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On the Unification of Process Semantics: Equational Semantics

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Abstract

The complexity of parallel systems has produced a large collection of semantics for processes, a classification of which is provided by Van Glabbeek's linear time-branching time spectrum; however, no suitable unified definitions were available. We have discovered the way to unify them, both in an observational framework and by means of a quite small set of parameterized (in)equations that provide a sound and complete axiomatization of the preorders that define them. In more detail, we have proved that we only need a generic simulation axiom (*NS*), which defines the family of constrained simulation semantics, thus covering the class of branching time semantics, and a generic axiom (*ND*) for reducing the non-determinism of processes, by means of which we introduce the additional identifications induced by each of the linear time semantics.

Keywords: processes, linear time-branching time spectrum, equational semantics, uniform presentation.

1 Introduction

In order to study the behavior of concurrent processes, many different semantics have been proposed. Most of them are defined starting from the interleaving model, where the essence of concurrent computation collapses into that of non-deterministic processes, modeled by means of labeled transition systems. Most of the popular semantics in that category appear in Van Glabbeek's linear time-branching time spectrum [13]. They were introduced by different authors, using different semantic frameworks; see Figure 1.

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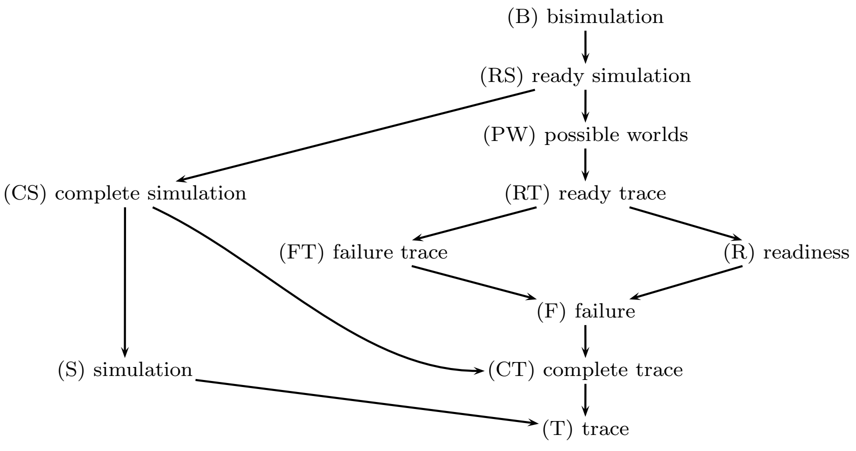

Fig. 1. Semantics in the linear time-branching time spectrum.

In his paper, Van Glabbeek aims for a uniform presentation of all these semantics by using three different approaches (the observational or testing framework, the logical one, and the algebraic characterization of the semantics) which provide sufficiently general frameworks to present them. Once in a uniform setting, it is much easier to compare the discriminating power of the different semantics, as is done in the lattice representation of the spectrum. All of these frameworks can be used to characterize not only the equivalences induced by the semantics but also the corresponding natural preorders, that in the logical presentation correspond to the "satisfy more formulas than" relation.

Even under a common framework, the semantics in the spectrum appear to be haphazardly defined and distributed in it. In this paper we aim to show that this is not the case: every process semantics can be understood as the combination of two "design decisions" that define what we have called the *dynamic* and the *static* behavior of processes.

By means of our unification we expect to cover the following three main goals:

- a new presentation of the spectrum of the process semantics justifying the special interest of some of these semantics and clarifying the relations among them;
- a uniform presentation of the semantics that allows to prove results simultaneously for all of them;
- finally, we complete the picture with some "lost" semantics which, either were proposed by other authors after the publication of Van Glabbeek's spectrum, or are introduced for the first time in this paper (as far as we know).

Our unification process has its roots in our coinductive characterizations [3,5] of the semantics in the spectrum by means of (bi)simulations up-to, that characterize both the equivalences and the preorders that define them. In particular, the dynamic component is closely related with simulations. In [6] we were able to unify the definitions of different simulation semantics, by means of *constrained* simulations, and to describe a generic axiom that characterizes all of them.

Similarly, there is also a single axiom defining the static or non-deterministic component of the semantics. In this case the number of choices for constraints is larger so that we obtain several semantics under the same simulation part. The unifying framework arises because two basic axioms parametrized conveniently suffice to define all the classic semantics in the spectrum, as well as many others.

We have also found a unified observational framework [7] where branching observations correspond to simulation semantics while linear ones capture the remaining. In order to emphasize the axiomatic aspect, we have chosen to devote the next three sections of this paper to relate the new equational framework with the known one, and then in Section 5 to briefly introduce the joint presentation of the two new unified frameworks.

2 Preliminaries

As is customary when studying algebraic characterizations of semantics, we will concentrate here on the study of finite processes. We will thus consider the basic process algebra BCCSP, that has repeatedly been used to algebraically represent that class of processes; in particular, in [13].

Definition 2.1 Given a set of actions Act, the set BCCSP(Act) of processes is defined by the following BNF-grammar:

$$p ::= \mathbf{0} \mid ap \mid p + q$$

where $a \in Act$; $\mathbf{0}$ represents the process that performs no action; for every action in Act, there is a prefix operator; and $+$ is a choice operator.

The operational semantics for BCCSP terms is defined by:

$$ap \xrightarrow{a} p \qquad \frac{p \xrightarrow{a} p'}{p + q \xrightarrow{a} p'} \qquad \frac{q \xrightarrow{a} q'}{p + q \xrightarrow{a} q'}$$

We write $p \xrightarrow{a}$ if there exists a process q such that $p \xrightarrow{a} q$ and extend it to define $\stackrel{\alpha}{\Longrightarrow}$ for arbitrary traces $\alpha \in Act^*$, as usual.

Many different semantics for these non-deterministic processes have been defined in the literature. The most important and popular semantics appear in Van Glabbeek's spectrum [13]. One indirect way to capture any semantics is by means of the equivalence relation induced by it: given a formal semantics $[\![\cdot]\!]$, we say that processes p and q are equivalent iff they have the same semantics, that is, $p \equiv q \Leftrightarrow [\![p]\!] = [\![q]\!]$. Also, these semantics can be defined by means of adequate observational scenarios, or by logical characterisations that introduce natural preorders whose kernels are the semantic equivalences.

Both equivalences and preorders have been axiomatized for most of these scenarios, as shown in [13], but in some cases only finite conditional axiomatization are possible, as discussed in [2]. In particular, bisimilarity can be axiomatized by means of the four simple axioms:

$$(B_1) \quad x + y \simeq y + x \qquad\qquad (B_3) \quad x + x \simeq x$$

$$(B_2) \quad (x + y) + z \simeq x + (y + z) \qquad (B_4) \quad x + \mathbf{0} \simeq x$$

These axioms state that the choice operator is commutative, associative and idempotent, having the empty process as identity element. These axioms also justify the use of the notation $\sum_a \sum_i ap_a^i$ for processes, where the commutativity and associativity of the choice operator is used to group together the summands whose initial action is a. We will also write $p|_a$ for the (sub)process we get by projecting all the a-summands of p; that is, if $p = \sum_a \sum_i ap_a^i$, then $p|_a = \sum_i ap_a^i$.

The initial offer of a process is the set $I(p) = \{a \mid a \in Act \text{ and } p \xrightarrow{a}\}$. This is a simple, but quite important observation function that plays a central role in the definition of the most popular semantics in the linear time-branching time spectrum. We will also denote by I the relation expressing the fact that two processes have the same initial offer: pIq (or $I(p, q)$) $\Leftrightarrow I(p) = I(q)$.

Throughout the paper there appear different order relations. We use $\sqsubseteq$ to denote semantic preorders (behavior preorders) and, for the sake of simplicity, we use the symbol $\sqsupseteq$ to represent the preorder relation $\sqsubseteq^{-1}$. With $\equiv$ we denote the corresponding equivalence (that is, $\sqsubseteq \cap \sqsupseteq$). To refer to a specific preorder in the linear time-branching time spectrum we shall append the initials of the intended semantics as subscripts to the symbol $\sqsubseteq$ ($\sqsubseteq_{RS}$ for ready simulation, $\sqsubseteq_F$ for failures and so on). A similar convention applies to the kernels of the preorders ($\equiv_{RS}$, $\equiv_F$, ...) and to the bisimulation equivalence $\equiv_B$. We use the symbols $\preceq$ and $\simeq$ for the inequalities and equations, respectively, of the algebraic calculus by means of which we axiomatize the semantics. We write $E \vdash t \preceq u$ or $E \vdash t \simeq u$ for the (in)equations that can be derived from the (in)equations in E using the standard rules of (in)equational logic, where the symmetry rule can be applied in the equational derivations, but not in the inequational ones.

Lastly we recall our definition of behavior preorder [3], which is the adequate notion to compare processes when all actions can be observed.

Definition 2.2 A preorder relation $\sqsubseteq$ over processes is a *behavior preorder* if

- it is weaker than bisimilarity, i.e. $p \equiv_B q \Rightarrow p \sqsubseteq q$, and
- it is a precongruence with respect to the prefix and choice operators, i.e. if $p \sqsubseteq q$ then $ap \sqsubseteq aq$ and $p + r \sqsubseteq q + r$.

3 A new axiomatization of the most popular semantics

As we have already hinted in the introduction, the dynamic part of the semantics is governed by the preorder generated by a simulation. On the one hand, bisimilarity is too strong for this purpose because its symmetric definition gives rise to an equivalence rather than a preorder; on the other hand, plain similarity is too weak because most popular semantics are not coarser than simulation equivalence. By contrast, ready similarity is a quite strong relation which can be easily defined as a simulation that only relates processes with the same set of initial actions.

Definition 3.1 (i) A simulation S is a relation between processes such that whenever pSq and $p \xrightarrow{a} p'$, there exists some $q \xrightarrow{a} q'$ such that $p'Sq'$. We say that process p is simulated by q, or that q simulates p, when there is a simulation S containing (p, q); then, we write $p \sqsubseteq_S q$.

(ii) An I-simulation S is a simulation included in I, that is, $S \subseteq I$. I-simulations are also called ready simulations and we write $p \sqsubseteq_{RS} q$ whenever there is some I-simulation S such that $(p, q) \in S$.

(iii) Given an arbitrary constraint N relating pairs of processes, an N-simulation, sometimes simply called constrained simulation if N is understood from the context, is a simulation which only relates pairs in N. We write $p \sqsubseteq_{NS} q$ whenever there is some N-simulation S such that $(p, q) \in S$.

Obviously, ordinary and ready simulations are particular cases of constrained simulations; other classes will be considered later in the paper.

Proposition 3.2 (i) *Plain similarity can be axiomatically defined by means of the axiom* (S) $x \preceq x + y$*, together with the axioms* B_1–B_4 *that define bisimilarity.*

(ii) *Ready similarity can be axiomatically defined by means of the conditional axiom* (RS) $I(x, y) \Rightarrow x \preceq x + y$*, together with* B_1–B_4*. It can also be axiomatized by means of the axiom scheme* $ax \preceq ax + ay$*, where* a *represents an arbitrary action.*

(iii) *Whenever* N *is a behavior preorder,* N*-similarity can be axiomatically defined by means of the conditional axiom* (NS) $N(x, y) \Rightarrow x \preceq x + y$*, together with* B_1–B_4*.*

Proof. See [13,6]. □

Let us now consider the diamond of semantics coarser than ready similarity in the spectrum. It consists of the failures, readiness, failure traces, and ready trace semantics. None of them is a simulation semantics, so their classic axiomatizations contain an additional component:

Failures:	(F)	$a(x + y) \preceq ax + a(y + w)$
Readiness:	(R)	$a(bx + by + u) \preceq a(bx + u) + a(by + v)$
Failure traces:	(FT)	$a(x + y) \preceq ax + ay$
Ready traces:	(RT)	$I(x) = I(y) \Rightarrow ax + ay \simeq a(x + y)$

Since we are interested in capturing the reduction of non-determinism, our first candidate for a general axiom covering all cases was (FT), which captures the fact that by delaying the choices we get "smaller" processes. However, since this axiom characterizes the failure trace semantics and this is finer than failure semantics, a more general axiom is needed: axiom (F) became our next proposal because failure semantics is the coarsest of the four semantics. More precisely, we expected to achieve the axiomatization of the four semantics in the diamond by means of concrete instances of the parameterized conditional axiom

$$(ND) \quad M(x,y,w) \Rightarrow a(x+y) \preceq ax + a(y+w)\,.$$

The conjecture turned out to be correct and the semantics in the diamond can be characterized by the following instances:

$$\begin{array}{lll} (ND^F) & M_F(x,y,w) \iff \text{true} \\ (ND^R) & M_R(x,y,w) \iff I(x) \supseteq I(y) \\ (ND^{FT}) & M_{FT}(x,y,w) \iff I(w) \subseteq I(y) \\ (ND^{RT}) & M_{RT}(x,y,w) \iff I(x) = I(y) \text{ and } I(w) \subseteq I(y) \end{array}$$

Since M_F is the universal relation containing all triples of processes, the corresponding instance of the conditional axiom (ND) is clearly equivalent to (F). Let us now prove that the remaining three semantics are also axiomatized by the corresponding instances of the axiom (ND) together with (RS).

Proposition 3.3 (i) *The readiness preorder* $\sqsubseteq_R$ *is axiomatized by* $\{B_1\text{–}B_4, (RS), (ND^R)\}$.

(ii) *The failure traces preorder* $\sqsubseteq_{FT}$ *is axiomatized by* $\{B_1\text{–}B_4, (RS), (ND^{FT})\}$.

(iii) *The ready traces preorder is axiomatized by the set* $\{B_1\text{–}B_4, (RS), (ND^{RT})\}$.

Proof.

(i) Let us show that the set $\{B_1\text{–}B_4, (RS), (ND^R)\}$ is logically equivalent to $\{B_1\text{–}B_4, (RS), (R)\}$. By taking $x ::= bx' + u$, $y ::= by$, and $w ::= v$ we have that (ND^R) implies (R). In the other direction, let x and y be arbitrary closed BCCSP terms with $I(y) \subseteq I(x)$: we will prove by structural induction on y that $\{B_1\text{–}B_4, (RS), (R)\} \vdash a(x+y) \preceq ax + a(y+w)$, for any term w.
- For $y = \mathbf{0}$, we have $a(x+y) \simeq ax \preceq ax + a(y+w)$, by application of (RS).
- For $y = by' + y''$, it must be $x = bx' + x''$ and taking $v ::= y'' + w$ in (R) we obtain $a(x+y) = a(bx' + by' + x'' + y'') \preceq a(x + y'') + a(y+w)$. We then have $I(y'') \subseteq I(x)$ and we can apply the induction hypothesis to get $\{B_1\text{–}B_4, (RS), (R)\} \vdash a(x+y) \preceq ax + a(y+w)$.

(ii) Let us show that the set $\{B_1\text{–}B_4, (RS), (ND^{FT})\}$ is logically equivalent to $\{B_1\text{–}B_4, (RS), (FT)\}$. The implication from left to right follows by taking $w ::= \mathbf{0}$. In the other direction, let w and y with $I(w) \subseteq I(y)$, so that $a(x+y) \preceq ax + ay$ using (FT) and, since $I(y) = I(y+w)$, we have $y \preceq y+w$ using (RS): hence, $a(x+y) \preceq ax + a(y+w)$.

(iii) Let us show that the set $\{B_1\text{–}B_4, (RS), (ND^{RT})\}$ is logically equivalent to $\{B_1\text{–}B_4, (RS), (RT)\}$. We first note that $\{B_1\text{–}B_4, (RS), (RT)\}$ is equivalent to $\{B_1\text{–}B_4, (RS), (RT_{\succeq})\}$, where $(RT_{\succeq})$ is the axiom $M_{RT}(x,y,w) \Rightarrow ax + ay \succeq a(x+y)$. This follows from the fact that, whenever $I(x) = I(y)$, we can use (RS) to get $x \preceq x+y$ and $y \preceq x+y$, and then $ax + ay \preceq a(x+y)$. Now, the implication from left to right follows by taking $w ::= \mathbf{0}$. From right to left, as above, whenever $I(w) \subseteq I(y)$ we have $y \preceq y + w$ and then, if $I(x) = I(y)$ we have $a(x + y) \preceq ax + ay$ and thus $a(x + y) \preceq ax + a(y + z)$.

□

Throughout this paper we are only considering ground complete axiomatizations for proving inequalities relating BCCSP terms. We leave for future work the study of the existence of ω-complete axiomatizations along the lines of [2].

Certainly, the use of arbitrary conditions in the axioms could be objected to since they can be used to hide the complexity of the semantics; however, the conditions needed to axiomatize the semantics in the spectrum are very simple. In any case, our main interest lies in obtaining a uniform presentation of the axiomatizations that simplifies their algebraic study.

Corollary 3.4 (i) *$\sqsubseteq_{FT}$ is axiomatized by the set $\{B_1–B_4, (RS), (ND_0^{FT})\}$, where (ND_0^{FT}) is the instance of (ND^{FT}) where w is $\mathbf{0}$.*

(ii) *$\sqsubseteq_{RT}$ is axiomatized by $\{B_1–B_4, (RS), (ND_0^{RT})\}$, where (ND_0^{RT}) is the instance of (ND^{RT}) where w is $\mathbf{0}$.*

Proof. Note that for the proof of Proposition 3.3 only the case $w = \mathbf{0}$ is needed.□

Even if the simplifications above are possible, we prefer to maintain the general forms of axioms (ND^{FT}) and (ND^{RT}) to keep all axiomatizations as similar as possible, which will come in handy when proving general properties of the semantics.

Corollary 3.5 (i) *$\sqsubseteq_F$ can be axiomatized by the axioms $\{B_1–B_4, (ND^F)\}$.*

(ii) *$\sqsubseteq_R$ can be axiomatized by the axioms $\{B_1–B_4, (ND^R)\}$.*

Proof. Note that (ND^F) implies (RS) and therefore (ND^R) implies (RS), by taking $y ::= \mathbf{0}$ and $w ::= y$. □

Note that the axiom controlling the reduction of non-determinism has been presented as an inequational axiom. Certainly, it cannot simply be replaced by the corresponding equation since, in general, it is not true that $ax + ay \simeq a(x + y)$. However, the two dimensions corresponding to (RS) and (ND^X) that control the "increase" of a process with respect to a preorder $\preceq$ are not orthogonal; for example, $a(x + y) \preceq a(x + y) + ax$ can be derived both by an application of (ND^{FT}) and of (RS). As a consequence of the relation between these two axioms, if (RS) is assumed then the inequational axiom (ND) can be substituted by its (stronger) equational form

$$(ND_{\equiv}) \quad M(x, y, w) \Rightarrow ax + a(y + w) + a(x + y) \simeq ax + a(y + w)\,.$$

As above, we write $(ND_{\equiv}^X)$ for the concrete instances of this axiom for $X \in \{F, R, FT, RT\}$.

Proposition 3.6 (i) *The set of axioms $\{B_1–B_4, (RS), (ND)\}$ is logically equivalent to $\{B_1–B_4, (RS), (ND_+)\}$, where (ND_+) is the axiom*

$$M(x, y, w) \Rightarrow ax + a(y + w) + a(x + y) \preceq ax + a(y + w)\,.$$

(ii) $\{B_1\text{–}B_4, (RS), (ND_+)\}$ *is logically equivalent to* $\{B_1\text{–}B_4, (RS), (ND_\equiv)\}$.

Proof.

(i) We only need to prove the implication from right to left, since the other follows from $\preceq$ being a precongruence. For that, from (RS) we get $a(x+y) \preceq a(x+y) + ax + a(y+w)$ whence, using (ND_+), $a(x+y) \preceq ax + a(y+w)$.

(ii) We only need to prove that, if $M(x, y, w)$, then

$$\{B_1\text{–}B_4, (RS), (ND_+)\} \vdash ax + a(y+w) \preceq ax + a(y+w) + a(x+y)\,,$$

which follows from (RS).

□

This result can be interpreted as saying that the only way to "enlarge" a process is by enhancing its possible behaviors by means of the "dynamic" simulation axioms; the static rules, (ND) and its variants, instead simply generate new identifications among processes.

Actually, any complete axiomatization of a preorder that contains the axiom (RS) can be turned into an equivalent axiomatization by replacing every inequality $u \preceq v$ by $u + v \simeq v$.

Proposition 3.7 *Let* $Q = \{B_1\text{–}B_4, (RS)\} \cup Q'$ *be an axiomatization of* $\sqsubseteq\, \subseteq I$. *Then, the equational variant of* Q, $Q^= = \{B_1\text{–}B_4, (RS)\} \cup \{M \Rightarrow u + v \simeq v \mid M \Rightarrow u \preceq v \in Q'\}$ *is also an axiomatization of* $\sqsubseteq$.

Proof. Analogous to the particular case considered in Proposition 3.6 above, which we have preferred to present before because it corresponds to the most important instance of the general result for this paper. □

4 The coarsest semantics in the spectrum

In the bottom part of the spectrum we find the simulation semantics coarser than ready simulation: plain and complete simulation, and the semantics coarser than these. For the simulation semantics we obtain the corresponding axiomatizations simply by considering the universal constraint for the case of plain simulations and the complete constraint for complete simulations:

Simulation	$U(x, y) ::= \mathit{true}$
Complete simulations	$C(x, y) ::= (x = \mathbf{0} \iff y = \mathbf{0})$

What about trace and completed trace semantics? It turns out that they can be defined by simply adding our axiom (ND^F)!

Proposition 4.1 (i) $\sqsubseteq_T$ *is axiomatized by the axioms*[6] $\{B_1\text{–}B_4, (S), (ND^F)\}$.

(ii) $\sqsubseteq_{CT}$ *is axiomatized by the axioms* $\{B_1\text{–}B_4, (CS), (ND^F)\}$ *where* (CS) *is the instantiation of* (NS) *taking* $C(x, y)$ *as* $N(x, y)$.

[6] Note that (S) is equivalent to (US), the instantation of (NS) with $N = U$.

Proof.

(i) The classic axiomatization of trace semantics is given by $\{B_1–B_4, (S), (T)\}$, where (T) is the axiom $ax + ay \simeq a(x + y)$. Note that $\{B_1–B_4, (S), (T)\}$ is logically equivalent to $\{B_1–B_4, (S), (T_\sqsubseteq)\}$, where $(T_\sqsubseteq)$ is the axiom $a(x+y) \preceq ax + ay$, because (S) can be used to obtain $ax \preceq a(x + y)$ and $ay \preceq a(x + y)$. And it is immediate that (ND^F) implies $(T_\sqsubseteq)$. Also, $\{(S), (T_\sqsubseteq)\} \vdash a(x + y) \preceq ax + a(y + w)$, since $a(x + y) \preceq ax + ay$ by $(T_\sqsubseteq)$ and $ax + ay \preceq ax + a(y + w)$ by (S).

(ii) Analogous to the previous case once we realize that the classic axiom for complete traces, (CT) $a(bx + u) + a(cy + v) \simeq a(bx + cy + u + v)$, is equivalent to the conditional axiom $C(x, y) \Rightarrow ax + ay \simeq a(x + y)$. This follows because $bx + u$ and $cy + v$ are two independent patterns describing non-null processes and when the condition is instantiated with x and y equal to $\mathbf{0}$ the identity is trivial: $a\mathbf{0} + a\mathbf{0} \simeq a\mathbf{0}$.

□

By an analogous argument to that in Proposition 3.6 we can obtain for $\sqsubseteq_T$ the axiomatization $\{B_1–B_4, (S), (ND^F_\equiv)\}$. Note that although $(ND^F_\equiv)$ is an equation, this axiomatization is not the classic one; obviously, (T) $ax + ay = a(x + y)$ implies $(ND^F_\equiv)$, but the converse is false.

It is easy to check that in the case of trace semantics, the particular instance (ND_0) of the axiom (ND) with w equal to $\mathbf{0}$ is powerful enough to generate the trace preorder. This was certainly not the case when we were under ready simulation, where (ND_0) just generates the failure trace preorder instead of the coarser failures preorder.

It is also interesting to note that for trace semantics the symmetric version of (ND),

$$(ND_{vw}) \quad a(x + y) \preceq a(x + v) + a(y + w)\,,$$

is also valid, so we can take both $\{B_1–B_4, (S), (ND_{vw})\}$ and $\{B_1–B_4, (S), (ND^\equiv_{vw})\}$, where

$$(ND^\equiv_{vw}) \quad a(x + v) + a(y + w) + a(x + y) \simeq a(x + v) + a(y + w)\,,$$

as alternative axiomatizations of the trace preorder.

Should we expect another diamond of "reasonable" semantics under plain simulation in the spectrum? Were that to be the case, why have we only found trace semantics?

In order to answer these questions, note that the diamond of semantics under ready simulation was completely governed by the function I, which appears in the constraints of the different instantiations of the axiom (ND). For plain simulations, however, the trivially true predicate $U(x, y)$ corresponds to the observation function that can see nothing. As a consequence, if we substitute U for I in each of the four constraints of the diamond they all collapse into a single one: trace semantics. Nevertheless, an alternative can be explored to obtain new semantics: let us keep the different axioms ND^X the way they stand and simply replace (RS) by (S).

Proposition 4.2 *$\{B_1$–$B_4, (S), (ND^{FT})\}$ is another axiomatization of trace semantics. Hence, under (S) the failures and the failure trace axioms generate the same preorder, namely the trace preorder.*

Proof. $\{B_1$–$B_4, (S), (ND_0)\}$ is a complete axiomatization of trace preorder, and (ND_0) is a particular case of (ND^{FT}). □

The axioms corresponding to readiness and ready traces, however, give rise to two new semantics that we shall name *extended ready* and *extended ready trace* semantics. They are defined taking into account the offers of the processes, either at the end of a trace or after each action within it: in order to have $p \sqsubseteq_{ER} q$, for each $p \overset{\alpha}{\Longrightarrow} p'$ with $I(p') = R$ we need some $q \overset{\alpha}{\Longrightarrow} q'$ with $I(q') \supseteq R$; the extended ready trace preorder $\sqsubseteq_{ERT}$ is defined analogously, but using ready traces.

Proposition 4.3 *(1) The set $\{B_1$–$B_4, (S), (ND^R)\}$ is an axiomatization of $\sqsubseteq_{ER}$; (2) the set $\{B_1$–$B_4, (S), (ND^{RT})\}$ is an axiomatization of $\sqsubseteq_{ERT}$.*

Let us now consider the versions of the axioms (ND^R), (ND^{FT}), (ND^{RT}) where the constraint I has been replaced by the completeness condition C defined by $C(x) \Leftrightarrow x = \mathbf{0}$:

$$
\begin{aligned}
&(C\text{-}ND^R) && M_{CR}(x, y, w) \iff (C(x) \Rightarrow C(y))\\
&(C\text{-}ND^{FT}) && M_{CFT}(x, y, w) \iff (C(y) \Rightarrow C(w))\\
&(C\text{-}ND^{RT}) && M_{CRT}(x, y, w) \iff (C(x) \iff C(y) \text{ and } C(y) \Rightarrow C(w))
\end{aligned}
$$

Once again, we only obtain three alternative axiomatizations of the complete trace semantics.

Proposition 4.4 *The following axiomatizations are equivalent: 1) $\{B_1$–$B_4, (CS), (ND^F)\}$; 2) $\{B_1$–$B_4, (CS), (C\text{-}ND^R)\}$; 3) $\{B_1$–$B_4, (CS), (C\text{-}ND^{FT})\}$; 4) $\{B_1$–$B_4, (CS), (C\text{-}ND^{RT})\}$.*

Proof. Clearly, (1) ⇒ (2) ⇒ (3) ⇒ (4) and therefore it is enough to prove that (4) ⇒ (1). If x and y are not $\mathbf{0}$ we can apply $(C\text{-}ND^{RT})$ to obtain the inequality in (ND^F). If x is $\mathbf{0}$ but y is not, we need to obtain $ay \preceq a\mathbf{0} + a(y+w)$. By (CS) we have $y \preceq y + w$ and then $ay \preceq a(y + w)$; applying (CS) again, $a(y + w) \preceq a(y + w) + a\mathbf{0}$ and thus $ay \preceq a\mathbf{0} + a(y+w)$. If y is $\mathbf{0}$ but x is not, we need to obtain $ax \preceq ax + aw$, which results from an immediate application of (CS). Finally, if both x and y are $\mathbf{0}$, $a\mathbf{0} \preceq a\mathbf{0} + aw$. □

As before, if we only consider the original axioms (ND^R), (ND^{FT}), and (ND^{RT}) we obtain, together with an alternative axiomatization of the complete trace semantics, two new semantics.

Proposition 4.5 *The axiomatization $\{B_1$–$B_4, (CS), (ND^{FT})\}$ is logically equivalent to $\{B_1$–$B_4,\ (CS), (ND^F)\}$. Hence, under (CS), the failures and the failure trace axioms generate the same semantics.*

Simulation axiom $\qquad N(x,y) \Rightarrow x \preceq x + y$

Non-determinism axiom $\qquad M(x,y,w) \Rightarrow a(x+y) \preceq ax + a(y+w)$

		$M(x,y,w)$				
$N(x,y)$		BCCSP[3]	$I(x) \supseteq I(y)$	$I(w) \subseteq I(y)$	$I(x) = I(y)$ $I(w) \subseteq I(y)$	$\exists b, y', x'.$ $y = by$ $x = bx' + w$
$true$	S	T	New	T	New	New
$x = \mathbf{0} \Leftrightarrow y = \mathbf{0}$	CS	CT	New	CT	New	New
$I(x) = I(y)$	RS	F	R	FT	RT	PW

Table 1
Axiomatization for semantics coarser than ready simulation

Proof. It is enough to prove that $(C\text{-}ND^{FT})$ can be derived from $\{B_1\text{–}B_4, (CS), (ND^F)\}$.

- If y is $\mathbf{0}$ we then have w equal to $\mathbf{0}$ and can apply (ND^{FT}).
- If y is not $\mathbf{0}$ we can apply (ND_0^{FT}) to obtain $a(x+y) \preceq ax + ay$ and then (CS) to conclude that $a(x+y) \preceq ax + a(y+w)$.

$\square$

By contrast, as happened also for plain simulations, under (CS) the axioms of the ready semantics generate two slightly different versions of the extended ready and extended ready trace semantics introduced before, that we call *extended complete ready* and *extended complete ready trace* semantics. In order to have $p \sqsubseteq_{ECR} q$, whenever $p \stackrel{\alpha}{\Longrightarrow} p'$ with $I(p') \neq \emptyset$ we require some $q \stackrel{\alpha}{\Longrightarrow} q'$ with $I(q') \supseteq I(p')$, but if $I(p') = \emptyset$ then the corresponding q' also has to satisfy $I(q') = \emptyset$. The extended complete ready trace preorder $\sqsubseteq_{ECRT}$ is defined in an analogous way, starting from the ready traces of the processes.

Table 1 presents a first snapshot of the generic axiomatization of the semantics coarser than ready simulation.

5 Relating the new observational and equational frameworks

In our companion paper [7] we have presented the new unified observational framework that allows us to characterize all the semantics in the spectrum by either branching or linear observations. They are defined as follows:

Definition 5.1 The sets L_N of *local observations* corresponding to each of the constrained simulations in the spectrum, and $L_N(p)$ of observations associated to a

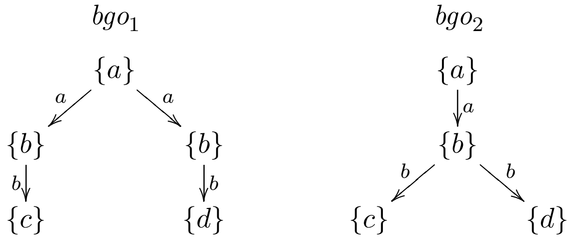

Fig. 2. Two branching observations

process p, are defined as follows:

- Plain simulation: $L_U = \{\cdot\}$, $L_U(p) = \cdot$.
- Ready simulation: $L_I = \mathcal{P}(Act)$, $L_I(p) = I(p)$.
- Complete simulation: $L_C = Bool$, $L_C(p)$ is *true* if $I(p) = \emptyset$ and *false* otherwise.
- Trace simulation [7]: $L_T = \mathcal{P}(Act^*)$, $L_T(p) = T(p)$, the set of traces of p.
- 2-nested simulation: $L_S = \{[\![p]\!]_S \mid p \in BCCSP\}$, $L_S(p) = [\![p]\!]_S$, where $[\![p]\!]_S$ represents the class of p with respect to the simulation semantics.

Definition 5.2 (i) Given a constraint N, a *branching general observation* of a process for that constraint is a finite, non-empty tree whose arcs are labeled with actions in Act and whose nodes are labeled with local observations from L_N; the corresponding set BGO_N is recursively defined as:
- $\langle l, \emptyset\rangle \in BGO_N$ for every $l \in L_N$.
- $\langle l, \{(a_i, bgo_i) \mid i \in 1..n\}\rangle \in BGO_N$ for every $n \in \mathbb{N}$, $a_i \in Act$ and $bgo_i \in BGO_N$.

(ii) The set $BGO_N(p)$ of branching general observations of p corresponding to the constraint N is

$$BGO_N(p) = \{\langle L_N(p), S\rangle \mid S \subseteq \{(a, bgo) \mid bgo \in BGO_N(p'), p \xrightarrow{a} p'\}\}\,.$$

(iii) We write $p \leq_N^b q$ if $BGO_N(p) \subseteq BGO_N(q)$.

In Figure 2 some simple examples of bgo's for $N = I$ are shown. We have represented bgo_1 as $\langle\{a\}, \{(a, \langle\{b\}, \{(b, \langle\{c\}, \emptyset\rangle)\}\rangle), (a, \langle\{b\}, \{(b, \langle\{d\}, \emptyset\rangle)\}\rangle)\}\rangle$ and bgo_2 as $\langle\{a\}, \{(a, \langle\{b\}, \{(b, \langle\{c\}, \emptyset\rangle), (b, \langle\{d\}, \emptyset\rangle)\}\rangle)\}\rangle$. We use braces for the set of children of a node, parentheses to represent a branch of the tree as a pair (initial arc, subtree below), and angle brackets to represent each tree as a pair $\langle$root, children$\rangle$.

We have already proved in [7] that N-simulation semantics can be characterized by means of branching observations in BGO_N. Since the natural way of defining the simulation semantics is by means of constrained simulations and we proved in [6] that they can be axiomatized by the corresponding instance of the generic simulation axiom (NS), we do not consider necessary to also present a direct proof of the axiomatizations in terms of these semantics.

[7] Trace simulations can be defined as T-simulations, with $T(x, y) ::= T(x) = T(y)$.

The observational characterization of the remaining semantics, except for possible futures, is achieved by means of linear observations:

Definition 5.3 (i) The set $lBGO_N$ of *linear branching general observations* for a local observer L_N is the subset of BGO_N defined as:
- $\langle l, \emptyset\rangle \in lBGO_N$ for each $l \in L_N$.
- $\langle l, \{(a, lbgo)\}\rangle$, whenever $a \in A$ and $lbgo \in lBGO_N$.

(ii) The set of *linear observations* of a process p with respect to the local observer L_N is $LGO_N(p) = BGO_N(p) \cap lBGO_N$.

Since lbgo's are linear they can be presented as decorated traces, avoiding the sets of descendants in the general bgo's. Therefore, we will consider them as elements of the set $L_N \times (Act \times L_N)^*$. Moreover, it would be easy to generate the set $LGO_N(p)$ by means of SOS rules; this is not possible, however, for $BGO_N(p)$, due to the branching nature of its elements.

Proposition 5.4 *The set $LGO_N(p)$ of linear general observations of a process p is recursively defined by $LGO_N(p) ::= \{\langle L_N(p)\rangle\} \cup \{\langle L_N(p), a\rangle \circ LGO_N(p') \mid p \xrightarrow{a} p'\}$, where $\circ$ is the extension to sets of sequence concatenation.*

Alternatively, and as usually done for decorated traces semantics, we could have described the set $LGO_N(p)$ as the computations defined by a structural operational semantics; this is an important difference with respect to the constrained simulation semantics due to their branching nature. For $N = I$ the set $LGO_I(p)$ defines the ready traces of p and the corresponding semantic order arises from set inclusion. Therefore, to characterize the related coarser semantics we need to introduce suitable loose orders.

Definition 5.5 For $\mathcal{T}, \mathcal{T}' \subseteq LGO_N$ we define the orders $\leq^l_N$, $\leq^{l\supseteq}_N$, $\leq^{lf}_N$, and $\leq^{lf\supseteq}_N$:
- $\mathcal{T} \leq^l_N \mathcal{T}' \iff \mathcal{T} \subseteq \mathcal{T}'$.
- $\mathcal{T} \leq^{l\supseteq}_N \mathcal{T}' \iff \forall X_0 a_1 X_1 \ldots X_n \in \mathcal{T}\ \exists Y_0 a_1 Y_1 \ldots Y_n \in \mathcal{T}'\ \forall i \in 0..n\ X_i \supseteq Y_i$.
- $\mathcal{T} \leq^{lf}_N \mathcal{T}' \iff \forall X_0 a_1 X_1 \ldots X_n \in \mathcal{T}\ \exists Y_0 a_1 Y_1 \ldots Y_n \in \mathcal{T}'\ X_n = Y_n$.
- $\mathcal{T} \leq^{lf\supseteq}_N \mathcal{T}' \iff \forall X_0 a_1 X_1 \ldots X_n \in \mathcal{T}\ \exists Y_0 a_1 Y_1 \ldots Y_n \in \mathcal{T}'\ X_n \supseteq Y_n$.

We will use $\leq^{lY}_N$ to denote in a generic way any of the orders obtained with $Y \in \{\langle empty\rangle, \supseteq, f, f\supseteq\}$. Then, we write $p \leq^{lY}_N q$ if $LGO_N(p) \leq^{lY}_N LGO_N(q)$.

By abuse of notation, we have used the superset inclusion symbol $\supseteq$ in the definitions above for all N. That is the right interpretation for the cases $N = I, T$; for $N = U, C$ the subset inclusions degenerate to equalities. For $N = S$, it should be interpreted as $[\![p]\!]_S \geq_S [\![q]\!]_S$. Then, it is easy to see that we could have used such an inequality $[\![p]\!]_N \geq_N [\![q]\!]_N$ in all cases. We have proved in [7] that the orders $\leq^{lY}_N$ with $Y \in \{\langle empty\rangle, \supseteq, f, f\supseteq\}$ characterize the semantics in the diamonds coarser than each of the corresponding N-simulations semantics; we then define $l(F) = lf\supseteq$, $l(FT) = l\supseteq$, $l(R) = lf$, and $l(RT) = l$.

To simply use set inclusion to compare the sets of observations of processes, appropriate closures can be defined.

Definition 5.6 For $\mathcal{T} \subseteq LGO_N$, the following three closures are defined:

- $\overline{\mathcal{T}}^{\supseteq} = \{X_0 a_1 X_1 \dots a_n X_n \mid \exists Y_0 a_1 Y_1 \dots a_n Y_n \in \mathcal{T}\ \forall i \in 0..n\ X_i \supseteq Y_i\}$.
- $\overline{\mathcal{T}}^{f} = \{X_0 a_1 X_1 \dots a_n X_n \mid \exists Y_0 a_1 Y_1 \dots a_n X_n \in \mathcal{T}\}$.
- $\overline{\mathcal{T}}^{f\supseteq} = \{X_0 a_1 X_1 \dots a_n X_n \mid \exists Y_0 a_1 Y_1 \dots a_n Y_n \in \mathcal{T}\ X_n \supseteq Y_n\}$.

Then, if $Y \in \{\supseteq, f, f\supseteq\}$, for $p \in BCCSP$ and N a constraint, we define $LGO_N^Y(p) = \overline{LGO_N(p)}^{Y}$.

Proposition 5.7 *For all* $Y \in \{\supseteq, f, f\supseteq\}$, $\mathcal{T} \leq_N^{lY} \mathcal{T}'$ *iff* $\overline{\mathcal{T}}^{Y} \subseteq \overline{\mathcal{T}'}^{Y}$.

Let us now see how, from this uniform definition of the linear semantics, the proofs of the correctness and completeness of the corresponding axiomatizations can be obtained in a generic way avoiding the case analyses of Sections 3 and 4. Although this could be done generically, with $N \in \{U, C, I, T\}$, we prefer to start with the particular case $N = I$, which corresponds to the most popular semantics, already studied in Section 3. To start with, we show how the axiomatizations can be synthetized from the observational characterizations.

Our general axiom (ND) for the reduction of non-determinism specifies the hypothesis $M(x, y, w)$ under which the process $ax + a(y + w)$ can be (syntactically) expanded by adding a new summand $a(x+y)$ without changing its semantics. Then, let us compare the two sides of our general axiom. Since $I(ax+a(y+w)) = I(ax) = I(a(y + w)) = \{a\}$, we have

$$\begin{aligned} LGO_I(ax + a(y + w)) &= LGO_I(ax) \cup LGO_I(a(y + w))\,, \\ LGO_I(a(y + w)) &= \{\langle\{a\}\rangle\} \cup \\ &\quad \{\langle\{a\}, a, I(y + w)\rangle \circ S \mid \langle\{a\}, a, I(y)\rangle \circ S \in LGO_I(ay) \vee \\ &\qquad \langle\{a\}, a, I(w)\rangle \circ S \in LGO_I(aw)\}\,. \end{aligned}$$

Notice then that the observations of $a(y + w)$ are exactly those of $ay + aw$ simply replacing $I(y)$ or $I(w)$, respectively, appearing in the expression above, by $I(y + w) = I(y) \cup I(w)$. Obviously, the same relation exists between the global observations of $a(x + y)$ and those of $ax + ay$.

Now, in order to get the adequate condition $M_X(x, y, w)$ for each of the semantics, let us examine the formulas that define the preorders $\leq_I^{lY}$:

- $\leq_I^l$. To have $LGO_I(a(x + y)) \subseteq LGO_I(ax) \cup LGO_I(a(y + w))$ it is enough to require $\{\langle\{a\}, a, I(x) \cup I(y)\rangle \circ S \mid \langle I(x)\rangle \circ S \in LGO_I(x)\} \subseteq \{\langle\{a\}, a, I(x)\rangle \circ S \mid \langle I(x)\rangle \circ S \in LGO_I(x)\}$ and $\{\langle\{a\}, a, I(x) \cup I(y)\rangle \circ S \mid \langle I(y)\rangle \circ S \in LGO_I(y)\} \subseteq \{\langle\{a\}, a, I(y) \cup I(w)\rangle \circ S \mid \langle I(y)\rangle \circ S \in LGO_I(y)\}$. Thus, a first proposal for M_{RT} would be

$$I(y) \subseteq I(x) \wedge I(x) = I(y) \cup I(w)\,.$$

However, due to the fact that this axiom will be used in combination with (RS), the following, more restrictive but simpler form, can be used instead:

$$M_{RT}(x, y, w) ::= I(x) = I(y) \wedge I(w) \subseteq I(y)\,.$$

Clearly, this form is stronger than the condition synthetized above. Reciprocally, $a(x + y) \preceq ax + a(y + w)$ can be proved from the assumptions $I(y) \subseteq I(x)$ and $I(x) = I(y) \cup I(w)$ using (RS) first to get $a(x+y) \preceq a(x+y+w)$, and then (ND) instantiated with M_{RT} to obtain $a(x + y + w) \preceq ax + a(y + w)$.

- $\leq_I^{l\supseteq}$. We need the inclusion $LGO_I(a(x + y))^{\supseteq} \subseteq LGO_I(ax + a(y + w))^{\supseteq}$ to hold. Since $I(x) \cup I(y) \supseteq I(x)$, the general observations in $a(x + y)$ that arise from x will be also in $LGO_I(ax)^{l\supseteq}$. For those that arise from y, it is required that $I(x)\cup I(y) \supseteq I(y)\cup I(w)$. Once again, (RS) can be used to simplify this condition into the simpler
$$M_{FT} ::= I(w) \subseteq I(y)\,.$$
The less restrictive variant of the axiom can be derived from the stronger one and (RS) as follows. Taking $w ::= \mathbf{0}$, since $I(\mathbf{0}) \subseteq I(y)$ we obtain $a(x + y) \preceq ax + ay$ from (ND^{FT}); in particular, $a(x + y + w) \preceq ax + a(y + w)$. Also, by (RS), $x + y \preceq (x + y) + (x + y + w)$, from where it follows $a(x + y) \preceq a(x + y + w)$.
- $\leq_I^{lf}$. We consider the inclusion $LGO_I(a(x + y))^f \subseteq LGO_I(ax + a(y + w))^f$. We only have to consider the lgo $\langle\{a\}, a, I(x) \cup I(y)\rangle$ in $LGO_I(a(x + y))^f$ and show that it also belongs to $LGO_I(ax + a(y + w))^f$, since all lgo's of length greater than 1 start with the prefix $\langle\{a\}, a\rangle$. For that, either $I(x) \cup I(y) = I(x)$ or $I(x) \cup I(y) = I(y) \cup I(w)$, that is, $I(y) \subseteq I(x)$ or $I(x) \cup I(y) = I(y) \cup I(w)$. Again, we can remove the second condition and define
$$M_R ::= I(y) \subseteq I(x)$$
since, whenever $I(x) \cup I(y) = I(y) \cup I(w)$, $a(x + y + w) \preceq ax + a(y + w)$ can be obtained by taking $x ::= y + w$, $y ::= x$, and $w ::= \mathbf{0}$, and then by applying (RS) we conclude $a(x + y) \preceq ax + a(y + w)$.
- $\leq_I^{lf\supseteq}$. An argument analogous to the previous one leads us to $I(x) \cup I(y) \supseteq I(x)$ or $I(x) \cup I(y) \supseteq I(y) \cup I(w)$, which is trivially true.

In order to prove the completeness of our axiomatizations we introduce the following notions of head normal forms.

Definition 5.8 For $p = \sum_{a\in X_0} \sum_{i\in I_a} ap_a^i$ and $X \in \{F, R, FT, RT\}$, its X-head normal form $hnf^X(p)$ is:

- For $a \in X_0$, $i \in I_a$, and $X_1 \subseteq \bigcup_{i\in I_a} I(p_a^i)$ such that $I(p_a^i) \subseteq X_1$, we define $hnf^X(p, a, i, X_1) = a(p_a^i + \sum\{p_a^j|_{X_1} \mid j \neq i, M_X(p_a^i, p_a^j|_{X_1}, p_a^j|_{X_1})\}$.
- $hnf^X(p) = p + \sum_{a\in X_0} \sum_{i\in I_a} \sum_{X_1\subseteq\bigcup I(p_a^i)} hnf^X(p, a, i, X_1)$.

It is clear that several redundancies arise in this definition: for example, if $X = RT$ then $hnf^X(p, a, i, X_1) = hnf^X(p, a, i, I(p_a^i))$, so that the argument X_1 would not be needed in this case. We prefer to maintain the generic definition in order to allow a homogeneous treatment of all the semantics.

Proposition 5.9 *If* $X \in \{RT, FT, R, F\}$, $\{B_1\text{–}B_4, (RS), (ND^X)\} \vdash hnf^X(p) \preceq p$.

Proof. Let $p = \sum_{a \in X_0} \sum_{i \in I_a} ap_a^i$. By the definition of $hnf^X(p, a, i, X_1)$, if $p_a^{j_1}$ is a summand that contributes to it then, since $M_X(p_a^i, p_a^{j_1}|_{X_1}, p_a^{j_1}|_{\overline{X_1}})$ holds, we have $\{B_1\text{–}B_4, (RS), (ND^X)\} \vdash a(p_a^i + p_a^{j_1}|_{X_1}) \preceq ap_a^i + ap_a^{j_1}$. Now, if $p_a^{j_2}$ is another summand, it is easy to check that $M_X(p_a^i + p_a^{j_1}|_{X_1}, p_a^{j_2}|_{X_1}, p_a^{j_2}|_{\overline{X_1}})$ also holds and then $\{B_1\text{–}B_4, (RS), (ND^X)\} \vdash a(p_a^i + p_a^{j_1}|_{X_1} + p_a^{j_2}|_{X_1}) \preceq a(p_a^i + p_a^{j_1}|_{X_1}) + ap_a^{j_2}$; combining it with the previous derivation, $\{B_1\text{–}B_4, (RS), (ND^X)\} \vdash a(p_a^i + p_a^{j_1}|_{X_1} + p_a^{j_2}|_{X_1}) \preceq ap_a^i + ap_a^{j_1} + ap_a^{j_2}$. By repeating this procedure for all summands we obtain $\{B_1\text{–}B_4, (RS), (ND^X)\} \vdash hnf^X(p, a, i, X_1) \preceq p|_a$. Finally, adding these inequalities leads us to $\{B_1\text{–}B_4, (RS), (ND^X)\} \vdash hnf^X(p) \preceq p$. □

In order to apply structural induction to prove the completeness of the axiomatizations we need that, whenever $p = \sum_{a \in X_0} \sum_{i \in I_a} ap_a^i$ and $p \leq_I^{l(X)} q$, there is a summand ah_a^k of $hnf^X(q)$ such that $p_a^i \leq_I^{l(X)} h_a^k$ for each $a \in Act$, $i \in I_a$.

Proposition 5.10 *Let* $X \in \{F, FT, R, RT\}$*, and let* $p = \sum_{a \in X_0} \sum_{i \in I_a} ap_a^i$*, and* $q = \sum_{a \in X_0} \sum_{j \in J_a} aq_a^j$*. If* $p \leq_I^{l(X)} q$ *then there exists a summand* ah_a^k *of* $hnf^X(q)$ *such that* $p_a^i \leq_I^{l(X)} h_a^k$.

Proof. We need to show that $\overline{LGO_I(p_a^i)}^{l(X)} \subseteq \overline{LGO_I(h_a^k)}^{l(X)}$ but, due to the fact that $\overline{(\_)}^{l(X)}$ is a closure operator [7], it is enough to prove just $LGO_I(p_a^i) \subseteq \overline{LGO_I(h_a^k)}^{l(X)}$. For $\langle I(p_a^i)\rangle \in LGO_I(p_a^i)$, since $p \leq_I^{l(X)} q$ there is some q_a^j such that $\langle I(p_a^i)\rangle \in \overline{LGO_I(q_a^j)}^{l(X)}$; we then consider $hnf^X(q, a, j, I(p_a^i)) = ah_a^k$.

If $t \in LGO_I(p_a^i)$ then $\langle I(p), a\rangle \circ t \in LGO_I(p) \subseteq \overline{LGO_I(q)}^{l(X)}$ and there exists j_t such that $t \in \overline{LGO_I(q_a^{j_t})}^{l(X)}$. In addition, $M_X(q_a^j, q_a^{j_t}|_{I(p_a^i)}, q_a^{j_t}|_{\overline{I(p_a^i)}})$:

- If $X = RT$, then $t \in LGO_I(q_a^{j_t})$ and therefore $I(q_a^{j_t}) = I(p_a^i) = I(q_a^j)$. Hence, $M_{RT}(q_a^j, q_a^{j_t}|_{I(p_a^i)}, \mathbf{0})$, and therefore $M_{RT}(q_a^j, q_a^{j_t}|_{I(p_a^i)}, q_a^{j_t}|_{\overline{I(p_a^i)}})$.
- If $X = FT$, from $t \in \overline{LGO_I(q_a^{j_t})}^{\supseteq}$ it follows that $I(q_a^{j_t}) \subseteq I(p_a^{j_t})$ and therefore $I(q_a^{j_t}|_{\overline{I(p_a^i)}}) = \emptyset \subseteq I(q_a^{j_t}|_{I(p_a^i)})$. Hence, $M_{FT}(q_a^j, q_a^{j_t}|_{I(p_a^i)}, q_a^{j_t}|_{\overline{I(p_a^i)}})$.
- If $X = R$, from $\langle I(p_a^i)\rangle \in \overline{LGO_I(q_a^j)}^{f}$ we have that $I(p_a^i) = I(q_a^j)$ and thus $I(q_a^{j_t}|_{\overline{I(p_a^i)}}) \subseteq I(q_a^j)$ and $M_R(q_a^j, q_a^{j_t}|_{I(p_a^i)}, q_a^{j_t}|_{\overline{I(p_a^i)}})$.
- For $X = F$ it is trivial since $M_F(x, y, w)$ is always true.

Therefore $q_a^{j_t}$ is one of the summands of h_a^k and, since $t \in \overline{LGO_I(q_a^{j_t})}^{l(X)}$, we have $p_a^i \leq_I^{l(X)} h_a^k$. □

Theorem 5.11 (Soundness and completeness) *For all* $X \in \{RT, FT, R, F\}$, $p \leq_I^{l(X)} q$ *iff* $\{B_1\text{–}B_4, (RS), (ND^X)\} \vdash p \preceq q$.

Proof. (Soundness) The axiomatizations are sound because of the way they have been derived.

(Completeness) By structural induction on p.

- If $p = \mathbf{0}$, then $p \leq_I^{l(X)} q$ implies that q is $\mathbf{0}$.
- If $p = \sum_{a \in X_0} \sum_{i \in I_a} ap_a^i$, then $p \leq_I^{l(X)} q$ implies that there exists a summand ah_a^k of $hnf^X(q)$ such that $p_a^i \leq_I^{l(X)} h_a^k$ as indicated above. By induction hypothesis, $\{B_1–B_4, (RS), (ND^X)\} \vdash p_a^i \preceq h_a^k$ and therefore $\{B_1–B_4, (RS), (ND^X)\} \vdash ap_a^i \preceq ah_a^k$; adding all these inequalities and using (RS), which is allowed because $I(p) = I(q)$, it follows that $\{B_1–B_4, (RS), (ND^X)\} \vdash p \preceq hnf^X(q)$ and, by Proposition 5.9, $\{B_1–B_4, (RS), (ND^X)\} \vdash p \preceq q$.

□

6 The semantics that are not coarser than ready simulation

Once we have a clear picture of the semantics that are coarser than ready simulation, it is time to consider the rest of the semantics in the spectrum. Let us start with the possible futures, already discussed in [13], and the impossible futures semantics [14].

Definition 6.1 (i) The possible futures semantics is defined as: $p \sqsubseteq_{PF} q$ if for all $p \overset{\alpha}{\Longrightarrow} p'$ there exists $q \overset{\alpha}{\Longrightarrow} q'$ with $T(p') = T(q')$.

(ii) The impossible futures semantics is defined as: $p \sqsubseteq_{IF} q$ if for all $S \subseteq \mathcal{P}(Act^*)$, if $p \overset{\alpha}{\Longrightarrow} p'$ with $T(p') \cap S = \emptyset$ then there exists $q \overset{\alpha}{\Longrightarrow} q'$ with $T(q') \cap S = \emptyset$.

The first definition above is that of readiness semantics but replacing the function I with T. Although less evident, the same is the case for impossible futures and the failures semantics (see [7]). We have in fact shown that they can be described by LGO_T observations so that they are defined by $\leq_T^{lf\supseteq}$ and $\leq_T^{lf}$ respectively.

Next we introduce the T-versions of our (ND^X) axioms: all of them are instances of our general axiom for reduction of non-determinism and therefore are defined by the adequate constraint $M_X^T(x, y, w)$. As expected, they are obtained by substituting every occurrence of I in $M_X(x, y, w)$ by the observer T defining the traces of processes.

Definition 6.2 The constraints M_X^T that characterize the semantics coarser than T-simulation semantics are:

$$
\begin{array}{ll}
(T\text{-}ND^F) & M_F^T(x, y, w) \iff \text{true} \\
(T\text{-}ND^R) & M_R^T(x, y, w) \iff T(x) \supseteq T(y) \\
(T\text{-}ND^{FT}) & M_{FT}^T(x, y, w) \iff T(w) \subseteq T(y) \\
(T\text{-}ND^{RT}) & M_{RT}^T(x, y, w) \iff T(x) = T(y) \text{ and } T(w) \subseteq T(y)
\end{array}
$$

Note that the last two semantics do not appear in Van Glabbeek's spectrum and, as far as we know, they have not been previously studied nor defined. We ignore whether they will be of practical interest in the future but, nonetheless, we have decided to include them for the sake of completeness.

By the same arguments as in Section 5 we can prove that $\leq_T^{l(X)}$ satisfies the axiom (T-ND^X) for $X \in \{RT, FT\}$. However this is not the case for $X \in \{R, F\}$ due to the following proposition.

Proposition 6.3 *($M_X^T(x, y, w)$ implies $T(a(x + y)) = T(ax + a(y + w))$) iff $X \in \{RT, FT\}$.*

Proof. M_{RT}^T implies M_{FT}^T, and therefore $T(y + w) = T(y)$, which leads to $T(ax + a(y + w)) = T(a(x + y))$. Neither M_R^T nor M_F^T refer to w and therefore, in general, $T(ax + a(y + w)) \neq T(a(x + y))$ in those cases. □

Note that when proving the correctness of the corresponding axiom (ND^X) for $\leq_I^{l(X)}$ we had in all cases $I(a(x + y)) = \{a\} = I(ax + a(y + w))$. Now we have $T(a(x+y)) = T(ax+a(y+w))$ only under the constraints corresponding to the finer semantics $\sqsubseteq_{FT}$ and $\sqsubseteq_{RT}$. The properties of the prefixes appearing in all the terms in both sides of the axiom (ND) are not used anymore in the proofs in Section 5, so they can be transferred to the T-semantics thus proving the correctness of (T-ND^X) for both $\leq_T^{l(RT)}$ and $\leq_T^{l(FT)}$.

The introduction of the equational versions ($ND_\equiv$) of the axiom (ND) now becomes crucial in order to preserve the genericity of our unifying study of the concurrency semantics. We saw that under (RS) these axioms were equivalent. However, since now we are observing the set of traces $T(x)$ of any process instead of just the initial offer $I(x)$, we consider T-simulations constrained by $T(x, y) ::= T(x) = T(y)$; under the corresponding axiom (TS), things are different.

Proposition 6.4 *$T(a(x + y) + ax + a(y + w)) = T(ax) \cup T(ay) \cup T(aw) = T(ax + a(y + w))$.*

As a consequence, for (T-ND_+^X) and (T-$ND_\equiv$), we can follow the same reasonings used in Section 5 to show that (ND^X) was satisfied by $\leq_X$.

Proposition 6.5 *For $X \in \{RT, FT, R, F\}$, the preorder $\leq_T^{l(X)}$ satisfies the axiom (T-ND_+^X) and also (T-$ND_\equiv^X$).*

Proof. To show that $\leq_T^{l(X)}$ satisfies (T-ND_+^X) we just need to apply Proposition 6.4 and follow the line of thought in the second bullet on page 15, substituting the observer T for I. For the other axiom, from $T(a(x + y)) \subseteq T(ax + a(y + w))$ it follows that $\{(TS)\} \vdash ax + a(y + w) \preceq (ax + a(y + w)) + a(x + y)$. □

Notice that for $X \in \{RT, FT\}$ we can also obtain the correctness of (T-$ND_\equiv^X$) from that of (T-ND^X) and vice versa, as a consequence of the following fact.

Proposition 6.6 *The axiomatization $\{B_1$–$B_4, (TS), (T\text{-}ND^X)\}$ is equivalent to the axiomatization $\{B_1$–$B_4, (TS), (T\text{-}ND_\equiv^X)\}$ for $X \in \{RT, FT\}$.*

Proof. Let us first show that $\{B_1$–$B_4, (TS), (T\text{-}ND^X)\}$ is equivalent to $\{B_1$–$B_4, (TS), (T\text{-}ND_+^X)\}$. This holds because ($T$-$ND^X$) implies ($T$-$ND_+^X$) and, since $T(w) \subseteq$

$T(y)$ implies $T(a(x+y)) = T(ax + a(y+w))$ and then we have $\{B_1\text{–}B_4, (TS)\} \vdash a(x+y) \preceq a(x+y) + (ax + a(y+w))$.

To prove $\{B_1\text{–}B_4, (TS), (T\text{-}ND_{+}^{X})\}$ equivalent to $\{B_1\text{–}B_4, (TS), (T\text{-}ND_{\equiv}^{X})\}$ we only need to show that $\{B_1\text{–}B_4, (TS), (T\text{-}ND_{+}^{X})\} \vdash (M_X^T(x,y,w) \Rightarrow ax+a(y+w) \preceq ax+a(y+w)+a(x+y))$, but we have that for $X \in \{RT, FT\}$, $M_X^T(x,y,w)$ implies $T(w) \subseteq T(y)$, so that $T(a(x+y)) = T(ax + a(y+w))$ and therefore $\{(TS)\} \vdash ax + a(y+w) \preceq (ax + a(y+w)) + a(x+y)$. □

The important fact about the obtained sets of correct axioms for the semantics $\leq_T^{l(X)}$ is that, although our proofs of completeness for the axiomatizations $\{B_1\text{–}B_4, (RS), (ND^X)\}$ considered the inequational axioms (ND^X), they were also valid for the axiomatizations $\{B_1\text{–}B_4, (RS), (ND_{\equiv}^{X})\}$.

The steps in the procedure that leads to the completeness of $\{B_1\text{–}B_4, (RS), (ND^X)\}$ can be adapted by substituting each reference to the observer I by T, thus obtaining a proof of the completeness of $\{B_1\text{–}B_4, (RS), (T\text{-}ND_{\equiv}^{X})\}$ for $\leq_T^{l(X)}$. However, the notion of head normal form for $N = I$ uses the fact that the summands $hnf^X(q, a, i, X_1)$ can be defined in terms of the offers $X_1 \subseteq \mathcal{P}(Act)$, which correspond to the values produced by the observer I. For an arbitrary N, a more general definition of hnf's, valid for every observer, is needed.

Definition 6.7 For $p = \sum_{a\in X_0}\sum_{i\in I_a} ap_a^i$, its totally expanded X-head normal form $tehnf_N^X(p)$ is:

- For $a \in X_0$, $i \in I_a$, and $K_a \subseteq I_a$ we consider a decomposition $p_a^k = p_a^{k_1} + p_a^{k_2}$ such that $M_X^N(p_a^i, p_a^{k_1}, p_a^{k_2})$. Then, $tehnf_N^X(p, a, i, \langle(p_a^{k_1}, p_a^{k_2})\rangle_{k\in K_a}) = a(p_a^i + \sum_{k\in K_a} p_a^{k_1})$.
- $tehnf_N^X(p) = \sum tehnf_N^X(p, a, i, \langle(p_a^{k_1}, p_a^{k_2})\rangle_{k\in K_a})$.

It is clear that for $K_a' \subseteq K_a$, or a decomposition $p_a^k = p_a^{k_3} + (p_a^{k_4} + p_a^{k_2})$ with $p_a^{k_1} = p_a^{k_3} + p_a^{k_4}$, the corresponding $tehnf_N^X(\ldots)$ is a subterm of $tehnf_N^X(p, a, i, \langle p_a^{k_1}, p_a^{k_2}\rangle_{k\in K_a})$ and thus contributes nothing to the expanded normal form. This is the reason why we preferred the more compact definition of $hnf^X(p)$ for semantics coarser than ready simulation.

Theorem 6.8 *For $X \in \{RT, FT, R, F\}$, $\{B_1\text{–}B_4, (TS), (T\text{-}ND_{\equiv}^{X})\} \vdash p \preceq q$ if and only if $p \leq_T^{l(X)} q$.*

The extended spectrum can be depicted as in Figure 3, where all implications are immediate from the axiomatizations of the corresponding semantics.

7 On the real diamond structure

Focusing on the diamonds coarser than each of the branching semantics in the extended spectrum, it would be natural to expect them to have the structure of a lattice. In particular, failure semantics would be the greatest lower bound of the readiness and failure traces semantics while ready traces semantics would be the

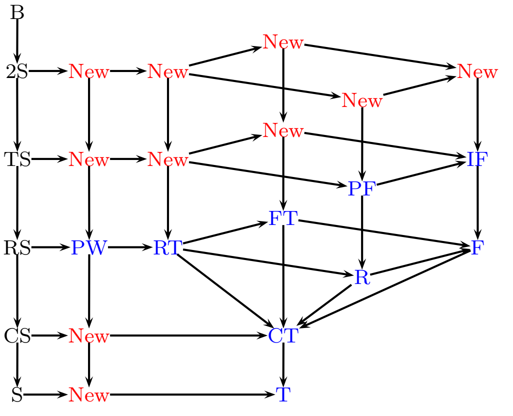

Fig. 3. Semantics in the new linear time-branching time spectrum.

corresponding lowest upper bound. Nevertheless, both intuitions are false and a semantics finer than failures and another coarser than ready traces can be found.

Let us first consider the case of the glb. We postulate the axiomatization of the corresponding semantics to be the conjuction of the two conditions M_R and M_{FT}, to obtain

$$M_{R\wedge FT}(x, y, w) \iff I(x) \supseteq I(y) \text{ and } I(w) \subseteq I(y).$$

We denote with $\sqsubseteq_{R\wedge FT}$ the order axiomatized by the corresponding axiom $(ND^{R\wedge FT})$.

Definition 7.1 The *readiness and failure traces semantics* is defined by the order $\sqsubseteq_{R\wedge FT}$ generated by the set of axioms $\{B_1–B_4, (RS), (ND^{R\wedge FT})\}$.

Proposition 7.2 *The ready traces semantics is strictly finer than the readiness and failure traces semantics.*

Proof. $\sqsubseteq_{RT} \subseteq \sqsubseteq_{R\wedge FT}$ is an immediate consequence of Proposition 3.3 and the fact that condition M_{RT} implies both M_R and M_{FT}, and hence also $M_{R\wedge FT}$. To show that $\sqsubseteq_{RT} \not\supseteq \sqsubseteq_{R\wedge FT}$, let us take $w ::= \mathbf{0}$, $y ::= b$, and $x ::= bB' + c$; then:

$$\underbrace{a(bB + \underbrace{bB' + c}_{p})} \sqsubseteq_{R\wedge FT} \underbrace{a(\underbrace{bB' + c}_{q}) + abB}$$

but, if $I(B) \neq I(B')$,

$$a(bB + bB' + c) \not\sqsubseteq_{RT} a(bB' + c) + abB$$

because $\{a\}a\{b, c\}bI(B) \in \mathit{ReadyTraces}(p) \setminus \mathit{ReadyTraces}(q)$. □

It is clear that the readiness and failure traces semantics is finer than both the readiness and the failure traces semantics; to show that it is actually the coarsest upper bound we need to prove that $\sqsubseteq_{R\wedge FT} = \sqsubseteq_R \cap \sqsubseteq_{FT}$, which cannot be easily done with algebraic arguments. Instead, it is trivial to obtain the observational characterization of the desired semantics by gathering together the failure traces and the ready observations. Based on Definition 5.5, we can define the corresponding order $\leq_N^{l\supseteq\wedge f}$ by

$$\mathcal{T} \leq_N^{l\supseteq\wedge f} \mathcal{T}' \iff \mathcal{T} \leq_N^{l\supseteq} \mathcal{T}' \text{ and } \mathcal{T} \leq_N^{lf} \mathcal{T}'.$$

A nicer characterization can be obtained as follows. We combine both kinds of observations into a single family of traces that we call *failure traces with final ready sets* by considering failure sets all along the trace but at the end of it, where we obtain the exact ready set. Once again, using the notation in Definition 5.5 we can present this characterization as

$$\mathcal{T} \leq_N^{l\supseteq\wedge f} \mathcal{T}' \iff$$

$$\forall X_0 a_1 \ldots X_n \in \mathcal{T}\ \exists Y_0 a_1 \ldots Y_n \in \mathcal{T}'\ (\forall i \in 0..n-1\ X_i \supseteq Y_i) \wedge X_n = Y_n.$$

Proposition 7.3 *The semantics defined by the order $\sqsubseteq_{R\wedge FT}$ coincides with that defined by $\leq_I^{l\supseteq\wedge f}$ and is thus the lub of the readiness and failure traces semantics.*

Proof. Similar to that of Theorem 5.11. □

The axiomatic characterization of the glb of the readiness and failure traces semantics is much simpler: we simply put together the axioms for the orders defining both semantics.

Definition 7.4 $\sqsubseteq_{R\vee FT}$ is the relation defined by the set of axioms $\{B_1\text{–}B_4, (RS), (ND^R), (ND^{FT})\}$.

If we define $M_{R\vee FT} ::= M_R \vee M_{FT}$, that is, $M_{R\vee FT}(x, y, w)$ holds if $I(x) \supseteq I(y)$ or $I(w) \subseteq I(y)$, we have the following characterization of $\sqsubseteq_{R\vee FT}$.

Proposition 7.5 *The order $\sqsubseteq_{R\vee FT}$ is generated by the set of axioms $\{B_1\text{–}B_4, (RS), (ND^{R\vee FT})\}$, where $(ND^{R\vee FT})$ is the instantiation of the generic axiom (ND) with $M_{R\vee FT}$.*

Proposition 7.6 *The semantics defined by the order $\sqsubseteq_{R\vee FT}$ is the finest semantics that is coarser than both the readiness and the failure traces semantics.*

Proof. Obvious, since any semantics coarser than the readiness semantics has to satisfy $\{B_1\text{–}B_4, (RS), (ND^R)\}$, any one coarser than failure traces must satisfy $\{B_1\text{–}B_4, (RS), (ND^{FT})\}$, and $M_{R\vee FT}$ is equivalent to $M_R \vee M_{FT}$. □

Once again the semantics defined by $\sqsubseteq^{R\vee FT}$ is not present in the ltbt spectrum and neither in the extended one; in particular, it is different from the failures semantics.

Proposition 7.7 $\sqsubseteq^{R\vee FT} \subsetneq \sqsubseteq^F$.

Proof. The inclusion is obvious since the failures semantics is coarser than both the readiness and the failure traces semantics. To show that the inclusion is strict, note that any two processes related by $\sqsubseteq^{R\vee FT}$ do not only have the same failures but also the same revivals, as defined by Reed, Roscoe, and Sinclair [11]. Revivals are sequences $a_1, \ldots, a_n(X, a)$ where $a_1, \ldots, a_n$ is a trace of the corresponding process after which the action a is offered, but the set of actions X is refused. This is indeed the case since all the axioms $u \preceq v$ in $\{B_1$–$B_4, (RS), (ND^R), (ND^{FT})\}$ preserve the revivals, which means $\mathit{Revivals}(\sigma(u)) \subseteq \mathit{Revivals}(\sigma(v))$ for every ground substitution σ, and the revivals order is a precongruence for the operators in BCCSP. For instance, for (ND^{FT}) we need to prove that $\mathit{Revivals}(\sigma(a(x+y))) \subseteq \mathit{Revivals}(\sigma(ax)) \cup \mathit{Revivals}(\sigma(a(y+w)))$ whenever $I(\sigma(w)) \subseteq I(\sigma(x))$. It is clear that the only non-trivial case occurs when $a(X,b) \in \mathit{Revivals}(\sigma(a(x+y)))$; then we have $(X,b) \in \mathit{Revivals}(\sigma(x+y))$ so that $X \in \mathit{Failures}(\sigma(x)) \cap \mathit{Failures}(\sigma(y))$ and $b \in I(\sigma(X))$ or $b \in I(\sigma(y))$. In the first case $a(X,b) \in \mathit{Revivals}(\sigma(ax))$ whereas, in the second, $X \in \mathit{Failures}(\sigma(x+y))$ and therefore $a(X,b) \in \mathit{Revivals}(\sigma(a(x+y)))$. The case for (ND^R) is simpler. Once we know that $\sqsubseteq^{R \vee FT}$ preserves the revivals we only need to observe that the revivals cannot be obtained from the failures of a process. In particular, we have $ab \sqsubseteq^F a + a(b+c)$, but $a(\{c\},b) \in \mathit{Revivals}(ab) \setminus \mathit{Revivals}(a + a(b+c))$. □

(We warmly thank Bill Roscoe for pointing out to us his works on the stable revivals semantics [11,12], where an endevor for an adequate presentation of the notion of responsiveness for a CSP-like language is made. Responsiveness had been previously studied by Fournet et al. in [9] under the name of stuck-freeness, for CCS.)

The semantics $\sqsubseteq^{R \vee FT}$, though not in the ltbt spectrum, is not completely new since it coincides with the revivals semantics (at least for BCCSP). To prove this, we first give a characterization of the revivals semantics in terms of our observational framework.

Definition 7.8 We define the order $\leq_N^{l\supseteq \vee f}$ by

$$\mathcal{T} \leq_N^{l\supseteq \vee f} \mathcal{T}' \iff$$
$$\forall X_0 a_1 \dots X_n \in \mathcal{T}\ \exists \{Y_0 a_1 Y_1 \dots Y_n^j \mid j \in J\} \subseteq \mathcal{T}' \text{ such that } X_n = \textstyle\bigcup_{j \in J} Y_n^j.$$

Proposition 7.9 *For all $p, q \in BCCSP$, $\mathit{Revivals}(p) \subseteq \mathit{Revivals}(q)$ if and only if $LGO_I(p) \leq_I^{l\supseteq \vee f} LGO_I(q)$.*

Proof. Note that $\leq_I^{l\supseteq \vee f}$ can be equivalently defined as

$$\mathcal{T} \leq_I^{l\supseteq \vee f} \mathcal{T}' \iff$$
$$\forall X_0 a_1 \dots X_n \in \mathcal{T}\ \forall a \in X_n\ \exists Y_0 a_1 \dots Y_n \in \mathcal{T}'\ (a \in Y_n \wedge Y_n \subseteq X_n).$$

Now, since $a_1 \dots a_n(X,a) \in \mathit{Revivals}(p)$ if and only if there exist $X_0 a_1 \dots X_n \in LGO_I(p)$ such that $a \in X_n$ and $X_n \cap X = \emptyset$, we immediately obtain the desired characterization. □

Definition 7.10 Given $\mathcal{T} \subseteq LGO_N$, $\mathcal{T}^{\supseteq \vee f}$ is defined as

$$\mathcal{T}^{\supseteq \vee f} ::= \{X_0 a_1 \dots X_n \mid \exists \{Y_0 a_1 \dots Y_n^j \mid j \in J\} \subseteq \mathcal{T} \text{ with } X_n = \bigcup_{j \in J} Y_n^j\}.$$

This clearly indicates that $\leq_I^{l\supseteq \vee f}$ is in between $\leq_I^{lf\supseteq}$, defining the failures semantics, and $\leq_I^{lf}$, defining readiness semantics. This is useful for the proof of the axiomatic characterization of the revivals semantics.

Theorem 7.11 *The revivals semantics defined by $\sqsubseteq_I^{l\supseteq\vee f}$ can be axiomatized by the set $\{B_1\text{–}B_4, (RS), (ND^{R\vee FT})\}$.*

Proof. It is similar to that of Theorem 5.11 for the case of failures semantics and, thus, also similar to the characterization of that semantics by means of acceptance trees [10] (and where the closure of the set of offers with respect to both union and convex closure is a critical argument). In connection to that, recall that the application of the particular case of (ND^F) corresponding to (ND^{FT}) allowed us to join arbitrary states after the same trace, while that corresponding to (ND^R) allowed us to obtain a common continuation after the same action at any state reachable by the same trace. All this can be done now using $(ND^{R\vee FT})$; however, we cannot add to an arbitrary state an action offered at another state reachable by the same trace since to do that we needed the unlimited strength of axiom (ND^F).□

It is clear that we can generalize most of the results above to any reasonable local observation function such as T or S, once we interpret $\subseteq$ as the corresponding order and $=$ as the induced equivalence. However, in order to define the adequate observational characterization of the revivals semantics for a local observation (or constraint) N, we should look for the adequate "elements" of the universe of observations. This leads us to traces when N is T, but it is not so clear how to define those "elements" for a non-extensional semantics such as that obtained when N is S.

Let us conclude this section with a look to the beautiful picture in Figure 4 showing the real structure of the full (bidimensional!) diamond, that should be included in all the upper levels of the extended ltbt spectrum.

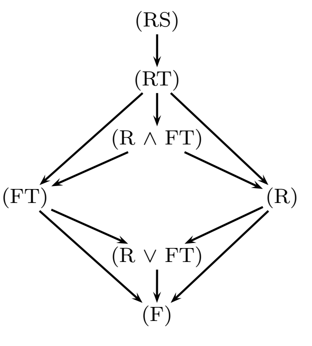

Fig. 4. The real diamond below ready simulation.

8 Conclusions

Following the algebraic approach, we have proved in this paper that the multiplicity of process semantics can be explained in quite a simple way as the combination of a simulation axiom governed by a constraint N and four versions of another axiom for reducing non-determinism. As already achieved in our first part of the study devoted to the observational semantics, the classification into branching and linear semantics has been clarified. More importantly, the similarities between all the semantics have been unearthed, allowing a generic study without the need to resort to case analysis proofs.

Using an approach based on the isolation of some simple properties, which are to be satisfied by any "natural" semantics (including, in particular, those in the spectrum), we showed in [5,6] how to to obtain a canonical preorder induced by a process equivalence and how to produce and axiomatization for this preorder from that of the equivalence itself. And combining both algebraic and coinductive approaches we have presented in [4,8] a generalization of the algorithm proposed in [1] to obtain an axiomatization of the equivalence induced by a preorder from that of the preorder itself.

As future work, it would be interesting to investigate the case in which recursive definitions of processes are available in the language, thus allowing for the possibility of infinite processes. Clearly, since all semantics (in the spectrum) are level continuous [3], we could use the "axiom" $p\downarrow_n \simeq q\downarrow_n \Rightarrow p \simeq q$ but, definitely, we would like to avoid using the equivalence we are trying to capture in the condition of the axiom. Another interesting direction is that of weak semantics, appropriately taking into account silent transitions, for which we already have some preliminary results.

Summing up, we expect that the universe of process semantics will be clarified by our work, making it easier in the future to develop generic studies with shorter, cleaner, and more elegant proofs.

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