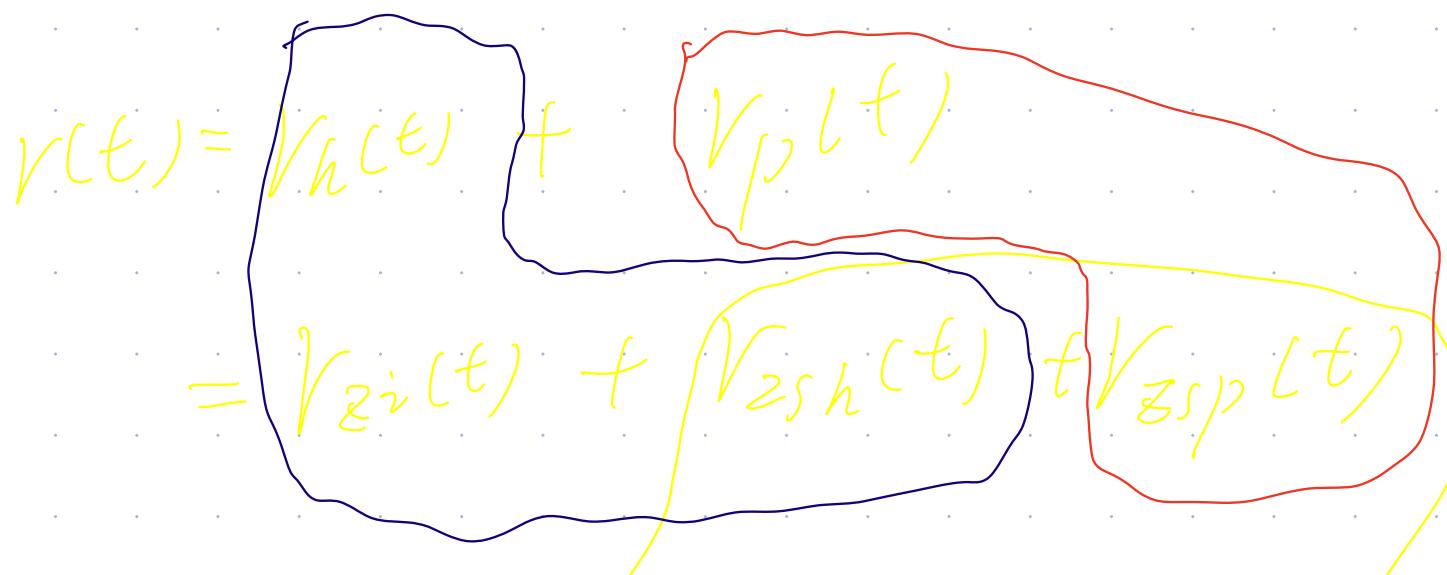
$$\therefore a = 2$$

$$\left\{ \begin{array}{l} -C_1 - 2C_2 = 0 + 2 \\ 3t C_1 + C_2 = 0 \end{array} \right. \quad V_{ZS}(0_-) = V_{ZS}(0_+) =$$

$$\left\{ \begin{array}{l} C_1 = -4 \\ C_2 = 1 \end{array} \right. \quad \begin{array}{l} -t \\ -2 + 1 \end{array}$$

$$V_{2S}(t) = (3 - 4e^{-t} + e^{-2t}) u(t)$$

$$\begin{aligned} y = V_{2S} + V_{zi} &= (4e^{-t} - 2e^{-2t}) u(t) + \\ &\quad (3 - 4e^{-t} + e^{-2t}) u(t) \\ &= (3 - e^{-2t}) u(t) \end{aligned}$$



$$= V_{2i}(t) + V_{2s}(t)$$



单位冲激响应 $h(t)$

激励原为 $\delta(t)$ 时的 y_{2s}

$$1. \quad c(t) = \delta(t)$$

$$2. \quad h(0_-) = 0$$

e.g. $r''(t) + 5r'(t) + 6r(t) = 3\delta'(t) + 2\delta(t)$, $h(t)$

解: $r''(t) + 5r'(t) + 6r(t) = 3\delta'(t) + 2\delta(t)$

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\lambda_1 = -2, \lambda_2 = -3$$

$$\therefore y_{2sh} = C_1 e^{-2t} + C_2 e^{-3t}$$

$$\delta(t) \neq y_{2sp} //$$

$$\therefore y_{2s} = y_{2sh} \quad h(t)(0_-) = 0$$

$$1 \quad \left\{ \begin{array}{l} v''(t) = a s(t) + b \delta(t) + c u(t) \end{array} \right.$$

$$5 \quad \left\{ \begin{array}{l} v'(t) = a s(t) + b u(t) \quad v'(0_+) = v'(0_-) + b \end{array} \right.$$

$$6 \quad \left\{ \begin{array}{l} v(t) = a u(t) \quad v(0_+) = v(0_-) + a \end{array} \right.$$

$$\left\{ \begin{array}{l} a = 3 \\ b + 5a = 2 \end{array} \right.$$

$$\left\{ \begin{array}{l} a = 3 \\ b = -13 \end{array} \right.$$

$$\therefore v'(0_+) = -13$$

$$v(0_+) = 3$$

$$\left\{ \begin{array}{l} C_1 + C_2 = 3 \\ -2C_1 e^{-2t} - 3C_2 e^{-3t} \end{array} \right.$$

$$\left. \begin{array}{l} \\ \end{array} \right\}_{t=0} = -13$$

$$\therefore h(t) = (-4e^{-2t} + 7e^{-3t})_{u(t)} - 2C_1 - 3C_2 = -13$$

$$-C_2 = -7$$

∴ ∴ ..

$$\begin{cases} c_1 = -4 \\ c_2 = 7 \end{cases}$$

e.g. $r(t) / f_2 r(t) = e(t) + 3e(t), f_{sh}(t)$

$$r(t) / f_2 r(t) = s(t) + 3\delta(t)$$

$$r f_2 = 0$$

$$r = -1$$

$$y_{esh} = C_1 e^{-2t}$$

--- $C_1 \in \mathbb{R}$

$\delta(t)$ និង γ_{2S}

$$y_{2S} = Ce^{-\alpha t}$$

$$1 \left\{ r(t) = a\delta(t) + b\delta'(t) + c u(t) \right.$$

$$2 \left\{ r(t) = a \underbrace{\delta(t)}_{} + b u(t) \right.$$

$$\begin{cases} a=1 \\ b+2a=3 \end{cases} \quad \begin{cases} a=1 \\ b=1 \end{cases}$$

$$\therefore r(0_+) = r(0_-) + b = 0 + 1 \approx 1$$

$$\therefore C_1 = 1 \quad \therefore y_{2S} = e^{-\alpha t} u(t) + \delta(t)$$

E.g. $r'(t) + 2r(t) = \overset{\text{CK}}{e''(t)} + 3e'(t) + 3e(t)$, $\underline{h(t)}$
 $e(t) = \delta(t)$ $\underline{y_{BS}(0_+)} = 0$

$$\begin{array}{l} r+2=0 \\ r=-2 \end{array}$$

$$\underline{\delta''(t) + 3\delta'(t) + 3\delta(t)}$$

$$y_{zsh} = Ce^{-2t}$$

$$y_{zs} = y_{zsh} = Ce^{-2t}$$

$$1 \left\{ \begin{array}{l} r'(t) = a\delta'(t) + b\delta(t) + c\delta(t) + d u(t) \end{array} \right.$$

$$2 \left\{ \begin{array}{l} r(t) = a\delta(t) + b\delta(t) + cu(t) \end{array} \right.$$

$$\begin{cases} a=1 \\ b+2a=3 \end{cases}$$

未配平

-1 -1 ...

1 1 ...

$$c+2b=3 \quad \text{因为左阶} < \text{右阶}$$

$$r^{(e)}_{(0+)}/r^{(e)}_{(0-)} = 0 + 1 = 1$$

$$y_{zs} = (e^{-2t} + g^{(e)} + f^{(e)} t) u(t)$$

阶跃响应 $g(t)$

1. $e(t) = u(t)$

2. y_{z_s}

$$\text{e.g. } r''(t) + 5r'(t) + 6r(t) = 3e^{2t} + 2e^{3t}$$

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\lambda_1 = -2, \lambda_2 = -3$$

$$y_{zh} = C_1 e^{-2t} + C_2 e^{-3t}$$

$$r''(t) + 5r'(t) + 6r(t) = 3\delta(t) + 2u(t)$$

$$y_{zp} = B$$

$$\therefore 6B = 2$$

$$B = \frac{1}{3}$$

$$\therefore y_z = y_{zh} + y_{zp} = C_1 e^{-2t} + C_2 e^{-3t} + \frac{1}{3}$$

$$\begin{cases} r''(t) = a\delta(t) + b u(t) \\ r(t) = n u(t) \end{cases}$$

$$6 \quad | \quad V(t) = atu(t)$$

$$\begin{cases} a = 3 \\ b + 5a = 2 \end{cases} \quad \begin{cases} a = 3 \\ b = -13 \end{cases}$$

$$V(t)_{(0+)} = V(t)_{(0-)} + a = 0 + 3 = 3$$

$$V(t)_{(0+)} = V(t)_{(0-)} = 0$$

$$\begin{cases} C_1 + C_2 + f = 0 \\ -2C_1 - 3C_2 = 3 \end{cases} \quad \begin{cases} C_1 = 2 \\ C_2 = -\frac{7}{3} \end{cases}$$

$$y_{2s} = \left(2e^{-2t} - \frac{7}{3}e^{-3t} + \frac{1}{3} \right) u(t)$$

$$\text{e.g. } r'(t) + 2r(t) = e^t + 3e^{2t} + 3e^{3t}$$

左-右 < 右 = 左 \therefore 雖已解不完全

$$r+2=0$$

$$r = -2$$

$$\therefore y_{2sh} = C e^{-2t}$$

$$\therefore e(t) = u(t)$$

$$\therefore r'(t) + 2r(t) = \delta(t) + 3\delta(t) + 3u(t)$$

$$\text{設 } y_{2sp} = B$$

$$\therefore 2B = 3$$

$$B = \frac{3}{2}$$

$$y_{2s} = y_{2sh} + y_{2sp} = Ce^{-2t} + \frac{3}{2}$$

r. r. $\sim r'(t) + 2r(t) + 3u(t) = 0$

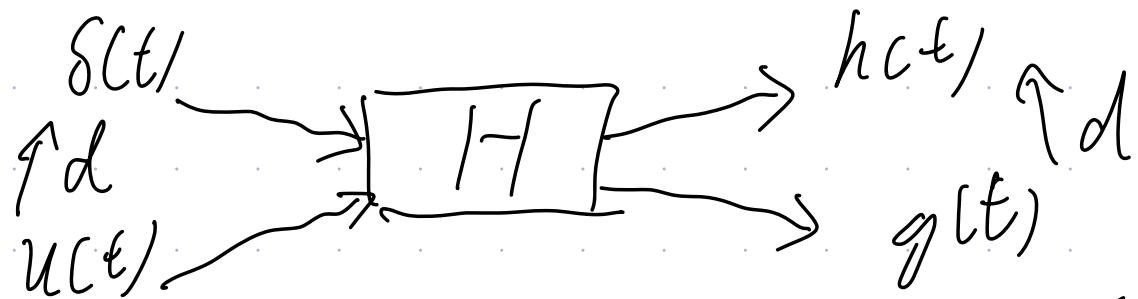
$$1 \quad \left\{ \begin{array}{l} r(t) = u \delta(t) + b \delta(t) + c(t) \\ r(t) = a \delta(t) + bu(t) \end{array} \right.$$

$$\begin{cases} a=1 \\ b+2a=3 \end{cases} \quad \begin{cases} a=1 \\ b=1 \end{cases}$$

$$r(t)_{(0+)} = r(t)_{(0-)} + b = 0 + 1 = 1$$

$$\therefore 1 = C + \frac{3}{2} \quad C = -\frac{1}{2}$$

$$\therefore g(t) = \left(-\frac{1}{2} e^{-2t} + \frac{3}{2} \right) u(t) + \delta(t)$$



$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau \Rightarrow g(t) = \int_{-\infty}^t h(\tau) d\tau$$

e.g. $\exists \delta(t) = (-4e^{-2t} + 7e^{-3t})u(t)$, & $g(t)$

$$g(t) = \int_{-\infty}^t (-4e^{-2\tau} + 7e^{-3\tau}) u(\tau) d\tau$$

$$= \left[\int_0^t (-4e^{-2\tau} + 7e^{-3\tau}) d\tau \right] u(t)$$

$$= \left[-4 \cdot \left(-\frac{1}{2}\right) \cdot e^{-2\tau} \Big|_0^t + 7 \cdot \left(-\frac{1}{3}\right) e^{-3\tau} \Big|_0^t \right] u(t)$$

$$= [2(e^{-2t} - 1) - \frac{7}{3}(e^{-3t} - 1)] u(t)$$

$$= \left(2e^{-2t} - \frac{7}{3} e^{-3t} + \frac{1}{3} \right) u(t)$$

e.g. $\mathcal{E}[g(t)] = e^{-2t} u(t-1)$, $\mathcal{E}[g(t)]$

$$g(t) = \int_{-\infty}^t e^{-2\tau} u(\tau-1) d\tau$$

$$= \int_1^t e^{-2\tau} d\tau u(t-1)$$

$$= -\frac{1}{2} e^{-2\tau} \Big|_1^t u(t-1)$$

$$= -\frac{1}{2} (e^{-2t} - e^{-2}) u(t-1)$$

卷积 (convolution)

1. definition: $f_1(t) * f_2(t) =$

$$f(t) * \delta(t) = f(t)$$

$$f(t) * \delta(t-t_0) = f(t-t_0)$$

$$f(t) * \delta'(t) = f'(t) \quad f(t) * \delta'(t-t_0) = f'(t-t_0)$$

$$f(t) * u(t) = \int_{-\infty}^t f(\tau) d\tau$$

e.g. $e^{-t} u(t) * \delta(3t+2)$

$$= e^{-t} u(t) * \frac{1}{3} \delta\left(t + \frac{2}{3}\right)$$

$$= \frac{1}{3} e^{-(t+\frac{2}{3})} u\left(t+\frac{2}{3}\right)$$

e.g. $e^{-2t} u(t) * \delta''(t) * u(t)$

$$= -2e^{-2t} u(t) + e^{-2t} \delta(t)$$

$$= -2e^{-2t} u(t) + \delta(t)$$

$$\text{e.g. } e^{-2t} u(t) * \delta''(t-1) * f u(t)$$

$$= e^{-2t} u(t) * \delta(t-1)$$

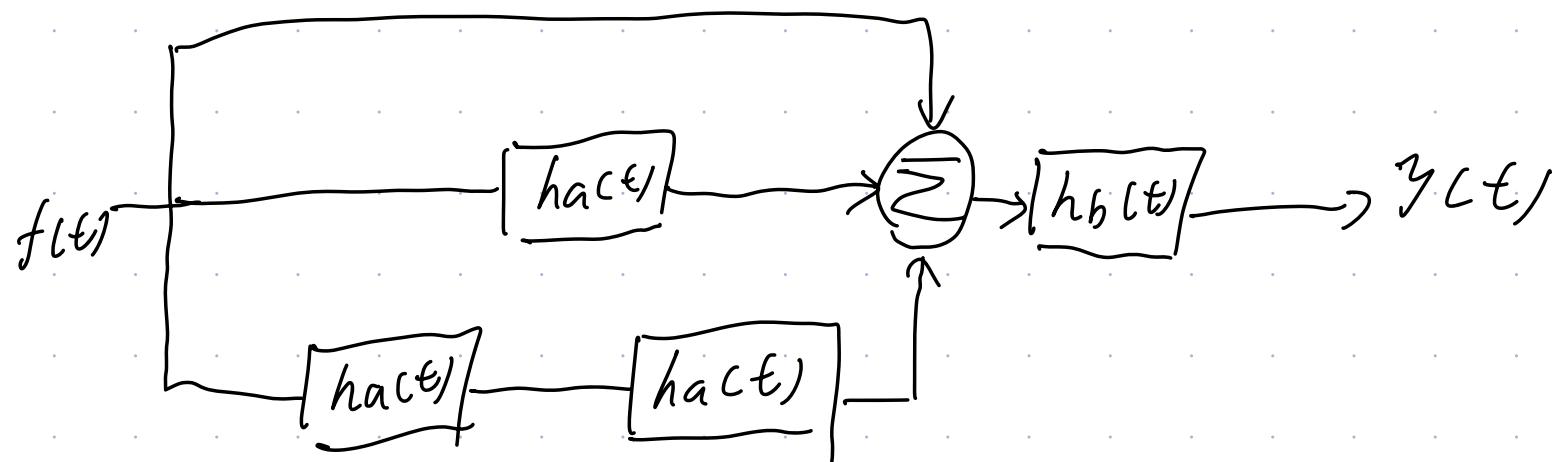
$$= e^{-2(t-1)} u(t-1)$$

$$\text{e.g. } \int_{-\infty}^{+\infty} e^{-2t} u(t-t) u(t) dt = e^{-2t} u(t) * u(-t)$$

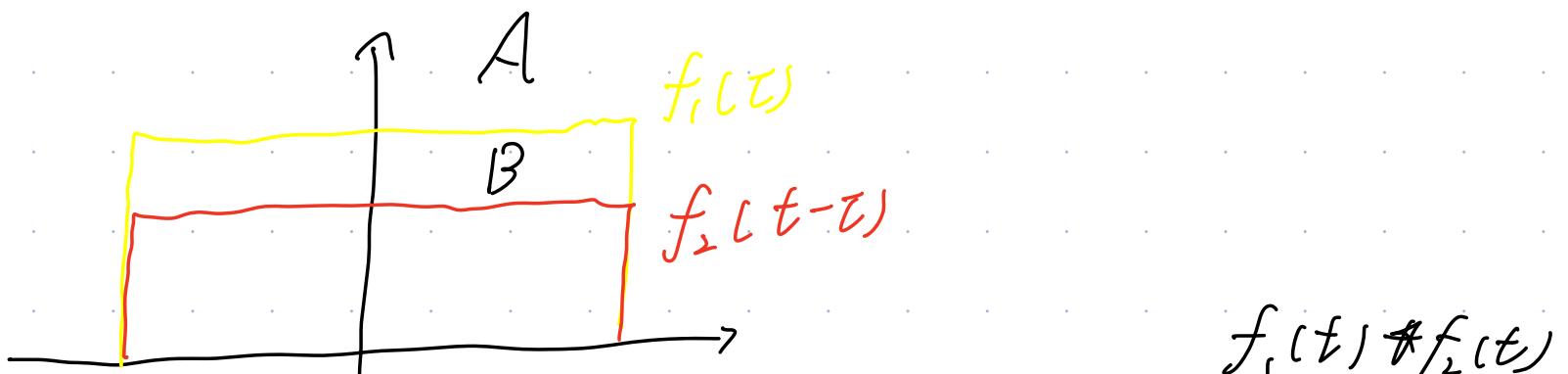
2. 应用 ($y_{zs} = c(t) * h(t)$)

系统级联 = 相卷积

系统并联 = 相加



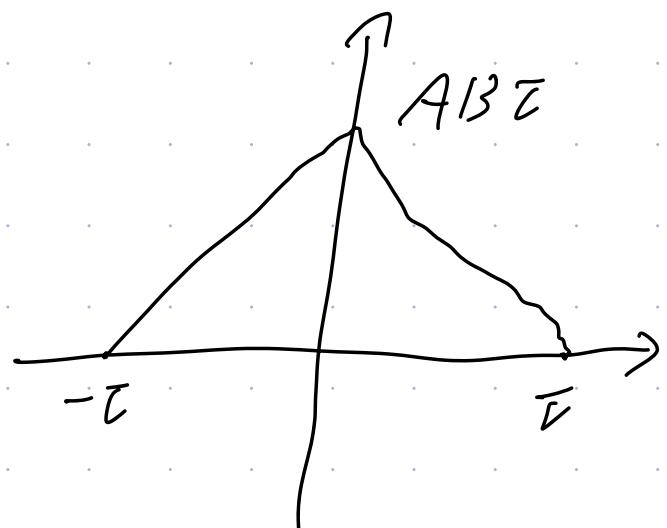
$$y(t) = [\delta(t) + h_a(t) + h_a(t) * h_a(t)] + h_b(t)$$



$$f_1(t) * f_2(t) = ABT$$

$$t = -T \quad f_1(t) * f_2(t) = 0$$

$$t = T \quad f_1(t) * f_2(t) = 0$$



$$\therefore G_1(t) = T \sin\left(\frac{\omega t}{2}\right)$$

$$\therefore f_1(t) =$$

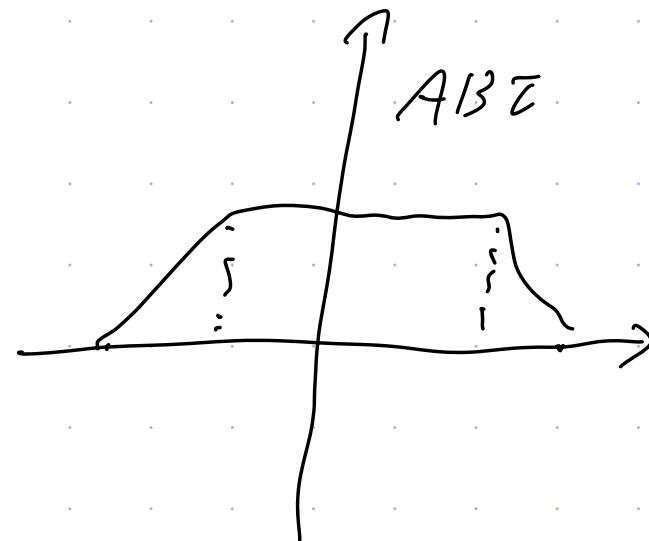
$$f_2(t) =$$



$$f_1(t) * f_2(t) = A B T$$

$$t = -T \quad f_1(t) * f_2(t) = 0$$

$$t = T \quad f_1(t) * f_2(t) = 0$$



$$\therefore G_T(t) = T \sin\left(\frac{\omega T}{2}\right)$$

$$f_1 f_2 =$$

.. $f(\omega)$

$$f_2(t) =$$

$$f_1(t) = 3e^{-2t} u(t), f_2(t) = 2u(t), f_3(t) = 2u(t-2)$$

$$1. f_1(t) * f_2(t) = \int_{-\infty}^{+\infty} 3e^{-2\tau} u(\tau) \cdot 2u(t-\tau) d\tau$$

$\because u(\tau), \tau > 0, u(t-\tau), t-\tau > 0, \tau < t$

$$\therefore = \int_0^t 3e^{-2\tau} \cdot 2 d\tau = -\frac{1}{2} \int_0^t 3e^{-2\tau} \cdot 2 d(-2\tau)$$

$$= -3e^{-2t} \Big|_0^t = -3(e^{-2t} - 1) u(t)$$

$$= 3(1 - e^{-2t}) u(t)$$

$u(t)$ 为黑大之 z , 上限 $>$ 下限

$$f_1(t) * f_3(t) = \int_{-\infty}^{+\infty} 3e^{-2t} u(\tau) \cdot 2u(t-\tau-2) d\tau$$

$$\because \tau > 0, t - \tau - 2 > 0$$

$$\therefore 0 < \tau < t - 2$$

$$\left[\int_0^{t-2} 3e^{-2\tau} d\tau \right] u(t-2)$$

$$= \left(-3e^{-2\tau} \Big|_0^{t-2} \right) u(t-2)$$

$$= -3(e^{-2(t-2)} - 1) u(t-2)$$

$$= 3(1 - e^{-2t+4}) u(t-2)$$

3. 已知 LT, $h(t) = u(t-1)$, ~~且~~ $f(t) = e^{-3t} u(t)$, 求 $y_{ZS}(t)$

$$y_{ZS}(t) = h(t) * f(t) = \int_{-\infty}^{t \wedge 1} e^{-3\tau} u(\tau) \cdot u(t-\tau-1) d\tau$$

$$\tau > 0, t - \tau - 1 > 0$$

$$0 < \tau < t - 1$$

$$\therefore = \int_0^{t-1} e^{-3\tau} d\tau \cdot u(t-1)$$

$$= \left(-\frac{1}{3} \cdot e^{-3\tau} \Big|_0^{t-1} \right) u(t-1)$$

$$= -\frac{1}{3} (e^{-3(t-1)} - 1) u(t-1)$$

$$= \frac{1}{3} (1 - e^{-3t+3}) u(t-1)$$

4. $f_1(t) = t u(t)$, $f_2(t) = u(t) - u(t-2)$, $\int f_1(t) f_2(t) dt$

$$f_1(t) * f_2(t) = \int_{-\infty}^{+\infty} t u(\tau) [u(t-\tau) - u(t-\tau-2)] d\tau$$

$$= \int_{-\infty}^{+\infty} t u(\tau) u(t-\tau) d\tau - \int_{-\infty}^{+\infty} t u(\tau) u(t-\tau-2) d\tau$$

$$= \left(\int_0^t \tau d\tau \right) u(t) - \left(\int_0^{t-2} \tau d\tau \right) u(t-2)$$

$$= \frac{t^2}{2} \Big|_0^t u(t) - \frac{\tau^2}{2} \Big|_0^{t-2} u(t-2)$$

$$= \frac{1}{2} t^2 u(t) - \frac{1}{2} (t-2)^2 u(t-2)$$