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Total opportunity cost matrix – Minimal total: A new approach to determine initial basic feasible solution of a transportation problem

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ABSTRACT

Transportation Problem (TP) deals with cost planning for delivering the product from the source to the destination and Initial Basic Feasible Solution (IBFS) is presented to find the way out in obtaining an optimal solution. IBFS is an important element to reach an optimal result. The previous methods related to it did not always provide the satisfied result all the time. Therefore a new method called Total Opportunity Cost Matrix – Minimal Total (TOCM-MT) to determine IBFS as a basic solution to solve TP was proposed. The objective is to achieve a total cost with similar or closer values to the optimal solution. TOCM for the initial matrix and a better mechanism are highly considered to obtain IBFS. Thirty-one numerical examples, in which twenty-five were selected from some journals and six were generated randomly, were used to evaluate the performance of it. The proposed method has been compared to Vogel's Approximation Method (VAM), Juman and Hoque Method (JHM), and Total Differences Method 1 (TDM1). TOCM-MT was proven to have twenty-four numerical examples with similar values and seven numerical examples with closer values to the optimal solution. The experiment results indicated that TOCM-MT obtained better minimal cost than that of VAM, JHM, and TDM1.

1. Introduction

The Transportation Problem (TP) deals with transporting some products from the source to the destination with the minimal total cost subject to satisfy the demand and the supply constraints [1]. The objective of TP is to achieve minimal total cost as an optimal solution. It was first developed by Hitchcock [2] and one of the linear programming problems [1,3]. It can be applied to the real-world problems such as personal assignment [4], task allocation [5], problems of flow shop scheduling [6,7], and vehicle routing [8,9].

There are two steps in obtaining the optimal solution of TP. The first step is finding an Initial Basic Feasible Solution (IBFS) and the second one is finding an optimal solution from IBFS [10]. It is necessary to start with IBFS to solve transportation problem [1,3,4,10]. Finding IBFS is important and significant because it is a basic solution in obtaining optimal solution [3]. The result of IBFS can be similar or closer to the values of the optimal solution. The better result of IBFS can decrease the number of iterations in obtaining an optimal solution [1]. This research focuses on finding IBFS to obtain the minimal total cost of the transportation problem.

Many studies related to IBFS have been done by several researchers and the three well-known ones are Northwest Corner Method (NCM), Least Cost Method (LCM), and Vogel's Approximation Method (VAM) [11]. Some methods to find IBFS based on LCM have been developed by some researchers such as Juman and Hoque [1], Babu [12], Juman and Hoque [13], Babu [14], Dhurai [15], Kousalya and Malarvizhi [16], Ahmed et al. [17,18], Deshmukh [19]. Juman and Hoque [1] proposed Juman Hoque Method (JHM) to obtain IBFS. It was different from the other ones because it started with an infeasible solution and leads to an efficient IBFS. The experiment showed that JHM led to the minimal total cost of 16 out of 18 transportation problem. Ahmed et al. [17] presented Incessant Allocation Method (IAM) to get the IBFS for transportation problem. It can be applied in a balanced and unbalanced transportation problem. They [18] developed the Allocation

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Table Method (ATM) which began from the least odd cost to obtain IBFS. It was similar to the one presented by Deshmukh [19].

VAM is the best well-known initial basic feasible solution in general [3]. It has been studied for a long time and modified by some researchers such as Hosseini [3], Soomro et al. [20,21], Rashid [22], Hakim [23]. Goyal [24], Das et al. [25,26], Azad and Hossain [27], Alkubaisi [28], Seethalakshmy and Srinivasan [29]. Hosseini [3] proposed Total Differences Method 1 (TDM1) to obtain IBFS for TP. VAM calculates penalties for all rows and columns, while TDM1 calculates penalty only for rows. Another difference is how to calculate the penalty. TDM1 penalty is more complete than that of VAM. It is because VAM penalty is the difference between two least costs and TDM1 penalty is the total differences between the least and other costs. Azad and Hossain [27] considered the average row and column penalties. Alkubaisi [28] used the median cost to find the penalty value. Seethalakshmy and Srinivasan [29] offered an alternative method to calculate the penalty value. The row penalty is the difference between the two highest costs and the column penalty is the difference between the max and the min cost. Soomro et al. [21] developed Modified Vogel's Approximation Method (MVAM) to find a basic feasible solution for the transportation problem.

Total Opportunity Cost Matrix (TOCM) was introduced by Kirca and Satir [30]. It transforms the original matrix of TP into an initial matrix by adding the row and the column opportunity cost matrix. The row/column opportunity cost matrix subtracts every element in it by the least cost. Khan et al. [31] proposed TOCM-SUM to obtain a feasible solution of the transportation problem. Dubey and Shrivastara [32] used TOCM to improve VAM. Islam et al. [33] presented Total Opportunity Cost Table (TOCT) to find the basic feasible solution (BFS) of the transportation problem. Khan et al. [34] specified the distribution indicator for each cell of the TOCM. Mathirajan and Meenakshi [35] combined TOCM and VAM to get the minimal total cost of the transportation problem.

Some literatures concerning with the TP and the various techniques have been developed. Maity and Roy [36] developed a mathematical model to solve a multi–objective nonlinear transportation problem with multi-choice demand. Their research [37] discussed the various mathematical models of multi-objective transportation problem (MOTP) such as goal programming (GP), weighted goal programming (WGP), and revised multi-choice goal programming (RMOGP). After that they developed a new approach of RMOGP and utility function of MOTP. Roy and Maity [38] considered a new way to minimize the cost and the time of transportation problem through a single objective function in the multi-choice environment with interval value. Their research [39] proposed the utility function to select goals of multi-objective transportation problem which has two stages and multi-choice grey number. Ali and Mustapha [40] compared five methods of transportation problem to find the best one. Roy et al. [41] introduced a conic scalarization approach to obtain a solution of the multi-objective transportation problem. Maity et al. [42] studied the multi-objective transportation problem with fuzzy multi-choice goals of an objective function.

Because of the intractability of carrying out calculations, some researchers implemented the methods using C++ program language, Java and Matlab. Imam et al. [43] and Sen et al. [44] used C++ program language to make the object-oriented model to solve the transportation problem. Juman and Hoque [1] implemented JHM in C++ program language. Lawal and Eberendu [45] implemented the Northwest, Least cost, VAM, Modified Distribution and the stepping stone in Java and Net Beans. Khan et al. [46] Implemented twelve methods in Matlab.

VAM, JHM and TDM1 were examined to find IBFS and they could not provide the optimal solution all the time. Then TDM1 was chosen because of the following reasons: the highest penalty (HP) is arbitrarily chosen and the maximum units are directly allocated to the least cost cell. Hence the optimal solution was not always obtained. Some studies used TOCM matrix as the initial matrix and produce a better IBFS. Therefore this research integrates TOCM and modified TDM1 called Total Opportunity Cost Matrix – Minimal Total (TOCM-MT) to determine IBFS of TP. The benefits of TOCM-MT over TDM1 can be expressed as follows:

1. TOCM-MT uses TOCM as the initial matrix due to the greater chance of being selected as the least cost while TDM1 uses the original one without any chances
2. TOCM-MT has the rules to select HP while TDM1 chooses HP arbitrarily when there are several HP with the same values. So that, TOCM-MT has a higher chance to obtain the optimal solution.
3. TOCM-MT has a mechanism to allocate the maximum units to the least cost cell when the least cost is equal to zero while TDM1 directly allocates maximum units to the least cost cell. So that, has a greater chance to obtain the minimal cost.

The remainder of this paper is organized as follows: Sections 2 and 3 present the mathematical formulation of the transportation problem and Total opportunity cost matrix. The existing methods are summarized in Section 4. Section 5 is the description of TOCM-MT and Section 6 is the illustration of the numerical samples. The experimental results showing the performance of the proposed method is discussed in Section 7. The conclusions of the experiment and future research are demonstrated in Section 8.

2. Mathematical formulation of transportation problem

Transportation problem (TP) can be represented by the network diagram in Fig. 1 [10,18] and by the formulation table in Table 1 [17]. The objective of the network diagram and formulation table is to determine the value of variable $X_{i,j}$ that will minimize the total cost of the transportation problem as shown in Eq. (1).

The following notation is used for the mathematical formulation [31].

m	Total number of supply
n	Total number of demand
S_i	Supply i
D_j	Demand j
$C_{i,j}$	Transportation cost from the supply i to demand j
$X_{i,j}$	Allocation made from the supply i and demand j

Using the notation, the objective TP can be formulated as follow:

$$\mathrm{Min} Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} \tag{1}$$

$$\begin{aligned} \text{Subject to} \quad & \textstyle\sum_{j=1}^{n} X_{ij} = S_i \quad \text{for } i = 1, 2, \ldots m \\ & \textstyle\sum_{i=1}^{m} X_{ij} = D_j \quad \text{for } j = 1, 2, \ldots, n \\ & \text{Where} \quad X_{ij} \geqslant 0 \quad \text{for all } i, j \end{aligned} \tag{2}$$

A transportation problem is balanced if the total supply is equal to the total demand as shown in Eq. (3).

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{m} D_j \tag{3}$$

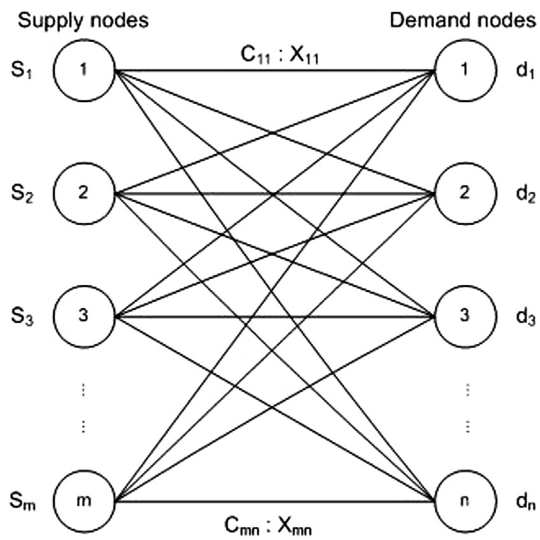

Fig. 1. Network diagram of the transportation problem.

Table 1
Formulation table of the transportation problem.

	Demand 1		Demand 2			Demand n		
Supply 1	C_{11}		C_{12}		...	C_{1n}		S_1
		X_{11}		X_{12}			X_{1n}	
Supply 2	C_{21}		C_{22}		...	C_{2n}		S_2
		X_{21}		X_{22}			X_{2n}	
	.		.		...	.		
	.		.			.		
	.		.			.		
Supply m	C_{m1}		C_{m2}		...	C_{mn}		S_m
		X_{m1}		X_{m2}			X_{mn}	
	D_1		D_2			D_n		

3. Total opportunity cost matrix

Total Opportunity Cost Matrix (TOCM) is introduced by Kirca and Satir [30]. It is transforming the matrix transportation problem from the original matrix into an initial matrix by adding the row and column opportunities. Table 2 is the matrix of the original transportation problem. The row opportunity subtracts every element in the row by the least cost in it. The column opportunity subtracts every element in the column by the least cost in it. TOCM is the sum of row and column opportunities as shown in Table 3.

4. The existing methods to find IBFS

Initial Basic Feasible Solution (IBFS) is an initial solution of transportation problem (TP) and is known as the starting solution of TP. In some cases, IBFS gets the optimal solution. Three existing methods will be discussed to find IBFS in this section.

The first method is Vogel's Approximation Method (VAM) which is better than Northwest method and the least cost method [1]. The steps of VAM can be described as follows: **Step 1** Construct the transportation problem matrix. If the total supply is not equal to the total demand, then the dummy row or column is added. **Step 2** Find the penalty for **each row** and **column**. The penalty is the difference between the two least costs. **Step 3** Select the highest penalty. **Step 4** Select the least cost. **Step 5** Allocate the maximum possible units to it. **Step 6** Adjust the supply and demand then cross out the satisfied row or column. **Step 7** Recalculate the penalty without considering the cross out rows and columns. **Step 8** Repeat steps 3–7 until all rows and columns are satisfied. **Step 9** Finally, calculate the total cost transportation problem. Total cost transportation problem is the multiplication of the cost and units allocated.

Table 2
Original transportation problem matrix.

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

Table 3
TOCM Matrix.

	D1	D2	D3	D4	Supply
S1	9	42	50	0	7
S2	91	22	10	80	9
S3	53	0	92	22	18
Demand	5	8	7	14	

The second method is the Juman and Hoque Method (JHM). JHM doesn't need balance transportation problem and penalty [1]. The steps of JHM can be described as follows: **Step 1** Construct the initial transportation problem matrix. **Step 2** For each column, identify the least cost cell, and then assign the demand there. **Step 3** Check each of the row whether the row sum is less than or equal to the supply quantity. If so, go to step 9. **Step 4** If there are a few unmet rows, determine the difference between the second least and the least unit costs, identify the smallest of them, and go to Step 5. If only one unmet row, go to step7. **Step 5** For each unmet row, check the cell without the second least unit cost in another unmet row. Check the previous row when such a row is found and go to step 7. **Step 6** Select any two unmet rows. For each of them, find differences between the second least and the least unit costs. **Step 7** Considering the identified unmet row in step 5 (or step 4or step 6) transfer the maximum amount of the excess supply from the least cost cell to the next least cost cell, and continue this transferring until no excess supply exists. **Step 8** Cross off the row that has been satisfied, and go to step 3. **Step 9** Stop, the current solution is the IBFS.

The third method is Total Difference Method 1 (TDM1). TDM1 calculates the penalty only for rows [3]. The steps of TDM1 can be described as follows: **Step 1** Construct the transportation problem matrix. If the total supply is not equal to the total demand, then the dummy row or column is added. **Step 2** Find the penalty for each row. The penalty is the total difference between the least and other costs. **Step 3** Select the highest penalty. **Step 4** Select the least cost. **Step 5** Allocate the maximum possible units to it. **Step 6** Adjust the supply and demand then cross out the satisfied row or column. **Step 7** Recalculate the penalty without considering the cross out rows and columns. **Step 8** Repeat steps 3–7 until all rows and columns are satisfied. **Step 9** Finally, calculate the total cost transportation problem. Total cost transportation problem is the multiplication of the cost and units allocated.

5. Total opportunity cost matrix – Minimal total

Total Opportunity Cost Matrix – Minimal Total (TOCM-MT) is the combination of TOCM and modified TDM1. The steps of TOCM-MT can be described as follows:

Step 1: Construct an original Transportation Problem (TP) matrix m × n with cost C_{ij}, supplies S_i; $i = 1..m$ and demands Dj; $j = 1..n$. If the total supply is not equal to the total demand, then the dummy row or column is added.

Step 2: Construct a row opportunity matrix from the original TP by finding the least cost of each row then subtract each cost in the row with the least cost.
Step 3: Construct a column opportunity matrix from original TP by finding the least cost of each column then subtract each cost in the column with the least cost.
Step 4: Construct the TOCM in which the entries are the sum of the row and column opportunity matrix.
Step 5: Find the penalty P_i for each row. The penalty P_i is the total difference between the least cost LC_i and other costs in the row, as shown in Eqs. (4) and (5).

$$LC_i = min(C_{ij}), \quad j = 1..n \tag{4}$$

$$P_i = \sum_{j=1}^{n}(C_{ij} - LC_i) \tag{5}$$

Step 6: Select the Highest Penalty (HP) as shown in Eq. (6). In case of a tie (i.e. equal HP), use the following tie-breakers in the given order: (i) Select HP with the smallest C_{ij}. (ii) In case of a tie in (i), select penalty with the greatest total cost TC_i as shown in Eq. (7). (iii) In case of a tie in (ii), select penalty with the max allocation of $X_{i,j}$.

$$HP = max(Pi), \; i = 1..m \tag{6}$$

$$TC_i = \sum_{j=1}^{n} C_{ij} \tag{7}$$

Step 7: Select the least cost (LC) from the highest penalty. In case of a tie (i.e. equal LC), select LC with the max allocation of $X_{i,j}$.
Step 8: Check the value of LC. If LC is not equal to zero, then go to step 9, else if LC equal to zero then select the HP from the first HP (HP_1) or second HP (HP_2). Select the HP by comparing each cost cell in HP_1 and each cost cell in HP_2. C_{1j} is the cost at HP_1 and C_{2j} is the cost at HP_2. The value of GV_{1j} is 1 if the cost in HP_1 is greater than the cost in HP_2 and 0 if the cost in HP_1 is smaller than the cost in HP_2. The value of GV_{2j} is 1 if the cost in HP_1 is smaller than the cost in HP_2 and 0 if the cost in HP_1 is greater than the cost in HP_2. $TotalGV_{1j}$ is the sum of GV_{1j} as shown in Eq. (8) and $TotalGV_{2j}$ is the sum of GV_{2j} as shown in Eq. (9). HP is HP_1 if $TotalGV_{1j}$ is greater than $TotalGV_{2j}$ and HP is HP_2 if $TotalGV_{1j}$ is smaller than $TotalGV_{2j}$ as shown in Eq. (10).

$$TotalGV_{1j} = \sum_{j=1}^{n} GV_{1j} \tag{8}$$

where

$$GV_{2j} = \begin{cases} 1 & if\ C_{1j} < C_{2j}, j = 1, 2, ..n \\ 0 & if\ C_{1j} \geqslant C_{2j}, j = 1, 2, ..n \end{cases}$$

$$TotalGV_{2j} = \sum_{j=1}^{n} GV_{2j} \tag{9}$$

where

$$HP = \begin{cases} HP_1 & if \quad TotalGV_{1j} \geqslant TotalGV_{2j} \\ HP_2 & if \quad TotalGV_{1j} < TotalGV_{2j} \end{cases} \tag{10}$$

Step 9: Allocate the maximum possible units X_{ij} to the least cost cell of HP.
Step 10: Adjust the supply and demand then cross out the satisfied row or column.
Step 11: Recalculate the penalty without considering the cross out rows and columns.

Table 4
Comparison of TOCM-MT and TDM1.

Step	Step Description	TOCM-MT	TDM1
1	Construct original TP	Yes	Yes
2	Construct a row opportunity matrix	Yes	No
3	Construct a column opportunity matrix	Yes	No
4	Construct the TOCM	Yes	No
5	Find the penalty Pi for each row	Yes	Yes
6	Rules to select the Highest Penalty	Yes	No
7	Select the least cost	Yes	Yes
8	Check the value of LC	Yes	No
9	Allocate the maximum possible units Xij	Yes	Yes
10	Adjust the supply and demand	Yes	Yes
11	Recalculate the penalty	Yes	Yes
12	Repeat step 6–11	Yes	Yes
13	Calculate TCTP	Yes	Yes

Step 12: Repeat steps 6–11 until all the rows and columns are satisfied.
Step 13: Finally, calculate the Total Cost Transportation Problem (TCTP) by combining TOCM-MT with the penalty and the original transportation problem as shown in equation (11).

$$TCTP = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} \tag{11}$$

There are three differences between TOCM-MT and TDM1. The first one is TOCM-MT uses TOCM matrix while TDM1 uses the original matrix. The second one is TOCM-MT has the rules to select the HP while TDM1 does not. The third one is TOCM-MT checks the value of the least cost before allocating the maximum units X_{ij} while TDM1 directly allocated the maximum units X_{ij} to the least cost. The comparison of TOCM-MT and TDM1 is shown in Table 4.

6. Computational experiment

This research used thirty-one numerical examples to illustrate the proposed method TOCM-MT. Twenty-five numerical examples were selected from journals and six numerical examples were generated randomly. The following sample from Deshmukh [19] is used to illustrate the proposed method. For Examples, a company has 3 supply plants which produce 7, 9 and 18 cars. The company supplies to four buyers whose demands are 5, 8, 7, 14 cars respectively. The transportation cost per piece of cars is given in Table 5. The goal is to find out the schedule of shifting cars from plants to buyers with the minimum total cost.

Step 1: Construct an original Transportation Problem (TP) as shown in Table 5.
Step 2: Construct a row opportunity matrix from the original TP; find the least cost of each row then subtract each cost in the row with the least cost, e.g. the least cost of row 1 is 10, then subtract each cost in the cell with 10.
Step 3: Construct a column opportunity matrix from the original TP; find the least cost of each column then subtract each

Table 5
An original transportation problem.

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

Table 6
TOCM Matrix of original TP.

	D1	D2	D3	D4	Supply
S1	9	42	50	0	7
S2	91	22	10	80	9
S3	53	0	92	22	18
Demand	5	8	7	14	

Table 7
Penalty 1 for each row and allocation 7 unit to the cell (2,3).

	D1	**D2**	**D3**	**D4**	**Supply**	**Penalty 1**
S1	9	42	50 x	0	**7**	101
S2	91	22	10 $\boxed{7}$	80	**2**	163
S3	53	0	92 x	22	**18**	167
Demand	**5**	**8**	**0**	**14**		

cost in the column with the least cost, e.g. the least cost of column 1 is 19, then subtract each cost in the cell with 19.
Step 4: Construct the TOCM in which the entries are the sum of the row and column opportunity matrix as shown in Table 6.
Step 5: Find the penalty for each row. The penalty is the total difference between the least cost and other costs in the row. Penalty 1 of row 1 is 101, row 2 is 163, and row 3 is 167 as shown in Table 7.
Step 6: Select the Highest Penalty (HP). The HP of Penalty 1 is 167 at row 3.
Step 7: Select the least cost (LC) from the highest penalty. The LC is 0.
Step 8: Check the value of LC. Because LC is equal to zero then select the HP from the first HP (HP_1) or second HP (HP_2). The first HP is 167 and second HP is 163.

Select the HP by comparing each cost cell in HP_1 and each cost cell in HP_2.

C_{11}, C_{12}, C_{13}, and C_{14} are cost at HP_1 where C_{11} is 53, C_{12} is 0, C_{13} is 92, and C_{13} is 22. C_{21}, C_{22}, C_{23}, and C_{24} are cost at HP_2 where C_{21} is 91, C_{22} is 22, C_{23} is 10, and C_{24} is 80.

The value of GV_{11} is 0 because 53 is smaller than 91. The value of GV_{12} is 0 because 0 is smaller than 22. The value of GV_{13} is 1 because 92 is greater than 10. The value of GV_{14} is 0 because 22 is smaller than 80. The value of GV_{21} is 1 because 53 is smaller than 91. The value of GV_22 is 1 because 0 is smaller than 22. The value of GV_{23} is 0 because 92 is greater than 10. The value of GV_{24} is 1 because 22 is smaller than 80. $TotalGV_{1j}$ is the sum of GV_{11}, GV_12, GV_{13}, and GV_{14}, and then the $TotalGV_{1j}$ is 1. $TotalGV_{2j}$ is the sum of GV_{21}, GV_{22}, GV_{23}, and GV_{24}, and then the $TotalGV_{2j}$ is 3. Because $TotalGV_{2j}$ is greater than $TotalGV_{1j}$, then HP is HP_2.

Step 9: Allocate the maximum possible units X_{ij} to the least cost cell of HP as shown in Table 7. Allocate 7 units (box) to the cell (3), (4). Adjust the supply and demand, supply S2 become 2 from 9 (it is remaining 2 cars in the Supply 2) and demand D3 become 0 from 7 (it means that the demand 3 has been satisfied)
Step 10: Adjust the supply and demand then cross out the satisfied row or column. Cross out the satisfied demand D3.
Step 11: Recalculate the penalty 2 without considering the cross out rows and columns as shown in Table 8. The penalty 2 of row 1 is 51, row 2 is 127, and row 3 is 75.
Step 12: Go to step 6 and repeat steps 6 – 11 until the rows and columns are satisfied. The final table is shown in Table 8.
Step 13: Finally, calculate the Total Cost Transportation Problem (TCTP) by combining TOCM-MT with the penalty and original transportation problem as shown in Table 9.

Hence the allocation units given from Table 9 are as follows: $X_{11} = 5$, $X_{14} = 2$, $X_{22} = 2$, X_{23} is 7, $X_{32} = 6$, $X_{34} = 12$, along with associated total cost 743, which is the optimal solution for this numerical example. Note that the total cost of this numerical example found by VAM and TDM1 is 779, which is not the optimal solution.

Based on the above description, TDM1 did not get the optimal solution which is different from TOCM-MT. The allocation units given from Table 10 are as follows: $X_{11} = 5$, $X_{14} = 2$, $X_{23} = 7$, $X_{24} = 2$, $X_{32} = 8$, $X_{34} = 10$, along with associated total cost 779.

7. Experimental result

This section provides the comparisons among the existing methods of VAM, JHM, TDM1, and the proposed one TOCM-MT. The comparison results are shown in Tables 11, 12, and in Figs. 2–5. The 25 numerical examples used in this experiment are taken from 20 different journals and also 6 numerical examples randomly generated. The detail data of 6 randomly generated samples are given in Appendix A. TOCM-MT, TDM1, and JHM have been coded in C++ programming language and run successfully for the solution of 31 numerical examples. TORA software was applied in VAM and optimal solution.

The experimental results of VAM, JHM, TDM1, and TOCM-MT for 25 numerical examples from the journal are shown in Table 11 and for 6 randomly generated numerical examples are shown in Table 12. Tables 11 and 12 show the improvement percentage of TOCM-MT over VAM, JHM and TDM1, in which the positive number means that TOCM-MT provides better results compared to VAM, JHM and TDM1, zero value indicates that the total cost of TOCM-MT is the same as VAM, JHM and TDM1, and the negative number means that VAM, JHM and TDM1 yields better results

Table 8
TOCM-MT with a penalty based on Deshmukh numerical example.

	D1	**D2**	**D3**	**D4**	**Supply**	**P*.1**	**P*.2**	**P*.3**	**P*.4**	**P*.5**
S1	9 $\boxed{5}$	42 x	50 x	0 $\boxed{2}$	7	101	51	51	9	9
S2	91 x	22 $\boxed{2}$	10 $\boxed{7}$	80 x	0	163	127	0	0	0
S3	53 x	0 $\boxed{6}$	92 x	22 $\boxed{12}$	0	167	75	75	31	0
Demand	5	0	0	2						

* P=Penalty

Table 9
Combining TOCM-MT with the penalty and original transportation problem based on Deshmukh numerical example.

	D1	D2	D3	D4	Supply
S1	19 $\boxed{5}$	30	50	10 $\boxed{2}$	7
S2	70	30 $\boxed{2}$	40 $\boxed{7}$	60	9
S3	40	8 $\boxed{6}$	70	20 $\boxed{12}$	18
Demand	5	8	7	14	

compared to TOCM-MT. The improvement percentage [1] is calculated using Eq. (12).

$$Ip = \frac{IBFS - Tm}{IBFS} x100 \tag{12}$$

Notation

Ip	Improvement
IBFS	initial basis feasible solution
Tm	TOCM-MT

Table 10
The final result of TDM1 based on Deshmukh numerical example.

	D1	D2	D3	D4	Supply	P*.1	P*.2	P*.3	P*.4	P*.5
S1	19 $\boxed{5}$	30	50	10 $\boxed{2}$	7	69	49	49	9	9
S2	70	30	40 $\boxed{7}$	60 $\boxed{2}$	~~9~~ ~~2~~	80	50	50	10	0
S3	40	8 $\boxed{8}$	70	20 $\boxed{10}$	~~18~~ ~~10~~	106	70	0	0	0
Demand	5	~~8~~	~~7~~	~~14~~ 4 ~~2~~						

* P=Penalty

Table 11
The experimental results of VAM, JHM, TDM1, and TOCM-MT for 25 numerical examples from the journal.

Problem chosen	Initial Basic Feasible Solution (IBFS)				Optimal (Op)	Improvement of TOCM-MT over VAM, JHM, and TDM1 (%) -(Ip)			Deviation percentage from optimal (%) (Dv)			
	VAM	JHM	TDM1	TOCM-MT (Tm)		VAM	JHM	TDM1	VAM	JHM	TDM1	TOCM-MT
Srinivasan [47]	955	880	880	880*	880	7.85	0	0	8.52	0	0	0
Sen et al. [44]	2,164,000	2,146,750	2,158,500	2,158,500	2,146,750	0.25	−0.55	0	0.8	0	0.55	0.55
Goyal [24]	1,745	1,650	1,650	1,650*	1,650	5.44	0	0	5.76	0	0	0
Deshmukh [19]	779	743	779	743*	743	4.62	0	4.62	4.85	0	4.85	0
Ramadan [48]	5,600	5,600	5,600	5,600*	5,600	0	0	0	0	0	0	0
Kulkarni [49]	880	840	980	980	840	−11.36	−16.67	0	4.76	0	16.67	16.67
Schrenk [50]	59	59	59	61	59	−3.39	−3.39	−3.39	0	0	0	3.39
Samuel [51]	28	28	28	28*	28	0	0	0	0	0	0	0
Imam et al. [43]	475	460	475	435*	435	8.42	5.43	8.42	9.2	5.75	9.2	0
Adlakha [52]	390	390	400	390*	390	0	0	2.5	0	0	2.56	0
Juman [1]	3,663	3,458	3,572	3,513	3,458	4.1	−1.59	1.65	5.93	0	3.3	1.59
Juman [1]	109	109	117	109*	109	0	0	6.84	0	0	7.34	0
Ahmed [18]	470	420	435	435	410	7.45	−3.57	0	14.63	2.44	6.1	6.1
Ahmed [18]	2,850	2,850	2,850	2,850*	2,850	0	0	0	0	0	0	0
Ahmed [17]	187	183	186	187	183	0	−2.19	−0.54	2.19	0	1.64	2.19
Uddin and Khan [10]	859	799	859	799*	799	6.98	0	6.98	7.51	0	7.51	0
Uddin and Khan [10]	273	273	273	273*	273	0	0	0	0	0	0	0
Das et al. [25]	1,220	1,170	1,160	1,160*	1,160	4.92	0.85	0	5.17	0.86	0	0
Khan et al. [31]	204	218	200	200*	200	1.96	8.26	0	2	9	0	0
Azad and Hossain [27]	248	240	248	240*	240	3.23	0	3.23	3.33	0	3.33	0
Morade [53]	820	820	820	820*	820	0	0	0	0	0	0	0
Jude [54]	190	190	190	190*	190	0	0	0	0	0	0	0
Jude [54]	92	83	83	83*	83	9.78	0	0	10.84	0	0	0
Hosseini [3]	3,520	3,460	3,570	3,460*	3,460	1.7	0	3.08	1.73	0	3.18	0
Hosseini [3]	650	610	650	610*	610	6.15	0	6.15	6.56	0	6.56	0

* Optimal solution.

Table 12
The experimental results of VAM, JHM, TDM1, and TOCM-MT 6 numerical examples randomly generated samples.

Sample No.	Initial Basic Feasible Solution (IBFS)				Optimal (Op)	Improvement of TOCM-MT over VAM, JHM, and TDM1 (%) -(Ip)			Deviation percentage from optimal (%) (Dv)			
	VAM	JHM	TDM1	TOCM-MT (Tm)		VAM	JHM	TDM1	VAM	JHM	TDM1	TOCM-MT
1	109	109	117	109*	109	0	0	6.84	0	0	7.34	0
2	990	960	990	910*	910	8.08	5.21	8.08	8.79	5.49	8.79	0
3	1,680	1,690	1,670	1,670*	1,670	0.6	1.18	0	0.6	1.2	0	0
4	2,400	2,340	2,400	2,400	2,280	0	−2.56	0	5.26	2.63	5.26	5.26
5	2,980	2,500	2,980	2,460*	2,460	17.45	1.6	17.45	21.14	1.63	21.14	0
6	327	327	291	291*	291	11.01	11.01	0	12.37	12.37	0	0

* Optimal solution.

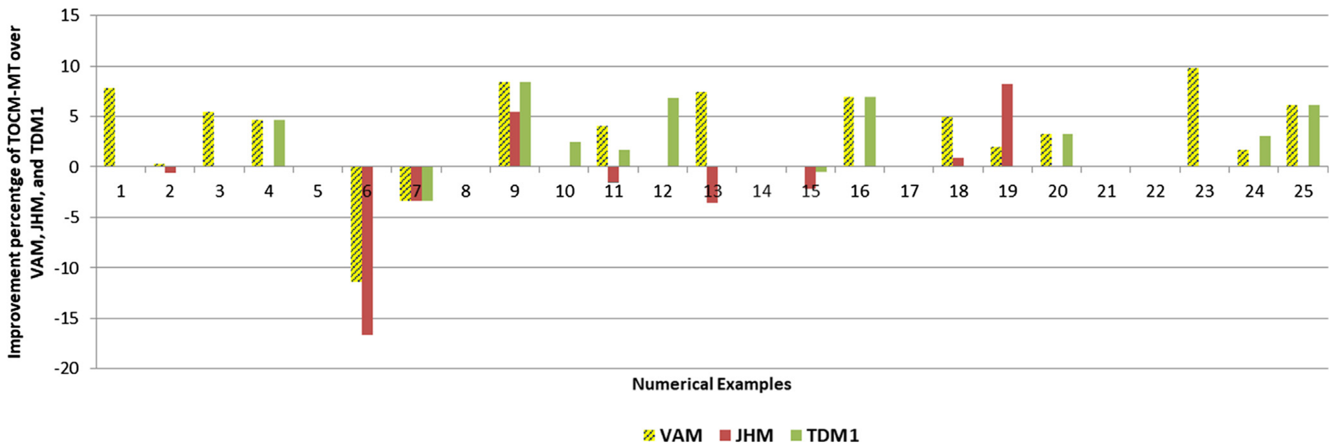

Fig. 2. The improvement percentage of TOCM-MT over VAM, JHM, and TDM1 for 25 numerical examples problem from the journal.

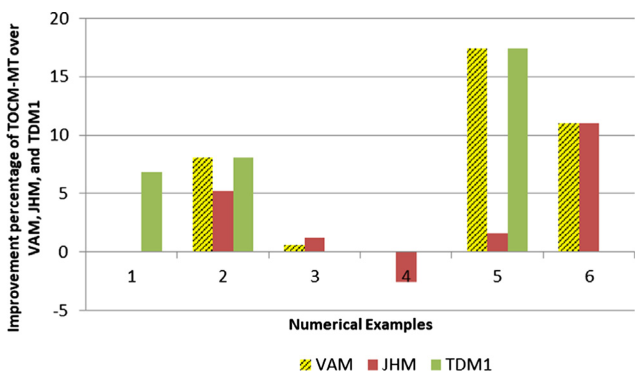

Fig. 3. The improvement percentage of TOCM-MT over VAM, JHM, and TDM1 for 6 numerical examples randomly generated samples.

Tables 11 and 12 show the TOCM-MT provides 18 better results compared to VAM and both have the same result in 11 numerical examples. In the remaining 2 numerical examples, VAM produces better result compared to TOCM-MT. The TOCM-MT obtains 7 better results compared to JHM and both have the same results in 17 numerical examples which the rest 7 numerical examples shown that JHM produces better result compared to TOCM-MT. The TOCM-MT also provides 12 better results compared to TDM1 and obtains the same results in 17 numerical examples for both methods. TDM1 produces better result compared to TOCM-MT in the rest 2 numerical examples.

Tables 11 and 12 also show the deviation percentage of VAM, JHM, TDM1, and TOCM-MT from the optimal solution, in which the zero value indicates that the total cost is the optimal solution and the positive number means that the total cost is larger than the optimal solution. The deviation percentage [1] is determined using Eq. (13).

$$Dv = \frac{IBFS - Op}{Op} x100 \tag{13}$$

Notation:

Dv	Deviation
IBFS	Initial basic feasible solution
Op	Optimal

Base on the deviation percentage, Tables 11 And 12 show that the TOCM-MT leads to the optimal solution 24 out of 31 numerical examples, whereas each VAM, JHM, and TDM1 leads to optimal solution 10, 22 and 14 out of 31 numerical examples, respectively. TOCM-MT leads to optimal solution 24 out of 31 numerical examples and obtains 77.42% accuracy. Otherwise, VAM leads to optimal solution 10 out of 31 numerical examples and obtains 32.26% accuracy. JHM leads to optimal solution 22 out of 31 numerical examples and obtains 70.96% accuracy. At last, TDM1 leads to the optimal solution 14 out of 31 numerical examples and obtains 45.16% accuracy.

The positive value of improvement percentage that shown in Figs. 2 and 3 indicated that TOCM-MT provides better result compare with VAM, JHM, and TDM1 in most cases. The zero value of

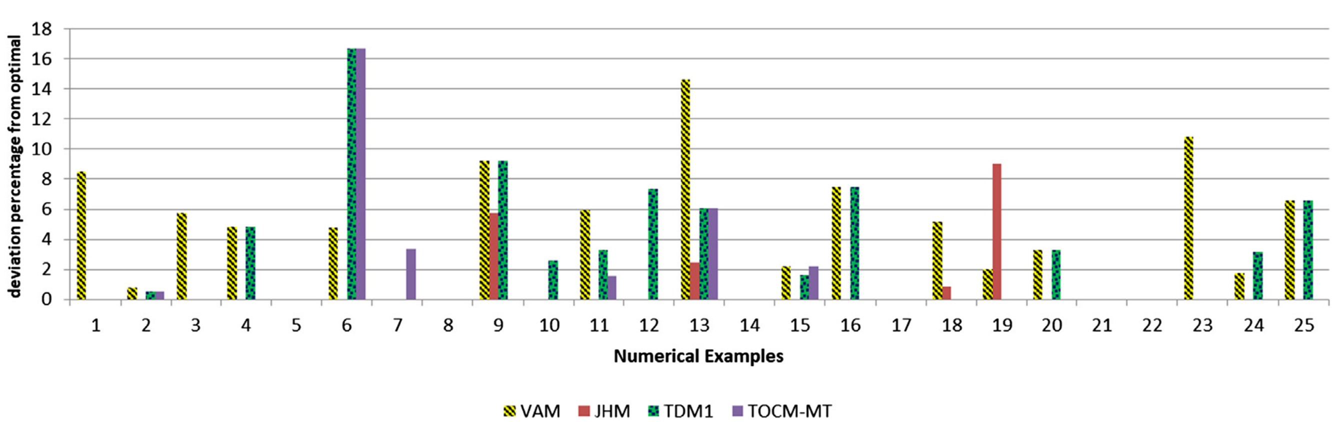

Fig. 4. The deviation percentage of VAM, JHM, TDM1 and TOCM-MT for 25 numerical examples from the journal.

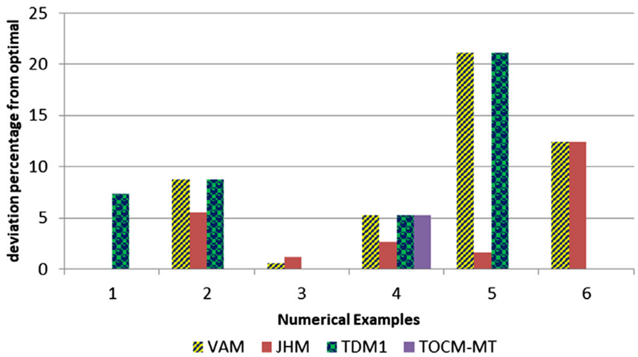

Fig. 5. The deviation percentage of VAM, JHM, TDM1 and TOCM-MT for 6 numerical examples randomly generated samples.

deviation percentage that shown in Figs. 4 and 5 indicated that TOCM-MT achieves the optimal solution in most cases.

The numerical examples for the benefits of TOCM-MT over TDM1 can be expressed as follows:

1. TOCM-MT uses TOCM as the initial matrix, while TDM1 uses the original matrix. The following sample from Hosseini [3] is used to illustrate the benefit of using TOCM as the initial matrix, and no ties exist.
   Table 13 shows the final result of TOCM-MT with the penalty. Table 14 shows the combining TOCM-MT with the penalty and original transportation problem. The final result of TDM1 which uses the original matrix is shown in Table 15.
   The allocation units given from Table 14 are as follows: $X_{11} = 60$, $X_{12} = 40$, $X_{13} = 20$, $X_{23} = 30$, $X_{24} = 40$, $X_{34} = 15$, along with associated total cost 3.460, which is the optimal solution for this numerical example.It
   could be seen that TOCM-MT is better than TDM1 in getting the optimal solution. The allocation units given from Table 15 are as follows: $X_{11} = 40$, $X_{14} = 40$, $X_{14} = 40$, $X_{24} = 70$, $X_{31} = 20$, $X_{33} = 30$, along with associated total cost 3.570.
2. TOCM-MT has rules to select HP while TDM1 chooses HP arbitrarily when there are several HP with the same values. The following sample from Juman [1] is used to illustrate the benefit of using TOCM-MT when there are several same values of HP.
   Table 16 shows the final result of TOCM-MT with the penalty. Table 17 shows the combining TOCM-MT with the penalty and original transportation problem. The final result of TDM1 which choose the HP arbitrarily when there are several same values of HP is shown in Table 18.
   The allocation units given from Table 17 are as follows: $X_{13} = 1$, $X_{16} = 1$, $X_{26} = 5$, $X_{31} = 1$, $X_{32} = 2$, $X_{33} = 3$, $X_{41} = 1$, $X_{44} = 4$, $X_{45} = 4$, along with associated total cost 109, which is the optimal solution for this numerical example.It

Table 14
Combining TOCM-MT with the penalty and original transportation problem based on Hosseini numerical example.

	D1	D2	D3	D4	Supply
S1	20 / $\boxed{60}$	22 / $\boxed{40}$	17 / x	4 / $\boxed{20}$	120
S2	24 / x	37 / x	9 / $\boxed{30}$	7 / $\boxed{40}$	70
S3	32 / x	37 / x	20 / x	15 / $\boxed{50}$	50
Demand	60	40	30	110	

   could be seen that TOCM-MT is better than TDM1 in getting the optimal solution. The allocation units given from Table 18 are as follows: $X_{13} = 2$, $X_{26} = 5$, $X_{32} = 2$, $X_{35} = 4$, $X_{41} = 2$, $X_{33} = 3$, $X_{43} = 2$, $X_{44} = 4$, $X_{46} = 1$, along with associated total cost 117.
3. TOCM-MT has a mechanism to allocate the maximum units to the least cost cell when the least cost equals to zero while TDM1 directly allocates maximum units to the least cost cell. The following sample from Deshmukh [19] is used to illustrate the benefit of using TOCM-MT when the least cost equal to zero. Table 8 shows the final result of TOCM-MT with the penalty. Table 9 shows the combining TOCM-MT with the penalty and original transportation problem. The final result of TDM1 when the least cost equal to zero is shown in Table 10. Hence the allocation units given from Table 9 are as follows: $X_{11} = 5$, $X_{14} = 2$, $X_{22} = 2$, $X_{23} = 7$, $X_{32} = 6$, $X_{34} = 12$, along with associated total cost 743, which is the optimal solution for this numerical example.It
   could be seen that TOCM-MT is better than TDM1 in getting the optimal solution. The allocation units given from Table 9 are as follows $X_{11} = 5$, $X_{14} = 2$, $X_{23} = 7$, $X_{24} = 2$, $X_{32} = 8$, $X_{34} = 10$, along with associated total cost 779.

8. Conclusion

Initial basic feasible solution (IBFS) is one of the main steps to achieve an optimal solution for TP. This research developed a new method called TOCM-MT to determine IBFS of TP. TOCM-MT is coded using C++ programming language. TOCM-MT can achieve a total cost which similar or closer values to the optimal solution.

TOCM-MT shows better performance than VAM, JHM, and TDM1 because TOCM-MT considers a total opportunity cost matrix for the initial matrix, provides a better mechanism if there are several HP with the same values and if the least cost of highest penalty is equal to zero. To evaluate the proposed method, thirty-one numerical examples were used in which twenty-five were selected from journals and six were generated randomly.

The comparative study indicated that TOCM-MT obtained eighteen better results than that of VAM, seven better results than that

Table 13
TOCM-MT with a penalty based on Hosseini numerical example.

	D1	D2	D3	D4	Supply	P*.1	P*.2	P*.3	P*.4
S1	16 / $\boxed{60}$	18 / $\boxed{40}$	21 / x	0 / $\boxed{20}$	120	55	34	34	34
S2	21 / x	45 / x	2 / $\boxed{30}$	3 / $\boxed{40}$	70	62	60	0	0
S3	29 / x	37 / x	16 / x	11 / $\boxed{50}$	50	49	44	44	0
Demand	60	40	30	110					

* P=Penalty

Table 15
The final result of TDM1 based on Hosseini numerical example.

	D1	D2	D3	D4	Supply	P*.1	P*.2	P*.3	P*.4	P*.5
S1	20 $\boxed{40}$	22 $\boxed{40}$	17	4 $\boxed{40}$	120	47	47	8	2	2
S2	24	37	9	7 $\boxed{70}$	70	49	0	0	0	0
S3	32 $\boxed{20}$	37	20 $\boxed{30}$	15	50	44	44	29	5	0
Demand	60	40	30	110						

* P=Penalty

Table 16
TOCM-MT with a penalty based on Juman numerical example.

	D1	D2	D3	D4	D5	D6	Supply	P*.1	P*.2	P*.3	P*.4	P*.5	P*.6	P*.7
S1	6	15	5 $\boxed{1}$	4	10	9 $\boxed{1}$	2	25	20	15	15	14	4	4
S2	5	0	4	9	5	2 $\boxed{5}$	5	25	16	11	11	6	2	2
S3	3 $\boxed{1}$	4 $\boxed{2}$	8 $\boxed{3}$	17	1	14	6	41	25	17	17	14	6	0
S4	4 $\boxed{1}$	11	13	0 $\boxed{4}$	0 $\boxed{4}$	13	9	41	41	25	0	0	0	0
Demand	2	2	4	4	4	6								

* P=Penalty

Table 17
Combining TOCM-MT with the penalty and original transportation problem based on Juman numerical example.

	D1	D2	D3	D4	D5	D6	Supply
S1	9	12	9 $\boxed{1}$	6	9	10 $\boxed{1}$	2
S2	7	3	7	7	5	5 $\boxed{5}$	5
S3	6 $\boxed{1}$	5 $\boxed{2}$	9 $\boxed{3}$	11	3	11	6
S4	6 $\boxed{1}$	8	11	2 $\boxed{4}$	2 $\boxed{4}$	10	9
Demand	2	2	4	4	4	6	

Table 18
The final result of TDM1 based on Juman numerical example.

	D1	D2	D3	D4	D5	D6	Supply	P*.1	P*.2	P*.3	P*.4	P*.5	P*.6
S1	9	12	9 $\boxed{2}$	6	9	10	2	19	16	4	1	1	1
S2	7	3	7	7	5	5 $\boxed{5}$	5	16	14	10	4	2	0
S3	6	5 $\boxed{2}$	9	11	3 $\boxed{4}$	11	6	27	17	11	0	0	0
S4	6 $\boxed{2}$	8	11 $\boxed{2}$	2 $\boxed{4}$	2	10 $\boxed{1}$	9	27	27	11	9	1	1
Demand	2	2	4	4	4	6							

* P=Penalty

of JHM, and twelve better results than that of TDM1. TOCM-MT was found to have 24 optimal solutions out of 31 numerical examples, thus this method gets 77.42% accuracy. Otherwise, VAM, JHM, and TDM1 obtain an accuracy level of 32.26%, 70.96%, and 45.16% respectively.

The future research might be carried out in developing the proposed IBFS method for real application in case of incomplete information about the parameters of the TP.

Appendix A

Six problem that are randomly generated.

Problem 1	Problem 4
$[C_{ij}]_{4x6}$ = [9 12 9 6 9 10; 7 3 7 7 5 5; 6 5 9 11 3 11; 6 8 11 2 2 10]	$[C_{ij}]_{4x4}$ = [21 5 9 11; 12 3 8 6; 9 8 10 5; 6 6 7 3]
$[S_i]_{4x1}$ = [2, 5, 6, 9]	$[S_i]_{4x1}$ = [120, 100, 80, 60]
$[D_j]_{1x6}$ = [2, 2, 4, 4, 4, 6]	$[D_j]_{1x4}$ = [120, 120, 80, 40]
Problem 2	**Problem 5**
$[C_{ij}]_{3x4}$ = [20 2 20 11; 24 7 9 20; 8 14 16 18]	$[C_{ij}]_{3x4}$ = [10 2 20 22; 12 7 9 40; 4 14 16 32]
$[S_i]_{3x1}$ = [30, 50, 20]	$[S_i]_{3x1}$ = [60, 100, 40]
$[D_j]_{1x4}$ = [10, 30, 30, 30]	$[D_j]_{1x4}$ = [20, 60, 60, 60]
Problem 3	**Problem 6**
$[C_{ij}]_{4x4}$ = [7 5 27 22; 4 3 24 12; 6 16 60 20; 2 6 21 6]	$[C_{ij}]_{3x3}$ = [7 8 7; 18 8 12; 8 12 12]
$[S_i]_{4x1}$ = [60, 50, 40, 30]	$[S_i]_{3x1}$ = [13, 14, 8]
$[D_j]_{1x4}$ = [60, 60, 40, 20]	$[D_j]_{1x3}$ = [14, 8, 13]

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