Electronic Notes in Theoretical Computer Science 53 (2001)
URL: http://www.elsevier.nl/locate/entcs/volume53.html 12 pages

The dynamics of information exchange dialogues

Balder ten Cate [1]

Institute for Logic, Language and Computation
University of Amsterdam
Amsterdam, The Netherlands

Abstract

A simple model of cooperative information exchange between multiple participants is defined, using propositional update semantics. The model is presented in the form of a reduction system. Several properties of the system are proven, concerning confluence, normalisation, properties of normal forms and reduction strategies. We end by discussing possible applications and extensions.

1 Introduction

Dynamic semantics (in its broad sense) is characterized by the fact that (1) the update effects of utterances plays a central role, and (2) there is a shift of attention from individual sentences to discourse. For this reason, it lends itself well as a basis for the formal analysis of the pragmatics of information exchange dialogues. In this paper, we discuss a simple model of the "game of cooperative information exchange", based on propositional update semantics. As was suggested in [10], the model is formulated as an abstract reduction system.

Our starting point is Stalnaker's theory of assertions. In making assertions, people take some of their private information and make it common ground. In terms of possible world semantics, they eliminate possibilities from the common ground, which they know not to be the case [9].

At any point in a dialogue, people make choices concerning which information to exchange (which possibilities to eliminate). Typically, there are many alternatives to choose from. Notwithstanding this apparent divergence, there is a clear direction in which the conversation proceeds. During the conversation, the information states of the participants grow more and more alike.

[1] Email: b.ten.cate@hum.uva.nl

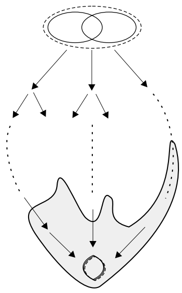


Fig. 1. Process of information exchange

This process converges to a (hypothetical) situation in which there is no information left to communicate, i.e., all available information is common ground (i.e., the information that used to be distributed between them). Of course, this situation is never reached in practice.[2]

If we assume for the moment that there are only two participants involved in the information exchange, then these considerations are reflected in Figure 1. The nodes of the graph represent states (i.e., Stalnakarian Contexts) and the arrows represent possible utterances, being transitions from one state to another.

The top three ovals together make up one state. In this state, the two agents each have certain information, and certain information is common ground between them.[3] At the "end state" (at the bottom of the picture) all three circles coincide. This corresponds to the hypothetical state that we discussed, in which each agent has the same information, and this information is common ground.[4]

In information exchange dialogues, the participants typically have a specific goal, e.g. to resolve a decision problem [15]. We can think of such a goal as a set of states. Then, the outlined area in the bottom half of the picture represents a possible goal (the goal being to reach one of these states).

Various pragmatic notions fit in this picture. For instance, *relevance* can be analysed as a strategy for achieving a goal. Intuitively, irrelevant utterances are utterances that will not help you to achieve your goal. This idea is further pursued in [11].

To formalise the picture of cooperative information exchange sketched here, we will combine notions from update semantics [16] and abstract reduction systems [8]. Abstract reduction systems (ARS's) form a field of research in

[2] This is an idealised model of information exchange. In practice, many complications arise (misunderstandings, mistakes, lying, etc.) that we will not address.
[3] The bigger circle corresponds to the common ground. This reflects the fact that any information that is common ground, is also private information of both agents.
[4] It is important to realise that Figure 1 contains only part of the general picture: only those states are depicted, that are reachable from the given starting state. The complete picture would contain all possible states as well as all possible utterances.Anticipating the discussion, in terms of reduction systems, Figure 1 gives the *reduction graph* of the state represented by the top three ovals.

theoretical computer science that is concerned with the reduction of terms or more abstract objects to a normal form. Combinatory logic and the lambda-calculus are the prime examples, but many other reduction systems have been devised, e.g. for *braids* and *knots*. For a general introduction, the reader is referred to [8] or [1].

The structure of the paper is as follows. First, we will formulate a model of cooperative information exchange, in the form of an ARS. Next, we will discuss some of its properties, concerning confluence, normalisation and normal forms. We will end with a discussion on possible applications and extensions.

2 Formulating the ARS

Defining an ARS involves two steps. First, one defines a set of objects and second, one defines a number of reduction relations over these objects. In our case, the objects will correspond to Stalnakerian "Contexts" or, as we will call them here, states. A state specifies the information that each agent has, as well as the information that is common ground between the agents.

We will keep things as simple as possible, and use ordinary propositional logic as our language. Let P be a propositional alphabet and $\mathcal{A}$ a finite set of agents. Let V be the set of all valuations over P. Then we define states as follows.

Definition 2.1 [States] A state is a function $\sigma : \mathcal{A} \cup \{c\} \to \wp(V)$ such that $\forall a \in \mathcal{A} : \sigma(a) \subseteq \sigma(c)$. Σ is the set of all states.

The c in the definition refers to the common ground. Notice that it is required that the common ground contains less information than each of the participants has, which is a very natural requirement.

We will write $\sigma \cap \tau$ for $\lambda a.(\sigma(a) \cap \tau(a))$ and $\sigma \subseteq \tau$ for $\forall a : \sigma(a) \subseteq \tau(a)$. [5]

In order to define the reduction relations, we need to introduce two central notions from update semantics: *support* and *update*. These notions are typically defined for individual information states. Here, we will reformulate them in terms of our multi-agent states.

Definition 2.2 [Support] $\sigma \Vdash_a \phi$ if $\forall v \in \sigma(a) : v \models \phi$

This definition states that agent a supports ϕ ("knows that ϕ") in state σ whenever ϕ is true in all the situations a considers possible (where ϕ is a propositional formula over the given alphabet P).

Definition 2.3 [Update] $\sigma + \phi = \lambda a.\{v \in \sigma(a) \mid v \models \phi\}$

According to this definition, when the agents update with a sentence ϕ, then all possibilities are eliminated in which ϕ is not the case (from the private

[5] This notation is justified by the fact that type-theoretically, the given definition is equivalent to one in which a state is a relation between agents and possibilities.

information states as well as from the common ground). It is easy to see that the set of all states Σ is closed under arbitrary updates.

Having *update* and *support* at our disposal, we can define the reduction relations of our ARS. Each reduction step will correspond to the assertion of a proposition by an agent.

Definition 2.4 [Reduction relations]

i. $\sigma \xrightarrow{a:\phi} \tau$ if $\sigma \Vdash_a \phi$ and $\sigma \nVdash_c \phi$ and $\tau = \sigma + \phi$

ii. $\sigma \xrightarrow{a} \tau$ if $\sigma \xrightarrow{a:\phi} \tau$ for some ϕ

iii. $\sigma \longrightarrow \tau$ if $\sigma \xrightarrow{a} \tau$ for some $a \in \mathcal{A}$

In a sense, these reduction relations specify the rules of the game. They specify when an agent can make an utterance and thereby reduce a state to another state: an agent can only make an utterance if he knows the content to be true, and it is not already common ground. The effect of the utterance is a public update with this information.[6]

The ARS that we will be concerned with in the rest of this paper is $\langle \Sigma, (\xrightarrow{a})_{a \in \mathcal{A}} \rangle$. Some notation conventions: let $\xrightarrow{\alpha}$ be any reduction relation. Then $\xrightarrow{\alpha}^{=}$ is the reflexive closure of $\xrightarrow{\alpha}$. $\xrightarrow{\alpha}\!\!\!\!\rightarrow$ is the reflexive, transitive closure of $\xrightarrow{\alpha}$. An object a is said to be in $\xrightarrow{\alpha}$-*normal form* if there is no b such that $a \xrightarrow{\alpha} b$. If $a \xrightarrow{\alpha}\!\!\!\!\rightarrow b$ and b is in normal form, then b is an $\xrightarrow{\alpha}$-normal form of a. Whenever we talk about normal forms without mentioning a specific reduction relation, then we mean $\longrightarrow$-normal forms.

Notice that the normal forms of our reduction relation $\longrightarrow$, being states that cannot be reduced any further, are precisely the hypothetical states that we discussed in the introduction (nobody has anything more to say).

3 Properties of the ARS

Having defined our ARS, we now turn to some of its properties. We will be mainly concerned with confluence, normalisation and properties of the normal forms (these are central concepts from the ARS theory).

Proposition 3.1

i. $\sigma \longrightarrow \tau \implies \tau \subset \sigma$

ii. The following diagrams hold.[7]

[6] As one can see, the role of the common ground c is to prevent the agents from repeating themselves. It is not possible to allow only utterances that are informative to some of the agents, for the speaker of a sentence does not know precisely the information state of the hearers. The only requirement that we can reasonably make is that when an utterance is made, the speaker considers it possible that the utterance is informative to some of the other agents. This is precisely what is done here, using the common ground.

[7] The diagrams should be read as follows: given any situation conform the solid arrows, we can extend it by drawing the dashed arrows. For instance, the second diagram says that

$$\begin{tikzcd} \cdot \arrow[r, "b:\psi"] \arrow[d, "a:\phi"] & \cdot \arrow[d, dashed, "a:\phi", "\equiv"'] \\ \cdot \arrow[r, dashed, "b:\psi", "\equiv"'] & \cdot \end{tikzcd}$$

(subcommutativity)

$$\begin{tikzcd} \cdot \arrow[r, dashed, "b:\phi\to\psi"] \arrow[d, "a:\phi"] & \cdot \arrow[d, dashed, "a:\phi"] \\ \cdot \arrow[r, "b:\psi"] & \cdot \end{tikzcd}$$

(swap)

$$\begin{tikzcd} \cdot \arrow[r, two heads, "a"] \arrow[d, two heads, "a"] & \cdot \arrow[d, dashed, two heads, "a"] \\ \cdot \arrow[r, dashed, two heads, "a"] & \cdot \end{tikzcd}$$

(confluence of $\xrightarrow{a}$)

$$\begin{tikzcd} \cdot \arrow[r, two heads] \arrow[d, two heads] & \cdot \arrow[d, dashed, two heads] \\ \cdot \arrow[r, dashed, two heads] & \cdot \end{tikzcd}$$

(confluence of $\longrightarrow$)

iii. If σ is in $\xrightarrow{a}$-normal form and $\sigma \twoheadrightarrow \tau$ then τ is in $\xrightarrow{a}$-normal form.

Proof. *(i)* and the first two diagrams are straightforward. The other two diagrams and *(iii)* follow by tiling with the first two diagrams. □

The last diagram of the above proposition tells us that $\longrightarrow$ is confluent (or, "has the Church-Rosser property"). This implies that any state has at most one normal form. We will now investigate which states have such a normal form, and what it looks like. If the propositional alphabet P is finite, then the answer is as follows.

Proposition 3.2

i. If P is finite, then $\longrightarrow$ is Strongly Normalising.

ii. If P is finite, then $nf(\sigma) = \lambda a. \bigcap_{b\in\mathcal{A}} \sigma(b)$

iii. If P is finite, then σ is in normal form iff $\forall a \in \mathcal{A} : \sigma(a) = \sigma(c)$

Proof.

i. If P is finite, then so is V. Then by Proposition 3.1(i), there can be no infinite reduction sequence.

ii. Let $\tau = nf(\sigma)$. We must show that $\forall a : \tau(a) = \bigcap_{b\in\mathcal{A}} \sigma(b)$.

[$\subseteq$] Suppose $v \notin \bigcap_{b\in\mathcal{A}} \sigma(b)$. Then for some b, $v \notin \sigma(b)$. Because P is finite, every valuation can be described completely by a finite formula. Let ϕ be the complete description of v. Then $\sigma \Vdash_b \neg\phi$, so $\tau \Vdash_b \neg\phi$. But we know that τ is a normal form, so it must hold that $\tau \Vdash_c \neg\phi$. So, $v \notin \tau(c)$, so $v \notin \tau(a)$.

[$\supseteq$] Suppose $v \in \bigcap_{b\in\mathcal{A}} \sigma(b)$. By induction on the length of the reduction sequence from σ to τ, we can show that $v \in \bigcap_{b\in\mathcal{A}} \tau(b)$. From this it follows that $v \in \tau(a)$ for any a.

iii. σ is in normal form iff $\sigma = nf(\sigma)$. If P is finite, that means that σ is in normal form iff $\forall a : \sigma(a) = \bigcap_{b\in\mathcal{A}} \sigma(b)$. This is equivalent to saying that $\forall a \in \mathcal{A} : \sigma(a) = \sigma(c)$.

□

In plain words, these results can be described as follows. If the propositional alphabet is finite, every state reduces to a normal form, and always in a finite number of steps. This normal form is precisely the state in which all all agents have the information that used to be distributed between them

---

for all states σ and τ, if $\sigma \xrightarrow{a:\phi} \xrightarrow{b:\psi} \nu$ then $\sigma \xrightarrow{b:\phi\to\psi} \xrightarrow{a:\phi} \tau$.

(i.e. the intersection of the old information states of the agents) and all this information is also common ground.

However, this is all under the assumption that the propositional alphabet is finite. In the general case, things are a bit more complicated. In general, we do not have Strong Normalisation (in fact, some states might not even *have* a normal form). This means that if the agents keep on exchanging information, this process might go on forever. And even if the process ends in a normal form, it is not guaranteed that in this final situation, all the agents have the same information. It might happen that one agent has more information then another, but he cannot communicate this information, because the language is not expressive enough (only finite sentences might be uttered). The following example illustrates this.

Example 3.3 Let $P = \{p_n \mid n \in \mathbb{N}\}$ and $\mathcal{A} = \{a, b\}$. Furthermore, let state σ be such that $\sigma(a) = \sigma(c) = V$ (i.e., all valuations) and $\sigma(b) = \{v \in V \mid v(p) = 1 \text{ for some } p \in P\}$. Then σ is in normal form, even though agent b has more information then agent a.

In order to give precise characterisations of the normal forms of our system, we need to introduce the auxiliary notion of *saturation*. This is defined in terms of ultrafilters, cf. [3].

Definition 3.4 [Saturation] $\hat{\sigma} = \lambda a.\{v \in V \mid \exists u \in \mathrm{Uf}(V) : \sigma(a) \in u \ \&\ \forall p \in P : \{w \mid w(p) = v(p)\} \in u\}$

Intuitively, $\hat{\sigma}$ is a copy of σ in which some information is lost (some worlds are added to the information states of the agents). The information that is lost, is precisely the inexpressible information that was causing us trouble. Consequently, in $\hat{\sigma}$, all information that is available to the agents is expressible in the language.

Formally, $\hat{\sigma}$ is the smallest saturated superset of σ. Furthermore, $\hat{\sigma}$ is equivalent to σ, in the sense that $\hat{\sigma} \Vdash_a \phi$ precisely if $\sigma \Vdash_a \phi$.

Proposition 3.5

i. $\sigma \subseteq \hat{\sigma}$

ii. *$\hat{\sigma}$ is saturated (i.e., "finite satisfiability implies satisfiability")*

iii. $\forall \tau : \sigma \subseteq \tau$ *&* τ *is saturated* $\Longrightarrow \hat{\sigma} \subseteq \tau$

iv. $\hat{\sigma} \Vdash_a \phi \iff \sigma \Vdash_a \phi$

Proof.

i. Suppose $v \in \sigma(a)$. Let π_v be the principal ultrafilter of v, i.e., $\{X \subseteq V \mid v \in X\}$. Then $\sigma(a) \in \pi_v$ and $\forall p \in P : \{w \mid w(p) = v(p)\} \in \pi_v$. Therefore, $v \in \hat{\sigma}(a)$.

ii. Suppose θ is finitely satisfiable in $\hat{\sigma}(a)$. Then $\{\{w \mid w \models \phi\} \mid \phi \in \theta\} \cup \{\sigma(a)\}$ has the finite intersection property and can therefore be extended to an ultrafilter u. Now let v be such that $v(p) = 1$ iff $\{w \mid w(p) = 1\} \in u$.

Then v and u meet the specified requirements, and therefore $v \in \hat{\sigma}(a)$. Furthermore, v satisfies θ (by induction on the length of the formulae in θ).

iii. Suppose $\sigma \subseteq \tau$ and τ is saturated. Suppose $v \in \hat{\sigma}(a)$. Then by definition, there is an ultrafilter $u \in Uf(V)$ such that $\sigma(a) \in u$ and $\forall p \in P : \{w \mid w(p) = v(p)\} \in u$. Let θ be the theory of v. Then θ is satisfiable in $\hat{\sigma}(a)$. Because we know that u has the finite intersection property, it follows by induction on the formulae in θ that θ is finitely satisfiable in $\sigma(a)$. But we know that $\sigma \subseteq \tau$, so then θ is also finitely satisfiable in $\tau(a)$. And by saturatedness of τ, θ must be satisfiable in τ. This can only be if $v \in \tau$.

iv. By induction on ϕ.

□

Using *saturation*, we can generalise our results to the case where P is infinite: if a state σ has a normal form, then it is $\lambda a.\big(\sigma(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}(b)\big)$. However, as was already noted, there can be states which do not have a normal form. In general, in the absence of normal forms, *cofinal reduction sequences* can play a similar role, functioning as a kind of infinitary version of normal forms (cf. [7]). A reduction sequence $a_1 \longrightarrow a_2 \longrightarrow a_3 \longrightarrow \ldots$ is called cofinal if it holds that $\forall b : a_1 \twoheadrightarrow b \implies \exists i : b \twoheadrightarrow a_i$. In our case, we will show that any cofinal reduction sequence starting with σ converges to $\lambda a.\big(\sigma(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}(b)\big)$.

Proposition 3.6

i. *If σ has a normal form, then $nf(\sigma) = \lambda a.\big(\sigma(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}(b)\big)$.*

ii. *σ is in normal form iff $\forall a \in \mathcal{A} : \hat{\sigma}(a) = \hat{\sigma}(c)$*

iii. *If $\sigma_1 \longrightarrow \sigma_2 \longrightarrow \ldots$ is a cofinal reduction sequence, then $\bigcap_i \sigma_i = \lambda a.\big(\sigma_1(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}_1(b)\big)$.*

Proof.

i. Let $\tau = nf(\sigma)$. I.e., $\sigma \twoheadrightarrow \tau$ and τ is in normal form. We need to show that $\forall a : \tau(a) = \sigma(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}(b)$.

[$\subseteq$] Suppose $v \in \tau(a)$. Then by Proposition 3.1(i), $v \in \sigma(a)$. Also $v \in \tau(c)$. Let θ be the theory of v. Then θ is satisfiable, and therefore also finitely satisfiable, in $\tau(c)$. Take any agent $b \in \mathcal{A}$. Because τ is in normal form, we know that $\forall \phi : \tau \Vdash_b \phi \implies \tau \Vdash_c \phi$. From this it follows that θ is also finitely satisfiable in $\tau(b)$. Then by Proposition 3.1(i), θ is also finitely satisfiable in $\sigma(b)$. So, θ is satisfiable in $\hat{\sigma}(b)$. This can only be iff $v \in \hat{\sigma}(b)$.

[$\supseteq$] By induction on the length of the reduction.

ii. [$\Rightarrow$] Suppose σ is in normal form and $v \in \hat{\sigma}(a)$. Let θ be the theory of v. Then θ is finitely satisfiable in $\hat{\sigma}(a)$ and therefore also in $\sigma(a)$. Because σ is in normal form, we have that $\forall \phi : \sigma \Vdash_a \phi \implies \sigma \Vdash_c \phi$. So, θ is also finitely satisfiable in $\sigma(c)$. But then, θ is finitely satisfiable in $\hat{\sigma}(c)$ and therefore also satisfiable in $\hat{\sigma}(c)$. This can only be if $v \in \hat{\sigma}(c)$.

[$\Leftarrow$] Suppose $\forall a \in \mathcal{A} : \hat{\sigma}(a) = \hat{\sigma}(c)$ and suppose $\sigma \Vdash_a \phi$. Then $\hat{\sigma} \Vdash_a \phi$. But then also $\hat{\sigma} \Vdash_c \phi$, and then $\sigma \Vdash_c \phi$. So, we can conclude that $\forall a, \phi : \sigma \Vdash_a \phi \implies \sigma \Vdash_c \phi$. So σ is in normal form.

iii. Suppose $\sigma_1 \longrightarrow \sigma_2 \longrightarrow \ldots$ is cofinal. We must show that $\forall a : (\bigcap_i \sigma_i)(a) = \sigma_1(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}_1(b)$.

[$\subseteq$] Suppose $v \notin \sigma(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}_1(a)$. Then there are two cases. The first case is that $v \notin \sigma_1(a)$. In that case, trivially $v \notin (\bigcap_i \sigma_i)(a)$. The second case is that $v \in \sigma_1(a)$ and $v \notin \bigcap_{b \in \mathcal{A}} \hat{\sigma}_1(b)$. Then there must be some $b \in \mathcal{A}$ such that $v \notin \hat{\sigma}_1(b)$. We know that $v \in \sigma_1(c)$. Let θ be the theory of v. θ is not satisfiable in $\hat{\sigma}_1(b)$ and therefore also not finitely satisfiable in $\hat{\sigma}_1(b)$. So, there is a finite set $\theta' \subseteq \theta$ such that θ' is not satisfiable in $\hat{\sigma}_1(b)$. Let ϕ be $\neg \bigwedge_{\psi \in \theta'} \psi$. Then $\hat{\sigma}_1 \Vdash_b \phi$ and therefore $\sigma_1 \Vdash_b \phi$. But $\sigma_1 \not\Vdash_c \phi$. So, $\sigma_1 \longrightarrow (\sigma_1 + \phi)$ and $v \notin (\sigma_1 + \phi)(a)$. By cofinality and Proposition 3.1(i), $v \notin \sigma_i(a)$ for some i. So, $v \notin \bigcap_i \sigma_i$.

[$\supseteq$] By induction on i it can be shown that $\forall i \forall a : \sigma_i(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}_i(b)) = \sigma_1(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}_1(b)$. Now suppose $v \in \sigma_1(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}_1(b)$. Then it follows that $\forall i : v \in \sigma_i(a)$. So, $v \in (\bigcap_i \sigma_i)(a)$.

□

Recapitulating, we have that if P is finite, then $\longrightarrow$ is cofinal and every state σ has normal form $\lambda a. \big(\sigma(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}(b)\big)$ (which is, in that case, equal to $\lambda a. \bigcap_{b \in \mathcal{A}} \sigma(b)$). Moreover, even if σ does not have a normal form, any *cofinal reduction sequence* starting with σ does converge to $\lambda a. \big(\sigma(a) \cap \bigcap_{b \in \mathcal{A}} \hat{\sigma}(b)\big)$.

As we will see next, if P is countable, then for every state there *is* such a cofinal reduction sequence. A fortiori, if the agents all follow certain strategies, the resulting reduction sequence is guaranteed to be cofinal (which means that any information that could be exchanged *is* exchanged at some point). To make this claim precise, consider the following standard definition from rewriting literature [8].

Definition 3.7 A sequential reduction strategy for $\xrightarrow{\alpha}$ is a map $\mathbb{F}$ such that

- $\sigma = \mathbb{F}(\sigma)$ if σ is in $\xrightarrow{\alpha}$-normal form, and
- $\sigma \xrightarrow{\alpha} \mathbb{F}(\sigma)$ otherwise.

Unlike the typical case in rewriting, we are concerned with several agents, each of which has their own strategy. Therefore, we need to define *strategies profiles* as tuples of strategies, one for each agent.

Definition 3.8 [Strategy profiles] A strategy profile $\mathcal{F}$ is a tuple $(\mathbb{F}_a)_{a \in \mathcal{A}}$ such that for each agent $a \in \mathcal{A}$, $\mathbb{F}_a$ is a sequential reduction strategy for $\xrightarrow{a}$.

We must also require that the strategy of one agent does not make reference to the private information of another agent. Strategy profiles that have this property are called *realistic*.

Definition 3.9 [Realistic strategy profiles] A strategy profile $\mathcal{F}$ is *realistic* if for all agents $a \in \mathcal{A}$ and states $\sigma, \tau \in \Sigma$ it holds that if $\sigma(a) = \tau(a)$ and $\sigma(c) = \tau(c)$ then $\mathbb{F}_a(\sigma) = \mathbb{F}_a(\tau)$

Essentially, a strategy profile is realistic if the individual stategies of the players do not distinguish between states that should indistinguishable to them (i.e., in which they have the same information).

Given that the agents follow a certain strategy profile and given a starting state, we can ask ourselves what happens if the agents start to communicate in accordance to their strategies. In general, the result will be a dialogue (that is, a reduction sequence), but the precise outcome depends on the particular system of turn taking that is used: although we know the strategies of the individual players, we don't know yet which player is at turn when.

At this point, it seems most sensible to make only the minimal requirement of *fairness*: every agent should have the oppurtunity to say something every once in a while. So, we define an $\mathcal{F}$-dialogue to be a reduction sequence that is both in accordance to $\mathcal{F}$ and fair.

Definition 3.10 [Dialogue] An $\mathcal{F}$-dialogue is a reduction sequence $\sigma_1 \longrightarrow \sigma_2 \longrightarrow \ldots$ such that

i. $\forall i \exists a \in \mathcal{A} : \sigma_{i+1} = \mathbb{F}_a(\sigma_i)$

ii. $\forall a \in \mathcal{A} \forall i \exists j \geq i$: either $\sigma_{j+1} = \mathbb{F}_a(\sigma_j)$ or σ_j is in $\xrightarrow{a}$-normal form.

The first of these two conditions specifies that the reduction sequence must be in accordance to the strategy profile: every step is in accordance with the strategy of some agent. The second condition expresses fairness: every agents should say something every once in a while (unless he has nothing left to say). Formally, the latter requirement says that for any agent and at any moment, there should be a later moment at which the agent either makes an utterance or has nothing left to say.

Let us call a strategy profile $\mathcal{F}$ *cofinal* if all $\mathcal{F}$-dialogues are cofinal. Then we can prove the following.

Proposition 3.11

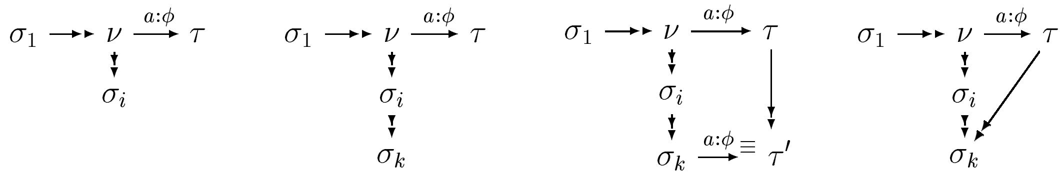
i. If P is finite, then every strategy profile is cofinal.

ii. If P is countable, then there are cofinal realistic strategy profiles.

Proof.

i. If P is finite, then $\longrightarrow$ is SN, so every reduction sequence is finite. But then any dialogue of any strategy profile must end in a normal form.

ii. If P is countable, then so is the set of all formulas over P. Let f be an injective mapping from formulae to natural numbers. Now define $\mathbb{F}_a(\sigma) = \sigma + \phi$, where ϕ is the f-lowest formula such that $\sigma \Vdash_a \phi$ and $\sigma \nVdash_c \phi$, or $\mathbb{F}_a(\sigma) = \sigma$ otherwise. Let $\mathcal{F} = (\mathbb{F}_a)_{a \in \mathcal{A}}$. We will show that every $\mathcal{F}$-dialogue is cofinal.



(a) (b) (c) (d)

Fig. 2. Steps in the proof of cofinality

Suppose $\sigma_1 \longrightarrow \sigma_2 \longrightarrow \ldots$ is an $\mathcal{F}$-dialogue. We must prove that for all τ, $\sigma_1 \longrightarrow\!\!\!\!\!\rightarrow \tau \implies \exists i : \tau \longrightarrow\!\!\!\!\!\rightarrow \sigma_i$. We do this by induction on the length of the reduction from σ_1 to τ. The base case (where $\sigma_1 = \tau$) is trivial. Now for the inductive step, suppose $\sigma_1 \longrightarrow\!\!\!\!\!\rightarrow \xrightarrow{a:\phi} \tau$. By applying the inductive hypothesis, we have the situation depicted in Figure 2(a).

We know that $\nu \Vdash_a \phi$, and so by monotonicity, $\forall j \geq i : \sigma_j \Vdash_a \phi$.

We also know that either there is a $j \geq i$ such that σ_j is in $\xrightarrow{a}$-normal form, or there are infinitely many $j \geq i$'s such that $\sigma_{j+1} = \mathbb{F}_a(\sigma_j)$. In the first case, let $k = j$. In the second case, as there are only finitely many formulae ψ with $f(\psi) \leq f(\phi)$, there must be a k such that $\sigma_k \Vdash_c \phi$. In either case, we have a k such that $\sigma_k \Vdash_c \phi$ (cf. Figure 2(b)).

By tiling with the subcommutativity-diagram, we come to situation 2(c). But by construction, $\sigma_k \Vdash_c \phi$, so it cannot be the case that $\sigma_k \xrightarrow{a:\phi} \tau'$. Therefore, $\sigma_k = \tau'$ (cf. Figure 2(d)). This concludes our proof.

□

As a corrolary, we have that if P is countable, then for every state, there is a cofinal reduction sequence: any fair dialogue in accordance with a cofinal strategy profile will do.[8]

4 Conclusion and discussion

A model of cooperative information exchange was introduced, based on update semantics, as well as the theory of abstract reduction systems. The main focus was on the formal properties of the model. We took a very simple update semantics, namely one for propositional logic, where the worlds are propositional valuations. Also, the update mechanism is very simple: even when an assertion is made that contradicts the private information of some agents, the agents will still update with the information, ending up in an absurd information state. The reason for choosing such a naive model, was to have a "technically clean" basic framework that has clear formal properties and which is relatively easy to grasp. The model will serve as a basis for future extensions that will turn it into a more realistic model of information exchange.

[8] As can be easily proven, $\mathcal{F}$-dialogues exist for any strategy profile $\mathcal{F}$ and starting state.

Various extensions present themselves. In [10] a start was already made with a similar system on the basis of [5]'s QL, which is a predicate logical language extended with questions. Also, attention has been directed to the theory of dynamic epistemic semantics and public announcements [4,2]. At first sight, it seems quite straightforward to take the notion of public announcement and turn it into an ARS similar to the one presented here. However, many complications arise. One of the problems is that the resulting ARS is in general not confluent. [9] This topic is presently being investigated further.

In [11], some preliminary results are reported on the application of the present framework to the analysis of *relevance*. To this extent, a notion of *communicative goal* is introduced, and a notion of success relating strategies (or rather strategy profiles) to goals. In principle, such an approach can also be generalised to apply not only to assertions but also to questions, cf. [10].

Another possible application could be in the analysis of structural properties of strategies for dealing with inconsistent information (which also relates to the use of *corrective utterances* in discourse, cf. [14,6,12]).

Finally, it is interesting to observe that the link between information exchange and rewriting becomes even more apparent when we shift from the compositional/set-theoretic perspective of dynamic semantics to a more representational (DRT-style) one. In the latter case, information state are no longer sets of possible worlds, but rather formulas or DRT's. If we would proceed in this way, then our ARS starts to look much more like an actual *term*-rewriting system.

References

[1] Baader, F. and T. Nipkow, "Term rewriting and all that," Cambridge University Press, Cambridge, UK, 1998.

[2] Baltag, A., L. S. Moss and S. Solecki, *The logic of public announcements, common knowledge and private suspicions*, Technical report, CWI (1999).

[3] Blackburn, P., M. de Rijke and Y. Venema, "Modal logic," Cambridge University Press, Cambridge, UK, To appear.

[4] Gerbrandy, J., "Bisimulations on planet Kripke," Ph.D. thesis, ILLC, University of Amsterdam (1999).

[5] Groenendijk, J., *The logic of interrogation: classical version*, in: T. Matthews and D. Strolovitch, editors, *Proceedings of the ninth conference on semantics and linguistics theory (SALT-9), Santa Cruz* (1999).

[6] Groenendijk, J., M. Stokhof and F. Veltman, *Coreference and modality in multi-speaker discourse*, in: H. Kamp and B. Partee, editors, *Context Dependence in the Analysis of Linguistic Meaning*, I.M.S., Stuttgart, 1997 pp. 195–216.

[9] [13] has shown that if we restrict ourselves to finite Kripke structures, we do have confluence, as well as strong normalisation.

[7] Klop, J., "Combinatory Reduction Systems," Mathematical centre tracts **127**, Mathematisch Centrum, Amsterdam, 1980.

[8] Klop, J. W., *Term rewriting systems*, Handbook of Logic in Computer Science **2**, Oxford University Press, Oxford, 1992 pp. 1–117.

[9] Stalnaker, R., *Assertion*, Syntax and semantics **9, *Pragmatics***, Academic Press, New York, 1978 pp. 315–332.

[10] ten Cate, B., "Dynamic and epistemic semantics of questions: the logic of consultation," Master's thesis, Vrije Universiteit, Amsterdam (2000), available from `http://home.student.uva.nl/balder.tencate/`.

[11] ten Cate, B., *Information exchange as reduction*, in: K. Striegnitz, editor, *Proceedings ESSLLI 2001 Student Session*, 2001.

[12] ten Cate, B. and M. Nilsenová, *Representing model conversation failures with belief networks*, in: *Proceedings of the Twelfth Belgium-Netherlands Artificial Intelligence Conference BNAIC '01*, Amsterdam, To appear.

[13] van Benthem, J., *Philosophy 298: Information update from a logical perspective* (2000), available from `http://turing.wins.uva.nl/~johan/298-2000.html`.

[14] van Leusen, N., *The interpretation of corrections*, Technical Report LP-94-21, ILLC, Amsterdam (1994).

[15] van Rooy, R., *Decision problems in pragmatics*, in: D. Traum and M. Poesio, editors, *Proceedings of GÖTALOG 2000*, 2000.

[16] Veltman, F., *Defaults in update semantics*, Journal of Philosophical Logic **25** (1996), pp. 221–261.