Applied Computing and Informatics (2017) 13, 66–78

Saudi Computer Society, King Saud University

Applied Computing and Informatics

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Weak signal acquisition enhancement in software GPS receivers – Pre-filtering combined post-correlation detection approach

G. Arul Elango [a,*], G.F. Sudha [a], Bastin Francis [b]

[a] *Department of Electronics and Communication Engineering, Pondicherry Engineering College, Pondicherry, India*
[b] *Tata Consultancy Services Limited, Bangalore, India*

Received 27 April 2014; revised 30 September 2014; accepted 28 October 2014
Available online 6 November 2014

KEYWORDS

GPS;
Singular Spectral Analysis;
Weak GPS signal acquisition

Abstract The Civilian Global Positioning System (GPS) receivers often encounter problems of interference and noise which degrade the receiver performance. The conventional methods of parallel code phase search acquisition with coherent, non-coherent and differential coherent detection for weak signal acquisition fail to enhance the signal for all conditions especially, when the Carrier to Noise ratio (C/N_0) falls below 15 dB-Hz. Hence, the GPS receiver has to employ sophisticated techniques to excise the noise and to improve the Signal-to-Noise Ratio (SNR) of the signal for further processing. In this paper, a pre-filtering technique of reduced rank Singular Spectral Analysis (SSA) is proposed for noise excision and is processed through coherent, non-coherent and differential detection postcorrelation methods to retrieve the signal embedded in noise. Monte Carlo simulations carried out to examine the acquisition sensitivity at various power levels with the different postcorrelation approaches indicate that the SSA combined with differential detection approach provides a significant performance improvement with lesser mean acquisition time. It has 96% probability of detection at a worst signal power level of −159 dBm (i.e. C/N_0 15 dB-Hz), compared to other conventional methods.

1. Introduction

Global Positioning System satellites use spread spectrum type of 'L' band carrier signal 'L1' with carrier frequency 1575.42 MHz and 'L2' with carrier frequency 1227.6 MHz which are modulated on 50-bps data stream. This is spread with a pseudorandom code called Coarse and Acquisition (C/A) code consisting of a 1023 chip sequence having a period of 1 ms and a chip rate of 1.023 MHz [1]. In comparison with traditional hardware GPS receivers, presently, software based

* Corresponding author.
E-mail addresses: arulelango2012@gmail.com (G. Arul Elango), gfsudha@pec.edu (G.F. Sudha), bastinfrancis@gmail.com (B. Francis).
Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.aci.2014.10.002

GPS receivers are gaining popularity not only for their reconfigurability [2] but also provide an excellent research tool for investigating and improving the receiver performance in a wide range of conditions. Also software based GPS receivers allow flexibility in dealing with noise excision. The exploitation of spectrum transforms and mathematical tools are more feasible in software and faster performance is achieved [3]. The signal acquired by a GPS receiver is deteriorated by interference and receiver noise giving false information about the visibility of satellite and is unable to provide a precise position of the user. This paper proposes an approach for signal acquisition enhancement based on software approach for a L1 GPS receiver. The typical C/N_0 value of an ideal GPS receiver ranges from 37 to 45 dB-Hz. Under noisy conditions, the GPS signal needs to be acquired with the power level ranging from -160 dBW to -200 dBW [4]. In tracking stage of ideal GPS receiver, signal strength up to 25 dB-Hz is allowed but acquisition is limited to about 35 dB-Hz [5], so acquisition sensitivity is more compared to tracking.

A variety of algorithms have been proposed for estimating and acquisition of the weak signal. The wavelet de-noising method coupled with differential coherent integration (DFC) proposed by Lei et al. [6] achieved a worst case improvement of 2 dB gain within 10 ms integration period at -176 dBW (i.e. 28 dB-Hz) compared to traditional DFC algorithm. An adapted acquisition algorithm depicted by Tian et al., based on DFC with coherent integration time of 1 ms with 60 DFC accumulation is able to detect the weak signal up to 30 dB-Hz. By increasing the integration time to 5 ms and 10 ms, the extremely feeble signal of 22 and 20 dB-Hz is detected with 90% detection [7]. The indoor and outdoor environment acquisition capability of DFC algorithm is mentioned by Ba et al. [4] with signal strength of -177 dBW (indoor) and -155 dBW (outdoor) acquired with 179 and 1 ms integration times. Yang and Tian [8] compared various weak signal acquisition algorithms and reported coherent correlation with DFC has larger output SNR 3–5 dB gain compared to other works but there seems a high complexity in implementation. The inertial navigation system (INS) aided acquisition algorithm can successfully capture the signal with C/N_0 as low as -150 dBm [9] while a Block Acquisition Method for C/N_0 as low as 21 dB-Hz [10]. A new peak finding algorithm [11] was able to locate the peak location accurately and provides faster performance in a software based acquisition for a C/N_0 of 19 dB-Hz. Similarly, a block average model based on the accumulation of synchronized and phase corrected signal blocks of Fast Fourier Transform (FFT) was found suitable for Gaussian noise, narrow band interference and weak signals [12]. Likewise, a Signal Existence Verification Process was proposed to detect and subsequently verify low power-received GPS signals even if the estimated code delay information had an offset of half chip from the correct one based on the time–frequency representation [13]. More recently, full bit acquisition algorithm for software GPS receiver in a weak signal environment was found capable of improving the C/N_0 by 2 dB-Hz when the noncoherent integration time equals 40 ms [14]. The earlier reported weakest signal that can be detected in 4 s of data using full bits method is with a C/N_0 of 19 dB-Hz [10]. Signal acquisition in the range of 15 dB-Hz can be accomplished by increasing the integration time up to 20 s. But when the C/N_0 falls below 15 dB-Hz, performing acquisition with coherent detection beyond 10 ms does not work due to data transition problem in the navigation data decoding. Performing noncoherent integration also goes in vain, since extending the integration time to 20 s to fulfill the required Processing Gain (G_p) to more than 20 dB, delays the time to first position fix during cold start conditions.

The present study investigates noise using Singular Spectral Analysis (SSA), as a pre-filtering approach which provides better weak signal detection without changing the traditional acquisition methods but with reduced mean acquisition time. The SSA is used as it is a powerful technique for noise reduction irrespective of the environment whether it is stationary or mobile, linear or non-linear, Gaussian or non-Gaussian and it does not require prior assumptions about the data [15].

When the GPS signal is corrupted by an additive white Gaussian noise (AWGN), the worst case signal power level lies in the range of -160 dBm. In order to boost up the processing gain at lower SNR, different postcorrelation detection techniques are used. The traditional postcorrelation detection methods use coherent, non-coherent and differential detection approaches. To address the conventional postcorrelation detection techniques failing to recover a noisy GPS signal as low as -159 dBm (C/No of 15 dB-Hz), the present study employs postcorrelation techniques, combined with SSA for weak signal acquisition enhancement (Fig. 1). In the presence of noise, the GPS trajectory matrix becomes full rank. When the rank is reduced to the minimum extent using reduced rank SVD process it decomposes the data matrix into signal subspace and noise subspace. Keeping only the signal subspace, the data matrix is said to be noise free. The residue matrix (noise) is found by subtracting the reduced rank matrix from the full rank trajectory matrix. However this rank reduction destroys the structure (Toeplitz/Hankel) of the matrix, thus demanding a step to re-establish the special structure of the matrix. The rank reduction process is repeated k times, followed by structure restoration till the minimum Frobenius norm is achieved. The reduced rank SVD is known as Singular Spectral Analysis (SSA) if the number of iteration is limited to 1, i.e., $k = 1$ [16].

The traditional postcorrelation detection techniques fail to recover a noisy GPS signal of signal strength as low as -159 dBm (C/No of 15 dB-Hz), therefore present investigation combines postcorrelation techniques with SSA for weak signal acquisition enhancement. The effectiveness of the SSA combined with differential coherent method is compared to other conventional methods in terms of processing gain and mean acquisition time for various power levels.

2. Simulation results and analyses

The received GPS 'L1' signal from the Radio Frequency (RF) front end is converted to Intermediate Frequency (IF) of 4.1304 MHz and sampled at a frequency of 16.367 MHz. For 1 ms of data, the number of samples can be found as 1/1000 of the sampling frequency i.e., $16.367 \times 10^6/1000 = 16367$ samples with a single bit resolution. To ensure good probability of successful acquisition, we have confined the N value as 16,367 samples. The digitized data obtained after analog to digital conversion are given as the input to the acquisition process to determine the code phase and Doppler frequency of visible satellites. The simulation parameters are mentioned in Table 1.

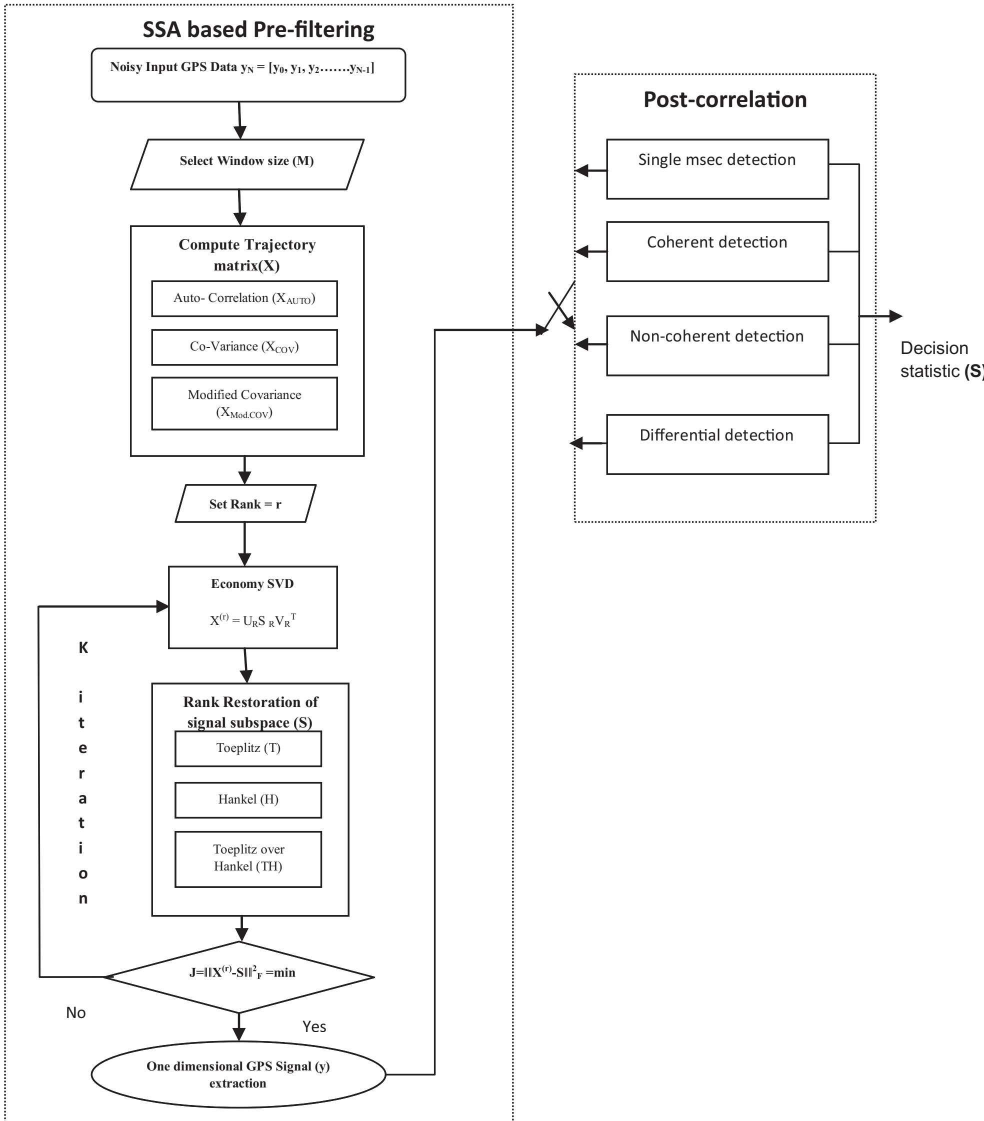


Figure 1 Flow chart for proposed pre-filtering combined postcorrelation detection in GPS receivers.

2.1. Choice of window size (M) and rank (r)

The choice of setting window size (M) is based on the W-correlation value $\rho^w = \frac{(x^{(r)}, x^{(n)})_w}{\|x^{(r)}\|\|x^{(n)}\|}$, where $(x^{(r)}, x^{(n)})_w = \sum_{i=1}^{n} w_i x_i^{(r)} x_i^{(n)}$. If $x^{(r)}, x^{(n)}$ are approximately separable and $\rho^w \approx 0$. Though there is no universal rule existing for the selection of the window length (M), it is generally accepted that $M \leqslant N/2$. For $(\|x_F^{(r)}\|) \gg \|x_F^{(n)}\|)$, a better separation is achieved with larger M. However, too large values of M lead to an undesirable decomposition which in turn yields mixing of residual components. Sometimes small transitions in M reduce the mixing and lead a better separation, hence SSA analysis was repeated several times using different values of M [17].

Table 1 Simulation parameters for GPS signal.	
Parameters	Values
Input sample data length	16,367
IF frequency	4.1304 MHz
Sampling frequency f_s	16.367 MHz
Doppler frequency search	± 10 kHz

At extreme noise level (−159 dBm) the de-noising is based on iterative SSA, initially the SVD is carried out with certain window size ($M1$) which splits the trajectory matrix into signal subspace (X^r) and noise subspace (X^n). The SVD is performed with another window size ($M2$) on noise subspace where it is decomposed into ($X_1{}^r$) and ($X_1{}^n$). This is continued up to several window sizes and finally all the signal subspace components (X^r) + ($X_1{}^r$) + ... (X_n^r) are added. This yields good separation of signals from noise. By repetitive trials, the value of M was chosen to be 13 for weak signal of −150 dBm and 250 for extremely weak signal of −159 dBm.

Normalized singular values for varying ranks are plotted as shown in Fig. 2a for a GPS signal of power level −150 dBm. A noise signal typically decreases with sequence of singular values. In our case a significant drop in a singular value occurs around the rank value of 2 which could be interpreted as a start of noise floor.

Similarly the extreme weak GPS signal of power level −159 dBm is tested and their normalized singular values for varying rank values are plotted in Fig. 2b. A rapid drop in the singular value occurs at the rank value of 2. So we have chosen rank (r) as 1 for both cases of GPS signal. To understand the effect of rank in restoration of weak GPS signals, the noisy GPS signal of power level −150 dBm is applied to the pre-filtering based SSA approach. As per the definition of SSA the number of iteration (k) is limited to 1. The reconstructed GPS signal with rank values of 1, 2 and 3 for three different trajectory matrices, autocorrelation, covariance and modified covariance are plotted in Fig. 3.

The autocorrelation method with rank 1 Hankel structure based restoration provides a better version of the reconstructed signal when compared other methods as shown in Fig. 3b. Hence, this is selected for further processing of GPS signal.

The spectrum for an actual input GPS data at −130 dBm (Fig. 4a) and noisy GPS data with −150 dBm power level (Fig. 4c) for a sampling frequency of 16.367 MHz with an IF of 4.13 MHz is shown in Fig. 4b and d respectively. The results after application of SSA on noisy GPS data are shown in Fig. 4e and f. From this we infer that the shape of the spectrum after SSA is preserved and the center frequency is approximately located around 4.13 MHz. If the power level goes below −159 dBm, i.e., for an extremely weak signal, the pre-filtering alone does not alleviate the detection of visible satellites. Hence further enhancement by postcorrelation analysis is required in the GPS receiver to fulfill the processing gain.

The simulation results for selection of the trajectory matrix depending on the three performance metrics namely Mean Square Error ($\hat{\varepsilon}$), maximum correlation peak and Processing Gain (G_p) are given in Table 7. Considering that the correlation peak is a measure of perfect alignment of the incoming GPS signal with the C/A code, higher correlation peak signifies maximum alignment between incoming signal and the local C/A code replica indicating better acquisition. Processing gain is the measure of total gain obtained after the signal acquisition and indicates how well the trajectory matrix is able to acquire a weak signal. Along with the processing gain G_p and correlation peak, the other performance metric chosen is Mean Square Error. The mean squared error $\hat{\varepsilon}$ between the actual and the estimated signal is given as $\hat{\varepsilon} = \frac{1}{M}\sum_{n=0}^{N-1}|e(n)|^2$.

For the three cases of trajectory matrices, autocorrelation, covariance and modified covariance, the Mean Square Error is given by the following expression

$$\begin{aligned}\hat{\varepsilon}_{\mathrm{aut}} &= \frac{1}{M}\sum_{n=0}^{N-1+M}|e(n)|^2\\ \hat{\varepsilon}_{\mathrm{cov}} &= \frac{1}{M}\sum_{n=M}^{N-1}|e(n)|^2\\ \hat{\varepsilon}_{\mathrm{mod.cov}} &= \frac{1}{M}\sum_{n=M}^{N-1}|e_+(n)|^2 - |e_-(n)|^2\end{aligned} \tag{1}$$

where $e_+(n)$ and $e_-(n)$ are the errors in the computation of upper Hankel and lower Toeplitz matrices respectively.

The results demonstrate that the autocorrelation type of trajectory matrix combined with differential detection in GPS acquisition, achieves the maximum postcorrelation processing gain of 20 dB at signal level of −150 dBm (Table 7) which is more compared to covariance (17.3 dB) and modified covariance (18.2 dB) methods. Similarly, autocorrelation type of trajectory matrix has produced the highest correlation peak. Though Covariance method is widely used in the SSA approach [18], the autocorrelation method has a full version of trajectory matrix that avoids the reacquisition of GPS data by making 5 satellites to be visible. Also, the Minimum Mean Square Error of 0.2346 is achieved by this method compared to others approaches. The perfect alignment of pseudorandom noise (PRN) code with the incoming signal has been achieved for SVN 31 with high G_p. From the simulation results, it is clear that this method is capable of correcting the code phase of satellites. The combination of autocorrelation with differential detection is the best choice among the others to enhance the signal in the worst case SNR level.

2.2. Performance comparison of SSA with other pre-filters

To demonstrate the efficacy of SSA pre-filtering, it is compared with other pre-filters such as Butterworth, Chebyshev and Wavelet techniques [6]. The filters are being tested with an extremely weak GPS signal having SNR of −29 dB, a center frequency of 4.13 MHz and sampling frequency of 16.367 MHz. A Butterworth filter of order 14 and Chebyshev filter of order 5 are designed with a pass band width of 2.2 MHz. For a wavelet based filtering, Haar wavelet with 8 level decomposition is used to de-noise the signal. A comparison of SNRs and achieved gain using the different filters shows that the Butterworth filter has obtained a gain of 5.8452 dB little more than Chebyshev filter and wavelet de-noising is able to get only an additional gain of 0.3438 dB. In contrast, the SSA pre-filter outperforms other methods with an impressive amount of 23 dB gain (see Table 2).

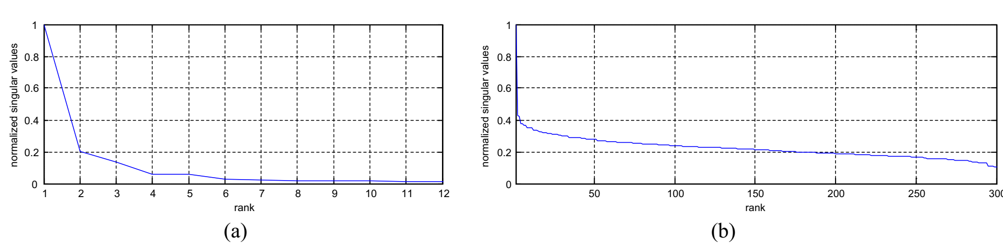


Figure 2 Selection of singular values (a) GPS signal of −150 dBm and (b) extremely weak signal of −159 dBm.

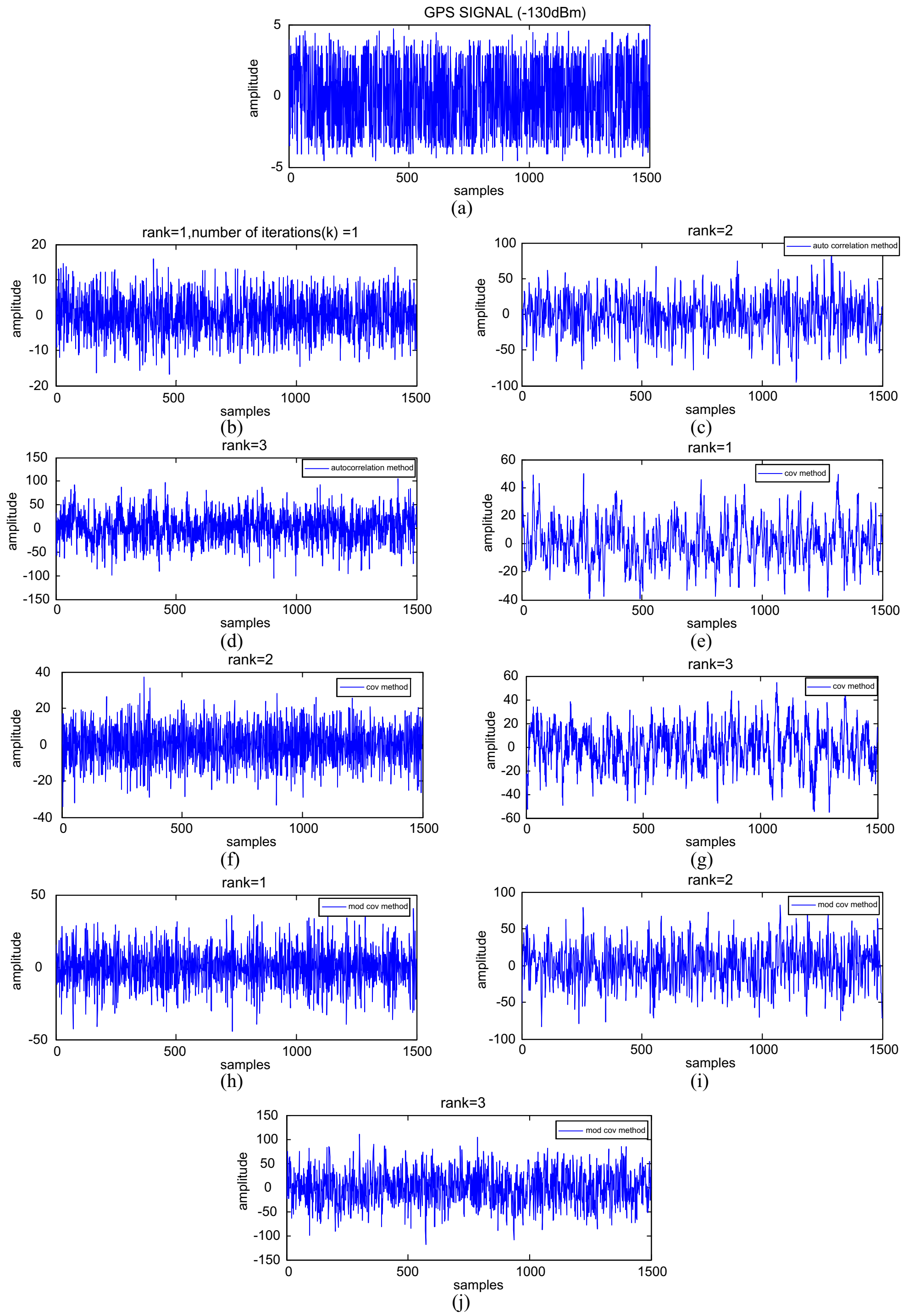


Figure 3 Reconstructed GPS signal for rank values 1, 2 and 3 (a) actual GPS signal. (b–d) – autocorrelation method (e–g) – covariance method (h–j) – modified covariance method.

2.3. Parallel code phase search acquisition scheme for single msec data

A weak GPS signal with SNR of -150 dBm (C/N_0 of 24 dB-Hz) is tested with the circular correlation based traditional FFT frequency domain technique. The Parallel Code Phase Peak search algorithm for determining the threshold is being used to determine the dominant peak [11] with the assumption that if the difference between maximum value and second maximum value is large enough, the probability of false alarm will be decreased.

The threshold condition for peak search is given in Eq. (2) [11]

$$\left.\begin{aligned} \max[S(n)] - \mathrm{mean}[S(n)] > VT_1(0.3) \\ \max[S(n)] - 2^{\mathrm{nd}}\max[S(n)] > VT_2(0.15) \end{aligned}\right\} \quad (2)$$

As per the conditions mentioned in Eq. (2), the difference between maximum and mean peak values and difference

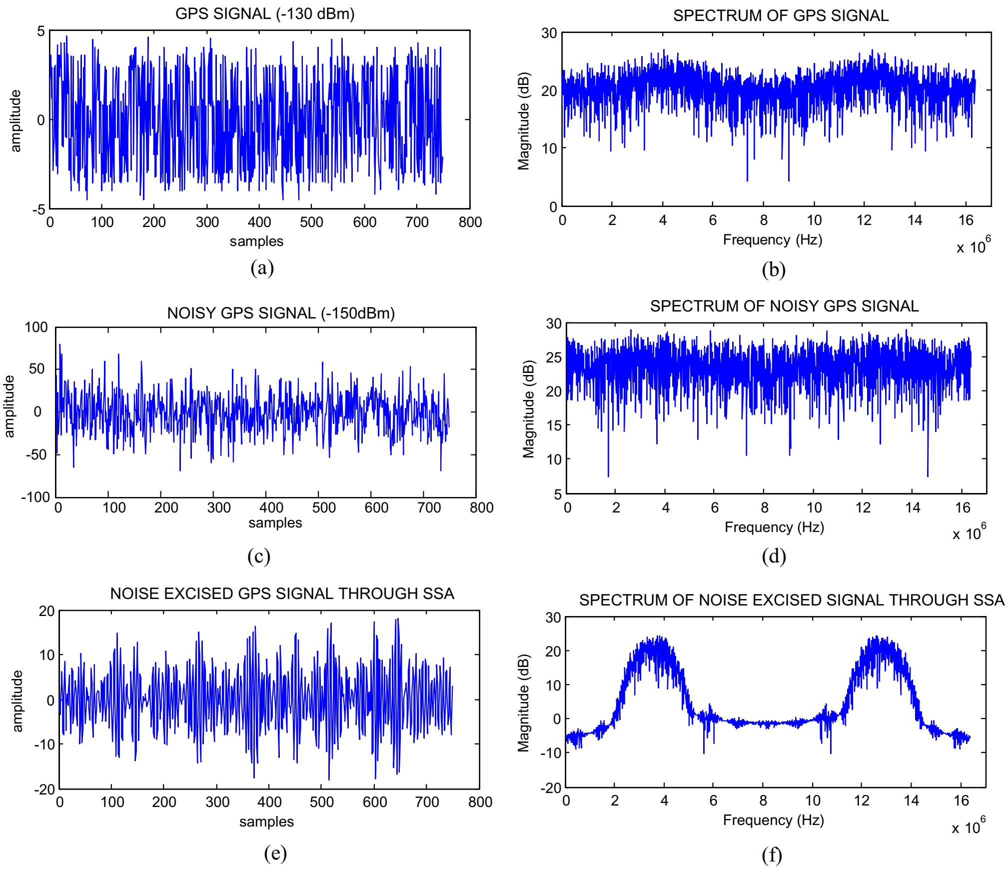


Figure 4 Simulation results of SSA on noisy GPS signal. (a) GPS signal at −130 dBm, (b) spectrum of the GPS signal, (c) noisy GPS signal of −150 dBm, (d) spectrum of noisy GPS signal. (e) Noise excised GPS signal and (f) spectrum of noise stripped GPS signal.

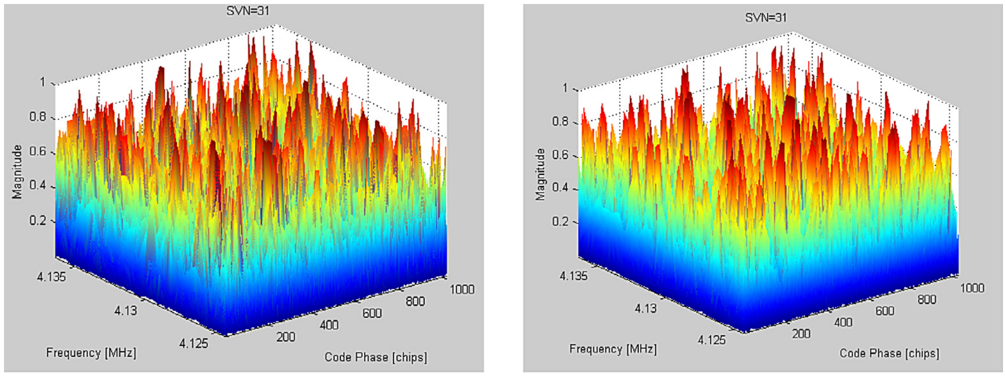
Table 2 Type of filters used for noise excision (SNR of weak signal before filtering = −29 dB).

Pre-filter type used	$\mathrm{SNR(dB)} = 10 * \log\left(\frac{\sum_{n=0}^{N-1} x(n)^2}{\sum_{n=0}^{N-1} (x(n)-\tilde{x}(n))^2}\right)$ After filtering	Gain (dB)
Butterworth filter	−23.1548	5.8452
Chebyshev filter	−23.3740	5.6260
Wavelet de-noising	−22.6816	6.1894
Singular spectral analysis	−6.3529	22.7335

between 1st maximum and 2nd maximum peak values should be greater than 0.3 and 0.15. In order to get a minimum probability of false alarm 10^{-2}, the thresholds VT_1 and VT_2 values are chosen as 0.99 and 0.25 respectively. In case, the correlation output crosses the above predetermined threshold with only one period time of C/A code, the GPS receiver cannot produce an SNR gain to make a reliable detection. It is observed that the weak signal detection with single msec data does not satisfy the condition. As it fails to give a maximum value of the decision statistic (S) and hence no satellites are visible. The simulation results of testing one msec data for Satellite Vehicle Number (SVN) 31 indicate that both with SSA and without SSA fail to excise the noise (Fig. 5a and b). For reasonable acquisition of signals around 35 dB-Hz with nominal amount of false alarm [5], single msec data is sufficient, still in order to meet the specified threshold values for weak signal (< 35 dB-Hz), longer duration data are needed.

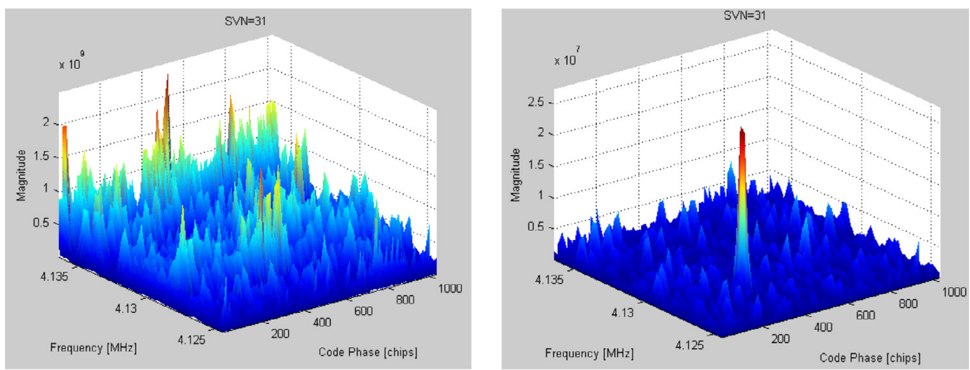
2.4. SSA combined coherent detection based post-correlation approach

The coherent detection method is tested for −150 dBm power level at 4 ms integration time. The method is unable to satisfy the maximum value of the decision statistic (S) and poor correlation performance of SVN 31 (Fig. 6a) and the data are tested with the SSA method. Among the locally generated 32-PRN codes correlated with the incoming signal, only SVN-31 crosses the peak condition (Fig. 6b) requiring minimum of 4 satellites to find out the user position. Increasing the acquisition data length beyond 10 ms leads to data transition. Longer duration coherent detection increases number of frequency bins to be searched which restricts extending the integration time during weak signal conditions. As coherent integration even when combined with SSA is not able to detect weak signals less than C/N_0 24 dB-Hz, therefore non-coherent integration is next to study.



(a) (b)

Figure 5 Correlation output of SVN-31 tested with 1 ms integration time for input power level of −150 dBm (C/N_0 of 24 dB-Hz). (a) Single msec data and (b) single msec processed using SSA.



(a) (b)

Figure 6 Correlation output of SVN-31 tested with coherent detection (4msec) for input power level of −150 dBm (C/N_0 of 24 dB-Hz). (a) Coherent integration time and (b) SSA combined with coherent integration time.

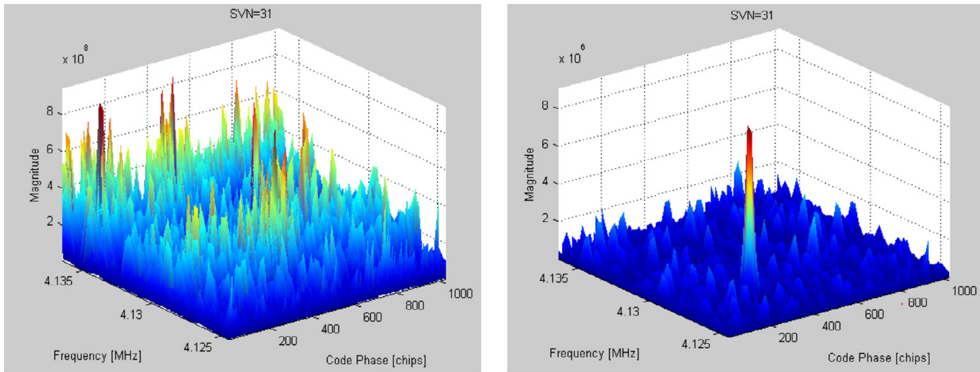
2.5. SSA combined noncoherent detection based post-correlation approach

The −150 dBm GPS data are tested for 40 ms duration noncoherent integration time. Fig. 7.a shows the non-coherent detection result of SVN 31 without SSA where the maximum value of the decision statistic (S) is not arrived, the required threshold conditions, and the signal is declared absent. While testing noncoherent integration with SSA, only SVN-31 is found with dominant peak (Fig. 7b). Studies have demonstrated that the Signal Sensitivity increases roughly with square root of integration time [19] and a 10 ms longer integration time gives only 5 dB increase in sensitivity. Hence it is concluded that 100 ms non-coherent integration correlation i.e. one tenth of second data are required to acquire a signal having C/N_0 up to 24 dB-Hz [5].

2.6. SSA combined differential coherent detection based postcorrelation approach

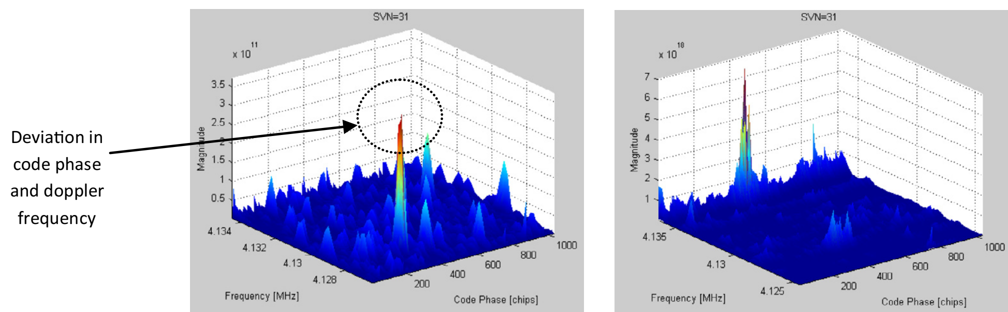
The 16 ms duration noisy data are tested for differential coherent detection approach at −150 dBm power level. The SVN-31 correlation performance is tested and it surpasses the threshold value. But at lower power level (below −159 dBm) the decision statistic (S) does not cross the predetermined threshold values for SVN-31.This is because the noise independent property has capability of removing the squaring loss up to certain SNR values only. On examining Fig. 8a and b it is inferred that differential coherent scheme is able to detect only one visible SVN-31 for SNR level of −150 dBm. When the signal level is weaker (−159 dBm), even an increase in integration time to 100 ms does not make any satellite to be visible.

In the proposed work, the SSA approach is combined with differential detection and five SVNs are visible and the decision threshold conditions have been satisfied without any constraints. Perfect correlation has been achieved and many false secondary peaks owing to noise are eliminated. The five SVNs (2, 17, 26, 27, and 31) meet the threshold condition for the signal range of C/N_0 24 dB-Hz and 15 dB-Hz (Table 3). Fig. 9a–e shows the five SVN's 3-D correlation and their corresponding code phase i.e., perfect alignment with the C/A code samples is plotted in Fig. 9f–j. Table 4 summarizes the comparison of the results of 15 dB-Hz weak GPS signal tested with different detection methods. The proposed method i.e. SSA combined with differential detection performs better in obtaining visible satellites for 16 ms weak GPS data and acquiring 5 SVNs to initiate the tracking process. Even though the differential detection scheme has dominant peak but the threshold condition 1 is not satisfied for all the SVNs. The code phase and Doppler frequency comparison between different detection techniques are listed in Table 5. The data show that the differ-



(a) (b)

Figure 7 Correlation output of SVN-31 tested with non-coherent detection (40 ms) for input power level of −150 dBm (C/N_0 of 24 dB Hz). (a) Non-coherent integration time and (b) SSA combined with Non-coherent integration time.



(a) (b)

Figure 8 Acquisition correlation output of SVN-31 tested with differential detection for input power level of −150 dBm (C/N_0 of 24 dB Hz) and −159 dBm (C/N_0 of 15 dB Hz). (a) 16 ms integration time at −150 dBm power level and (b) 100 ms integration time at −159 dBm power level.

Table 3 Comparison of different detection techniques at two power levels.

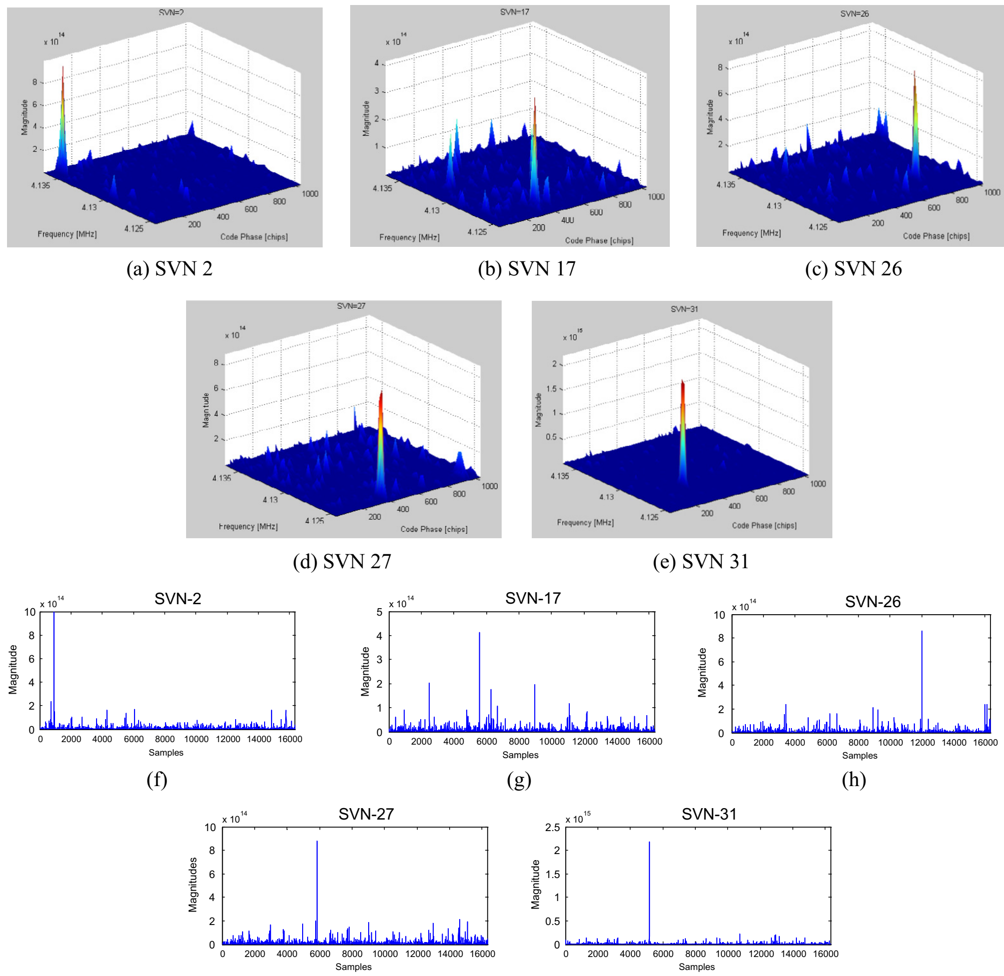
Power level	Peak search condition = Max peak-mean peak > 0.99 and Max peak-secondary peak > 0.25							
	Detection type							
	1 ms data		Coherent post-correlation		Noncoherent post-correlation		Differential post-correlation	
	Without pre-filter	Pre-filter combined	Without pre-filter	Pre-filter combined	Without pre-filter	Pre-filter combined	Without pre-filter	Pre-filter combined
−150 dBm	No visible SVN	No visible SVN	No visible SVN	1 SVN (31)	No visible SVN	1 SVN (31)	1 SVN (31)	5 SVN (2, 17, 26, 27 and 31)
−159 dBm	No visible SVN	No visible SVN	No visible SVN	No visible SVN	No visible SVN	No visible SVN	No visible SVN	5 SVN (2, 17, 26, 27 and 31)

ential coherent detection technique with pre-filtering approach corrects the deviation in code phase of 22.1 chips and Doppler frequency of 164 Hz of the SVN-31 caused by noise at −150 dBm when compared to differential detection.

3. Acquisition sensitivity analysis

3.1. Processing Gain (G_p) estimation in software GPS receiver

The nominal signal strength of a typical C/A code receiver is −130 dBm [20]. The noise floor for 1 kHz is at −144 dBm; thus the corresponding SNR is 14 dB (−130 + 144). The input power level of incoming GPS signal varies from −130 dBm to −159 dBm and the locally generated PRN-31 is correlated with incoming signal for the probability of false alarm (p_{fa}) of 10^{-2}. The probability of detection and the G_p are computed for different integration periods. The Monte Carlo simulation is carried out for 10,000 trials to ensure the effectiveness of the performance comparison between the three detection methods. A weak GPS signal of −140 dBm (34 dB-Hz) i.e. SNR of (−140 + 144) = 4 dB was set as power level. In order to achieve nominal SNR of 14 dB, a gain of 10 dB is required,



(a) SVN 2 (b) SVN 17 (c) SVN 26

(d) SVN 27 (e) SVN 31

(f) (g) (h)

(i) (j)

Figure 9 Correlation outputs of visible SVN tested with GPS data of input power level of −159 dBm (C/N_0 15 dB-Hz) using SSA combined differential coherent approach. (a–e) – 3D search correlation results for SVN 2, 17, 26, 27 and 31 (f–j) – code phase for SVN 2, 17, 26, 27 and 31.

Table 4 Determination of visible SVN based on peak search condition tested for different detection techniques processed through SSA at −159 dBm power level.

SVN	Pre-filter combined coherent				Pre-filter combined non-coherent				Differential [6–8]				Pre-filter combined differential detection			
	Max peak (e^{+07})	2nd max peak (e^{+07})	Avg peak (e^{+07})	S	Max peak (e^{+07})	2nd max peak (e^{+07})	Avg peak (e^{+07})	S	Max peak (e^{+19})	2nd max peak (e^{+18})	Avg Peak (e^{+18})	S	Max peak (e^{+15})	2nd max peak (e^{+14})	Avg peak (e^{+12})	S
2	3.364	0.795	0.183	×	4.205	0.993	0.229	×	0.195	0.265	0.114	×	0.9945	0.555	5.007	√
17	2.166	0.715	0.161	×	2.708	0.894	0.201	×	0.167	1.003	0.126	×	0.4122	0.449	3.808	√
26	3.127	1.545	0.193	×	3.909	1.931	0.242	×	0.208	0.389	0.133	×	0.8596	2.098	5.680	√
27	3.167	1.559	0.228	×	3.959	1.948	0.285	×	2.180	0.087	1.057	×	0.8814	2.135	7.442	√
31	4.983	1.206	0.202	×	6.229	1.508	0.253	×	0.702	3.139	0.302	×	2.182	1.279	6.414	√

S – Whether detection threshold condition satisfied – √ – Yes and × – No.

hence signal must be averaged over a sufficiently long non-coherent integration time (20 ms) to build up the SNR to a reliable level (10 dB) (Fig. 10a). In a similar fashion, the power is varied at different levels and the corresponding probability of detection and the G_p are examined. The detection metrics for the three detection methods have been presented in Table 6 for different values of signal quality. SNR after correlation is also determined with longer integration times. The SNR computation in the software receiver is defined as the ratio between the accumulated and averaged in phase arm ($\hat{I}_{\text{arm}}$) power to the

Table 5 Acquisition output for different detection techniques processed through SSA pre-filtering.

Power level	SVN	Pre-filter combined coherent		Pre-filter combined non-coherent		Differential detection		Pre-filter combined differential detection	
		Code phase (chips)	Doppler frequency (Hz)	Code phase (chips)	Doppler frequency (Hz)	Code phase (chips)	Doppler frequency (Hz)	Code phase (chips)	Doppler frequency
−150 dBm	2	×	×	×	×	×	×	56.5	−4800
	17	×	×	×	×	×	×	347.8	4500
	26	×	×	×	×	×	×	750.4	2700
	27	×	×	×	×	×	×	365.9	5400
	31	322.9	2400	322.9	2400	345	2564	322.9	2400
−159 dBm	2	×	×	×	×	×	×	56.5	−4800
	17	×	×	×	×	×	×	347.8	4500
	26	×	×	×	×	×	×	750.4	2700
	27	×	×	×	×	×	×	365.9	5400
	31	×	×	×	×	×	×	322.9	2400

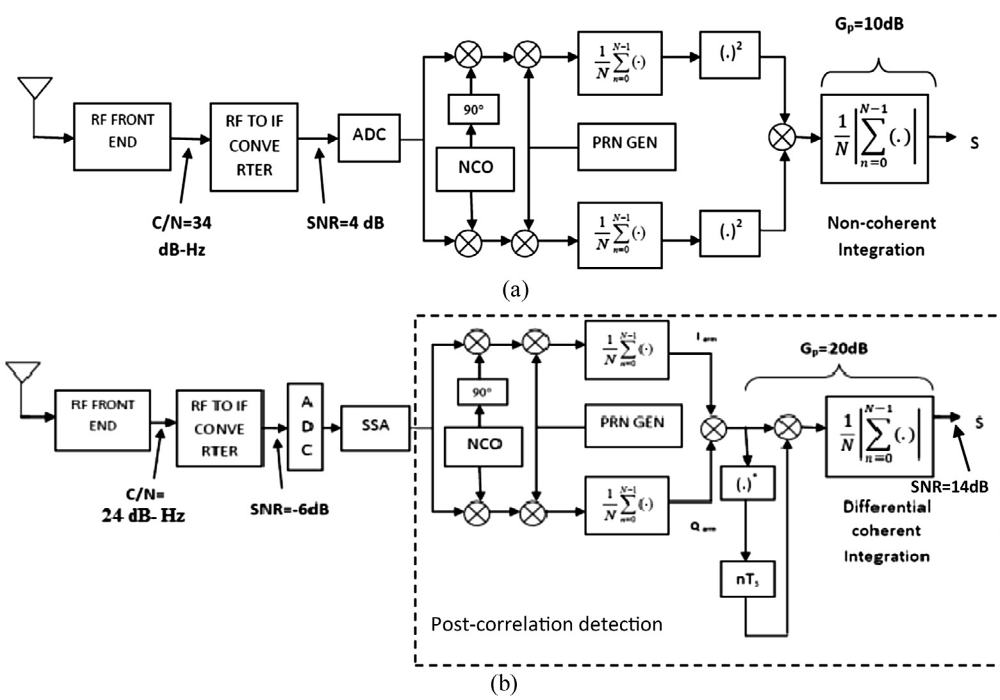


Figure 10 Acquisition sensitivity improvement at different power levels for PRN-31. (a) G_p improvement of 10 dB through 50 ms non-coherent integration at 34 dB-Hz power level and (b) G_p improvement of 20 dB through 16 ms pre-filtering combined differential coherent integration at 24 dB-Hz power level.

accumulated and averaged noise power ($\hat{Q}_{\text{arm}}$) given by Eq. (3) [21]

$$SNR = 10\log_{10}\left(\frac{\hat{I}^2_{\text{arm}}}{2\hat{Q}^2_{\text{arm}}}\right)\text{dB}$$
$$\hat{I}^2_{\text{arm}} = \frac{(f_s\tau)^2 c}{2} \text{ and } \hat{Q}^2_{\text{arm}} = f_s\tau\sigma_y^2 \tag{3}$$

where τ – integration time, C – carrier power, σ_y^2 – noise variance.

Theoretically noncoherent integration time of 20 ms is enough to obtain the gain of 10 dB at −140 dBm, however, 50 ms is required to get the highest probability of detection during the simulation. More than a second of data is required to get the 100% of detection at the power level of −150 dBm. As the required gain goes beyond 20 dB in the incredibly lower SNR, an increment of 1000 ms integration time results only in1.5 dB improvement in gain. So a group of data i.e. 20 s is required to attain the gain of 25 dB which is practically impossible. Processing non-coherent integration of 20 s data on a Personal Computer (PC) of 1.46 GHz processor with 1 GB memory runs 14 h time which delays the time to fix the first position of the satellite in the acquisition stage. Such expensive computations will be feasible only for off-line applications [5]. The differential detection scheme shows appreciable performance compared to non-coherent detection with lower acqui-

Table 6 Required integration time, Processing Gain (G_p) and mean acquisition time (T_{acq}) at different power levels of GPS signal.

Detection type	Input C/N_0 (dB-Hz)	Integration time (ms)	Probability of detection (%)	G_p (dB)	Mean acquisition time (s)
Noncoherent detection	44 (−130 dBm)	1	100	–	23.39
	34 (−140 dBm)	2	1.8	2.20	46.78
		10	3.62	6.71	233.92
		20	10.53	8.46	467.85
		50	97.30	10.67	$1.16e^{+03}$
	24 (−150 dBm)	50	26.66	10.67	$1.16e^{+03}$
		100	34.54	12.29	$2.33e^{+03}$
		500	92.76	15.94	$1.16e^{+04}$
		1000	98.45	17.48	$2.33e^{+04}$
		1500	100	18.37	$3.50e^{+04}$
	19 (−155 dBm)	2000	93.23	19.01	$4.67e^{+04}$
		10,000	100	22.54	$2.33e^{+05}$
	15 (−159 dBm)	20,000	91.22	24.05	$4.67e^{+05}$
Differential detection [7]	44	4	100	–	25.07
	34	8	100	9.30	40.09
	24	16	100	19.72	78.56
	19	32	98.56	23.42	124.76
	15	64	68.24	24.32	153.23
SSA combined with differential detection	44	2	100	–	70.93
	34	2	100	10.03	70.93
	24	4	100	19.22	112.90
	19	8	99.44	24.50	131.28
	15	16	96.11	29	269.88

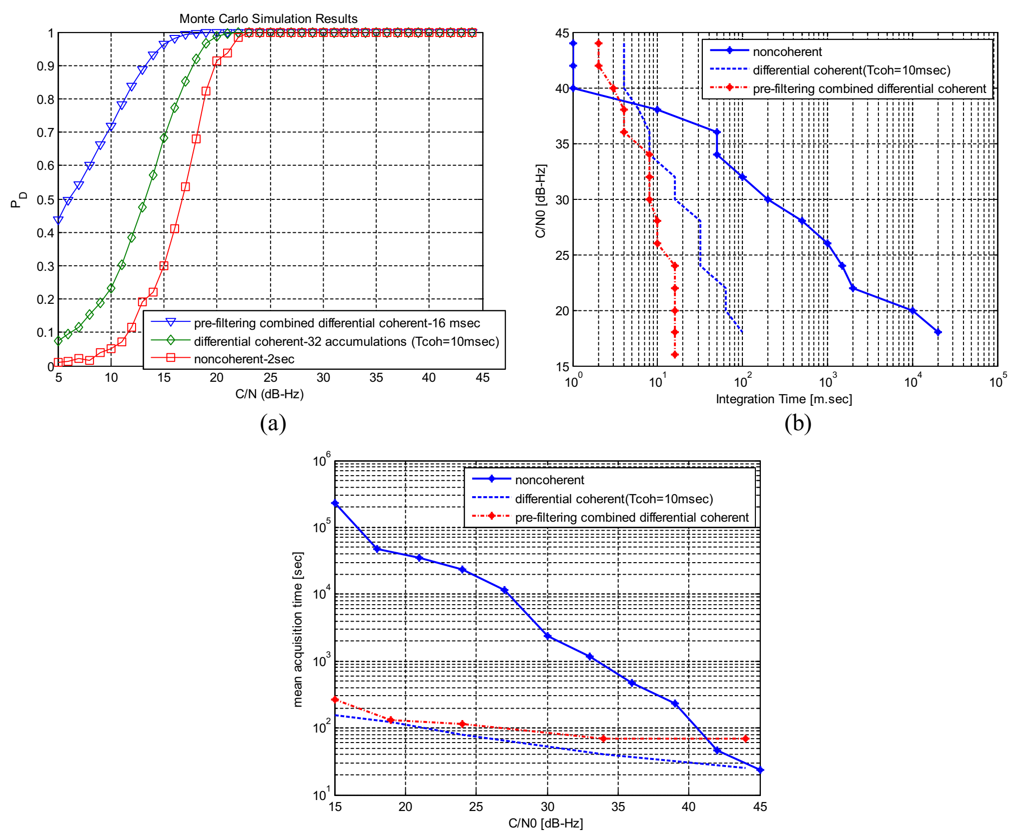


Figure 11 Performance comparison of various detection techniques. (a) C/N_0 versus probability of detection, (b) integration time versus C/N_0 and (c) C/N_0 versus mean acquisition time.

Table 7 Selection of trajectory matrix tested for SVN-31 at −150 dBm.

Parameters	Type of trajectory matrix		
	Autocorrelation	Autocovariance	Modified covariance
Mean square error ($\hat{\varepsilon}$)	0.2346	0.7789	0.6356
Maximum correlation peak	2.182e^{15}	2.8549e^{13}	4.5627e^{13}
G_p (dB)	20	17.3	18.2

sition time, however, for dealing with15 dB-Hz weak signal, even after extending the integration time to 100 ms is not sufficient enough to meet the decision statistic and the detection probability is around 0.68 only. On the other hand, the SSA combined differential detection does well in the lower SNR level. Fig. 10b shows the G_p improvement of 20 dB under the power level 24 dB-Hz while performing 16 ms differential integration time combined with SSA. The pre-filtering combined differential coherent approach provides a significant G_p of 29 dB within the integration time of 16 ms having 96% of probability of detection under the signal level of −159 dBm. An additional gain of 3 dB with a detection probability of 0.96 is being provided by the pre-filtering combined differential coherent method (Fig. 11) compared with the differential detection approach in the signal range lower than C/N_0 20 dB-Hz. Integration Time versus C/N_0 is plotted where C/N_0 goes below the nominal level (Fig. 11b), the required non-coherent integration time is in the order of 10^4 ms whereas performing pre-filtering combined differential coherent detection the required integration is about 20 ms.

3.2. Mean acquisition time computation

Mean acquisition time (T_{acq}) for coarse acquisition (C/A) code is computed for a parallel code phase search acquisition system [22].

$$T_{\text{acq}} = \frac{(2-P_d)(1+kP_{\text{fa}})}{2P_d}(q\tau_d),\ q = \frac{2046 * 41}{(1/T_{\text{coh}})} \tag{4}$$

where τ_d is total GPS integration period, kP_{fa} is false alarm penalty time (by keeping $k = 10$), q is the total number of cells to be searched in the bin and $T_{\text{coh}} = 1$ ms for coherent, noncoherent and 10 ms for DFC. Using the Eq. (4), T_{acq} is computed for different integration times. Fig. 11c shows that processing 20 s non-coherent data takes the computing time of 4.6785e+05 s whereas utilizing 16 ms pre-filtering combined differential integration method, the required gain is attained within 269.88 s of mean acquisition time.

3.3. Computation complexity

The initial step of formation of trajectory matrix requires complexity of O (N^2) operations. The multiplication of left singular matrix with size $N \times (N-r)$, diagonal matrix of size $(r \times r)$ and the right singular matrix with size $M \times M$ in the economy SVD rank reduction step requires O (N^3) operations. The final step of diagonal averaging in rank restoration requires O (N^2) computations. In total, a worst case complexity of O (N^2) + O (N^3) + O (N^2) = O (N^3) operations are required for SSA algorithm computation. While running the SSA algorithm on 1.46 GHz processor with 1 GB memory in MATLAB simulator, the average computation time of SVD stage is only around 9.5773 s, rank reduction stage is 7.883210 s and reconstruction stage is 6.767183 s. Hence total computation time of SSA is only 24.227 s. The 16 ms DFC based parallel code phase search acquisition algorithm takes 241.2868 s. Therefore the total mean acquisition time for the SSA based differential scheme for 15 dB-Hz C/N_o is 269.88 s as shown in Table 6. Hence the inclusion of the SSA stage has significantly increased the overall computation time. Therefore with the proposed approach, the extremely weak signal has been acquired with less acquisition time without much increase in computational time.

4. Conclusion and future work

The SSA based de-noising approach relies on finding the economy SVD of the autocorrelation trajectory matrix of noisy input samples and maintaining the structure of the matrix by applying suitable rank reconstruction (Toeplitz/Hankel) methods. On testing with noisy GPS signal, this method combined with DFC efficiently handles lower power signal level of −159 dBm with shorter integration time and achieves 3 dB gain improvement within 269.88 s of mean acquisition time. The detection of the number of visible satellites is increased and also the re-acquisition of GPS data is avoided. From the simulation results, the differential coherent detection technique when combined with pre-filtering corrects the deviation in code phase and Doppler frequency of the visible SVN compared to other conventional methods. The SSA based de-noising coupled with DFC has an ability to recover the weak signals only up to −159 dBm which is the limitation of current approach. Therefore for an indoor environment, Independent Component analysis can be used in the SSA signal decomposition step for finding proper rotations of eigen triples in weak signal separability.

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