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Electronic Notes in Theoretical Computer Science

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Electronic Notes in Theoretical Computer Science 270 (1) (2011) 75–79

www.elsevier.com/locate/entcs

The Thermodynamic Arrow-of-time and Quantum Mechanics

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Abstract

I give an explanation of the thermodynamic arrow-of-time (namely entropy increases with time) within a quantum mechanical framework. This entails giving a solution to the Loschmidt paradox, i.e. showing how an irreversible macro-dynamics can arise from a reversible micro-dynamics. I argue that, in accordance to the reversible dynamics, both entropy-increasing and entropy-decreasing transformations take place, but entropy-decreasing transformations cannot leave any information of their having happened. This is indistinguishable from their not having happened at all. The second law of thermodynamics is then reduced to a tautology: the only transformations that can be seen are those where entropy does not decrease. However, typicality arguments seem to prevent this argument to be used as a complete solution to the arrow-of-time dilemma: it might still be necessary to postulate a low entropy initial state for the system under consideration.

Keywords: Thermodynamic entropy, von Neumann entropy, arrow-of-time, second law, quantum and classical mutual information.

1 Introduction

"Time" is the most often used English noun, but it is a concept notoriously difficult to define accurately. Moreover, many of the intuitive properties we assign to time are not at all mirrored by the notion of time we derive from Physics [1]. Here we will focus on one of the most important aspects of the "time problem" in physics, i.e. on the second law of thermodynamics: entropy in a closed system does not decrease with time. Even though thermodynamic entropy can be axiomatically introduced in many different ways (e.g. see [2]), it is basically a measure of how the usable energy of a system gets degraded into heat. Once this has happened, the energy cannot be converted any more, unless a lower temperature heat-sink is present. In

[1] I thank Lloyd, Giovannetti, Beretta, Zeh, and D'Ariano for stimulating discussions and feedback.

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1571-0661
doi:10.1016/j.entcs.2011.01.007

this sense, entropy is then a measure of irreversibility in physical processes. Few years after Boltzmann proposed his statistical interpretation of entropy, Loschmidt pointed out that it is definitely paradoxical that a time reversible micro-dynamics gives rise to a non reversible macro-dynamics [3]. In fact, following Boltzmann's derivation, it is not at all surprising that entropy tends to increase in the past-to-future direction, but rather it is quite surprising that it decreases in the future-to-past direction [4]. Boltzmann (and many after him) argued that this problem can be solved by postulating a very low entropy initial-state of the universe. He suggested that this might be a result of a random fluctuation. A recent estimate by Penrose gives the likelihood of such a fluctuation as 1 over $10^{10^{123}}$ [5]. Even though this fantastically improbable fluctuation may be justified *a posteriori* with the anthropic principle, a most important criticism to this view is that it is vastly more probable that the random fluctuation created the universe as it was a fraction of a second ago (including our memories of having lived longer than a fraction of a second) rather than the even more fantastically-improbable state of 13 billion years ago [6] (as cited in [7]). Many other solutions to the Loschmidt paradox have been proposed (e.g. see [1,4,8,9,10,12]), but none are completely satisfactory.

The following arguments are based on the fact that the entropy of a system with state ρ can be equated (when the system is in equilibrium) with the von Neumann entropy $S(\rho) = -\text{Tr}[\rho \log_2 \rho]$. This equivalence can be derived from general principles [13,14]. However, all that is necessary for the following argument is the fact that bits of von Neumann entropy can be exchanged with bits of thermodynamic entropy. This can be achieved through a Maxwell demon or a Szilard engine [15,16].

Here I describe a solution to Loschmidt's paradox that rests on the fact that ultimately entropy is an observer-dependent quantity (although in basically *all* practical situations it can be considered as an objective quantity). In fact, suppose that two boxes full of gas are prepared in a state in which the trajectories of the gas particles are perfectly correlated, but where each box appears in thermal equilibrium. Clearly an observer that has knowledge of the correlations can extract energy from the two boxes. On the contrary, an observer ignorant of the correlations would just see the two boxes in a maximum entropy thermal equilibrium state [3], from which no energy can be extracted (without using an external heat sink). If quantum mechanics is taken into account, this situation is even more evident. In fact, supposing that the state of the whole universe is a pure state, then any quantum system can be extended, by taking its purification, in such a way that the global state of system plus purification is a zero entropy pure state (the quantum entropy of a system is lower or equal to than that of its subsystems). Then, by suitably augmenting any system, it is possible to extract the whole energy from a system: extending the system, the entropy is reduced. Of course, in basically all practical situations this is completely unfeasible, as the correlations between subsystems will in general be extremely complicated and the purification will involve a huge system.

[3] One may object that the ignorant observer is ignorant also of the *true* value of the entropy. However, there is no operative way that any observer can exclude that any box is correlated with some unknown system, unless the state of the whole universe is known. Hence, any observer *must* by necessity assume that no correlations exist, unless it is known otherwise. An alternative definition of thermodynamic entropy that tests for all possible correlations with all other systems of the universe would be quite useless in practice!

Thus, for all practical purposes, macroscopic systems can always be considered uncorrelated, so that the subjectivity of the entropy is not of any practical concern. However, from a fundamental point of view, it can be used to obtain a solution to Loschmidt's paradox, by considering any observer and its memory as a physical system. In fact, any memory (i.e. information about a physical system or an event) must be encoded into some physical degree of freedom. Then, if a transformation decorrelates these degrees of freedom from the system or the event, the observer will find itself without *any* information about the system or the event. The situation in which all information about an event has been decorrelated from it is completely indistinguishable from the event not having happened at all. Now, suppose that this event was a transformation that increased the entropy, its "erasure" through the above mechanism can (and in fact, will) decrease the entropy without violating any physical laws.

The basic idea is quite simple. Suppose that the system A is under observation, and that the observer is the system O (which describes also the degrees of freedom of the observer's memory). It is easy to derive (see below) that

$$\Delta S(A) + \Delta S(O) - \Delta S(R) - \Delta S(A:O) = 0 \,, \tag{1}$$

where $\Delta S(X) \equiv S_t(\rho_X) - S_0(\rho_X)$ is the difference between the entropies at the final time t and at the initial time 0 of the system X, R is an auxiliary system (reservoir), and $\Delta S(A:O)$ is the difference between the initial and final quantum mutual information $S(A:O) \equiv S(\rho_A) + S(\rho_O) - S(\rho_{AO})$ between systems A and O (ρ_{AO} being the joint state of the two systems). The quantum mutual information measures the shared correlations (quantum and classical) between A and O [17]. The interpretation of Eq. (1) is then straightforward: the entropy of systems A and O *can* decrease, by either augmenting the entropy of a reservoir (which is unsurprising), or by reducing the quantum mutual information between them. Now, suppose that the reservoir is not employed to change S, then the entropy of the system A can actually be reduced by destroying correlations that may have built up with the observer O or with the system's environment, namely by reducing the quantum information that the observer possesses about the system A.

Equation (1) is concerned only with the situation at an initial time 0 and at a final time t. If we want to consider the situation at an intermediate time t_i, we can simply employ Eq. (1) twice: between time 0 and t_i, and again between time t_i and t. It is then clear that the entropy of the system A may easily be higher at time t_i than at the final time t. However, in that case, it is also clear that the decrease in entropy must be obtained either by dumping the entropy in another system (the observer, or the reservoir), or by reducing the quantum mutual information with respect to its value at t_i. This means that the correlations that intervening events may have built up at time t_i are deleted, so that the memories of these events are erased (or, at least, the maximum available mutual information on them is reduced). A couple of thought experiments that may clarify the above points are given in Ref. [18].

We now give the simple proof of Eq. (1), which basically consists in choosing the reservoir R such that the combined system AOR is in a pure state both at the initial time 0 and at the final time t (i.e. the reservoir is a purification for AO, the system and the observer). This implies that at both times,

$$S_0(\rho_{AO}) - S_0(\rho_R) = S_t(\rho_{AO}) - S_t(\rho_R) = 0 \;\Rightarrow\; \Delta S(R) = \Delta S(AO) \,, \tag{2}$$

where the suffixes refers to the time. By introducing Eq. (2) into the left-hand-side of Eq. (1), we see that this term is null, as required.

Can the mechanism described above be used to give a complete solution to Loschmidt's paradox? It effectively points out why we see only entropy-increasing transformations (the entropy-decreasing ones cannot leave any trace of their having happened) *and* it points out why the initial state of the universe should be lower entropy than today's (because of the above mechanism, as any observer looks into its past, it will see entropy diminishing for decreasing time). However, it seems that typicality arguments would suggest that any observer would see a high entropy state, basically a thermal equilibrium state, in its present. In fact, without prior assumptions on the state of the universe, one should assume it is in a highly-probable high-entropy state [11]. This implies that to *any* observer *inside* the universe, the state of any (sufficiently small) subsystem will appear as a thermal equilibrium high-entropy state [11]. This is not what we observe.

Then, it seems necessary to be forced to separately postulate a low entropy initial state, or, rather, an initial pure state that is sufficiently symmetric [19] that any subsystem can be seen in a low entropy state (e.g. the vacuum state [20]). This is analogous to Boltzmann's initial-state postulate.

In usual everyday situations, recoherence events that involve an observer are extremely rare (or, rather, practically impossible). This implies that, although correct in principle, my argument is inconsequential in practice: most entropy-increasing events can be explained solely from the fact that for macroscopic systems decoherence is much more probable than recoherence. This asymmetry can be seen as stemming from asymmetric boundary conditions *à la* Boltzmann (more precisely, from the fact that in the initial state of the universe most subsystems were factorized and highly symmetric [19]). The mechanism I describe in my paper becomes relevant only when recoherence events are comparable in probability with decoherence events, namely in a pure-state universe near a heat-death state where subsystems are almost maximally correlated.

This implies that my argument cannot be seen as a complete solution for the arrow-of-time dilemma: even though it might explain how entropy-increasing events are singled out from any observer's point of view, it seems it cannot entirely account for the humongous difference between initial and current entropy of the universe.

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