On Well-Foundedness and Expressiveness of Promoted Tyft

(Being Promoted Makes a Difference)

MohammadReza Mousavi [a,b,1] and Michel Reniers [a]

[a] *Department of Computer Science, Eindhoven University of Technology, P.O. Box 513, NL-5600 MB Eindhoven, The Netherlands*

[b] *Department of Computer Science, Reykjavík University, Kringlan 1, IS-103 Reykjavík, Iceland*

Abstract

In this paper, we solve two open problems posed by Karen L. Bernstein regarding her promoted tyft format for structured operational semantics. We show that, unlike formats with closed terms as labels, such as the tyft format, the well-foundedness assumption cannot be dropped for the promoted tyft format while preserving the congruence result. We also show that the well-founded promoted tyft format is incomparable to the tyft format with closed terms as labels, i.e., there are transition relations that can be specified by the promoted tyft format but not by the tyft format, and vice versa.

Keywords: Structural Operational Semantics (SOS), SOS Rule Formats, Promoted Tyft, Tyft.

1 Introduction

In [3], Bernstein proposed the promoted tyft format which is an elegant framework for specifying the operational semantics of higher-order processes. She proved that the *well-founded* promoted tyft format guarantees strong bisimilarity to be a congruence. The semantics of the lazy lambda-calculus [1] and the Calculus of Higher-Order Communicating Systems (CHOCS) [9] were specified in the promoted tyft format of [3] and using the congruence meta-theorem, it was shown that applicative bisimilarity for the lazy-lambda calculus and strong bisimilarity for CHOCS are both congruences. The conclusions of [3] reads as follows.

"In this paper, we have described a rule format that is a simple but expressive generalization of Groote and Vaandrager's tyft/tyxt rule format. ... There are

---

several open questions related to the work in this paper. It is not clear that the well-foundedness property is necessary for the congruence result. We are not sure how the extensions to tyft/tyxt format that allow negative premises are compatible with our extensions. It is not clear whether promoted tyft/tyxt format is strictly more expressive than tyft/tyxt format."

We touched upon the second open question in another publication [8]. In this paper, we answer the first and the third questions as follows.

- We show that the promoted tyft format does not necessarily induce congruence of strong bisimilarity if the well-foundedness assumption is omitted;

- We show that the well-founded subset of the promoted tyft format is incomparable, in its expressiveness, with the tyft format. In other words, we give two counter-examples witnessing that there exist transition relations that can be specified by one rule format but not by the other.

The rest of this paper is organized as follows. In Section 2, we give some basic definitions. Section 3 addresses the well-foundedness concept, shows that it cannot be dropped for the promoted tyft format while preserving the congruence result. Section 4 addresses the expressiveness of the promoted tyft format and proves it incomparable to the tyft format. The paper is concluded in Section 5.

2 Preliminaries

Definition 2.1 (Signature, Term and Substitution) *Assume a countable set of variables V (with typical members x, y, x', y', x_i, y_i ...). A signature Σ is a set of function symbols (operators, with typical members f, g, ...) with fixed arities $ar : \Sigma \to \mathbb{N}$. Functions with zero arity are called constants. Terms $s, t, t_i, \ldots \in \mathcal{T}$ are constructed inductively using variables and function symbols. A list of terms is denoted by $\overrightarrow{t}$. When we write $f(\overrightarrow{t})$, we assume that $\overrightarrow{t}$ has the right size, i.e., $ar(f)$. All terms are considered open terms. Closed terms $p, q, \ldots \in \mathcal{C}$ are terms that do not mention a variable and are typically denoted by $p, q, p', q', p_i, \ldots$. A substitution σ replaces variables in a term with terms. The set of variables appearing in term t is denoted by $vars(t)$.*

Definition 2.2 (Transition System Specification (TSS)) *A TSS is a pair (Σ, D) where Σ is a signature and D is a set of deduction rules. A deduction rule $dr \in D$, is defined as a pair (H, c) where H is a set of formulae and c is a formula. For all $t, t', t'' \in \mathcal{T}$ we define that $t \xrightarrow{t'} t''$ is a formula. The formula c is called the conclusion of dr and the formulae from H are called its premises. A deduction rule (H, c) is mostly denoted by $\dfrac{H}{c}$.*

The concept of closed-ness and substitution are lifted to formulae and sets of formulae in the natural way (i.e., a substitution applied to a formula, applies to all three terms). We refer to t as the source, t' as the label and t'' as the target of the transition. We may also write $vars(\phi)$ and $vars(H)$ to denote variables appearing in a formula and in a set of formulae, respectively.

In this paper, we only deal with positive TSS's. Hence, the semantics of TSS's is straight-forward and is defined as the set of all closed provable formulae using instances of deduction rules.

Definition 2.3 (Tyft [6] and Promoted Tyft [3]) *A deduction rule is in* tyft *format if and only if it has the following form*

$$\frac{\{t_i \xrightarrow{t_i'} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{t'} t''},$$

where variables in $\overrightarrow{x}$ and y_i's are all distinct variables, all labels, i.e., t' and t_i''s, are closed terms and I is a (possibly infinite) set of indices.

A rule of the above form is in the promoted tyft *format if the source and targets of all formulae in it conform to the constraints of the* tyft *format and further t_i''s contain at least one function symbol (i.e., are not variable), t' is of the form $g(\overrightarrow{z})$ where variables in $\overrightarrow{z}$ are all distinct and different from variables in $\overrightarrow{x}$ and y_i's.*

A TSS is in tyft *(*promoted tyft*) format if and only if all its deduction rules are.*

A subset of the tyft format is the one using constants (instead of closed terms) as labels which is also considered in this paper and compared to the promoted tyft format in Section 4. Arguably, this subset can be considered the original definition of the tyft format as defined by [6]. The generalization to *closed* terms as labels (if at all considered a generalization) is entirely safe and orthogonal to all existing results (e.g., congruence, conservativity and commutativity meta-theorems [2,5]).

The transition relation induced by a TSS (in the above two formats) is the set of all provable formulae as defined below.

Definition 2.4 *A* proof *of a closed formula ϕ is a well-founded upwardly branching tree whose nodes are labelled by closed formulae such that*

- *the root node is labelled by ϕ, and*
- *if ψ is the label of a node and $\{\psi_i \mid i \in I\}$ is the set of labels of the nodes directly above this node, then there are a deduction rule $\dfrac{\{\chi_i \mid i \in I\}}{\chi}$ and a substitution σ such that $\sigma(\chi) = \psi$, and for all $i \in I$, $\sigma(\chi_i) = \psi_i$.*

Definition 2.5 (Strong (Bi)similarity) *A relation $R \subseteq \mathcal{C} \times \mathcal{C}$ is a strong simulation relation when $\forall_{p,q \in \mathcal{C}} \ pRq \Rightarrow \forall_{p',p'' \in \mathcal{C}} \ p \xrightarrow{p'} p'' \Rightarrow \exists_{q'' \in \mathcal{C}} \ q \xrightarrow{p'} q'' \wedge (p'', q'') \in R$. A symmetric strong simulation relation is a strong bisimulation relation. Closed terms p and q are (bi)similar, denoted by $p \lesssim q$ ($p \leftrightarrow q$) if there is a strong (bi)simulation relation R such that $p \ R \ q$.*

For a relation $R \subseteq \mathcal{C} \times \mathcal{C}$, we write $\overrightarrow{p} \ R \ \overrightarrow{q}$ and by that we mean $\overrightarrow{p}$ and $\overrightarrow{q}$ have the same size (possibly zero) and for all p_i and q_i at the same position in the two lists $p_i \ R \ q_i$.

Lemma 2.6 (Substituting Bisimilar Labels Under Context [3]) *For a TSS*

*in the **promoted tyft** format,* $\forall_{p,q,\overrightarrow{p'},\overrightarrow{q'},p'' \in \mathcal{C}} \forall_{f \in \Sigma}\quad p \overset{f(\overrightarrow{p'})}{\to} p'' \wedge p \underleftrightarrow{\ } q \wedge \overrightarrow{p'} \underleftrightarrow{\ } \overrightarrow{q'} \Rightarrow$
$\exists_{q'' \in \mathcal{C}} q \overset{f(\overrightarrow{q'})}{\to} q'' \wedge p'' \underleftrightarrow{\ } q''.$

Definition 2.7 ((Pre-)Congruence) *An equivalence (a pre-order) $R \subseteq \mathcal{C} \times \mathcal{C}$ is a* (pre-)congruence *when $\forall_{f \in \Sigma} \forall_{\overrightarrow{p},\overrightarrow{q} \in \mathcal{C}}\ \overrightarrow{p}\ R\ \overrightarrow{q} \Rightarrow f(\overrightarrow{p})\ R\ f(\overrightarrow{q}).$*

3 Well-Foundedness

In [3], Bernstein proposes a definition of well-foundedness which coincides with the following and proves that for the well-founded subset of the **promoted tyft** format, bisimilarity is a congruence.

Definition 3.1 *The variable dependency graph of a deduction rule is a graph of which the nodes are variables and there is an edge from x to y when y appears in the target of a premise and x in its source or label. A deduction rule is well-founded when there is no backward chain of infinite length in the variable dependency graph. A TSS is well-founded when all its deduction rules are.*

Note that this definition coincides with that of [6] in case of TSS's with closed terms as labels. An alternative definition of well-foundedness is the one that treats the labels in the same way as the targets of formulae (while in the above definition labels are treated like sources). This alternative definition, called *p-well-foundedness* in [8], is not useful for proving congruence of strong bisimilarity (while it is useful for proving congruence of higher-order bisimilarity) and in fact, as shown below, there are p-well-founded TSS's in the **promoted tyft** format for which bisimilarity is not a congruence.

Theorem 3.2 (Congruence for Well-founded (Promoted [3]) Tyft [6])
*For a well-founded TSS in the (*promoted*) **tyft** format, strong bisimilarity is a congruence.*

Theorem 3.3 (Tyft Reduces to Well-founded Tyft [4]) *For an arbitrary TSS in the **tyft** format, there exists a well-founded TSS in the **tyft** format which induces the same transition relation.*

In the following three examples, we show that the congruence result for bisimilarity can be ruined if the TSS's in the **promoted tyft** format do not satisfy the well-foundedness assumption. The first example violates the well-foundedness assumption by having a self-loop on a variable which appears both in the label and the target of a premise.

Example 3.4 *Consider the following set of deduction rules defined on a signature with 0 and 1 and f as a unary function symbol.[2] The following TSS is in the **promoted tyft** format. Note that the last deduction rule is not well-founded due to the occurrence of y both in the target and the label of the premise. (This deduction*

---

[2] In the coming examples we omit stating the precise signature as it is clear from the symbols used in the deduction rules.

rule is indeed p-well-founded.)

$$\frac{}{0 \xrightarrow{0} 1} \quad \frac{}{1 \xrightarrow{0} 0} \quad \frac{x \xrightarrow{0} y}{1 \xrightarrow{f(x)} x} \quad \frac{x \xrightarrow{0} y}{0 \xrightarrow{f(x)} y} \quad \frac{x \xrightarrow{f(y)} y}{f(x) \xrightarrow{1} y}$$

The following is the transition relation induced by the above TSS.

$$\{0 \xrightarrow{0} 1, \quad 1 \xrightarrow{0} 0,$$
$$1 \xrightarrow{f(0)} 0, \quad 1 \xrightarrow{f(1)} 1, \quad 0 \xrightarrow{f(0)} 1, \quad 0 \xrightarrow{f(1)} 0,$$
$$f(1) \xrightarrow{1} 0, \quad f(1) \xrightarrow{1} 1\}$$

Note that for the above transition relation it holds that $0 \leftrightarrow 1$, but it does not hold that $f(0) \leftrightarrow f(1)$. Therefore, bisimilarity is not a congruence.

In the following two examples, the same exercise is repeated, i.e., it is shown that although the TSS is in the promoted tyft format and $0 \leftrightarrow 1$, it does not hold that $f(0) \leftrightarrow f(1)$. In the next example, the TSS is not well-founded since a variable in the target of a premise also appears in the source of the same premise and thus has a self-loop in the variable dependency graph.

Example 3.5 *Consider the following TSS in the promoted tyft format. The last deduction rule is not well-founded.*

$$\frac{}{0 \xrightarrow{0} 0} \quad \frac{}{1 \xrightarrow{0} 0} \quad \frac{x \xrightarrow{0} y}{0 \xrightarrow{f(x)} x} \quad \frac{x \xrightarrow{0} y}{1 \xrightarrow{f(x)} y} \quad \frac{y \xrightarrow{f(x)} y}{f(x) \xrightarrow{1} y}$$

The following is the transition relation induced by the above TSS.

$$\{0 \xrightarrow{0} 0, \quad 1 \xrightarrow{0} 0, \quad 0 \xrightarrow{f(0)} 0, \quad 0 \xrightarrow{f(1)} 1, \quad 1 \xrightarrow{f(0)} 0, \quad 1 \xrightarrow{f(1)} 0, \quad f(0) \xrightarrow{1} 0\}$$

The last example violates well-foundedness (and congruence of bisimilarity) by having a non-trivial cycle concerning target, label and source of two premises.

Example 3.6 *Consider the following TSS in the promoted tyft format. The last deduction rule is not well-founded.*

$$\frac{}{0 \xrightarrow{0} 1} \quad \frac{}{1 \xrightarrow{0} 0} \quad \frac{}{0 \xrightarrow{1} 0} \quad \frac{}{1 \xrightarrow{1} 1} \quad \frac{x \xrightarrow{0} y}{1 \xrightarrow{f(x)} x} \quad \frac{x \xrightarrow{0} y}{0 \xrightarrow{f(x)} y} \quad \frac{x \xrightarrow{f(y)} y' \quad y' \xrightarrow{1} y}{f(x) \xrightarrow{1} y}$$

The following is the transition relation induced by the above TSS.

$$\{0 \xrightarrow{0} 1, \quad 1 \xrightarrow{0} 0, \quad 0 \xrightarrow{1} 0, \quad 1 \xrightarrow{1} 1,$$
$$0 \xrightarrow{f(0)} 1, \quad 0 \xrightarrow{f(1)} 0, \quad 1 \xrightarrow{f(0)} 0, \quad 1 \xrightarrow{f(1)} 1, \quad f(1) \xrightarrow{1} 0, \quad f(1) \xrightarrow{1} 1\}$$

The essence of all counter-examples given before is the presence of a cycle in the variable dependency graph. Such cycles may allow for checking syntactic equivalence of terms (e.g., comparing the argument in the target of a premise against a constant) and hence ruin the congruence result. An interesting question is whether there exists a subset of non-well-founded promoted tyft which indeed guarantees congruence, we conjecture that the *safe* subset of the promoted tyft format, as defined below, is the desired subset which guarantees congruence.

Definition 3.7 (Safe Cycles) *Consider a cycle $u_0 \to \ldots \to u_n \to u_0$ in the variable dependency graph of a deduction rule of the following form:*

$$\frac{\{t_i \xrightarrow{t_i'} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{g(\overrightarrow{z})} t''} ,$$

Such a cycle is called safe if in the variable dependency graph, there is no path $u \to \ldots \to u_i$ for all i, $0 \le i \le n$ such that u is among $\overrightarrow{x}$ or among $\overrightarrow{z}$. A deduction rule (TSS) is safe when all cycles in its variable dependency graph (all its deduction rules) are safe.

The following deduction rule contains a safe cycle in its premise.

$$\frac{c \xrightarrow{f(y)} y}{f(x) \xrightarrow{g(x)} y}$$

4 Expressiveness

4.1 Well-Founded Promoted Tyft *does not reduce to* Tyft

Consider the following TSS in the promoted tyft format.

$$\overline{0 \xrightarrow{0} 0} \quad \overline{1 \xrightarrow{0} 0} \quad \overline{0 \xrightarrow{0} 1} \quad \overline{1 \xrightarrow{0} 1}$$

$$\overline{0 \xrightarrow{1} 0} \quad \overline{1 \xrightarrow{1} 0} \quad \overline{0 \xrightarrow{1} 1} \quad \overline{1 \xrightarrow{1} 1}$$

$$\frac{x \xrightarrow{0} y}{0 \xrightarrow{f(x)} 1} \quad \frac{x \xrightarrow{0} y}{1 \xrightarrow{f(x)} 0} \quad \frac{x \xrightarrow{f(x)} y}{0 \xrightarrow{f(x)} y} \quad \frac{x \xrightarrow{f(x)} y}{1 \xrightarrow{f(x)} y} \quad \frac{x \xrightarrow{f(x)} y}{f(x) \xrightarrow{1} y}$$

The transition relation induced by the above TSS is as follows.

$$\{0 \xrightarrow{0} 0; 1 \ , \ 1 \xrightarrow{0} 0; 1 \ , \ 0 \xrightarrow{1} 0; 1 \ , \ 1 \xrightarrow{1} 0; 1 \ ,$$
$$0 \xrightarrow{f(0)} 1 \ , \ 0 \xrightarrow{f(1)} 0; 1 \ , \ 1 \xrightarrow{f(0)} 0; 1 \ , \ 1 \xrightarrow{f(1)} 0 \ , \ f(0) \xrightarrow{1} 1 \ , \ f(1) \xrightarrow{1} 0\}$$

where $p \xrightarrow{p'} p''; q''$ means $p \xrightarrow{p'} p''$ and $p \xrightarrow{p'} q''$. We claim that the above transition relation cannot be specified by any TSS in the tyft format.

If there is such a TSS, then there is a TSS in the pure well-founded tyft format which induces the same transition relation as above [4].

Consider the pure well-founded TSS in the tyft format that (purportedly) induces the same transition relation as above. Assume, without loss of generality that the proof of $f(0) \xrightarrow{1} 1$ from such a TSS does not depend on the proof for $f(1) \xrightarrow{1} 0$ (otherwise, a similar assumption should hold for the transition of $f(1) \xrightarrow{1} 0$ and one can swap 0's and 1's in the sources, labels and targets of the transitions in the remainder of the proof and the argument remains valid). The last deduction rule applied to derive the proof for $f(0) \xrightarrow{1} 1$ should be of the following form.

$$(\mathbf{dr}) \frac{\{t_i \xrightarrow{p_i'} y_i | i \in I\}}{f(x) \xrightarrow{1} t''},$$

and there is a substitution σ such that $\sigma(x) = 0$, $\sigma(t'') = 1$ and all $\sigma(t_i \xrightarrow{p_i'} y_i)$ have a proof tree.

Definition 4.1 (Distance of a Variable) *Given the above deduction rule, define the* distance *of variable x as 0 and a variable y_i to be the maximum of distances of variables appearing in t_i plus 1. The distance of a premise is the distance of the variable of its target.*

Term t'' can either be a variable or the constant 1 (otherwise, if it contains a function symbol other than 1, t'' cannot be unified with 1). Since **(dr)** is pure, it can only contain variables x or y_i's ($i \in I$) and thus t'' can be either 1, or x or y_i (for some $i \in I$).

(i) If t'' is 1, i.e., if the deduction rule is of the following form

$$\frac{\{t_i \xrightarrow{p_i'} y_i | i \in I\}}{f(x) \xrightarrow{1} 1},$$

then we define substitutions σ_k' inductively (on the rank of the premises) maintaining $\sigma(x) \leftrightarrow \sigma_k'(x)$ for all variables x in the domain of σ_k'. First, define σ_0' with $\sigma_0'(x) = 1$ and note that indeed $\sigma(x) = 0 \leftrightarrow 1 = \sigma_0'(x)$. Substitution σ_{k+1}' is obtained from σ_k' as follows: select a premise $t_i \xrightarrow{p_i'} y_i$ (or all such premises) for which the variables of the source are in the domain of σ_k'. Then, as $\sigma(t_i) \leftrightarrow \sigma_k'(t_i)$ (this follows from the fact that $\sigma(x) \leftrightarrow \sigma_k'(x)$ for all variables x from the domain of σ_k' and the fact that for a TSS in the tyft format, bisimilarity is a congruence) and $\sigma(t_i) \xrightarrow{p_i'} \sigma(y_i)$ we obtain the existence of q_i' such that $\sigma_k'(t_i) \xrightarrow{p_i'} q_i'$ and $\sigma(y_i) \leftrightarrow q_i'$. Then define $\sigma_{k+1}'(y_i) \doteq q_i'$.

Define σ' to be the supremum of the chain of premises $\sigma_0', \sigma_1', \ldots$ (which

is increasing with respect to the subset ordering on their domains). Then, all premises of the deduction rule are derivable with respect to substitution σ'. Thus providing us with a proof for $f(1) \xrightarrow{1} 1$ (which is not supposed to be provable according to the above transition relation).

(ii) If t'' is x, then $\sigma(x) = \sigma(t'') = 0$ which is contradictory to the target of the transition $f(0) \xrightarrow{1} 1$.

(iii) Thus, it only remains to consider the case where t'' is a variable y_c, for some $c \in I$, i.e., the deduction rule is of the following form

$$\frac{\{t_i \xrightarrow{p_i'} y_i | i \in I\}}{f(x) \xrightarrow{1} y_c}.$$

Take an arbitrary variable y_j such that $\sigma(y_j) = 1$ and define σ_0' and σ_0'' to be the following partial substitutions:

$$\sigma_0'(x) = \sigma_0'(y_j) = 0 \qquad \text{and} \qquad \sigma_0''(x) = \sigma_0''(y_j) = 1.$$

Then, using an induction on the distance of y_j, we show that we can complete either σ_0' or σ_0'' to a substitution σ' such that the range of σ' is $\{0, 1\}$ and for all $k \in I$, $\sigma'(t_k' \xrightarrow{p_k'} y_k)$ is provable.

Then, it follows that for the particular case of y_c, since $\sigma(y_c) = 1$, that we can prove either $f(0) \xrightarrow{1} 0$ or $f(1) \xrightarrow{1} 1$ which is contradictory to the transition relation that should be induced by the TSS.

- (Base case) If the distance of y_j is 1, i.e., y_j is the target of a premise of which the source only contains x as variable or is a closed term, then the premise $t_j \xrightarrow{p_j'} y_j$ can be of one of the following eleven shapes (for all other transitions in the above transition relation, the target of the transition is 0 and thus cannot match with 1).

$$0; x \xrightarrow{0;1;f(0);f(1)} y_j \qquad \text{or} \qquad 1 \xrightarrow{0;1;f(0)} y_j,$$

where we have abused the ; notation to avoid writing all eleven cases explicitly.

For each of these eleven cases both substitutions σ_0' and σ_0'' are complete. Furthermore, for each of the cases, at least one of these substitutions gives a transition that actually belongs to the transition relation induced by the TSS.

Assuming that σ_0' is the substitution that proves the premise $t_j \xrightarrow{p_j'} y_j$, as before, one can complete the definition to a substitution σ' inductively on the distance of the premises.

- (Induction step) Consider a rule in which y_j has distance $n + 1$ for $n \geq 1$. As the distance of y_j is $n + 1$, it cannot be the case that t_j is a closed term or the variable x, since then the distance of y_j would have been 1. Hence, t_j

is a term containing at least a variable. Our previous assumption that the proof of $f(0) \xrightarrow{1} 1$ does not depend on a proof for $f(1) \xrightarrow{1} 0$ and the fact that all other transitions in the transition relation have a left-hand side 0 or 1 indicates that t_j has to be a variable, say y_k. Now, suppose that p'_j is 0, 1, or $f(1)$. Then, define the substitution σ' to be $\sigma'(y_j) = 0$ and $\sigma'(v) = \sigma(v)$ for all other variables v and this way we have a proof for all the premises using σ' which is an extension of σ'_0. Thus it only remains to check the case where $p'_j = f(0)$. Therefore, the premise $t_j \xrightarrow{p'_j} y_j$ is of the form $y_k \xrightarrow{f(0)} y_j$ for some $k \in I$ where y_k has distance n. Note that necessarily $\sigma(y_k) = 0$ since otherwise the substitution σ' with $\sigma'(v) = \sigma(v)$ for all variables v with distance smaller than the distance of y_j and $\sigma'(y_j) = 0$ can be completed inductively on the rank of the premises to a substitution that extends σ'_0 and proves all the premises.

Based on a similar reasoning we must conclude that the premise $t_k \xrightarrow{p'_k} y_k$ should be of the form $y_l \xrightarrow{f(1)} y_k$ for some $l \in I$ where y_l has distance $n-1$ and $\sigma(y_l) = 1$.

Thus we have a deduction rule of the following form:

$$\frac{y_l \xrightarrow{f(1)} y_k \qquad y_k \xrightarrow{f(0)} y_j \qquad \{t_i \xrightarrow{p'_i} y_i | i \in I - \{j,k\}\}}{f(x) \xrightarrow{1} y_c}.$$

By the induction step, we can complete the definition of one of the two following substitutions:

$$\sigma'_0(x) = \sigma'_0(y_l) = 0 \qquad \text{and} \qquad \sigma''_0(x) = \sigma''_0(y_l) = 1$$

to a substitution σ' or σ'' such that all the premises with a distance of $n-1$ or less find a proof. If σ'_0 can be completed, then we define $\sigma'(y_k) = 1$ and $\sigma'(y_j) = 0$ and complete the definition of σ' for all premises with distance n or more, as before. If σ''_0 can be completed, we define $\sigma''(y_k) = 0$ and $\sigma''(y_j) = 1$ and complete the definition of σ''.

This concludes the proof as in all of the above cases, we can construct a proof for either $f(0) \xrightarrow{1} 0$ or $f(1) \xrightarrow{1} 1$ (or both) none of which are supposed to be in the induced transition relation.

4.2 **Tyft** *does not reduce to* **Promoted Tyft**

Example 4.2 *Consider the following TSS in the* tyft *format. The signature of the TSS consists of* 0, 1 *and* 2 *as constants and* f *as a unary function symbol.*

$$\frac{}{2 \xrightarrow{f(0)} 2}$$

The transition relation induced by it is $\{2 \overset{f(0)}{\to} 2\}$. We claim that there is no TSS in the **promoted tyft** *format which can induce the same transition relation. It trivially holds that $0 \leftrightarrow 1$ and from $2 \overset{f(0)}{\to} 2$ and Lemma 2.6 that (for a TSS in the* **promoted tyft** *format) $2 \overset{f(1)}{\to} 2$ is also in the induced transition relation.*

If one restricts the tyft format to the subset with only constants as labels, then it trivially conforms to all requirements of the promoted tyft format and thus, the promoted tyft format (taking the first example in Section 4.1) is strictly more expressive than the tyft format with constants as labels.

4.3   (Promoted) Tyft *reduces to* Promoted PANTH

In [8], we introduced the promoted PANTH format which generalizes promoted tyft with negative premises. But even restricted to positive TSS's, the promoted PANTH format generalizes both the promoted tyft and the tyft format. To define the promoted PANTH format, we need the following notion of volatile operators.

Definition 4.3 *(Volatile Operators) Given a TSS (Σ, D) an operator $f \in \Sigma$ is called* volatile *when there exists a rule $d \in D$ of the following form:*

$$\frac{\{t_i \overset{t_i'}{\to} t_i'' \mid i \in I\}}{t \overset{t'}{\to} t''}$$

and $f(\overrightarrow{t_k})$ is a subterm of t_i' for some $i \in I$ such that $vars(\overrightarrow{t_k}) \cap vars(t) \neq \emptyset$ or $\exists_{i \in I} vars(\overrightarrow{t_k}) \cap vars(t_i') \neq \emptyset$.

Note that for a TSS in the tyft format, no operator is volatile as the set $vars(\overrightarrow{t_k})$ is always empty. Arguments of transition labels under a volatile operator should be replaceable by bisimilar terms (i.e., given a transition with a volatile operator as the outermost operator in the label, another transition with the label containing bisimilar arguments under the same volatile operator should be provable to a bisimilar target). In the definition of the promoted tyft format, all operators are assumed to be volatile and this assumption has resulted in a less expressive rule format, as we prove shortly.

The following is a simplified definition of the promoted PANTH format (restricted to positive TSS's and without predicates and lists of terms as labels) that suffices for our purposes.

Definition 4.4 (Positive Promoted PANTH) *A deduction rule is in the* positive promoted PANTH *format when it is of the following form*

$$\frac{\{t_i \overset{t_i'}{\to} y_i \mid i \in I\}}{f(\overrightarrow{x}) \overset{t'}{\to} t''}$$

and first, all x_i and y_j variables ($0 \leq i < ar(f)$ and $j \in I$) and variables in t' are
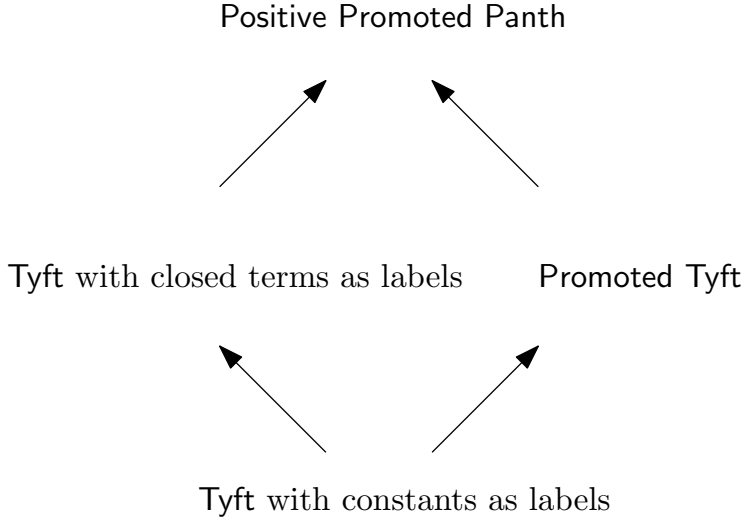
Fig. 1. Comparison of the expressiveness of rule formats.

pairwise distinct, second, if a component of t_i ($i \in I$) is a variable (i.e., does not have any function symbol) then it is not among x_i's and y_j's and third,

(i) *if t' contains a volatile $g \in \Sigma$ then t' is of the form $g(\overrightarrow{z})$ where all z_i's are distinct variables and for all $j \in I$, all t_i containing a variable among $\overrightarrow{z}$ are of the form $g_i(\overrightarrow{t'_i})$ where g_i is volatile,*

(ii) *if there is a volatile operator in the signature and if t' is a variable z then for all $i \in I$, t_i containing z are either z itself or are of the form $g_i(\overrightarrow{t'_i})$ where g' is volatile.*

It follows immediately from the above definition that any TSS in the tyft format is in the positive promoted PANTH format since a TSS in the tyft format contains no volatile operator. On the other extreme resides the promoted tyft format which is a subset of positive promoted PANTH in which all operators are considered volatile (regardless of whether or not they actually are volatile). Thus, we conclude that positive promoted PANTH is strictly more expressive than both tyft and promoted tyft since it includes TSS's of examples of Section 4.1 and has both formats as its (proper) subsets.

Figure 1 summarizes the result of our comparison. Each arrow shows strict inclusion of the sets of definable transition relations.

5   Conclusions

In this paper we studied issues related to the well-foundedness of premises and expressiveness for (the set of transition relation that can be specified by) TSS's in the promoted tyft format. We showed that well-foundedness cannot be dropped while preserving the congruence property for bisimilarity. Furthermore, we compared the expressiveness of the tyft, the promoted tyft, and the positive subset of the

promoted PANTH formats and showed that while the tyft format with closed terms is incomparable to the promoted tyft format, the positive subset of the promoted PANTH format is strictly more expressive than both.

Regarding well-foundedness, we are currently studying the congruence meta-theorem for the safe subset of the promoted tyft format. The techniques used in [4] are not directly applicable to this setting as the open terms on the labels (containing at least one function symbol) cannot be trivially resolved to variables. Regarding expressiveness, it is interesting to compare the safe promoted tyft format with the promoted tyft format. We do not yet know the answer but expect the two formats to be equally expressive.

References

[1] S. Abramsky, The lazy lambda calculus. in: *Research topics in functional programming*, Addison-Wesley, Boston, MA, USA, 1990, pp. 65–116.

[2] L. Aceto, W. J. Fokkink, C. Verhoef, Structural Operational Semantics, *Chapter 3 of Handbook of Process Algebra*, Elsevier Science, Dordrecht, The Netherlands, 2001, pp. 197–292.

[3] K. L. Bernstein, A congruence theorem for structured operational semantics of higher-order languages, in: *Proceedings of the 13th IEEE Symposium on Logic In Computer Science (LICS'98)*, IEEE Computer Society, Los Alamitos, CA, USA, 1998, pp. 153–164.

[4] W. J. Fokkink, R. J. van Glabbeek, Ntyft/ntyxt rules reduce to ntree rules, Information and Computation (I&C) 126 (1) (1996) 1–10.

[5] J. F. Groote, M.R. Mousavi, M. A. Reniers, A Hierarchy of SOS Rule Formats. in: *Proceedings of the 2nd Workshop on Structural Operational Semantics (SOS'05)*, Electronic Notes in Theoretical Computer Science 156 (1) (2006) 3–25.

[6] J. F. Groote, F. W. Vaandrager, Structured operational semantics and bisimulation as a congruence, *Information and Computation (I&C)* 100 (2) (1992) 202–260.

[7] D. J. Howe, Proving congruence of bisimulation in functional programming languages, *Information and Computation (I&C)* 124 (1996) 103–112.

[8] M.R. Mousavi, M. J. Gabbay, M. A. Reniers, SOS for higher order processes, in: *Proceedings of the 16th International Conference on Concurrency Theory (CONCUR'05)*, Lecture Notes in Computer Science, Springer-Verlag, Berlin, Germany, 2005, pp. 308–322.

[9] B. Thomsen, A Theory of Higher Order Communicating Systems", *Information and Computation (I& C)* 116 (1) (1995) 38–57.