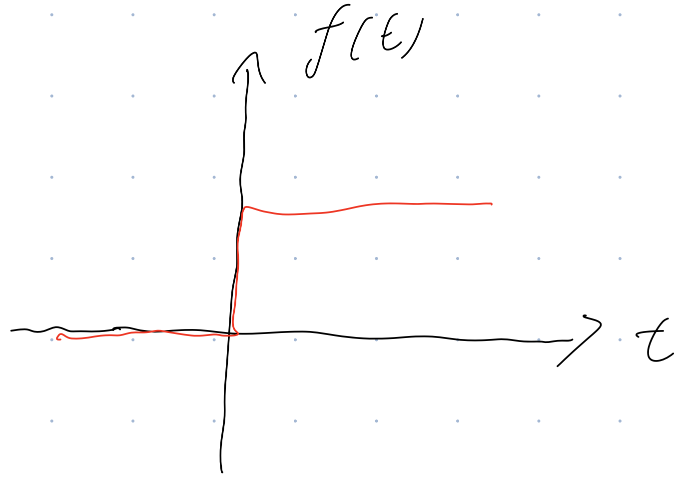
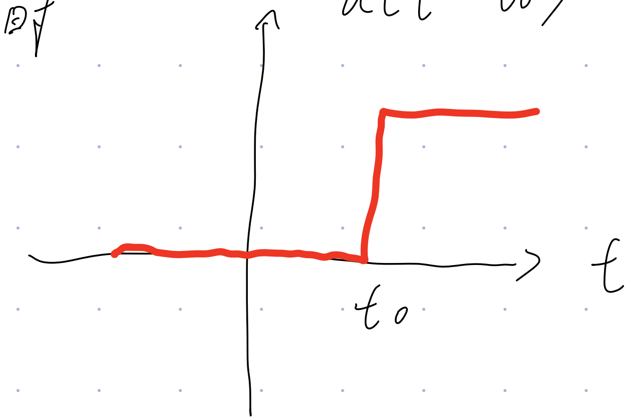
Signal

1、单位阶跃信号 $u(t)$

$$u(t) = \begin{cases} 0 & (t<0) \\ 1 & (t>0) \end{cases}$$



延时 $u(t-t_0)$



2、单位冲激信号 $\delta(t) = \frac{d}{dt} u(t)$

$$\begin{cases} \int_{-\infty}^{+\infty} \delta(t)\,dt = 1 & \text{强度(面积)} = 1 \\ \delta(t) = 0 \quad (t \neq 0) & \text{宽度无穷小} \end{cases}$$

property

1. 筛分性质 $x(t)\delta(t) = x(0)\delta(t)$

$$\int_{-\infty}^{+\infty} x(t)\delta(t)\,dt = x(0)$$

2. 尺度运算 $\delta(at) = \frac{1}{|a|}\delta(t)$

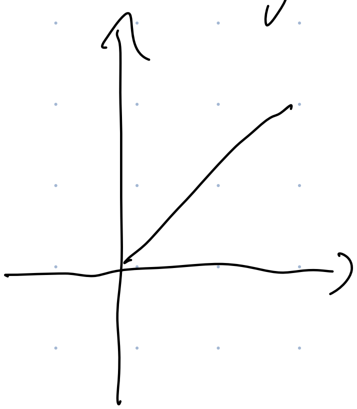
3. even func. $\delta(-t) = \delta(t)$

3、冲激偶信号 $\delta'(t) = \frac{d}{dt}\delta(t)$

$$\int_{-\infty}^{+\infty} \delta'(t) f(t) dt = -f'(0)$$ 筛选性质

$$\int_{-\infty}^{+\infty} \delta'(t) dt = 0$$ $S = 0$

$y = r(t)$ 斜变信号



$$r(t) \xrightarrow{\frac{d}{dt}} u(t) \xrightarrow{\frac{d}{dt}} \delta(t) \xrightarrow{\frac{d}{dt}} \delta'(t)$$

continuous

周期 $E=\lim\limits_{T\to\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}}|f(t)|^2dt$

非周期 $E=\int_{-\infty}^{+\infty}|f(t)|^2dt$

$P=\lim\limits_{T\to\infty}\frac{E}{T}=\lim\limits_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|f(t)|^2dt$

discrete

$E=\sum\limits_{n=-\infty}^{+\infty}|x[n]|^2$

$P=\lim\limits_{N\to\infty}\frac{1}{2N+1}\sum\limits_{k=-N}^{N}|x[k]|^2$

Fourier series

周期信号为$f(t)$，周期$=T_1$，$w_1=\frac{2\pi}{T_1}$

$$a_0=\frac{1}{T}\int_{-\infty}^{+\infty} f(t)\,dt$$ 面积/一个T

$$a_n=\frac{2}{T}\int_T \cos(n w_0 t) f(t)\,dt$$

$$b_n=\frac{2}{T}\int_T \sin(n w_0 t) f(t)\,dt$$

FT

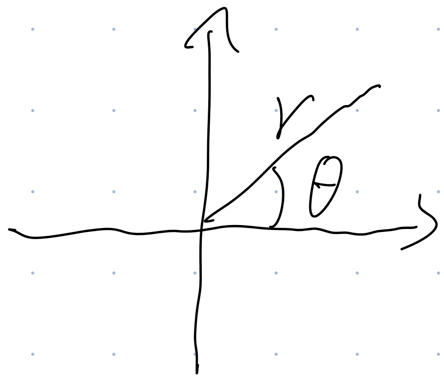
$$F(\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{j\omega t}dt$$

$f(t)\longleftrightarrow F(\omega)$

$F(\omega)$可写为$|F(\omega)|e^{j\varphi(\omega)}$

$|F(\omega)|-\omega$，$\varphi(\omega)-\omega$曲线称为非周期信号的幅度谱和相位谱



$1Hz = 1s$

$\omega = 2\pi f$

$\sin\theta = \sin\omega t$

冲激函数匹配法

e.g. $y''(t)+2y(t)+y(t)=f''(t)+2f(t)$

$y(0_-)=1,\ y'(0_-)=-1,\ f(t)=u(t)$，求 $y(0_+),\ y'(0_+)$

解：

$f''(t)+2f(t)=\delta'(t)+2u(t)$

积分　　积分直到出现 $u(t)$ 为止

1　$y''(t)=a\delta'(t)+b\delta(t)+cu(t)$

2　$y'(t)=a\delta(t)+bu(t)+\cancel{ctu(t)}$　　$y'(0_+)=y'(0_-)+b$

1　$y(t)=au(t)$　　$y(0_+)=y(0_-)+a$

($a=1$

$$\begin{cases} b+2a=0 \\ c+2b+a=2 \end{cases}$$

1、 $r''(t)+3r'(t)+2r(t)=2e'(t)+6e(t)$

已知 $r(0_-)=2$, $r'(0_-)=0$, $e(t)=u(t)$

解：

$r^2+3r+2=0$

$r_1=-1$, $r_2=-2$

$r_h(t)=C_1e^{-t}+C_2e^{-2t}$

$\therefore y(t)=C_1e^{-t}+C_2e^{-2t}+3$

$$\begin{array}{l} 1 \\ 3 \\ 2 \end{array}\begin{cases} r''(t)=a\delta(t)+bu(t) \\ r'(t)=au(t) & r'(0_+)=r'(0_-)+a \\ r(t)=\underbrace{atu(t)}_{\text{无跳变}} & r(0_+)=r(0_-) \end{cases}$$

$\therefore a=2$

$2e'(t)+6e(t)=2\delta(t)+6u(t)$

δ(t) 无特解

∴设 $r_p(t)=B$

$0+0+2B=6$

∴ $B=3$

∵ $r'(0_+)=r'(0_-)+2$

$r(0_+)=r(0_-)=0$

$$\begin{cases} -C_1-2C_2=0+2 \\ C_1+C_2+3=2 \end{cases} \qquad \begin{cases} C_1=0 \\ C_2=-1 \end{cases}$$

∴ $r(t)=(-e^{-2t}+3)u(t)$

最后 $+u(t)$

2. $y''(t)+4y'(t)+3y(t)=f(t)$,

$y(0_-)=y'(0_-)=1$, $f(t)=u(t)$

解: $r^2+4r+3=0$

解：

$r_1=-1, r_2=-3$

$y_h(t)=C_1e^{-t}+C_2e^{-3t}$

$f(t)=u(t)$，设 $y_p(t)=B$

$0+0+3B=1$

$\therefore B=\frac{1}{3}$

$y=C_1e^{-t}+C_2e^{-3t}+\frac{1}{3}$

$f(t)=u(t)$ 无跳变

则 $y(0_+)=y(0_-)=1$

$y'(0_+)=y'(0_-)=1$

$y(0_+) = y(0_-) = 1$

代入 y

零输入/零状态响应

y_{zs}: zero status, $y_{zs}^{(k)}(0_-) = 0$

y_{zi}: zero input, $y_{zi}^{(k)}(0_-) = y_{zi}^{(k)}(0_+)$

1、 $r''(t)+3r'(t)+2r(t)=2e'(t)+6e(t)$

已知 $r(0_-)=2,\ r'(0_-)=0,\ e(t)=u(t)$

解：

$y=y_{zi}+y_{zs}$

求 y_{zi} ⇒ 齐次解

$r^2+3r+2=0$

$r_1=-1,\ r_2=-2$

$r_{zi}(t)=C_1e^{-t}+C_2e^{-2t}$

$r_{zi}(0_+)=r_{zi}(0_-)=2$

$r'_{zi}(0_+)=r'_{zi}(0_-)=0$

$$\begin{cases} C_1 + C_2 = 2 \\ -C_1 - 2C_2 = 0 \end{cases} \qquad \begin{cases} C_1 = 4 \\ C_2 = -2 \end{cases}$$

$$\therefore V_{zi}(t) = (4e^{-t} - 2e^{-2t})u(t)$$

2、

$r''(t)+3r'(t)+2r(t)=2e'(t)+6e(t)$

已知 $r(0_-)=2,\ r'(0_-)=0,\ e(t)=u(t)$

求 $y_{zs} \Rightarrow$ 齐 + 特

$r^2+3r+2=0$

$r_1=-1,\ r_2=-2$

$r_{zi}(t)=C_1e^{-t}+C_2e^{-2t}$

$\because 2e'(t)+6e(t)=2\delta(t)+6u(t)$

$\therefore$ 设 $y_{zsp}=B$

$0+0+2B=6$

$B=3$

$r_{zs}(t)=3+C_1e^{-t}+C_2e^{-2t}$

$$\begin{cases} 1 \quad r_{zs}'' = a\delta(t) + bu(t) \\ 3 \quad r_{zs}' = au(t) \quad r_{zs}'(0_+) = r_{zs}'(0_-) + a \\ 2 \quad r_{zs} = atu(t) \quad r_{zs}(0_+) = r_{zs}(0_-) \end{cases}$$

0

$\therefore a = 2$

$$\begin{cases} -C_1 - 2C_2 = 0 + 2 \\ 3 + C_1 + C_2 = 0 \end{cases}$$

$r_{zs}(0_-) = r_{zs}(0_+) =$

$$\begin{cases} C_1 = -4 \\ C_2 = 1 \end{cases}$$

$-t \quad -2t$

$$v_{zs}(t) = (3 - 4e^{-t} + e^{-2t})u(t)$$

$$y = v_{zs} + v_{zi} = (4e^{-t} - 2e^{-2t})u(t) + (3 - 4e^{-t} + e^{-2t})u(t)$$

$$= (3 - e^{-2t})u(t)$$

$$v(t) = v_h(t) + v_p(t)$$

$$= v_{zi}(t) + v_{zsh}(t) + v_{zsp}(t)$$

$$= y_{zi}(t) + y_{zs}(t)$$

单位冲激响应 $h(t)$

激励为$\delta(t)$输出的y_{zs}

1、$e(t)=\delta(t)$

2、$h^{(k)}(0_-)=0$

e.g. $r''(t)+5r'(t)+6r(t)=3e'(t)+2e(t)$，求$h(t)$

解：$r''(t)+5r'(t)+6r(t)=3\delta'(t)+2\delta(t)$

$r^2+5r+6=0$

$r_1=-2, r_2=-3$

$\therefore y_{zsh}=C_1e^{-2t}+C_2e^{-3t}$

$\delta(t)$无y_{zsp}

$\therefore y_{zs}=y_{zsh}$ // $h(t)(0_-)=0$

$$
\begin{cases}
1 & r''(t) = a\delta'(t) + b\delta(t) + c\,u(t) \\
5 & r'(t) = a\delta(t) + b\,u(t) \qquad r'(0_+) = r'(0_-) + b \\
6 & r(t) = a\,u(t) \qquad r(0_+) = r(0_-) + a
\end{cases}
$$

$$\therefore \begin{cases} a = 3 \\ b + 5a = 2 \end{cases} \qquad \begin{cases} a = 3 \\ b = -13 \end{cases}$$

$$\therefore r'(0_+) = -13$$

$$r(0_+) = 3$$

$$\begin{cases} C_1 + C_2 = 3 \\ -2C_1 e^{-2t} - 3C_2 e^{-3t} \Big|_{t=0} = -13 \end{cases}$$

$$-2C_1 - 3C_2 = -13$$

$$-C_2 = -7$$

$$\therefore h(t) = (-4e^{-2t} + 7e^{-3t})u(t)$$

$$\begin{cases} c_1 = -4 \\ C_2 = 7 \end{cases}$$

e.g. $r'(t) + 2r(t) = e'(t) + 3e(t)$, 求 $h(t)$

$r'(t) + 2r(t) = \delta'(t) + 3\delta(t)$

$r + 2 = 0$

$r = -2$

$y_{zsh} = C_1 e^{-2t}$

$\delta(t)$无 [illegible]

$y_{zs} = C_1 e^{-2t}$

$$\begin{cases} 1 \quad r'(t) = a\delta'(t) + b\delta(t) + c\,u(t) \\ 2 \quad r(t) = a\delta(t) + b\,u(t) \end{cases}$$

$$\therefore \begin{cases} a = 1 \\ b + 2a = 3 \end{cases} \quad \begin{cases} a = 1 \\ b = 1 \end{cases}$$

$\therefore r(0_+) = r(0_-) + b = 0 + 1 = 1$

$\therefore C_1 = 1 \quad \therefore y_{zs} = e^{-2t}u(t) + \delta(t)$

e.g. $r'(t)+2r(t)=e''(t)+3e'(t)+3e(t)$，求 $h(t)$

$e(t)=\delta(t)$ $\quad y_{zs}^{(k)}(0_-)=0$

$r+2=0$

$r=-2$

$\delta''(t)+3\delta'(t)+3\delta(t)$

$y_{zsh}=Ce^{-2t}$

$y_{zs}=y_{zsh}=Ce^{-2t}$

$$\begin{cases} r'(t)=a\delta''(t)+b\delta'(t)+c\delta(t)+d\,u(t) & (1) \\ r(t)=a\delta'(t)+b\delta(t)+c\,u(t) & (2) \end{cases}$$

未配平

$$\begin{cases} a=1 \\ b+2a=3 \end{cases}$$

$\lfloor\ c+2b=3$ 因为左1阶 < 右2阶

$$r^{(t)}(0_+) = r^{(t)}(0_-) + C = 0 + 1 = 1$$

$$y_{zs} = \left(e^{-2t} + \delta'(t) + \delta(t)\right)u(t)$$

阶跃响应 $g(t)$

1、$e(t) = u(t)$

2、y_{zs}

e.g. $r''(t)+5r'(t)+6r(t)=3e'(t)+2e(t)$

$$r^2+5r+6=0$$

$$r_1=-2,\ r_2=-3$$

$$y_{zsh}=C_1e^{-2t}+C_2e^{-3t}$$

$$r''(t)+5r'(t)+6r(t)=3\delta(t)+2u(t)$$

$$y_{zsp}=B$$

$$\therefore\ 6B=2$$

$$B=\frac{1}{3}$$

$$\therefore\ y_{zs}=y_{zsh}+y_{zsp}=C_1e^{-2t}+C_2e^{-3t}+\frac{1}{3}$$

$$\begin{array}{l} 1 \\ 5 \end{array}\begin{cases} r''(t)=a\delta(t)+bu(t) \\ r'(t)=au(t) \end{cases}$$

$$r'(t) = a\,u(t)$$

6 $r(t) = a\,t\,u(t)$

$$\begin{cases} a = 3 \\ b + 5a = 2 \end{cases} \quad \begin{cases} a = 3 \\ b = -13 \end{cases}$$

$$r'(t)_{(0+)} = r'(t)_{(0-)} + a = 0 + 3 = 3$$

$$r(t)_{(0+)} = r(t)_{(0-)} = 0$$

$$\begin{cases} C_1 + C_2 + \frac{1}{3} = 0 \\ -2C_1 - 3C_2 = 3 \end{cases} \quad \begin{cases} C_1 = 2 \\ C_2 = -\frac{7}{3} \end{cases}$$

$$y_{zs} = \left(2e^{-2t} - \frac{7}{3}e^{-3t} + \frac{1}{3}\right)u(t)$$

e.g. $r'(t) + 2r(t) = e''(t) + 3e'(t) + 3e(t)$

左一阶 < 右二阶 ∴ 匹配不完全

$r + 2 = 0$

$r = -2$

$\therefore y_{zsh} = Ce^{-2t}$

$\because e(t) = u(t)$

$\therefore r'(t) + 2r(t) = \delta'(t) + 3\delta(t) + 3u(t)$

设 $y_{zsp} = B$

$\therefore 2B = 3$

$B = \frac{3}{2}$

$y_{zs} = y_{zsh} + y_{zsp} = Ce^{-2t} + \frac{3}{2}$

$$
\begin{cases}
1 \quad r(t) = a\,\delta(t) + b\,\delta(t) + c\,u(t) \\
2 \quad r(t) = a\,\delta(t) + b\,u(t)
\end{cases}
$$

$$
\begin{cases} a = 1 \\ b + 2a = 3 \end{cases} \qquad \begin{cases} a = 1 \\ b = 1 \end{cases}
$$

$$
r(t)_{(0+)} = r(t)_{(0-)} + b = 0 + 1 = 1
$$

$$
\therefore \ 1 = C + \frac{3}{2} \qquad C = -\frac{1}{2}
$$

$$
\therefore \ g(t) = \left(-\frac{1}{2}e^{-2t} + \frac{3}{2} + \delta(t)\right)u(t)
$$

卷积 (convolution)

1. definition: $f_1(t) * f_2(t) =$

$f(t) * \delta(t) = f(t)$

$f(t) * \delta(t-t_0) = f(t-t_0)$

$f(t) * \delta'(t) = f'(t)$ $\quad$ $f(t) * \delta'(t-t_0) = f'(t-t_0)$

$f(t) * u(t) = \int_{-\infty}^{t} f(\tau)d\tau$

e.g. $e^{-t}u(t) * \delta(3t+2)$

$= e^{-t}u(t) * \frac{1}{3}\delta(t+\frac{2}{3})$

$= \frac{1}{3}e^{-(t+\frac{2}{3})}u(t+\frac{2}{3})$

e.g. $e^{-2t}u(t) * \delta''(t) * u(t)$

$= -2e^{-2t}u(t) + e^{-2t}\delta(t)$

$= -2e^{-2t}u(t) + \delta(t)$

e.g. $e^{-2t}u(t) * \delta''(t-1) * tu(t)$

$$= e^{-2t}u(t) * \delta(t-1)$$

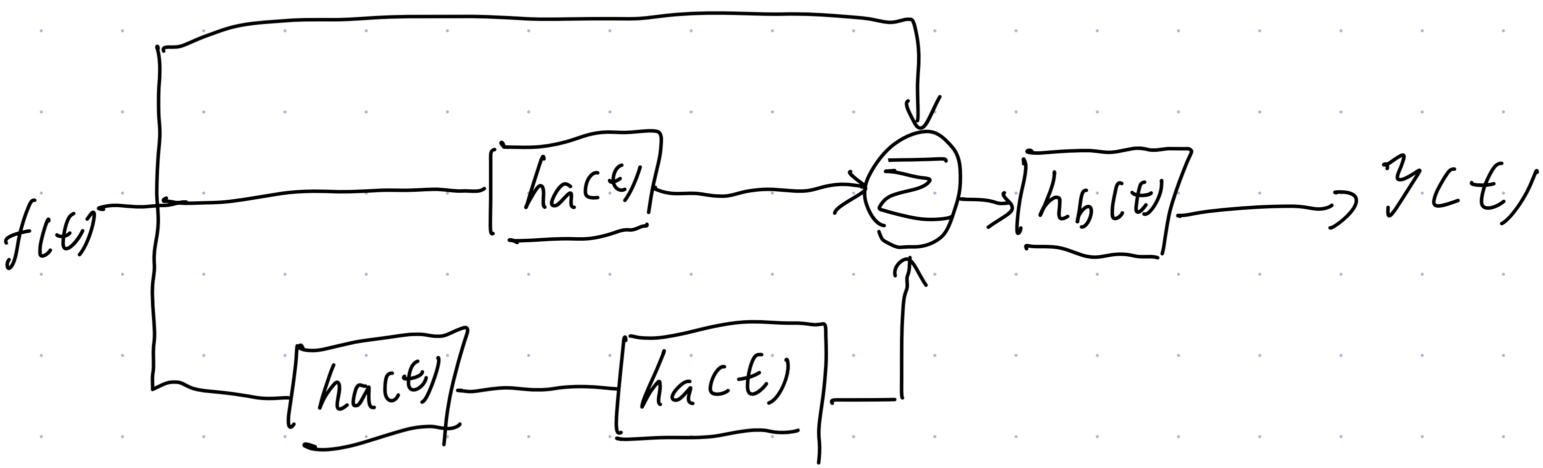
$$= e^{-2(t-1)}u(t-1)$$

e.g. $\int_{-\infty}^{+\infty} e^{-2\tau} u(\tau-t) u(\tau) d\tau = e^{-2t} u(t) * u(-t)$

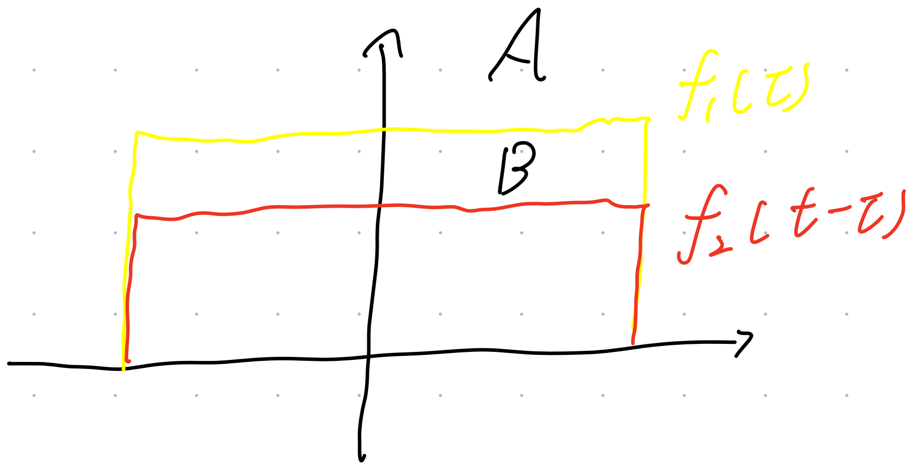
2. 应用 ($y_{zs} = e(t) * h(t)$)

系统级联 = 相卷积

系统并联 = 相加



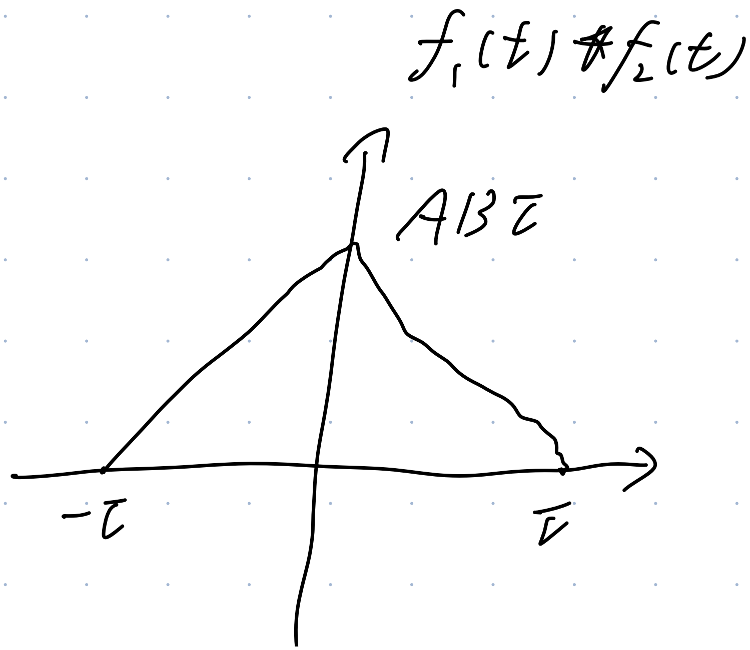
$y(t) = [\delta(t) + h_a(t) + h_a(t) * h_a(t)] + h_b(t)$



$f_1(t)*f_2(t) = AB\tau$

$t = -\tau \quad f_1(t)*f_2(t) = 0$

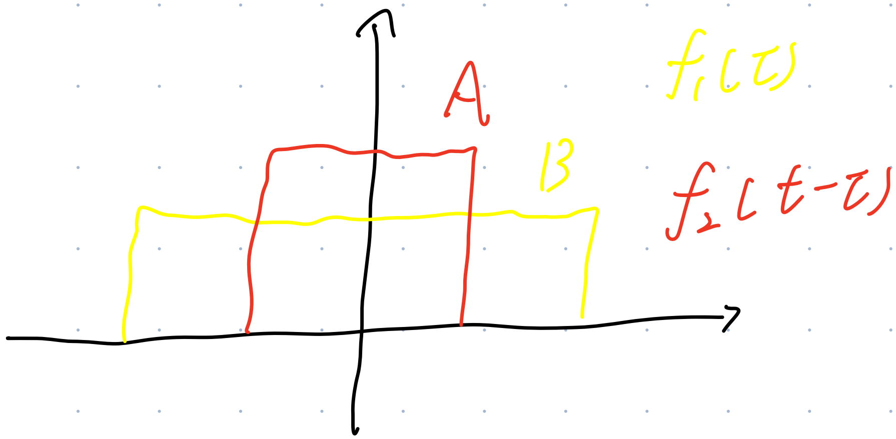
$t = \tau \quad f_1(t)*f_2(t) = 0$

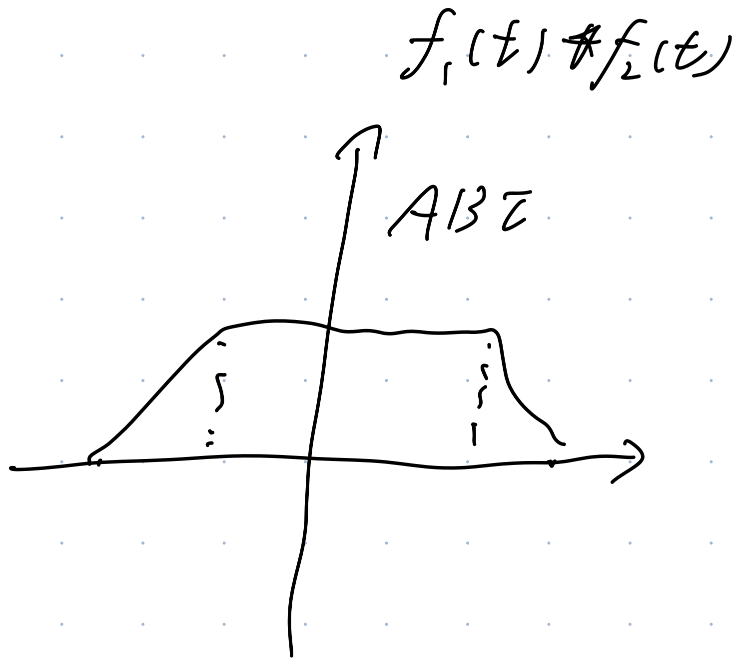


$\because G_\tau(t) = \tau Sa\left(\frac{\omega\tau}{2}\right)$

$\therefore f_1(t) =$

$f_2(t) =$





$f_1(t) * f_2(t) = AB\tau$

$t = -\tau \quad f_1(t) * f_2(t) = 0$

$t = \tau \quad f_1(t) * f_2(t) = 0$

$\because G_\tau(t) = \tau Sa(\frac{\omega\tau}{2})$

$\cdot \ f_1(f) =$

$\therefore f_1(t)$

$f_2(t) =$

$f_1(t)=3e^{-2t}u(t)$, $f_2(t)=2u(t)$, $f_3(t)=2u(t-2)$

1. $f_1(t)*f_2(t)=\int_{-\infty}^{+\infty}3e^{-2\tau}u(\tau)\cdot 2u(t-\tau)\,d\tau$

$\because u(\tau)$, $\tau>0$, $u(t-\tau)$, $t-\tau>0$, $\tau<t$

$\therefore =\int_0^t 3e^{-2\tau}\cdot 2\,d\tau=-\frac{1}{2}\int_0^t 3e^{-2\tau}\cdot 2\,d(-2\tau)$

$=-3e^{-2\tau}\Big|_0^t=-3(e^{-2t}-1)u(t)$

$=3(1-e^{-2t})u(t)$

u(t)的默认取，上限>下限

2、$f_1(t) * f_3(t) = \int_{-\infty}^{+\infty} 3e^{-2\tau} u(\tau) \cdot 2u(t-\tau-2)\, d\tau$

$\because \tau > 0,\ t-\tau-2 > 0$

$\therefore\ 0 < \tau < t-2$

$$\left[\int_0^{t-2} 3e^{-2\tau} \cdot 2\, d\tau\right] u(t-2)$$

$$= \left(-3e^{-2\tau}\Big|_0^{t-2}\right) u(t-2)$$

$$= -3\left(e^{-2(t-2)} - 1\right) u(t-2)$$

$$= 3\left(1 - e^{-2t+4}\right) u(t-2)$$

3. 已知LTI, $h(t)=u(t-1)$, 输入 $f(t)=e^{-3t}u(t)$, 求 $y_{zs}(t)$

$$y_{zs}(t)=h(t)*f(t)=\int_{-\infty}^{+\infty} e^{-3\tau}u(\tau)\cdot u(t-\tau-1)\,d\tau$$

$\tau>0,\ t-\tau-1>0$

$0<\tau<t-1$

$$\therefore =\int_0^{t-1} e^{-3\tau}d\tau \cdot u(t-1)$$

$$=\left(-\frac{1}{3}\cdot e^{-3\tau}\Big|_0^{t-1}\right)u(t-1)$$

$$=-\frac{1}{3}\left(e^{-3(t-1)}-1\right)u(t-1)$$

$$=\frac{1}{3}\left(1-e^{-3t+3}\right)u(t-1)$$

4. $f_1(t)=tu(t)$, $f_2(t)=u(t)-u(t-2)$，求 $f_1(t)*f_2(t)$

$$f_1(t)*f_2(t)=\int_{-\infty}^{+\infty}\tau u(\tau)[u(t-\tau)-u(t-\tau-2)]d\tau$$

$$=\int_{-\infty}^{+\infty}\tau u(\tau)u(t-\tau)\,d\tau-\int_{-\infty}^{+\infty}\tau u(\tau)u(t-\tau-2)d\tau$$

$$=\left(\int_0^t\tau\,d\tau\right)u(t)-\left(\int_0^{t-2}\tau\,d\tau\right)u(t-2)$$

$$=\frac{\tau^2}{2}\Big|_0^t\cdot u(t)-\frac{\tau^2}{2}\Big|_0^{t-2}\cdot u(t-2)$$

$$=\frac{1}{2}t^2u(t)-\frac{1}{2}(t-2)^2u(t-2)$$