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Default Theories Over Monadic Languages

(Extended Abstract)

Michael Kaminski[1]

Department of Computer Science
Technion – Israel Institute of Technology
Haifa, Israel

Julia Mosin[2]

IBM Haifa Labs
Haifa University Campus
Haifa, Israel

Abstract

In this paper we compare the semantical and syntactical definitions of extensions for open default theories. We prove that, over monadic languages, these definitions are equivalent and do not depend on the cardinality of the underlying *infinite* world. We also show that, under the *domain closure assumption*, one free variable open default theories are decidable.

Keywords: Default logic, Open defaults, Herbrand interpretations, Monadic languages

1 Introduction

Non-monotonic logics are intended to simulate the process of human reasoning by providing a formalism for deriving consistent conclusions from an incomplete description of the world.

Reiter's *default logic* ([11]) is one of the widely used non-monotonic formalisms and maybe the only non-monotonic formalism that has a clearly useful

[1] Email: kaminski@cs.technion.ac.il
[2] Email: mjulia@il.ibm.com

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contribution to the wider field of computer science through logic programming and database theory. This logic deals with rules of inference called *defaults* which are expressions of the form

$$\delta(\boldsymbol{x}) = \frac{\alpha(\boldsymbol{x}) : M\beta_1(\boldsymbol{x}), \ldots, M\beta_m(\boldsymbol{x})}{\gamma(\boldsymbol{x})}, \tag{1}$$

where $\alpha(\boldsymbol{x})$, $\beta_1(\boldsymbol{x}), \ldots, \beta_m(\boldsymbol{x})$, $m \geq 1$, and $\gamma(\boldsymbol{x})$ are formulas of first-order logic whose free variables are among $\boldsymbol{x} = x_1, \ldots, x_n$. A default is *closed* if none of $\alpha, \beta_1, \ldots, \beta_m$, and γ contains a free variable. Otherwise it is *open*. Roughly speaking, the intuitive meaning of a default is as follows. For every n-tuple of objects $\boldsymbol{t} = t_1, \ldots, t_n$, if $\alpha(\boldsymbol{t})$ is believed, and the $\beta_i(\boldsymbol{t})$s are consistent with one's beliefs, then one is permitted to deduce $\gamma(\boldsymbol{t})$ and add it to the "belief set." Thus, an open default can be thought of as a kind of "default scheme," where free variables $\boldsymbol{x}$ can be replaced by any of the theory's objects. Various examples of deduction by defaults can be found in [11].

Whereas closed defaults have been quite thoroughly investigated, very little is known about open ones. However, interesting cases of default reasoning usually deal with open defaults, because the intended use of defaults is to determine whether an object possesses a given property, rather than accepting or rejecting a "fixed statement."

It was pointed out in [7] that when applying open defaults one must specify *all* the objects of the underlying theory. Also, it was argued in [3] that one must distinguish between objects defined explicitly (closed terms) and objects introduced implicitly (by existential formulas, say).

In this paper we use the semantical definition of extensions for open default theories proposed in [7] and [3], where, in contrast to the syntactical definitions in [10] and [11], free variables are treated as object variables, rather than meta-variables for the closed terms of the theory. The reason for choosing a semantical definition of extensions is that, on the one hand, it provides a complete description of the theory objects, and, on the other hand, it distinguishes between explicitly and implicitly defined objects.

Since the semantical treatment of open default theories allows one to describe all the elements of the domain under consideration, it has no syntactical counterpart within the ordinary first-order default logic, unless the domain is explicitly defined by the *domain closure assumption*, i.e., the axiom

$$\forall x \bigvee_{i=1}^{m} x = t_i, \tag{2}$$

where $t_1, \ldots, t_m$ are closed terms ([5]). Under the domain closure assumption, extensions can be described syntactically by extending the underlying language of default theory with an infinite set of new constant symbols and replacing each open default with the set of all its closed instances.

It was shown in [6] that extensions for open default theories depend on the domain cardinality (cf. [4]) and that over countable[3] or finite domains, extensions for open default theories can be described syntactically in first-order logic extended with an infinitary Carnap rule of inference

$$\frac{\{\varphi(t)\}_{t\in\boldsymbol{T}_{\mathcal{L}}}}{\forall x\varphi(x)}, \tag{3}$$

denoted by $\boldsymbol{C}$. Here and hereafter $\boldsymbol{T}_{\mathcal{L}}$ denotes the set of all closed terms of language $\mathcal{L}$.

In this paper we show that, when the underlying language of default theory is monadic, the semantical definition and the above syntactical description of extensions for open default theories are equivalent and do not depend on the cardinality of the underlying infinite domain. That is, extensions for open default theories over monadic languages can be (equivalently) described syntactically in first-order logic extended with the Carnap rule of inference. Like in the case of explicitly defined finite domains, the syntactical definition treats an open default as the set of all its closed instances over the underlying language of default theory, extended with an infinite set of new constant symbols. We prove then, that in this syntactical definition, it is sufficient to extend the underlying language with a countable set of new constant symbols. As a corollary we obtain that the original semantical definition of extension for open default theories over monadic languages can always be restricted to a countable base.

It should be pointed out that, even though monadic languages are rather restrictive, many (if not most) examples and case studies of open default deal with monadic languages.

In addition, we show that, under the domain closure assumption (2), for *uniterm* default theories introduced in [1], we may restrict ourselves to a *computable* finite base. Therefore, uniterm default theories are decidable.

The paper is organized as follows. In the next section we recall the notation and some basic results used throughout this paper. In Section 3 we show that extensions for default theories over monadic languages do not depend on the cardinality of the underlying infinite domain. Finally, in Section 4 we show that under the domain closure assumption, extensions for uniterm default theories do not depend on the cardinality of the underlying domain and, therefore, we may restrict ourselves to an explicitly defined finite domain.

[3] In this paper, “countable” means *infinite* countable.

2 Background

In this section we briefly recall the definitions of default theories and the Herbrand semantics of first-order logic. We assume that the reader is acquainted with classical first-order logic.

2.1 Default theories

Reiter's *default logic* ([11]) deals with rules of inference called *defaults* which are expressions of the form (1).

A *default theory* is a pair (D, A), where D is a set of defaults and A is a set of first-order sentences (axioms). A default theory is *closed*, if all its defaults are closed. Otherwise it is *open*.

2.2 Extensions for closed default theories

In this section we recall the syntactical and semantical definitions of extensions for closed default theories.

Recall that closed defaults are expressions of the form

$$\frac{\alpha : M\beta_1, \ldots, M\beta_m}{\gamma},$$

where α, $\beta_1, \ldots, \beta_m$, $m \geq 1$, and γ are closed formulas.

Definition 2.1 ([11]) Let (D, A) be a closed default theory. For any set of sentences S let $\Gamma_{(D,A)}(S)$ be the smallest set of sentences B (beliefs) that satisfies the following three properties.

D1. $A \subseteq B$.

D2. $\boldsymbol{Th}(B) = B$, i.e., B is deductively closed.

D3. If $\frac{\alpha : M\beta_1, \ldots, M\beta_m}{\gamma} \in D$, $\alpha \in B$, and $\neg\beta_1, \ldots, \neg\beta_m \notin S$, then $\gamma \in B$.

A set of sentences E is an *extension* for (D, A) if $\Gamma_{(D,A)}(E) = E$, i.e., if E is a fixed point of the operator $\Gamma_{(D,A)}$.

Next, we present a semantical definition of extension for closed default theories. Here and hereafter, for any class of interpretations W, by $\boldsymbol{Th}_{\mathcal{L}}(W)$ we mean the set of all closed formulas over $\mathcal{L}$ satisfied by all elements of W.

Definition 2.2 ([2]) Let (D, A) be a closed default theory over $\mathcal{L}$. For any class of interpretations W, let $\Sigma_{(D,A)}(W)$ be the largest class V of models of A that satisfies the following condition.

If $\frac{\alpha : M\beta_1, \ldots, M\beta_m}{\gamma} \in D$, $\alpha \in \boldsymbol{Th}_{\mathcal{L}}(V)$, and $\neg\beta_1, \ldots, \neg\beta_m \notin \boldsymbol{Th}_{\mathcal{L}}(W)$, then $\gamma \in \boldsymbol{Th}_{\mathcal{L}}(V)$.

It is known from [2] that the definition of extensions as the theories of the fixed points of the operator Σ is equivalent to Reiter's original definition (Definition 2.1). That is, a set of sentences E is an extension for a closed default theory (D, A) if and only if $E = \boldsymbol{Th}_{\mathcal{L}}(W)$ for some fixed point W of $\Sigma_{(D,A)}$.

2.3 Herbrand semantics of first-order logic

In this section we define Herbrand semantics of first-order logic that is the basis of the semantical approach to open default theories.

We denote by $\mathcal{L}$ the language of the underlying first-order logic. Let b be a set that contains no symbols of $\mathcal{L}$. We denote by $\mathcal{L}_b$ the language obtained from $\mathcal{L}$ by augmenting its set of constants with all elements of b. The set of all closed terms of the language $\mathcal{L}_b$ is called the *Herbrand universe* of $\mathcal{L}_b$. A *Herbrand b-interpretation* is a set of ground (closed) atomic formulas of $\mathcal{L}_b$. Note that closed formulas over $\mathcal{L}_b$ are of the form $\varphi(t_1, \ldots, t_n)$, where $t_1, \ldots, t_n$ are closed terms of language $\mathcal{L}_b$ and $\varphi(x_1, \ldots, x_n)$ is a formula over $\mathcal{L}$ whose free variables are among $x_1, \ldots, x_n$. The set b is called the *base* of Herbrand b-interpretation.

Let w be a Herbrand b-interpretation and let φ be a closed formula over $\mathcal{L}_b$. We say that w *satisfies* φ, denoted $w \models \varphi$, if the following holds.

- If φ is an atomic formula, then $w \models \varphi$ if and only if $\varphi \in w$;
- $w \models \varphi \supset \psi$ if and only if $w \not\models \varphi$ or $w \models \psi$;
- $w \models \neg\varphi$ if and only if $w \not\models \varphi$; and
- $w \models \forall x \varphi(x)$ if and only if for each $t \in \boldsymbol{T}_{\mathcal{L}_b}$, $w \models \varphi(t)$.

For a Herbrand b-interpretation w we define the $\mathcal{L}$-*theory* ($\mathcal{L}_b$-*theory*) of w, denoted $\boldsymbol{Th}_{\mathcal{L}}(w)$ ($\boldsymbol{Th}_{\mathcal{L}_b}(w)$), as the set of all closed formulas of $\mathcal{L}$ ($\mathcal{L}_b$) satisfied by w. For a set of Herbrand b-interpretations W we define the $\mathcal{L}$-*theory* ($\mathcal{L}_b$-*theory*) of W, denoted $\boldsymbol{Th}_{\mathcal{L}}(W)$ ($\boldsymbol{Th}_{\mathcal{L}_b}(W)$), as the set of all closed formulas of $\mathcal{L}$ ($\mathcal{L}_b$) satisfied by all elements of W. That is, $\boldsymbol{Th}_{\mathcal{L}}(W) = \bigcap_{w \in W} \boldsymbol{Th}_{\mathcal{L}}(w)$ ($\boldsymbol{Th}_{\mathcal{L}_b}(W) = \bigcap_{w \in W} \boldsymbol{Th}_{\mathcal{L}_b}(w)$). Finally, let X be a set of closed formulas over $\mathcal{L}_b$. We say that w is a *Herbrand b-model*, denoted by $w \models X$, if $X \subseteq \boldsymbol{Th}_{\mathcal{L}_b}(w)$.

Remark 2.3 It is well-known that for an infinite set of new constant symbols b, Herbrand b-interpretations are complete and sound for first-order logic. That is, for a set of formulas X over $\mathcal{L}$ and a formula φ over $\mathcal{L}$, $X \vdash \varphi$ if and only if φ is satisfied by all Herbrand b-interpretations which satisfy X. In particular, Herbrand b-interpretations with an infinite base naturally arise in the Henkin proof of the completeness theorem ([9, Lemma 2.16, p. 70]).

2.4 Extensions for open default theories

In this section, departing from Definition 2.2 and following [7] and [3] we present a definition of extensions for *open* default theories. It is known from [3] (see also Remark 2.5 below) that for closed default theories this definition is equivalent to the original Reiter's definition (Definition 2.1).

We start with the intuition underlying the definition. There are two types of objects in the domain of a default theory. One type consists of the *fixed* built-in objects which belong to $\boldsymbol{T}_{\mathcal{L}}$ and must be present in any Herbrand interpretation, and the other type consist of implicitly defined *unknown* objects which may vary from one Herbrand interpretation to other, e.g., objects introduced by existentially quantified formulas. These objects generate other unknown objects by means of the function symbols of $\mathcal{L}$. Thus, it seems natural to assume that the theory domain is a Herbrand universe of the original language augmented with a set of new (unknown) objects, cf. [8, Chapter 1, §3].

The following definition of extensions for open default theories is a relativization of Definition 2.2 to Herbrand b-interpretations with an infinite set of new constant symbols b. The reason for passing to a semantical definition is that, in general, it is impossible to describe a Herbrand universe by means of the standard proof theory. The only exception is the cases when the theory domain is explicitly finite ([5]), i.e., contains axiom (2).

Definition 2.4 ([3]) Let b be a set of new constant symbols and let (D, A) be a default theory. For any set of Herbrand b-interpretations W let $\Delta^b_{(D,A)}(W)$ be the largest set V of Herbrand b-models of A that satisfies the following condition.

For any default $\dfrac{\alpha(\boldsymbol{x}) : M\beta_1(\boldsymbol{x}), \ldots, M\beta_m(\boldsymbol{x})}{\gamma(\boldsymbol{x})} \in D$ and any tuple $\boldsymbol{t}$ of elements of $\boldsymbol{T}_{\mathcal{L}_b}$ if $\alpha(\boldsymbol{t}) \in \boldsymbol{Th}_{\mathcal{L}_b}(V)$ and $\neg\beta_1(\boldsymbol{t}), \ldots, \neg\beta_m(\boldsymbol{t}) \notin \boldsymbol{Th}_{\mathcal{L}_b}(W)$, then $\gamma(\boldsymbol{t}) \in \boldsymbol{Th}_{\mathcal{L}_b}(V)$.

A set of sentences E is called a b-*extension* for (D, A) if $E = \boldsymbol{Th}_{\mathcal{L}}(W)$ for some fixed point W of $\Delta^b_{(D,A)}$.

We will also refer to the set b as the *base* of E.

Remark 2.5 It follows from the Löwnheim-Skolem theorem that, for a closed default theory (D, A) and an infinite base b, a set of sentences is a b-extension for (D, A) if and only if it is an "ordinary" Reiter's extension for (D, A).

From now on, unless we state otherwise, we deal with infinite bases, because the cardinality of a finite base b can be extracted from the b-extension, which is undesirable in the general case.

Remark 2.6 Note that for two bases b and b' of different cardinality the sets of b- and b'-extensions for an open default theory do not necessarily coincide, see [6, Example 7.1].

2.5 Syntactical description of extensions for open default theories

This section contains a syntactical definition of extensions for open default theories. The basic idea of the syntactical definition is, roughly speaking, as follows. Following [10], we treat an open default as the set of all its closed instances over the language $\mathcal{L}_b$ - the original language $\mathcal{L}$ extended with the base b of H_b.

Whereas over explicitly defined finite domains, "completeness" of the set of all closed instances of a set of defaults follows from the *domain closure assumption* (2), completeness in the case of infinite domains is a more delicate issue. The infinite domain counterpart of the domain closure assumption is the Carnap rule of inference $\boldsymbol{C}$ (3).

Definition 2.7 below is a relativization of Definition 2.1 to first-order logic extended with $\boldsymbol{C}$. We shall need one more bit of notation.

For a set of formulas X we denote by $\boldsymbol{Th}^{\boldsymbol{C}}(X)$ the set of all formulas deducible from X in first-order logic extended with $\boldsymbol{C}$. We say that a set of formulas X is $\boldsymbol{C}$-*consistent* if $\boldsymbol{Th}^{\boldsymbol{C}}(X)$ is consistent in the usual first-order sense.

Definition 2.7 ([6]) Let (D, A) be a closed default theory. For any set of sentences S let $\Gamma^{\boldsymbol{C}}_{(D,A)}(S)$ be the smallest set of sentences B (beliefs) that satisfies the following three properties.

CD1. $A \subseteq B$.

CD2. $\boldsymbol{Th}^{\boldsymbol{C}}(B) = B$, i.e., B is "$\boldsymbol{C}$-deductively" closed.

CD3. If $\dfrac{\alpha : M\beta_1, \ldots, M\beta_m}{\gamma} \in D$, $\alpha \in B$, and $\neg\beta_1, \ldots, \neg\beta_m \notin S$, then $\gamma \in B$.

A set of sentences E is a $\boldsymbol{C}$-*extension* for (D, A) if $\Gamma^{\boldsymbol{C}}_{(D,A)}(E) = E$, i.e., if E is a fixed point of the operator $\Gamma^{\boldsymbol{C}}_{(D,A)}$.

To define $\boldsymbol{C}$-extensions for *open* default theories we need the notion of a closed instance of an open default.

Definition 2.8 Let $\delta(\boldsymbol{x}) = \dfrac{\alpha(\boldsymbol{x}) : M\beta_1(\boldsymbol{x}), \ldots, M\beta_m(\boldsymbol{x})}{\gamma(\boldsymbol{x})}$ be an open default. An *instance* of $\delta(\boldsymbol{x})$ is a closed default $\delta(\boldsymbol{t}) = \dfrac{\alpha(\boldsymbol{t}) : M\beta_1(\boldsymbol{t}), \ldots, M\beta_m(\boldsymbol{t})}{\gamma(\boldsymbol{t})}$, where $\boldsymbol{t} = t_1, \ldots, t_n$ is a tuple of closed terms of the underlying language. For an open default δ, the set of all closed instances of δ is denoted by $\bar{\delta}$, and for a set of defaults D, $\bar{D}_{\mathcal{L}} = \bigcup_{\delta \in D} \bar{\delta}$ is the set of all closed instances (over $\mathcal{L}$) of all defaults of D.

Theorem 2.9 below shows that, in contrast with Remark 2.6, restrictions of $\boldsymbol{C}$-extensions for $(\bar{D}_{\mathcal{L}_b}, A)$ to $\mathcal{L}$ do not depend on the cardinality of the (infinite) base b. This theorem is our *new* result, but it naturally belongs to this section of the background. It is used for the proofs of the results stated in Section 3.

Theorem 2.9 (Cf. Remark 2.6) *Let (D, A) be an open default theory and let b and b' be infinite sets of new constant symbols. Then for any $\boldsymbol{C}$-extension E for $(\bar{D}_{\mathcal{L}_b}, A)$ there is a $\boldsymbol{C}$-extension E' for $(\bar{D}_{\mathcal{L}_{b'}}, A)$, such that $E \cap \boldsymbol{Fm}_{\mathcal{L}} = E' \cap \boldsymbol{Fm}_{\mathcal{L}}$.* [4]

2.6 Extensions over countable bases

This section deals with extensions over a countable base b. We start with the "$\boldsymbol{C}$-completeness" theorem for which we shall need the following definition.

Definition 2.10 Herbrand b-interpretations (models) with the empty base b are called a *term* interpretations (models).

Obviously, term interpretations are sound for the Carnap rule. Theorem 2.11 below shows that if $\mathcal{L}$ is countable, then term interpretations are also complete.

Theorem 2.11 ([6]) *Let $\mathcal{L}$ be countable. If X is $\boldsymbol{C}$-consistent, then X has a term model.*

Remark 2.12 It is well-known that Theorem 2.11 does not hold for uncountable languages, e.g., see [6, Example 6.7].

Theorem 2.13 below shows that, for countable bases, semantical and syntactical definitions of extensions are equivalent.

Theorem 2.13 ([6]) *Let (D, A) be an open default theory and let b be a countable set of new constant symbols. Then E is a $\boldsymbol{C}$-extension for $(\bar{D}_{\mathcal{L}_b}, A)$ if and only if there is a fixed point W of $\Delta^b_{(D,A)}$ such that $E = \boldsymbol{Th}_{\mathcal{L}_b}(W)$.*

[4] We denote by $\boldsymbol{Fm}_{\mathcal{L}}$ the set of all closed formulas over $\mathcal{L}$.

Corollary 2.14 ([6]) *Let (D, A) be an open default theory and let b be a countable set of new constant symbols. Then E is a b-extension for (D, A) if and only if there is a $\boldsymbol{C}$-extension E' for $(\bar{D}_{\mathcal{L}_b}, A)$ such that $E = E' \cap \boldsymbol{Fm}_{\mathcal{L}}$.*

Remark 2.15 Note that the above corollary does not hold in the general case, see [6, Example 6.9]. Thus, as it was pointed out in [6], in order to define syntactically extensions over uncountable domains, we have, in addition to the Carnap rule, to use *infinitary languages* which allow to express *set-theoretic* rules of inference. It seems that infinitary logic is a too high price for a syntactical equivalent of the domain closure assumption.

3 Default theories over monadic languages

This section deals with the main subject of our paper – open default theories over monadic languages. We show that for monadic languages b-extensions do not depend on the cardinality of the (infinite) base b.

We start with the completeness theorem for first-order logic with the Carnap rule over monadic languages.

Theorem 3.1 (Cf. Theorem 2.11.) *Let $\mathcal{L}$ be a monadic language. If X is a $\boldsymbol{C}$-consistent theory, then X has a term model.*

Theorem 3.1 can be equivalently restated as follows.

Theorem 3.2 *Let $\mathcal{L}$ be a monadic language and let X be a $\boldsymbol{C}$-deductively closed set of closed formulas over $\mathcal{L}_b$. Then $X = \boldsymbol{Th}_{\mathcal{L}_b}(V)$, where V is the set of all term models of X.*

Combining Theorem 3.2 with Definition 2.4 we obtain that the set of $\boldsymbol{C}$-extensions for $(\bar{D}_{\mathcal{L}_b}, A)$ coincide with the theories of fixed points of $\Delta^b_{(D,A)}$.

Theorem 3.3 (Cf. Corollary 2.14) *Let $\mathcal{L}$ be a monadic language, (D, A) be an open default theory over $\mathcal{L}$, and let b be a set of new constant symbols. Then E is a b-extension for (D, A) if and only if there is a $\boldsymbol{C}$-extension E' for $(\bar{D}_{\mathcal{L}_b}, A)$ such that $E = E' \cap \boldsymbol{Fm}_{\mathcal{L}}$.*

Finally, Theorem 3.4 below, that states that for monadic languages b-extensions do not depend on the cardinality of base b, is an immediate consequence of Theorems 2.9 and 3.3.

Theorem 3.4 *Let $\mathcal{L}$ be a monadic language, (D, A) be an open default theory over $\mathcal{L}$, and let b and b' be infinite sets of new constant symbols. Then E is a b-extension for (D, A) if and only if it is a b'-extension for (D, A).*

In particular, it follows from Theorem 3.4 that when dealing with open default theories over monadic languages we may restrict ourselves to a countable base b, which is not true in the general case, see Remark 2.6.

4 Uniterm default theories

This section deals with *uniterm default theories* introduced in [1], see Definition 4.2 below. We show that under the *domain closure assumption*, when dealing with uniterm default theories we may restrict ourselves to finite bases.

For the general case, it is known from [5] that under the *domain closure assumption* b-extensions for an open default theory (D, A) coincide with the restrictions of extensions of $(\bar{D}_{\mathcal{L}_b}, A)$ to $\mathcal{L}$.

Theorem 4.1 ([5]) *Let (D, A) be an open default theory such that for some constants $a_1, \ldots, a_m$, $A \vdash \forall x \bigvee_{i=1}^{m} x = a_i$,* [5] *and let b be an infinite set of new constant symbols. Then E is an extension for $(\bar{D}_{\mathcal{L}_b}, A)$ if and only if there is a fixed point W of $\Delta^b_{(D,A)}$ such that $E = \boldsymbol{Th}_{\mathcal{L}_b}(W)$.*

Next we recall the definition of uniterm default theories.

Definition 4.2 (Cf. [1, Definitions 6 and 10]) Let $\mathcal{L}$ be a finite monadic language. A propositional (boolean) combination of atomic formulas over the same variable x is called a *uniterm formula over x*. A default theory (D, A) is called *uniterm* if for every default $\delta \in D$, all formulas which appear in δ are uniterm formulas over the same variable.

The main result of this section is that for a uniterm default theory (D, A) the restrictions of extensions for $(\bar{D}_{\mathcal{L}_b}, A)$ to $\mathcal{L}$ do not depend on the cardinality of base b (Theorem 4.3). Thus, under the domain closure assumption, extensions for uniterm default theories do not depend on the base cardinality either.

Theorem 4.3 *Let (D, A) be a uniterm default theory and let b be an infinite set of new constant symbols. There exist a finite set of new constant symbols b', such that there is an extension E for $(\bar{D}_{\mathcal{L}_b}, A)$ if and only if there is an extension E' for $(\bar{D}_{\mathcal{L}_{b'}}, A)$, such that $E \cap \boldsymbol{Fm}_{\mathcal{L}} = E' \cap \boldsymbol{Fm}_{\mathcal{L}}$.*

Now it follows from Theorems 4.1 and 4.3 that, under the *domain closure assumption*, when dealing with uniterm default theories we may restrict ourselves to finite bases.

[5] Cf. (2).

Theorem 4.4 *Let (D, A) be a uniterm default theory, such that for some constants $a_1, \ldots, a_m$, $A \vdash \forall x \bigvee_{i=1}^{m} x = a_i$. There exist a finite set of new constant symbols b', such that for each infinite set of new constant symbols b and each $E \subseteq \boldsymbol{Fm}_{\mathcal{L}}$, E is a b-extension for (D, A) if and only if it is a b'-extension for (D, A).*

An immediate corollary to Theorem 4.4 is that under the domain closure assumption extensions for uniterm default theories are computable.

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