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## Position Estimation Method for Prism Based Stereovision System

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### Abstract

In this paper, a novel position estimation method of prism was proposed for single-lens stereovision system. The prism with multi faces was considered as a single optical system composed of some refractive planes. A transformation matrix which can express the relationship between an object point and its image by the refraction of prism was derived based on geometrical optics, and a mathematical model was introduced which can denote the position of prism with arbitrary faces only by 7 parameters. This model can extend the application of single-lens stereovision system using prism to a more widely area. Experimentation results are presented to prove the effectiveness and robustness of our proposed model.

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### 1. Introduction

Stereo vision is an important branch of computer vision. It aims to recover the depth information of the object and environment from two or more image taken from different viewpoints[1]. The stereo process can be summarized by the following steps: 1) camera calibration, which determines the value of internal and external parameters of the vision system; 2) corresponding of features between the images under certain geometric and other constraints; and 3) reconstruction[2-4].

Unlike normal stereovision system, prism based single-lens stereovision system use only one camera and a prism which be placed in front of the camera to capture different views of the same scene[5-8]. Obviously, it has many advantages compared with traditional stereovision. Because only one camera required, it can reduce

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the cost and eliminate the multi-camera synchronization problem automatically [9,10]. It can also decrease the size of the whole system so that it can be used in narrow space. These advantages enable this system with a good potential application and it develops rapidly in recent decade [11-14].

## 2. Virtual image of a multi-ocular prism

According to [15], a 3D point and one of its virtual image by the twice refraction of prism can be expressed by way of combine these two process together as:

$$A'' = M_2 M_1 A = M_p A \quad (1)$$

Where  $M_p$  is the transformation matrix which can be used to calculate the virtual image for the refraction of prism.

## 3. Position estimation of prism

### 3.1. Parameters of a multi-ocular prism

As well as camera, the parameters of a prism used in stereovision system can also be divided into intrinsic and extrinsic parameters. The intrinsic parameters which including refractive index, the angle and the thickness denoted the shape of a prism; the extrinsic parameters which including the surface normal and point coordinates of each plane expressed the position of the prism in camera coordinate.

The numbers of intrinsic parameters for any prism were fixed in 3, while the extrinsic parameters of a multi-ocular prism with one back plane and  $m$  inclined planes have  $m+1$  normal vectors and  $m+1$  points, each normal vector and point get three variables, so if we want to express this prism, there should be  $6*(m+1) + 3$  variables (for example, the number of parameters for a two-ocular prism is 21, for a three-ocular prism is 27). Obviously, it was overload and would hardly get accuracy results.

### 3.2. Simplification of surface normal

If a vector  $V_0 = [v_{ox}, v_{oy}, v_{oz}]^T$  rotated around a vector  $V = [v_x, v_y, v_z]^T$  to a new vector  $V_n = [v_{nx}, v_{ny}, v_{nz}]^T$  with angle  $\delta$ , the transformation can be written as:

$$V_n = M_V^\delta V_0 \quad (2)$$

Where  $M_V^\delta$  denote the rotation matrix with angle  $\delta$  around the rotation axis  $V$ . If we considered the surface normal of each inclined plane as a vector which rotated around a vector with the same angle, the result would be simple than before. For a multi-ocular prism with  $m$  inclined planes, if the surface normal of back plane is denoted by  $V_b$ , the direction vector of edge  $l_i (i=1, 2, \dots, m)$  are  $V_i (i=1, 2, \dots, m)$ . According to equation (2), we have:

$$V_i = M_{V_b}^\alpha V_{i-1} (i=2, 3, \dots, m) \quad (3)$$

where  $\alpha = (m-2)\pi/m$ .

Moreover, the surface normal of inclined planes can be taken as  $V_b$  rotated around  $V_i$  with angle  $\theta$ , and then we have:

$$N_i = M_{V_i}^\theta V_b (i=1, 2, \dots, m) \quad (4)$$

Where  $\theta$  is the angle between back plane and inclined plane, which is one of the intrinsic parameters. This equation denoted that for any multi-ocular prism, all the surface normal of extrinsic parameters can be

expressed only by  $V_b$  and  $V_1$ . If  $V_b = [v_{bx}, v_{by}, v_{bz}]^T$  and  $V_1 = [v_{1x}, v_{1y}, v_{1z}]^T$ . Using the knowledge that  $V_b$  and  $V_1$  are orthonormal, we have:

$$V_b \cdot V_1 = 0 \quad (5)$$

$$\|V_b\| = \|V_1\| = 1 \quad (6)$$

Therefore, the two fundamental constraints (19) and (20) can reduce  $V_b$  and  $V_1$  to four variables which could be denoted by  $V = [v_{bx}, v_{by}, v_{bz}, v_{1x}]$ .

### 3.3. Point selection

An obvious advantage of multi-ocular prism was all the inclined plane must intersect at one common point; this character can easily reduce the point number from  $m+1$  to 2. If this common point is denoted by  $P = [p_x, p_y, p_z]^T$ , the perpendicular line from  $P$  to the back plane intersected back plane with point  $P_b = [p_{bx}, p_{by}, p_{bz}]^T$ , we have:

$$P = P_b + tV_b \quad (7)$$

$$\|P - P_b\| = d \quad (8)$$

where  $t$  is a scalar,  $d$  is the distance from  $P$  to  $P_b$  as well as the thickness of prism intrinsic parameters. From (7) and (8), the variables of points for a prism with  $m$  inclined planes could be expressed by:

$$D = \{v_{bx}, v_{by}, v_{bz}, v_{1x}, p_x, p_y, p_z\} \quad (9)$$

### 3.4. Homographic between a 3D point and its image

From equation (4), we have:

$$s\tilde{x} = M_{\text{int}} M_p M_{\text{ext}} \tilde{X} \quad (10)$$

Where  $s$  is an arbitrary scale factor,  $M_{\text{int}}$  is camera intrinsic matrix,  $M_{\text{ext}}$  is camera extrinsic matrix. Moreover, if  $X^c$  stand for the 3D points in camera coordinate system, then we have

$$X^c = M_{\text{ext}} \tilde{X} \quad (11)$$

And equation (11) can be written as

$$s\tilde{x} = M_{\text{int}} M_p X^c \quad (12)$$

### 3.5. Maximum likelihood estimation

We are given  $n$  3D object points and  $m$  inclined plane of prism. There should be  $n*m$  points in image plane. Assume that the image points are corrupted by independent and identically distributed noise. The maximum likelihood estimation of prism position can be obtained by minimizing the following functional:

$$\sum_{i=1}^n \sum_{j=1}^m \|x_{ij} - \hat{m}(M_{\text{int}}, M_p, M_{\text{ext}}, X_{ij})\| \quad (13)$$

Where  $\hat{m}(M_{\text{int}}, M_p, M_{\text{ext}}, X_{ij})$  is the projection of point in view  $j$ , according to equation (13). The intrinsic and extrinsic matrix can be solved by single camera calibration, combine (9) and (12) into (13), the function changed to:

$$\sum_{i=1}^n \sum_{j=1}^m \|x_{ij} - \hat{m}(D, X_{ij}^c)\| \quad (14)$$

Where  $D$  is the parameters of prism,  $x_{ij}$  is the 2D points in the camera image plane and  $X_{ij}^c$  stand for the coordinate of 3D point  $X_{ij}^c$  in camera coordinate system, and each point  $X_{ij}^c$  must correspond to  $m$   $x_{ij}$  by equation (28) as a process of perspective projection. For the position estimation of prism, the first step is camera calibration. When the intrinsic and extrinsic parameters of camera were known, we can work out  $X_{ij}^c$  by equation (27). Then we could get  $n \times m$  corresponding points between  $x_{ij}$  and  $X_{ij}^c$ . Minimizing (30) is a nonlinear minimization problem, which could be solved with the Levenberg-Marquardt Algorithm [16]. The initial guess could use the position of standard coordinate, because we always want to set the prism as similar as the standard coordinate.

## 4. Experiment and analysis

### 4.1. Experimentation Devices

Camera and prism are mounted on a mechanical stand which installs vernier calipers in X, Y, Z axis as well as rotational stage. The relative positions between the camera and prism are known and adjustable, and a high-precision laser be used for measuring the distance from camera image plane to object, as shown in Fig 1.



Fig 1 System set up and multi-ocular prism

### 4.2. Experiment results

The images captured by our stereovision system were divided into two, three and four segments respectively according to the corresponding prisms. In order to explain it clearly, we used to express them, as shown in Fig 2.

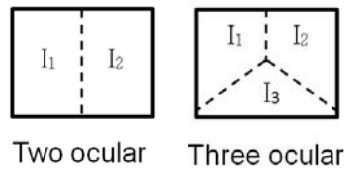


Fig 2 The segments of image plane for two and three ocular prism

Table 1 listed the reprojection errors of two ocular prism which calculated by our proposed method. The system setup and parameters for these data were introduced as follow: the focal length of camera is 8mm, the resolution of camera image plane is 1024\*768, pixel size is 0.00465\*0.00645mm, the camera optical center is taken as the original point of camera coordinate, and the angel of the prism is both 6.4 degree. The circle board was placed at some fitted positions which paralleled with the camera image plane and the sampling interval is 500mm.

Table 1 Reprojection errors of two ocular prism

Distance range(mm)	Area of image plane	Coordinates of captured image points	Coordinates of reprojective points	Distance error(pixel)
1000	I <sub>1</sub>	324,121	323.4,122.7	1.8027
		468,608	468.9,609.3	1.58114
	I <sub>2</sub>	736,120	737.4,121.7	2.2023
		871,611	870.1,612.3	1.5811
1500	I <sub>1</sub>	455,99	452.9,99.7	2.2136
		408,697	406.9,698.6	1.9416,
	I <sub>2</sub>	865,101	865.2,99.9	1.1180
		925,700.6	925.6,698.9	1.8027
2000	I <sub>1</sub>	308,270	307.8,268.9	1.1180
		497,576	498.1,576.1	1.1045
	I <sub>2</sub>	822,271	823.4,271.1	1.4035
		1007,578	1007.8,578.4	0.8944
2500	I <sub>1</sub>	467,155	456.8,155	1.2
		87,680	87.1,681.2	1.2041
	I <sub>2</sub>	779,159	781.2,159.4	2.2360
		603,688	603,687.1	0.9
3000	I <sub>1</sub>	268,114	267.8,115.2	1.2165
		488,685	490.3,685.7	2.4042
	I <sub>2</sub>	669,110	667.8,111.1	1.6279
		997,686	998.7,686.4	1.7464
Average				1.5650

As shown in these two tables, the accuracy of our proposed method is sufficient for a large range of prism based stereovision system. As well as multi-camera stereovision system, multi-ocular prism system can provide more comprehensive information on the environment and the object, so the reprojection errors of four-ocular prism are less than that of two-ocular prism. However, the more faces a prism with, the more time will be spent for solving the equations.

## 5. Conclusions

In this paper, we introduced a new method for position estimation of stereovision system using prism. The parameters of multi-ocular prism were reduced to only 7 extrinsic parameters which could express the position of the prism and 3 intrinsic parameters which could denote the shape of the prism. Our method is based on optical geometry, and could be used in stereovision system with any coordinate. The experiments with both three prisms show that the method is efficient, robust and has good property of convergence and small reprojection errors.

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