Egyptian Informatics Journal (2013) 14, 147–155

Cairo University

Egyptian Informatics Journal

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ORIGINAL ARTICLE

An extension of fuzzy decision maps for multi-criteria decision-making

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Received 7 October 2012; revised 6 May 2013; accepted 19 May 2013
Available online 17 June 2013

KEYWORDS

Multi-Criteria Decision-Making (MCDM);
Analytic network process(ANP);
Analytical hierarchical process(AHP);
Fuzzy cognitive maps (FCM);
Fuzzy Decision Maps (FDM)

Abstract This paper presents a new extension to Fuzzy Decision Maps (FDMs) by allowing use of fuzzy linguistic values to represent relative importance among criteria in the preference matrix as well as representing relative influence among criteria for computing the steady-state matrix in the stage of Fuzzy Cognitive Map (FCM). The proposed model is called the Linguistic Fuzzy Decision Networks (LFDNs). The proposed LFDN provides considerable flexibility to decision makers when solving real world Multi-Criteria Decision-Making (MCDM) problems. The performance of the proposed LFDN model is compared with the original FDM using a previously published case study. The result of comparison ensures the ability to draw the same decisions with a more realistic decision environment.

1. Introduction

Multi-Criteria Decision-Making (MCDM) problems are common in everyday life. In personal context, a house or a car one buys may be characterized in terms of criteria such as: price, size, style, safety, and comfort. In business context, MCDM problems are more complicated and usually of large scale. Sometimes, MCDM problems are referred to as Multi-Criteria Decision Analysis (MCDA), Multi-Criteria Decision Aid (MCDA), or Multi-Attribute Decision-Making (MADM) [1,2].

MCDM refers to making preference decisions (such as evaluation, prioritization, selection, and so on) over the available alternatives that are characterized by multiple, usually conflicting criteria [1,3].

According to many researchers, there are two distinctive types of MCDM problems, namely, Multi-Objective Decision-Making (MODM) and Multi-Attribute Decision-Making (MADM) [1–3]. When the feasible set of alternatives of a decision consists of a finite number of elements that are explicitly known at the beginning of the solution process, these problems are called Multi-Attribute Decision-Making (MADM) [1,2].

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Peer review under responsibility of Faculty of Computers and Information, Cairo University.

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http://dx.doi.org/10.1016/j.eij.2013.05.001

When the number of alternatives of a decision is uncountably infinite, the alternatives are not specified directly but are defined in terms of decision variables as is usually done in single optimization problems like linear programming [1,2]. This type of problem is called a Multi-Objective Decision-Making (MODM).

Many different methods have been developed to solve MCDM. The Analytic Hierarchy Process (AHP) was first introduced by Thomas Saaty in the 1971, and it is one of the most famous methods of MCDM because it is simple and easy to use [4,5]. AHP has been applied to solve many decision problems in political, economic, social, and management sciences [5]. The Analytic Network Process (ANP) was developed by Saaty in 1996 to overcome the problem of dependence and feedback among criteria in AHP method [6]. ANP is considered as the general form of the AHP, which has been used for MCDM. The ANP has been applied in many practical decision-making problems, such as project selection, product planning, green supply chain management, and optimal scheduling problems [7].

Fuzzy Decision Maps (FDMs) method was proposed in 2006 to deal with the structure of MCDM with dependency and feedback to overcome the complexity drawback of ANP [7]. FDM method is a good technique for MCDM in real situations. In FDM, human judgments for performing the pairwise comparison and the causal relationship between criteria are usually given by exact numerical values. However, in many practical cases, numerical values are not suitable in the real world problems. Human judgments with preferences are often unclear and hard to be estimated by exact numerical values. A more sensible method is to use linguistic assessments instead of numerical values, in which all assessments of criteria in the problem are evaluated by means of linguistic values.

Fuzzy set theory was first proposed by Lotfi Zadeh in the 1965 [8]. It is a methodology to tackle uncertainty and to handle imprecise information in real situations. With fuzzy sets theory, information can be expressed by linguistic or fuzzy values rather than by numbers [9]. In this paper, we propose a methodology that extends the FDM method by applying linguistic variables and a fuzzy aggregation method. We call this method Linguistic Fuzzy Decision Networks (LFDNs).

This paper is organized as follows: Section 2 presents fuzzy numbers and linguistic variables. Fuzzy Cognitive Map (FCM) is discussed in Section 3. Section 4 presents the FDM method. The proposed LFDN is introduced in Section 5. A case study is given in Section 6. Discussing the results is given in Section 7, and Section 8 concludes the paper.

2. Fuzzy numbers and linguistic variables

A Fuzzy Number (FN) is a fuzzy set which has some properties such as being: convex and normal, its membership function is piecewise continuous and it is defined over $\Re$ [9]. FN can take various forms such as Triangular Fuzzy Number (TFN), Trapezoidal FN, and Gaussian FN. TFN is the most popular form due to its simple membership function which is represented by three parameters as (l, m, u). The parameters l, m, and u denote the smallest possible value, the most promising value, and the largest possible value of the fuzzy number, respectively. Let $\tilde{A} = (l, m, u)$ be a triangular fuzzy number, then its membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\text{Triangle}(x : l, m, u) = \begin{cases} 0 & x < l \\ \frac{(x-l)}{(m-l)} & l \le x \le m \\ \frac{(u-x)}{(u-m)} & m \le x \le u \\ 0 & x > u \end{cases} \tag{1}$$

The fuzzy arithmetic operations of two TFNs $\tilde{A}_1 = (l_1, m_1, u_1), \tilde{A}_2 = (l_2, m_2, u_2)$, and λ is positive real numbers are defined as follows [9].

$$\tilde{A}_1^{-1} = 1/\tilde{A} = (1/u_1, \quad 1/m_1, \quad 1/l_1) \quad \text{Where} \quad l_1, m_1, u_1 > 0 \tag{2}$$

$$\tilde{A}_1 + \tilde{A}_2 = (l_1 + l_2, \quad m_1 + m_2, \quad u_1 + u_2) \tag{3}$$

$$\tilde{A}_1 - \tilde{A}_2 = (l_1 - u_2, \quad m_1 - m_2, \quad u_1 - l_2) \tag{4}$$

$$\tilde{A}_1 / \tilde{A}_2 = (l_1/u_2, \quad m_1/m_2, \quad u_1/l_2) \tag{5}$$

$$\tilde{A}_1 \times \tilde{A}_2 = (l_1 \cdot l_2, \quad m_1 \cdot m_2, \quad u_1 \cdot u_2) \tag{6}$$

$$\lambda \times \tilde{A}_1 = (\lambda \cdot l_1, \quad \lambda \cdot m_1, \quad \lambda \cdot u_1), \quad \lambda > 0 \tag{7}$$

In some situations, it is required to reduce a given fuzzy number into a single crisp representative value. This is called defuzzification operation. There are several available defuzzification methods for this purpose such as: Mean Of Maximum (MOM) and centroid method (Center Of Gravity (COG) or Center Of Area (COA)) [9].

Lotfi Zadeh introduced the concept of linguistic variable in 1975, which allows computation with words instead of numbers [10,11]. Computing with words can be achieved when linguistic variables take linguistic values (i.e., values in the form of words or sentences from natural language). The concept of linguistic variables is very useful to describe situations that are too complex or not well defined in conventional quantitative expression. Furthermore, Linguistic variables allow the translation of natural language into logical or numerical statements, i.e., computing with words, which provides the tools for further theoretical application, such as fuzzy control and approximate reasoning which is also referred to as human reasoning [9,11].

3. Fuzzy Cognitive Map (FCM)

Fuzzy Cognitive Map (FCM) was originally introduced by Kosko in 1986 [12] as an extension of cognitive map model. The most significant enhancement lies in the way of reflecting causal relationships. Instead of using only the sign (+ or −),

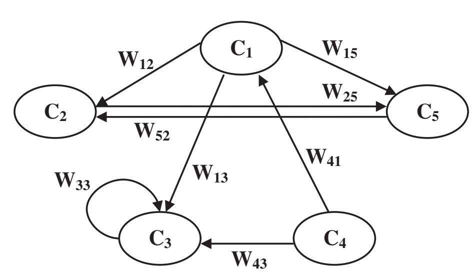

Figure 1 Fuzzy cognitive map [12].

each edge is associated with a number (weight) that determines the degree of considered causal relation between each two concepts. Fig. 1 illustrates a graphical representation of a FCM. It could be considered as a signed weighted graph that consists of nodes and weighted arcs (or edges) with feedback. Each node of the graph stands for a concept that describes one of the behaviors of the system. The value of node (concept) changes over time, usually over the interval $[0, 1]$. Concepts are connected by signed and weighted arcs (Edges) representing the causal relationships between the concepts. Edges or arcs weight usually take values over the fuzzy causal interval $[-1, 1]$ or the interval $[0, 1]$. According to [13], expert describes the causal relationship between concept by using linguistic variables or fuzzy terms that are defined over a bipolar intervals, i.e., $[-1, 1]$, rather than the usual binary one, i.e., $[0, 1]$. The linguistic value for each interconnection between two concepts is reduced to a single crisp value through a defuzzification process. Thus, the weight for any interconnection is simply crisp number that ranges over $[-1, 1]$. Consequently, three possible states arise for each edge weight, first when ($w_{ij} = 0$), which indicates no relationship between C_i and C_j, second, when ($w_{ij} > 0$) indicates direct causality between concepts C_i and C_j, that is, the increase (decrease) in the value of C_i leads to the increase (decrease) in the value of C_j, third, when ($w_{ij} < 0$) indicates inverse (negative) causality between concepts C_i and C_j. That is, the increase (decrease) in the value of C_i leads to the decrease (increase) in the value of C_j.

The influence of a specific concept to other concepts can be calculated using the following updating equation [7,14,15]:

$$C^{(t+1)} = f(C^{(t)} \cdot E), C^{(0)} = I_{n\times n} \tag{8}$$

where $I_{n\times n}$ denotes the identity matrix, $E = [W_{ij}]$ is $n \times n$ weight matrix, which gathers the values of causal edge weight between concepts C_i and C_j, $C^{(t+1)}$ and $C^{(t)}$ are the state matrices at iterations ($t + 1$) and (t), respectively, $C^{(0)}$ is the initial matrix, and f is a threshold transformation function that is used to normalize concept values to a certain binary or bipolar ranges [13]. There are several formulas used as threshold transformation functions such as [7,14,16].

$$\text{Hard limit function} \qquad f(x) = \begin{cases} 1 \ \mathit{if}\ x \geq 1 \\ 0 \ \mathit{if}\ x < 1 \end{cases}, \quad x \in [0, 1] \tag{9}$$

$$\text{Hyperbolic-tangent function} \qquad f(x) = \tan h(x) = (1 - e^{-x})/(1 + e^{-x}), \quad x \in [-1, 1] \tag{10}$$

$$\text{Logistic function} \quad f(x) = 1/(1 + e^{-x}), \quad x \in [0, 1] \tag{11}$$

When FCM starts execution, i.e., using the updating of Eq. (8), the resulting transformed state matrix is then repeatedly multiplied by FCM weight matrix and is transformed by a threshold function until the system (FCM) settles down to steady state which is one of the following cases[7,14,15].

- *A hidden pattern or fixed point attractor*: This case is reached when the FCM state matrix remains fixed for successive iterations, for example, $A_1 \to A_2 \to A_3 \to A_3$, where the matrix A_3 is known as the fixed point attractor.
- *A limit cycle*: A sequence of FCM state matrix keeps repeating forming a cycle, for example, $A_1 \to A_2 \to A_3 \to A_4 \to A_5 \to A_3 \to A_4 \to A_5$ where the three matrix $A_3 \to A_4 \to A_5$ forming a cycle is known as the limit cycle.

FCMs have advantages such as simplicity, flexibility, and efficacy for analyzing and modeling the real world as a collection of concept and causal relationships. This simplicity support experts or decision makers to better understand the underlying formal model and its execution [17]. The main disadvantage of FCMs is the possibility to reach or to settle down in an undesired steady state [18]. FCM have been applied in many applications such as, geographic information systems (GISs), fault detection, decision-making, business management, text categorization, industrial analysis, and system control [7,16,19].

4. Fuzzy Decision Maps (FDMs)

FDM was proposed in 2006 for solving the MCDM problem with dependence and feedback. It incorporates the eigenvalue method, FCM, and the weighting equation to overcome the shortcoming of AHP and ANP method [7]. The step of FDM method to derive the priorities of criteria can be summarized as follows [7].

Step 1: Compare the importance among criteria to derive the local weight vector using the eigenvalue approach.
Step 2: Depict the fuzzy cognitive map to indicate the influence among criteria by the expert.
Step 3: Calculate the updating Eq. (8) to obtain the steady-state matrix.
Step 4: Derive the global weight vector. We should first normalize the local weight vector (V) and the steady-state matrix (M) as follows:

$$V_n = \frac{1}{k} V, \quad M_n = \frac{1}{c} M \tag{12}$$

where k is the largest element of V and c is the largest row sum of M. Then, the global weight vector (W) can be computed as follows:

$$W = V_n + M_n V_n \tag{13}$$

where V_n is the normalization of the local weight vector, and M_n is the normalization of the steady-state matrix. Finally, normalize the global weight (W).

The advantages of FDM [7,14] are overcoming the drawbacks of the AHP and ANP, employing the different threshold functions to indicate the various kinds of relationship among criteria, dealing with the direct and indirect influences and the ability to solve both the compound and the interaction effects. However, in the FDM, decision makers or experts must quantify a precise value to judge the ratios of weights between criteria, i.e., the process ignores the problem of uncertainty. Since, in practice, problems are usually complicated and uncertain, it is even hard for experts to quantify the precise weight values.

5. The proposed Linguistic Fuzzy Decision Networks (LFDNs)

The proposed LFDN uses linguistic values instead of crisp values in computing of both of the pairwise comparison and the FCM stage in the original FDM model. Therefore, LFDN handles the practical uncertainty usually exists in real world MCDM problems in a more efficient and humanistic way rather than that of the FDM model.

Table 1 Fuzzy preference scale.

Intensity of fuzzy scale	Definition of linguistic values	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
$\tilde{1}$	Equal importance (EI)	$(1, 1, 1)$	$(1, 1, 1)$
$\tilde{2}$	Equally moderate importance	$(1, 2, 3)$	$(1/3, 1/2, 1)$
$\tilde{3}$	Moderate importance (MI)	$(1, 3, 5)$	$(1/5, 1/3, 1)$
$\tilde{4}$	Moderately strong importance	$(2, 4, 6)$	$(1/6, 1/4, 1/2)$
$\tilde{5}$	Strong importance (SI)	$(3, 5, 7)$	$(1/7, 1/5, 1/3)$
$\tilde{6}$	Strongly very strong importance	$(4, 6, 8)$	$(1/8, 1/6, 1/4)$
$\tilde{7}$	Very strong importance (VSI)	$(5, 7, 9)$	$(1/9, 1/7, 1/5)$
$\tilde{8}$	Very strongly extreme importance	$(6, 8, 9)$	$(1/9, 1/8, 1/6)$
$\tilde{9}$	Extreme importance (EXI)	$(7, 9, 9)$	$(1/9, 1/9, 1/7)$

Let us first introduce the concept of local division. If $\widetilde{A} = (l_1, m_1, u_1)$, $\widetilde{B} = (l_2, m_2, u_2)$ are two triangular positive fuzzy numbers, then we define the local division operation as follows:

$$\widetilde{A}/\widetilde{B} = (l_1/l_2, m_1/m_2, u_1/u_2) \tag{14}$$

We introduce this concept to be used only for the case of normalizing a given vector (or matrix) of TFN values. Otherwise, the division formula given in Eq. (5) is the standard one.

We use a centroid method as a defuzzification for ranking a TFN. Thus, if $\widetilde{A} = (l, m, u)$ is a TFN, then it is defuzzified by computing its center of area as follows:

$$D(\widetilde{A}) = (l + m + u)/3 \tag{15}$$

Now, the steps of LFDN method can be described as follows:

5.1. Step 1: Derive the fuzzy local weight vector

Elicit the fuzzy judgment matrix of pairwise comparison among the criteria of the decision system. This should be done by domain experts. TFNs are used in pairwise comparison to express the expert preference in the form of linguistics values. Let the preference matrix $\widetilde{P} = [a_{ij}]$ where $\widetilde{P}$ represents $n \times n$ matrix and a_{ij} is the importance of criterion C_i w.r.t. criterion C_j according to the fuzzy preference scale shown in Table 1. The corresponding membership functions can be depicted as shown in Fig. 2.

The reciprocals, such as 1/3, 1/5, 1/7, and 1/9, indicate the opposite respectively of the values 3, 5, 7, and 9.

The fuzzy pairwise comparison matrix $\tilde{P}$ with TFNs appears as follows:

$$\widetilde{P} = \begin{bmatrix} (1,1,1) & (l_{11}, m_{11}, u_{11}) & \cdots & (l_{1n}, m_{1n}, u_{1n}) \\ (\frac{1}{u_{11}}, \frac{1}{m_{11}}, \frac{1}{l_{11}}) & (1,1,1) & \cdots & (l_{2n}, m_{2n}, u_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\frac{1}{u_{1n}}, \frac{1}{m_{1n}}, \frac{1}{l_{1n}}) & (\frac{1}{u_{2n}}, \frac{1}{m_{2n}}, \frac{1}{l_{2n}}) & \cdots & (1,1,1) \end{bmatrix} \tag{16}$$

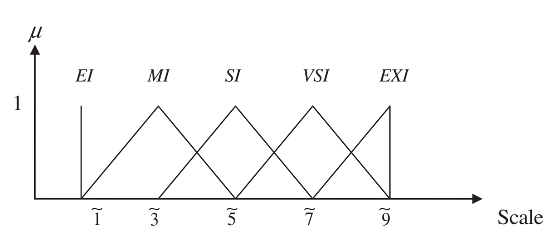

Figure 2 Membership functions of linguistic values of the fuzzy preference scale.

After preparing the fuzzy pairwise comparison matrix, the approximate fuzzy eignvalue method is applied, in order to derive the fuzzy local weight vector ($\widetilde{V}$) by:

- Sum up each row of fuzzy pairwise comparison matrix $\tilde{P}$ to get the vector of fuzzy numbers that represents the priorities among criteria without dependence or feedback
-

$$\widetilde{V}_i = \sum_{j=1}^{n} a_{ij} = \left(\sum_{j=1}^{n} l_{ij}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} u_{ij} \right), \quad i = 1, \ldots, n \tag{17}$$

$$\widetilde{V} = [\widetilde{V}_1, \widetilde{V}_2, \ldots, \widetilde{V}_n]^T \tag{18}$$

- Defuzzifiy the obtained fuzzy vector using Eq. (15) to get the highest rank fuzzy value $\tilde{r}$ of the fuzzy local weight vector $\widetilde{V}$

$$D(\tilde{r}) = \max(D(\widetilde{V})) \tag{19}$$

- Normalize the fuzzy local weight vector using the local division formula in Eq. (14)

$$\widetilde{V}_n = \frac{1}{\tilde{r}} \widetilde{V} \tag{20}$$

where $\widetilde{V}_n$ is the normalization of the fuzzy local weight vector.

5.2. Step 2: Derive of the fuzzy influence weight matrix

In this step, we introduce the modification of the updating Eq. (8) by using linguistic values in the form of TFNs, so the influence among criteria can be calculated using Eq. (21) as follows:

$$\widetilde{C}^{(t+1)} = \widetilde{F}(\widetilde{C}^{(t)} \cdot \widetilde{E}), \widetilde{C}^{(0)} = \widetilde{I}_{n \times n} \tag{21}$$

where $\widetilde{I}_{n\times n}$ denotes the identity matrix with TFNs, $\widetilde{E} = (\widetilde{W}_{ij})$ is $n \times n$ weight matrix with TFNs, which gathers the values of causal edge weight between concepts $\widetilde{C}_i$ and $\widetilde{C}_j$, $\widetilde{C}^{(t+1)}$ and $\widetilde{C}^{(t)}$ are the state matrices with TFNs at iterations $(t + 1)$ and (t), respectively, $\widetilde{C}^{(0)}$ is the initial matrix with TFNs and $\widetilde{F}$ is a threshold transformation function with TFNs.

It is important to introduce the modified threshold transformation function using linguistic values in the form of TFNs. For example, the hyperbolic-tangent function in Eq. (10) is modified to have the form given in Eq. (22):

$$\widetilde{F}(x_l, x_m, x_u) = (1 - e^{-x_l}/1 + e^{-x_l}, 1 - e^{-x_m}/1 + e^{-x_m}, \\ 1 - e^{-x_u}/1 + e^{-x_u}) \tag{22}$$

Table 2 Linguistic values for causal relationships.

Linguistic values	TFN
Extremely low (EL)	(0, 0.1, 0.2)
Very low (VL)	(0.1, 0.2, 0.3)
Low (L)	(0.2, 0.3, 0.4)
Medium low (ML)	(0.3, 0.4, 0.5)
Medium (M)	(0.4, 0.5, 0.6)
Medium high (MH)	(0.5, 0.6, 0.7)
High (H)	(0.6, 0.7, 0.8)
Very high (VH)	(0.7, 0.8, 0.9)
Extremely high (EH)	(0.8, 0.9, 1)

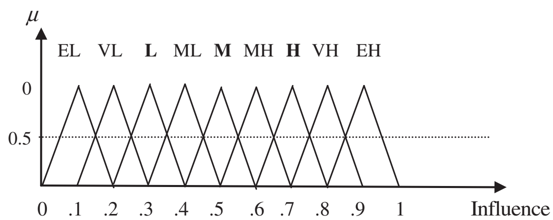

Figure 3 Membership functions of linguistic values for causal relationships.

The FCM can now be constructed with linguistic values to indicate the influence among criteria. The relationships between concepts are described using the degree of influence. Experts describe this degree of influence using linguistic values as TFNs, see Table 2, and the corresponding membership functions are shown in Fig. 3.

Thus, the FCM model can be applied using the updating Eq. (21) until it reaches a fuzzy steady-state matrix ($\widetilde{C}$), which represents the causal relationship among criteria. Then, normalize the fuzzy steady-state matrix as follows:

- Sum up each row of the fuzzy steady-state matrix $\widetilde{C}$ to get the fuzzy number vector $\widetilde{S}$

$$\widetilde{S}_i = \sum_{j=1}^{n} w_{ij} = \left(\sum_{j=1}^{n} l_{ij}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} u_{ij}\right), \quad i = 1, \ldots, n \tag{23}$$

$$\widetilde{S} = \left[\widetilde{S}_1, \widetilde{S}_2, \ldots, \widetilde{S}_n\right]^T \tag{24}$$

- Defuzzifiy the fuzzy vector $\widetilde{S}$ using Eq. (15) to get the highest rank fuzzy value $\tilde{k}$ of fuzzy number vector $\widetilde{S}$.

$$D(\tilde{k}) = \max(D(\widetilde{S})) \tag{25}$$

- Normalize the fuzzy steady-state matrix using Eq. (14)

$$\widetilde{C}_n = \frac{1}{\tilde{k}}\widetilde{C} \tag{26}$$

where $\widetilde{C}_n$ is the normalization of the fuzzy steady-state matrix.

5.3. Step 3: Derive the fuzzy global weight vector

The fuzzy global weight $\widetilde{W}$ can be driven by the following fuzzy weighting equation

$$\widetilde{W} = \widetilde{V}_n + \widetilde{C}_n \widetilde{V}_n \tag{27}$$

where $\widetilde{V}_n$ is the normalization of the fuzzy local weight vector and $\widetilde{C}_n$ is the normalization of the fuzzy steady-state matrix, and finally, normalize the fuzzy global weight vector as follows:

- Defuzzifiy the fuzzy global weight vector $\widetilde{W}$ by using Eq. (15) to get the maximum fuzzy value $\tilde{\lambda}$ of the fuzzy global weight vector $\widetilde{W}$

$$D(\tilde{\lambda}) = \max(D(\widetilde{W})) \tag{28}$$

- Normalize the fuzzy steady-state matrix using Eq. (14)

$$\widetilde{W}_n = \frac{1}{\tilde{\lambda}}\widetilde{W} \tag{29}$$

where $\widetilde{W}_n$ is the normalization of the fuzzy global weight vector. Now, $\widetilde{W}_n$ can be defuzzified using Eq. (15) to get the corresponding rank for each criterion.

6. Case study

In this section, the proposed LFDN model is tested using the same example given in [7] for testing the FDM model. In this example, a decision maker tries to select the best alternative according to five criteria. The original pairwise comparison matrix is given as shown in Table 3. The Original FCM of the five criteria is shown in Fig. 4, while the original matrix of influence among criteria is given as shown in Table 4.

Now, we start to solve the above case study using our proposed LFDN model as follows:

– *The first step*: Relax the hard formulation values of the original pairwise comparison matrix, using the fuzzy scale, given in Table 1, as TFNs linguistic values. The obtained comparison matrix with linguistic values is given in Table 5. Empty cells correspond to locations of inverse linguistic values which will be substituted automatically as reciprocal

Table 3 Original pairwise comparison matrix (P) [7].

	Criterion1	Criterion2	Criterion3	Criterion4	Criterion5
Criterion1	1	3	1	5	1/3
Criterion2	1/3	1	1/3	3	3
Criterion3	1	3	1	5	1/3
Criterion4	1/5	1/3	1/5	1	1/2
Criterion5	3	1/3	3	2	1

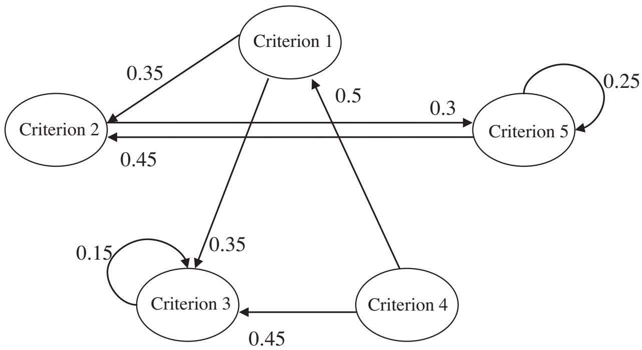

Figure 4 Original fuzzy cognitive map for case study [7].

fuzzy numbers. After substituting the corresponding TFNs and reciprocal TFNs, the final comparison matrix appears as shown in Table 6.

By using Table 1, we can rewrite pairwise comparison matrix $\tilde{P}$ with TFN as.

By using Eq. (17), we can obtain the fuzzy local weights vector of each criterion:

$$\widetilde{V}_i = \sum_{j=1}^{5} a_{ij} = \left(\sum_{j=1}^{5} l_{ij}, \sum_{j=1}^{5} m_{ij}, \sum_{j=1}^{5} u_{ij} \right), \; i = 1, \ldots, n$$

$$\widetilde{V}_1 = (1+1+1+3+1/5, \;\; 1+3+1+5+1/3, \;\; 1+5+1+7+1) = (6.2, 10.33, 15)$$

$$\widetilde{V}_2 = (1/5+1+1/5+1+1, \;\; 1/3+1+1/3+3+3, \;\; 1+1+1+5+5) = (3.4, 7.66, 13)$$

$$\widetilde{V}_3 = (1+1+1+3+1/5, \;\; 1+3+1+5+1/3, \;\; 1+5+1+7+1) = (6.2, 10.33, 15)$$

$$\widetilde{V}_4 = (1/7+1/5+1/7+1+1/3, \;\; 1/5+1/3+1/5+1+1/2, \;\; 1/3+1+1/3+1+1) = (1.816, 2.23, 3.66)$$

$$\widetilde{V}_5 = (1+1/5+1+1+1, \;\; 3+1/3+3+2+1, \;\; 5+1+5+3+1) = (4.2, 9.33, 15)$$

So the fuzzy local weight vector ($\widetilde{V}$) is:

$$\widetilde{V} = ((6.2, 10.33, 15), (3.4, 7.66, 13), (6.2, 10.33, 15), (1.816, 2.23, 3.66), (4.2, 9.33, 15))^T$$

Then, using Eq. (20) to normalize the fuzzy local weight vector ($\widetilde{V}$), we have:

$$\widetilde{V}_n = ((1,1,1), (0.5483, 0.7415, 0.8666), (1,1,1), (0.2929, 0.2158, 0.244), (0.6774, 0.9031, 1))^T$$

– *The second step*: The original hard formulation values of the influence interaction among various criteria, as given in Fig. 4 and Table 4, are relaxed with the linguistic values of causal relationships and their corresponding TFNs forms as given in Table 2. The FCM for the considered case study is shown in Fig. 5. The obtained FCM matrix with influence linguistic values ($\widetilde{E}$), and its form after substitution of the corresponding TFNs values are given in Tables 7 and 8.

Then, we calculate the updating Eq. (21). The hyperbolic-tangent transformation function with TFN as given in Eq. (22) is used. After eleven iteration cycles of FCM model, i.e., $\widetilde{C}^{11}$, a fuzzy steady-state matrix, of the type hidden pattern or fixed point attractor, is reached. Table 9 illustrates the obtained fuzzy steady-state matrix ($\widetilde{C}$) with TFNs.

Table 4 Original FCM matrix with exact numerical values influence (E) [7].

	Criterion1	Criterion2	Criterion3	Criterion4	Criterion5
Criterion1	0	0.35	0.35	0	0
Criterion2	0	0	0	0	0.3
Criterion3	0	0	0.15	0	0
Criterion4	0.5	0	0.45	0	0
Criterion5	0	0.45	0	0	0.25

Table 5 Pairwise comparison matrix with linguistic values ($\widetilde{P}$).

	Criterion1	Criterion2	Criterion3	Criterion4	Criterion5
Criterion1	Equal	Moderate	Equal	Strong	
Criterion2		Equal		Moderate	Moderate
Criterion3	Equal	Moderate	Equal	Strong	
Criterion4				Equal	
Criterion5	Moderate		Moderate	Equally moderate	Equal

Table 6 Pairwise comparison matrix with TFN ($\widetilde{P}$).

	Criterion1	Criterion2	Criterion3	Criterion4	Criterion5
Criterion1	(1, 1, 1)	(1, 3, 5)	(1, 1, 1)	(3, 5, 7)	(1/5, 1/3, 1)
Criterion2	(1/5, 1/3, 1)	(1, 1, 1)	(1/5, 1/3, 1)	(1, 3, 5)	(1, 3, 5)
Criterion3	(1, 1, 1)	(1, 3, 5)	(1, 1, 1)	(3, 5, 7)	(1/5, 1/3, 1)
Criterion4	(1/7, 1/5, 1/3)	(1/5, 1/3, 1)	(1/7, 1/5, 1/3)	(1, 1, 1)	(1/3, 1/2, 1)
Criterion5	(1, 3, 5)	(1/5, 1/3, 1)	(1, 3, 5)	(1, 2, 3)	(1, 1, 1)

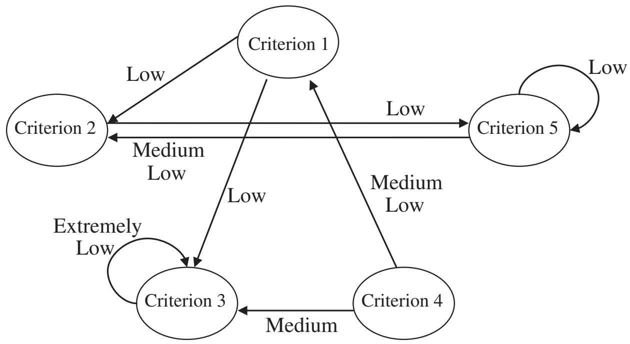

Figure 5 A fuzzy cognitive map with linguistic values for case study.

Eq. (26) is used to normalize the fuzzy steady-state matrix ($\widetilde{C}$). Table 10 shows the obtained normalization of fuzzy steady-state matrix ($\widetilde{C}_n$) with TFNs.

– The final step: We use Eq. (27) to derive the fuzzy global weights vector as follows:

$$\begin{aligned}\widetilde{W} =&((1.3629, 1.4799, 1.5697), (0.8583, 1.1828, 1.3905),\\ &(1, 1.0811, 1.1307), (1.1077, 1.0296, 1.0688),\\ &(1.3023, 1.7392, 1.9438))^T\end{aligned}$$

Then, we use Eq. (29) to normalize the fuzzy global weight vector ($\widetilde{W}$) as follows:

$$\begin{aligned}\widetilde{W}_n =&((0.2420, 0.2272, 0.2209), (0.1524, 0.1816, 0.1957),\\ &(0.1775, 0.1660, 0.1591), (0.1967, 0.1580, 0.1504),\\ &(0.2312, 0.2670, 0.2736))^T\end{aligned}$$

Then, Eq. (15) is used to defuzzifiy the normalization of fuzzy global weights vector to get the ranking among criteria as follows:

$$D(\widetilde{W}_n) = (0.2300, 0.1766, 0.1675, 0.1683, 0.2573)^T$$

Table 11 illustrates the ranking of fuzzy global weights vector.

From Table 11, it is clear that the highest priority is criterion 5, while the lowest one is criterion 3.

Table 7 FCM matrix with linguistic values influence ($\widetilde{E}$).

	Criterion1	Criterion2	Criterion3	Criterion4	Criterion5
Criterion1	None	Low	Low	None	None
Criterion2	None	None	None	None	Low
Criterion3	None	None	Extremely low	None	None
Criterion4	Medium	None	Medium low	None	None
Criterion5	None	Medium low	None	None	Low

Table 8 FCM matrix with TFN ($\widetilde{E}$).

	Criterion1	Criterion2	Criterion3	Criterion4	Criterion5
Criterion1	(0, 0, 0)	(0.2, 0.3, 0.4)	(0.2, 0.3, 0.4)	(0, 0, 0)	(0, 0, 0)
Criterion2	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0.2, 0.3, 0.4)
Criterion3	(0, 0, 0)	(0, 0, 0)	(0, 0.1, 0.2)	(0, 0, 0)	(0, 0, 0)
Criterion4	(0.4, 0.5, 0.6)	(0, 0, 0)	(0.3, 0.4, 0.5)	(0, 0, 0)	(0, 0, 0)
Criterion5	(0, 0, 0)	(0.3, 0.4, 0.5)	(0, 0, 0)	(0, 0, 0)	(0.2, 0.3, 0.4)

Table 9 Fuzzy steady-state matrix with TFN ($\widetilde{C}$).

	Criterion1	Criterion2	Criterion3	Criterion4	Criterion5
Criterion1	(0, 0, 0)	(0.1025, 0.1570, 0.2157)	(0.0996, 0.1565, 0.2182)	(0, 0, 0)	(0.0192, 0.0418, 0.0768)
Criterion2	(0, 0, 0)	(0.0284, 0.0543, 0.0908)	(0, 0, 0)	(0, 0, 0)	(0.1899, 0.2718, 0.3645)
Criterion3	(0, 0, 0)	(0, 0, 0)	(0, 0.0525, 0.1106)	(0, 0, 0)	(0, 0, 0)
Criterion4	(0.1973, 0.2449, 0.2913)	(0.0203, 0.0387, 0.0638)	(0.1681, 0.2439, 0.3284)	(0, 0, 0)	(0.0038, 0.0103, 0.0228)
Criterion5	(0, 0, 0)	(0.1891, 0.2689, 0.3561)	(0, 0, 0)	(0, 0, 0)	(0.2762, 0.3786, 0.4899)

Table 10 The normalization of fuzzy steady-state matrix with TFN ($\widetilde{C}_n$).

	Criterion1	Criterion2	Criterion3	Criterion4	Criterion5
Criterion1	(0, 0, 0)	(0.2202, 0.2425, 0.2549)	(0.2141, 0.2417, 0.2579)	(0, 0, 0)	(0.0412, 0.0646, 0.0908)
Criterion2	(0, 0, 0)	(0.0611, 0.0838, 0.1073)	(0, 0, 0)	(0, 0, 0)	(0.4080, 0.4197, 0.4307)
Criterion3	(0, 0, 0)	(0, 0, 0)	(0, 0.0811, 0.1307)	(0, 0, 0)	(0, 0, 0)
Criterion4	(0.4241, 0.3782, 0.3442)	(0.0436, 0.0598, 0.0754)	(0.3612, 0.3766, 0.3881)	(0, 0, 0)	(0.0081, 0.0159, 0.0269)
Criterion5	(0, 0, 0)	(0.4064, 0.4153, 0.4209)	(0, 0, 0)	(0, 0, 0)	(0.5935, 0.5846, 0.5790)

Table 11 The ranking of the normalized fuzzy global weights vector.

Global weights	LFDN method		
	Fuzzy value	Crisp value (COA)	Ranking
Criterion1	(0.2420, 0.2272, 0.2209)	0.2300	2
Criterion2	(0.1524, 0.1816, 0.1957)	0.1766	3
Criterion3	(0.1775, 0.1660, 0.1591)	0.1675	5
Criterion4	(0.1967, 0.1580, 0.1504)	0.1683	4
Criterion5	(0.2312, 0.2670, 0.2736)	0.2573	1

Table 12 The comparison of proposed method and FDM method.

Global weights	FDM method[7]		LFDN method	
	Crisp value	Ranking	Crisp value (COA)	Ranking
Criterion1	0.2238	2	0.2300	2
Criterion2	0.1842	3	0.1766	3
Criterion3	0.1516	5	0.1675	5
Criterion4	0.1654	4	0.1683	4
Criterion5	0.2751	1	0.2573	1

7. Discussions

In this paper, we introduce the proposed Linguistic Fuzzy Decision Networks (LFDNs) that combines approximate fuzzy eigenvalue method, FCM with linguistic values and fuzzy weighting equation to deal with MCDM problems. The fuzzy local weights are first derived by using the approximate fuzzy eigenvalue method. Then, the FCM with linguistic values is verified to describe the influence among criteria. Next, the fuzzy steady-state matrix is calculated by using the modification of the updating equation. Finally, the fuzzy global weights vector is derived by using the fuzzy weighting equation.

Table 12 shows the comparison of the results obtained by both of the original FDM method [7], and the proposed LFDN method when solving the considered case study. It is clear that we have the same result but with more ability to handle uncertainty expressed as linguistic values which provides great flexibility for decision makers when dealing with real MCDM problems. Thus, to summarize, the proposed LFDN is a direct extension of the FDM model with the ability to handle uncertainties in the form of linguistic values.

8. Conclusion

Fuzzy Decision Maps (FDMs) approaches have been developed to overcome the difficulties of Analytical Network process (ANP) for solving MCDM problems with dependence and feedback. The construction of inner influences among criteria in ANP represents a hard task for decision makers. FDM adopt the methodology of fuzzy cognitive maps (FCM) to simplify getting the final preferential matrix of criteria with inner influences. However, FDM request the decision maker to use crisp weight values to indicate the relative influences among criteria which is not only difficult but also impractical. In this paper, the proposed Linguistic Fuzzy Decision Networks approach (LFDNs) is a direct extension of the original FDM to allow handling uncertainties in the form of linguistic values. The proposed model is tested using a case study that is also used for testing FDM in [7]. The results give the same ranking of criteria which ensures the ability of LFDN to solve MCDM problems with dependence and feedback in a more flexible and humanistic manner.

References

[1] Yoon K, Hwang CL. Multiple attribute decision making: an introduction. Thousand Oaks (CA): Sage Publications; 1995.

[2] Vincke P. Multi criteria decision aid. New York: John Wiley and Sons; 1992.

[3] Hwang CL, Yoon K. Multiple attribute decision making: methods and applications. Berlin: Springer-Verlag; 1981.

[4] Saaty TL. The analytic hierarchy process. New York: McGraw-Hill; 1980.

[5] Luo Z, Zhou J, Zheng L-P, Mo L, He Y. A TFN–ANP based approach to evaluate virtual research center comprehensive performance. Expert Syst Appl 2010;37(12):8379–86.

[6] Saaty TL. Decision making with dependence and feedback: the analytic network process. Pittsburgh (PA): RWS Publications; 1996.

[7] Yu R, Tzeng GH. A soft computing method for multi-criteria decision making with dependence and feedback. Appl Math Comput 2006;180:63–75.

[8] Zadeh LA. Fuzzy set. Inform Control 1965;8(3):338–53.

[9] Hanss M. Applied fuzzy arithmetic: an introduction with engineering applications. Berlin, Heidelberg: Springer-Verlag; 2005.

[10] Zadeh LA. The concept of a linguistic variable and its application to approximate reasoning-1. Inform Sci 1975;8(3):199–249.

[11] Engelbrecht P. Computational intelligence: an introduction. 2nd ed. John Wiley & Sons; 2007.

[12] Kosko B. Fuzzy cognitive maps. Int J Man–Mach Stud 1986;24(1):65–75.

[13] Stylios CD, Groumpos PP. Modeling complex systems using fuzzy cognitive maps. IEEE Trans Syst Man Cybern Part A 2004;34(1):155–62.

[14] Tzeng GH, Chen WH, Yu R, Shih ML. Fuzzy decision maps: a generalization of the DEMATEL methods. Soft Comput 2010;14(11):1141–50.
[15] Tzeng GH, Huang JJ. Multiple attribute decision making: methods and applications. New York: CRC Press, Taylor & Francis Group; 2011.
[16] Stach W, Kurgan LA, Pedrycz W. A survey of fuzzy cognitive map learning methods. In: Grzegorzewski P, Krawczak M, Zadrozny S, editors. Issues in soft computing: theory and applications; 2005.
[17] Akgun I, Kandakoglu A, Ozok AF. Fuzzy integrated vulnerability assessment model for critical facilities in combating the terrorism. Expert Syst Appl 2010;37(5):3561–73.
[18] Kyriakarakos G, Dounis AI, Arvanitis KG, Papadakis G. A fuzzy cognitive maps-petri nets energy management system for autonomous polygeneration microgrids. Appl Soft Comput 2012;12(12):3785–97.
[19] Papageorgiou EI, Lamia G. A review of fuzzy cognitive maps research and their applications during the last decade. In: Glykas M, editor. Business process management. Berlin Heidelberg: Springer-Verlag; 2013.