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## Image Denoising Method with Adaptive Bayes Threshold in Nonsubsampled Contourlet Domain

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### Abstract

In this paper, an improved image denoising method based on a nonlinear thresholding function with adaptive bayes threshold in Nonsubsampled Contourlet Transform (NSCT) domain. In overcoming the shortcomings of the same threshold, the noise deviation of the different sub-band are estimated based on the coefficients of different directional and level sub-bands in NSCT domain, and the thresholds of every sub-band is estimated by Bayesian threshold estimation method. After choosing the thresholds, a nonlinear thresholding function was chosen to overcome the shortcomings of the soft and the hard thresholding function. The simulation results show that the proposed method in this paper can remove Gaussian white noise more effectively, and get a higher PSNR value and keep image texture and detail information more clearly, which also has a better visual effect.

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**Keywords:** Image Denoising; Bayes Threshold; Nonsubsampled Contourlet Transform; Nonlinear Thresholding Function.

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## 1. Introduction

With the rise in the number of image sensors (or pixels) per unit area of a chip, modern image capturing devices are increasingly sensitive to noise. Camera manufacturers, therefore, depend on image denoising algorithms to reduce the effects of such noise artifacts in the resultant image.<sup>[1]</sup> Recently proposed denoising methods use different approaches to address the problem. Image denoising is used to remove the additive noise while retaining as much as possible the important signal features. Denoising images corrupted by Gaussian noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. Wavelet transform is good at energy compaction, the small coefficient are more likely due to noise and large coefficient due to important signal feature. So, Wavelet denoising is an important technique in the area of image noise reduction. Over the last decade, it has been widely used in images denoising.<sup>[2-6]</sup>

Wavelet transform is an efficient image denoising algorithm, but it lacks shift and orientation invariance, and is poor in directional selectivity. Aiming at the shortcomings, Martin Vetterli and M. N. Do<sup>[7]</sup> put forward a kind of “Real” two-dimension signal sparse representation method: Contourlet transform, which has multi-resolution, local time-frequency and the opposite directions. Then the contourlet transform is used widely in images denoising, and have a well performance.<sup>[8-9]</sup> However, Contourlet transform lacks the shift invariance which due to the sampling process, the image will produce false Gibbs distortion and invariance which due to the sampling process. In 2006, the nonsubsampling contourlet transform (NSCT) was proposed, and it is a fully shift-invariant, multiscale, and multidirection expansion<sup>[10]</sup>. The performances of denoising methods in NSCT domain have proven to be very efficient in image denoising.<sup>[11-13]</sup>

In this paper, an improved image denoising method based on a nonlinear thresholding function with adaptive bayes threshold in NSCT domain. In overcoming the shortcomings of the same threshold, the noise deviation of the different sub-band are estimated based on the coefficients of different directional and level sub-bands in NSCT domain, and the thresholds of every sub-band is estimated by Bayesian threshold estimation method. After choosing the thresholds, a nonlinear thresholding function was chosen to overcome the shortcomings of the soft and the hard thresholding function. The simulation results show that the proposed method in this paper can remove Gaussian white noise more effectively, and get a higher PSNR value and keep image texture and detail information more clearly, which also has a better visual effect.

### Nomenclature

$\delta$	the signal standard variance estimation
$\sigma$	the noise standard variance estimation
$w_{i,j}$	the coefficient of the original signal
$S_{m,n}$	the NSCT sub-band
$T_{m,n}$	the threshold of sub-band $S_{m,n}$
$\hat{w}_{i,j}$	the coefficient of the restore signal
$\alpha_{i,j}$	the weighing factor of the coefficient $w(i,j)$
$M \times N$	the size of sub-band
PSNR	the peak signal-to-noise ratio

## 2. Nonsampled Contourlet Transform

In contourlet transform, the Laplacian Pyramid (LP) and the direction filter bank (DFB) are employed for multiscale decomposition and directional decomposition, respectively. To get rid of the frequency aliasing of the contourlet transform and to achieve shift-invariance, the nonsampled contourlet transform (NSCT) was proposed based on nonsampled pyramid filter bank (NSPFB) and nonsampled direction filter bank (NSDFB). Fig.1 shows the decomposition framework of NSCT. Fig.1 (a) displays an overview of the NSCT. The structure consists in a bank of filters that splits the 2-D frequency plane in the sub-bands illustrated in Fig.1(b). NSCT is a redundant transform which has shift invariance, multi-scale and multi-resolution, it samples on the filter, then analysis and synthesize the filter, the design and reconstruction of this filter is easy to be realized, which could better collect frequency and more regularity.

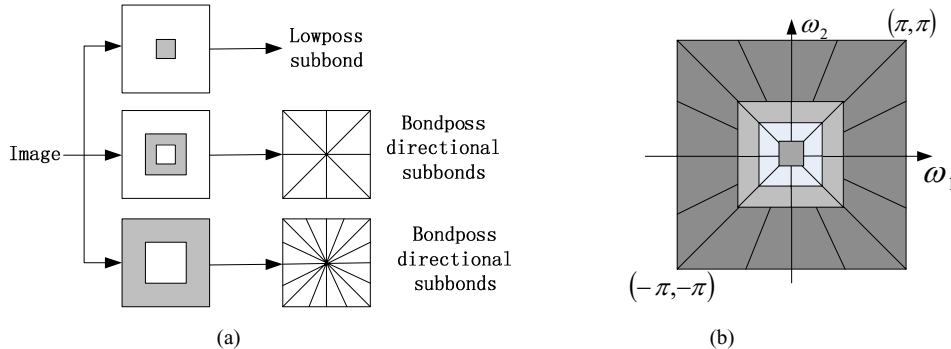


Figure 1 Nonsampled Contourlet Transform (a) NSFB structure that implements the NSCT; (b) Idealized frequency partitioning obtained with the proposed structure

## 3. Proposed Denoising Method

The noisy images may contain additive noise and multiplicative noise, and the most common White Gaussian noise is chosen as the noise model in this paper. NSCT is linear transformation, so its coefficients are divided into two parts: coefficients of original image and coefficients of noise. The coefficients of original image is large and concentrated, while the coefficients of noise is small and dispersive. Therefore, a proper threshold method can be set to deal with them. The image signal can be recovered after reconstruction with the new NSCT coefficients. The basic principle of the shrinking threshold denoising algorithm in NSCT domain is comparing NSCT sub-band coefficients with the thresholds, the coefficient which is less than the threshold is set to zero, while the greater one retained or modified. So threshold denoising algorithm mainly involves two key problems: the estimation of thresholds and the choice of thresholding function. In this paper, the threshold is estimated by the adaptive Bayesian threshold method, and a nonlinear thresholding function is chosen to deal with NSCT sub-band coefficients.

### 3.1. Adaptive Bayesian Threshold Estimation

The threshold estimation is one core of the shrinking threshold denoising algorithm. There is many threshold selection methods exist, such as VisuShrink, SURE and Bayesian threshold estimation methods. Bayesian threshold estimation is obtained under Bayes standards, set the ideal threshold getting in the condition of Bayes minimal risk is:

$$T = \arg \min r_{\text{Bayes}} \quad (1)$$

Where  $r_{\text{Bayes}}$  is the Bayesian risk function. It is very difficult to solve the analytical expressions. So the approximate solution usually is calculated by numerical methods in the simulation.

$$T = \frac{\sigma^2}{\delta} \quad (2)$$

Where  $\sigma^2$  is the noise variance estimation, and  $\delta^2$  is the signal variance estimation. The estimate noise variance  $\sigma^2$  and the estimated signal variance  $\delta^2$  can be obtained by equation (3), (4):

$$\sigma^2 = \left( \frac{\text{median}(|w_{i,j}|)}{0.6745} \right)^2 \quad (3)$$

$$\delta^2 = \max \left( \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N w_{i,j}^2 - \sigma^2, 0 \right) \quad (4)$$

Where  $w_{i,j}$  is the lowest frequency coefficient after the transformation,  $M \times N$  is the size of sub-band.

In Bayesian threshold estimation, the noise variance  $\sigma^2$  is estimated as a same value in each directional and each level sub-bands. However, NSCT is not an orthogonal transform, the noise variance of sub-bands in each directional and each level are different in the NSCT domain. So, the noise variance of every sub-band is estimated independently. In this paper the noise variances are estimated based on the sub-band coefficients of each directional and each level.

For the NSCT sub-band  $S_{m,n}$  ( $m$  is the level and  $n$  is the direction), its noise variance  $\sigma_{m,n}^2$  is obtained by equation (3), and  $w_{i,j}$  is the coefficients of sub-band  $S_{m,n}$ .

$$\sigma_{m,n}^2 = \left( \frac{\text{median}(|w_{i,j}|)}{0.6745} \right)^2 \quad (5)$$

Then, the threshold  $T_{m,n}$  of sub-band  $S_{m,n}$  is estimated by equation (2).

### 3.2. New Thresholding Function

The most common thresholding functions are the hard thresholding function and the soft-thresholding function. The hard threshold denoising is as follow:

$$\hat{w}_{i,j} = \begin{cases} w_{i,j}, & |w_{i,j}| \geq T \\ 0, & |w_{i,j}| < T \end{cases} \quad (6)$$

The soft threshold denoising method is as follow:

$$\hat{w}_{i,j} = \begin{cases} \text{sign}(w_{i,j})(|w_{i,j}| - T), & |w_{i,j}| \geq T \\ 0, & |w_{i,j}| < T \end{cases} \quad (7)$$

Where  $T$  is the threshold,  $w_{i,j}$  is the coefficient of the original signal,  $\hat{w}_{i,j}$  is the coefficient of the restore signal.

The soft and hard threshold denoising methods are to compares the threshold with the decomposition coefficient of the original signal. In these methods, the nonimportant coefficients are set to zero. In hard thresholding, the important coefficients remain unchanged. In soft thresholding, the important coefficients are reduced by the absolute threshold value. However, there is deviation between the coefficients and the original ones. So, a nonlinear thresholding function is applied in the NSCT domain for denoising in this paper. The thresholding function is as follows:

$$\hat{w}(i, j) = \begin{cases} \text{sign}(w(i, j))(|w(i, j)| - \alpha_{i,j} \cdot T_{m,n}) & |w(i, j)| \geq T_{m,n} \\ 0, & |w(i, j)| < T_{m,n} \end{cases} \quad (8)$$

Where,  $T_{m,n}$  is the threshold of sub-band  $S_{m,n}$ ,  $w(i, j)$  and  $\hat{w}(i, j)$  is the original and denoised coefficients of sub-band  $S_{m,n}$ , the weighing factor  $\alpha_{i,j}$  in the coefficient  $w(i, j)$  is defined as:

$$\alpha_{i,j} = \frac{T_{m,n}}{|w(i, j)| \cdot \exp(|w(i, j)| - T_{m,n})} \quad (9)$$

### 3.3. Process of Proposed Denoising Method

The specific procedure of proposed denoising method based on adaptive Bayes threshold and nonlinear thresholding function in NSCT domain is as follows:

- Performs multiscale decomposition of the noisy image in NSCT domain, obtain the sub-bands coefficients of the noisy image in different directional and levels;
- Calculates the noise variance  $\sigma_{m,n}^2$  and threshold  $T_{m,n}$  in different sub-bands based on the coefficients in different directional and level sub-bands by equation (2)-(5);
- Estimates the denoised coefficients of high-frequency sub-bands based on the threshold  $T_{m,n}$  and sub-bands coefficients by equation (8) and (9);
- After denoising procedure, does the nonsubsampled contourlet inverse transform and rebuild the image after the process of sub-band coefficients, gets the restore image.

## 4. Experiments

To evaluate the performance of the proposed approach, this section compares the proposed scheme with other popular denoising schemes in Wavelet, Contourlet and NSCT domain based hard-thresholding. The three benchmark images named Lena, Barbara and Boat (8-bit grayscale, size  $512 \times 512$ ) are used for the experiments. The noisy images are obtained by adding Gaussian white noise (noise standard deviations ranging from 10 to 40) to the noise free image. We adopt the peak signal-to-noise ratio (PSNR) to test the denoising effect. The PSNR results of various are shown in the Table.1. The denoised results of the standard image Lena with noise standard deviation  $\sigma = 40$  are shown in the Fig.4. The peak signal-to-noise ratio is defined as:

$$\text{PSNR} = 10 * \log_{10} \frac{I_{\max} \times I_{\max}}{\frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N (I_{i,j} - \hat{I}_{i,j})^2} \quad (dB) \quad (10)$$

Where  $I_{\max}$  is the maximum gray level value of the image,  $I_{i,j}$  is the gray value of pixel at  $(i, j)$  of the noise free image  $I$ ,  $\hat{I}_{i,j}$  is the gray value of pixel at  $(i, j)$  of the denoised image  $\hat{I}$ , and  $M$  and  $N$  are the numbers of row and column of the test image, respectively.

Table.1 PSNR (dB) of different denoised images with different algorithm

Image	$\sigma$	10	15	20	25	30	35	40
Lena	Noisy	28.14	24.62	22.09	20.16	18.60	17.22	16.09
	Wavelet	30.92	28.28	26.20	24.56	23.08	21.82	20.78
	Contourlet	30.00	28.62	27.70	26.95	26.40	25.97	25.56
	NSCT	31.24	27.73	25.08	23.04	21.38	19.95	18.78
	Proposed	34.54	32.77	31.64	30.81	30.14	29.47	28.80
Barbara	Noisy	28.13	24.62	22.09	20.16	18.60	17.22	16.09
	Wavelet	28.79	26.23	24.31	22.86	21.65	20.60	19.69
	Contourlet	26.73	25.25	24.25	23.53	23.06	22.70	22.38
	NSCT	31.48	28.41	25.76	23.59	21.80	20.32	19.01
	Proposed	31.05	28.95	27.74	27.03	26.30	25.63	25.04
Boat	Noisy	28.13	24.60	22.10	20.17	18.60	17.27	16.09
	Wavelet	29.71	27.28	25.48	23.91	22.54	21.41	20.44
	Contourlet	27.14	26.11	25.38	24.77	24.33	23.96	23.61
	NSCT	30.39	27.48	25.02	23.04	21.44	20.02	18.83
	Proposed	32.61	30.92	29.72	28.68	27.81	27.15	26.63

Table.1 shows the PSNR results of the four methods on the experimented images corrupted by different noise variance of additive Gaussian noise. The results show the proposed method gives highest PSNR for all the experimented images. In addition, it consistently provides better results than other denoising methods for different images and noise levels.



Fig. 2. (a) Original image; (b) Noisy image (16.09 dB); (c) Denoised with Wavelet (20.78dB); (d) Denoised with Contourlet (25.56 dB); (e) Denoised with NSCT(18.78 dB); (f) Denoised with the proposed method (28.80 dB)

Fig. 2 shows the denoising performance of the above mentioned methods using the standard image Lena. Fig. 2(a) and (b) show the noise free and the noisy images, respectively. The denoised images obtained using different denoising methods and the proposed method in the DWT domain are illustrated in Fig. 2(c)–(f), respectively, and have PSNR values of 20.78, 25.56, 18.78 and 28.80 dB, respectively. From the figures in Fig. 2, it can be observed that the proposed denoising approach is better than the other denoising methods.

## 5. Conclusions

In this paper, an improved image denoising method based on a nonlinear thresholding function with adaptive bayes threshold in NSCT domain. In overcoming the shortcomings of the same threshold, the noise deviation of the different sub-band are estimated based on the coefficients of different directional and level sub-bands in NSCT domain, and the thresholds of every sub-band is estimated by Bayesian threshold estimation method. After choosing the thresholds, a nonlinear thresholding function was chosen to overcome the shortcomings of the soft and the hard thresholding function. The simulation results show that the proposed method in this paper can remove Gaussian white noise more effectively, and get a higher PSNR value and keep image texture and detail information more clearly, which also has a better visual effect.

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