

2014 AASRI Conference on Sports Engineering and Computer Science (SECS 2014)

## A New Algorithm for Tone Detection

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### Abstract

This study presents a new algorithm for detecting dual-tone multi-frequency (DTMF) signals. The discrete multi-frequency transform (DMFT) method with optimal phase was employed to reduce the DTMF detection computations. With the DMFT approach, the pass-band ripple is employed as a cost function, and the optimal phase can be obtained using the open-loop search method. Compared to the traditional discrete Fourier transform (DFT) approach, not only is a computational saving of 75% to 83% achieved, but spectrum efficiency is also maintained. These experimental results demonstrate the excellent performance of the DMFT method.

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Peer-review under responsibility of Scientific Committee of American Applied Science Research Institute

**Keywords:** Tone Detection, Goertzel Algorithm, DFT, FFT, DTMF.

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### 1. Introduction

In a public switched telecommunications network (PSTN), dual-tone multi-frequency (DTMF) signaling is widely employed for analog telephone dialing, data entry, voicemail systems, and remote control of various consumer electronics. DTMF signaling is used to transfer digital numbers. The PSTN system also defines additional tones to indicate the status of the calling process.

The International Telecommunications Union (ITU) specifications allow a frequency tolerance of approximately  $\pm 1.5\%$  for valid DTMF tones. Thus, tones with an offset of  $\pm 3.5\%$  must be rejected. However,

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frequencies with an offset ranging between  $\pm 1.5\%$  and  $\pm 3.5\%$  can be accepted or rejected. An example of tone detection at a frequency of 1477 Hz is shown in Fig 1. Frequencies within 1455 to 1499 Hz must be accepted, whereas frequencies below 1425 Hz or higher than 1529 Hz must be rejected. Frequencies within 1425 to 1455 Hz or 1499 to 1529 Hz can be accepted or rejected.

Previously, the discrete Fourier transform (DFT) algorithm [1, 3] was commonly employed to detect DTMF signals. The DFT approach converts time-domain signals into a frequency-domain spectrum. In Fig 1, the DFT size  $N$  with 512 was employed. The spectrum index  $k$  at 93 to 96 must be calculated for a frequency of 1477 Hz. The spectrum index  $k$  at 92 and 97 can also be considered. By contrast, the spectrum index  $k$  at less than 91 and more than 98 must be rejected. Compared to 1-point DFT and following the ITU specifications, at least four computations of 1-point DFT at  $k = 93$  to 96 are required for a frequency of  $1477 \pm 1.5\%$  Hz. The 1-point DFT requires  $2N$  multiplies and  $2(N-1)$  adders for computation. Thus, the computation for a frequency of 1477 Hz requires at least  $8N$  multiplies and  $8(N-1)$  adders.

For the Goertzel algorithm [2, 4], a recursive equation was proposed for detecting DTMF signals. The Goertzel approach was derived from the DFT approach. However, compared to the DFT approach, the Goertzel approach saves a constant table. Furthermore, in complex input signals, similar to the inverse DFT (IDFT), the Goertzel approach requires only  $2(N+2)$  multipliers and  $4(N+1)$  adders. However, the Goertzel algorithm does not contribute significantly to computational savings.

In this study, the discrete multi-frequency transform (DMFT) was proposed for detecting DTMF signals. Using DMFT, compared to the DFT approach, a computational saving of 75% to 83% was achieved. The DMFT algorithm is described in Section II, and Section III provides the experimental results. The final section presents a conclusion.

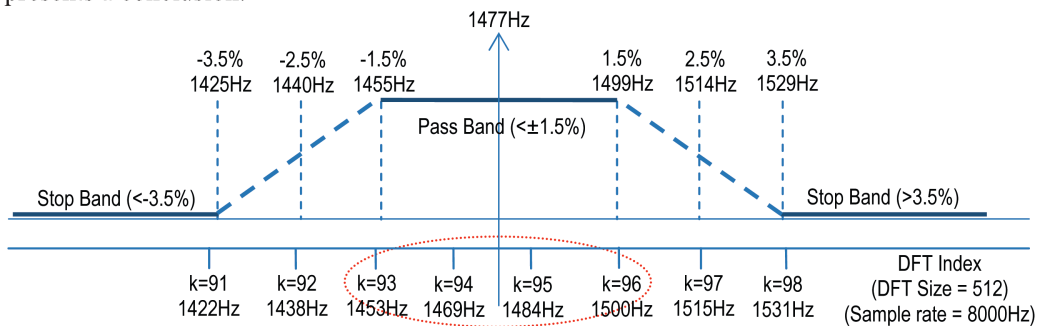


Fig.1. Tone detection at a frequency of 1477 Hz

## 2. Tone Detection Algorithm

In this study, DMFT with a different phase was employed to detect DTMF signals. The following equations describe the DFT and DMFT algorithms. Equation 1 expresses the general DFT. The basis frequency component  $W_N^{nk}$ , which is equal to  $e^{-j\frac{2\pi nk}{N}}$ , was employed to extract/detect the signal at a frequency of  $f = \frac{k \cdot SR}{N}$  Hz. The value of  $SR$  is the sample rate. Equation 2 expresses the DMFT algorithm,

where the basis frequency component  $e^{-j\frac{2\pi nk}{N}}$  is replaced by the multi-frequency signal  $\sum_{m=0}^{M-1} e^{-j(\frac{2\pi m(k+m)}{N} + m\theta)}$ ,

$\theta$  is the phase of each basis frequency component, and  $M$  is the number of basis frequencies. If  $M$  equals 1, the DMFT equation equals the DFT equation. However, regardless of the value of  $L$ , the multi-frequency

signal  $\tilde{W}_{N,\theta}^{nk}$  of DMFT is similar to the basis frequency component  $W_N^{nk}$  of DFT, and is a constant table and independent of input signal  $x[n]$ . Thus, the computation of DFT and DMFT employs the same approach. However, the frequency response of DFT and DMFT differs. The frequency response of the DMFT equation can be controlled to fit the specific requirements of DTMF detection.

$$\text{DFT: } X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk} = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi nk}{N}} \quad (1)$$

$$\text{DMFT: } X[k] = \sum_{n=0}^{N-1} x[n] \tilde{W}_{N,\theta}^{nk} = \sum_{n=0}^{N-1} x[n] \sum_{m=0}^{M-1} e^{-j \left( \frac{2\pi m(k+m)}{N} + m\theta \right)} \quad (2)$$

A detailed performance analysis of the DFT and DMFT algorithms is shown in Fig 2. Fig 2(a) shows the frequency response of the general DFT approach with a frequency index  $k = 11$  (344 Hz). The DFT size  $N$  equals 256, and the sample rate  $SR$  equals 8000 Hz. Fig 2(b) to 2(d) show the frequency responses of DMFT at a frequency index of  $k = 11$  and  $L = 2$ . In Fig 2(b), a phase  $\theta = 0^\circ$  was set. The frequency response between  $k = 11$  and 12 was extremely poor. In Fig 2(c), a phase  $\theta = 50^\circ$  was set. However, the frequency response between  $k = 11$  and 12 remained poor. In Fig 2(d), a phase  $\theta = 102^\circ$  was set. The frequency response between  $k = 11$  and 12 was reasonably good. In the Fig 2(e), the frequency response of DMFT with  $L = 3$  is shown. The frequency responses is reasonable good. These analysis results of Fig 2 confirm that the phase  $\theta$  of DMFT equation can be well controlled to match the requirement of DTMF detection. The detail description on the estimation process of optimal phase of DMFT algorithm will be given as following.

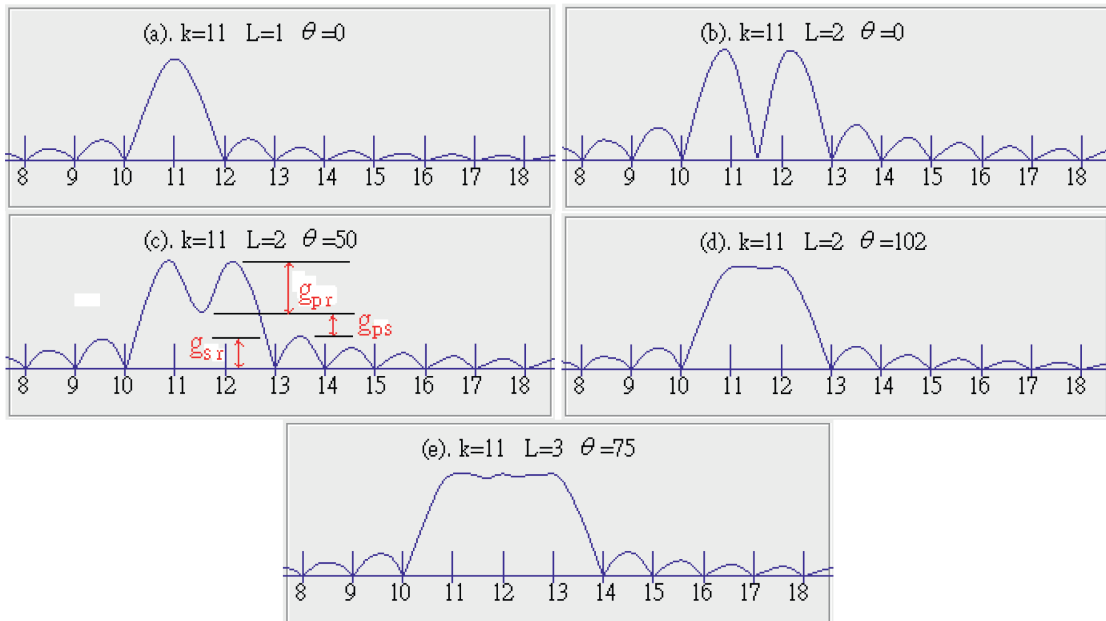


Fig.2. The frequency response of DMFT with different  $L$  and  $\theta$

Two cost functions, as shown in Fig 2(c) and defined below, were identified to determine the optimal phase  $\hat{\theta}$ .  $\hat{\theta} = \{ \min(g_{pr}) \mid 0 \leq \theta \leq 360^\circ \}$ , The  $g_{pr}$  is the pass-band ripple, which is the frequency response range between frequency indices  $k$  and  $k + (L - 1)$ .  $g_{pr}$  is also shown in Fig 2(c).

### 3. Experiment Results

The DMFT method was proposed to improve DTMF detection computations. The computational requirements of the DFT method, Goertzel algorithm, and DMFT method are listed in Table 1. The computational savings provided by DMFT are superior to those of the traditional DFT and Goertzel approach. In general cases of DFT-based DTMF detection, the basis frequencies number approximately 4 to 6. Thus, using the DMFT approach, the computational saving is approximately 75% to 83%; this is a reasonably satisfactory result.

Table 1. The computations (\*, +) of each method

	DFT	Goertzel	DMFT
$L = 1$	(2N,2N-2)	(N+2,2N+1)	(2N,2N-2)
$L = 2$	(4N,4N-4)	(2N+4,4N+2)	(2N,2N-2)
$L = 3$	(6N,6N-6)	(3N+6,6N+3)	(2N,2N-2)
$L = 4$	(8N,8N-8)	(4N+8,8N+4)	(2N,2N-2)
$L = 5$	(10N,10N-10)	(5N+10,10N+5)	(2N,2N-2)
$L = 6$	(12N,12N-12)	(6N+12,12N+6)	(2N,2N-2)

The frequency responses of the DMFT approach were also thoroughly analyzed. The stop-band ripple  $g_{sr}$ , pass-band ripple  $g_{pr}$ , and differential gain  $g_{ps}$  between the pass-band and stop-band and the optimal phase  $\hat{\theta}$  are listed in Table 2. A frequency index of  $k = 11$  and an FFT size of 256 were set. When  $L = 1$ , a general DFT approach was indicated. The stop-band ripple  $g_{sr}$  was -9.89 dB. The differential gain  $g_{ps}$  between the stop-band and pass-band equaled 6.88 dB. When  $L = 2$ , the stop-band ripple  $g_{sr}$  was -9.75 dB. The differential gain  $g_{ps}$  between the stop-band and pass-band equaled 6.70 dB. As shown in Table 4, the stop-band ripple, pass-band ripple, and differential gain for each  $L$  value are almost identical. This indicates that the frequency response performances of DFT and DMFT are almost the same. However, the computational savings provided by DMFT are greater, as shown in Table 1.

Table 2. The stop-band ripple, pass-band ripple, differential gain, and optimal phase for each  $L$  value

	$g_{sr}$	$g_{pr}$	$g_{ps}$	$\theta$
$k = 11, L = 1$	-9.89 dB		6.88 dB	
$k = 11, L = 2$	-9.75 dB	0.07 dB	6.70 dB	$102^\circ$
$k = 11, L = 3$	-9.55 dB	0.19 dB	6.38 dB	$75^\circ$
$k = 11, L = 4$	-10.62 dB	0.26 dB	7.55 dB	$129^\circ$
$k = 11, L = 5$	-10.45 dB	0.26 dB	7.32 dB	$109^\circ$
$k = 11, L = 6$	-10.28 dB	0.38 dB	7.13 dB	$264^\circ$

#### 4. Conclusions

The DMFT method with an optimal phase was proposed to reduce DTMF detection computations. A computational saving of 75% to 83% was achieved without declination of spectrum efficiency. The results showed that the performance of the DMFT method was superior to that of the DFT and Goertzel approaches. In the future, the phase and gain of the basis frequency components of the DMFT approach can be controlled more precisely to further improve DTMF detection.

#### Acknowledgements

These and the Reference headings are in bold but have no numbers. Text below continues as normal.

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