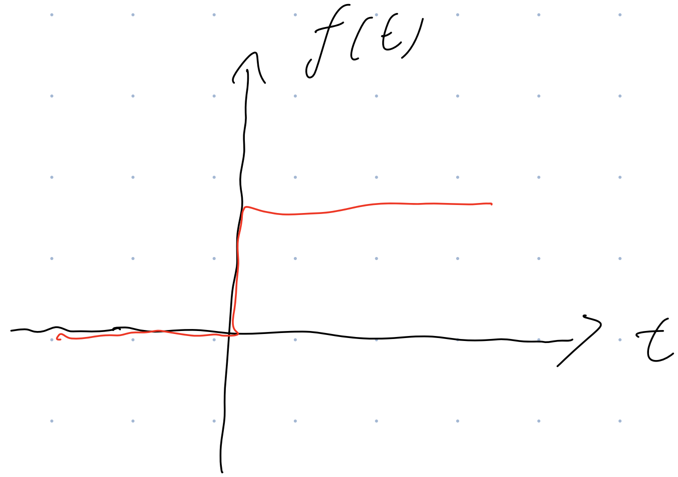
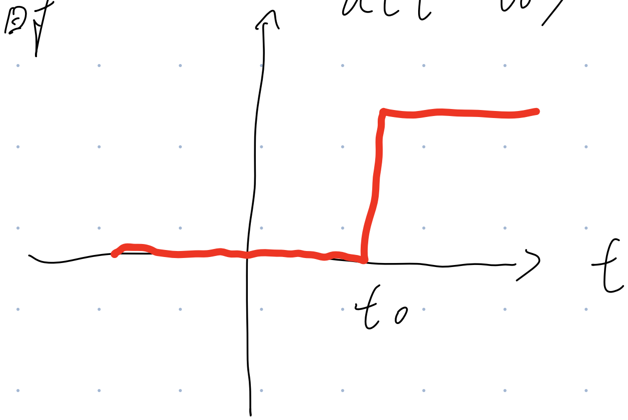
Signal

1、单位阶跃信号 $u(t)$

$$u(t) = \begin{cases} 0 & (t<0) \\ 1 & (t>0) \end{cases}$$



延时 $u(t-t_0)$



2、单位冲激信号 $\delta(t) = \frac{d}{dt} u(t)$

$\int_{-\infty}^{+\infty} \delta(t)dt = 1$ 强度(面积)=1

$\delta(t) = 0 \quad (t \neq 0)$ 宽度无穷小

property

1. 筛分性质 $x(t)\delta(t) = x(0)\delta(t)$

$\int_{-\infty}^{+\infty} x(t)\delta(t)dt = x(0)$

2. 尺度运算 $\delta(at) = \frac{1}{|a|}\delta(t)$

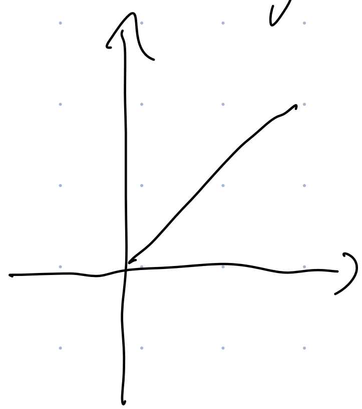
3. even func. $\delta(-t) = \delta(t)$

3、冲激偶信号 $\delta'(t) = \frac{d}{dt}\delta(t)$

$$\int_{-\infty}^{+\infty} \delta'(t) f(t) dt = -f'(0)$$ 筛选性质

$$\int_{-\infty}^{+\infty} \delta'(t) dt = 0$$ $S = 0$

$y = r(t)$ 斜变信号



$$r(t) \xrightarrow{\frac{d}{dt}} u(t) \xrightarrow{\frac{d}{dt}} \delta(t) \xrightarrow{\frac{d}{dt}} \delta'(t)$$

continuous

$$\begin{cases} \text{周期} \quad E=\lim\limits_{T\to\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}}|f(t)|^2dt \\ \text{非周期} \quad E=\int_{-\infty}^{+\infty}|f(t)|^2dt \\ P=\lim\limits_{T\to\infty}\frac{E}{T}=\lim\limits_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|f(t)|^2dt \end{cases}$$

discrete

$$\begin{cases} E=\sum\limits_{n=-\infty}^{+\infty}|x[n]|^2 \\ P=\lim\limits_{N\to\infty}\frac{1}{2N+1}\sum\limits_{k=-N}^{N}|x[k]|^2 \end{cases}$$

Fourier series

周期信号为$f(t)$，周期$=T_1$，$w_1=\frac{2\pi}{T_1}$

$$a_0=\frac{1}{T}\int_{-\infty}^{+\infty}f(t)\,dt$$ 面积/一个T

$$a_n=\frac{2}{T}\int_T \cos(nw_0t)f(t)\,dt$$

$$b_n=\frac{2}{T}\int_T \sin(nw_0t)f(t)\,dt$$

FT

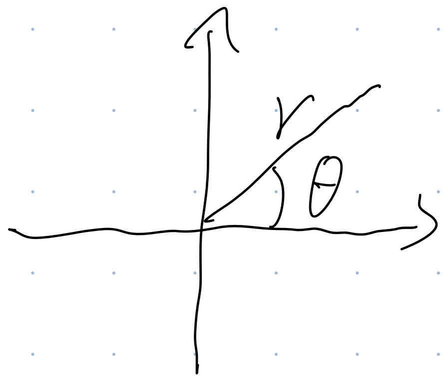
$$F(\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{j\omega t}dt$$

$f(t)\longleftrightarrow F(\omega)$

$F(\omega)$可写为$|F(\omega)|e^{j\varphi(\omega)}$

$|F(\omega)|-\omega$，$\varphi(\omega)-\omega$曲线称为非周期信号的幅度谱和相位谱



$1Hz = 1s$

$\omega = 2\pi f$

$\sin\theta = \sin\omega t$

冲激函数匹配法:

e.g. $y''(t)+2y'(t)+y(t)=f''(t)+2f(t)$

$y(0_-)=1,\ y'(0_-)=-1,\ f(t)=u(t)$, 求 $y(0_+),\ y'(0_+)$

解:

$f''(t)+2f(t)=\delta'(t)+2u(t)$

积分　　积分直到出现 $u(t)$ 为止

1 $\quad y''(t)=a\delta'(t)+b\delta(t)+cu(t)$

2 $\quad y'(t)=a\delta(t)+bu(t)\ \cancel{+ctu(t)}$　　$y'(0_+)=y'(0_-)+b$

1 $\quad y(t)=au(t)$　　$y(0_+)=y(0_-)+a$

$\big(\ a=1$

$$\begin{cases} b+2a=0 \\ c+2b+a=2 \end{cases}$$

$$r''(t)+3r'(t)+2r(t)=2e'(t)+6e(t)$$

已知 $r(0_-)=2,\ r'(0_-)=0,\ e(t)=u(t)$

解:

$r^2+3r+2=0$

$r_1=-1,\ r_2=-2$

$r_h(t)=C_1e^{-t}+C_2e^{-2t}$

$2e'(t)+6e(t)=2\delta(t)+6u(t)$

$\delta(t)$ 无特解

$\therefore$ 设 $r_p(t)=B$

$0+0+2B=6$

$\therefore B=3$

$\therefore r(t)=C_1e^{-t}+C_2e^{-2t}+3$

$$\begin{array}{l} 1 \\ 3 \\ 2 \end{array}\left\{\begin{array}{lll} r''(t) & = a\delta(t)+bu(t) & \\ r'(t) & = au(t) & r'(0_+)=r'(0_-)+a \\ r(t) & = atu(t) & r(0_+)=r(0_-) \end{array}\right.$$

无跳变

$\therefore a=2$

$\therefore r'(0_+)=r'(0_-)+2$

$r(0_+)=r(0_-)=0$

$$\begin{cases} -C_1-2C_2=0+2 \\ C_1+C_2+3=2 \end{cases} \quad \begin{cases} C_1=0 \\ C_2=-1 \end{cases}$$

$\therefore r(t)=(-e^{-2t}+3)u(t)$

最后$+u(t)$

$$r''(t)+3r'(t)+2r(t)=2e'(t)+6e(t)$$

已知 $r(0_-)=2$, $r'(0_-)=0$, $e(t)=u(t)$

解:

$$y=y_{zi}+y_{zs}$$

求 y_{zi} $\Rightarrow$ 齐次解

$$r^2+3r+2=0$$

$$r_1=-1,\ r_2=-2$$

$$r_{zi}(t)=C_1e^{-t}+C_2e^{-2t}$$

$$r_{zi}(0_+)=r_{zi}(0_-)=2$$

$$r_{zi}'(0_+)=r_{zi}'(0_-)=0$$

$$\begin{cases} c_1 + c_2 = 2 \\ -c_1 - 2c_2 = 0 \end{cases} \qquad \begin{cases} c_1 = 4 \\ c_2 = -2 \end{cases}$$

$$\therefore r_{zi}(t) = (4e^{-t} - 2e^{-2t})u(t)$$

$$r''(t) + 3r'(t) + 2r(t) = 2e'(t) + 6e(t)$$

已知 $r(0_-) = 2$, $r'(0_-) = 0$, $e(t) = u(t)$

求 $y_{zs} \Rightarrow$ 齐+特

$$r^2 + 3r + 2 = 0$$

$$r_1 = -1,\ r_2 = -2$$

$$r_{zi}(t) = c_1 e^{-t} + c_2 e^{-2t}$$

$$\therefore 2e'(t) + 6e(t) = 2\delta(t) + 6u(t)$$

$\therefore$ 代入 $y_{zsp} = 12$

$0 + 0 + 2B = 6$

$B = 3$

$r_{zs}(t) = 3 + C_1 e^{-t} + C_2 e^{-2t}$

$$\begin{cases} r_{zs}'' = a\delta(t) + bu(t) \\ r_{zs}' = au(t) \quad r_{zs}'(0_+) = r_{zs}'(0_-) + a \\ r_{zs} = atu(t) \quad r_{zs}(0_+) = r_{zs}(0_-) \end{cases}$$

1

3

2

$\therefore a = 2$

$$\begin{cases} -C_1 - 2C_2 = 0 + 2 \end{cases}$$

$$3 + C_1 + C_2 = 0 \qquad V_{zs}(0_-) = V_{zs}(0_+) = 0$$

$$\begin{cases} C_1 = -4 \\ C_2 = 1 \end{cases}$$

$$V_{zs}(t) = (3 - 4e^{-t} + e^{-2t})u(t)$$

$$y = V_{zs} + V_{zi} = (4e^{-t} - 2e^{-2t})u(t) + (3 - 4e^{-t} + e^{-2t})u(t)$$

$$= (3 - e^{-2t})u(t)$$

$$r(t) = r_h(t) + r_p(t)$$

$$= r_{zi}(t) + r_{zs,h}(t) + r_{zs,p}(t)$$

$$= r_{zi}(t) + r_{zs}(t)$$

in: $e(t)=\delta(t)$

out: $h(t)$ 且为 y_{zs} $\therefore h^{(k)}(0_-)=0$

e.g.

$y(0_+)$

$$y(0^-) = y_{zi}(0^-) = y_{zi}(0^+)$$

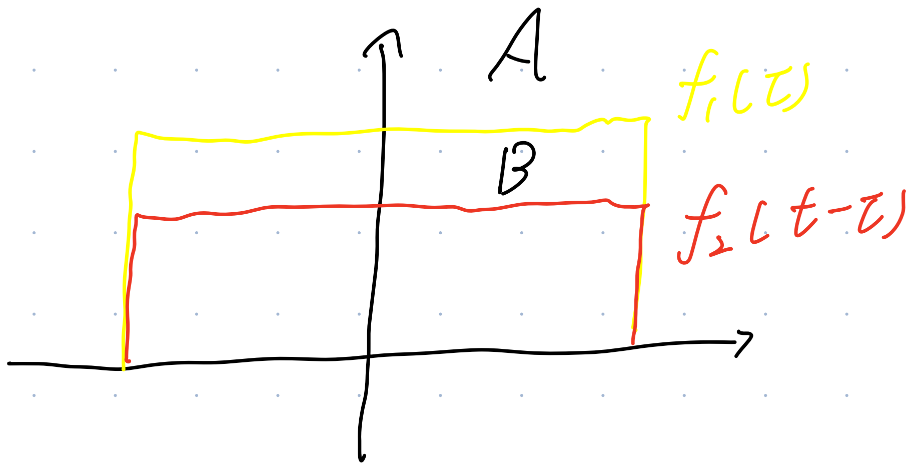
$$y(0^+)$$

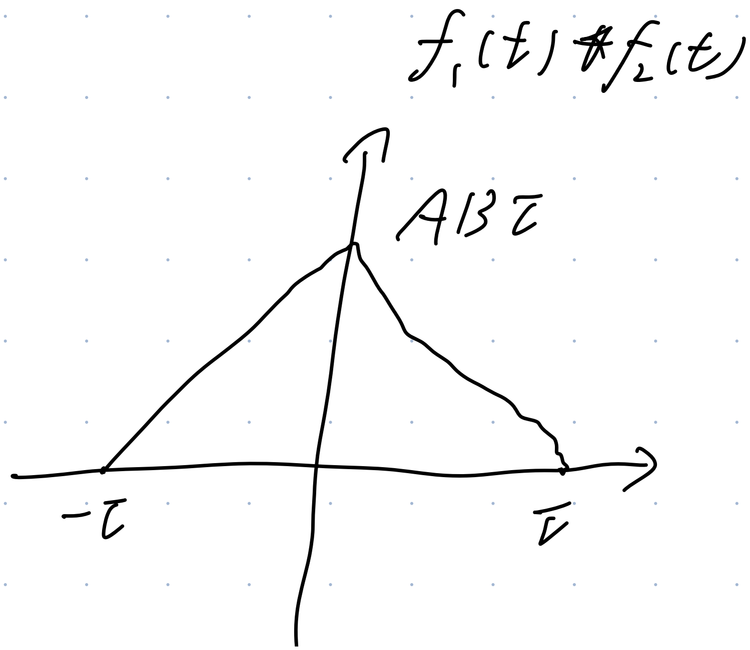
$$s^2 Y(s) + 5sY(s) + 6Y(s) = sX(s) + 4X(s)$$

$$\frac{Y(s)}{} = \frac{4+s}{}$$

$X(S)$ $\int \xi$

convolution =





$f_1(t)*f_2(t) = AB\tau$

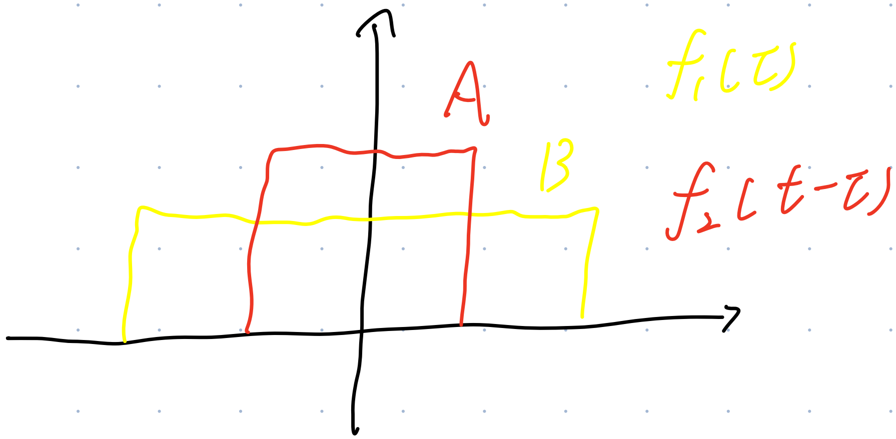
$t = -\tau \quad f_1(t)*f_2(t) = 0$

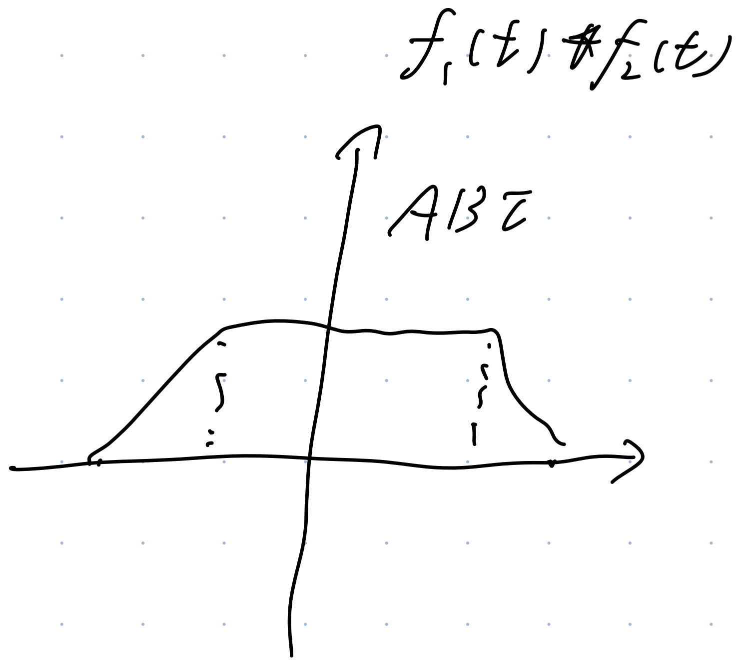
$t = \tau \quad f_1(t)*f_2(t) = 0$

$\because G_\tau(t) = \tau Sa\left(\frac{\omega\tau}{2}\right)$

$\therefore f_1(t) =$

$f_2(t) =$





$f_1(t) * f_2(t) = AB\tau$

$t = -\tau \quad f_1(t) * f_2(t) = 0$

$t = \tau \quad f_1(t) * f_2(t) = 0$

$\because G_\tau(t) = \tau Sa(\frac{\omega\tau}{2})$

· $f_1(f) =$

$\therefore f_1(t)$

$f_2(t) =$