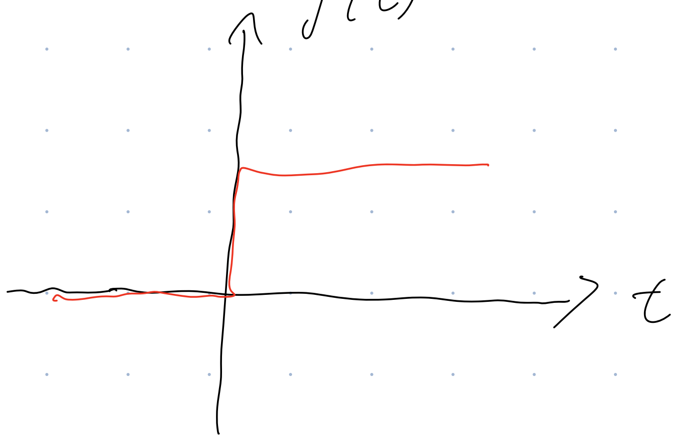
Signal

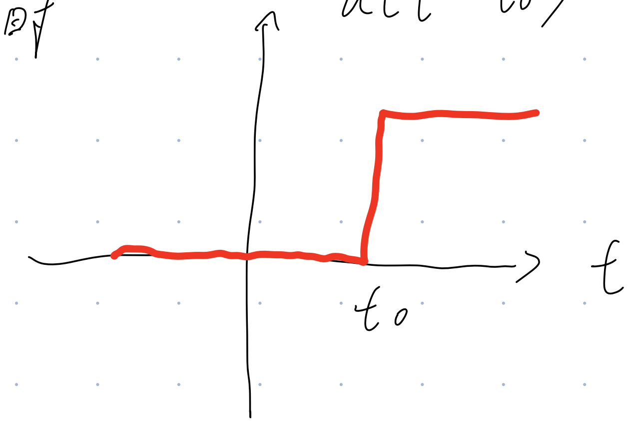
四种奇异信号

1、单位阶跃信号 $u(t)$

$$u(t)=\begin{cases}0 & (t<0)\\ 1 & (t>0)\end{cases}$$



延时 $u(t-t_0)$



2、单位冲激信号 $\delta(t) = \frac{d}{dt} u(t)$

$$\begin{cases} \int_{-\infty}^{+\infty} \delta(t) dt = 1 & \text{强度(面积)=1} \\ \delta(t) = 0 \quad (t \neq 0) & \text{宽度无穷小} \end{cases}$$

property

1. 筛分性质 $x(t)\delta(t) = x(0)\delta(t)$

$$\int_{-\infty}^{+\infty} x(t)\delta(t) dt = x(0)$$

2. 尺度运算 $\delta(at) = \frac{1}{|a|}\delta(t)$

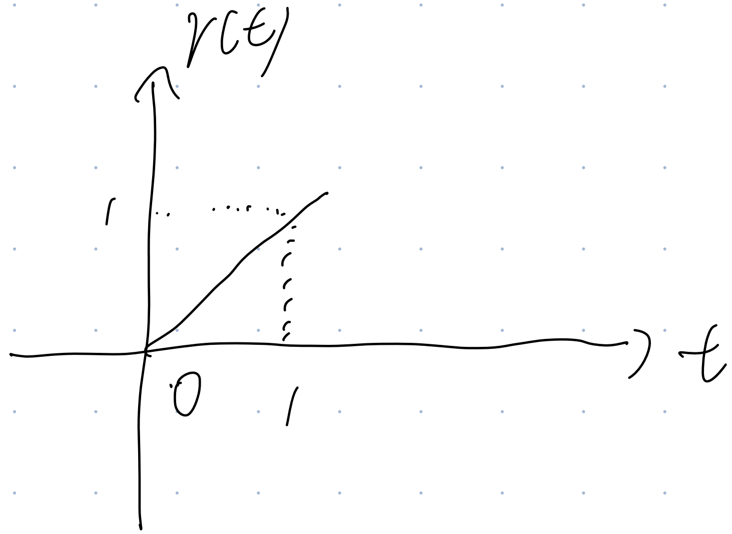
3. even func. $\delta(-t) = \delta(t)$

3、冲激偶信号 $\delta'(t) = \frac{d}{dt}\delta(t)$

$$\int_{-\infty}^{+\infty} \delta'(t) f(t) dt = -f'(0)$$ 筛选性质

$$\int_{-\infty}^{+\infty} \delta'(t) dt = 0$$ $S = 0$

4、单位斜变信号 $r(t)=tu(t)$



$r(t)=tu(t)$

$$2r(t)=\begin{cases}2t, & t\geq 0\\ 0, & t<0\end{cases}$$

$$r(t) \xrightarrow{\frac{d}{dt}} u(t) \xrightarrow{\frac{d}{dt}} \delta(t) \xrightarrow{\frac{d}{dt}} \delta'(t)$$

Energy Signal / Power signal

continuous

- 周期 $E = \lim_{T \to \infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt$
- 非周期 $E = \int_{-\infty}^{+\infty} |f(t)|^2 dt$
- $P = \lim_{T \to \infty} \frac{E}{T} = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt$

discrete

- $E = \sum_{n=-\infty}^{+\infty} |x[n]|^2$
- $P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{k=-N}^{N} |x[k]|^2$

周期信号 $\Rightarrow P$

时限信号 $\Rightarrow E$

单个信号是否为周期信号

- Continous → $\frac{2\pi}{T}$
  - π和 有理数 → 周期
  - 无理数 → 非周期
- Discrete → $\frac{2\pi}{w}$
  - 无理数 → 非周期
  - integer → $N = \frac{2\pi}{w}$
  - 有理分数 → $\frac{2\pi}{w} = \frac{P}{Q}$, $N = P$

多个信号是否为周期信号

$$LCM(T_1, T_2)$$

e.g. $f(t) = \sin \pi t + 2\cos 2t$

$$T_1 = \frac{2\pi}{\pi} = 2 \quad T_2 = \frac{2\pi}{2} = \pi$$

$$\frac{2}{\pi} = \text{无理数}$$

$\therefore$ 非周期

e.g. $f(t) = \cos 2t + \sin 3t$

$$T_1 = \frac{2\pi}{2} = \pi \quad T_2 = \frac{2\pi}{3} = \frac{2}{3}\pi$$

$$\frac{\pi}{\frac{2}{3}\pi} = \frac{3}{2}$$

$T = 2\pi$

$\therefore T = 2\pi$

e.g. $f(k) = 2\sin 4\pi k$

$$\frac{2\pi}{4\pi} = \frac{1}{2}$$

$$T = 1$$

e.g. $f(k) = \cos\frac{3}{4}\pi k + \sin\frac{2}{3}\pi k$

$$\frac{2\pi}{\frac{3}{4}\pi} = \frac{8}{3} \qquad T_2 = \frac{2\pi}{\frac{2}{3}\pi} = 3$$

$$T_1 = 8$$

$$\therefore T = 24$$

冲激函数匹配法

e.g. $y''(t)+2y(t)+y(t)=f''(t)+2f(t)$

$y(0_-)=1,\ y'(0_-)=-1,\ f(t)=u(t)$, 求 $y(0_+),\ y'(0_+)$

解:

$f''(t)+2f(t)=\delta'(t)+2u(t)$

积分　积分直到出现 $u(t)$ 为止

$$
\begin{array}{ll}
1 & y''(t)=a\delta'(t)+b\delta(t)+cu(t) \\
2 & y'(t)=a\delta(t)+bu(t)+\cancel{ctu(t)} \qquad y'(0_+)=y'(0_-)+b \\
1 & y(t)=au(t) \qquad y(0_+)=y(0_-)+a
\end{array}
$$

($a=1$

$$\begin{cases} b+2a=0 \\ c+2b+a=2 \end{cases}$$

1、 $r''(t)+3r'(t)+2r(t)=2e'(t)+6e(t)$

已知 $r(0_-)=2,\ r'(0_-)=0,\ e(t)=u(t)$

解:

$r^2+3r+2=0$

$r_1=-1,\ r_2=-2$

$r_h(t)=C_1e^{-t}+C_2e^{-2t}$

$\therefore y(t)=C_1e^{-t}+C_2e^{-2t}+3$

$$\begin{cases} r''(t)=a\delta(t)+bu(t) & 1 \\ r'(t)=au(t) \quad r'(0_+)=r'(0_-)+a & 3 \\ r(t)=atu(t) \quad r(0_+)=r(0_-) & 2 \end{cases}$$

无跳变 $\therefore a=2$

$\therefore e'(t)+6e(t)=2\delta(t)+6u(t)$

$\delta(t)$ 无特解

$\therefore$ 设 $r_p(t) = B$

$0 + 0 + 2B = 6$

$\therefore B = 3$

$\because r'(0_+) = r'(0_-) + 2$

$r(0_+) = r(0_-) = 0$

$$\begin{cases} -C_1 - 2C_2 = 0 + 2 \\ C_1 + C_2 + 3 = 2 \end{cases} \quad \begin{cases} C_1 = 0 \\ C_2 = -1 \end{cases}$$

$\therefore r(t) = (-e^{-2t} + 3)u(t)$

最后 $+ u(t)$

2. $y''(t) + 4y'(t) + 3y(t) = f(t)$,

$y(0_-) = y'(0_-) = 1$, $f(t) = u(t)$

解. $r^2 + 4r + 3 = 0$

解

$r_1=-1, r_2=-3$

$y_h(t)=C_1e^{-t}+C_2e^{-3t}$

$f(t)=u(t)$，设 $y_p(t)=B$

$0+0+3B=1$

$\therefore B=\frac{1}{3}$

$y=C_1e^{-t}+C_2e^{-3t}+\frac{1}{3}$

$f(t)=u(t)$ 无跳变

得 $y(0_+)=y(0_-)=1$

$y'(0_+)=y'(0_-)=1$

$y(0_+) = y(0_-) - 1$

相小 y

零输入/零状态响应

y_{zs}: zero status, $y_{zs}^{(k)}(0_-) = 0$

只与输入有关

不关心题目给的 $y(0_-)$，自己的 $y_{zs}(0_-) = 0$

y_{zi}: zero input, $y_{zi}^{(k)}(0_+) = y^{(k)}(0_-)$

只与初始状态有关

给 $y(0_+)$ 可求全响应

1、 $r''(t)+3r'(t)+2r(t)=2e'(t)+6e(t)$

已知 $r(0_-)=2,\ r'(0_-)=0,\ e(t)=u(t)$

解：

$y=y_{zi}+y_{zs}$

求 y_{zi} ⇒ 齐次解

$r^2+3r+2=0$

$r_1=-1,\ r_2=-2$

$r_{zi}(t)=C_1e^{-t}+C_2e^{-2t}$

$r_{zi}(0_+)=r_{zi}(0_-)=2$

$r'_{zi}(0_+)=r'_{zi}(0_-)=0$

$$\begin{cases} c_1 + c_2 = 2 \\ -c_1 - 2c_2 = 0 \end{cases} \qquad \begin{cases} c_1 = 4 \\ c_2 = -2 \end{cases}$$

$$\therefore v_{zi}(t) = (4e^{-t} - 2e^{-2t})u(t)$$

2.

$r''(t)+3r'(t)+2r(t)=2e'(t)+6e(t)$

已知 $r(0_-)=2$, $r'(0_-)=0$, $e(t)=u(t)$

求 y_{zs} ⇒ 齐 + 特

$r^2+3r+2=0$

$r_1=-1$, $r_2=-2$

$r_{zi}(t)=C_1e^{-t}+C_2e^{-2t}$

$\because 2e'(t)+6e(t)=2\delta(t)+6u(t)$

$\therefore$ 设 $y_{zsp}=B$

$0+0+2B=6$

$B=3$

$$r_{zs}(t) = 3 + C_1 e^{-t} + C_2 e^{-2t}$$

$$\begin{cases} r_{zs}'' = a\delta(t) + bu(t) & 1 \\ r_{zs}' = au(t) \quad r_{zs}'(0_+) = r_{zs}'(0_-) + a & 3 \\ r_{zs} = atu(t) \quad r_{zs}(0_+) = r_{zs}(0_-) & 2 \end{cases}$$

$$\therefore a = 2$$

$$\begin{cases} -C_1 - 2C_2 = 0 + 2 \\ 3 + C_1 + C_2 = 0 \end{cases}$$

$$\begin{cases} C_1 = -4 \\ C_2 = 1 \end{cases} \qquad r_{zs}(0_-) = r_{zs}(0_+) = 0$$

$$r_{zs}(t) = (3 - 4e^{-t} + e^{-2t})u(t)$$

$$r = r_{zs} + r_{zi} = (4e^{-t} - 2e^{-2t})u(t) +$$

$$(3-4e^{-t}+e^{-2t})u(t)$$

$$= (3-e^{-2t})u(t)$$

$$r(t) = r_h(t) + r_p(t)$$

$$= r_{zi}(t) + r_{zsh}(t) + r_{zsp}(t)$$

$$= r_{zi}(t) + r_{zs}(t)$$

e.g. 求系统的全响应,指出自由响应,强迫响应, $y_{zi}(t)$, $y_{zs}(t)$

$y''(t)+4y'(t)+4y(t)=f'(t)+3f(t)$, $y(0_-)=1$, $y'(0_-)=2$, $f(t)=e^{-t}u(t)$

解: $r^2+4r+4=0$

$$r_1=r_2=-2$$

$$\therefore y_{zi}=(C_1+C_2t)e^{-2t}$$

$$\begin{cases} y_{zi}(0_+)=y_{zi}(0_-)=1 \\ y_{zi}'(0_+)=y_{zi}'(0_-)=2 \end{cases} \quad \begin{cases} C_1=1 \\ C_2e^{-2t}-2(C_1+C_2t)e^{-2t}\big|_{t=0}=2 \end{cases} \quad \begin{cases} C_1=1 \\ C_2=4 \end{cases}$$

$$\therefore y_{zi}=\left[(1+4t)e^{-2t}\right]u(t)$$

$y_{zsh}(t) = (C_1 + C_2 t)e^{-2t}$

$t>0$时, $f'(t) + 3f(t) = -e^{-t}u(t) + e^{-t}\delta(t) + 3e^{-t}u(t)$

$= 2e^{-t}u(t) + \delta(t)$

设 $y_{zsp}(t) = Be^{-t}$

$\because Be^{-t} - 4Be^{-t} + 4Be^{-t} = 2e^{-t}$

$\therefore B = 2$

$y_{zs}(t) = [(C_1 + C_2 t)e^{-2t} + 2e^{-t}]u(t)$

$$\begin{cases} y''(t) = a\delta(t) + bu(t) \\ y'(t) = au(t) & y_{zs}'(0_+) = y_{zs}(0_-) + a = 0 + 1 = 1 \\ y(t) = atu(t) & y_{zs}(0_+) = y_{zs}(0_-) = 0 \end{cases}$$

1

4

4

$$\begin{cases} C_1 + 2 = 0 \\ \end{cases} \qquad \begin{cases} C_1 = -2 \\ \end{cases}$$

$$C_2 - 2C_1 - 2 = 1 \qquad C_2 = -1$$

$$y_{zs}(t) = [(-2-t)e^{-2t} + 2e^{-t}]u(t)$$

$$y(t) = [\underbrace{(3t-1)e^{-2t}}_{\text{自由}} + \underbrace{2e^{-t}}_{\text{强迫}}]u(t)$$

单位冲激响应 $h(t)$

∴激励为$\delta(t)$输出的y_{zs}

1、$e(t)=\delta(t)$

2、$h^{(k)}(0_-)=0$

e.g. $r''(t)+5r'(t)+6r(t)=3e'(t)+2e(t)$，求$h(t)$

解：$r''(t)+5r'(t)+6r(t)=3\delta'(t)+2\delta(t)$

$r^2+5r+6=0$

$r_1=-2, r_2=-3$

$\therefore y_{zsh}=C_1e^{-2t}+C_2e^{-3t}$

$\delta(t)$无y_{zsp}

$\therefore y_{zs}=y_{zsh}$ // $h(t)(0_-)=0$

$$
\begin{array}{l}
1 \\ 5 \\ 6
\end{array}
\begin{cases}
r''(t) = a\delta'(t) + b\delta(t) + c\,u(t) \\
r'(t) = a\delta(t) + b\,u(t) & r'(0_+) = r'(0_-) + b \\
r(t) = a\,u(t) & r(0_+) = r(0_-) + a
\end{cases}
$$

$$
\therefore \begin{cases} a = 3 \\ b + 5a = 2 \end{cases} \qquad \begin{cases} a = 3 \\ b = -13 \end{cases}
$$

$$
\therefore r'(0_+) = -13 \qquad \begin{cases} C_1 + C_2 = 3 \\ -2C_1e^{-2t} - 3C_2e^{-3t}\Big|_{t=0} = -13 \end{cases}
$$

$$
r(0_+) = 3
$$

$$
\therefore h(t) = (-4e^{-2t} + 7e^{-3t})u(t) \qquad -2C_1 - 3C_2 = -13
$$

$$
-C_2 = -7
$$

$$\begin{cases} C_1 = -4 \\ C_2 = 7 \end{cases}$$

e.g. $r'(t) + 2r(t) = e'(t) + 3e(t)$, $h(t)$

$r'(t) + 2r(t) = \delta'(t) + 3\delta(t)$

$r + 2 = 0$

$r = -2$

$y_{zsh} = C_1 e^{-2t}$

$\delta(t)$元 [illegible]

$y_{zs}=C_1e^{-2t}$

$$\begin{cases} r'(t) = a\delta'(t) + b\delta(t) + c\,u(t) & (1) \\ r(t) = a\delta(t) + b\,u(t) & (2) \end{cases}$$

$$\therefore \begin{cases} a = 1 \\ b + 2a = 3 \end{cases} \quad \begin{cases} a = 1 \\ b = 1 \end{cases}$$

$\therefore r(0_+) = r(0_-) + b = 0 + 1 = 1$

$\therefore C_1 = 1 \quad \therefore y_{zs} = e^{-2t}u(t) + \delta(t)$

e.g. $r'(t) + 2r(t) = e''(t) + 3e'(t) + 3e(t)$, 求 $h(t)$

$e(t) = \delta(t)$ $\quad y_{zs}(0_-) = 0$

$r + 2 = 0$

$r = -2$

$\delta''(t) + 3\delta'(t) + 3\delta(t)$

$y_{zsh} = Ce^{-2t}$

$y_{zs} = y_{zsh} = Ce^{-2t}$

1 $\begin{cases} r'(t) = a\delta''(t) + b\delta'(t) + c\delta(t) + d\,u(t) \\ \\ r(t) = a\delta'(t) + b\delta(t) + c\,u(t) \end{cases}$ 2

$\begin{cases} a = 1 \\ b + 2a = 3 \end{cases}$

未配平

$\{ c+2b=3$ 因为左1阶<右2阶

$r^{(t)}(0_+) = r^{(t)}(0_-) + C = 0+1 = 1$

$y_{zs} = \left(e^{-2t} + \delta'(t) + \delta(t)\right)u(t)$

阶跃响应 $g(t)$

1、$e(t)=u(t)$

2、$y_{zs}(t)$

3、

e.g. $r''(t)+5r'(t)+6r(t)=3e'(t)+2e(t)$

$$r^2+5r+6=0$$

$$r_1=-2,\ r_2=-3$$

$$y_{zsh}=C_1e^{-2t}+C_2e^{-3t}$$

$$r''(t)+5r'(t)+6r(t)=3\delta(t)+2u(t)$$

$$y_{zsp}=B$$

$$\therefore 6B=2$$

$$B=\frac{1}{3}$$

$$\therefore y_{zs}=y_{zsh}+y_{zsp}=C_1e^{-2t}+C_2e^{-3t}+\frac{1}{3}$$

$$\begin{array}{ll} 1 & \left\{\begin{array}{l} r''(t)=a\delta(t)+b\,u(t) \\ r'(t)=a\,u(t) \end{array}\right. \\ 5 & \end{array}$$

$$r'(t) = a u(t)$$

6 $$r(t) = a t u(t)$$

$$\begin{cases} a = 3 \\ b + 5a = 2 \end{cases} \quad \begin{cases} a = 3 \\ b = -13 \end{cases}$$

$$r'(t)_{(0+)} = r'(t)_{(0-)} + a = 0 + 3 = 3$$

$$r(t)_{(0+)} = r(t)_{(0-)} = 0$$

$$\begin{cases} C_1 + C_2 + \frac{1}{3} = 0 \\ -2C_1 - 3C_2 = 3 \end{cases} \quad \begin{cases} C_1 = 2 \\ C_2 = -\frac{7}{3} \end{cases}$$

$$y_{zs} = \left(2e^{-2t} - \frac{7}{3}e^{-3t} + \frac{1}{3}\right)u(t)$$

e.g. $r'(t)+2r(t)=e''(t)+3e'(t)+3e(t)$

左一阶 < 右二阶 ∴ 配平不完全

$r+2=0$

$r=-2$

$\therefore y_{zsh}=Ce^{-2t}$

$\because e(t)=u(t)$

$\therefore r'(t)+2r(t)=\delta'(t)+3\delta(t)+3u(t)$

设 $y_{zsp}=B$

$\therefore 2B=3$

$B=\frac{3}{2}$

$y_{zs}=y_{zsh}+y_{zsp}=Ce^{-2t}+\frac{3}{2}$

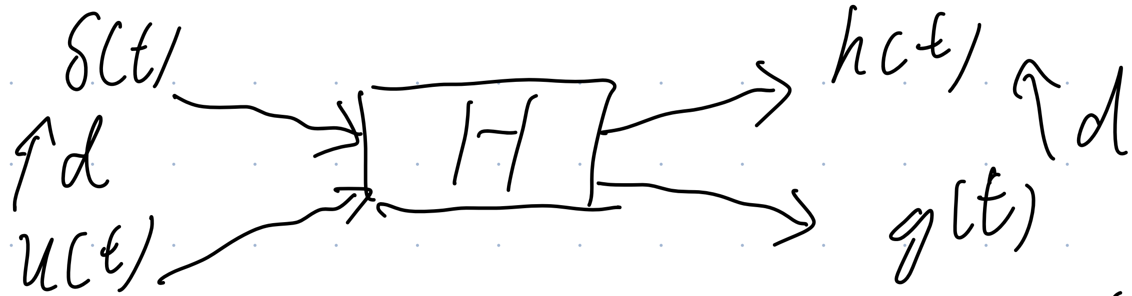
$$
\begin{cases}
1 \quad r(t) = a\,\delta(t) + b\,\delta(t) + c\,u(t) \\
2 \quad r(t) = a\,\delta(t) + b\,u(t)
\end{cases}
$$

$$
\begin{cases} a = 1 \\ b + 2a = 3 \end{cases} \qquad \begin{cases} a = 1 \\ b = 1 \end{cases}
$$

$$r(t)_{(0+)} = r(t)_{(0-)} + b = 0 + 1 = 1$$

$$\therefore \ 1 = C + \frac{3}{2} \qquad C = -\frac{1}{2}$$

$$\therefore \ g(t) = \left(-\frac{1}{2}e^{-2t} + \frac{3}{2}\right)u(t) + \delta(t)$$



$$u(t)=\int_{-\infty}^{t}\delta(\tau)d\tau \Rightarrow g(t)=\int_{-\infty}^{t}h(\tau)d\tau$$

e.g. 已知 $h(t)=(-4e^{-2t}+7e^{-3t})u(t)$，求 $g(t)$

$$g(t)=\int_{-\infty}^{t}(-4e^{-2\tau}+7e^{-3\tau})u(\tau)d\tau$$

$$=\left[\int_{0}^{t}(-4e^{-2\tau}+7e^{-3\tau})d\tau\right]u(t)$$

$$=\left[-4\cdot\left(-\frac{1}{2}\right)\cdot e^{-2\tau}\Big|_{0}^{t}+7\cdot\left(-\frac{1}{3}\right)e^{-3\tau}\Big|_{0}^{t}\right]u(t)$$

$$=\left[2(e^{-2t}-1)-\frac{7}{3}(e^{-3t}-1)\right]u(t)$$

$$= \left(2e^{-2t} - \frac{7}{3}e^{-3t} + \frac{1}{3}\right) u(t)$$

e.g. 已知 $h(t) = e^{-2t} u(t-1)$, 求 $g(t)$

$$g(t) = \int_{-\infty}^{t} e^{-2\tau} u(\tau - 1) d\tau$$

$$= \int_{1}^{t} e^{-2\tau} d\tau \; u(t-1)$$

$$= -\frac{1}{2} e^{-2\tau} \Big|_{1}^{t} u(t-1)$$

$$= -\frac{1}{2}\left(e^{-2t} - e^{-2}\right) u(t-1)$$

e.g.

$$\frac{d\,r(t)}{dt}+5r(t)=\int_{-\infty}^{\infty}e(\tau)f(t-\tau)d\tau-e(t)$$

$f(t)=e^{-t}u(t)+3\delta(t)$, 求 $h(t)$

解: $h'(t)+5h(t)=\int_{-\infty}^{\infty}\delta(\tau)f(t-\tau)d\tau-\delta(t)$

$$h'(t)+5h(t)=f(t)-\delta(t)$$

$$h'(t)+5h(t)=e^{-t}u(t)+2\delta(t)$$

$r+5=0$

$r=-5$

设 $y_{zsh}(t)=Ce^{-5t}$

$y_{zsp}=Be^{-t}$

$$-\beta e^{-t}+5\beta e^{-t}=e^{-t}$$

$$(-\beta+5\beta)e^{-t}=e^{-t}$$

$$\beta=\frac{1}{4}$$

$$y_{zsp}=\frac{1}{4}e^{-t}$$

$$\therefore h(t)=Ce^{-5t}+\frac{1}{4}e^{-t}$$

$$\begin{array}{l}1\\5\end{array}\begin{cases}h'(t)=a\delta(t)+bu(t)\\h(t)=au(t)\end{cases}$$

$$\begin{cases}a=2\\b=e^{-t}-10\end{cases}$$

$$\therefore h(0^+)=h(0^-)+2=0+2=2$$

$$\therefore 2=C+\frac{1}{4}$$

$$C=\frac{7}{4}$$

$$h(t)=(\frac{7}{4}e^{-5t}+\frac{1}{4}e^{-t})u(t)$$

卷积 (convolution)

1. definition: $f_1(t) * f_2(t) =$

$f(t) * \delta(t) = f(t)$

$f(t) * \delta(t-t_0) = \int_{-\infty}^{+\infty} f(\tau)\delta(t-\tau-t_0)d\tau = f(t-t_0)$

$f(t) * \delta'(t) = f'(t)$ $\quad f(t) * \delta'(t-t_0) = f'(t-t_0)$

$f(t) * u(t) = \int_{-\infty}^{t} f(\tau)d\tau$

e.g. $e^{-t}u(t) * \delta(3t+2)$

$= e^{-t}u(t) * \frac{1}{3}\delta(t+\frac{2}{3})$

$= \frac{1}{3}e^{-(t+\frac{2}{3})}u(t+\frac{2}{3})$

e.g. $e^{-2t}u(t) * \delta''(t) * u(t)$

$= -2e^{-2t}u(t) + e^{-2t}\delta(t)$

$= -2e^{-2t}u(t) + \delta(t)$

e.g. $e^{-2t}u(t) * \delta''(t-1) * tu(t)$

$$= e^{-2t}u(t) * \delta(t-1)$$

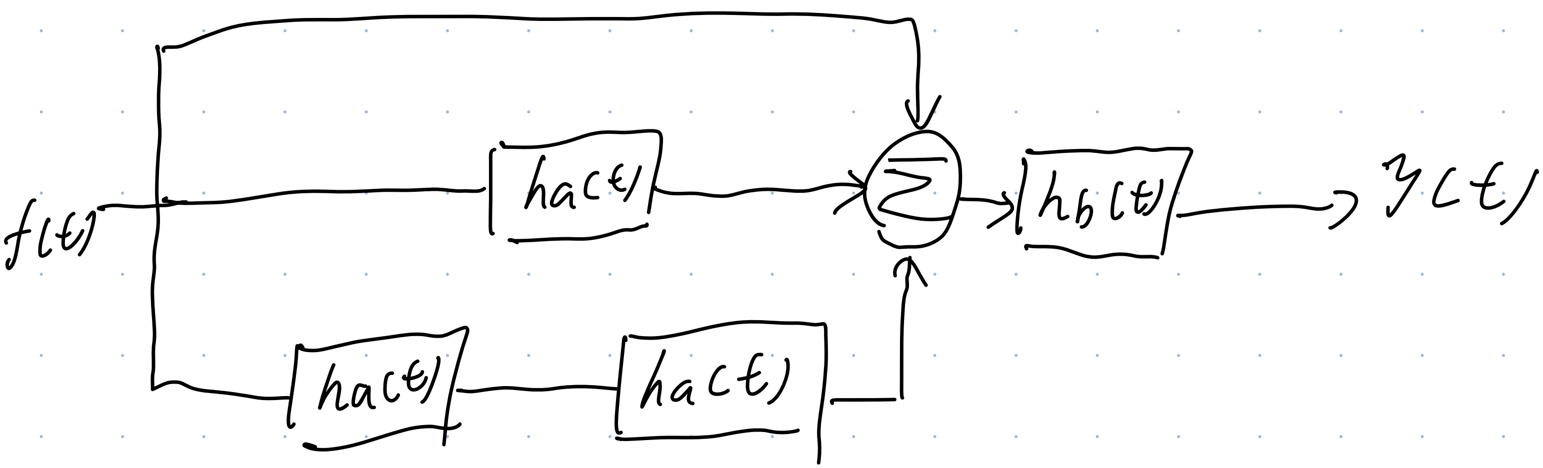
$$= e^{-2(t-1)}u(t-1)$$

e.g. $\int_{-\infty}^{+\infty} e^{-2\tau} u(\tau-t) u(\tau) d\tau = e^{-2t} u(t) * u(-t)$

2. 应用 ($y_{zs} = e(t) * h(t)$)

系统级联 = 相卷积

系统并联 = 相加



$y(t) = [\delta(t) + h_a(t) + h_a(t) * h_a(t)] + h_b(t)$

$u(t)$无默认项

视题目上下限而定 e.g. $u(t-1)$

作用是表示上限 > 下限

$f_1(t)=3e^{-2t}u(t),\ f_2(t)=2u(t),\ f_3(t)=2u(t-2)$

1. $f_1(t)*f_2(t)=\int_{-\infty}^{+\infty}3e^{-2\tau}u(\tau)\cdot 2u(t-\tau)d\tau$

$\because u(\tau),\ \tau>0,\ u(t-\tau),\ t-\tau>0,\ \tau<t$

$\therefore=\int_0^t 3e^{-2\tau}\cdot 2\,d\tau=-\frac{1}{2}\int_0^t 3e^{-2\tau}\cdot 2\,d(-2\tau)$

$=-3e^{-2\tau}\Big|_0^t=-3(e^{-2t}-1)u(t)$

$=3(1-e^{-2t})u(t)$

$$2、f_1(t)*f_3(t)=\int_{-\infty}^{+\infty}3e^{-2\tau}u(\tau)\cdot 2u(t-\tau-2)\,d\tau$$

$$\because \tau>0,\ t-\tau-2>0$$

$$\therefore\ 0<\tau<t-2$$

$$\left[\int_0^{t-2}3e^{-2\tau}\cdot 2\,d\tau\right]u(t-2)$$

$$=\left(-3e^{-2\tau}\Big|_0^{t-2}\right)u(t-2)$$

$$=-3\left(e^{-2(t-2)}-1\right)u(t-2)$$

$$=3\left(1-e^{-2t+4}\right)u(t-2)$$

3. 已知LTI，$h(t)=u(t-1)$，输入 $f(t)=e^{-3t}u(t)$，求 $y_{zs}(t)$

$$y_{zs}(t)=h(t)*f(t)=\int_{-\infty}^{+\infty}e^{-3\tau}u(\tau)\cdot u(t-\tau-1)\,d\tau$$

$\tau>0,\ t-\tau-1>0$

$0<\tau<t-1$

$$\therefore =\int_{0}^{t-1}e^{-3\tau}d\tau\cdot u(t-1)$$

$$=\left(-\frac{1}{3}\cdot e^{-3\tau}\Big|_{0}^{t-1}\right)u(t-1)$$

$$=-\frac{1}{3}\left(e^{-3(t-1)}-1\right)u(t-1)$$

$$=\frac{1}{3}\left(1-e^{-3t+3}\right)u(t-1)$$

4. $f_1(t)=tu(t)$, $f_2(t)=u(t)-u(t-2)$，求 $f_1(t)*f_2(t)$

$$f_1(t)*f_2(t)=\int_{-\infty}^{+\infty}\tau u(\tau)[u(t-\tau)-u(t-\tau-2)]d\tau$$

$$=\int_{-\infty}^{+\infty}\tau u(\tau)u(t-\tau)\,d\tau-\int_{-\infty}^{+\infty}\tau u(\tau)u(t-\tau-2)d\tau$$

$$=\left(\int_0^t \tau\, d\tau\right)u(t)-\left(\int_0^{t-2}\tau\, d\tau\right)u(t-2)$$

$$=\frac{\tau^2}{2}\Big|_0^t\cdot u(t)-\frac{\tau^2}{2}\Big|_0^{t-2}\cdot u(t-2)$$

$$=\frac{1}{2}t^2u(t)-\frac{1}{2}(t-2)^2u(t-2)$$

e.g. $y(t)=\int_{t-2}^{\infty} e^{t-\tau} f(\tau-1)d\tau$

$=\int_{-\infty}^{\infty} e^{t-\tau} f(\tau-1)\,u(\tau-t+2)d\tau$

$= e^{t}u(2-t) * f(t-1)$

$= e^{t}u(2-t) * f(t) * \delta(t-1)$

$h(t)= e^{t}u(2-t) * \delta(t-1)$

$= e^{t}u(3-t)$

$y(t)=\int_{-\infty}^{t-1} e^{\lambda-t} f(\lambda-1)(t-\lambda)d\lambda$ $\qquad \lambda < t-1$

$=\int_{-\infty}^{+\infty} e^{\lambda-t} f(\lambda-1)(t-\lambda)\,u(t-\lambda-1)\,d\lambda$

$= e^{-t}t u(t-1) * f(t-1)$

$= te^{-t}u(t-1) * f(t) * \delta(t-1)$

$$h(t) = (t-1)e^{1-t}u(t-2)$$

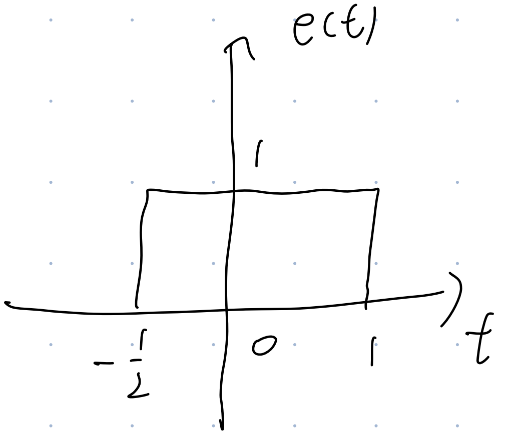
e.g. $e^{-2t}u(t+3) * u(t-5)$

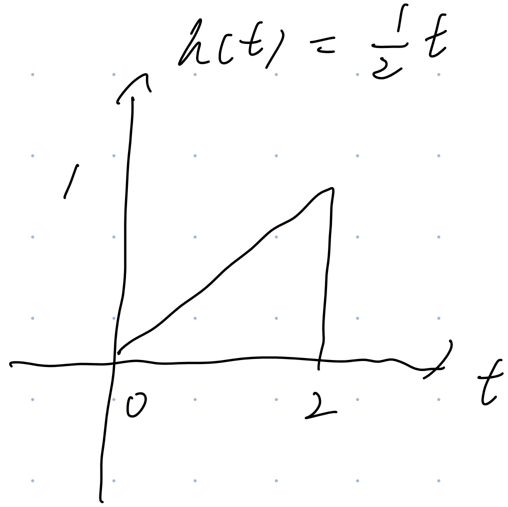
$= e^{6}e^{-2(t+3)}u(t+3) * u(t-5)$

$= e^{6}e^{-2t}u(t) * \delta(t+3) * u(t) * u(t-5)$

$=$ 用LT变换

图解法，求 $e(t) * h(t)$

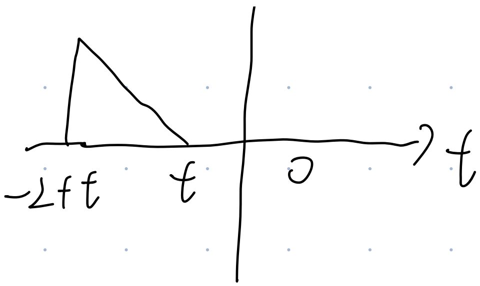




1. $e(t) \to e(\tau)$ $h(t) \to h(\tau)$

2. $h(\tau) \to h(-\tau) \to h(t-\tau)$

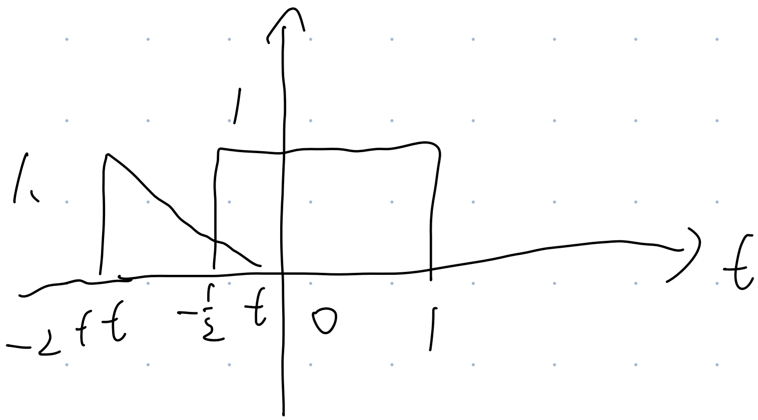
$\uparrow h(t-\tau)$



$\int_{-\infty}^{+\infty} f_1(\tau) f_2(t-\tau)\,d\tau$ 找重叠不为0的区域

3.

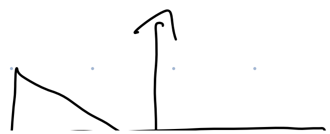
1.



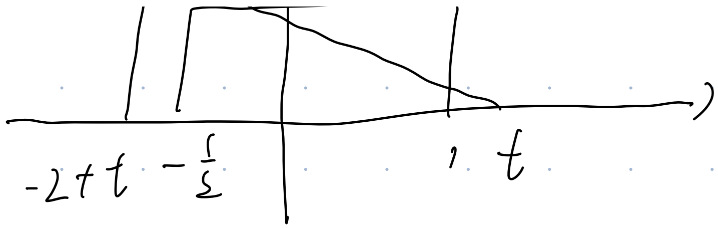
$$\begin{cases} -\frac{1}{2} < t < 1 \\ -2+t < -\frac{1}{2} \\ t < \frac{3}{2} \end{cases}$$

$\int_{-\frac{1}{2}}^{t} 1 \times \frac{1}{2}(t-\tau)\,d\tau$ $\qquad -\frac{1}{2} \le t < 1$

2.



$$\begin{cases} t > 1 \\ -2+t < -\frac{1}{2} \end{cases}$$



$$t < \frac{3}{2}$$

$$1 < t < \frac{3}{2}$$

$$\int_{-\frac{1}{2}}^{1} 1 \times \frac{1}{2}(t-\tau)\,d\tau$$

3.

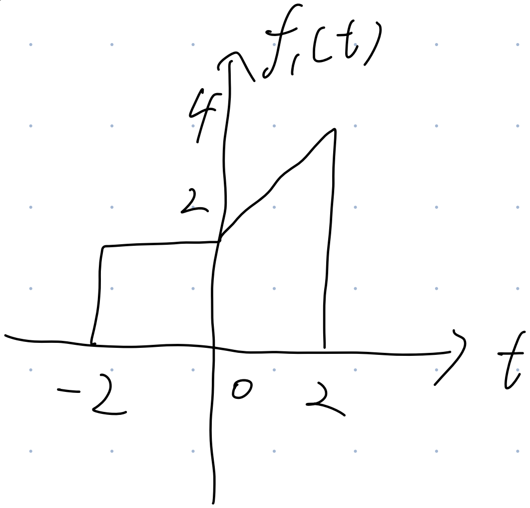
$$\begin{cases} -\frac{1}{2} \le -2+t < 1 \\ t > 1 \end{cases}$$

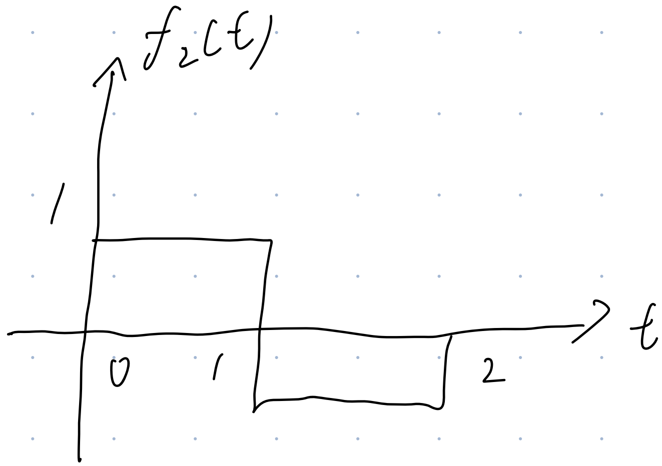
$$\frac{3}{2} \le t < 3$$

$$\frac{3}{2} < t < 3$$

$$\int_{t-2}^{1} 1 \times \frac{1}{2}(t-\tau)\,d\tau$$

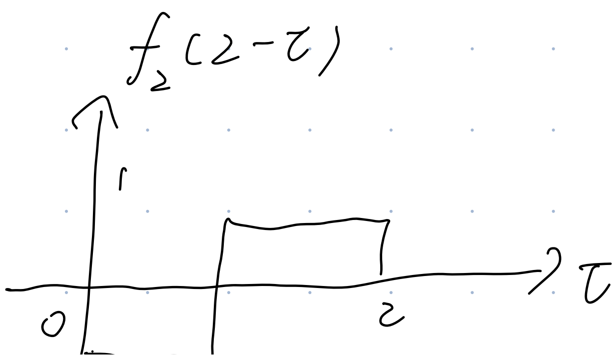
e.g.

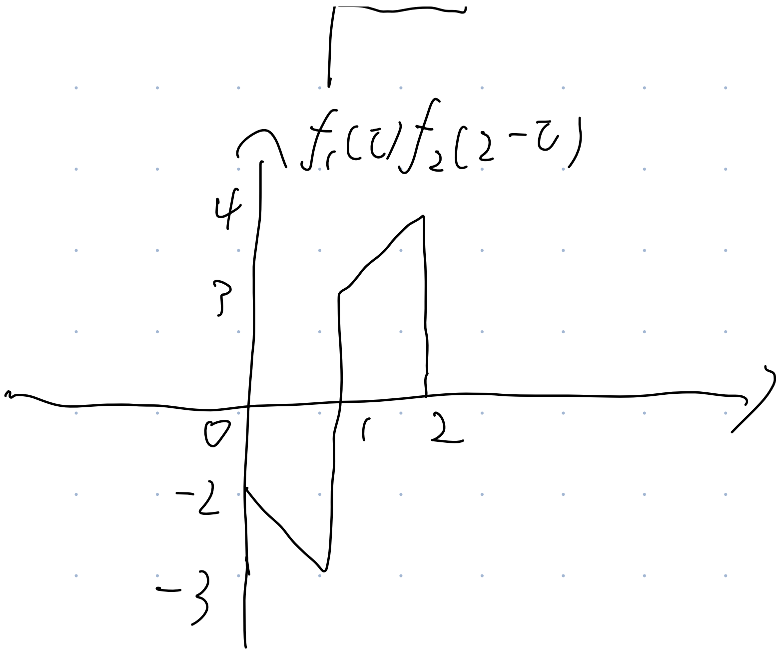




已知 $f_1(t)$, $f_2(t)$, $f(t)=f_1(t)*f_2(t)$, 求 $f(2)$

解: $f(2)=\int_{-\infty}^{+\infty} f_1(\tau) f_2(2-\tau)\, d\tau$





$$\int = -(2+3)\times 1\times\frac{1}{2} + (3+4)\times 1\times\frac{1}{2}$$

$$= -\frac{5}{2} + \frac{7}{2} = 1$$

e.g.

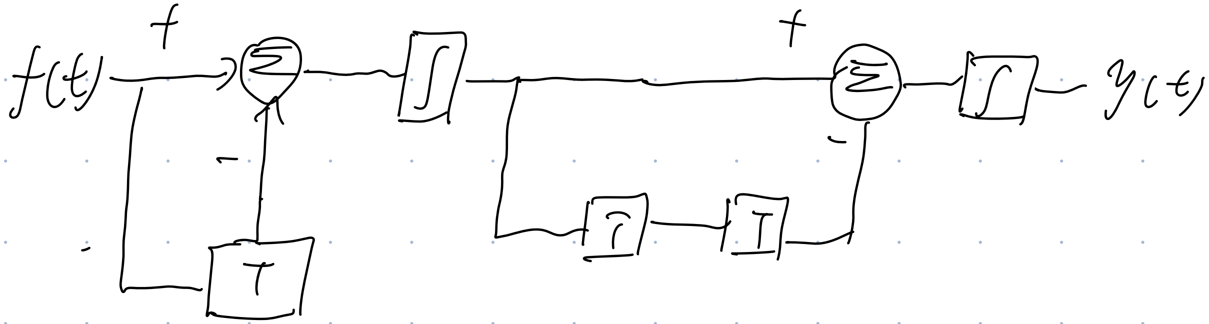
$$u(t) * u(t) = t u(t)$$

$$u(t+3) * u(t-5) = (t-2) u(t-2)$$

$$\int_{-\infty}^{t-2} f(\tau) d\tau = f(t) * u(t-2)$$

$$\int_0^t f(\tau)d\tau = f(t)u(t) * u(t)$$

e.g.



$$(\delta(t) - \delta(t-T)) * u(t) * [\delta(t) - \delta(t-2T)] * u(t)$$

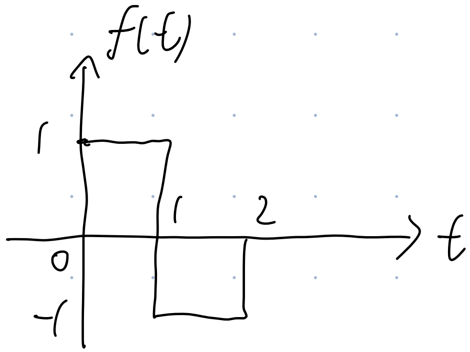
$$= (\delta(t) - \delta(t-2T) - \delta(t-T) + \delta(t-3T)) * tu(t)$$

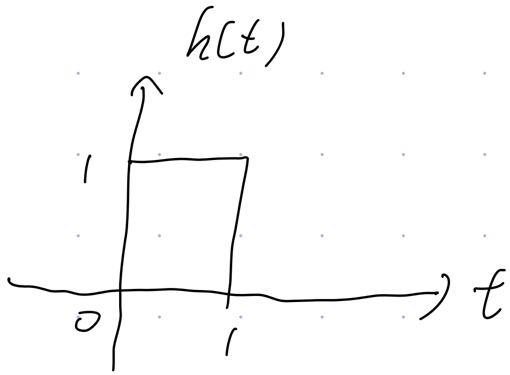
$$= tu(t) - (t-2T)u(t-2T) - (t-T)u(t-T) + (t-3T)u(t-3T)$$

微积分图解法

想办法出 $\delta(t)$

e.g. 求 $g(t)=f(t)*h(t)$

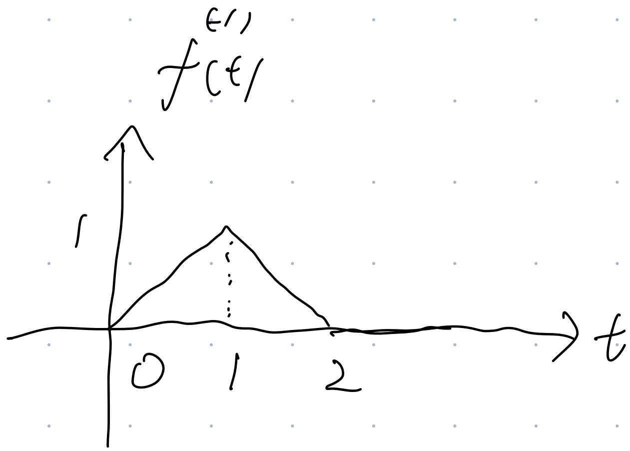


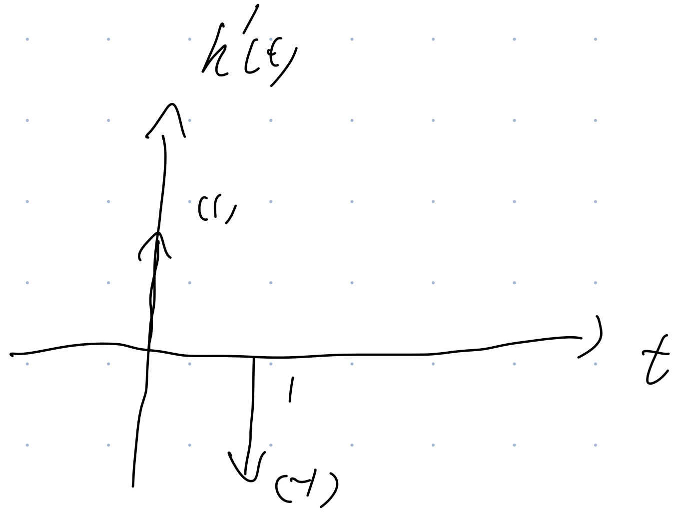


$$g(t)=f(t)*h(t)=f^{(-1)}(t)*h'(t)$$

$$=\left(\int_{-\infty}^{t} u(\tau)-2u(\tau+1)+u(\tau+2)\,d\tau*\big(u(t)-u(t-1)\big)\right)'$$

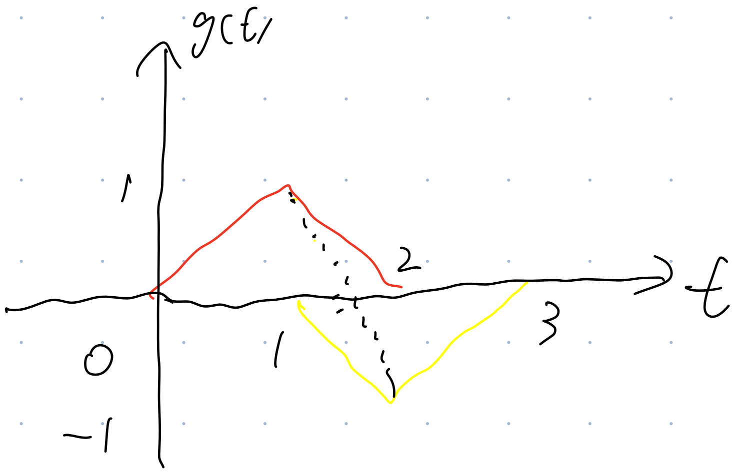
$$[t\,u(t)-2(\tau+1)u(\tau+1)+(\tau+2)u(\tau+2)]*[\delta(t)-\delta(t-1)]$$



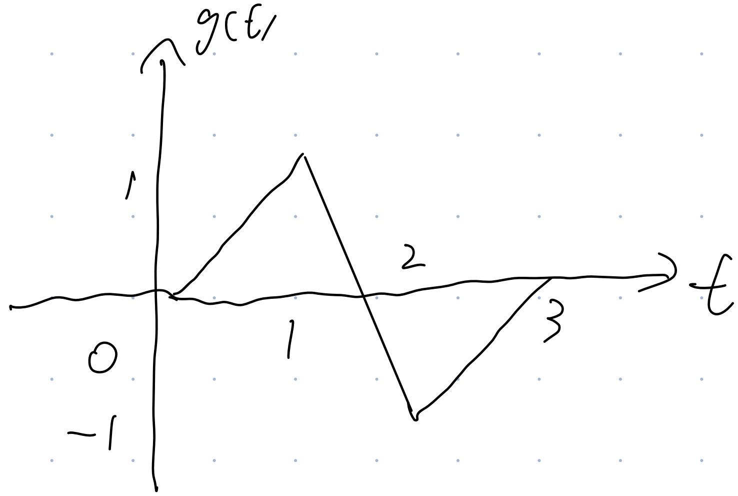


$$原式 = f^{(-1)}(t) * [\delta(t) - \delta(t-1)]$$

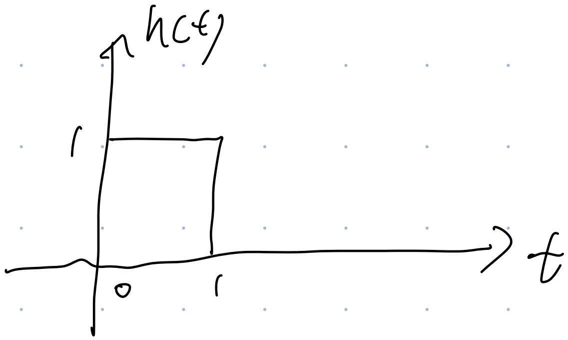
$$= f^{(-1)}(t) - f^{(-1)}(t-1)$$



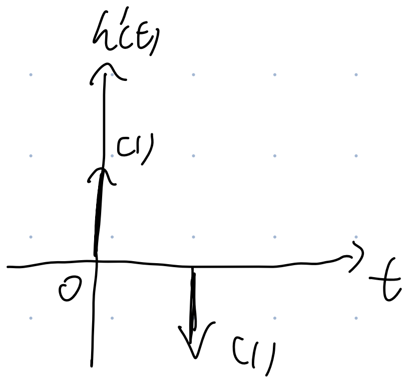
$\Rightarrow$

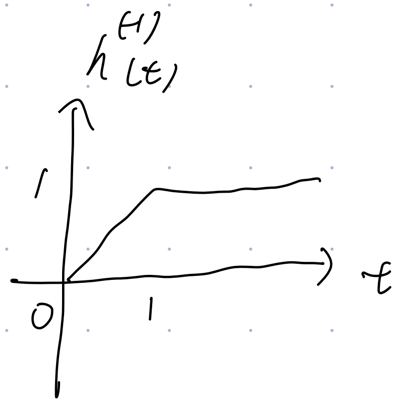


e.g. 已给$h(t)$，求$g(t)=h(t)*h(t)$



解：$h(t)=u(t)-u(t-1)$

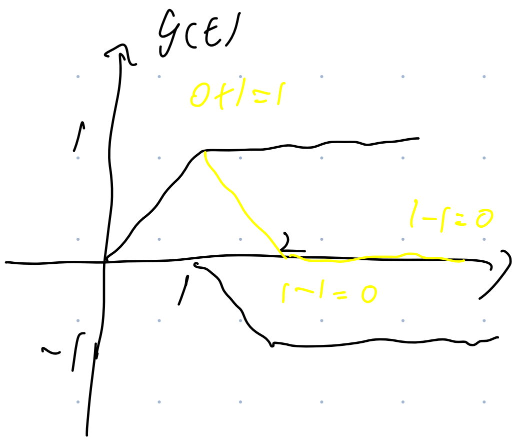




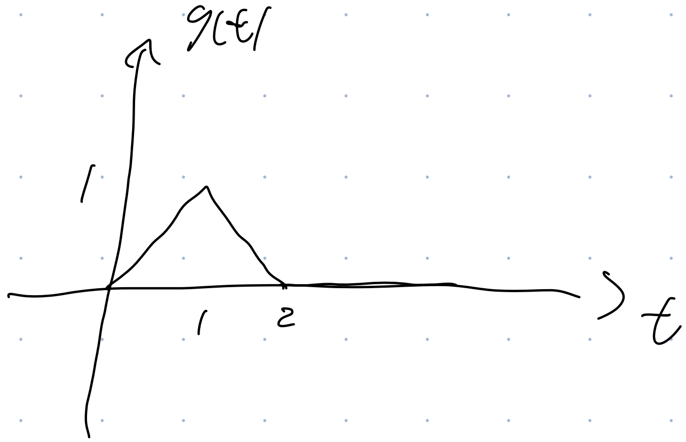
$$g(t)=h'(t)*h^{(-1)}(t)$$

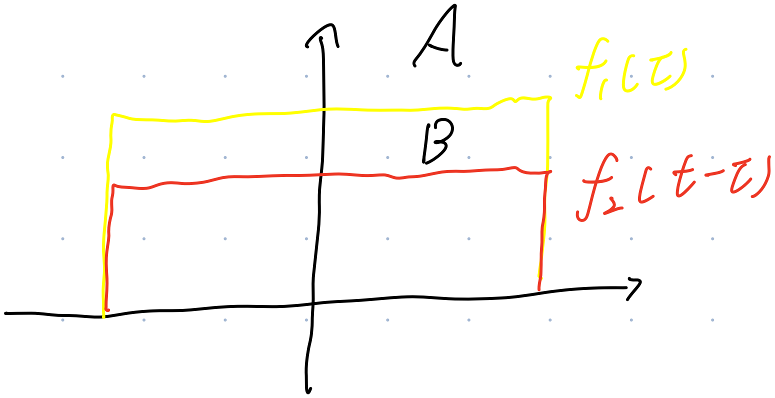
$$=(\delta(t)-\delta(t-1))*(t\,u(t)-(t-1)u(t-1))$$

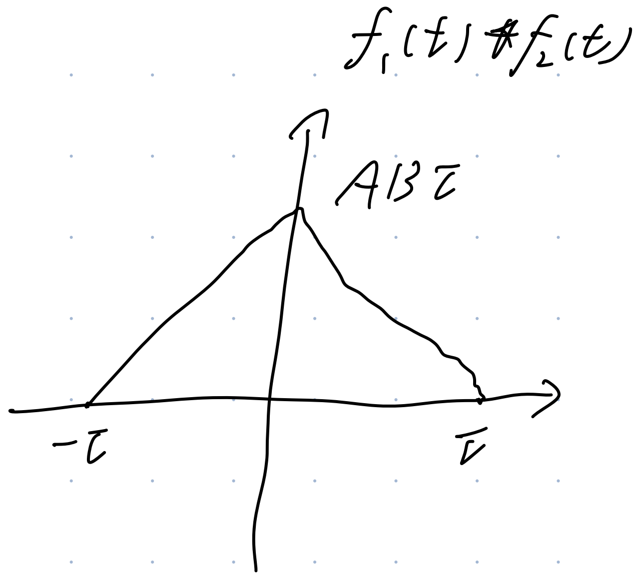
$= h(t) - h(t-1)$



$\Rightarrow$







$f_1(t)*f_2(t) = AB\tau$

$t=-\tau$ $\quad f_1(t)*f_2(t) = 0$

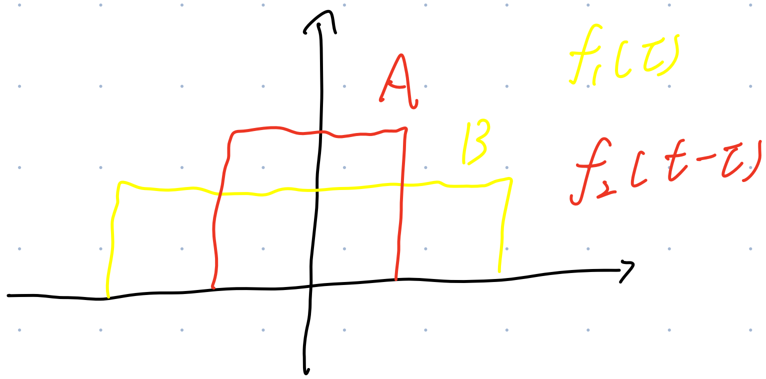
$t=\tau$ $\quad f_1(t)*f_2(t) = 0$

$\because G_\tau(t) = \tau Sa(\frac{\omega\tau}{2})$

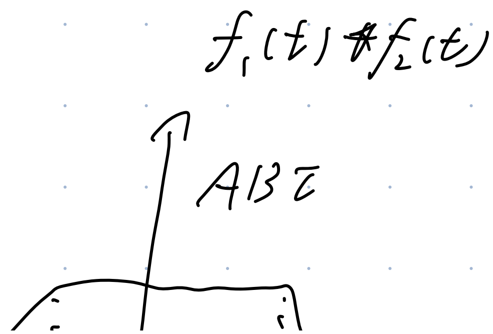
$\therefore f_1(t) =$

$f\ (t) =$

$f_2(t)$

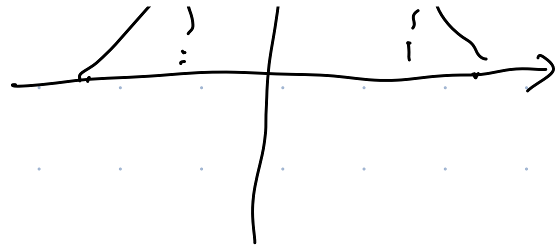


$$f_1(t) * f_2(t) = AB\tau$$



$t=-\tau \quad f_1(t)*f_2(t)=0$

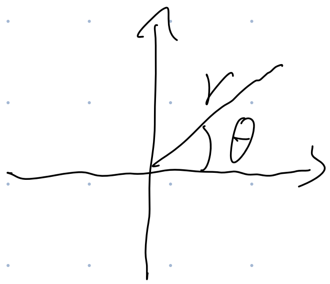
$t=\tau \quad f_1(t)*f_2(t)=0$



$\because G_\tau(t)=\tau Sa(\frac{\omega\tau}{2})$

$\therefore f_1(t)=$

$f_2(t)=$



$1Hz = 1s$

$\omega = 2\pi f$

$\sin\theta = \sin\omega t$

欧拉公式

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\sin t = \cos(t - \frac{\pi}{2})$$

sin超前cos $\frac{\pi}{2}$ 个相位

Fourier series

周期信号为$f(t)$，周期$=T_1$，$w_1=\frac{2\pi}{T_1}$

$$a_0=\frac{1}{T}\int_{-\infty}^{+\infty}f(t)\,dt$$ 面积/一个T

$$a_n=\frac{2}{T}\int_T \cos(nw_0t)f(t)\,dt$$

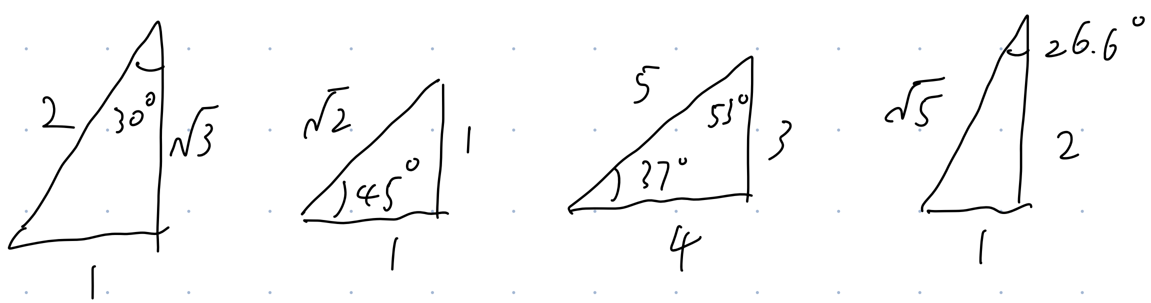
$$b_n=\frac{2}{T}\int_T \sin(nw_0t)f(t)\,dt$$

指数型

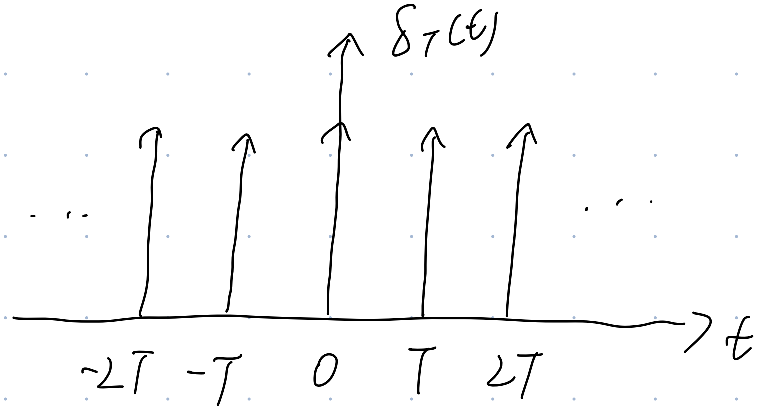
$$x(t) = \sum_{k=-\infty}^{+\infty} X_k e^{jk\omega t}$$

$$X_k = \frac{1}{T}\int_T x(t) e^{-jk\omega t} dt$$

$$x(t) \xleftrightarrow{\mathcal{F}} X_k$$



e.g. 求 $\delta_T(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$ 的FS



解: $x(t)=\sum_{k=-\infty}^{+\infty}X_k e^{jk\omega t}$

$$X_k=\frac{1}{T}\int_T x(t)e^{-jk\omega t}dt$$

$$=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\delta(t)e^{-jk\omega t}dt$$

$$=\frac{1}{T},\ k=0,\pm1,\pm2\cdots$$

$$x(t)=\sum_{k=-\infty}^{+\infty}\frac{1}{T}e^{jk\frac{2\pi}{T}t}$$

FT

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$$