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Research on Ruin Problems with Two-type by-claims and Delay Period Based on Compound Binomial Risk Model

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Abstract

In this paper, we propose a new a new risk model with two-type by-claims and delay period based on compound binomial distribution, in which every main claim induces two kinds of by-claims but either of the by-claims maybe delayed to the next period. The nature of the risk model with time-correlated claims is studied based on recursion of joint distribution functions. Through introducing three submodels, we obtain the expression of the joint distribution $f(u,x,y)$ of the surplus just before ruin and deficit at ruin when initial surplus is 0 and the recursive formula of the joint distribution when initial surplus is u . In addition, when initial surplus is 0 the ruin probability of the risk model is given. At last, we provide the method to compute the ruin probability when initial surplus is u .

*Keywords:*Two-type by-claims; Compound binomial risk model; Delay Period ; Joint ditribution; Ruin probability

1. Introduction

In the classical risk model, assumption of independence among claims is an important condition to the study of risk models. However, in many practical situations, this assumption is often inconsistent with the operation of insurance companies. Sometimes, claims are time-correlated, and it is necessary to study risk

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models that are more realistic. Ambagaspitiya, 1998,1999 and Partrat,1999 studied the probability distribution, moment generating function and ruin probability of main claims under the condition of correlated aggregated claims; Yuen and Guo, 2001 studied the compound binomial model with time-correlated claims, in which each claim causes one kind of by-claim that maybe delayed, and obtained the finite survive probability; Xiao and Guo, 2007 obtained the recursive formula of joint distribution $f(u,x,y)$ of the surplus just before ruin and the deficit at ruin, but still only one kind by-claim. Unlike their models above, we studied a new risk model with practical significance. Consider the claims in a car accident, the vehicle damage may induce bodily injury or property damage at the same time or afterwards, so in this risk model each main claim causes two kinds of by-claims, and both kinds have the probability being delayed to the next time period. In our study, we give the recursive formula of joint distribution $f(u,x,y)$ of the surplus just before ruin and the deficit at ruin, the ruin probability is also obtained.

2. Depict of the model

In this risk model, we assume each claim produces two kinds of by-claims. Denote the discrete time units by $k=0,1,2,\ldots$.In time period $(k-1,k]$, a main claim happens with a probability of $q(0<q<1)$, thus $p=1-q$ is the probability of no main claims. We assume that the occurrences of main claims in different time periods are independent. The first by-claim and its associated main claim may occur in the same time period with probability r_1, or the occurrence of the by-claim maybe delayed to the next time period with probability $1-r_1$; the other by-claim and its associated main claim may occur concurrently with probability r_2, or the by-claim maybe delayed and occur in the next time period with probability $1-r_2$. All claim amounts are independent, positive and integer valued. Main claim amounts $X_1, X_2, \cdots$ are independent and identically distributed random variables with common probability $h_l = P\{X=l\}(l=1,2,\cdots)$, probability generating function $\tilde{h}(z) = \sum_{l=1}^{\infty} h_l z^l$, and expectation $\mu_X = \sum_{l=1}^{\infty} l h_l$. The two types of by-claims $Y_1, Y_2, \cdots$ and $Z_1, Z_2, \cdots$ are independent and identically distributed random variables with common probability $g_m = P\{Y=m\}(m=1,2,\cdots)$ and $k_n = P\{Z=n\}(n=1,2,\cdots)$, probability generating function $\tilde{g}(y)$, $\tilde{k}(z)$ and expectation μ_Y , μ_Z respectively.

We assume initial surplus u is a non-negative integer. W_k^X , W_k^Y and W_k^Z are the total amount of main claims and two kinds of by-claims respectively. With the assumption that the premium collected per period is 1, the surplus process is defined as :

$$U_k^{(1)} = u + k - W_k^X - W_k^Y - W_k^Z \ (k=0,1,2,\cdots) \tag{1}$$

where $U_0^{(1)} = u$ is the initial surplus. Define $T^{(1)} = \inf\{k : U_k^{(1)} < 0\}$ as the ruin time, and $\phi(u) = P\{T^{(1)} < \infty\}$ as the ruin probability.

3. Joint distribution of the surplus just before ruin and deficit at ruin in simple case $u=0$

There are four different cases when claims occur: a main claim and its two by-claims occur concurrently in some period, then the surplus process gets renewed but the initial value; a main claim and its first by-claim occur simultaneously in some period but its second by-claim is delayed and occurs in the next period; a main claim and its second by-claim occur simultaneously in some period but its first by-claim is delayed and occurs in the next period; a main claim occurs in some period but the two types of by-claims are both delayed to the next period. Then in the last three cases the surplus processes behave differently because of the delayed by-claim occurring in the next period. Thus we define the following three new surplus processes:

$$U_k^{(2)} = u + k - W_k^X - W_k^Y - W_k^Z - Y \tag{2}$$

$$U_k^{(3)} = u + k - W_k^X - W_k^Y - W_k^Z - Z \tag{3}$$

$$U_k^{(4)} = u + k - W_k^X - W_k^Y - W_k^Z - Y - Z \tag{4}$$

where $U_0^{(2)} = U_0^{(3)} = U_0^{(4)} = u$.

Accordingly, let $T^{(2)} = \inf\left\{k : U_k^{(2)} < 0\right\}$, $T^{(3)} = \inf\left\{k : U_k^{(3)} < 0\right\}$, $T^{(4)} = \inf\left\{k : U_k^{(4)} < 0\right\}$ be the ruin time of model (2), (3), (4). Define respectively the joint distributions of the surplus just before ruin and the deficit at ruin for model (1), (2), (3), (4) as follows:

$$f(u,x,y) = P\left\{T < \infty, U_{T-1}^{(1)} = x, U_T^{(1)} = -y / U_0^{(1)} = u\right\}$$

$$f^{(2)}(u,x,y) = P\left\{T < \infty, U_{T-1}^{(2)} = x, U_T^{(2)} = -y / U_0^{(2)} = u\right\}$$

$$f^{(3)}(u,x,y) = P\left\{T < \infty, U_{T-1}^{(3)} = x, U_T^{(3)} = -y / U_0^{(3)} = u\right\}$$

$$f^{(4)}(u,x,y) = P\left\{T < \infty, U_{T-1}^{(4)} = x, U_T^{(4)} = -y / U_0^{(4)} = u\right\}$$

In $(0,1]$, along with the case when no claim occurs, there are five different cases altogether for surplus process *(1)*, thus when $u \neq x$, by the law of full probability we have:

$$\begin{aligned} f(u,x,y) &= pf(u+1,x,y) + qr_1r_2 \sum_{l+m+n \le u+1} f(u+1-l-m-n,x,y)h_1g_mk_n \\ &+ qr_1(1-r_2) \sum_{l+m \le u+1} f^{(2)}(u+1-l-m,x,y)h_lg_m + qr_2(1-r_1) \sum_{l+n \le u+1} f^{(3)}(u+1-l-n,x,y)h_lk_n \\ &+ q(1-r_1)(1-r_2) \sum_{l \le u+1} f^{(4)}(u+1-l,x,y)h_l \end{aligned} \tag{5}$$

The case $u = x$ is special. When the total amount of claims is greater than $u+1$, ruin occurs. However, because the surplus just before ruin is x, the relevant terms still exist. Thus:

$$\begin{aligned} f(x,x,y) &= pf(x+1,x,y) + qr_1r_2 \sum_{l+m+n \le x+1} f(x+1-l-m-n,x,y)h_1g_mk_n \\ &+ qr_1(1-r_2) \sum_{l+m \le x+1} f^{(2)}(u+1-l-m,x,y)h_lg_m + qr_2(1-r_1) \sum_{l+n \le x+1} f^{(3)}(x+1-l-n,x,y)h_lk_n \\ &+ q(1-r_1)(1-r_2) \sum_{l \le x+1} f^{(4)}(x+1-l,x,y)h_l + D_1 \end{aligned} \tag{6}$$

where D_1 is the case when ruin occurs at time 1 :

$$\begin{aligned} D_1 &= qr_1r_2 \sum_{l+m+n = x+y+1} h_lg_mk_n + qr_1(1-r_2) \sum_{l+m = x+y+1} h_lg_m \\ &+ q(1-r_1)r_2 \sum_{l+n = x+y+1} h_lk_n + q(1-r_1)(1-r_2)h_{x+y+1} \end{aligned} \tag{7}$$

By the same reasoning, to model (2), (3), (4), when $u \neq x$, we have:

$$\begin{aligned} f^{(2)}(u,x,y) &= p \sum_{s \le u+1} f(u+1-s,x,y)g_s + qr_1r_2 \sum_{l+m+n+s \le u+1} f(u+1-l-m-n-s,x,y)h_1g_mk_ng_s \\ &+ qr_1(1-r_2) \sum_{l+m+s \le u+1} f^{(2)}(u+1-l-m-s,x,y)h_lg_mg_s + qr_2(1-r_1) \sum_{l+n+s \le u+1} f^{(3)}(u+1-l-n-s,x,y)h_lk_ng_s \\ &+ q(1-r_1)(1-r_2) \sum_{l+s \le u+1} f^{(4)}(u+1-l-s,x,y)h_lg_s \end{aligned} \tag{8}$$

$$\begin{aligned} f^{(3)}(u,x,y) &= p \sum_{t \le u+1} f(u+1-t,x,y)k_t + qr_1r_2 \sum_{l+m+n+t \le u+1} f(u+1-l-m-n-t,x,y)h_1g_mk_nk_t \\ &+ qr_1(1-r_2) \sum_{l+m+t \le u+1} f^{(2)}(u+1-l-m-t,x,y)h_lg_mk_t + qr_2(1-r_1) \sum_{l+n+t \le u+1} f^{(3)}(u+1-l-n-t,x,y)h_lk_nk_t \\ &+ q(1-r_1)(1-r_2) \sum_{l+t \le u+1} f^{(4)}(u+1-l-t,x,y)h_lk_t \end{aligned} \tag{9}$$

$$f^{(4)}(u,x,y)=p\sum_{s+t\le u+1}f(u+1-s-t,x,y)g_sk_t+qr_1r_2\sum_{l+m+n+s+t\le u+1}f(u+1-l-m-n-s-t,x,y)h_lg_mk_ng_sk_t$$
$$+qr_1(1-r_2)\sum_{l+m+s+t\le u+1}f^{(2)}(u+1-l-m-s-t,x,y)h_lg_mg_sk_t+qr_2(1-r_1)\sum_{l+n+s+t\le u+1}f^{(3)}(u+1-l-n-s-t,x,y)h_lk_ng_sk_t$$
$$+q(1-r_1)(1-r_2)\sum_{l+s+t\le u+1}f^{(4)}(u+1-l-s-t,x,y)h_lg_sk_t \tag{10}$$

When $u=x$, we have:

$$f^{(2)}(x,x,y)=p\sum_{s\le x+1}f(x+1-s,x,y)g_s+qr_1r_2\sum_{l+m+n+s\le x+1}f(x+1-l-m-n-s,x,y)h_lg_mk_ng_s$$
$$+qr_1(1-r_2)\sum_{l+m+s\le x+1}f^{(2)}(x+1-l-m-s,x,y)h_lg_mg_s+qr_2(1-r_1)\sum_{l+n+s\le x+1}f^{(3)}(x+1-l-n-s,x,y)h_lk_ng_s$$
$$+q(1-r_1)(1-r_2)\sum_{l+s\le x+1}f^{(4)}(x+1-l-s,x,y)h_lg_s+D_2 \tag{11}$$

$$f^{(3)}(x,x,y)=p\sum_{t\le x+1}f(x+1-t,x,y)k_t+qr_1r_2\sum_{l+m+n+t\le x+1}f(x+1-l-m-n-t,x,y)h_lg_mk_nk_t$$
$$+qr_1(1-r_2)\sum_{l+m+t\le x+1}f^{(2)}(x+1-l-m-t,x,y)h_lg_mk_t+qr_2(1-r_1)\sum_{l+n+t\le x+1}f^{(3)}(x+1-l-n-t,x,y)h_lk_nk_t$$
$$+q(1-r_1)(1-r_2)\sum_{l+t\le x+1}f^{(4)}(x+1-l-t,x,y)h_lk_t+D_3 \tag{12}$$

$$f^{(4)}(x,x,y)=p\sum_{s+t\le x+1}f(x+1-s-t,x,y)g_sk_t+qr_1r_2\sum_{l+m+n+s+t\le x+1}f(x+1-l-m-n-s-t,x,y)h_lg_mk_ng_sk_t$$
$$+qr_1(1-r_2)\sum_{l+m+s+t\le x+1}f^{(2)}(x+1-l-m-s-t,x,y)h_lg_mg_sk_t+qr_2(1-r_1)\sum_{l+n+s+t\le x+1}f^{(3)}(x+1-l-n-s-t,x,y)h_lk_ng_sk_t$$
$$+q(1-r_1)(1-r_2)\sum_{l+s+t\le x+1}f^{(4)}(x+1-l-s-t,x,y)h_lg_sk_t+D_4 \tag{13}$$

Where

$$D_2=pg_{x+y+1}+qr_1r_2\sum_{l+m+n+s=x+y+1}h_lg_mk_ng_s+qr_1(1-r_2)\sum_{l+m+s=x+y+1}h_lg_mg_s$$
$$+q(1-r_1)r_2\sum_{l+n+s=x+y+1}h_lk_ng_s+q(1-r_1)(1-r_2)\sum_{l+s=x+y+1}h_lg_s \tag{14}$$

$$D_3=pk_{x+y+1}+qr_1r_2\sum_{l+m+n+t=x+y+1}h_lg_mk_nk_t+qr_1(1-r_2)\sum_{l+m+t=x+y+1}h_lg_mk_t$$
$$+q(1-r_1)r_2\sum_{l+n+t=x+y+1}h_lk_nk_t+q(1-r_1)(1-r_2)\sum_{l+t=x+y+1}h_lk_t \tag{15}$$

$$D_4=p\sum_{s+t=x+y+1}g_sk_t+qr_1r_2\sum_{l+m+n+s+t=x+y+1}h_lg_mk_ng_sk_t+qr_1(1-r_2)\sum_{l+m+s+t=x+y+1}h_lg_mg_sk_t$$
$$+q(1-r_1)r_2\sum_{l+n+s+t=x+y+1}h_lk_ng_sk_t+q(1-r_1)(1-r_2)\sum_{l+s+t=x+y+1}h_lg_sk_t \tag{16}$$

Assume that $\sum_u^\infty f(u,x,y)<\infty,\sum_u^\infty f^{(2)}(u,x,y)<\infty,\sum_u^\infty f^{(3)}(u,x,y)<\infty,\sum_u^\infty f^{(4)}(u,x,y)<\infty$. By summing (5) and (6), (8) and (11) , (9) and (12) , (10) and (13) on u from 0 to ∞ respectively, and solving the equation group, we can easily obtain:

Theorem 1. *When $u=0$, the joint distribution of the surplus just before ruin and deficit at ruin of surplus process (1) is*

$$f(0,x,y)=\frac{(p+qr_1r_2)D_1+qr_1(1-r_2)D_2+q(1-r_1)r_2D_3+q(1-r_1)(1-r_2)D_4}{p(p+qr_1r_2)} \tag{17}$$

Where D_1, D_2, D_3, D_4 *are given by(7), (14), (15), (16), respectively.*

Through summing on $P\{T<\infty|u=0\}=\sum_{x=0}^{\infty}\sum_{y=1}^{\infty}f(0,x,y)$, we have:

Theorem 2. *When* $u=0$ *, the ruin probability is*

$$P\{T<\infty|u=0\}=\frac{q}{p(p+qr_1r_2)}[\mu_X+\mu_Y+\mu_Z-1-p(1-r_1r_2)]$$

4. Recursive formula in complex case $u>0$

Now with the initial value $f(0,x,y)$, we are to find the recursive formula for $f(u,x,y)$, When $0\le z<1$， consider the generating functions of f , $f^{(2)}$, $f^{(3)}$ and $f^{(4)}$:

$$\tilde{M}_1(z)=\sum_{u=0}^{\infty}f(u,x,y)z^u，\tilde{M}_2(z)=\sum_{u=0}^{\infty}f^{(2)}(u,x,y)z^u，\tilde{M}_3(z)=\sum_{u=0}^{\infty}f^{(3)}(u,x,y)z^u，\tilde{M}_4(z)=\sum_{u=0}^{\infty}f^{(4)}(u,x,y)z^u\cdot$$

Multiply both sides of (5) and (6), (8) and (11), (9) and (12), (10) and (13) by z^u , and sum every equation over u from 0 to ∞, we obtain four equations. By combining these equations, it holds that:

$$\begin{aligned}&[z-p-qr_1r_2\tilde{h}(z)\tilde{g}(z)\tilde{k}(z)-qr_1(1-r_2)\tilde{h}(z)\tilde{g}^2(z)-qr_2(1-r_1)\tilde{h}(z)\tilde{k}^2(z)-q(1-r_1)(1-r_2)\tilde{h}(z)\tilde{g}(z)\tilde{k}(z)]\tilde{M}_1(z)\\&=-pf(0,x,y)+qr_1(1-r_2)\tilde{h}(z)\tilde{g}(z)[\frac{1}{z\tilde{k}(z)}pf(0,x,y)+D_2z^x-\tilde{g}(z)D_1z^x]\\&+qr_2(1-r_1)\tilde{h}(z)\tilde{k}(z)[\frac{1}{z\tilde{g}(z)}pf(0,x,y)+D_3z^x-\tilde{k}(z)D_1z^x]\\&\quad+q(1-r_1)(1-r_2)\tilde{h}(z)[\frac{1}{z}pf(0,x,y)+D_4z^x-\tilde{g}(z)\tilde{k}(z)D_1z^x]+D_1z^{x+1}\end{aligned}\tag{18}$$

Then compare the coefficients of z^u of both sides of (18), we have:

Theorem 3. *Let* u *(*u *is a positive valued integer) be the initial reserve of the surplus process (1), then the joint distribution of the surplus just before ruin and deficit at ruin satisfies the following recursive formula:*

$$\begin{aligned}&pf(u,x,y)=f(u-1,x,y)-qr_1r_2\sum_{v=3}^{u}\sum_{l+m+n=v}f(u-t,x,y)h_lg_mk_n\\&-qr_1(1-r_2)\sum_{v=3}^{u}\sum_{l+m+s=v}f(u-t,x,y)h_lg_mg_s-qr_2(1-r_1)\sum_{v=3}^{u}\sum_{l+n+t=v}f(u-t,x,y)h_lk_nk_t\\&-q(1-r_1)(1-r_2)\sum_{v=3}^{u}\sum_{l+m+n=v}f(u-t,x,y)h_lg_mk_n-pqf(0,x,y)r_1(1-r_2)\sum_{l+m+n=u+1}\frac{h_lg_m}{k_n}\\&-pqf(0,x,y)r_2(1-r_1)\sum_{l+m+n=u+1}\frac{h_lk_n}{g_m}-pqf(0,x,y)(1-r_1)(1-r_2)h_{u+1}\\&+D_1[qr_1(1-r_2)\sum_{l+m+s=u-x}h_lg_mg_s\,\mathrm{I}\{u\ge 3+x\}+qr_2(1-r_1)\sum_{l+n+t=u-x}h_lk_nk_t\,\mathrm{I}\{u\ge 3+x\}\\&+q(1-r_1)(1-r_2)\sum_{l+m+n=u-x}h_lg_mk_n\,\mathrm{I}\{u\ge 3+x\}-\mathrm{I}\{u=1+x\}]\\&-qr_1(1-r_2)\sum_{l+m=u-x}h_lg_m\,\mathrm{I}\{u\ge 2+x\}D_2-qr_2(1-r_1)\sum_{l+n=u-x}h_lk_n\,\mathrm{I}\{u\ge 2+x\}D_3\\&\quad-q(1-r_1)(1-r_2)h_{u-x}\,\mathrm{I}\{u\ge 1+x\}D_4\end{aligned}\tag{19}$$

With Theorem 1 and Theorem 2, we can compute the joint distribution of the surplus just before ruin and deficit at ruin when u is of any value. Furthermore, sum $f(u,x,y)$ over x from 0 to ∞ and y from 1 to ∞, we can finally have the ruin probability $\phi(u)$ when initial is u .

5. Conclusion

Considering the claims in a car accident, and the vehicle damage may induce bodily injury or property damage at the same time or afterwards, we studied a new risk model with two-type by-claims and delay Period based on compound binomial distribution. The nature of the risk model with time-correlated claims is studied based on recursion joint distribution functions. The recursive formula of joint distribution $f(u,x,y)$ of the surplus just before ruin and the deficit at ruin, and the ruin probability are obtained. The results are helpful to research the other risk models from such as [6,7].

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