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## Full Length Article

# Exact travelling wave solutions of the coupled nonlinear evolution equation via the Maccari system using novel $(G'/G)$ -expansion method

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## ABSTRACT

In this article, the novel  $(G'/G)$ -expansion method is used to construct exact travelling wave solutions of the coupled nonlinear evolution equation. This technique is uncomplicated and simple to use, and gives more new general solutions than the other existing methods. Also, it is shown that the novel  $(G'/G)$ -expansion method, with the help of symbolic computation, provides a straightforward and vital mathematical tool for solving nonlinear evolution equations. For illustrating its effectiveness, we apply the novel  $(G'/G)$ -expansion method for finding the exact solutions of the  $(2 + 1)$ -dimensional coupled integrable nonlinear Maccari system.

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## 1. Introduction

The investigation of exact travelling wave solutions to nonlinear evolution equation plays an important role in the study of nonlinear physical phenomena for various fields of

science and engineering, especially in mathematical physics, plasma physics, fluid dynamics, quantum field theory, biophysics, chemical kinematics, geochemistry, propagation of shallow water waves, high-energy physics and so on. The analytical solutions of such equations are of fundamental

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importance since a lot of mathematical-physical models are described by nonlinear evolution equations (NLEEs). Many powerful and direct methods have been developed to find explicit solutions to the NLEEs, such as, wave of translation [1], the inverse scattering transform [2], the Hirota's bilinear method [3], the Darboux transformation method [4], the Backlund transformation method [5], the tanh method [6], the tanh-sech method [7], the symmetry method [8], the Painleve expansion method [9], the Exp-function method [10–14], the Adomian decomposition method [15], the homogeneous balance [16] and so on to construct exact solution of NLEEs. Lately, Wang et al. [17] introduced an expansion technique called the  $(G'/G)$ -expansion method, and they verified that it is a simple technique look for analytic solutions of NLEEs. In order to show the efficiency of the  $(G'/G)$ -expansion method and to extend the range of its applicability, further research has been carried out by several researchers, such as, Zhang et al. [18] proposed a generalization of the  $(G'/G)$ -expansion method for solving the evolution equations with variable coefficients. Zhang et al. [19] also presented an improved  $(G'/G)$ -expansion method to seek general traveling wave solutions. Zayed [20] obtainable a new approach of the  $(G'/G)$ -expansion method where  $G(\xi)$  satisfies the Jacobi elliptical equation  $[G'(\xi)]^2 = e_2 G^4(\xi) + e_1 G^2(\xi) + e_0$ . Zayed [21] again proposed an alternative approach of this method in which  $G(\xi)$  satisfies the Riccati equation  $G'(\xi) = A + BG^2(\xi)$ , where  $A$  and  $B$  are arbitrary constants. Akbar et al. [22] proposed a generalized and improved  $(G'/G)$ -expansion method which give more new solutions than the improved  $(G'/G)$ -expansion method [19]. Recently, Alam et al. [23] further improved the  $(G'/G)$ -expansion method known as novel  $(G'/G)$ -expansion method. They have solved only single NLEEs using this method.

The nonlinear Maccari system is an important mathematical model in physics. Currently, Lee et al. [24], Hafez et al. [25], and Manafian et al. [26,27] have solved the Maccari system using the Kudryashov method, the  $\exp(-(\Phi(\xi)))$ -expansion method and the Exp-function method respectively. Therefore, the aim of this article is to investigate new exact travelling wave solutions to the Maccari system by use of the novel  $(G'/G)$ -expansion method, which is more effective than others methods.

## 2. Description of the novel $(G'/G)$ -expansion method

Let us consider the nonlinear evolution equation

$$P(u, u_t, u_x, u_y, u_{xx}, u_{yy}, u_{tt}, u_{tx}, \dots), \quad (1)$$

where  $P$  is a polynomial in  $u(x, y, t)$  and its partial derivatives wherein the highest order partial derivatives and the nonlinear terms are concerned. The most important steps of the method are as follows:

**Step 1:** Combine the real variables  $x, y$  and  $t$  by a complex variable  $\xi$ , we suppose that

$$u(x, y, t) = u(\xi), \quad \xi = x + y + ct, \quad (2)$$

where  $c$  denotes the speed of the traveling wave. By use of Eq. (2), Eq. (1) is converted into an ODE for  $u = u(\xi)$ :

$$Q(u, u', u'', u''', \dots) = 0, \quad (3)$$

where,  $Q$  is a function of  $u(\xi)$  and its derivatives wherein prime stands for derivative with respect to  $\xi$ .

**Step 2:** Assume the solution of Eq. (3) can be expressed in powers  $\psi(\xi)$ :

$$u(\xi) = \sum_{j=-N}^N \alpha_j (\psi(\xi))^j \quad (4)$$

where

$$\psi(\xi) = (d + \Phi(\xi)) \quad (5)$$

$$\text{and } \Phi(\xi) = \frac{G'(\xi)}{G(\xi)}.$$

Here  $\alpha_{-N}$  or  $\alpha_N$  may be zero, but both of them could not be zero simultaneously.  $\alpha_j$  ( $j=0, \pm 1, \pm 2, \dots, \pm N$ ) and  $d$  are constants to be determined later, and  $G=G(\xi)$  satisfies the second order nonlinear ODE:

$$GG'' = \lambda GG' + \mu G^2 + \nu (G')^2 \quad (6)$$

where prime denotes the derivative with respect to  $\xi$ ;  $\lambda, \mu$ , and  $\nu$  are real parameters.

The Cole-Hopf transformation  $\Phi(\xi) = \ln(G(\xi))_\xi = \frac{G'(\xi)}{G(\xi)}$  reduces the Eq. (6) into Riccati equation:

$$\Phi'(\xi) = \mu + \lambda \Phi(\xi) + (\nu - 1)\Phi^2(\xi) \quad (7)$$

Eq. (7) has individual twenty five solutions (see Zhu [28] for details).

**Step 3:** The value of the positive integer  $N$  can be determined by balancing the highest order linear terms with the nonlinear terms of the highest order come out in Eq. (3). If the degree of  $u(\xi)$  is  $D[u(\xi)] = n$ , then the degree of the other expressions will be as follows:

$$D\left[\frac{d^p u(\xi)}{d\xi^p}\right] = n + p, \quad D\left[u^p \left(\frac{d^q u(\xi)}{d\xi^q}\right)^s\right] = np + s(n + q).$$

**Step 4:** Substitute Eq. (4) including Eqs. (5) and (6) into Eq.

(3), we obtain polynomials in  $\left(d + \frac{G'(\xi)}{G(\xi)}\right)^j$  and  $\left(d + \frac{G'(\xi)}{G(\xi)}\right)^{-j}$ , ( $j=0, 1, 2, \dots, N$ ). Collect each coefficient of the resulted polynomials to zero, yields an over-determined set of algebraic equations for  $\alpha_j$  ( $j=0, \pm 1, \pm 2, \dots, \pm N$ ),  $d$  and  $V$ .

**Step 5:** Suppose the value of the constants can be obtained by solving the algebraic equations obtained in Step 4. Substituting the values of the constants together with the solutions of Eq. (6), we will obtain new and comprehensive exact traveling wave solutions of the nonlinear evolution Eq. (1).

**Discussion 1:** It is noteworthy to examine that if we replace  $\lambda$  by  $-\lambda$  and  $\mu$  by  $-\mu$  and put  $\nu=0$  in Eq. (6), then the novel  $(G'/G)$ -expansion method coincide with Akbar et al.'s [19] generalized

and improved  $(G'/G)$ -expansion method. If we put  $d=0$  in Eq. (5) and  $\nu=0$  in Eq. (6), this method is identical to the improved  $(G'/G)$ -expansion method presented by Zhang et al. [19]. Again if we put  $d=0$ ,  $\nu=0$  and negative the exponents of  $(G'/G)$  are zero in Eq. (4), then this method turn out into the basic  $(G'/G)$ -expansion method introduced by Wang et al. [17]. Finally, if we put  $\nu=0$  in Eq. (6) and  $\alpha_j$  ( $j=1, 2, 3, \dots, N$ ) are functions of  $x$ ,  $y$  and  $t$  instead of constants then the this method is transformed into the generalized the  $(G'/G)$ -expansion method developed by Zhang et al. [18]. Therefore we observe that the methods mentions in the refs. [17,19,22,29,30] are only special cases of the novel  $(G'/G)$ -expansion method.

### 3. New exact travelling wave solutions of the $(2 + 1)$ -dimensional Maccari system

Let us consider the  $(2 + 1)$ -dimensional coupled integrable nonlinear system in the following form

$$\left\{ \begin{array}{l} \alpha_{-1} = \alpha_{-1}, \alpha_0 = -\frac{(2\nu d - \lambda - 2d)\alpha_{-1}}{2(\mu - \lambda d + \nu d^2 - d^2)}, a_1 = 0, \quad p = p, q = q, d = d, r = 2\mu\nu - \frac{\lambda^2}{2} + p^2 - 2\mu, \\ c = -\frac{(-4\mu d^2 + 2\nu^2 d^4 - 4\nu d^4 + 2\lambda^2 d^2 + 2\mu^2 - 4\lambda d^3\nu + 4\mu\nu d^2 + 4\lambda d^3 - 4\mu\lambda d + 2d^4 - a_{-1}^2)}{2(\mu - \lambda d + \nu d^2 - d^2)^2} \end{array} \right\} \quad (14)$$

$$\left\{ \begin{array}{l} iu_t + u_{xx} + uv = 0 \\ v_t + v_y + (|u|^2)_x = 0 \end{array} \right\}. \quad (8)$$

$$\left\{ \begin{array}{l} \alpha_{-1} = \alpha_{-1}, \alpha_0 = 0, \alpha_1 = 0, \quad p = p, q = q, d = \frac{\lambda}{2(\nu - 1)}, r = 2\mu\nu - \frac{\lambda^2}{2} + p^2 - 2\mu, \\ c = -\frac{(\lambda^4 + 8\lambda^2\mu - 8\lambda^2\nu\nu - 8\nu^2 a_{-1}^2 - 8a_{-1}^2 + 16a_{-1}^2\nu + 16\mu^2\nu^2 + 16\mu^2 - 32\mu^2\nu)}{(\lambda^4 + 8\lambda^2\mu - 8\lambda^2\nu\nu + 16\mu^2\nu^2 + 16\mu^2 - 32\mu^2\nu)} \end{array} \right\} \quad (15)$$

If we apply the following transformation

$$u(x, y, t) = e^{i\omega t} U(\xi), \quad v(x, y, t) = V(\xi), \quad (9)$$

where  $\omega = px + qy + rt$  and  $\xi = x + y + ct$ , the Maccari system in (8) can be reduced to a system of ODE form as follows:

$$\left\{ \begin{array}{l} U'' - (r + p^2)U + UV = 0 \\ (c + 1)V' + (U^2)' = 0 \end{array} \right\} \quad (10)$$

Integrating the second equation in (10) and neglecting the constant of integration we find

$$V = -\frac{1}{(c + 1)}U^2 \quad (11)$$

Substituting (11) into the first equation of system in Eq. (10), we obtain

$$(c + 1)U'' - (c + 1)(r + p^2)U - U^3 = 0, \quad (12)$$

where primes denotes the differentiation with regard to  $\xi$ . Inserting (4) and (6) and considering the homogeneous balance between  $U''$  and  $U^3$  in Eq. (12), we obtain  $3N = N + 2$ . i. e.  $N = 1$ . Therefore, we have,

$$U(\xi) = \alpha_{-1}(\psi(\xi))^{-1} + \alpha_0 + \alpha_1(\psi(\xi)). \quad (13)$$

Substituting Eq. (13) into Eq. (12), the left hand side is transformed into polynomials in  $\left(d + \frac{G'(\xi)}{G(\xi)}\right)^j$ , ( $j=0, 1, 2, \dots, N$ ) and  $\left(d + \frac{G'(\xi)}{G(\xi)}\right)^{-j}$ , ( $j=0, 1, 2, \dots, N$ ). Equating the coefficients of similar power of these polynomials to zero, we obtain a system of algebraic equations for  $\alpha_{-1}, \alpha_0, \alpha_1, d, p, q, r$  and  $c$ . Solving the obtaining system of algebraic equations by use of the symbolic computation software, such as Maple 13, we obtain

Set 1:

Set 2:

Set 3:

$$\left\{ \begin{array}{l} \alpha_{-1} = 0, \alpha_0 = -\frac{(2\nu d - \lambda - 2d)\alpha_1}{2(\nu - 1)}, \alpha_1 = \alpha_1, \quad p = p, q = q, d = d, \\ r = 2\mu\nu - \frac{\lambda^2}{2} + p^2 - 2\mu, \quad c = \frac{a_1^2 - 2 + 4\nu - 2\nu^2}{2(\nu^2 + 1 - 2\nu)} \end{array} \right\} \quad (16)$$

Set 4:

$$\left\{ \begin{array}{l} \alpha_{-1} = \frac{(4\mu\nu - \lambda^2 - 4\mu)\alpha_1}{4(\nu - 1)^2}, \alpha_0 = 0, \alpha_1 = \alpha_1, \quad p = p, q = q, d = \frac{\lambda}{2(\nu - 1)}, \\ r = 4\mu + p^2 + \lambda^2 - 4\mu\nu, c = \frac{a_1^2 - 2 + 4\nu - 2\nu^2}{2(\nu^2 + 1 - 2\nu)} \end{array} \right\} \quad (17)$$

Set 5:

$$\left\{ \begin{array}{l} \alpha_{-1} = -\frac{(4\mu\nu - \lambda^2 - 4\mu)\alpha_1}{4(\nu-1)^2}, \alpha_0 = 0, \alpha_1 = \alpha_1, p = p, q = q, d = \frac{\lambda}{2(\nu-1)}, \\ r = -8\mu + p^2 - 2\lambda^2 + 8\mu\nu, c = \frac{a_1^2 - 2 + 4\nu - 2\nu^2}{2(\nu^2 + 1 - 2\nu)} \end{array} \right\} \quad (18)$$

By substituting Eqs. 14–18 to the Eq. (13), we get

$$U_1(\xi) = a_{-1} \left\{ (d + (G'/G))^{-1} - \frac{(2\nu d - \lambda - 2d)}{2(\mu - \lambda d + \nu d^2 - d^2)} \right\} \quad (19)$$

$$U_2(\xi) = a_{-1} \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right)^{-1} \quad (20)$$

$$U_3(\xi) = \alpha_1 \left\{ -\frac{(2\nu d - \lambda - 2d)}{2(\nu-1)} + (d + (G'/G)) \right\} \quad (21)$$

$$U_4(\xi) = \alpha_1 \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right)^{-1} + \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right) \right\} \quad (22)$$

$$U_5(\xi) = \alpha_1 \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right)^{-1} + \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right) \right\} \quad (23)$$

Therefore, with the help of Eqs. (9), (11) and (19), the travelling wave solution of the Maccari system is given by

$$\begin{aligned} u_1(\xi) &= a_{-1} e^{i(px+qy+rt)} \times \left\{ (d + (G'/G))^{-1} - k \right\} \\ v_1(\xi) &= -\frac{a_{-1}^2}{(c+1)} \times \left\{ (d + (G'/G))^{-1} - k \right\}^2 \end{aligned} \quad (24)$$

where,

$$\xi = x + y - \frac{(-4\mu d^2 + 2\nu^2 d^4 - 4\nu d^4 + 2\lambda^2 d^2 + 2\mu^2 - 4\lambda d^3 \nu + 4\mu\nu d^2 + 4\lambda d^3 - 4\mu\lambda d + 2d^4 - a_1^2)}{2(\mu - \lambda d + \nu d^2 - d^2)} t,$$

$k = \frac{(2\nu d - \lambda - 2d)}{2(\mu - \lambda d + \nu d^2 - d^2)}$ ,  $r = 2\mu\nu - \frac{\lambda^2}{2} + p^2 - 2\mu$  and  $a_{-1}, \mu, \nu, \lambda, d, p$  and  $q$  are arbitrary constants.

By substituting the value of  $(G'/G)$  into Eq. (24), we obtain the following: when  $\Omega = \lambda^2 - 4\mu\nu + 4\mu > 0$  and  $\lambda(\nu-1) \neq 0$  (or  $\mu(\nu-1) \neq 0$ ), we get that

$$\begin{aligned} u_{1_1}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-1} - k \right\} \\ v_{1_1}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-1} - k \right\}^2 \end{aligned} \quad (25)$$

$$\begin{aligned} u_{1_2}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \coth\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-1} - k \right\} \\ v_{1_2}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \coth\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-1} - k \right\}^2 \end{aligned} \quad (26)$$

$$u_{1_3}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left\{ \lambda + \sqrt{\Omega} \left( \tanh(\sqrt{\Omega}\xi) \pm i \sec h(\sqrt{\Omega}\xi) \right) \right\} \right)^{-1} - k \right\} \quad (27)$$

$$v_{1_3}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left\{ \lambda + \sqrt{\Omega} \left( \tanh(\sqrt{\Omega}\xi) \pm i \sec h(\sqrt{\Omega}\xi) \right) \right\} \right)^{-1} - k \right\}^2$$

$$u_{1_4}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left\{ \lambda + \sqrt{\Omega} \left( \coth(\sqrt{\Omega}\xi) \pm \csc h(\sqrt{\Omega}\xi) \right) \right\} \right)^{-1} - k \right\} \quad (28)$$

$$v_{1_4}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left\{ \lambda + \sqrt{\Omega} \left( \coth(\sqrt{\Omega}\xi) \pm \csc h(\sqrt{\Omega}\xi) \right) \right\} \right)^{-1} - k \right\}^2$$

$$u_{1_5}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{1}{4(\nu-1)} \left\{ 2\lambda + \sqrt{\Omega} \left( \tanh\left(\frac{1}{4}\sqrt{\Omega}\xi\right) + \coth\left(\frac{1}{4}\sqrt{\Omega}\xi\right) \right) \right\} \right)^{-1} - k \right\} \quad (29)$$

$$v_{1_5}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{1}{4(\nu-1)} \left\{ 2\lambda + \sqrt{\Omega} \left( \tanh\left(\frac{1}{4}\sqrt{\Omega}\xi\right) + \coth\left(\frac{1}{4}\sqrt{\Omega}\xi\right) \right) \right\} \right)^{-1} - k \right\}^2$$

$$u_{1_6}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm\sqrt{\Omega(A^2+B^2)} - A\sqrt{\Omega} \cosh(\sqrt{\Omega}\xi)}{A \sinh(\sqrt{\Omega}\xi) + B} \right\} \right)^{-1} - k \right\} \quad (30)$$

$$v_{1_6}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm\sqrt{\Omega(A^2+B^2)} - A\sqrt{\Omega} \cosh(\sqrt{\Omega}\xi)}{A \sinh(\sqrt{\Omega}\xi) + B} \right\} \right)^{-1} - k \right\}^2$$

$$\begin{aligned}
 u_{17}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm \sqrt{\Omega(A^2+B^2)} + A\sqrt{\Omega} \cosh(\sqrt{\Omega}\xi)}{A \sinh(\sqrt{\Omega}\xi) + B} \right\} \right)^{-1} - k \right\} \\
 v_{17}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm \sqrt{\Omega(A^2+B^2)} + A\sqrt{\Omega} \cosh(\sqrt{\Omega}\xi)}{A \sinh(\sqrt{\Omega}\xi) + B} \right\} \right)^{-1} - k \right\}^2
 \end{aligned} \tag{31}$$

where, A and B are real non-zero constants.

$$\begin{aligned}
 u_{18}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{2\mu \cosh\left(\frac{1}{2}\sqrt{\Omega}\xi\right)}{\sqrt{\Omega} \sinh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) - \lambda \cosh\left(\frac{1}{2}\sqrt{\Omega}\xi\right)} \right)^{-1} - k \right\} \\
 v_{18}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{2\mu \cosh\left(\frac{1}{2}\sqrt{\Omega}\xi\right)}{\sqrt{\Omega} \sinh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) - \lambda \cosh\left(\frac{1}{2}\sqrt{\Omega}\xi\right)} \right)^{-1} - k \right\}^2
 \end{aligned} \tag{32}$$

$$\begin{aligned}
 u_{19}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{2\mu \sinh\left(\frac{1}{2}\sqrt{\Omega}\xi\right)}{\sqrt{\Omega} \cosh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) - \lambda \sinh\left(\frac{1}{2}\sqrt{\Omega}\xi\right)} \right)^{-1} - k \right\} \\
 v_{19}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{2\mu \sinh\left(\frac{1}{2}\sqrt{\Omega}\xi\right)}{\sqrt{\Omega} \cosh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) - \lambda \sinh\left(\frac{1}{2}\sqrt{\Omega}\xi\right)} \right)^{-1} - k \right\}^2
 \end{aligned} \tag{33}$$

$$\begin{aligned}
 u_{110}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{2\mu \cosh(\sqrt{\Omega}\xi)}{\sqrt{\Omega} \sinh(\sqrt{\Omega}\xi) - \lambda \cosh(\sqrt{\Omega}\xi) \pm i\sqrt{\Omega}} \right)^{-1} - k \right\} \\
 v_{110}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{2\mu \cosh(\sqrt{\Omega}\xi)}{\sqrt{\Omega} \sinh(\sqrt{\Omega}\xi) - \lambda \cosh(\sqrt{\Omega}\xi) \pm i\sqrt{\Omega}} \right)^{-1} - k \right\}^2
 \end{aligned} \tag{34}$$

$$\begin{aligned}
 u_{111}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{2\mu \sinh(\sqrt{\Omega}\xi)}{\sqrt{\Omega} \cosh(\sqrt{\Omega}\xi) - \lambda \sinh(\sqrt{\Omega}\xi) \pm \sqrt{\Omega}} \right)^{-1} - k \right\} \\
 v_{111}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{2\mu \sinh(\sqrt{\Omega}\xi)}{\sqrt{\Omega} \cosh(\sqrt{\Omega}\xi) - \lambda \sinh(\sqrt{\Omega}\xi) \pm \sqrt{\Omega}} \right)^{-1} - k \right\}^2
 \end{aligned} \tag{35}$$

when  $\Omega = \lambda^2 - 4\mu\nu + 4\mu < 0$  and  $\lambda(\nu-1) \neq 0$  (or  $\mu(\nu-1) \neq 0$ ), we get that

$$\begin{aligned}
 u_{112}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-1} - k \right\} \\
 v_{112}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-1} - k \right\}^2
 \end{aligned} \tag{36}$$

$$\begin{aligned}
 u_{113}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{-\Omega} \cot\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-1} - k \right\} \\
 u_{113}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{-\Omega} \cot\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-1} - k \right\}^2
 \end{aligned} \tag{37}$$

$$\begin{aligned}
 u_{114}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \sqrt{-\Omega} \left( \tan(\sqrt{-\Omega}\xi) \pm \sec(\sqrt{-\Omega}\xi) \right) \right\} \right)^{-1} - k \right\} \\
 v_{114}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \sqrt{-\Omega} \left( \tan(\sqrt{-\Omega}\xi) \pm \sec(\sqrt{-\Omega}\xi) \right) \right\} \right)^{-1} - k \right\}^2
 \end{aligned} \tag{38}$$

$$u_{115}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left\{ \lambda + \sqrt{-Q} \left( \cot(\sqrt{-Q}\xi) \pm \csc(\sqrt{-Q}\xi) \right) \right\} \right)^{-1} - k \right\}$$

$$v_{115}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{1}{2(\nu-1)} \left\{ \lambda + \sqrt{-Q} \left( \cot(\sqrt{-Q}\xi) \pm \csc(\sqrt{-Q}\xi) \right) \right\} \right)^{-1} - k \right\}^2$$
(39)

$$u_{116}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{1}{4(\nu-1)} \left\{ -2\lambda + \sqrt{-Q} \left( \tan\left(\frac{1}{4}\sqrt{-Q}\xi\right) - \cot\left(\frac{1}{4}\sqrt{-Q}\xi\right) \right) \right\} \right)^{-1} - k \right\}$$

$$v_{116}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{1}{4(\nu-1)} \left\{ -2\lambda + \sqrt{-Q} \left( \tan\left(\frac{1}{4}\sqrt{-Q}\xi\right) - \cot\left(\frac{1}{4}\sqrt{-Q}\xi\right) \right) \right\} \right)^{-1} - k \right\}^2$$
(40)

$$u_{117}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm\sqrt{-Q}(A^2-B^2) - A\sqrt{-Q}\cos(\sqrt{-Q}\xi)}{A\sin(\sqrt{-Q}\xi) + B} \right\} \right)^{-1} - k \right\}$$

$$v_{117}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm\sqrt{-Q}(A^2-B^2) - A\sqrt{-Q}\cos(\sqrt{-Q}\xi)}{A\sin(\sqrt{-Q}\xi) + B} \right\} \right)^{-1} - k \right\}^2$$
(41)

$$u_{118}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm\sqrt{-Q}(A^2-B^2) + A\sqrt{-Q}\cos(\sqrt{-Q}\xi)}{A\sin(\sqrt{-Q}\xi) + B} \right\} \right)^{-1} - k \right\}$$

$$v_{118}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{1}{2(\nu-1)} \left\{ -\lambda + \frac{\pm\sqrt{-Q}(A^2-B^2) + A\sqrt{-Q}\cos(\sqrt{-Q}\xi)}{A\sin(\sqrt{-Q}\xi) + B} \right\} \right)^{-1} - k \right\}^2$$
(42)

where, A and B are constants such that  $A^2 - B^2 > 0$ .

$$u_{119}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{2\mu\cos(\sqrt{-Q}\xi/2)}{\sqrt{-Q}\sin(\sqrt{-Q}\xi/2) + \lambda\cos(\sqrt{-Q}\xi/2)} \right)^{-1} - k \right\}$$

$$v_{119}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{2\mu\cos(\sqrt{-Q}\xi/2)}{\sqrt{-Q}\sin(\sqrt{-Q}\xi/2) + \lambda\cos(\sqrt{-Q}\xi/2)} \right)^{-1} - k \right\}^2$$
(43)

$$u_{120}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{2\mu\sin(\sqrt{-Q}\xi/2)}{\sqrt{-Q}\cos(\sqrt{-Q}\xi/2) - \lambda\sin(\sqrt{-Q}\xi/2)} \right)^{-1} - k \right\}$$

$$v_{120}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{2\mu\sin(\sqrt{-Q}\xi/2)}{\sqrt{-Q}\cos(\sqrt{-Q}\xi/2) - \lambda\sin(\sqrt{-Q}\xi/2)} \right)^{-1} - k \right\}^2$$
(44)

$$u_{121}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{2\mu\cos(\sqrt{-Q}\xi)}{\sqrt{-Q}\sin(\sqrt{-Q}\xi) + \lambda\cos(\sqrt{-Q}\xi) \pm \sqrt{-Q}} \right)^{-1} - k \right\}$$

$$v_{121}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{2\mu\cos(\sqrt{-Q}\xi)}{\sqrt{-Q}\sin(\sqrt{-Q}\xi) + \lambda\cos(\sqrt{-Q}\xi) \pm \sqrt{-Q}} \right)^{-1} - k \right\}^2$$
(45)

$$u_{122}(\xi) = a_{-1} e^{i(px+qy+rt)} \left\{ \left( d + \frac{2\mu\sin(\sqrt{-Q}\xi/2)}{\sqrt{-Q}\cos(\sqrt{-Q}\xi/2) - \lambda\sin(\sqrt{-Q}\xi/2) \pm \sqrt{-Q}} \right)^{-1} - k \right\}$$

$$v_{122}(\xi) = -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d + \frac{2\mu\sin(\sqrt{-Q}\xi/2)}{\sqrt{-Q}\cos(\sqrt{-Q}\xi/2) - \lambda\sin(\sqrt{-Q}\xi/2) \pm \sqrt{-Q}} \right)^{-1} - k \right\}^2$$
(46)

when  $\mu=0$  and  $\lambda(\nu-1) \neq 0$ , we get that

$$\begin{aligned} u_{123}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda \xi) - \sinh(\lambda \xi)\}} \right)^{-1} - k \right\} \\ v_{123}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda \xi) - \sinh(\lambda \xi)\}} \right)^{-1} - k \right\}^2 \end{aligned} \quad (47)$$

$$\begin{aligned} u_{124}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{\lambda \{ \cosh(\lambda \xi) + \sinh(\lambda \xi) \}}{(\nu-1) \{k + \cosh(\lambda \xi) + \sinh(\lambda \xi)\}} \right)^{-1} - k \right\} \\ v_{124}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{\lambda \{ \cosh(\lambda \xi) + \sinh(\lambda \xi) \}}{(\nu-1) \{k + \cosh(\lambda \xi) + \sinh(\lambda \xi)\}} \right)^{-1} - k \right\}^2 \end{aligned} \quad (48)$$

when  $(\nu-1) \neq 0$  and  $\lambda=\mu=0$ , we get that

$$\begin{aligned} u_{125}(\xi) &= a_{-1} e^{i(px+qy+rt)} \left\{ \left( d - \frac{1}{(\nu-1) \xi + c_1} \right)^{-1} - k \right\} \\ v_{125}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \left\{ \left( d - \frac{1}{(\nu-1) \xi + c_1} \right)^{-1} - k \right\}^2 \end{aligned} \quad (49)$$

Again, by use of (20) and the solutions  $G(\xi)$  of Eq. (6), the travelling wave solutions of the Maccari system are obtained in the following form:

$$\begin{aligned} u_2(\xi) &= a_{-1} \exp\{i(px+qy+rt)\} \times \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right)^{-1} \\ v_2(\xi) &= -\frac{a_{-1}^2}{(c+1)} \times \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right)^{-2} \end{aligned}$$

where,  $\xi = x + y - \frac{(\lambda^4 + 8\lambda^2\mu - 8\lambda^2\nu - 8\nu^2 a_{-1}^2 - 8a_{-1}^2 + 16a_{-1}^2\nu + 16\mu^2\nu^2 + 16\mu^2 - 32\mu^2\nu)}{(\lambda^4 + 8\lambda^2\mu - 8\lambda^2\nu + 16\mu^2\nu^2 + 16\mu^2 - 32\mu^2\nu)} t$ ,

$r = 2\mu\nu - \frac{\lambda^2}{2} + p^2 - 2\mu$  and  $a_{-1}, \mu, \nu, \lambda, p$  and  $q$  are arbitrary constants.

when  $\Omega = \lambda^2 - 4\mu\nu + 4\mu > 0$  and  $\lambda(\nu-1) \neq 0$  (or  $\mu(\nu-1) \neq 0$ ),

$$\begin{aligned} u_{21}(\xi) &= a_{-1} \exp\{i(px+qy+rt)\} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-1} \\ v_{21}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-2} \end{aligned} \quad (50)$$

$$\begin{aligned} u_{22}(\xi) &= a_{-1} \exp\{i(px+qy+rt)\} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \coth\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-1} \\ v_{22}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \coth\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-2} \end{aligned} \quad (51)$$

Other families are ignored for convenience. when  $\Omega = \lambda^2 - 4\mu\nu + 4\mu < 0$  and  $\lambda(\nu-1) \neq 0$  (or  $\mu(\nu-1) \neq 0$ ),

$$\begin{aligned} u_{212}(\xi) &= a_{-1} \exp\{i(px+qy+rt)\} \times \left( \frac{\lambda}{2(\nu-1)} + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-1} \\ v_{212}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \times \left( \frac{\lambda}{2(\nu-1)} + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-2} \end{aligned} \quad (52)$$

$$\begin{aligned} u_{213}(\xi) &= a_{-1} \exp\{i(px+qy+rt)\} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{-\Omega} \cot\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-1} \\ v_{213}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{-\Omega} \cot\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-2} \end{aligned} \quad (53)$$

Other families are ignored for convenience.

when  $\mu=0$  and  $\lambda(\nu-1) \neq 0$ ,

$$\begin{aligned}
 u_{2\ 23}(\xi) &= a_{-1} \exp\{i(px + qy + rt)\} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda \xi) - \sinh(\lambda \xi)\}} \right)^{-1} \\
 v_{2\ 23}(\xi) &= -\frac{a_{-1}^2}{(c+1)} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda \xi) - \sinh(\lambda \xi)\}} \right)^{-2}
 \end{aligned} \tag{54}$$

Other families are ignored for convenience.

Again, by use of (21) and the solutions  $G(\xi)$  of Eq. (6), the travelling wave solutions of the Maccari system are obtained in the following form:

$$\begin{aligned}
 u_3(\xi) &= \alpha_1 \exp\{i(px + qy + rt)\} \times \{k_1 + (d + (G'/G))\} \\
 v_3(\xi) &= -\frac{\alpha_1^2}{(c+1)} \times \{k_1 + (d + (G'/G))\}^2
 \end{aligned}$$

where,  $\xi = x + y + \frac{a_1^2 - 2 + 4\nu - 2\nu^2}{2(\nu^2 + 1 - 2\nu)} t$ ,  $k_1 = -(2\nu d - \lambda - 2d)/2(\nu - 1)$ ,  $r = 2\mu\nu - \lambda^2/2 + p^2 - 2\mu$  and  $a_1, \mu, \nu$ ,  $\lambda, p$  and  $q$  are arbitrary constants. when  $\Omega = \lambda^2 - 4\mu\nu + 4\mu > 0$  and  $\lambda(\nu - 1) \neq 0$  (or  $\mu(\nu - 1) \neq 0$ ),

$$\begin{aligned}
 u_{3\ 1}(\xi) &= \alpha_1 \exp\{i(px + qy + rt)\} \times \left\{ k_1 + \left( d - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega} \xi\right) \right) \right) \right\} \\
 v_{3\ 1}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \times \left\{ k_1 + \left( d - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega} \xi\right) \right) \right) \right\}^2
 \end{aligned} \tag{55}$$

Other families are ignored for convenience. when  $\Omega = \lambda^2 - 4\mu\nu + 4\mu < 0$  and  $\lambda(\nu - 1) \neq 0$  (or  $\mu(\nu - 1) \neq 0$ ),

$$\begin{aligned}
 u_{3\ 12}(\xi) &= \alpha_1 \exp\{i(px + qy + rt)\} \times \left\{ k_1 + \left( d + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega} \xi\right) \right) \right) \right\} \\
 v_{3\ 12}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \times \left\{ k_1 + \left( d + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega} \xi\right) \right) \right) \right\}^2
 \end{aligned} \tag{56}$$

Other families are ignored for convenience. when  $\mu = 0$  and  $\lambda(\nu - 1) \neq 0$ ,

$$\begin{aligned}
 u_{3\ 23}(\xi) &= \alpha_1 \exp\{i(px + qy + rt)\} \times \left\{ k_1 + \left( d - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda \xi) - \sinh(\lambda \xi)\}} \right) \right\} \\
 v_{3\ 23}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \times \left\{ k_1 + \left( d - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda \xi) - \sinh(\lambda \xi)\}} \right) \right\}^2
 \end{aligned} \tag{57}$$

Other families are ignored for convenience. when  $(\nu - 1) \neq 0$  and  $\lambda = \mu = 0$ ,

$$\begin{aligned}
 u_{3\ 25}(\xi) &= \alpha_1 \exp\{i(px + qy + rt)\} \times \left\{ k_1 + \left( d - \frac{1}{(\nu-1) \xi + c_1} \right) \right\} \\
 v_{3\ 25}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \times \left\{ k_1 + \left( d - \frac{1}{(\nu-1) \xi + c_1} \right) \right\}^2
 \end{aligned} \tag{58}$$

Again, by use of (22) and the solutions  $G(\xi)$  of Eq. (6), the travelling wave solutions of the Maccari system are obtained in the following form:

$$\begin{aligned}
 u_4(\xi) &= \alpha_1 e^{i(px + qy + rt)} \times \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right)^{-1} + \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right) \right\} \\
 v_4(\xi) &= -\frac{\alpha_1^2}{(c+1)} \times \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right)^{-1} + \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right) \right\}^2
 \end{aligned}$$

where,  $\xi = x + y + \frac{a_1^2 - 2 + 4\nu - 2\nu^2}{2(\nu^2 + 1 - 2\nu)} t$ ,  $r = 4\mu + p^2 + \lambda^2 - 4\mu\nu$  and  $a_1, \mu, \nu$ ,  $\lambda, p$  and  $q$  are arbitrary constants. when  $\Omega = \lambda^2 - 4\mu\nu + 4\mu > 0$  and  $\lambda(\nu - 1) \neq 0$  (or  $\mu(\nu - 1) \neq 0$ ),



$$\begin{aligned}
u_{4_1}(\xi) &= \alpha_1 e^{i(px+qy+rt)} \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right) \right\} \\
v_{4_1}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right) \right\}^2
\end{aligned} \tag{59}$$

Other families are ignored for convenience.

when  $\Omega = \lambda^2 - 4\mu\nu + 4\mu < 0$  and  $\lambda(\nu-1) \neq 0$  (or  $\mu(\nu-1) \neq 0$ ),

$$\begin{aligned}
u_{4_{12}}(\xi) &= \alpha_1 e^{i(px+qy+rt)} \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right) \right\} \\
v_{4_{12}}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right) \right\}^2
\end{aligned} \tag{60}$$

Other families are ignored for convenience.

when  $\mu=0$  and  $\lambda(\nu-1) \neq 0$ ,

$$\begin{aligned}
u_{4_{23}}(\xi) &= \alpha_1 e^{i(px+qy+rt)} \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda\xi) - \sinh(\lambda\xi)\}} \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda\xi) - \sinh(\lambda\xi)\}} \right) \right\} \\
v_{4_{23}}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda\xi) - \sinh(\lambda\xi)\}} \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda\xi) - \sinh(\lambda\xi)\}} \right) \right\}^2
\end{aligned} \tag{61}$$

Other families are ignored for convenience. when  $(\nu-1) \neq 0$  and  $\lambda=\mu=0$ ,

$$\begin{aligned}
u_{4_{25}}(\xi) &= \alpha_1 e^{i(px+qy+rt)} \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{(\nu-1) \xi + c_1} \right)^{-1} + \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{(\nu-1) \xi + c_1} \right) \right\} \\
v_{4_{25}}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \left\{ \frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{(\nu-1) \xi + c_1} \right)^{-1} + \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{(\nu-1) \xi + c_1} \right) \right\}^2
\end{aligned} \tag{62}$$

Finally, by use of (23) and the solutions  $G(\xi)$  of Eq. (6), the travelling wave solutions of the Maccari system are obtained in the following form:

$$\begin{aligned}
u_5(\xi) &= \alpha_1 e^{i(px+qy+rt)} \times \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right)^{-1} + \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right) \right\} \\
v_5(\xi) &= -\frac{\alpha_1^2}{(c+1)} \times \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right)^{-1} + \left( \frac{\lambda}{2(\nu-1)} + (G'/G) \right) \right\}^2
\end{aligned}$$

where,  $\xi = x + y + \frac{\alpha_1^2 - 2 + 4\nu - 2\nu^2}{2(\nu^2 + 1 - 2\nu)} t$ ,  $r = -8\mu + p^2 - 2\lambda^2 + 8\mu\nu$  and  $\alpha_1, \mu, \nu, \lambda, p$  and  $q$  are arbitrary constants.  
when  $\Omega = \lambda^2 - 4\mu\nu + 4\mu > 0$  and  $\lambda(\nu-1) \neq 0$  (or  $\mu(\nu-1) \neq 0$ ),

$$\begin{aligned}
u_{51}(\xi) &= \alpha_1 e^{i(px+qy+rt)} \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right) \right\} \\
v_{51}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{2(\nu-1)} \left( \lambda + \sqrt{\Omega} \tanh\left(\frac{1}{2}\sqrt{\Omega}\xi\right) \right) \right)^2 \right\}
\end{aligned} \tag{63}$$

Other families are ignored for convenience.

when  $\Omega = \lambda^2 - 4\mu\nu + 4\mu < 0$  and  $\lambda(\nu-1) \neq 0$  (or  $\mu(\nu-1) \neq 0$ ),

$$\begin{aligned}
u_{512}(\xi) &= \alpha_1 e^{i(px+qy+rt)} \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \times \left( \frac{\lambda}{2(\nu-1)} + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right) \right\} \\
v_{512}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \times \left( \frac{\lambda}{2(\nu-1)} + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} + \frac{1}{2(\nu-1)} \left( -\lambda + \sqrt{-\Omega} \tan\left(\frac{1}{2}\sqrt{-\Omega}\xi\right) \right) \right)^2 \right\}
\end{aligned} \tag{64}$$

Other families are ignored for convenience.

when  $\mu=0$  and  $\lambda(\nu-1) \neq 0$ ,

$$\begin{aligned}
u_{523}(\xi) &= \alpha_1 e^{i(px+qy+rt)} \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda\xi) - \sinh(\lambda\xi)\}} \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda\xi) - \sinh(\lambda\xi)\}} \right) \right\} \\
v_{523}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \times \left( \frac{\lambda}{2(\nu-1)} - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda\xi) - \sinh(\lambda\xi)\}} \right)^{-1} \right. \\
&\quad \left. + \left( \frac{\lambda}{2(\nu-1)} - \frac{\lambda k}{(\nu-1) \{k + \cosh(\lambda\xi) - \sinh(\lambda\xi)\}} \right)^2 \right\}
\end{aligned} \tag{65}$$

Other families are ignored for convenience.

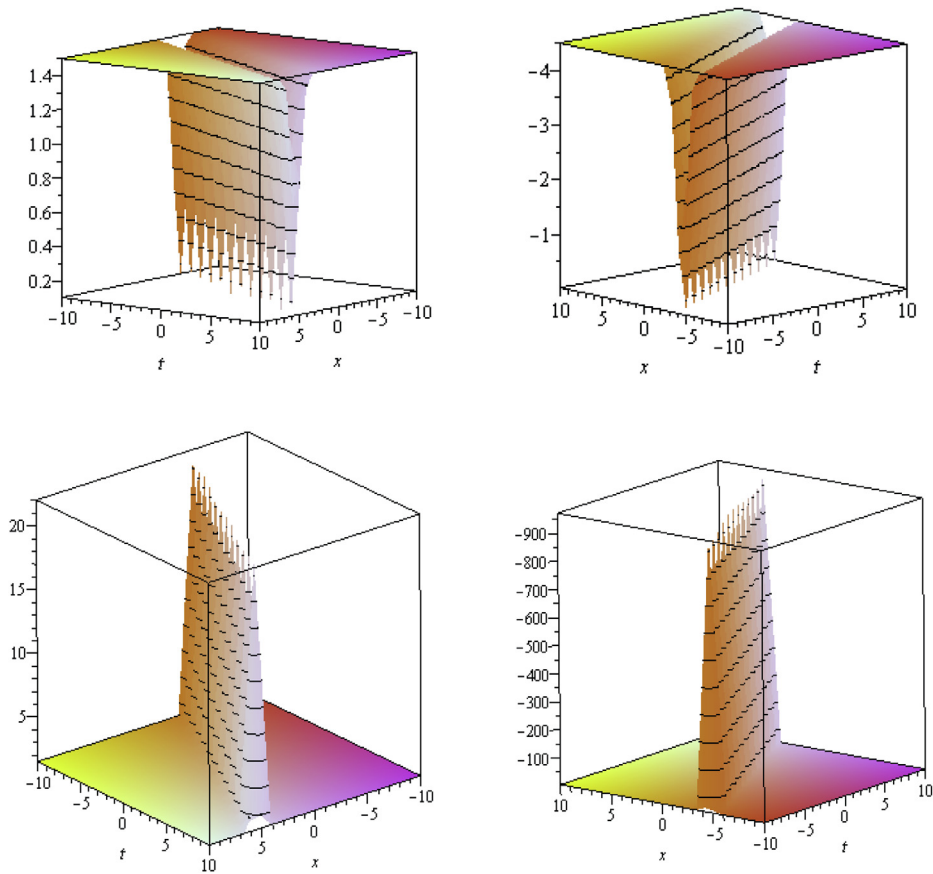
when  $(\nu-1) \neq 0$  and  $\lambda=\mu=0$ ,

$$\begin{aligned}
u_{525}(\xi) &= \alpha_1 e^{i(px+qy+rt)} \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{(\nu-1)\xi + c_1} \right)^{-1} + \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{(\nu-1)\xi + c_1} \right) \right\} \\
v_{525}(\xi) &= -\frac{\alpha_1^2}{(c+1)} \left\{ -\frac{(4\mu\nu - \lambda^2 - 4\mu)}{4(\nu-1)^2} \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{(\nu-1)\xi + c_1} \right)^{-1} + \left( \frac{\lambda}{2(\nu-1)} - \frac{1}{(\nu-1)\xi + c_1} \right)^2 \right\}
\end{aligned} \tag{66}$$

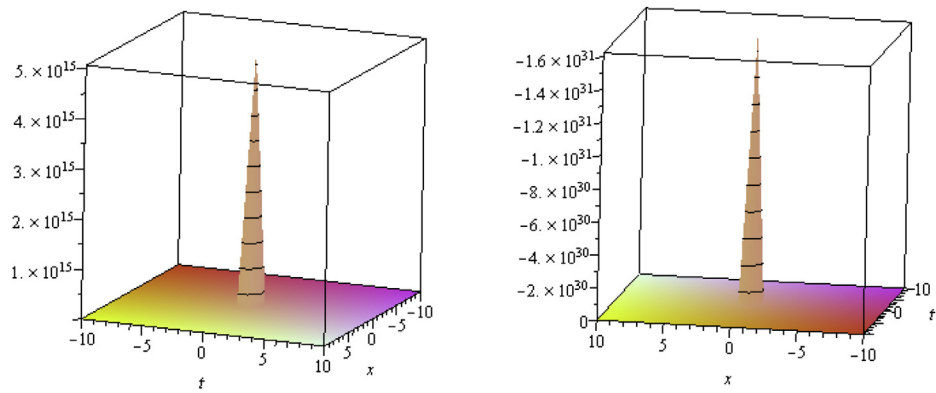
#### 4. Physical explanation

Solitons are everywhere in the nature. Solutions  $u_{11}, v_{11}, u_{12}, v_{12}, u_{14}, v_{14}, u_{16}, v_{16}, u_{17}, v_{17}, u_{18}, v_{18}, u_{19}, v_{19}, u_{111}, v_{111}, u_{31}, v_{31}, u_{412}, v_{412}, u_{512}, v_{512}, u_{523}, v_{523}$  and  $u_{525}, v_{525}$  of the Maccari system (8) are described the soliton. Solitons are special kinds of solitary waves. The soliton solution is a specially localized solution,

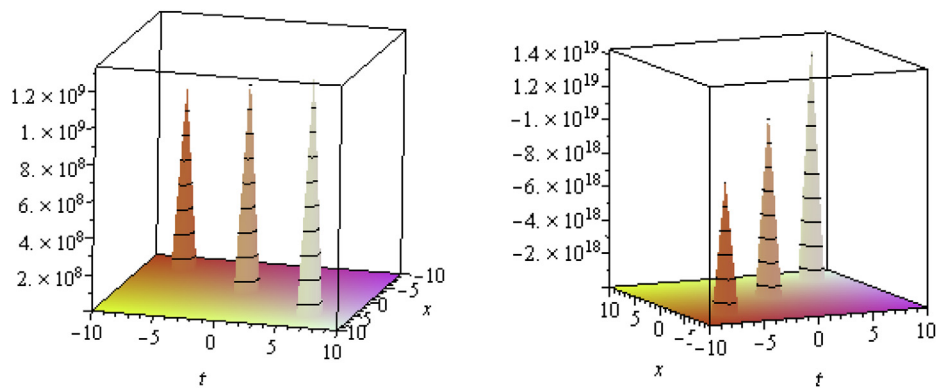
hence  $u'(\xi), u''(\xi), u'''(\xi) \rightarrow 0$  as  $\xi \rightarrow \pm\infty$ . Solitons have a remarkable property– it keeps its identity upon interacting with other solitons. Soliton solutions also give rise to particle-like structures, such as magnetic monopoles etc. Fig. 1 presented the soliton obtained from solutions  $u_{11}, v_{11}, u_{12}$  and  $v_{12}$  with  $p=-c/2, q=1, \lambda=1, \mu=-1, d=1.5, \nu=3, \alpha_{-1}=2$  and  $-10 \leq x, t \leq 10, y=0$  respectively.



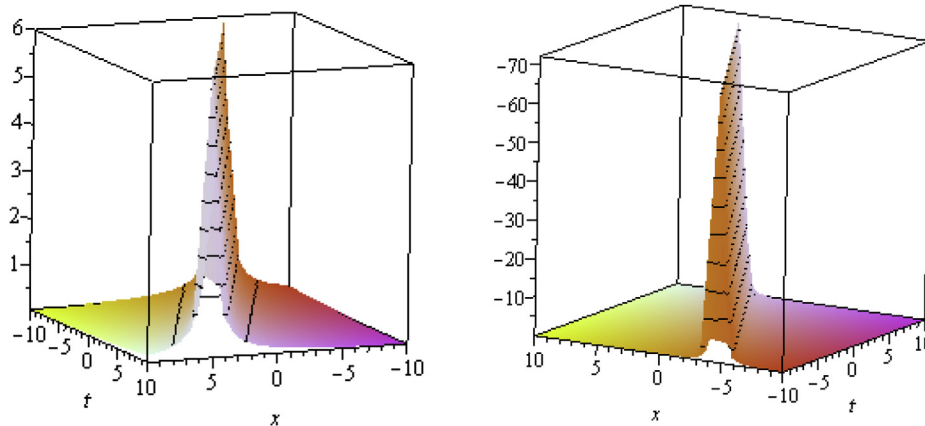
**Fig. 1 – 3D plot of the soliton traveling wave solutions of  $u_{1_1}$ ,  $v_{1_1}$ ,  $u_{1_2}$  and  $v_{1_2}$  with  $-10 \leq x, t \leq 10, y=0$  respectively.**



**Fig. 2 – 3D plot of the single soliton traveling wave solutions of  $u_{2_1}$  and  $v_{2_1}$  with  $-10 \leq x, t \leq 10, y=0$  respectively.**



**Fig. 3 – 3D plot of the multiple soliton traveling wave solutions of  $u_{4_1}$  and  $v_{4_1}$  with  $-10 \leq x, t \leq 10, y=0$  respectively.**



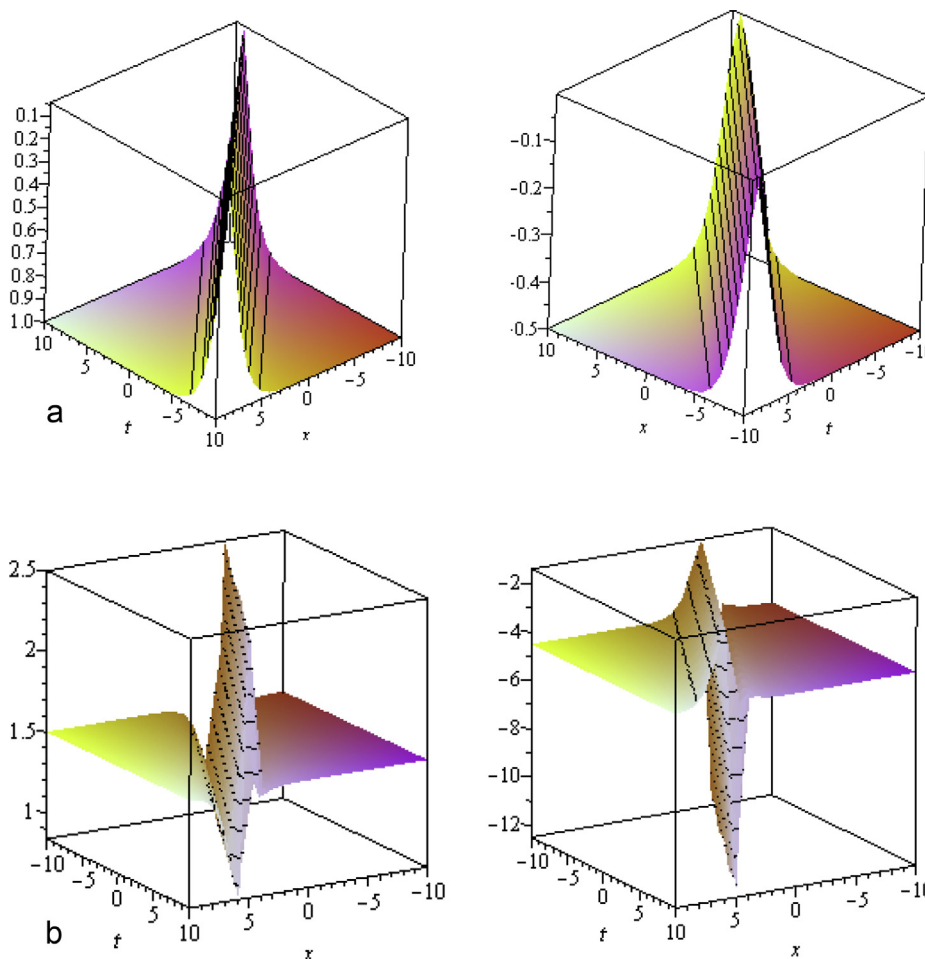
**Fig. 4 – 3D plot of cuspon solution of  $u_{125}$  and  $v_{125}$  with  $-10 \leq x, t \leq 10, y=0$  respectively.**

Solutions of  $u_{21}, v_{21}, u_{212}$  and  $v_{212}$  represents the single soliton solution. In Fig. 2, we have presented the single soliton solutions of  $u_{21}$  and  $v_{21}$  for  $p=-c/2, q=1, \lambda=1, \mu=-1, \nu=3, \alpha_{-1}=2$  with  $-10 \leq x, t \leq 10, y=0$  respectively.

Solutions  $u_{31}, u_{41}, v_{41}, u_{425}, v_{425}, u_{51}, v_{51}, u_{525}$  and  $v_{525}$  describes the multiple soliton solutions. In Fig. 3, we have

presented the multiple soliton solutions of  $u_{41}, v_{41}$  for  $p=-c/2, q=1, \lambda=1, \mu=-1, \nu=3, \alpha_1=1$  with  $-10 \leq x, t \leq 10, y=0$  respectively.

Solutions of  $u_{125}, v_{125}, u_{325}, v_{325}, u_{423}$  and  $v_{423}$  are Cuspon of the Maccari system (8). Cuspons are other forms of solitons where solution exhibits cusps at their crests. Unlike peakons where the derivatives at the peak differ only by a sign, the



**Fig. 5 – a) 3D plot of Bell-shape  $\text{sech}^2$  solitary traveling wave solution of  $u_{123}$  and  $v_{123}$  with  $-10 \leq x, t \leq 10$ . b) 3D plot of the singular Kink traveling wave solution of  $u_{15}$  and  $v_{15}$  with  $-10 \leq x, t \leq 10, y=0$  respectively.**

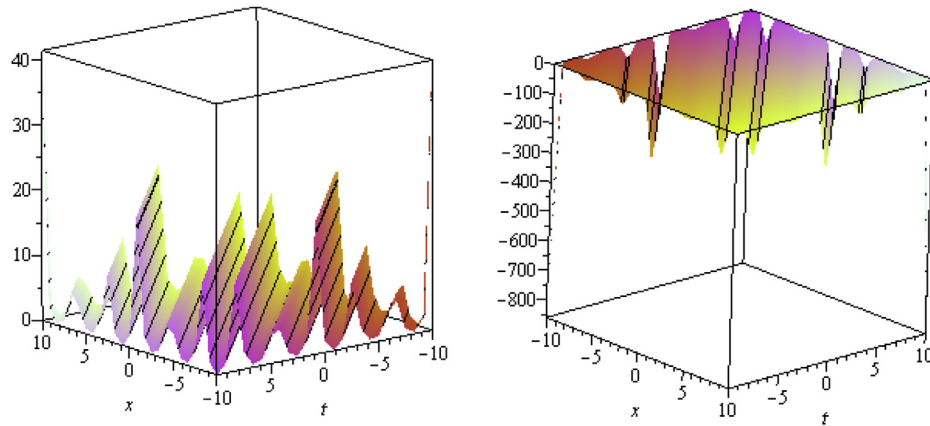


Fig. 6 – 3D plot of the periodic traveling wave solution of  $u_{3/2}$  and  $v_{3/2}$  with  $-10 \leq x, t \leq 10, y=0$ .

derivatives at the jump of a cuspon diverge. The statement is that cuspon can be represented as  $u(x, t) = e^{-|x-a|^{1/2}}$ ,  $n > 1$ . It can easily be shown that  $u_\xi \rightarrow \infty$  at the cusp, and  $u_\xi, u_{\xi\xi} \dots \rightarrow 0$  to distinguish the soliton property. In Fig. 4, we presented the shape of the cuspon, obtain from solutions  $u_{1/25}$  and  $v_{1/25}$  of the Maccari system (8) for  $\lambda=0, \mu=0, \nu=3, d=1, \alpha_{-1}=2, c_1=0.5$  with  $-10 \leq x, t \leq 10, y=0$  respectively.

Solutions  $u_{1/23}, v_{1/23}, u_{1/24}, v_{1/24}, u_{3/23}$  and  $v_{3/23}$  are bell-shape sec  $h^2$  solitary traveling wave solution. The Fig. 5 shows the shape of bell-shaped sec  $h^2$  solitary traveling wave solution (only shows the shape of solution of  $u_{1/23}$  and  $v_{1/23}$  only for  $p=-c/2, p=1, k=5, \lambda=1, \mu=0, \nu=3, d=1, \alpha_{-1}=2$  with  $-10 \leq x, t \leq 10, y=0$ ).

Singular kink solution is another kind of travelling wave solution which comes from infinity as in trigonometry. The solution of  $u_{1/5}, v_{1/5}, u_{2/23}$  and  $v_{2/23}$  comes infinity as in trigonometry, are singular kink solution. The Fig. 5 shows the shape of the exact singular kink-type solution (only shows the shape of solution of  $u_{1/5}, v_{1/5}$  for  $p=-c/2, q=1, \lambda=1, \mu=-1, \nu=3, d=1.5, \alpha_{-1}=2$  with  $-10 \leq x, t \leq 10, y=0$ ).

Solutions of  $u_{3/12}$  and  $v_{3/12}$  represent the exact periodic traveling wave solutions. Periodic solutions are traveling wave solutions that are periodic such as  $\cos(x-t)$ . In Fig. 6, we have

represented the periodic solution of  $u_{3/12}$  and  $v_{3/12}$  for  $p=-c/2, q=1, \lambda=-1, \mu=1, \nu=2, d=1, \alpha_{-1}=1$  with  $-10 \leq x, t \leq 10$ .

Solutions of  $u_{1/12}, v_{1/12}$  to  $u_{1/22}, v_{1/22}$  are represented the exact soliton periodic traveling wave solutions of the Maccari system (8). In Fig. 7, we have presented soliton periodic traveling wave solution of  $u_{1/13}, v_{1/13}$  for  $p=-c/2, q=1, \lambda=-1, \mu=1, \nu=3, d=1, \alpha_{-1}=2$  with  $-10 \leq x, t \leq 10, y=0$  respectively.

Solutions of  $u_{1/3}, v_{1/3}, u_{1/10}$  and  $v_{1/10}$  are represented the exact singular kink periodic traveling wave solutions of the Maccari system (8). In Fig. 8, we have presented singular kink periodic traveling wave solution of  $u_{1/3}$  and  $v_{1/3}$  for  $p=-c/2, q=1, \lambda=1, \mu=-1, \nu=3, d=1.5, \alpha_{-1}=2$  with  $-10 \leq x, t \leq 10, y=0$  respectively.

## 5. Conclusion

In this article, we have investigated a system of complex coupled equation. The novel  $(G'/G)$ -expansion method has been successfully applied to find more general travelling wave solutions of the coupled complex system. From the above solutions, we observe that if we take the particular values for the physical parameters, then these solutions are identical with the some particular solutions obtained by

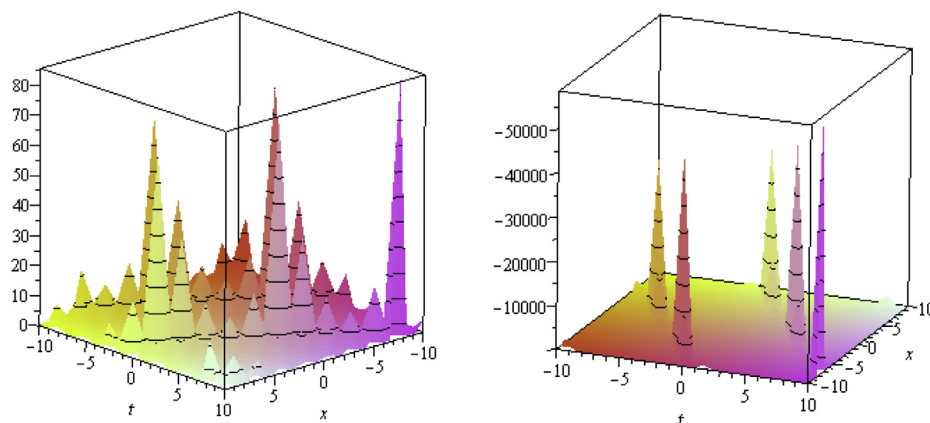


Fig. 7 – 3D plot of the soliton periodic traveling wave solution of  $u_{1/13}$  and  $v_{1/13}$  with  $-10 \leq x, t \leq 10, y=0$ .

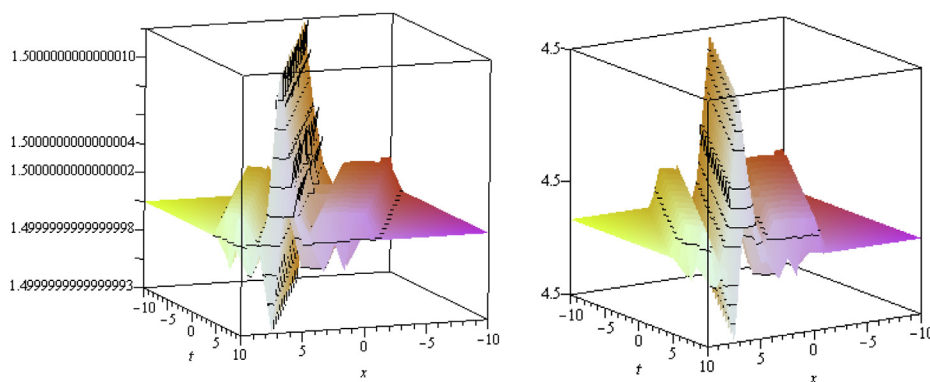


Fig. 8 – 3D plot of the singular kink periodic traveling wave solution of  $u_{13}$ ,  $v_{13}$  with  $-10 \leq x, t \leq 10, y=0$ .

other methods and give us more new exact solutions than the other existing methods. A variety of distinct physical structures such as soliton solution, singular soliton solution, cusp, kink type solution, singular kink solution, periodic solution, bell type solitary wave solution and solitary wave solutions are formally derived. The various types of exact travelling wave solutions provide the mathematical foundation in physics and engineering. Therefore, it is examined that the novel  $(G'/G)$ -expansion method would be a vital mathematical tool for solving not only a single NLEEs but also the coupled NLEEs.

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