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Multi-sourcing under supply uncertainty and Buyer's risk aversion

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ABSTRACT

We address the combined problem of supplier (or vendor) selection and ordering decision when a buyer can choose to procure from multiple suppliers whose yields are uncertain and potentially correlated. We model this problem as a stochastic program with recourse in which the buyer purchases from the suppliers in the first period and, if needed, chooses to purchase from the spot market or from the suppliers with excess supply, whichever is beneficial, in the second period in order to meet the target procurement quantity. We solve the above problem using *sample average approximation* (SAA) technique that enables us to solve the problem easily in practice. We compare the performance of our solution with the certainty equivalent problem, which is practiced widely and which we use as the benchmark, to evaluate the efficacy of our approach. Next, we extend our model to incorporate buyer's risk aversion with respect to the quantity procured. We reformulate the multi-sourcing problem as a mixed integer linear program (MILP) and adopt a statistical approach to account for buyer's risk aversion. Thus, we design a simple computational technique that provides an optimal sourcing policy from a set of suppliers when each supplier's yield is uncertain with a generic probability distribution.

1. Introduction

Procurement (or sourcing) is one of the major activities of any supply chain because it is essential to ensure sufficient raw materials and other auxiliary inputs of the firms in the supply chain in order to produce their intended output goods and services. Moreover, since procurement happens as one of the early operations in a production system, it largely determines the throughput of the system, by being a potential bottleneck.

Among the procurement activities of a firm, supplier (or vendor) selection and decision of order quantities is crucial. For this reason, managers exercise utmost care when selecting vendors and placing orders. Furthermore, the supply uncertainty that a manager faces from each supplier compounds the procurement process. In this paper, we develop a simple approach to manage supplier selection and order placement process when supply is uncertain. We use the term "*supplier yield uncertainty*", which is commonly used in the OM literature, to refer to the idea of supply uncertainty.

Unlike many of the earlier methods in the extant literature that assume independent supply uncertainties among suppliers, our method can also be used when supply uncertainty is correlated (either positively or negatively) among suppliers. Correlated supplier yield uncertainties are not only important but also commonly observed when upstream supply of a raw material to the suppliers is from a common pool. For instance, the supply uncertainty among different steel-sheet suppliers in a specific geographical area could be highly correlated because the upstream supplier of steel (or ore) to the steel-sheet suppliers may be common. The same can be observed in many other commodities like coal and other minerals, fresh produce, human-labor and the like. Therefore, it is important for a procurement manager to account for supplier-yield correlations when making supplier selection and order decisions. In addition to the common external upstream uncertainties faced by the suppliers, their individual process uncertainties further add to the final uncertainty of the supply that they can provide to the downstream buying firm, which we refer to as "*buyer*" in this paper.

First, we develop a simple OR-based technique to address this supplier-selection and order-management problem. We compare our method with the *certainty-equivalent problem* approach that is often practiced in order to assess the efficacy of our model. Later, we extend our model to incorporate buyer's risk aversion, which captures buyer's behavioral aspects and his sensitivity to shortfall in procured quantity that are not entirely captured in our initial model. Given the complexity of the problem, we propose a statistical-based procedure to solve the multi-sourcing problem under supply uncertainty and varying supplier selling prices. Thus, we provide a simple and yet scalable method that can be used well to solve multi-sourcing problem under supply uncertainty.

Our paper is organized as follows. In Section 2 we provide a review of the relevant literature. In Section 3, we model and analyze a single-supplier scenario and extend it to incorporate buyer's risk aversion. In Section 4, we develop and analyze the multi-supplier model to solve the multi-sourcing problem under supply uncertainty. We also extend this model to incorporate buyer's risk aversion towards the shortfall in the

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procured quantity. We provide some discussion of the model and its results along with a few numerical examples in Section 5, and we conclude our paper in Section 6. We provide all the proofs in the appendix.

2. Literature review

Procurement and inventory management under supplier yield uncertainty is a well-researched topic in the OM literature. Pan (1989) is one of the earlier papers that developed a *linear-programming* based mathematical model to solve the supplier selection problem depending on the dimensions of price, quality, and service. He et al. (2009) extended the above model to a stochastic setting where the supplier's service and quality are random. The authors developed a chance-constrained model, which they solved using a genetic algorithm, to solve the supplier selection problem. Among the other papers that deal with supplier yield uncertainty, many focus on inventory management under single-supplier models. For example, Gerchak et al. (1988), Ciarallo et al. (1994), Wang and Gerchak (1996), Parlar et al. (1995), Özekici and Parlar (1999), Erdem and Özekici (2002), Khang and Fujiwara (2000), Mazzola et al. (1987) are a few important papers to mention that discuss single supplier models. In a more generic setting, Kasilingam and Lee (1996) addressed a single-period supplier-selection problem through a mixed-integer linear program (MILP) model when the supplier quality in terms of fraction of defects is deterministic and the retailer's demand is uncertain. They used chance constraints to handle demand uncertainty.

With respect to multi-supplier models, Anupindi and Akella (1993) analyzed dual sourcing (i.e., 2 suppliers) in the presence of yield uncertainty. They proved that, with the newsvendor objective function (with back-ordering), the optimal policy is a two-threshold policy: order from both the suppliers, or order from the cheaper supplier, or order from neither, depending on the initial inventory position. Giri (2011) addressed a single-product single-period inventory problem of a risk-averse buyer, where the first supplier is cheaper but unreliable, whereas the second supplier is perfectly reliable with a finite capacity but is costlier. In the same vein, Parlar and Wang (1993) and Parlar et al. (1995) also analyzed EOQ models in the presence of 2 suppliers with different yield distributions. They identified conditions when dual sourcing is preferred to single sourcing. Our paper differs from the above papers mainly because it primarily addresses the problem of multi-sourcing where there are more than 2 suppliers who can either "oversupply" or "under-supply" the product depending on their respective yields.

Next, among the papers that analyze multi supplier models (i.e., sourcing from more than 2 suppliers), Kasilingam and Lee (1996) addressed a single-period supplier-selection problem through an MILP model when the supplier quality in terms of fraction of defects is deterministic and the retailer's demand is uncertain. They used chance constraints to handle demand uncertainty. Fadıloğlu et al. (2008) analyzed EOQ model with binomial supplier yield uncertainty and different purchasing costs across suppliers. They showed that supplier diversification may not always be beneficial. Erdem et al. (2006) examined an EOQ type model under multi-supplier yield uncertainty with same unit purchasing costs, and they showed that the quantity of expected unsatisfied order is same for each supplier. In a newsvendor setting Dada et al. (2007) analyzed supplier-selection when there are many suppliers each of which is either perfectly reliable or unreliable, and the yield uncertainties across suppliers are independent (and extended it to *associated* random yields[1]). They showed that although the aggregate order quantity of the buyer is higher when suppliers are unreliable than when they are perfectly reliable, the service level experienced by consumers is lower when suppliers are unreliable. Yan et al. (2012) analyzed the multi-sourcing problem under both the cases when yields are independent and correlated. For independent supplier yields they showed that the wholesale price takes precedence over reliability; however, they showed that this may not be true for correlated supplier yields. In a more recent paper, Xue et al. (2016) developed a mean–variance model when manufacturer is risk-sensitive and suppliers are unreliable. The authors identified the optimal diversification strategy and drew insights about trade off between cost and reliability. We refer the reader to Ho et al. (2010) for a comprehensive review of the literature on supplier selection under supply uncertainty, based on various analytical techniques used. A few other more recent and relevant papers that discuss multi-sourcing include (Balakrishnan and Natarajan, 2014; Bohner and Minner, 2017; Şen et al., 2014; Dong et al., 2021; Kirschstein and Meisel, 2019; Manerba et al., 2018; Manerba and Perboli, 2019; Zhang et al., 2012). Zhang et al. (2012) develop a hybrid solution technique for supplier choice and order quantity decision problem. They do not consider any uncertainties in their model. Şen et al. (2014) formulates a scenario-based multi-stage stochastic optimization model to address multi-item-multi-sourcing problem in the presence of random supplier discounts and random spot-market prices. They propose certainty-equivalent heuristics for the problem. Bohner and Minner (2017) formulate an MILP problem and offer a solution for the supplier selection and order quantity problem when suppliers offer quantity discounts and are prone to failures (i.e., yield is zero). Manerba et al. (2018) and Manerba and Perboli (2019) also formulate a 2-stage stocastic program with recourse where products price or only the products demand are stochastic. Kirschstein and Meisel (2019) address the problem of selecting suppliers and storage facilities as well as determining order quantities and transport flows when suppliers offer discounts. In a recent paper, Dong et al. (2021) address the multi-sourcing problem when supplier yields are correlated and when each supplier can produce only a fraction of the bubyer's order quantity (i.e., when yield < 1). Thus, all these papers address the case when supplier can meet only a portion of the buyer's order due to yield uncertainty (i.e., supplier's yield is always less than 1) but do not consider random yields so that the final quantity available with a supplier can be either more or less than the order quantity. Accounting for suppliers' yield uncertainties when making procurement decisions is important because they can affect the buyer's production operations, capacity, and inventory costs significantly, and supplier's yield can often be higher than 1 thereby resulting in overproduction of the product, which the buyer can take advantage of in order to reduce his procurement costs.

Our paper differs from the above discussed papers in the following important ways. First, unlike many of the above models that focus primarily on short-supply of product by suppliers, we account for yields that cause both short-supply as well as excess supply. However, as mentioned by Dada et al. (2007), the buyer will procure from a supplier at most the quantity he orders with the supplier, although the supplier has an oversupply of product due to a higher yield.

Second, in case of oversupply, we allow the buyer to choose between the excessive supply from the suppliers or the spot market, whichever is beneficial, to procure additional quantity in order to meet the target procurement quantity, if required. Doing so accounts for the recourse actions of the buyer when the total initial quantity procured from the suppliers is less than the target quantity, and if there are a few suppliers who have their product supply higher than the quantity ordered by the buyer with the corresponding supplier. Procuring in advance and re-procuring later in order to meet the target quantity in case of short supply is a common practice adopted by procurement managers.

Third, we extend our model to account for buyer's risk-aversion so that our solution technique can capture other aspects, which are not explicitly captured in the initial model, through buyer's risk aversion. For instance, we can incorporate the impact of shortage cost through buyer's risk aversion by noting that a buyer is more risk averse whenever the shortage cost is high. Fourth, we do not impose any restriction on the stochastic structure of the random supplier yields. Our technique can be

[1] See Esary et al. (1967) for the definition and properties of associated random variables.

used under all types of yield uncertainties: (i) independent yields, (ii) positively correlated yields, and (iii) negatively correlated yields.

3. Single supplier model

First, we analyze the case of a buyer sourcing from a single supplier to meet the former's target procurement quantity, which we denote by Q. The buyer places an order for x units with the supplier, who charges a unit price of c. Let the random variable Z denote the supply uncertainty or "*yield*" of the supplier so that the net supply available will be $z \cdot x$ units, where z is a realization of the random variable Z. We assume that $Z \geqslant 0$ and, without loss of generality, that $\mathbb{E}[Z] = 1$.[2] We denote the distribution and density functions of Z by $F(\cdot)$ and $f(\cdot)$, respectively.

While many papers in the existing literature assume that $Z \in [0, 1]$, so that the supplier always falls short of the order quantity x, we allow $Z \in [0, \infty)$ to incorporate the scenario when the supplier will be able to supply more than the quantity x that has been asked for.[3]

Within the supply chain literature, *procurement* is characterized by the following two fundamental features, which distinguish it from *production* (Dada et al., 2007; Deo and Corbett, 2009; Li and Zheng, 2006; Parlar et al., 1995):

1. A buyer pays for only the quantity that his supplier delivers, and
2. The quantity delivered by a supplier is at most the order quantity the buyer places with the supplier (Dada et al., 2007).

Therefore, since we analyze *procurement* from a supplier in the presence of supply uncertainty in a supply chain context, we assume the following in our model:

Assumption 1. Although a supplier will be able to supply more than the ordered quantity whenever $z > 1$, we assume that the buyer will initially procure at most the quantity x ordered from the supplier.[4]

Assumption 2. In the event that the initially procured quantity is less than the target procurement quantity Q, the buyer will top-up the procured quantity by procuring either from the supplier, whenever the supplier has excess supply, or from the spot market, whichever is beneficial.

Let the random variable $Y(x, Z) = \min\{x, Z \cdot x\}$ denote the initial quantity procured by the buyer from the supplier. We compute the distribution of $Y(x, Z)$ in the following lemma.

Lemma 1. *The distribution of $Y(x, Z)$ for a given order quantity x is given by:*

$$\mathbb{P}(Y(x, Z) \geqslant y) = \begin{cases} 0 & \text{if } x < y, \\ 1 - F\left(\frac{y}{x}\right) & \text{if } x \geqslant y. \end{cases} \tag{1}$$

Depending on the net quantity $Y(x, Z)$ procured from the supplier, the buyer will procure the remaining quantity $[Q - Y(x, Z)]^+$ from the spot market at unit price $s > c$ or form the supplier at unit price c, if the latter has excess yield (i.e., $z > 1$).

Actually, in many contexts, the supplier will prefer selling the extra produce to the buyer at unit price c to selling it in the spot market at unit price $s(> c)$ due to a high unit travel (or inconvenience) cost t incurred in transporting the produce to the spot market. In fact, supplier will choose the wholesale price c such that $s - t < c < s$ so that both the buyer and the supplier are better-off through a direct transaction than through a transaction in the spot market. Fafchamps and Hill (2005) find out that while the likelihood of a farmer selling to the market decreases as the market gets farther, the likelihood of selling at the farm gate increases. Aayog (2017) also mentions that directly buying from farmers in India helps farmers and buyers in fetching better prices and in doing away with middlemen.

For convenience of modeling, we divide the time horizon into two periods, where the quantity procured by the buyer in the first period is $Y(x, Z)$ and the remaining quantity, if required, to meet the target Q, is procured in the second period, as shown in Fig. 1. Next, we use a 2-period dynamic programming formulation to solve problem.

3.1. Second-period problem

For any order quantity x in the first period, the buyer procures the remaining quantity, either from the spot market or from the supplier, in the second period based on the following rules, for any realization z of the yield:

1. If $z < 1$, then the quantity purchased from the supplier during the first period is $z \cdot x$, and
   (a) If $z \cdot x < Q$, then the buyer procures $Q - z \cdot x$ at unit cost s from the spot market in the second period.
   (b) If $z \cdot x \geqslant Q$, then the buyer does not procure anything from the spot market in the second period.
2. If $z \geqslant 1$, then the quantity purchased from supplier during the first period is x, and
   (a) If $z \cdot x < Q$, then the buyer procures $z \cdot x - x$ from the supplier at unit cost $c(< s)$, and procures $Q - z \cdot x$ at unit cost s from the spot market in the second period.
   (b) If $z \cdot x \geqslant Q$, then the buyer procures the remaining quantity $[Q - x]^+$ from the supplier at unit cost $c(< s)$ in the second period.

Now, using the above rules, if the ordered quantity x is higher than the target quantity Q (i.e., $x > Q$), then for any realization $z < Q/x(< 1)$, we have the second period's procurement cost to be $s \cdot [Q - zx]^+$. Next, for any yield realization z such that $Q/x \leqslant z < 1$, the quantity that is available with the supplier is $z \cdot x$, which is at least the target quantity Q but less than the order quantity x (i.e., $Q \leqslant z \cdot x < x$). Therefore, the buyer procures nothing during the second period, because the total quantity $Y(x, z) = \min\{z \cdot x, x\} = z \cdot x$ that has been procured during the first period would satisfy the quantity target constraint $Y(x, z) \geqslant Q$. Lastly, for any other yield realization (i.e., $z \geqslant 1$), the buyer again procures nothing during the second period, because the total quantity $Y(x, z) = \min\{z \cdot x, x\} = x(> Q)$.

On the other hand, if the order quantity $x \leqslant Q$, then for any z such that $z \leqslant 1(\leqslant Q/x)$, the first period's procurement quantity is $Y(x, z) = z \cdot x(< Q)$ so that the buyer's procurement cost during the second period is $s \cdot (Q - z \cdot x)$. Next, for any realized $1 < z \leqslant Q/x$, the buyer can purchase up to $z \cdot x$ units at c from the supplier at total cost $c(z \cdot x - x)$ and the remaining units to meet the target Q at a cost of $s(Q - z \cdot x)$ from the spot market. Lastly, if $z \geqslant Q/x$, then the supplier will purchase $Q - x$ units at a total purchase cost of $c(Q - x)$ from the supplier who has a total of $(z - 1) \cdot x$ units left to sell.

Therefore, the expected procurement cost incurred in the second period is given by:

$$J_2(x) = \begin{cases} \int_0^{Q/x} s \cdot (Q - zx) f(z) dz, & \text{if } Q < x \\ \int_0^1 s \cdot (Q - zx) f(z) dz + \int_{Q/x}^{\infty} c \cdot (Q - x) f(z) dz + \int_1^{Q/x} [c(zx - x) + s(Q - zx)] f(z) dz, & \text{if } Q \geqslant x \end{cases}$$

3.2. First-period problem

By using (2), the total expected cost-to-go in the first period is given by:

$$J_1(x) = \mathbb{E}[c \cdot Y(x, Z)] + J_2(x) = \mathbb{E}[c \cdot \min\{x, Zx\}] + J_2(x), \tag{3}$$

[2] If not, we can scale x accordingly to set $\mathbb{E}[Z] = 1$.

[3] Such circumstances are often encountered in agricultural, food, chemical, and other such industries, where the process yield can be higher than the output quantity that has been planned for. For instance, there could be higher crop yield due to a bounty crop in the case of agricultural products.

[4] This formulation of procurement quantity is same as the one adopted in Dada et al. (2007).

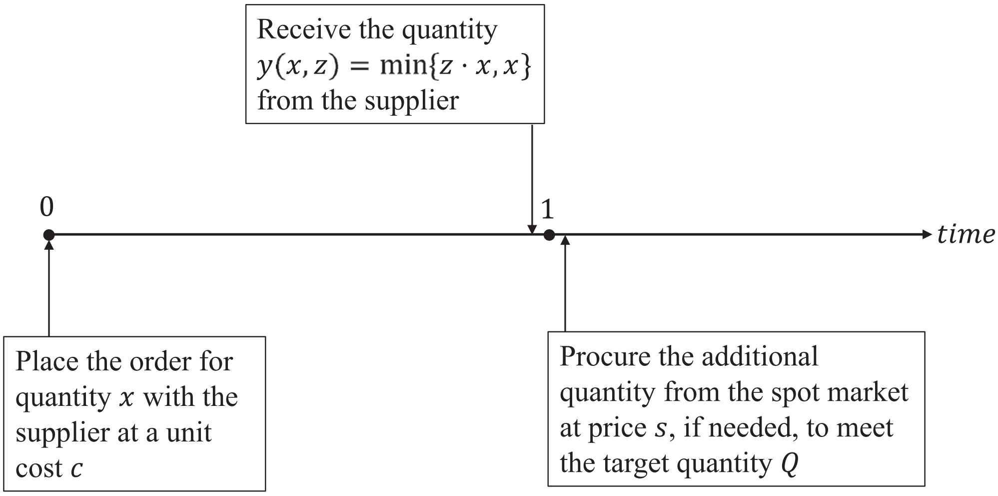


Fig. 1. Sequence of events in the buyer's procurement.

and the optimization problem of the buyer in the first period is:

$$\mathbf{DP}: \min_{x \geq 0} J_1(x) \tag{4}$$

The following lemma proves that the expected procurement cost $\mathbb{E}[c \cdot Y(x, Z)]$ in the first period is a linear function of x.

Lemma 2. $\mathbb{E}[Y(x, Z)] = x\left[\int_0^1 zf(z)\,dz + 1 - F(1)\right]$, *where* $f(\cdot)$ *and* $F(\cdot)$ *are the density and distribution functions of* Z*, respectively.*

The optimal order quantity x^* of the problem **DP** is always greater than the target quantity Q; that is, it is always optimal to over-order during the first period, as one would have intuitively guessed. We formalize this result in the following lemma that also gives the value of the optimal order quantity x^* during the first period:

Lemma 3. *Let* x^* *denote the optimal solution of* **DP** *and let* $c_0 = c\left[1 + \frac{1-F(1)}{\int_0^1 zf(z)dz}\right]$. *Then:*

1. $x^* \geqslant Q$.
2. *If* s *is "low" (i.e.,* $s \leqslant c_0$*), then* $x^* = Q$.
3. *If* s *is "high" (i.e.,* $s > c_0$*), then* x^* *is the unique solution of the following equation:*

$$s \int_0^{Q/x} zf(z)dz = c\left[\int_0^1 zf(z)dz + 1 - F(1)\right] = c_0 \int_0^1 zf(z)dz. \tag{5}$$

Clearly, the optimal first-period order quantity x^* is increasing in the future spot price s and decreasing in the suppliers price c, which is as expected.

3.3. Risk averse buyer

In order to develop a chance-constraint model, we add the constraint that the buyer initially intends to procure the target quantity Q from the supplier with a probability of at least α.[5]

That is, we obtain:

$$\mathbb{P}(Y(x, Z) \geqslant Q) \geqslant \alpha \Leftrightarrow 1 - F\left(\frac{Q}{x}\right) \geqslant \alpha \Leftrightarrow Q \leqslant F^{-1}(1-\alpha) \cdot x, \tag{6}$$

where $x(\geqslant Q \geqslant 0)$ is the actual order quantity placed by the buyer with the supplier and the second statement in (6) is obtained by using Lemma 1.

Therefore, for a risk averse buyer, the problem **DP** given in (4) can be rewritten as the following chance constrained problem:

$$\mathbf{DP-RA}: \min_{x \geq 0} J_1(x) \text{ s.t. } Q \leqslant F^{-1}(1-\alpha) \cdot x, \tag{7}$$

whose solution is given by:

$$x^*_{RA} = \max\left\{x^*, \frac{Q}{F^{-1}(1-\alpha)}\right\}, \tag{8}$$

where x^* is given in Lemma 3. Clearly, the optimal order quantity x^*_{RA} is (weakly) increasing in the buyer's risk aversion (i.e., $\frac{\partial x^*_{RA}}{\partial \alpha} \geqslant 0$).

4. Multi-supplier model preliminaries

Let $\mathcal{S}$ denote the set of all suppliers from which the commodity can be sourced. We use index i to refer to the suppliers in $\mathcal{S}$. Let $c_i(< s)$ denote the unit price charged by supplier i and the random variable Z_i denote the supply uncertainty or "*yield*" of supplier $i \in \mathcal{S}$. We let the decision variable x_i denote the buyer's initial order quantity with supplier i and, for any order quantity x_i, supplier i will be able to supply $z_i \cdot x_i$ units at the time of procurement during the first period, where z_i is the realized yield of supplier $i \in \mathcal{S}$.

Therefore, the random variable $Y_i(x_i, Z_i) = \min\{x_i, Z_i \cdot x_i\}$ denotes the initial quantity procured by the buyer from supplier $i \in \mathcal{S}$. As earlier, we can compute the distribution of $Y_i(x_i, Z_i)$, for all $i \in \mathcal{S}$ as:

$$\mathbb{P}\left(Y_i(x_i, Z_i) \geqslant y_i\right) = \begin{cases} 0 & \text{if } x_i < y_i, \\ 1 - F_i\left(\frac{y_i}{x_i}\right) & \text{if } x_i \geqslant y_i, \end{cases} \tag{9}$$

where $F_i(\cdot)$ is the distribution of Z_i, $i \in \mathcal{S}$.

Next, by using Lemma 2 for each $i \in \mathcal{S}$, the expected total cost of procurement incurred by the buyer, after procuring from all the suppliers $i \in \mathcal{S}$ is given by:

$$\mathbb{E}\left[\sum_{i \in \mathcal{S}} c_i \cdot Y_i(x_i, Z_i)\right] = \sum_{i \in \mathcal{S}} c_i \cdot \mathbb{E}\left[Y_i(x_i, Z_i)\right] = \sum_{i \in \mathcal{S}} c_i x_i \left[\int_0^1 uf_i(u)\,du + 1 - F_i(1)\right] \tag{10}$$

and the expected total quantity procured by the buyer in period 1 is:

$$\mathbb{E}\left[\sum_{i \in \mathcal{S}} Y_i(x_i, Z_i)\right] = \sum_{i \in \mathcal{S}} x_i \cdot \left[\int_0^1 uf_i(u)\,du + 1 - F_i(1)\right].$$

Next, by using the expected values of the supply yields, we can approximate the buyer's optimization problem by the following *certainty-equivalent* problem (Bertsekas, 1995; Şen et al., 2014; Tang and Girotra, 2010; Tomlin, 2009) with suppliers $\mathcal{S}$:

$$\mathbf{CEP}(\mathcal{S}): \min_{\mathbf{x}} \left\{\mathbb{E}\left[\sum_{i \in \mathcal{S}} c_i \cdot Y_i(x_i, Z_i)\right] = \sum_{i \in \mathcal{S}} \left(c_i\left[\int_0^1 uf_i(u)\,du + 1 - F_i(1)\right]\right) \cdot x_i\right\}$$
$$\text{s.t.}$$
$$\mathbb{E}\left[\sum_{i \in \mathcal{S}} Y_i(x_i, Z_i)\right] = \sum_{i \in \mathcal{S}} \left[\int_0^1 uf_i(u)\,du + 1 - F_i(1)\right] \cdot x_i \geqslant Q, \tag{11}$$

where Q is the target quantity that the buyer needs to procure. We obtain the above constraint in $\mathbf{CEP}(\mathcal{S})$ using Lemma 2, for each $i \in \mathcal{S}$. We use

[5] α can be thought of as a measure of risk-aversion of the buyer towards shortfall of the initial procurement quantity.

CEP($\mathcal{S}$) as the benchmark model and compare the performance of our approach with the solution of **CEP**($\mathcal{S}$).[6] It should be noted that **CEP**($\mathcal{S}$) is an LP in the decision variables x_i ($i \in \mathcal{S}$), whose coefficients are always some constants.

4.1. Exact solution

Now, we formulate the multi-sourcing problem as a 2-period dynamic program and obtain the optimal solution through backward recursion.

4.1.1. Second period problem

In the second period problem, the vectors $\mathbf{x} = [x_i]_{i\in\mathcal{S}}$, which denotes the initial order quantities placed by the buyer during the first period, and $\mathbf{z} = [z_i]_{i\in\mathcal{S}}$, which denotes the yields of suppliers $i \in \mathcal{S}$, form the state variables and are perfectly known. The total quantity procured in the first period is $\sum_{i\in\mathcal{S}} y_i(x_i, z_i)$, where $y_i(x_i, z_i) = \min\{x_i, z_i x_i\}, i \in \mathcal{S}$ so that the remaining quantity yet to be procured during the second period is $\left[Q - \sum_{i\in\mathcal{S}} y_i(x_i, z_i)\right]^+$ so that the procurement quantity target Q is met. The supply available from supplier $i \in \mathcal{S}$ in the second period is $(z_i - 1)^+ x_i$, because only suppliers with excess yield (i.e., with $z_i > 1$) can supply during the second period. Hence, the buyer's procurement problem in the second period is given by:[7]

$$H_2(\mathbf{x}, \mathbf{z}) = \min \sum_{i\in\mathcal{S}} c_i w_i + s w_s \tag{12}$$

s.t.

$$w_i \leqslant (z_i - 1)^+ x_i,\ \forall i \in \mathcal{S}, \tag{13}$$

$$\sum_{i\in\mathcal{S}} w_i + w_s \geqslant Q - \sum_{i\in\mathcal{S}} y_i(x_i, z_i), \tag{14}$$

$$w_s \geqslant 0, w_i \geqslant 0,\ \forall i \in \mathcal{S}. \tag{15}$$

4.1.2. First period problem

Through backward recursion we can compute the expected cost-to-go in the first-period problem as:

$$\begin{aligned} H_1(\mathbf{x}) &= \mathbb{E}_{\mathbf{Z}}\left[\sum c_i \cdot Y_i\big(x_i, Z_i\big)\right] + \mathbb{E}_{\mathbf{Z}}\left[H_2(\mathbf{x}, \mathbf{Z})\right] \\ &= \sum_{i\in\mathcal{S}} c_i \left[\int_0^1 u f_i(u) du + 1 - F_i(1)\right] \cdot x_i + \mathbb{E}_{\mathbf{Z}}\left[H_2(\mathbf{x}, \mathbf{Z})\right], \end{aligned} \tag{16}$$

so that the buyer's problem is:

$$\mathbf{DP}(\mathcal{S}) : \min_{\mathbf{x}\geqslant 0} H_1(\mathbf{x}), \tag{17}$$

which, despite being convex in $\mathbf{x}$, could be difficult to solve if $|\mathcal{S}|$ is large due to the *curse-of-dimensionality* and the complex structure of $H_2(\mathbf{x}, \mathbf{z})$.

4.2. Sample average approximation (SAA) based solution

Given the aforementioned difficulty in solving **DP** given in (17), we solve the problem by using *sample average approximation* (SAA) Monte Carlo technique with an N-sized sample $\vec{\mathbf{z}} = [\mathbf{z}_1, \ldots, \mathbf{z}_N]$, where $\mathbf{z}_j = [z_{ij}]_{i\in\mathcal{S}}$ is the jth supplier-yield vector (and z_{ij} denotes the yield of supplier $i \in \mathcal{S}$ in the jth sample). We formulate the following approximately-equivalent, deterministic (large) linear program (LP) that is easier to solve:[8]

$$\begin{aligned} \mathbf{SAA-LP}\left(\mathcal{S}, \vec{\mathbf{z}}\right) : \min &\sum_{i\in\mathcal{S}} c_i \left[\int_0^1 u f_i(u) du + 1 - F_i(1)\right] x_i \\ &+ \tfrac{1}{N} \sum_{j=1}^N \left[\sum_{i\in\mathcal{S}} c_i w_{ij} + s w_{sj}\right] \end{aligned} \tag{18}$$

s.t.

$$w_{ij} \leqslant (z_{ij} - 1)^+ x_i \quad \forall i \in \mathcal{S},\ \forall j = 1, \ldots, N, \tag{19}$$

$$\sum_{i\in\mathcal{S}} w_{ij} + w_{sj} \geqslant Q - \sum_{i\in\mathcal{S}} \min\{z_{ij}, 1\} \cdot x_i \quad \forall j = 1, \ldots, N, \tag{20}$$

$$x_i \geqslant 0, w_{ij} \geqslant 0, w_{sj} \geqslant 0, \quad \forall i \in \mathcal{S}, j = 1, \ldots, N, \tag{21}$$

where w_{ij} denotes the quantity sourced from supplier $i \in \mathcal{S}$ in the second period, and w_{sj} is the spot purchase quantity when the suppliers' yields are $\mathbf{z}_j = [z_{ij}]_{i\in\mathcal{S}}$, for all $j = 1, \ldots, N$.

Whenever applicable, suppliers' capacities K_i, $i \in \mathcal{S}$, can be incorporated through the following capacity constraints:

$$x_i \leqslant K_i,\ \forall i \in \mathcal{S}, \tag{22}$$

so that the buyer cannot order a quantity larger than the capacity of the respective supplier.[9]

4.3. Multi-sourcing by a risk averse buyer

A risk averse buyer, as mentioned in Section 3.3, will intend to initially procure the target quantity Q from the suppliers with a probability of at least α, during the first period. That is:

$$\mathbb{P}\left(\sum_{i\in\mathcal{S}} Y_i(x_i, Z_i) = \sum_{i\in\mathcal{S}} \min\{1, Z_i\} \cdot x_i \geqslant Q\right) \geqslant \alpha, \tag{23}$$

which requires the distribution of the random variable $\sum_{i\in\mathcal{S}} Y_i(x_i, Z_i)$. The distribution of $\sum_{i\in\mathcal{S}} Y_i(x_i, Z_i)$ involves an N-fold convolution of the distributions of $Y_i(x_i, Z_i)$ that are given in (9) and, hence, could be difficult to compute. Therefore, to obviate this difficulty we adopt a statistical approach by using the N-sized sample $\vec{\mathbf{z}}$ of supplier yields by adding the following constraints to **SAA-LP**($\mathcal{S}, \vec{\mathbf{z}}$):

$$\sum_{i\in\mathcal{S}} \min\{1, z_{ij}\} \cdot x_i \geqslant Q \cdot v_j \quad \forall j = 1, \ldots, N \tag{24}$$

$$\sum_{j=1}^N v_j \geqslant \alpha \cdot N, \tag{25}$$

$$v_j \in \{0, 1\} \quad \forall j = 1, N, \tag{26}$$

[6] Actually, by setting the random variables $Y_i(x_i, Z_i)$ at their respective expected values $\mathbb{E}\left[Y_i(x_i, Z_i)\right]$, for all $i \in \mathcal{S}$, the total procurement cost for the two periods in the *certainty-equivalent problem* is obtained as $\sum_{i\in\mathcal{S}} c_i \mathbb{E}\left[Y_i(x_i, Z_i)\right] + s\left[Q - \sum_{i\in\mathcal{S}} \mathbb{E}\left[Y_i(x_i, Z_i)\right]\right]^+$, so that the constraint $Q \leqslant \sum_{i\in\mathcal{S}} \mathbb{E}\left[Y_i(x_i, Z_i)\right]$ holds true for optimal solution since $s > c_i\ \forall i \in \mathcal{S}$. Hence, the total cost is $\sum_{i\in\mathcal{S}} c_i \mathbb{E}\left[Y_i(x_i, Z_i)\right]$, if $\mathbf{x}$ is optimal. Therefore, we can rewrite the following problem as **CEP**($\mathcal{S}$):

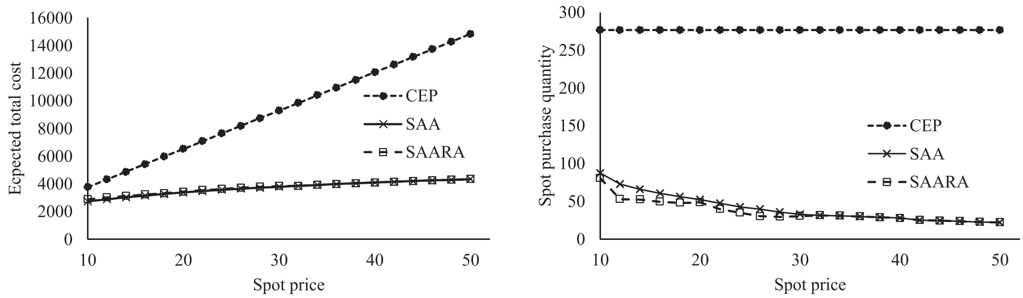
$$\min \sum_{i\in\mathcal{S}} c_i \mathbb{E}\left[Y_i(x_i, Z_i)\right] + s\left[Q - \sum_{i\in\mathcal{S}} \mathbb{E}\left[Y_i(x_i, Z_i)\right]\right]^+$$
s.t. $x_i \geqslant 0\ \forall i \in \mathcal{S}$,

where the constraint $Q \leqslant \sum_{i\in\mathcal{S}} \mathbb{E}\left[Y_i(x_i, Z_i)\right]$ is added in (11).

[7] Note that the constraint (14) can be used in lieu of $\sum_{i\in\mathcal{S}} w_i + w_s \geqslant [Q - \sum_{i\in\mathcal{S}} y_i(x_i, z_i)]^+$ because $w_s, w_i \geqslant 0,\ \forall i \in \mathcal{S}$.

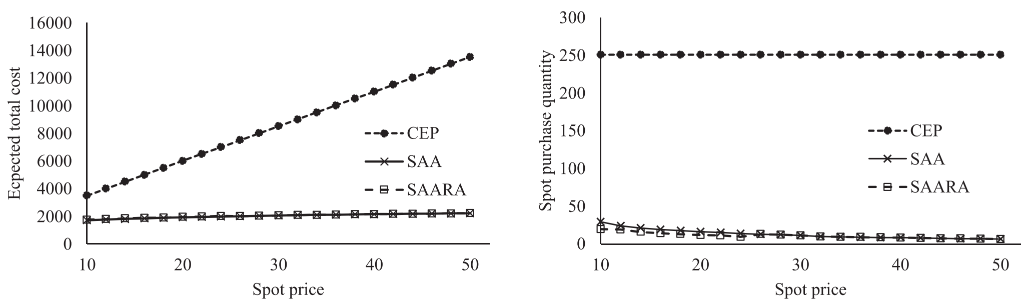
[8] **SAA-LP**($\mathcal{S}, \vec{\mathbf{z}}$) is an LP because all the coefficients of its variables x_i, w_{ij} and w_{sj} are some constants for any realized values of the yield matrix $\vec{\mathbf{z}}$.

[9] It should be noted that the actual output may be higher than the capacity depending on the yield value (i.e., when z_i is higher than 1); such scenarios often occur in practice, for example, in agriculture, chemical industries, metallurgy, etc. Nevertheless the supplier will not accept orders higher than its capacity, which could be the number of units that the supplier can produce when yield is equal to 1 (i.e., $z_i = 1$).



(a) Optimal cost with recourse
(b) Spot purchase quantity

Fig. 2. Optimal cost and spot purchase quantity for type i cost structure.



(a) Optimal cost with recourse
(b) Spot purchase quantity

Fig. 3. Optimal cost and spot purchase quantity for type ii cost structure.

which gives the following problem:

$$\textbf{SAARA}-\textbf{MILP}\left(\mathcal{S},\vec{\mathbf{z}}\right):$$

$$\min \sum_{i\in\mathcal{S}} c_i\left[\int_0^1 u f_i(u)du + 1 - F_i(1)\right]\cdot x_i + \frac{1}{N}\sum_{j=1}^{N}\left[\sum_{i\in\mathcal{S}} c_i w_{ij} + s w_{sj}\right]$$
s.t. (19), (20), (21), (24), (25), (26).

As in the case of **SAA-LP**($\mathcal{S},\vec{\mathbf{z}}$), supplier capacity constraints can be imposed in **SAARA-MILP**($\mathcal{S},\vec{\mathbf{z}}$) by adding (22) to the above problem.

We can easily conclude that a buyer will procure more aggressively during the first period as his risk-aversion increases. We formalize and prove this finding in the following result:

Lemma 4. *The average quantity procured during the first period increases, and so the average quantity purchased in the spot market during the second period decreases, as the risk aversion α increases.*

5. Discussion and numerical study

In this section, we examine the performance of the solution of the multi-sourcing problem obtained using SAA method (i.e., **SAA-LP**($\mathcal{S},\vec{\mathbf{z}}$)) against the base model **CEP**($\mathcal{S}$) in order to draw some practical insights. Later, we observe the computational efficiency of our models **SAA-LP**($\mathcal{S},\vec{\mathbf{z}}$) and **SAARA-MILP**($\mathcal{S},\vec{\mathbf{z}}$) especially with respect to the size of the problem (i.e., number of suppliers available). Next, specifically for the **SAARA-MILP**($\mathcal{S},\vec{\mathbf{z}}$) problem, which solves the multi-sourcing problem for a risk averse-buyer, we set the MIP gap to 0.02 so that the solution obtained is within 2% of the optimal solution and the risk-aversion factor of the buyer $\alpha = 80\%$. Moreover, for solving **SAA-LP**($\mathcal{S},\vec{\mathbf{z}}$) and **SAARA-MILP**($\mathcal{S},\vec{\mathbf{z}}$) we choose a sizable sample size of $N = 1000$.

We perform all the computations using programs written in Python 3.8 by using Gurobi 9.0.1 as the backend optimizer. All programs are executed on 64-bit Windows 10 operating system on a 11th Gen Intel i7 core machine at 2.8 GHz with 16 GB RAM.

5.1. Cost improvement on **CEP**($\mathcal{S}$)

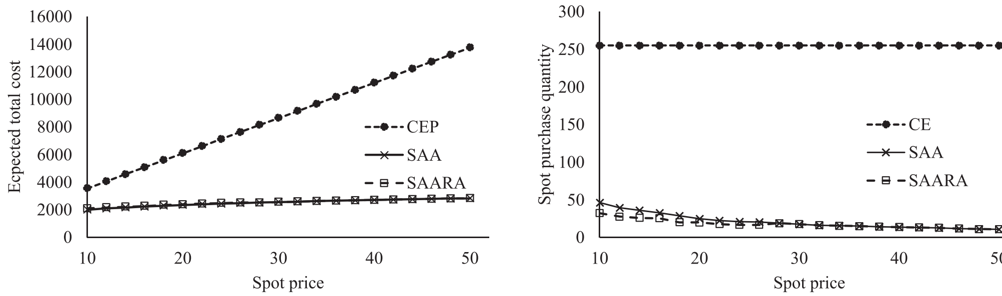
Now, we compare the reduction in cost that our models offer against the more commonly used **CEP**($\mathcal{S}$). We perform this comparison for 3 types of cost structure, which we discuss next.

In our first numerical example, we assume there are 10 suppliers in $\mathcal{S}$, with yields independently and normally distributed with average yields 1 (i.e., $Z_i \sim \mathcal{N}(1,\sigma_i), i \in \mathcal{S}$).[10] Furthermore, we assume that a supplier with low yield variability quotes a higher wholesale price, that is $\sigma_i \geqslant \sigma_j \Leftrightarrow c_i \leqslant c_j, \forall i,j \in \mathcal{S}$. We set the suppliers' yield standard deviations $\boldsymbol{\sigma} = [1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1]$ and their unit prices follow three different price structures: (i) the unit price c is linearly decreasing in σ (i.e., $\frac{c_i - c_j}{\sigma_i - \sigma_j} = constant$), (ii) the unit price c is inversely proportional to σ (i.e., $c \propto \sigma^{-1}$) and (iii) the unit price c compounds with decrease in σ (i.e., $c_{i+1} = kc_i, k > 1$ and $\sigma_{i+1} < \sigma_i$). We use the following unit price vectors that correspond to scenarios (i), (ii), and (iii), respectively:

(i) $\mathbf{c}_{(\mathrm{i})} = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$,
(ii) $\mathbf{c}_{(\mathrm{ii})} = [1, 1.11, 1.25, 1.43, 1.67, 2, 2.5, 3.33, 5, 10]$, and
(iii) $\mathbf{c}_{(\mathrm{iii})} = [1, 1.29, 1.67, 2.15, 2.78, 3.59, 4.64, 6, 7.74, 10]$.

We set the target procurement quantity as $Q = 1000$. Figs. 2–4 provide the comparison of (a) the buyer's total expected procurement cost and (b) the average spot market purchase obtained from our SAA-based

[10] Though we use Normal distribution for supply yields as they are widely used in OM literature (Agrawal and Nahmias, 1997; Anupindi and Akella, 1993; Dong et al., 2021; Gurnani et al., 2000; Schmitt and Snyder, 2012), our model accommodates any probability distribution for supply yields.



(a) Optimal cost with recourse (b) Spot purchase quantity

Fig. 4. Optimal cost and spot purchase quantity for type iii cost structure.

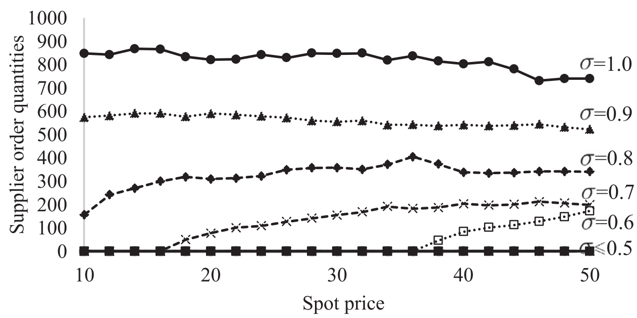
solutions against the benchmark certainty equivalent problem for both risk-neutral and risk-averse buyer cases.

We observe that the average cost incurred through the SAA-based models is substantially lower than the CEP model, which is as expected because while CEP ignores the recourse procurement actions, the SAA-based models account for these and utilizes the information gained through the realized supply yields, which is provided by the N-sized sample $\vec{\mathbf{z}}$.

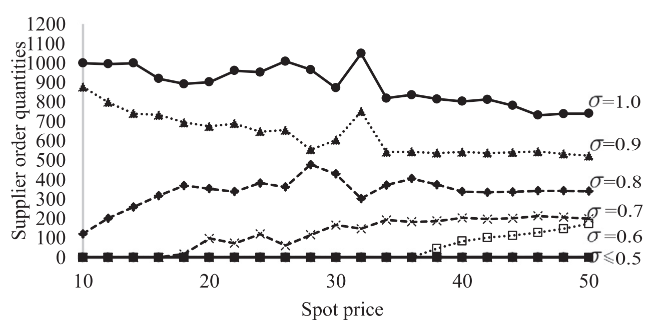
The figures also show that the average quantity procured in the spot market for the SAA-based models is remarkably lower than the CEP model. Furthermore, the average spot procured quantity in the SAA-based models is decreasing in the spot market price s. Thus, compared to the CEP technique that is commonly practiced and that we use as benchmark, the SAA-based models are substantially more efficient and are yet easily implementable in practice.

As observed in Figs. 2–4, the quantity purchased in spot market by a risk-averse buyer is always lower than that purchased by a risk-neutral buyer. This conforms with our intuition because a risk-averse buyer over-orders with the suppliers in the first period because the spot-price in the second period is higher than the prices quoted by the sellers during the first period. This obviates the need for the buyer to buy more in the spot-market during the second period.

Furthermore, we observe that buyer orders from a supplier with a higher price (i.e., lower yield variance) if, and only if, the spot price is sufficiently high. This largely conforms with the finding made by Yan et al. (2012) that the wholesale price takes precedence over reliability when supplier yields are independent. Specifically, from Fig. 5 (which is for linear unit price structure $\mathbf{c}_{(i)}$) we observe that there is a threshold value $\overline{\sigma}$ such that the order quantity corresponding to all suppliers with $\sigma_i > \overline{\sigma}$ is decreasing in the spot price s, while the order quantities with all suppliers with $\sigma_i < \overline{\sigma}$ is increasing in s. Furthermore,

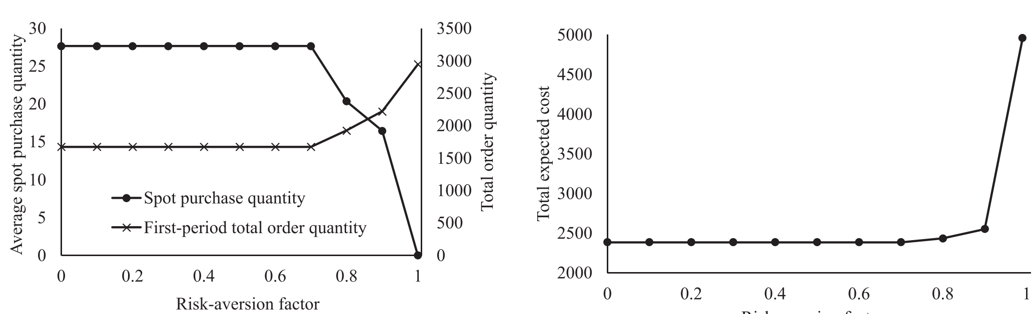


(a) When buyer is risk-neutral



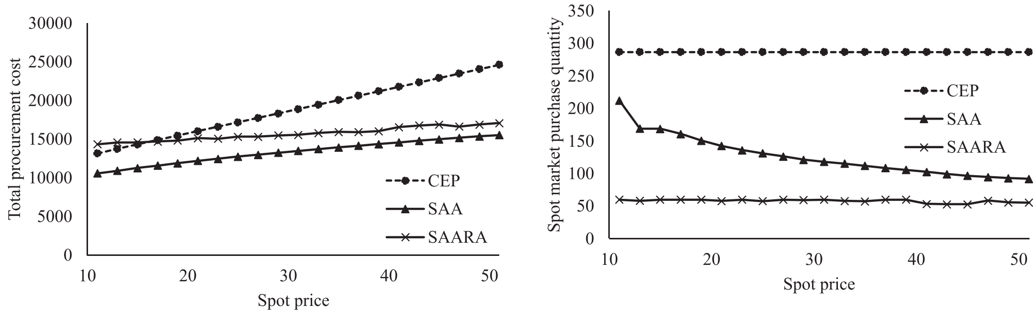
(b) When buyer is risk-averse with $\alpha = 0.8$

Fig. 5. Buyer's ordering pattern to suppliers with different yield uncertainties.



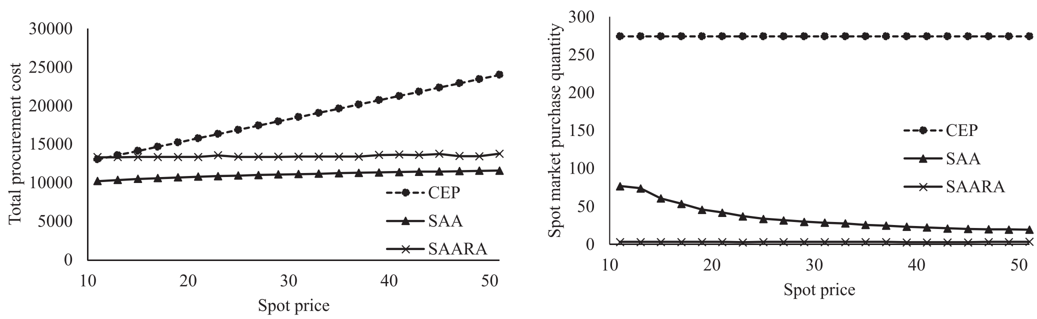
(a) Impact of α on spot-procurement and first-period total order quantities (b) Impact of α on total expected cost

Fig. 6. Impact of risk-aversion factor α.



(a) Optimal cost with recourse (b) Spot purchase quantity

Fig. 7. Optimal cost and spot purchase quantity when supplier yield covariance is given by Σ_1.



(a) Optimal cost with recourse (b) Spot purchase quantity

Fig. 8. Optimal cost and spot purchase quantity when supplier yield covariance is given by Σ_2.

the rate of decrease in order quantity is higher for suppliers with higher yield variance. A similar trend is observed for wholesale price structures $\mathbf{c}_{(ii)}$ and $\mathbf{c}_{(iii)}$ also.

Next, we observe the impact of risk-aversion factor α on buyer's cost and spot-procurement quantity by solving **SAARA-MILP**$(\mathcal{S},\vec{\mathbf{z}})$ for different values of $\alpha \in [0,1]$. Earlier in Lemma 4 we showed that the average total order quantity in the first period is increasing in α. Therefore, we can conclude that the average spot purchase quantity will decrease, for any target quantity Q, which is illustrated in Fig. 6a. Furthermore, Fig. 6a shows that buyer tends to order more quantity during the first period as his risk-aversion α increases. Next, Fig. 6b shows that the total expected cost is also increasing in α due to over-ordering by the buyer as his risk-aversion increases.

Lastly, we examine the case of correlated supplier yields by considering two examples where the unit wholesale price is the same across all suppliers, which we set to $c = 10$, and the suppliers have correlated yields but have the same individual yield variance. For the first and second examples, we use the following covariance matrices, respectively:

$$\Sigma_1 = \begin{bmatrix} 1.0 & -0.9 & 0.8 \\ -0.9 & 1.0 & 0.5 \\ 0.8 & 0.5 & 1.0 \end{bmatrix} \text{ and } \Sigma_2 = \begin{bmatrix} 1.0 & -0.9 & 0.0 \\ -0.9 & 1.0 & 0.5 \\ 0.0 & 0.5 & 1.0 \end{bmatrix}.$$

The corresponding suplier order quantities are given in Tables 1 and 2.

In the first example, when the covariance matrix is Σ_1, we observe that the supplier selection and ordering policy is to largely order only from suppliers 1 and 2 because their yields are negatively correlated, while the yields of suppliers 1 and 3, and 2 and 3 are positively correlated. Thus, the optimal policy is to diversify across buyers with negatively correlated yields. This is evident from Table 1. Likewise, in the second example, when the supplier 3's yield is uncorrelated with supplier 1's, but the yields of suppliers 1 and 2 are negatively correlated and of suppliers 2 and 3 are positively correlated, it is optimal to diversify across suppliers with negative correlation, as expected. Hence, the buyer diversifies his risk across the suppliers and sources from Suppliers 1 and 2 as shown in Table 2. The corresponding average procurement costs (at 5% MIPGAP for the risk-averse model) and spot purchase quantities are given in Figs. 7 and 8. Our observations concur with the results discussed by Dong et al. (2021).

However, the certainty-equivalent based model does not take the supplier yield correlations into account and prescribes to suggest any quantity from any supplier (since all the suppliers are identical in this example with respect to price, yield mean, and yield variance) such that the total order quantity is equal to 1461.19 (one such solution is given in Tables 1 and 2 where the buyer sources all the quantity from Supplier 1).

5.2. Computational performance

After discussing the reduction in total cost that our models offer in comparison with **CEP**$(\mathcal{S})$ in the previous subsection, we now discuss the computational performance of **SAA-LP**$(\mathcal{S},\vec{\mathbf{z}})$ and **SAARA-MILP**$(\mathcal{S},\vec{\mathbf{z}})$ models as the number of suppliers which the buyer can source from increases and when the supplier yields are independently distributed. We make our observations through an elaborate numerical study. Fig. 9a provides the computational times (i.e., CPU-time) taken for the **SAA-LP**$(\mathcal{S},\vec{\mathbf{z}})$ solution. Empirically, we can observe that the CPU-time increases linearly in the size of the problem. The CPU-times for capacity-constrained version of **SAA-LP**$(\mathcal{S},\vec{\mathbf{z}})$ are also very similar to those given in Fig. 9a. From the figure, it is evident that **SAA-LP**$(\mathcal{S},\vec{\mathbf{z}})$ can be efficiently solved even for large instances of the problem (i.e., a large number of suppliers).

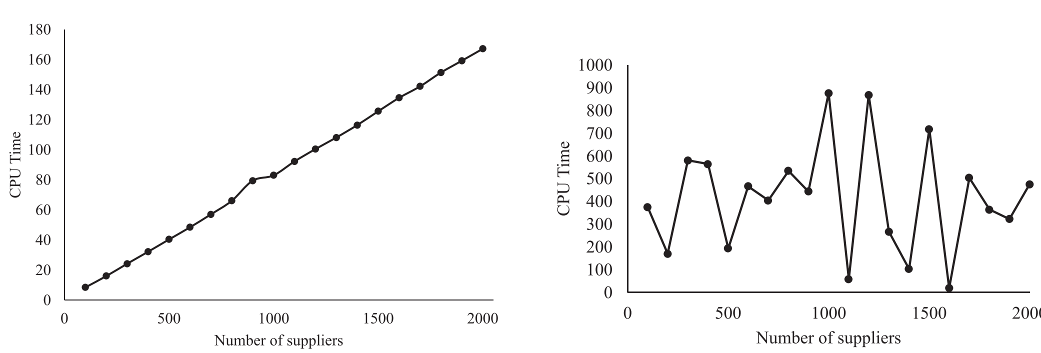
However, the CPU time requirement of **SAARA-LP**$(\mathcal{S},\vec{\mathbf{z}})$ does not exhibit an increasing trend as the number of suppliers increases because the problem is an MILP and its computational time is highly dependent on the structure of the branch-and-bound tree, which is a common fea-

Table 1
Supplier order quantities when supplier yield covariance is given by Σ_1 and when MIPGAP = 5%. S1, S2 and S3 denote suppliers 1, 2, and 3, respectively.

	CEP			SAA			SAARA (MIPGAP = 5%)		
Spot price	S1	S2	S3	S1	S2	S3	S1	S2	S3
11	1461.19	0	0	394.05	440.35	36.26	1000	1000	0
13	1461.19	0	0	449.08	491.3	59.62	982.89	1000	33.56
15	1461.19	0	0	455.17	487.37	57.45	1000	1000	0
17	1461.19	0	0	450.75	495.93	82.89	1000	1000	0
19	1461.19	0	0	463.26	504.42	103.12	1000	1000	0
21	1461.19	0	0	460.14	515.76	131.96	1020.71	1000	12.01
23	1461.19	0	0	472.28	526.3	141.44	1000	1000	0
25	1461.19	0	0	469.1	532.49	164.71	980.78	1001.95	42.14
27	1461.19	0	0	478.52	547.1	167.65	1000	1000	0
29	1461.19	0	0	488.73	566.09	167.69	1012.33	1000	0
31	1461.19	0	0	493.46	572.68	178.48	1000	1000	0
33	1461.19	0	0	503.73	580.12	179.72	1000	998.37	28.18
35	1461.19	0	0	516.73	582.72	187.35	1000	1002.69	30.46
37	1461.19	0	0	523.19	593.69	196.3	1000	1000	0
39	1461.19	0	0	530.51	605.12	199.48	1000	1000	0
41	1461.19	0	0	544.29	617.85	198.75	971	1001.95	122.78
43	1461.19	0	0	565.25	623.92	202.19	1000	996.5	117.52
45	1461.19	0	0	574.21	632.85	207.81	1000	1002.69	115.56
47	1461.19	0	0	580.68	633.55	219.82	1028.26	1000	0
49	1461.19	0	0	590.27	638.19	221.12	1000	1023.67	44.92
51	1461.19	0	0	594.32	647.82	219.49	1000	1040.74	40.89

Table 2
Supplier order quantities when supplier yield covariance is given by Σ_2 and when MIPGAP = 5%. S1, S2 and S3 denote suppliers 1, 2, and 3, respectively.

	CEP			SAA			SAARA (MIPGAP = 5%)		
Spot price	S1	S2	S3	S1	S2	S3	S1	S2	S3
11	1461.19	0	0	492.14	507.86	0	920.3	1000	0
13	1461.19	0	0	495	514.17	0	920.3	1000	0
15	1461.19	0	0	524.16	530.98	0	920.3	1000	0
17	1461.19	0	0	539.46	544.35	0	920.3	1000	0
19	1461.19	0	0	559.11	560.79	0	920.3	1000	0
21	1461.19	0	0	568.69	572.93	0	920.3	1000	0
23	1461.19	0	0	584.13	586.42	0	1000	960.02	0
25	1461.19	0	0	592.83	599.55	0	920.3	1000	0
27	1461.19	0	0	600.64	604.38	0	920.3	1000	0
29	1461.19	0	0	611.66	608.45	0	920.3	1000	0
31	1461.19	0	0	618.51	611.81	0	920.3	1000	0
33	1461.19	0	0	622.64	615.66	0	920.61	1000	0
35	1461.19	0	0	626.91	627.67	0	920.3	1000	0
37	1461.19	0	0	628.97	635.42	0	920.3	1000	0
39	1461.19	0	0	633.8	644.98	0	1000	960.02	0
41	1461.19	0	0	641.42	648.34	0	971.38	1000	0
43	1461.19	0	0	644.18	658.98	0	1000	960.02	0
45	1461.19	0	0	649.19	662.77	0	1000	990.68	0
47	1461.19	0	0	649.62	667.16	0	925.09	1001.49	0
49	1461.19	0	0	651.26	669.43	0	920.3	1000	0
51	1461.19	0	0	652.24	670.76	0	1004.01	969.37	0



(a) **SAA-LP**$(\mathcal{S}, \vec{\mathbf{z}})$ (b) **SAARA-LP**$(\mathcal{S}, \vec{\mathbf{z}})$

Fig. 9. Computational performance of SAA-based models against problem size (i.e., number of suppliers).

ture of MILPs. However, the actual clock times are considerably lower for the CPU-times that are given in Fig. 9b when MIPGAP is 5%. We notice that the longest time taken to solve the problem is approximately 5 minutes, when the number of suppliers is 1200.

6. Conclusions

In this paper, we modeled and analyzed a single buyer's problem of supplier (or vendor) selection and ordering placement under supply uncertainty, when the buyer can procure from multiple suppliers, whose yields can be potentially correlated. In our model, a supplier can supply either less than or more than the quantity that the buyer orders, due shortage or excess supply caused by yield uncertainty. However, the buyer initially procures at most the quantity that he orders with each supplier and later chooses to procure up-to the target procurement quantity, either from the suppliers with excess supply or from the spot market, if the total quantity procured initially from the suppliers is lower than the target procurement quantity. We modeled the problem as a stochastic program with recourse.

In our analysis, we first proved that the expected cost of procurement from the suppliers during the first period is linear in the buyer's order quantities. Then, using this fact, we developed a simple solution using SAA (sample average approximation) technique to solve the multi-sourcing problem in an efficient manner. Later, we extended the model to incorporate buyer's risk-aversion towards a shortage in the initial quantity procured. Although we could initially incorporate buyer's risk aversion through a chance constraint, we noted that handling the chance-constraint was difficult due to (i) the complexity involved in computing the distribution of the total quantity procured during the first period and (ii) the complex structure of the second period's solution as a function of the first period's decisions. Hence, we modeled buyer's risk aversion in a statistical manner by reformulating the original LP as an MILP.

Using a few numerical examples, we evaluated the efficacy of our solution method against the certainty-equivalent problem, which is widely practiced and which we used as a benchmark. We used examples with both independent supplier yields and correlated (both positively and negatively) supplier yields and observed that our procedure performs substantially superior to the certainty-equivalent problem. Later, we also observe and discuss the computational efficiency of our SAA-based models as the size of the problem (i.e., the number of suppliers) increases.

Thus, in the above manner, we formulated the multi-sourcing problem under supply uncertainty and designed a simple and yet scalable procedure to solve the problem, even in the presence of buyer's risk aversion.

Declaration of Competing Interest

We assure that there is no conflict of interest

Appendix A. Proofs

Proof of Lemma 1:

$$\mathbb{P}(Y(x,z)\geqslant y)=\mathbb{P}(\min\{x,z\cdot x\}\geqslant y)=\mathbb{P}(x\geqslant y, z\cdot x\geqslant y)$$
$$=\begin{cases}0 & \text{if } x<y,\\ \mathbb{P}\left(z\geqslant \frac{y}{x}\right) & \text{if } x\geqslant y,\end{cases} \tag{A.1}$$

which is as given in (1).

Proof of Lemma 2:

For an order quantity x, the expected procurement quantity from supplier is:

$$\mathbb{E}[Y(x,Z)]=\int_0^\infty \mathbb{P}(Y(x,z)>y)\,dy$$

$$=\int_0^\infty \mathbb{P}(\min\{x,zx\}>y)\,dy=\int_0^x\left[1-F\left(\frac{y}{x}\right)\right]dy, \tag{A.2}$$

where the last term is obtained by using (1). Now,

$$\frac{d\mathbb{E}[Y(x,Z)]}{dx}=\int_0^x f\left(\frac{y}{x}\right)\cdot\frac{y}{x^2}\,dy+1-F(1)=\int_0^1 uf(u)\,du+1-F(1),$$

where the last integral is obtained through the variable substitution $u=\frac{y}{x}$. Thus, $\frac{d\mathbb{E}[Y(x,Z)]}{dx}$ is constant (i.e., $\int_0^1 uf(u)\,du+1-F(1)$) for all values of x, which shows that $\mathbb{E}[Y(x,Z)]$ is linear in x. That is, $\mathbb{E}[Y(x,Z)]=x\cdot\left[\int_0^1 uf(u)\,du+1-F(1)\right]+C$, where C is the constant of integration (with respect to x). But, by using the base condition that $\mathbb{E}[Y(x=0,Z)]=0$, we can conclude that $C=0$, which completes the proof of the lemma. ■

Proof of Lemma 3: By differentiating $J_1(x)$ for $x<Q$ and rearranging the terms, we obtain:

$$\frac{dJ_1}{dx}=-(s-c)\left[\int_0^1 zf(z)dz+\int_0^{Q/x} zf(z)dz\right]-c\cdot F(1)<0, \tag{A.3}$$

which indicates that the optimal value $x^*\geqslant Q$.

Now, by differentiating $J_1(x)$ with respect to x when $x\geqslant Q$, we obtain the first order condition as:

$$\frac{dJ_1}{dx}=c\left[\int_0^1 zf(z)dz+1-F(1)\right]-s\int_0^{Q/x} zf(z)dz=0.$$

It is easy to observe that if $s\leqslant c_0$, which is defined in the lemma statement, then $\frac{dJ_1}{dx}>0$ indicating that $x^*=Q$. On the other hand, if $s>c_0$, then the optimal solution x^* is given by the above first order condition, which when rearranged gives (5). ■

Proof of Lemma 4: First, we note that as α increases, the vector $\mathbf{v}=[v_j]_{j=1}^N$ is increasing in α due to constraint (25). We say a vector $\mathbf{u}$ is more than $\mathbf{w}$ if, and only if, $u_k\geqslant w_k\ \forall k$ and $u_j>w_j$ for some j. Therefore, for a sufficiently large increase in risk-aversion (i.e., α), the vector $\mathbf{v}$ is strictly increasing.

Let $\alpha_1<\alpha_2$ be such that $\mathbf{v}(\alpha_1)<\mathbf{v}(\alpha_2)$, that is, $0=v_k(\alpha_1)<v_k(\alpha_2)=1$ for some $j\in\{1,\dots,N\}$. Now, let k be the sample (with the corresponding supplier-yield vector $\mathbf{z}_k=[z_{ik}]_{i\in\mathcal{S}}$) where the solution of **SAA-LP**$(\mathcal{S},\bar{\mathbf{z}})$ at $\alpha=\alpha_1$ violates the constraint (24) at $\alpha=\alpha_2$, i.e., $\sum_{i\in\mathcal{S}}\min\{1,z_{ik}\}\cdot x_i(\alpha_1)\leqslant Q\cdot v_k(\alpha_2)$, while the solution of **SAA-LP**$(\mathcal{S},\bar{\mathbf{z}})$ at $\alpha=\alpha_2$ satisfies the constraint, i.e., $\sum_{i\in\mathcal{S}}\min\{1,z_{ik}\}\cdot x_i(\alpha_2)\geqslant Q\cdot v_k(\alpha_2)$. Therefore, the average procurement quantity during the first period, which is given by $\frac{1}{N}\cdot\sum_{j=1}^N\sum_{i\in\mathcal{S}}\min\{1,z_{ij}\}\cdot x_i(\alpha_1)$, when $\alpha=\alpha_1$ is lower than the average procurement quantity during the first period, which is given by $\frac{1}{N}\cdot\sum_{j=1}^N\sum_{i\in\mathcal{S}}\min\{1,z_{ij}\}\cdot x_i(\alpha_2)$, when $\alpha=\alpha_2$, because $\sum_{i\in\mathcal{S}}\min\{1,z_{ik}\}\cdot x_i(\alpha_1)\leqslant\sum_{i\in\mathcal{S}}\min\{1,z_{ik}\}\cdot x_i(\alpha_2)$, and the problem is a minimization problem with positive objective function coefficients and the constraint (24) is a $\geqslant$ type constraint in $\mathbf{x}$ so that

$$\sum_{j=1,j\neq k}^N\sum_{i\in\mathcal{S}}\min\{1,z_{ij}\}\cdot x_i(\alpha_1)\leqslant\sum_{j=1,j\neq k}^N\sum_{i\in\mathcal{S}}\min\{1,z_{ij}\}\cdot x_i(\alpha_2).\ ■$$

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