

# Measurements on domains and topology

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## Abstract

The concept of a measurement on a continuous partial order has been recently introduced by Keye Martin. Measurement provides a uniform degree of approximation for elements of the kernel, i.e., those elements with measure zero. Measurement also induces a Scott topology for elements “near” the kernel. The results below are joint work with Keye Martin.

I.  $X$  is developable and  $T_1$  iff it is the kernel of a measurement on a continuous poset,  $X$  is developable  $T_1$  and choquet complete iff it is the kernel of a measurement on a continuous cpo.

II. For each developable  $T_1$ -space  $X$ , there exists a developable  $T_1$ -space  $M(X)$  with a poset order  $<$  such that (1)  $X$  is the kernel of a measurement. (2) the topology on  $M(X)$  is exactly the topology induced by the measurement. (3) If  $X$  is  $T_2$ , then  $M(X)$  is a Moore space. (4) If  $X$  is a complete (or even semi-complete) Moore space then  $(M(X), <)$  is a cpo. (5) If  $X$  is the real line, then  $M(X)$  is a non-normal Moore space. (6) ( $MA \neq CH$ ) If  $X$  is a subspace of the real line and  $\omega < \text{card}(X) < c$ , then  $M(X)$  is a normal nonmetrizable Moore space.

III. The countable ordinals with the order topology is the top of a Scott domain, and is a  $G_\delta$  set with respect to the Scott topology on the domain. [This answers several questions open questions in the area.]

IV. Finally, we give a new recursion induction theorem for cpo's using measurement theory.

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