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Existence and Nonexistence of Positive Periodic Solution for Impulsive Nicholson's Blowflies Model

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Abstract

In this paper, impulsive Nicholson's blowflies model is studied. By using Leray-Schauder fixed point theorem , we obtain some sufficient conditions for the existence of positive periodic solution. In addition, the nonexistence of positive periodic solution is also investigated. Our results extend and improve the previous literatures.

Keywords: Impulsive Nicholson's blowflies model; Periodic solution; Existence; Nonexistence; Leray-Schauder fixed point theorem

1. Introduction

As we know, population dynamics have received great attention from many authors. In the natural ecological systems, there exist a lot of discontinuous, impulsive phenomena, for instance, people release or harvest a species at fixed time, many species are given birth instantaneously and seasonally, and so on. Introducing the impulsive effect to ecology model can describe the species and the ecological systems more truly and reasonablly. Impulsive differential equation has become a hot and important study topic[1-3,5,11-14].

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A very basic and important ecological problem in the study of population dynamics concerns the existence of positive periodic solutions. Recently, many authors investigated the existence of positive periodic solution by using Krasnoselskii cone fixed point theorem and Mawhin's coincidence degree theory[9-14].

In this paper,we consider impulsive Nicholson's blowflies model

$$\begin{cases} x'(t) = -a(t)x(t) + b(t)x(t-\tau)e^{-\beta(t)x(t-\tau)}, & t \neq t_k \\ \Delta x(t_k) = I_k(x(t_k^-)), & t = t_k \end{cases} \tag{1.1}$$

where $a(t) \in C(R,R), b(t), \beta(t) \in C(R,(0,+\infty))$, $a(t), b(t), \beta(t)$ are all ω-periodic functions, $\int_0^\omega a(t)dt > 0$, $I_k(u) \in C([0,+\infty),[0,+\infty))$, τ and ω are positive constants, $t_k \in R, t_{k+1} > t_k, k \in Z$, $\lim_{k\to\pm\infty} t_k = \pm\infty, \Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = x(t_k)$, there exists $q \in Z^+$,such that $I_{k+q}(u) = I_k(u)$, $t_{k+q} = t_k + \omega$.

In [7][8],the oscillation and global attractivity of Nicholson's blowflies model were studied. The purpose of this paper is to obtain sufficient conditions for the existence of positive periodic solutions of system (1.1) by using Leray-Schauder fixed point theorem. Furthermore, we also investigate the nonexistence of positive periodic solutions of system (1.1).

2. Preliminaries

Let $X = \{x(t) \mid x \in PC(R,R), x(t+\omega) = x(t)\}$, where $PC(R,R) = \{x : R \to R \mid x(t)$ is continuous for $t \neq t_k$, $x(t_k^+)$ and $x(t_k^-)$ exist, $x(t_k^+) = x(t_k)\}$. For $x \in X$,we define $\| x \| = \sup_{t\in[0,\omega]} |x(t)|$, then X is Banach space.

Let
$$G(t,s) = \frac{\exp\{\int_t^s a(\xi)d\xi\}}{\exp\{\int_0^\omega a(\xi)d\xi\} - 1},$$

$H = [t, t+\omega], H_1 = \{t \in H \mid a(t) \geq 0\}, H_2 = \{t \in H \mid a(t) < 0\}, \qquad a^+(t) = \max\{0, a(t)\}, a^-(t) = \min\{0, a(t)\}.$

Then

$$G(t,s) = \frac{\exp\{(\int_{[t,s]\cap H_1} a(\xi)d\xi + \int_{[t,s]\cap H_2} a(\xi)d\xi)\}}{\exp\{\int_0^\omega a(\xi)d\xi\} - 1} \leq \frac{\exp\{\int_{[t,s]\cap H_1} a(\xi)d\xi\}}{\exp\{\int_0^\omega a(\xi)d\xi\} - 1} \leq \frac{\exp\{\int_0^\omega a^+(\xi)d\xi\}}{\exp\{\int_0^\omega a(\xi)d\xi\} - 1} := B,$$

we also have

$$G(t,s) \geq \frac{\exp\{\int_{[t,s]\cap H_2} a(\xi)d\xi\}}{\exp\{\int_0^\omega a(\xi)d\xi\} - 1} \geq \frac{\exp\{\int_0^\omega a^-(\xi)d\xi\}}{\exp\{\int_0^\omega a(\xi)d\xi\} - 1} := A,$$

So we get $\quad 0 < A \leq G(t,s) \leq B, \quad s \in [t, t+\omega].$

It is easy to verify that $x(t)$ is the ω-periodic solution of equation (1.1) if and only if $x(t)$ is the ω-periodic solution of the below integral equation

$$x(t) = \int_t^{t+\omega} G(t,s)b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds + \sum_{t<t_k<t+\omega} G(t,t_k)I_k(x(t_k^-)).$$

We define operator $T : X \to X$,

$$(Tx)(t) = \int_t^{t+\omega} G(t,s)b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds + \sum_{t<t_k<t+\omega} G(t,t_k)I_k(x(t_k^-)).$$

Obviously, $x(t) \in PC(R,R)$ is the ω-periodic solution of equation (1.1) if and only if x is the fixed point of operator T. We can check that operator T is completely continuous.

The following Leray-Schauder fixed point theorem (see Theorem 6.5.4 in [4]) is an important tool in our proofs.

Lemma 1 .[4] **(Leray-Schauder)** Let Ω be a closed convex subset of Banach space X, $0\in\Omega$, Let $T:\Omega\to\Omega$ be a completely continuous operator.Then , either the set $\{x\in\Omega \mid x=\lambda Tx, 0<\lambda<1\}$ is unbounded or the operator T has at least one fixed point in Ω.

3. Existence of Positive Periodic Solution

In this paper, we use notations: $\tilde{b}=\int_0^{\omega} b(t)dt$, $\underline{\beta}=\inf_{t\in[0,\omega]}\beta(t)$.

Theorem 1. Assume that $\lim_{u\to\infty}\frac{I_k(u)}{u}=0(\forall k)$, then equation (1.1) has at least one positive ω-periodic solution.

Proof. Let $f(t,u)=b(t)ue^{-\beta(t)u}$, then we have $\lim_{u\to+\infty}\max_{t\in[0,\omega]}\frac{f(t,u)}{u}=0$.

For $\varepsilon>0$ and $\varepsilon\le\frac{1}{2B(\omega+q)}$, there exists $L>0$, such that $f(t,u)<\varepsilon u, I_k(u)<\varepsilon u$ for $u>L$.

Choose $Q\ge L+2+2B\omega\max_{0\le t\le\omega,0\le u\le L} f(t,u)+2Bq\max_{1\le k\le q}\{\max_{0\le u\le L} I_k(u)\}$,

Let $\Omega=\{x\mid x\in X, \|x\|\le B, x(t)\ge\delta\|x\|\}$, $\delta=\frac{A}{B}$, then Ω is the closed convex subset of X.

For $x\in\Omega$, $(Tx)(t)\le B\int_t^{t+\omega} b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds+B\sum_{t<t_k<t+\omega} I_k(x(t_k^-))=B\int_0^{\omega} f(s,x(s-\tau))ds+B\sum_{0<t_k<\omega} I_k(x(t_k^-))$

$$=B\int_{J_1=\{s\in[0,\omega],x(s-\tau)>L\}} f(s,x(s-\tau))ds+B\int_{J_2=\{s\in[0,\omega],x(s-\tau)\le L\}} f(s,y(s-\tau))ds$$

$$+B\sum_{0<t_k<\omega,x(t_k^-)>L} I_k(x(t_k^-))+B\sum_{0<t_k<\omega,x(t_k^-)\le L} I_k(x(t_k^-))$$

$$\le B\int_0^{\omega}\varepsilon x(s-\tau)ds+B\omega\max_{0\le t\le\omega,0\le u\le L} f(t,u)+B\sum_{0<t_k<\omega,x(t_k^-)>L}\varepsilon x(t_k^-)+Bq\max_{1\le k\le q}\{\max_{0\le u\le L} I_k(u)\}$$

$$\le B\omega\varepsilon\|x\|+B\omega\max_{0\le t\le\omega,0\le u\le L} f(t,u)+Bq\varepsilon\|x\|+Bq\max_{1\le k\le q}\{\max_{0\le u\le L} I_k(u)\}$$

$$\le B\omega\varepsilon Q+B\omega\max_{0\le t\le\omega,0\le u\le L} f(t,u)+Bq\varepsilon Q+Bq\max_{1\le k\le q}\{\max_{0\le u\le L} I_k(u)\}$$

$$=BQ(\omega+q)\varepsilon+B\omega\max_{0\le t\le\omega,0\le u\le L} f(t,u)+Bq\max_{1\le k\le q}\{\max_{0\le u\le L} I_k(u)\}\le\frac{1}{2}Q+\frac{1}{2}Q=Q.$$ Thus $\|Tx\|\le Q$.

On the other hand, $\|Tx\|\le B[\int_0^{\omega} f(s,x(s-\tau))ds+\sum_{0<t_k<\omega} I_k(x(t_k^-))]$,

$$(Tx)(t)\ge A[\int_0^{\omega} f(s,x(s-\tau))ds+\sum_{0<t_k<\omega} I_k(x(t_k^-))]\ge\tfrac{A}{B}\|Tx\|=\delta\|Tx\|,$$

hence we have $T\Omega\subset\Omega$. For $x\in\Omega$ and $x=\lambda Tx, 0<\lambda<1$, we have $x(t)=\lambda(Tx)(t)<(Tx)(t)\le Q$, which implies $\|x\|\le Q$. So $\{x\in\Omega\mid x=\lambda Tx, 0<\lambda<1\}$ is bounded. By lemma 1 ,we know operator T has at least one fixed point in Ω,which implies equation (1.1) has at least one positive ω-periodic solution. The proof of Theorem 1 is complete.

Theorem 2. Assume that $B\tilde{b}<1$, and there exists $\rho_1>0$, such that $I_k(u)\le\frac{\rho_1(1-B\tilde{b})}{Bq}$ $(\forall k)$ for $0<u\le\rho_1$, then equation (1.1) has at least one positive ω-periodic solution.

Proof. Let $\Omega=\{x\mid x\in X, \|x\|\le\rho_1, x(t)\ge\delta\|x\|\}$, $\delta=\frac{A}{B}$, then Ω is the closed convex subset of X.

For $x\in\Omega$, $(Tx)(t)\le B\int_t^{t+\omega} b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds+B\sum_{t<t_k<t+\omega} I_k(x(t_k^-))=B\int_0^{\omega} b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds+B\sum_{0<t_k<\omega} I_k(x(t_k^-))$

$$\le B\int_0^{\omega} b(s)x(s-\tau)ds + B\sum_{0<t_k<\omega}\frac{\rho_1(1-B\tilde{b})}{Bq} = B\int_0^{\omega} b(s)x(s-\tau)ds + Bq\frac{\rho_1(1-B\tilde{b})}{Bq}$$

$$\le B\|x\|\int_0^{\omega} b(s)ds + \rho_1(1-B\tilde{b}) = B\|x\|\tilde{b} + \rho_1(1-B\tilde{b}) \le B\rho_1\tilde{b} + \rho_1(1-B\tilde{b}) = \rho_1$$

Thus $\|Tx\|\le \rho_1$. On the other hand, $\|Tx\|\le B[\int_0^{\omega} f(s,x(s-\tau))ds + \sum_{0<t_k<\omega} I_k(x(t_k^-))]$,

$$(Tx)(t)\ge A[\int_0^{\omega} f(s,x(s-\tau))ds + \sum_{0<t_k<\omega} I_k(x(t_k^-))] \ge \tfrac{A}{B}\|Tx\| = \delta\|Tx\|,$$

hence we have $T\Omega\subset\Omega$. For $x\in\Omega$ and $x=\lambda Tx, 0<\lambda<1$, we have $x(t)=\lambda(Tx)(t)<(Tx)(t)\le\rho_1$, which implies $\|x\|\le\rho_1$. So $\{x\in\Omega \mid x=\lambda Tx, 0<\lambda<1\}$ is bounded. By lemma 1 , we know operator T has at least one fixed point in Ω ,which implies equation (1.1) has at least one positive ω-periodic solution. The proof of Theorem 2 is complete.

Theorem 3. Assume that $I_k(u)\le\frac{1}{2Bq}u \quad (\forall k)$ for $u>0$, then equation (1.1) has at least one positive ω-periodic solution.

Proof. Choose constant $d>1+\frac{2B\tilde{b}}{\beta e}$, Let $\Omega=\{x \mid x\in X, \|x\|\le d, x(t)\ge\delta\|x\|\}$, $\delta=\frac{A}{B}$, then Ω is the closed convex subset of X .

We notice that $g(u)=ue^{-\beta u}$ reaches its maximum $\frac{1}{\beta e}$ at $u=\frac{1}{\beta}$.

For $x\in\Omega$, $(Tx)(t)\le B\int_t^{t+\omega} b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds + B\sum_{t<t_k<t+\omega} I_k(x(t_k^-))$

$$= B\int_0^{\omega} b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds + B\sum_{0<t_k<\omega} I_k(x(t_k^-)) \le B\int_0^{\omega} b(s)x(s-\tau)e^{-\underline{\beta}x(s-\tau)}ds + B\sum_{0<t_k<\omega} I_k(x(t_k^-))$$

$$\le B\int_0^{\omega} b(s)\frac{1}{\underline{\beta}e}ds + B\sum_{0<t_k<\omega}\frac{1}{2Bq}x(t_k^-) = \frac{B\tilde{b}}{\underline{\beta}e} + B\sum_{0<t_k<\omega}\frac{1}{2Bq}x(t_k^-) \le \frac{B\tilde{b}}{\underline{\beta}e} + B\sum_{0<t_k<\omega}\frac{1}{2Bq}d = \frac{B\tilde{b}}{\underline{\beta}e} + B\frac{1}{2Bq}qd \le \frac{d}{2}+\frac{d}{2} = d$$

Thus $\|Tx\|\le d$. The rest of proof steps are the same as that of Theorem 1, so the proof of Theorem 3 is complete.

4. Nonexistence of Positive Periodic Solution

Theorem 4. Assume that there exist constants $\alpha_k>0$, such that $I_k(u)\le\alpha_k u$ for u>0, and $B(\tilde{b}+\sum_{k=1}^{q}\alpha_k)<1$, then equation (1.1) has no positive ω-periodic solution.

Proof. Suppose equation (1.1) has positive ω-periodic solution $x(t)$, then $x(t)=(Tx)(t)$. Hence we have

$$\|x\| = \max_{t\in[0,\omega]}|x(t)| = \max_{t\in[0,\omega]}|(Tx)(t)| = \max_{t\in[0,\omega]}\left(\int_t^{t+\omega} G(t,s)b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds + \sum_{t<t_k<t+\omega} G(t,t_k)I_k(x(t_k^-))\right)$$

$$\le \max_{t\in[0,\omega]}\left(B\int_t^{t+\omega} b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds + B\sum_{t<t_k<t+\omega} I_k(x(t_k^-))\right) = \max_{t\in[0,\omega]}\left(B\int_0^{\omega} b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds + B\sum_{0<t_k<\omega} I_k(x(t_k^-))\right)$$

$$= B\int_0^{\omega} b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds + B\sum_{0<t_k<\omega} I_k(x(t_k^-)) \le B\int_0^{\omega} b(s)x(s-\tau)ds + B\sum_{0<t_k<\omega}\alpha_k x(t_k^-)$$

$$\le B\|x\|\int_0^{\omega} b(s)ds + B\|x\|\sum_{0<t_k<\omega}\alpha_k = B(\tilde{b}+\sum_{k=1}^{q}\alpha_k)\|x\| < \|x\|,$$

which is a contradiction.Therefore equation (1.1) has no positive ω-periodic solution.

Theorem 5. Assume that there exist constants $\gamma_k > 0$, such that $I_k(u) \ge \gamma_k u$ for $u > 0$, and $A\delta\sum_{k=1}^{q}\gamma_k > 1$, then equation (1.1) has no positive ω-periodic solution.

Proof. Suppose equation (1.1) has positive ω-periodic solution $x(t)$, then $x(t) = (Tx)(t)$. It is easy to know $x(t) \ge \delta \| x \|$. Hence we have

$$\| x \| = \max_{t\in[0,\omega]} | x(t) | = \max_{t\in[0,\omega]} | (Tx)(t) | = \max_{t\in[0,\omega]}\left(\int_t^{t+\omega} G(t,s)b(s)x(s-\tau)e^{-\beta(s)x(s-\tau)}ds + \sum_{t<t_k<t+\omega} G(t,t_k)I_k(x(t_k^-)) \right)$$

$$\ge \max_{t\in[0,\omega]}\left(\sum_{t<t_k<t+\omega} G(t,t_k)I_k(x(t_k^-)) \right) \ge \max_{t\in[0,\omega]}\left(A\sum_{t<t_k<t+\omega} I_k(x(t_k^-)) \right) = \max_{t\in[0,\omega]}\left(A\sum_{0<t_k<\omega} I_k(x(t_k^-)) \right) = A\sum_{0<t_k<\omega} I_k(x(t_k^-)) = A\sum_{k=1}^{q} I_k(x(t_k^-))$$

$$\ge A\sum_{k=1}^{q}\gamma_k x(t_k^-) \ge A\sum_{k=1}^{q}\delta \| x \| \gamma_k = A\delta \| x \| \sum_{k=1}^{q}\gamma_k > \| x \|,$$

which is a contradiction.Therefore equation (1.1) has no positive ω-periodic solution.

Remark . In the study of periodic solution, most of the previous literatures required that f is superlinear or sublinear. However, in this paper, $f(t,u) = b(t)ue^{-\beta(t)u}$,which is neither superlinear nor sublinear.We obtain sufficient conditions for the existence of positive periodic solution in case that f is neither superlinear nor sublinear ,which extend and improve the results of previous literatures.In addition, we also obtain sufficient conditions for the nonexistence of positive periodic solution.

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