

A Non-Topological View of Dcpo's as Convergence Spaces (Extended Abstract)

Reinhold Heckmann¹

*AbsInt Angewandte Informatik GmbH, Stuhlsatzenhausweg 69, D-66123
Saarbrücken, England, code UK
{heckmann}@absint.com*

Abstract

Abstract The category TOP of topological spaces is not cartesian closed, but can be embedded into the cartesian closed category CONV of *convergence spaces*. “It is well-known that the category DCPO of dcpo's and Scott continuous functions can be embedded into TOP, and so into CONV, by considering the Scott topology. We propose a different, “*cotopological*” embedding of DCPO into CONV, which, in contrast to the topological embedding, preserves products. If X is a cotopological dcpo, i.e. a dcpo with the cotopological CONV-structure, and Y is a topological space, then $[X \rightarrow Y]$ is again topological, and conversely, if X is a topological space, and Y a cotopological complete lattice, then $[X \rightarrow Y]$ is again a cotopological complete lattice. For a dcpo D , the topological and the cotopological convergence structures coincide if and only if D is a continuous dcpo. Moreover, cotopological dcpo's still enjoy some of the properties which characterise continuous dcpo's. For instance, all cotopological complete lattices are injective spaces (in CONV) w.r.t. topological subspace embeddings.

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