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## **ORIGINAL ARTICLE**

# Performance comparison of fuzzy and non-fuzzy classification methods



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#### **KEYWORDS**

k-means; Fuzzy c-means; Gustafson-Kessel; Classification; Partition based clustering Abstract In data clustering, partition based clustering algorithms are widely used clustering algorithms. Among various partition algorithms, fuzzy algorithms, Fuzzy c-Means (FCM), Gustafson–Kessel (GK) and non-fuzzy algorithm, k-means (KM) are most popular methods. k-means and Fuzzy c-Means use standard Euclidian distance measure and Gustafson–Kessel uses fuzzy covariance matrix in their distance metrics. In this work, a comparative study of these algorithms with different famous real world data sets, liver disorder and wine from the UCI repository is presented. The performance of the three algorithms is analyzed based on the clustering output criteria. The results were compared with the results obtained from the repository. The results showed that Gustafson–Kessel produces close results to Fuzzy c-Means. Further, the experimental results demonstrate that k-means outperforms the Fuzzy c-Means and Gustafson–Kessel algorithms. Thus the efficiency of k-means is better than that of Fuzzy c-Means and Gustafson–Kessel algorithms.

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#### 1. Introduction

Many organizations generate and store large volume of data in their databases. The methods to extract the most useful knowledge from the databases are known as Data mining or

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knowledge discovery in databases (KDD). Data mining is an analytic process of discovering valid, unsuspected relationships among datasets and transforms the data into a structure that are both understandable and useful to the users.

Data analysis contains several techniques and tools for handling the data. Classification or clustering is well known method in data analysis. It is a multivariate analysis technique to partition the dataset into groups (classes or clusters) in a dataset such that the most indiscernible objects belong to the same group while the discernible objects in different groups. Clustering methods are used as a common technique in many fields such as pattern recognition, machine learning, image segmentation, medical diagnostics and bio-informatics [5].

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The two important features in clustering are partition-based clustering and hierarchical-based clustering. Partition-based clustering algorithms have the capable of discovering underlying structures of clusters by using appropriate objective function [15]. The algorithms k-means (KM), Fuzzy c-Means (FCM) and Gustafson–Kessel (GK) clustering algorithms are widely used partition-based clustering algorithms. The algorithms k-means and Fuzzy c-Means are proposed based on Euclidean distance measure and an adaptive distance measure was proposed in Gustafson–Kessel (GK) clustering algorithm.

Several comparisons are carried out by the following researchers: Jaindong, Hongzan, Jaiwen, Qiyong [16] analyzed the performance of k-means and Fuzzy c-Means algorithms and reported that the k-means method is preferable to FCM for Arterial Input Function (AIF) detection using both clinical and simulated data. Velmurugun [14] has compared the clustering performance of k-means and Fuzzy c-Means algorithms using different shapes of arbitrary distributed data points and reported that the k-means performs better than FCM. Simhachalam and Ganesan [12] analyzed the performance of Fuzzy c-Means and Gustafson-Kessel algorithms on medical diagnostics systems and reported that the performance of GK method is better than the FCM method. Wang and Garibaldi [17] have compared the performance of k-means and Fuzzy c-Means algorithms on Infrared spectra collected from auxiliary lymph node tissue section. Mousumi Gupta [8] proposed data scaling method in Gustafson-Kessel algorithm for target detection on scaled data and compared with FCM method. Neha and Seema [9] examined the performance between FCM and GK using cluster validity measures. Dibya Joyti and Anil kumar Gupta [3] evaluated the performance between k-means and Fuzzy c-Means algorithms based on time complexity. Soumi Gosh and Sanjay Kumar Dubey [13] evaluated the clustering performance of k-means and Fuzzy c-Means algorithms on the basis of the efficiency of the clustering output and the computational time and reported that k-means is superior to FCM. Bharati and Gohokar [1] compared the color image segmentation performance between k-means and Fuzzy c-Means algorithms.

The work in this paper aimed to compare the performance of the three clustering techniques, k-means (KM), Fuzzy c-Means (FCM) and Gustafson–Kessel (GK). The most popular real world date sets such as Liver Disorders and Wine are applied to test the performance of these algorithms and a comparative analysis is presented in this work. The rest of this work is organized as follows: In Section 2, concise details of data sets and the three algorithms are presented. In Section 3, results and discussion are presented and the conclusions are in Section 4.

#### 2. Materials and methods

Clustering is an unsupervised data analysis which is used to partition a set of records or objects into clusters or classes with similar characteristics. The partition is done in such a fashion that most similar (or related) objects are placed together, while dissimilar (or unrelated) objects are placed in different classes or groups.

The desired characteristics of clustering methods are ability to deal with different types of attributes with high dimensionality, effective handling of outliers and noise with minimum knowledge, ability to discover the underlying shapes and structures of the data, scalability, usability and interpretability. Clustering methods are categorized into five different methods: partitioning method, hierarchical method, data density based method, grid based method and model based or soft computing methods. Among these five methods partition based methods, k-means (KM), Fuzzy c-Means (FCM) and Gustafson–Kessel (GK) clustering algorithms are implemented using two well known data sets liver disorders and wine to generate two clusters and three clusters respectively.

#### 2.1. The dataset

The real world data sets Liver Disorder and Wine were obtained from the UCI Machine Learning Repository donated by Richard [11] and Forina [4] respectively. The Liver data set contains 341 samples with 6 attributes or blood tests each. These blood tests are capable of detecting liver disorders which might arise due to excessive alcohol consumption. The attributes are the measurements of the blood tests namely mean corpuscular volume (mcv), alkaline phosphatase (alkphos), alanine aminotransferase (sgpt), aspartate aminotransferase (sgot), gamma-glutamyl transpeptidase (gammagt) and the number of half-pint equivalents of alcoholic beverages drunk per day (drinks). The 341 samples are clustered into two different classes according to the liver disorders: Class 1 containing 142 samples and Class 2 containing 199 samples. The Wine data set contains 178 samples and each sample has 13 attributes or chemical analysis of the wine derived from three different cultivars but grown in the same region in Italy. The samples are grouped into three different classes according to the cultivars: Cultivar 1 containing 59 samples, Cultivar 2 containing 71 samples and Cultivar 3 containing 48 samples. The attributes are the values of chemical analysis of Alcohol, Malic acid, Ash, Alkalinity of ash, Magnesium, Total phenols, Flavonoids, Nonflavonoid phenols, Proanthocyanins, Color intensity, Hue, OD280/OD315 of diluted wines and Proline.

#### 2.2. k-means clustering

MacQueen [7] introduced the k-means or Hard C-Means algorithm in 1967. It is a partitioning algorithm applied to classify data into  $c(1 \le c \le N)$  clusters and each object (observation) can only belong to one cluster at any one time. Consider a dataset Z with N observations. Each observation is an n-dimensional row vector,  $z_k = [z_{k1}, z_{k2}, \ldots z_{kn},] \in \Re^n$ . The dataset Z is represented as  $N \times n$  matrix. The rows of Z represent samples (observations) and the columns are measurements for these samples (objects). k-means model achieves its partitioning by the iterative optimization of its objective function (a squared error function) given as

$$J(V) = \sum_{i=1}^{c} \sum_{k=1}^{N} ||z_k - v_i||^2$$
 (1)

where  $||z_k - v_i||^2$  is the Euclidean distance calculated between kth object,  $z_k$  and ith centroid,  $v_i$ . The algorithm comprises the following basic steps:

- Step 1: Initial the desired number of clusters, c.
- Step 2: Place c cluster centroids.
- Step 3: Assign each sample to a cluster by determining the closest distance between the sample and centroid.
- Step 4: Update the cluster centroid using  $v_i = \frac{1}{c_i} \sum_{i=1}^{c_i} z_i$ , where  $c_i$  is the number of objects in the *i*th cluster.
- Step 5: Determine the closest distances between the objects and centroids.
- Step 6: Update the samples in the clusters.
- Step 7: Repeat from Step 3 until stopping criterion has been met.

k-means algorithm is an iterative method. This algorithm can be run several times to reduce the sensitivity caused by initial random selection of centroids.

#### 2.3. Fuzzy c-Mean clustering

Fuzzy c-Means algorithm (FCM) is one of the most popular fuzzy clustering methods. FCM is developed based on fuzzy theory. In this method it uses membership function to assign membership values ranged from 0 to 1 to each object. The feature in FCM is that every object belongs to every cluster with different membership values. The partition of the dataset Z into c clusters is represented by the fuzzy partition matrix  $U = [\mu_{ik}]_{c \times N}$ . The fuzzy partitioning space for Z is the set

$$M_{fc} = \left\{ U \in \Re^{c \times N} / \mu_{ik} \in [0, 1], \ \forall i, k; \sum_{i=1}^{c} \mu_{ik} = 1, \ \forall k; 0 < \sum_{k=1}^{N} \mu_{ik}, \ \forall i \right\}$$
(2)

Fuzzy c-Mean model achieves its partitioning by the iterative optimization of its objective function given as

$$\min_{U,V} \left\{ J(Z; \ U, \ V) = \sum_{i=1}^{c} \sum_{k=1}^{N} (\mu_{ik})^{m} ||z_{k} - v_{i}||_{A}^{2} \right\} \quad \text{where } U \in M_{fc}$$
(3)

Here  $m \in [1, \infty)$  is a weighting parameter that determines the degree of fuzziness,  $V = [\nu_1, \nu_2, \dots, \nu_c]$  where  $\nu_i \in \Re^n$  is a vector of (unknown) cluster prototypes (centers). The prototypes, the membership functions and the distance metric are calculated by the Eqs. (3)–(5) respectively.

$$v_i = \frac{\sum_{k=1}^{N} (\mu_{ik})^m z_k}{\sum_{k=1}^{N} (\mu_{ik})^m} \tag{4}$$

$$\mu_{ik} = \left(\sum_{j=1}^{c} \left(\frac{D_{ikA}}{D_{jkA}}\right)^{\frac{2}{m-1}}\right)^{-1}$$
 (5)

$$D_{ikA}^{2} = \|z_{k} - v_{i}\|_{A}^{2} = (z_{k} - v_{i})^{T} A(z_{k} - v_{i})$$
(6)

where  $1 \le i \le c$ ,  $1 \le k \le N$ .

When the objective function converges to a local minimum the iteration terminates. Bezdek et al. [6] proposed the detailed algorithm. The algorithm comprises of the following basic steps:

Step 1: Randomly initialize  $U^{(0)}, c, m$  and the termination tolerance  $\varepsilon > 0$ .

- Step 2: With  $U^{(k)}$  determine the centroids vector  $V = [v_1, v_2, \dots, v_c]$  by using Eq. (4).
- Step 3: Update  $U^{(k)}$ ,  $U^{(k+1)}$  by using Eq. (5).
- Step 4: If  $||U^{(k+1)} U^{(k)}|| < \varepsilon$  then stop, else repeat from step 2 by increasing the step value k.

Although FCM is a popular clustering method it has some drawbacks. For example, it creates noise points when the method is applied to partition two clusters with an object having equidistance from two cluster's centers. FCM uses standard Euclidean distance norm.

#### 2.4. Gustafson-Kessel clustering

Another fuzzy iterative algorithm GK (extended FCM) was initially proposed by Gustafson and Kessel [2] and later improved by Babuska et al. [10]. Babuska et al. introduced an adaptive distance norm, in order to detect different geometrical shapes of the clusters in one data set when the covariance matrix  $F_i$  fails to be non-singular by the choice of the matrix  $A_i$ . The distance metric in this algorithm is given by

$$D_{ikA_i}^2 = \|z_k - v_i\|_A^2 = (z_k - v_i)^T A_i (z_k - v_i), \quad 1 \le i$$
  
 
$$\le c, \quad 1 \le k \le N$$
 (7)

The GK algorithm objective functional is defined as

$$\underbrace{\min}_{U,V} \left\{ J(Z; \ U, \ V, \ \{A_i\}) = \sum_{i=1}^{c} \sum_{k=1}^{N} (\mu_{ik})^m D_{ikA_i}^2 \right\} \tag{8}$$

To obtain a feasible solution the norm inducing matrix is constrained as  $|A_i| = \rho_i$ ,  $\rho_i > 0$ ,  $\forall i$ .

The expression for  $A_i$  is defined as

$$A_i = [\rho_i \det(F_i)]^{n-1} F_i^{-1}, \quad 1 \le i \le c$$
 (9)

where the *i*th cluster's fuzzy covariance matrix  $F_i$  is given by

$$F_i = [\phi_{i1} \dots \phi_{in}] diag(\lambda_{i1} \dots \lambda_{in}) [\phi_{i1} \dots \phi_{in}]^{-1} \quad \text{with}$$
 (10)

$$F_{i} = \frac{\sum_{k=1}^{N} (\mu_{ik})^{m} (z_{k} - v_{i}) (z_{k} - v_{i})^{T}}{\sum_{k=1}^{N} (\mu_{ik})^{m}}, 1 \leqslant i \leqslant c,$$

$$F_i = (1 - \gamma)F_i + \gamma \det(F_0)^{1/n}I$$

and the eigenvalues and eigenvectors are set as  $\lambda_{ij} = \max_{j} \lambda_{ij} / \beta$  for all j for which  $\max_{j} \lambda_{ij} / \lambda_{ij} > \beta$  respectively.

The algorithm comprises of the following basic steps:

- Step 1: Randomly initialize  $U^{(0)}, c$ , the termination tolerance  $\varepsilon > 0$ , the cluster volumes  $\rho_i > 0$  (generally 1),  $\beta = 10^{15}$  and weighting parameter  $\gamma \in [0, 1]$ .
- Step 2: Determine the centroids  $v_i^{(k)}$  by using Eq. (4).
- Step 3: Calculate the cluster covariance matrices  $F_i$  by using Eq. (10).
- Step 4: Obtain the distances by using Eq. (7).
- Step 5: Update  $U^{(k)}$ ,  $U^{(k+1)}$  by using Eq. (4).
- Step 6: If  $||U^{(k+1)} U^{(k)}|| < \varepsilon$  then stop, else repeat from step 2 by increasing the step value k.

The same parameters which are used in FCM are used in GK algorithm. The constraint  $\rho_i$  is used to find the clusters

of approximately equal volumes and the parameters  $\beta$ ,  $\gamma$  help to avoid the covariance matrix to become singular.

#### 3. Results and discussion

#### 3.1. Results

The algorithms were implemented in MATLAB version R2010a. To achieve good clustering results authors considered the maximum of 100 iterations and 15 independent test runs. The threshold value is  $\varepsilon = 0.00001$  and the weighting exponent in FCM and GK is m = 2.

The liver disorder data set contains 341 samples classified as two different classes. Each sample is characterized by 6 attributes and all the samples are labeled by numbers 1 to 341. The samples from 1 to 142 are classified as class 1 and from 143 to 341 are classified as class 2. The algorithms KM, FCM and GK are applied to generate two clusters. GK generates two clusters corresponding to class 1 and class 2 containing 67 and 274 samples respectively. 45 samples that belong to class 2 are wrongly assigned to class 1 and 120 samples associated with class 1 are wrongly assigned to class 2. FCM generates two clusters corresponding to class 1 and class 2 containing 53 and 288 samples respectively. 36 samples that belong to class 2 are wrongly grouped into class 1 and 125 samples that belong to class 1 are wrongly grouped into class 2.

The method KM generates two clusters containing 38 and 303 samples corresponding to class 1 and class 2 respectively.

24 samples that belong to class 2 are wrongly grouped into class 1 and 128 samples that belong to cluster 1 are wrongly grouped into class 2.

The data set of wine contains 178 samples classified into three different clusters according to their cultivars. Each sample is characterized by 13 attributes and all the samples are labeled by numbers 1 to 178. The samples from 1 to 59 are classified as cultivar 1, from 60 to 130 are classified as cultivar 2 and from 131 to 178 are classified as cultivar 3. The algorithms KM, FCM and GK are applied to cluster the data set into three different clusters namely Cultivar 1, Cultivar 2 and Cultivar 3. GK generates three clusters corresponding to cultivar 1, cultivar 2 and cultivar 3 containing 32, 92 and 54 samples respectively. 17 samples associated with cultivar 2 are wrongly assigned to cultivar 1. 43 samples associated with cultivar 1 and the samples numbered 141, 142 that belong to cultivar 3 are wrongly grouped into cultivar 2. The samples numbered 42 of cultivar 1 and 7 samples of cultivar 2 are wrongly assigned to cultivar 3. FCM generates three clusters corresponding to cultivar 1, cultivar 2 and cultivar 3 containing 46, 71 and 61 samples respectively. The sample numbered 74 that belongs to cultivar 2 is wrongly assigned to cultivar 1. 21 samples that belong to cultivar 3 wrongly grouped into cultivar 2. 14 samples associated with cultivar 1 and 20 samples associated with cultivar 2 are wrongly grouped into cultivar 3.

The method KM classified the data set into three clusters namely cultivar 1, cultivar 2 and cultivar 3 containing 47, 69

Clustering method		Liver data set (2 clusters)		Wine data set (3 clusters)			
		Class 1	Class 2	Cultivar 1	Cultivar 2	Cultivar 3	
k-means (KM)	Correct	14	175	46	50	29	
	Incorrect	24	128	1	19	33	
	Total	38	303	47	69	62	
Fuzzy c-Means (FCM)	Correct	17	163	45	50	27	
	Incorrect	36	125	1	21	34	
	Total	53	288	46	71	61	
Gustafson-Kessel (GK)	Correct	22	154	15	47	46	
	Incorrect	45	120	17	45	8	
	Total	67	274	32	92	54	

**Table 2** Comparison of performance of the clustering results obtained by the algorithms FCM, FPCM and PFCM for the liver disorder and wine data sets.

Clustering method	Liver da	ata set (2	clusters)	Wine data set (3 clusters)			
	Correctness %		Classification performance %	Correctness %			Classification performance %
	Class 1	Class 2		Cultivar 1	Cultivar 2	Cultivar 3	
k-means (KM)	9.85	87.94	55.43	77.69	70.42	60.41	70.22
Fuzzy c-Means (FCM)	11.97	81.91	52.79	76.27	70.42	56.25	68.54
Gustafson–Kessel (GK)	15.49	77.38	51.62	25.42	66.19	95.83	60.68

and 62 samples respectively. The sample numbered 74 that belongs to cultivar 2 is assigned to cultivar 1 wrongly. 19 samples associated with cultivar 3 are wrongly assigned to cultivar 2. 13 samples of cultivar 1 and 20 samples of cultivar 2 are wrongly grouped into cultivar 3.

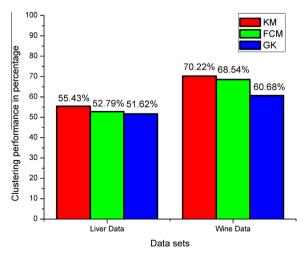
The results of the clustering methods containing number of samples that are classified properly and improperly into the respected clusters of the data sets are summarized and shown in Table 1.

#### 3.2. Discussions

According to the results of the k-means algorithm obtained for the liver disorder data set, out of 142 samples of the class 1 cluster, 14 samples were properly grouped. 128 samples of the class 1 cluster were grouped incorrectly as the class 2 samples. These frequencies are equal to 17 and 125 samples respectively if Fuzzy c-Means is applied and 22 and 120 samples respectively if Gustafson–Kessel algorithm is applied. Further, out of 199 samples of the cluster class 2, 175 samples were correctly classified. Only 24 samples of the cluster class 2 were wrongly classified as the class 1 samples. These frequencies are equal to 163 and 36 samples respectively if Fuzzy c-Means is applied and 154 and 45 samples respectively if Gustafson–Kessel algorithm is applied.

For the liver data set the algorithm k-means achieved accuracy of about 9.85% for the cluster class 1 and 87.94% for the cluster class 2. In comparison, FCM method achieved accuracy of about 11.97% and 81.91% and GK method achieved accuracy of about 15.49% and 77.38% respectively.

According to the results of the k-means algorithm obtained for the wine data set, out of 59 samples of the cluster cultivar 1, 46 samples were assigned properly. 13 samples of the cultivar 1 cluster were grouped wrongly as the cultivar 3 samples. These frequencies are equal to 45 and 14 samples respectively if Fuzzy c-Means is used. When Gustafson–Kessel algorithm is implemented, out of 59 samples of cultivar 1, 15 samples are properly classified. In the remaining 44 samples that belong to cultivar 1, 43 samples were assigned to cultivar 2 and only one sample was assigned to cultivar 3 wrongly. Further, out



**Figure 1** Performance comparison between KM, FCM and GK algorithms.

of 71 samples of the cluster cultivar 2, 50 samples were assigned properly. Only one sample was wrongly classified as cultivar 1 sample and 20 samples were grouped wrongly as cultivar 3 samples. These frequencies are equal to 50, 1 and 20 samples respectively if Fuzzy c-Means is used and 47, 17 and 7 samples respectively if Gustafson–Kessel algorithm is implemented. For the cluster cultivar 3 with 48 samples, 29 samples are correctly classified. 19 samples of cultivar 3 were incorrectly assigned to cultivar 2. These frequencies are equal to 27 and 21 samples respectively if Fuzzy c-Means is implemented and 46 and 2 samples respectively if Gustafson–Kessel algorithm is applied.

For the wine data set the algorithm k-means achieved accuracy of about 77.96% for the cultivar 1, 70.42% for the cultivar 2 and 60.41% for the cultivar 3. In comparison, FCM method achieved accuracy of about 76.27%, 70.42% and 56.25% and GK method achieved accuracy of about 25.42%, 66.19% and 95.83% respectively.

According to the results obtained for the three methods the classification performance of k-means yields its best with 55.43% comparing to the method FCM and GK which yield 52.79% and 51.62% respectively in case of liver disorder data set. The classification performance of k-means yields its best with 70.22% comparing to the method FCM and GK which yield 68.54% and 60.68% respectively in case of wine data set

The correctness and the classification performance in percentage of the three methods are summarized in Table 2.

The classification performance of the algorithms KM, FCM and GK is shown in Fig. 1. In Fig. 1, x-axis represents the data sets and the performance percentages of the algorithms are represented by y-axis.

#### 4. Conclusion

Cluster analysis is used to partition a dataset into several clusters. The data from the same cluster have most similar characteristics, which could be distinguishable for those of other clusters. In this work, among several different algorithms of cluster analysis, three popular clustering algorithms, k-means (KM), Fuzzy c-Means (FCM) and Gustafson-Kessel (GK) algorithms have been used for comparative study. Authors tested the classification performance of these algorithms with two well known data sets such as liver disorder and wine from UCI machine learning repository. Although fuzzy clustering has its own advantages, the experimental results showed that the classification performance of Hard c-means i.e. k-means had better results than FCM and GK algorithms. Authors also presented the comparable results of correctness obtained from the experiments. As a result, the k-means algorithm yields more accurate compared to Fuzzy c-Means and Gustafson-Kessel (GK) clustering algorithms. Further, as a future study, the hybridization of these algorithms with evolutionary algorithms can be implemented to improve the classification performance.

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