

signal

1.2 判断下列信号是否为功率信号或能量信号。

(1) $x(t) = 5\sin(2t)$;

(2) $x(t) = 10t, t \geq 0$;

(3) $x[n] = (-0.5)^n, n \geq 0$;

(4) $x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2-t, & 1 \leq t \leq 2 \\ 0, & \text{其他} \end{cases}$

(5) $x(t) = \begin{cases} 5\cos(t), & -1 \leq t \leq 1 \\ 0, & \text{其他} \end{cases}$;

(6) $x[n] = \begin{cases} n, & 0 \leq n < 4 \\ 8-n, & 4 \leq n \leq 8 \\ 0, & \text{其他} \end{cases}$

1.2

$$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

1. $x(t) = 5 \sin(2t)$

$$E = \lim_{T \rightarrow \infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} |5 \sin(2t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} 25 \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{1 - \cos 4t}{2} dt$$

$$= \lim_{T \rightarrow \infty} \frac{25}{2} \left[t + \frac{1}{4} \cos 4t \right]_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$= \lim_{T \rightarrow \infty} \frac{25}{2} \left[\frac{T}{2} + \frac{1}{4} \cos(2T) + \frac{T}{2} - \frac{1}{4} \cos(2T) \right]$$

$$= \lim_{T \rightarrow \infty} \frac{25}{2} T = \infty$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \frac{25}{2} T = \frac{25}{2} \text{ W}$$

功率有限, 能量无穷, 为功率信号

2. $x(t) = 10t, t \geq 0$

$$E = \int_{-\infty}^{+\infty} 100t^2 dt$$

$$= 100 \left. \frac{t^3}{3} \right|_{-\infty}^{+\infty}$$

$$= \infty$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} 100 \frac{T^3}{3T} = \lim_{T \rightarrow \infty} \frac{100}{3} T^2$$

$$= \infty$$

\therefore 为非功率信号

4.

$$E = \int_{-\infty}^{+\infty} |f(t)|^2 dt$$

$$= \int_0^1 t dt + \int_1^2 2-t dt$$

$$= \left. \frac{t^2}{2} \right|_0^1 + \left. 2t - \frac{t^2}{2} \right|_1^2$$

$$= \frac{1}{2} + 4 - 2 - \left(2 - \frac{1}{2} \right)$$

$$= 1$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} = 0 \text{ W}$$

\therefore 能量有限, 功率为 0

\therefore 为能量信号

$$5. E = \int_{-\infty}^{+\infty} |f(t)|^2 dt$$

$$= \int_{-1}^1 25 \cos^2 t dt$$

$$= 25 \int_{-1}^1 \frac{1 + \cos 2t}{2} dt$$

$$= \frac{25}{2} \left[t + \frac{1}{2} \sin 2t \right]_{-1}^1$$

$$= 25 + 25 \sin 2$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{25 + 25 \sin 2}{T}$$

$$= 0$$

∴ 能量有限, 功率为 0
为能量信号

6.

$$E = \sum_{n=-\infty}^{+\infty} |f[n]|^2$$

$$= \sum_{n=0}^4 n^2 + \sum_{n=4}^8 (8-n)^2$$

$$= \frac{4 \times 5 \times 9}{2} \times 2$$

$$= 180$$

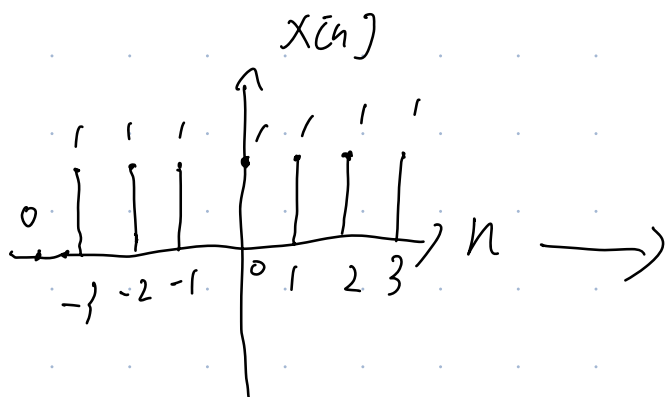
$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{k=-\infty}^{\infty} (f[k])^2$$

$$= \lim_{N \rightarrow \infty} \frac{180}{2N+1} = 0$$

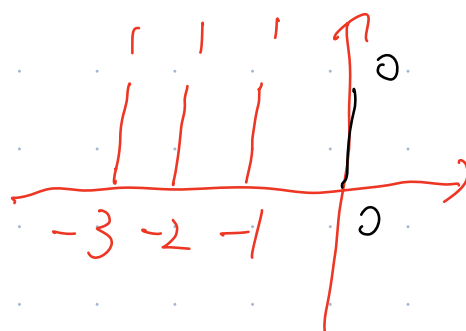
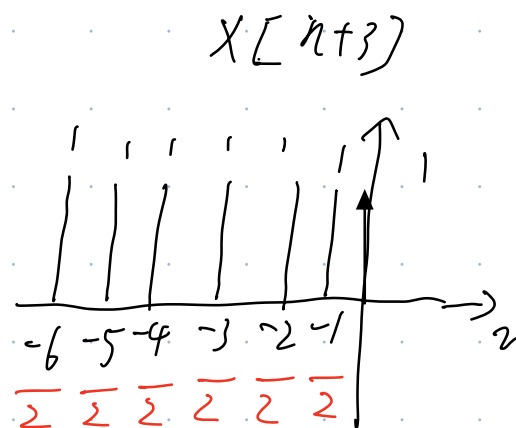
\therefore 能量有限, 功率为 0

\therefore 为能量信号

1.3 考虑离散时间信号 $x[n] = \begin{cases} 1, & |n| \leq 3 \\ 0, & |n| > 3 \end{cases}$, 求 $y[n] = x[2n+3]$ 。



$x[2n+3]$



1.16* 下列命题哪些正确?

- (1) 两个周期信号之和一定是周期信号;
- (2) 所有非周期信号都是能量信号;
- (3) 两个线性时不变系统级联构成的系统仍是线性时不变的;
- (4) 两个非线性系统级联构成的系统是非线性的。

1. X $y_1 = \sin 2t$ $y_2 = \sin \pi t$
 $T_1 = \pi$ $T_2 = 2$

$$T_{1+2} = \frac{\pi}{2} \text{ 为无理数}$$

2.

1.18* 求下列积分。

(1) $\int_0^{\infty} e^{j\omega t} \delta(t+1) dt$;

(2) $\int_{-\infty}^{\infty} e^{j\omega t} \delta(t_0 - t) dt$;

(3) $\int_{-\infty}^3 (2t^2 + 3t) \delta(0.5t - 2) dt$;

(4) $\int_{-\infty}^{\infty} u(2t-2)u(4-2t) dt$;

(5) $\int_{-\infty}^t 2 \sin \tau \delta(\tau - \pi/3) d\tau$ 。

1.20* 画出信号 $x(t) = \text{sgn}(\cos(\pi t / 2))$ 的波形。

1.21* 如图 1.41, 已知 $x(5-2t)$, 画出 $x(t)$ 。

1.22* 如图 1.42, 已知 $x(t)$, 画出 $x(-0.5t-1)$ 和 $x(2t+2)*\delta(t-3)$ 的波形。

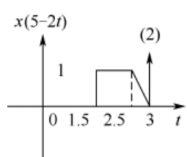


图 1.41 习题 1.21

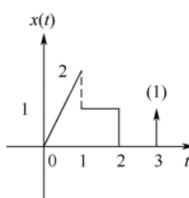


图 1.42 习题 1.22

2.1 计算下列连续时间 LTI 系统的转移算子及零输入响应。

(1) $y''(t) + 5y'(t) + 4y(t) = 2x'(t) + 5x(t)$, $y(0^-) = 1$, $y'(0^-) = 5$;

(2) $y''(t) + 4y'(t) + 4y(t) = 3x'(t) + 2x(t)$, $y(0^-) = -2$, $y'(0^-) = 3$;

(3) $y''(t) + 2y'(t) + 5y(t) = 4x'(t) + 3x(t)$, $y(0^-) = 1$, $y'(0^-) = 3$;

$$1. \lambda^2 + 5\lambda + 4 = 0$$

$$\lambda_1 = -1, \lambda_2 = -4$$

$$y_{zi} = C_1 e^{-t} + C_2 e^{-4t}$$

$$\therefore y_{zi}(0_+) = y_{zi}(0_-) = 1$$

$$y'_{zi}(0_+) = y'_{zi}(0_-) = 5$$

$$\therefore \begin{cases} C_1 + C_2 = 1 \\ -C_1 - 4C_2 = 5 \end{cases} \quad \begin{cases} C_1 = 3 \\ C_2 = -2 \end{cases}$$

$$\therefore y_{zi} = (3e^{-t} - 2e^{-4t})u(t)$$

$$2. \lambda^2 + 4\lambda + 4 = 0$$

$$\lambda_1 = \lambda_2 = -2$$

$$y_{zi} = (C_1 + C_2 t)e^{-2t}$$

$$y_{zi}(0_+) = y_{zi}(0_-) = -2$$

$$y'_{zi}(0_+) = y'_{zi}(0_-) = 3$$

$$\begin{cases} C_1 = -2 \\ C_2 - 2C_1 = 3 \end{cases} \quad \begin{cases} C_1 = -2 \\ C_2 = -1 \end{cases}$$

$$\therefore y_{zi} = [(-2 - t)e^{-2t}] u(t)$$

$$3. r^2 + 2r + 5 = 0$$

$$r_{1,2} = \frac{-2 \pm \sqrt{4 - 20}}{2} = -1 \pm 4i$$

$$\text{设 } y_{zi} = e^{-t} [C_1 \cos(4t) + C_2 \sin(4t)]$$

$$y_{zi}(0_+) = y_{zi}(0_-) = 1$$

$$y'_{zi}(0_+) = y'_{zi}(0_-) = 3$$

$$\begin{cases} C_1 = 1 \\ 4C_2 - C_1 = 3 \end{cases} \quad \begin{cases} C_1 = 1 \\ C_2 = 1 \end{cases}$$

$$\therefore y_{zi} = \left[e^{-t} [\cos(4t) + \sin(4t)] \right] u(t)$$

2.3 确定下列系统的单位冲激响应。

(1) $y''(t) + 4y'(t) + 4y(t) = 2x'(t) + 5x(t)$;

(2) $y''(t) + 5y'(t) + 6y(t) = 2x''(t) + 7x'(t) + 4x(t)$;

(3) $y''(t) + 3y'(t) + 2y(t) = x'(t) + x(t)$ 。