

2012 AASRI Conference on Computational Intelligence and Bioinformatics

## A Differential Evolutionary Algorithm for Flatness Error Evaluation

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### Abstract

A differential evolutionary algorithm (DE) is proposed to implement the minimum zone evaluation of flatness errors. It is a heuristic evolutionary algorithm based on population optimization. Compared with other methods, it is efficient and robust. Then, the objective function calculation approaches of planar error are developed, which directly originate from the definition of minimum zone solution and conform to the ISO standard. Finally, the experimental results evaluated by different methods confirm the effectiveness of the proposed DE. Compared to conventional evaluation methods, it has the advantages of algorithm simplicity and good flexibility.

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Selection and/or peer review under responsibility of American Applied Science Research Institute

Keywords: Intelligent Computation; Differential Evolutionary; Flatness Error; Minimum Zone Solution

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### 1. Introduction

The plane feature is one of the most basic geometric primitives which contributes significantly to fundamental machinery parts to achieve intended functionalities. Flatness error is of prime importance to<sup>†</sup> quality and lifetime of mechanical products. In the current industrial practice, standards are followed for representation and interpretation of geometric tolerances. The ANSI Dimensioning and Tolerance Standard

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Y14.5 [1] specifies that the form tolerances on a component must be evaluated with reference to an ideal geometric feature, ISO Standards [2] recommends the form tolerance being evaluated based on the concept of minimum zone. As for the flatness tolerance, it can be defined as the least possible value of the normal distance between two parallel planes that enclose all measurement points.

One of the most widely used techniques for geometric error evaluation is the least squares method (LSM) and. It is easy for implementation, efficient in computation, and commonly used in coordinate measuring machines (CMMs). However, the LSM does not guarantee the minimum zone solution specified in the ISO standard and can result in overestimation of form tolerances and the rejection of good parts.

In order to obtain the minimum zone solution, Murthy and Abdin [3] proposed a simplex search technique. Fukuda and Shimokohbe [4] suggested a minimax approximation method, by which a subset of the measured points that defines a zone of the feature is sequentially replaced until the minimum zone is achieved. The nonlinear optimization is used by Wang [5], Kanada and Suzuki [6]. Weber et al. [7] proposed a unified linear approximation technique for use in evaluating the forms of straightness, flatness, circularity and cylindricity. Non-linear equation for each form was linearized using Taylor expansion, and then it was solved as a linear program. Zhu and Ding [8] established the equivalence between the width of a point set and the inner radius of the convex hull of the Minkowski difference between the point set and itself. The minimum zone tolerance was formulated as a linear programming problem. Hermann [9] proposed the algorithms based on techniques borrowed from computational geometry.

The minimum zone association of flatness error evaluation is a non-convex problem, and it can be formulated as nonlinear optimal problem with complex constraint. Therefore, this evaluation method is virtually the optimization problem. Intelligent computations, such as the genetic algorithm (GA), improved genetic algorithm (IGA), hybrid optimization algorithm and particle swarm optimization technique, were used to solve the nonlinear problem of flatness evaluation [10-13]. Examples had proved that above methods were effective to compute the minimum zone flatness error, but to most methods above, some parameters need to be decided by experience in advance. This paper suggests the evaluation algorithm of roundness of a novel differential evolution (DE) algorithm to find the value of flatness error according to the minimum zone criterion. DE is another very effective intelligent computation method for solving optimization problems with non-convex and nonlinear characteristics as it does not require derivative information. DE for flatness error is a simple yet powerful optimizer with fewer parameters.

The rest of this paper is organized as follows: The mathematical model of flatness error is presented in Section 2. The DE is developed in section 3. Examples are presented in section 4. Finally, conclusions are given in Section 5.

## 2. Mathematical model of flatness error evaluation

Assuming  $P_i(x_i, y_i, z_i)$  ( $i = 1, 2, \dots, n$ ) is the measured point extracted by measuring a plane part. A flatness tolerance specifies a tolerance zone defined by two parallel planes within which the surface must lie [14]. If all extracted data points  $P_i(x_i, y_i, z_i)$  are between two parallel planes, the minimum separation between these two parallel planes is called the minimum zone solution (MZS) of flatness error. Assuming one of the two parallel plane equations of MZS is

$$z = ax + by + c \quad (1)$$

The distance  $d_i$  from data points  $P_i(x_i, y_i, z_i)$  to the parallel plane is

$$d_i = \frac{z_i - ax_i - by_i - c}{\sqrt{1 + a^2 + b^2}} \quad (2)$$

The minimum separation  $f$  between these two parallel planes is

$$f = \min (\max(d_i) - \min(d_i)) = \min \left( \max \left( \frac{z_i - ax_i - by_i}{\sqrt{1 + a^2 + b^2}} \right) - \min \left( \frac{z_i - ax_i - by_i}{\sqrt{1 + a^2 + b^2}} \right) \right) \quad (3)$$

Obviously, the minimum separation  $f$  is a function of  $(a, b)$ . Consequently, evaluating the minimum zone flatness error is translated into searching the values of  $(a, b)$ , so that the separation  $f(a, b)$  is the minimum and this minimum value is just the flatness error. It is a non-linear optimization problem.

### 3. DE

#### 3.1. DE ALGORITHM.

DE algorithm aims at evolving a population of  $NP$   $D$ -dimensional parameter vectors, so-called individuals, which encode the candidate solutions, i.e.  $X_{i,G} = \{x_{i,G}^1, \dots, x_{i,G}^D\}$ ,  $i=1, \dots, NP$ , as a population for each generation  $G$ , towards the global optimum.  $NP$  does not change during their initialization process. The initial vector population is chosen randomly and should cover the entire parameter space. As a rule, we will assume the initial population should better cover the entire search space as much as possible by uniformly randomizing individuals within the search space constrained by the prescribed minimum and maximum parameter bounds  $X_{\min} = \{x_{\min}^1, \dots, x_{\min}^D\}$  and  $X_{\max} = \{x_{\max}^1, \dots, x_{\max}^D\}$ .

#### 3.2. Mutation Operation.

After initialization, DE employs the mutation operation to produce a mutant vector  $V_{i,G}$  with respect to each individual  $X_{i,G}$ , so-called target vector, in the current population. For each target vector  $X_{i,G}$  at the generation  $G$ , its associated mutant vector  $V_{i,G} = \{v_{i,G}^1, v_{i,G}^2, \dots, v_{i,G}^D\}$ , can be generated via certain mutation strategy. For example, the five most frequently used mutation strategies implemented in the DE codes are following: “DE/rand/1”, “DE/best/1”, “DE/rand-to-best/1”, “DE/best/2”, “DE/rand/2” [15].

#### 3.3. Crossover Operation.

After the mutation phase, crossover operation is applied to each pair of the target vector  $X_{i,G}$  and its corresponding mutant vector  $V_{i,G}$  to generate a trial vector:  $U_{i,G} = (u_{i,G}^1, u_{i,G}^2, \dots, u_{i,G}^D)$ . In the basic version, DE employs the binomial (uniform) crossover defined as follows:

$$u_{i,G}^j = \begin{cases} v_{i,G}^j, & \text{if } (\text{rand}_j[0,1] \leq CR) \text{ or } (j = j_{\text{rand}}) \\ x_{i,G}^j, & \text{otherwise} \end{cases} \quad j=1, 2, \dots, D \quad (4)$$

In (4), the crossover rate  $CR$  is a user-specified constant within the range  $[0,1]$ , which controls the fraction of parameter value copied from the mutant vector.  $j_{\text{rand}}$  is a randomly chosen integer in the range  $[1,D]$ . The binomial crossover operator copies the  $j$ th parameter of the mutant vector  $V_{i,G}$  to the corresponding element in the trial vector  $U_{i,G}$ , if  $\text{rand}_j[0,1] \leq CR$ , or  $j = j_{\text{rand}}$ . Otherwise, it is copied from the corresponding target vector  $X_{i,G}$ . There exists another exponential crossover operator, in which the parameters of trial vector  $U_{i,G}$  are inherited from the corresponding mutant vector  $V_{i,G}$  starting from a randomly chosen parameter index till the first time  $\text{rand}_j[0,1] > CR$ . The remaining parameters of trial vector  $U_{i,G}$  are copied from the corresponding target vector  $X_{i,G}$ . The condition  $j = j_{\text{rand}}$  is introduced to ensure that the trial vector  $U_{i,G}$  will differ from its corresponding target vector  $X_{i,G}$  by at least one parameter.

### 3.4. Selection Operation

If the values of some parameters of a newly generated trial vector exceed the corresponding upper and lower bounds, we randomly and uniformly reinitialize them within the prespecified range. Then the objective function values of all trial vectors are evaluated. After that, a selection operation is performed. The objective function value of each trial vector  $f(U_{i,G})$  is compared to that of its corresponding target vector  $f(X_{i,G})$  in the current population. If the trial vector has less or equal objective function value than the corresponding target vector, the trial vector will replace the target vector and enter the population of the next generation. Otherwise, the target vector will remain in the population for the next generation. The selection operation can be expressed as follows:

$$X_{i,G}^j = \begin{cases} U_{i,G}, & \text{iff}(U_{i,G}) \leq f(X_{i,G}) \\ X_{i,G}, & \text{otherwise} \end{cases} \quad (5)$$

The above 3 steps are repeated generation after generation until some specific termination criteria are satisfied.

### 3.5. DE for flatness error evaluation.

Input: Parameters of the algorithm for flatness error evaluation.

Step1. Creating initial population

Set the generation number  $G=0$ ,  $P_G = \{X_{1,G}, \dots, X_{NP,G}\}$ , and  $X_{i,G} = \{x_{i,G}^1, \dots, x_{i,G}^D\}$ ,  $i=1, 2, \dots, NP$

Step 2. Evaluate  $P_{G=0}$  and let the generation counter  $t=0$ .

Step 3. While (the stopping criterion is not satisfied) do

{For each individual  $X_{i,G=t}$ ,  $i=1, 2, \dots, NP$ , its offspring  $X_{i,G=t+1}$  is generated by mutation, crossover and selection operations. Evaluate  $P_{G=t+1}$  and let  $t = t + 1$ .

} End While

Output: The best solution  $X_{best,G=t}$ , the objective function  $f$  from  $P_{G=t}$ .

## 4. Practical examples

In this study, the strategy “DE/best/1 is selected as the best performance from candidate mutation strategies and we use the following parameter values:  $D=2$ ; the population size  $NP=10*D$ , the scale factor  $F=0.85$  and the crossover probability  $CR=0.9$ . Using the DE to evaluate flatness error, the termination condition is set the maximum iteration 100. For comparative purpose, two typical examples available are selected.

**Example 1.** The sampling data is available in literature [11]. The results are tabulated in Table 1. From Table 1 it is observed that the flatness error of LSC and IGA are 0.0219 and 0.01839, respectively in literature [11], and the flatness error obtained by canonical GA with elitist selection (EGA) are 0.02161, While the flatness error obtained by the DE is 0.01838. It is well in agreement with the literature [11].

**Example 2.** The measurement data from the plane surface are given in literature [8]. The results are tabulated in Table 2. It is observed that the flatness error of LSC is 0.00303, and the results obtained by the optimization procedure in literature [8] is 0.002627, and the results obtained by the EGA are 0.002739, while the flatness error obtained by the DE is 0.002627. It is well in agreement with the literature [8].

Table1 Evaluation results of data Example 1

Calculation method	LSM	EGA	Literature [11]	DE
Flatness error(mm)	0.0219	0.02161	0.01839	0.01838

Table2 Evaluation results of data Example 2

Calculation method	LSM	EGA	Literature [8]	DE
Flatness error(mm)	0.00303	0.002739	0.002627	0.002627

In the EGA, the population was set 40, the possibilities of crossover and mutation was set 0.90 and 0.20, respectively, and the maximum iteration was 200. The presented method for two examples not only provided a more accurate and stable solution, but also it took less time than EGA. Figs.1-2 showed the searching processes of the minimum zone flatness error of two examples.

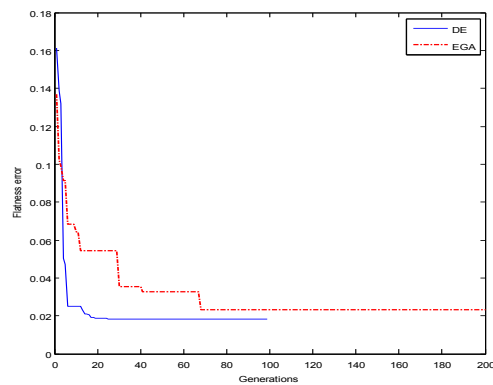


Fig. 1. Optimizing process of Example 1. by EGA and DE

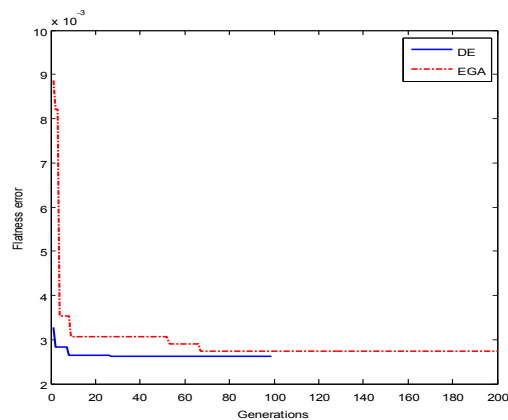


Fig. 2. Optimizing process of Example2. by EGA and DE

## 5. Conclusions

In this paper, an effective DE is presented to implement the flatness error evaluation of minimum zone method. The objective function mathematical model of minimum zone method is formulated and the initial population generated methods is given. Compared with conventional evaluation methods, the proposed algorithm has the advantages of algorithm simplicity and good flexibility. The typical practical examples are representative in industrial application and their results verified that the proposed method could search the optimal solution efficiently. And it is well suited for form error evaluation in CMMs.

## Acknowledgements

The research was supported by National Natural Science Foundation of China under Grant No. 51075198 and Innovation Foundation of Nanjing Institute of Technology under Grant No. CKJ2011004.

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