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Fragments and Chemical Organisations

Peter Kreyssig, Peter Dittrich [1]

Bio Systems Analysis
Friedrich Schiller University of Jena
Jena, Germany
peter.kreyssig@uni-jena.de, dittrich@minet.uni-jena.de

Abstract

We describe some aspects of the relationship between the chemical organisations of a reaction network and the chemical organisations of the fragments of this network. The conjecture is that fragments embeddable in a (feasible) organisation of molecules is a (feasible) organisation of fragments. We show this for a certain type of reaction network. However the fragmentation can always be altered such that the result becomes valid for all networks. Of course, this comes at a cost, namely a less coarse-grained fragment system. For the case that we do not change the fragmentation, we give examples for which the statement is true and counterexamples for which it is not. Furthermore we present some interesting examples illustrating our findings and further questions.

Keywords: fragments, rule-based models, model reduction, reaction networks, reaction network, chemical organisation theory, constructive dynamical systems

1 Introduction

By a *reaction network* we mean a pair (M, R) where M is a set and R is a subset of $\mathcal{P}_{mult}(M) \times \mathcal{P}_{mult}(M)$. Here $\mathcal{P}_{mult}(M)$ is the set of multisets over M. We call the elements of M *molecular species* and the elements of R *reactions* resembling the notions of chemistry. Reaction networks can also be described by Petri nets or multiset rewriting. Often the reactions are not explicitly given, but implicitly as *rules* meaning that similar reactions are grouped together. For this we write $(M, \overline{R})$ and refer to it as *rule-based* model in contrast to the *reaction-based* model (M, R). Our $\overline{R}$ can be seen as a set of subsets of R which cover R. We then say that an element in $\overline{R}$ *matches* the reactions it contains.

The map yielding (M, R) for a corresponding $(M, \overline{R})$ is called t. We use the κ–language [2] to formalise rule-based models and to write down examples in a concise manner. In general one would think that the rule-based model is preferable

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since it allows to not write down all the molecular species and reactions, but only to give patterns [1]. However the theory of chemical organisations is formulated for reaction-based models, whereas the notion of fragments is defined for rule-based models. We will switch between the models freely when they describe the same reaction network.

The notion of *fragments* was introduced by Feret *et al.* in [5]. The idea behind it is to use techniques from the *abstract interpretation framework* to derive a *reduced* or *coarse-grained* model out of the rule-based model $(M, \overline{R})$. It consists of a set of *observables* $\mathcal{F}$ and a *differential system*, i.e. a set of ODEs, describing their concentrations. This new model carries a lot of information about the original set of ODEs given by the law of mass action to (M, R), but is possibly much smaller. The elements of $\mathcal{F}$ are called fragments because they can be considered fractions or components of molecules in M. The distinctive feature of fragments is their *soundness* for the differential semantics, which can roughly be summarised as follows. Assuming mass action kinetics for (M, R) the concentration of a fragment is given as a linear combination of concentrations of molecular species. In particular, the soundness means that fixpoints of the molecular species dynamics give rise to fixpoints for the fragments dynamics.

In [3] the definition for *chemical organisations* is given. Roughly, a subset O of M is an organisation if no reaction in O produces molecules not contained in O (*closure* condition) and positive reaction rates can be chosen such that the amount of molecules in O does not decline (*self-maintenance* condition). By Theorem 42 of [3] the molecules of M present in a fixpoint of the molecular species dynamics form an organisation.

We are interested in the case that the reduced dynamics of the fragments is a set of *chemical ODEs*, since we are then able to compute organisations of fragments. So let $(\mathcal{F}, \widetilde{R})$ be the reaction network of fragments together with the reactions $\widetilde{R}$ between fragments yielding the reduced dynamics. As mentioned earlier the soundness of fragments implies that the fragments that appear in an organisation found in a fixpoint form an organisation of $(\mathcal{F}, \widetilde{R})$. Thus we conjecture that this does not only hold for organisations found in fixpoints, but for all organisations. This conjecture is proven here for a certain class of reaction networks. We also give a counterexample to the conjecture and an example showing that our result could be extended.

Sections 2 and 3 shortly outline the parts of the theory of organisations and fragments we need. Section 4 contains a more formal version of our conjecture and its proof in the special case. In Section 5 we give some examples. Finally we state some benefits of our results and mention open questions.

2 Chemical Organisation Theory

We summarise the needed concepts of chemical organisation theory [3], [6].

Definition 2.1 Let M be a set and R be a subset of $\mathcal{P}_{mult}(M) \times \mathcal{P}_{mult}(M)$. The pair (M, R) is called *reaction network* and we call M the set of *molecules* and R

the set of *reactions.*

We fix a reaction network (M, R) for the rest of the paper. By *applying* a reaction $(l, r) \in R$ to a multiset over M we mean replacing the subset l by the subset r. We assume that the multiset considered is always large enough.

For $(l, r) \in R$ we also write $l \longrightarrow r$ or

$$\sum_{m \in M} l_m m \longrightarrow \sum_{m \in M} r_m m$$

where we denote by $l_m, r_m \in \mathbb{N}_0$ the multiplicity of m in l, r respectively. This resembles notation from chemistry. Furthermore the *support* and the *product* of (l, r) are

$$\operatorname{supp}(l, r) := \{m \in M \mid l_m > 0\}, \ \ \operatorname{prod}(l, r) := \{m \in M \mid r_m > 0\}.$$

Let A be a subset of M. We define R_A by setting

$$R_A := \{(l, r) \in R \mid \operatorname{supp}(l, r) \subseteq A\}.$$

The *stoichiometric matrix* $S_A \in \mathbb{R}^{|A| \times |R_A|}$ for A is given by

$$(S_A)_{a,(l,r)} = r_a - l_a, \quad a \in A, (l, r) \in R.$$

Definition 2.2 A subset A of M is *closed* if for all reactions $(l, r) \in R_A$ we have $\operatorname{prod}(l, r) \subseteq A$, i.e. if (A, R_A) is a reaction network.

A being closed means that by applying reactions from R_A to multisets over A we do not get molecules outside A.

Definition 2.3 A subset A of M is *semi-self-maintaining* if for every $a \in A$ and $(l, r) \in R_A$ with $l_a - r_a > 0$ there is a $(l', r') \in R_A$ with $r'_a - l'_a > 0$.

A being semi-self-maintaining means that if a reaction application destroys a species, there is a reaction producing this species.

Definition 2.4 A subset A of M is *self-maintaining* if there is a vector $v \in \mathbb{R}^{|R_A|}$ with strictly positive entries such that $S_A v \in \mathbb{R}^{|A|}$ has only non-negative entries.

A being self-maintaining means that applying reactions from R_A at certain rates to a multiset over M does not reduce the number of molecules of any species of A.

Definition 2.5 A subset of M is a *chemical (semi-)organisation* if it is closed and (semi-)self-maintaining. The set of organisations is called $\mathcal{O}$. The map assigning to a reaction network its set of organisations is called org.

When assuming the law of mass action for the dynamics of the reaction network (M, R), we can give a more refined notion of chemical organisations, called *feasible organisations.* Let us denote the *flux vector function* by $v_{M,\mathbf{k}} : \mathbb{R}_{\geq 0}^{|M|} \to \mathbb{R}_{\geq 0}^{|R|}$. It is subject to mass action kinetics with rate constants $\mathbf{k} \in \mathbb{R}^{|R|}$. This kinetic law fulfils the "Feinberg Condition" [4], i.e. a component $v_{M,\mathbf{k}}(x)_{(l,r)}$ (with $(l,r) \in R$) in a state $x \in \mathbb{R}^{|M|}_{\geq 0}$ is positive if and only if all molecules of l are present in the state x otherwise it is zero. The same is true and can be formulated for closed subsets of (M,R), in particular for organisations.

Definition 2.6 An organisation O is called a $\mathbf{k}$-*feasible organisation* if there is a vector of concentrations $x \in \mathbb{R}^{|O|}_{>0}$ (and therefore also $v_{O,\mathbf{k}}(x) > 0$ holds) such that $S_O v_{O,\mathbf{k}}(x) \geq 0$. If an organisation is $\mathbf{k}$-feasible for all $\mathbf{k} \in \mathbb{R}^{|O_R|}$, we call it *feasible*.

3 Fragments

Here we outline the main aspects of the concept of fragment [1], [5].

We use, without defining them, some notions of κ, e.g. *agent*, *binding (site)*, *phosphorylation site*, *(modified) site*, *pattern* and *pattern matching*. The parts of a pattern are called *components*. Any pattern that matches a fragment is called a *subfragment*. A rule can be defined as mapping a pattern to a pattern. The application of a rule consists of the four steps *binding deletion*, *agent creation*, *agent deletion* and *binding creation*.

We do not describe the construction process for the fragments nor for the reactions but only give some distinct properties we need later on and mention that fragments are directly constructable from the so-called *annotated contact map (aCM)*. The aCM includes a *cover of the set of sites* of each agent.

Now we can formulate the four following statements, which hold for the fragments [5] and [1], Propositions VI.4 to VI.7.

Q1: No fragment strictly overlaps with a left hand side component on a modified site.

Q2: Any left hand side component matches a fragment, i.e. it is a subfragment.

Q3: The concentration of any subfragment can be expressed as a linear combination of fragment concentrations.

Q4: Fragments are closed under rule action.

For a rule-based model $(M, \overline{R})$ we denote its set of fragments by $\mathcal{F}$. When representing expressions in κ as graphs one uses more naturally the notion of *embedding* instead of matching. The set of possible embeddings of a fragment F in a molecular species $m \in M$ is denoted by $[F, m]$, i.e. this set describes all the possible ways of matching F to m.

Whenever we consider the case that the reduced dynamics is a set of chemical ODEs and therefore is given by a set of reactions $\widetilde{R}$, we write $(\mathcal{F}, \widetilde{R})$. This happens for example if all the covers included in the aCM are not only covers but actually partitions of the set of sites of the agents. The reactions in $\widetilde{R}$ are then given by matching the fragments to the rules $\overline{R}$. The map assigning to a rule-based model $(M, \overline{R})$ its reaction network of fragments $(\mathcal{F}, \widetilde{R})$ is called $\overline{\text{frag}}$. Since $(\mathcal{F}, \widetilde{R})$ is a reaction network, we can compute its organisations. For a better distinguishability we call the organisations of $(\mathcal{F}, \widetilde{R})$ *f–organisations* and write $\mathcal{O}_{\mathcal{F}}$ for the set containing them. Analogously to the last part of Section 2 we can define a flux vector function

$v_{\mathcal{F},\widetilde{\mathbf{k}}} : \mathbb{R}_{\geq 0}^{|\mathcal{F}|} \to \mathbb{R}_{\geq 0}^{|\widetilde{R}|}$ for the fragment network. It is subject to mass action kinetics with rate constants $\widetilde{\mathbf{k}} \in \mathbb{R}^{|\widetilde{R}|}$. The rates $\widetilde{\mathbf{k}}$ depend on the rates $\mathbf{k}$ of the reactions R and are given by the process of constructing the reduced system, cf. Appendix of [5].

The *abstraction* by the fragments is denoted by $T : \mathbb{R}_{\geq 0}^{|M|} \to \mathbb{R}_{\geq 0}^{|\mathcal{F}|}$. It maps concentrations of molecules to concentrations of fragments (therefore $Tx > 0$ if $x > 0$) and commutes with the differential system, this property is also called *soundness* [1].

4 Main Result

We restrict our considerations to the case where M and R are finite.

We also assume that all the covers included in the aCM are not only covers but actually partitions of the set of sites of the agents. In this case the reduced model is a set of chemical ODEs and the reactions yielding these equations (assuming mass action kinetics) are given by applying (or matching) the rules in $\overline{R}$ to the fragments.

If the aCM does not fulfill this requirement, we can try to use two other sets of reactions on the fragments for the computation of organisations. Firstly, there are the reactions given by matching the fragments to the rules $\overline{R}$, but the ODEs they yield are not sound. Then our conjecture is true for some cases and false for others, cf. Section 5. But since the soundness does not hold the meaning of the conjecture and its relevance to these systems is very limited. Secondly, there are the reactions given by the reduced system in case it consists of chemical ODEs. If these are not given by $\overline{R}$, they are likewise of limited use.

On the other hand, the aCM can always be changed by joining cover classes together. However this comes at the price of a more finely grained fragmentation.

The objects defined so far can be summarised in the following diagram.

$$\begin{tikzcd}
& (\mathcal{F}, \widetilde{R}) \arrow[r, mapsto, "\mathrm{org}"] & \mathcal{O}_{\mathcal{F}} \\
(M, \overline{R}) \arrow[ur, mapsto, "\overline{\mathrm{frag}}"] \arrow[dr, mapsto, "t"] & & \\
& (M, R) \arrow[r, mapsto, "\mathrm{org}"] & \mathcal{O} \arrow[uu, dotted, "\Theta", "?"']
\end{tikzcd}$$

Where we define $\Theta : \mathcal{O} \to \mathcal{O}_{\mathcal{F}}$ by its action on $O \in \mathcal{O}$

$$\Theta(O) := \{F \in \mathcal{F} \mid \exists m \in O : \; [F, m] \neq \emptyset\}.$$

We also note that Θ is monotonic, i.e. if $O_1 \subseteq O_2$ then $\Theta(O_1) \subseteq \Theta(O_2)$. Actually we do not know yet whether Θ maps into $\mathcal{O}_{\mathcal{F}}$. This is exactly the statement of our conjecture.

Conjecture 4.1 *The map Θ is properly defined, i.e. every organisation is mapped to an f–organisation by Θ.*

Lemma 4.2 *Every closed set is mapped to a closed set of fragments by Θ.*

Proof. We fix a closed set $C \subseteq M$ and show that $\Theta(C) \subseteq \mathcal{F}$ is closed in $(\mathcal{F}, \widetilde{R})$. Take a reaction $(\tilde{l}, \tilde{r}) \in \widetilde{R}$ such that $\text{supp}(\tilde{l}, \tilde{r}) \subseteq \Theta(C)$. We need to show that $\text{prod}(\tilde{l}, \tilde{r}) \subseteq \Theta(C)$. Let be $\text{supp}(\tilde{l}, \tilde{r}) = \{F_1, \ldots, F_k\}$ and $m_i \in C \subseteq M$ such that $F_i \in \Theta(\{m_i\})$ with $i = 1, \ldots, k$. There is a reaction $(l, r) \in R_C$ with $\text{supp}(l, r) = \{m_1, \ldots, m_k\}$ and $\text{prod}(\tilde{l}, \tilde{r}) \subseteq \Theta(\text{prod}(l, r))$ by Q1 and Q2. Since C is closed we have $\text{prod}(l, r) \subseteq C$ and therefore $\Theta(\text{prod}(l, r)) \subseteq \Theta(C)$. This shows $\text{prod}(\tilde{l}, \tilde{r}) \subseteq \Theta(C)$. □

The following intuition is a guide for the next lemma. For a large enough multiset over a self-maintaining set G we can apply all possible reactions at a suitable rate such that after the application there is the same or a higher amount of each molecular species of G present. Counting the possible embeddings of fragments before and after the application shows obviously not a decrease in number. So we would like to use the rates chosen for G to construct rates for the reactions of $\Theta(G)$. In our special case, for every reaction in R there is only one reaction in $\widetilde{R}$ matching it. Then the reaction rates for $\Theta(G)$ can be chosen to be the summed up rates of G.

Lemma 4.3 *Every self-maintaining set is mapped to a self-maintaining set of fragments by Θ.*

Proof. We fix a self-maintaining set $G \subseteq M$ and show that $\Theta(G) \subseteq \mathcal{F}$ is self-maintaining in $(\mathcal{F}, \widetilde{R})$. By our assumption the set of reactions R_G can be partitioned by the set of reactions $\widetilde{R}_{\Theta(G)}$. By this we mean that any $(\tilde{l}, \tilde{r}) \in \widetilde{R}_{\Theta(G)}$ matches all the elements of the set

$$R(\tilde{l}, \tilde{r}) := \{(l_1, r_1), \ldots, (l_k, r_k)\} \subseteq R_G$$

and

$$\dot{\bigcup_{(\tilde{l},\tilde{r}) \in \widetilde{R}_{\Theta(G)}}} R(\tilde{l}, \tilde{r}) = R_G.$$

Since G is self-maintaining there is a $v \in \mathbb{R}^{|R_G|}$ with strictly positive entries such that $S_G v$ has only non-negative entries. If we set

$$\tilde{v}_{(\tilde{l},\tilde{r})} := \sum_{(l,r) \in R(\tilde{l},\tilde{r})} v_{(l,r)}$$

then $S_{\Theta(G)} \tilde{v}$ has only non-negative entries. □

It is not obvious that feasible organisations are mapped to feasible f–organisations by Θ. Here is a proof for this statement.

Lemma 4.4 *If O is a $\mathbf{k}$-feasible organisation then $\Theta(O)$ is a $\widetilde{\mathbf{k}}$-feasible organisation.*

Proof. There is an $x \in \mathbb{R}^{|O|}_{>0}$ with $S_O v_{O,\mathbf{k}}(x) \geq 0$. We also know that $Tx > 0$ and that $T(S_O v_{O,\mathbf{k}}(x)) \geq 0$ because of $S_O v_{O,\mathbf{k}}(x) \geq 0$. From the soundness of the abstraction map T we have $0 \leq T(S_O v_{O,\mathbf{k}}(x)) = S_{\mathcal{F}} v_{\mathcal{F},\widetilde{\mathbf{k}}}(Tx)$. It follows that $\Theta(O)$ is $\widetilde{\mathbf{k}}$-feasible. □

5 Examples

We use the κ–notation [2] to write down some examples illustrating various interesting effects.

Simple Example

In our first example the number of fragments is the same as the number of molecules, but the organisational structure differs. We define our example network by taking two agents A and B with one binding site a, b respectively and one phosphorylation site x, y respectively: $A(a,x)$, $B(b,y)$. The rules are $A(a), B(b) \rightarrow A(a!1)B(b!1)$; $A(x_u) \leftrightarrow A(x_p)$ and $B(y_u) \leftrightarrow B(y_p)$. The fragments of this network are $A(a)$, $A(b)$, $A(a!)$, $A(b!)$, $A(x_u)$, $A(x_p)$, $B(y_u)$ and $B(y_p)$ with the reactions $A(a) + B(b) \rightarrow A(a!) + B(b!)$; $A(x_u) \leftrightarrow A(x_p)$ and $B(y_u) \leftrightarrow B(y_p)$.

In the following Hasse diagram, see Fig. 1, the subset relations of organisations are shown. Only newly occurring species are depicted in order to keep it concise. Firstly we see that Lemmata 4.2 and 4.3 say that the sets $\emptyset$, $\{B(b), B(y_u), B(y_p)\}$, $\{A(a), A(x_u), A(x_p)\}$, $\{A(a!), A(x_u), A(x_p),\ B(b!), B(y_u), B(y_p)\}$, $\{A(a!), A(x_u), A(x_p),\ B(b!), B(y_u), B(y_p), B(b)\}$ and $\{A(a!), A(x_u), A(x_p),\ B(b!), B(y_u),\ B(y_p), A(a)\}$ should be f–organisations which is easily verified. But there are plenty more. On the whole we found 48 f–organisations.

If we change the rules to $A(a), B(b) \leftrightarrow A(a!1)B(b!1)$; $A(x_u) \rightarrow A(x_p)$ and $B(y_u) \rightarrow B(y_p)$, the complete set of molecules is not an organisation any more, but still a semi-organisation. From the f–reactions $A(a) + B(b) \leftrightarrow A(a!) + B(b!)$; $A(x_u) \rightarrow A(x_p)$ and $B(y_u) \rightarrow B(y_p)$ we see that the full set of fragments is not semi-self-maintaining. This shows that Θ does not preserve the semi-self-maintenance.

Polymerisation

Strictly speaking our considerations do not apply here because the set of molecules and rules is infinitely large. Still, the conclusions of the Lemmata 4.2 and 4.3 are correct for the polymerisation as well.

In this example the number of fragments is very small compared to the number of molecules. We define a simple polymerisation as follows. Take one agent A with two binding sites a and b and one rule $A(a), A(b) \rightarrow A(a!1)A(b!1)$. The fragments of this reaction network are $A(a)$, $A(b)$, $A(a!)$ and $A(b!)$ with the reaction $A(a) + A(b) \rightarrow A(a!) + A(b!)$. A ring is a molecule of the form

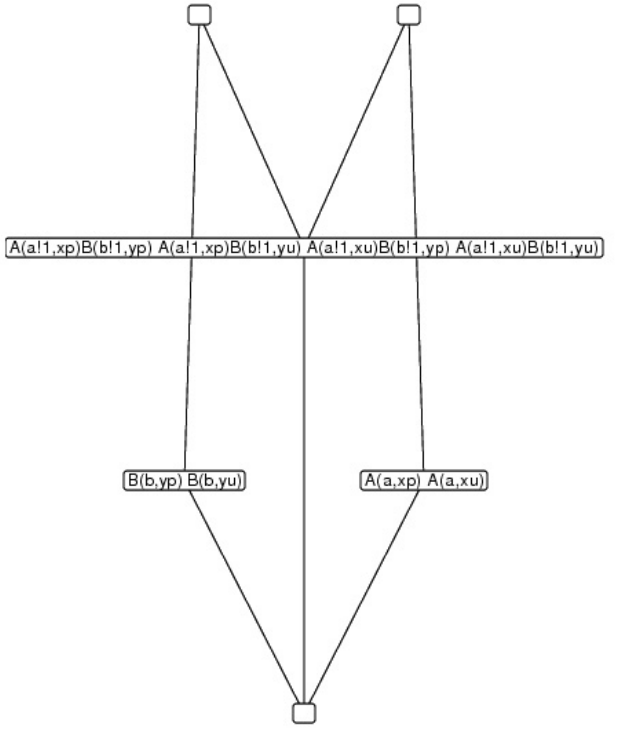

Fig. 1. Hasse diagram of organisations for the simple example. Only newly occurring species are shown. The unlabelled terminal node stands for the empty set.

$A_1(a_1!1, b_1!k), A_2(a_2!2, b_2!1), \ldots, A_k(a_k!k, b_k!(k-1))$ with $k \in \mathbb{N}$. The organisations are all the sets consisting of rings only, since any molecule not being a ring can react to a ring and would therefore violate the self-maintaining condition.

Firstly, we see that the conclusion of the Lemmata 4.2 and 4.3 say that the set $\{A(a!), A(b!)\}$ should be an f–organisation, which is true. But there are more, see Fig. 2.

Secondly and more interestingly, except the mentioned $\{A(a!), A(b!)\}$ no other f–organisation can be realised as a set of molecules at all. For example there is no $A \subseteq M$ such that $\Theta(A) = \{A(a)\}$ since $\Theta(A)$ then would have to include $A(b)$ as well.

If we include the backward rule $A(a!1)A(b!1) \to A(a), A(b)$, the only non-trivial organisation is the whole set. It is mapped to $\{A(a), A(b), A(a!), A(b!)\}$ by Θ. The complete Hasse diagram of f–organisations is shown in Fig. 3.

EGF Model

We analysed the organisations of the simple EGF model also described in [5]. The definition of this network as well as two Hasse diagrams of the (f-)organisations are provided in the appendix.

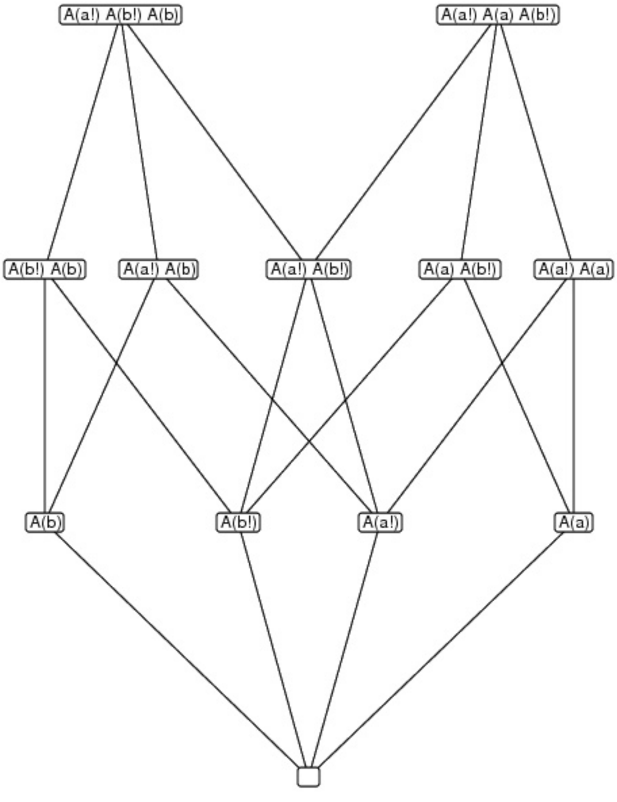

Fig. 2. Hasse diagram of f–organisations of our polymerisation example. The unlabelled terminal node stands for the empty set.

Since here we have an overlap in the cover classes of the aCM, our proof does not apply, but still the result holds. This was already mentioned in Section 4. Here the map Θ is even injective but not surjective.

In the set of 356 species we found 32 organisations and 66 semi-organisations. Figure A.3 in the appendix shows the Hasse diagram of organisations with their images under Θ. The set of fragments consists of 38 elements. It has 56 f–organisations and 126 semi-f–organisations. Figure A.4 in the appendix shows the Hasse diagram of f–organisations.

Counterexample

As mentioned in Section 4 there is a counterexample[2] to the conjecture in the case that there is an overlap of cover classes in the aCM, i.e. we have indeed a cover and not a partition. This leads to an over counting since several fragment reaction match a single reaction of molecules. That means there are more fragments used up when applying the reactions then actual molecules are used up. This gives an intuition why there are organisations not mapped to f–organisations by Θ. Actually

[2] We owe this example to Jérôme Feret. Thanks very much.

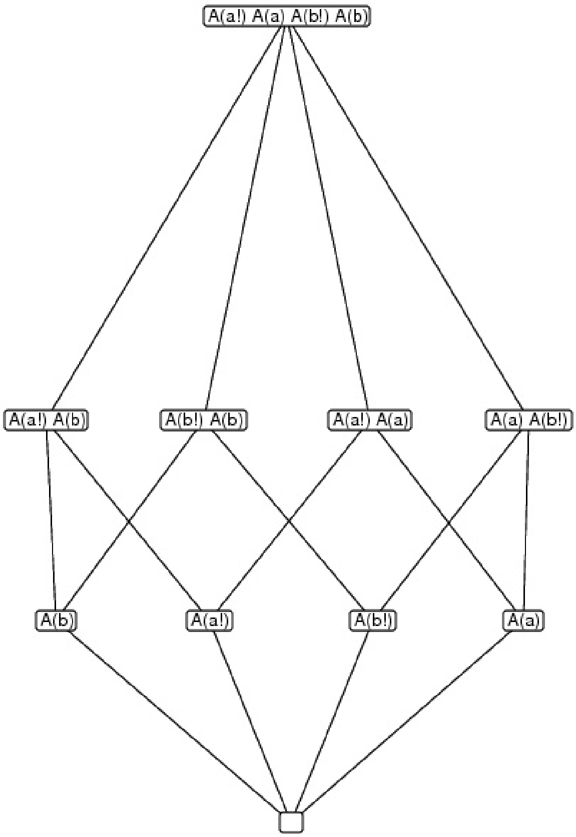

Fig. 3. Hasse diagram of f–organisations of our polymerisation example when including the backward reaction. The unlabelled terminal node stands for the empty set.

the construction of the reduced model takes exactly this over counting phenomenon into account.

The definition of the counterexample network as well as two Hasse diagrams of the (f-)organisations are provided in the appendix.

As an example we mention that the biggest organisation is mapped to a set of fragments which is not an organisation.

6 Conclusion and Outlook

For the special case considered here we showed that the set of fragments embeddable in a (feasible) organisation of molecules is a (feasible) organisation of fragments. The case described is presumably the only situation where it is possible to find a sensible notion of organisations on the fragments. Additionally, we can always modify the aCM so that our results hold. This comes, of course, at a cost since it increases the amount of fragments.

If (M, R) is *consistent*, we know that the set of organisations $\mathcal{O}$ has a structure of a lattice [3]. Does this imply that $(\mathcal{F}, \widetilde{R})$ is also consistent and that the f–

organisations therefore also have a lattice structure?

We showed that every chemical organisation corresponds to an f–organisation and that there can be several organisations corresponding to the same f–organisation. However, our examples also show that there are f–organisations that cannot be realised as a set of molecules, e.g. that there are some that are not in the image of Θ. Let us call them *unrealisable* and all others *realisable*. This immediately raises a lot of questions, e.g. is the set of realisable f–organisations a lattice, is there a unique smallest realisable f–organisation, under what condition do we not have any unrealisable f–organisations, etc.

Fragment organisations should be particularly helpful when looking for organisations. When checking whether a subset of $A \subseteq M$ is an organisation we have a new necessary criterion, since $\Theta(A)$ needs to be an f–organisation. Whether this additional criterion is helpful in practise, we need to evaluate.

Our results immediately inspire the definition of a classification of organisations by looking at the preimages of our map Θ. We say $O \in \mathcal{O}$ and $O' \in \mathcal{O}$ are in the same *f–class* if $\Theta(O) = \Theta(O')$. Thereby we get a partition of our set of organisations. What common properties organisations in the same class have and what distinguishes them from organisations in other classes is to be analysed further.

Another interesting question is whether we can use the fragments in $\Theta(O)$ in order to describe and structure O further.

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A Listings

A.1 Counterexample

Here we list the data for the counterexample.

Agents, Sites and Rules

```
   E(a,b)
   R(a,b,c)

0  E(a~u) <-> E(a~p)
1  R(a),E(b) -> R(a!1)E(b!1)
2  E(a~p,b!1),R(a!1,b~u) -> E(a~p,b!1)R(a!1,b~p),R(a,b~u,c~u),E(a~u,b)
3  E(a~p,b!1),R(a!1,c~u) -> E(a~p,b!1)R(a!1,c~p)
```

Species

number and κ-expression:

```
0  R(a,b~u,c~u)
1  E(a~u,b)
2  E(a~p,b)
3  R(a!1,b~u,c~u)E(a~u,b!1)
4  R(a!1,b~u,c~u)E(a~p,b!1)
5  R(a!1,b~u,c~p)E(a~u,b!1)
6  R(a!1,b~u,c~p)E(a~p,b!1)
7  R(a!1,b~p,c~u)E(a~u,b!1)
8  R(a!1,b~p,c~u)E(a~p,b!1)
9  R(a!1,b~p,c~p)E(a~u,b!1)
10 R(a!1,b~p,c~p)E(a~p,b!1)
11 R(a,b~u,c~p)
12 R(a,b~p,c~u)
13 R(a,b~p,c~p)
```

Organisations

number and set:

```
0  { }
1  { 13 }
2  { 12 }
3  { 11 }
4  { 0 }
5  { 12 13 }
6  { 11 13 }
7  { 11 12 }
8  { 0 13 }
9  { 0 12 }
10 { 0 11 }
11 { 9 10 }
12 { 1 2 }
13 { 11 12 13 }
14 { 0 12 13 }
15 { 0 11 13 }
16 { 0 11 12 }
17 { 9 10 13 }
18 { 9 10 12 }
19 { 9 10 11 }
20 { 0 9 10 }
21 { 0 11 12 13 }
22 { 9 10 12 13 }
23 { 9 10 11 13 }
24 { 9 10 11 12 }
25 { 0 9 10 13 }
26 { 0 9 10 12 }
27 { 0 9 10 11 }
28 { 1 2 9 10 }
29 { 9 10 11 12 13 }
30 { 0 9 10 12 13 }
31 { 0 9 10 11 13 }
32 { 0 9 10 11 12 }
33 { 0 9 10 11 12 13 }
34 { 0 1 2 3 4 5 6 7 8 9 10 }
```

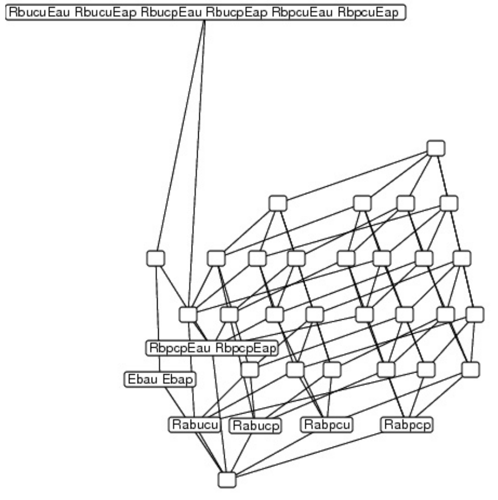

Fig. A.1. Hasse diagram of organisations of the counterexample. Only newly occurring species are shown. The unlabelled terminal node stands for the empty set.

Fragments

number and κ-expression:

```
0  E(a~u,b)
1  E(a~p,b)
2  R(a!1,b~u)E(a~u,b!1)
3  R(a,c~u)
4  R(a,c~p)
5  R(a,b~u)
6  R(a,b~p)
7  R(a!1,b~p)E(a~u,b!1)
8  R(a!1,b~p)E(a~p,b!1)
9  R(a!1,c~p)E(a~u,b!1)
10 R(a!1,c~p)E(a~p,b!1)
11 R(a!1,b~u)E(a~p,b!1)
12 R(a!1,c~u)E(a~u,b!1)
13 R(a!1,c~u)E(a~p,b!1)
```

F–organisations

number and set:

```
0 {  }
1 { 6 }
2 { 5 }
3 { 4 }
4 { 3 }
5 { 5 6 }
6 { 4 6 }
7 { 4 5 }
```

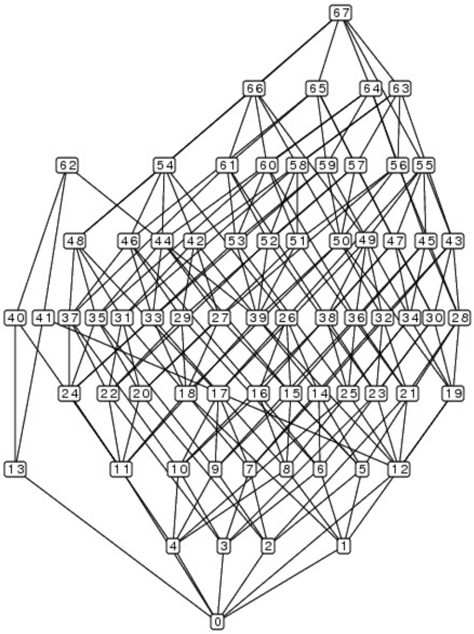

Fig. A.2. Hasse diagram of f–organisations of the counterexample. The unlabelled terminal node stands for the empty set.

```
8 { 3 6 }
9 { 3 5 }
10 { 3 4 }
11 { 9 10 }
12 { 7 8 }
13 { 0 1 }
14 { 4 5 6 }
15 { 3 5 6 }
16 { 3 4 6 }
17 { 3 4 5 }
18 { 6 9 10 }
19 { 6 7 8 }
20 { 5 9 10 }
21 { 5 7 8 }
22 { 4 9 10 }
23 { 4 7 8 }
24 { 3 9 10 }
25 { 3 7 8 }
26 { 3 4 5 6 }
27 { 5 6 9 10 }
28 { 5 6 7 8 }
29 { 4 6 9 10 }
30 { 4 6 7 8 }
31 { 4 5 9 10 }
32 { 4 5 7 8 }
33 { 3 6 9 10 }
34 { 3 6 7 8 }
35 { 3 5 9 10 }
36 { 3 5 7 8 }
37 { 3 4 9 10 }
38 { 3 4 7 8 }
39 { 7 8 9 10 }
```

```
40 { 0 1 9 10 }
41 { 0 1 7 8 }
42 { 4 5 6 9 10 }
43 { 4 5 6 7 8 }
44 { 3 5 6 9 10 }
45 { 3 5 6 7 8 }
46 { 3 4 6 9 10 }
47 { 3 4 6 7 8 }
48 { 3 4 5 9 10 }
49 { 3 4 5 7 8 }
50 { 6 7 8 9 10 }
51 { 5 7 8 9 10 }
52 { 4 7 8 9 10 }
53 { 3 7 8 9 10 }
54 { 3 4 5 6 9 10 }
55 { 3 4 5 6 7 8 }
56 { 5 6 7 8 9 10 }
57 { 4 6 7 8 9 10 }
58 { 4 5 7 8 9 10 }
59 { 3 6 7 8 9 10 }
60 { 3 5 7 8 9 10 }
61 { 3 4 7 8 9 10 }
62 { 0 1 7 8 9 10 }
63 { 4 5 6 7 8 9 10 }
64 { 3 5 6 7 8 9 10 }
65 { 3 4 6 7 8 9 10 }
66 { 3 4 5 7 8 9 10 }
67 { 3 4 5 6 7 8 9 10 }
```

A.2 EGF

Here we list the data for the EGF example.

Agents, Sites and Rules

```
   E(r)
   R(l,r,y4,y6)
   G(a,b)
   S(y,c)
   O(d)

0  E(r),R(l,r) <-> E(r!1)R(l!1,r)
1  E(r!2)R(l!2,r),E(r!1)R(l!1,r) <-> E(r!3)E(r!2)R(l!3,r!1)R(l!2,r!1)
2  E(r!3)E(r!2)R(l!3,r!1)R(l!2,r!1,y6~u) -> E(r!3)E(r!2)R(l!3,r!1)R(l!2,r!1,y6~p)
3  R(y6~p) -> R(y6~u)
4  E(r!3)E(r!2)R(l!3,r!1)R(l!2,r!1,y4~u) -> E(r!3)E(r!2)R(l!3,r!1)R(l!2,r!1,y4~p)
5  R(y4~p) -> R(y4~u)
6  R(y4~p!1,r!)S(y~u,c!1) -> R(y4~p!1,r!)S(y~p,c!1)
7  S(y~p,c!) -> S(y~u,c!)
8  S(y~p,c) -> S(y~u,c)
9  G(a,b),R(y6~p) <-> G(a!1,b)R(y6~p!1)
10 G(a,b!),R(y6~p) <-> G(a!1,b)R(y6~p!1)
11 G(a!1,b)R(y6~p!1),O(d) <-> G(a!2,b!1)R(y6~p!2)O(d!1)
12 G(a,b),O(d) <-> G(a,b!1)O(d!1)
13 G(a!1,b)S(y~p!1,c),O(d) <-> G(a!2,b!1)S(y~p!2,c)O(d!1)
14 G(a!1,b)S(y~p!1,c!),O(d) <-> G(a!2,b!1)S(y~p!2,c!)O(d!1)
15 R(y4~p),S(y~u,c) -> R(y4~p!1)S(y~u,c!1)
16 R(y4~p!1)S(y~u,c!1) -> R(y4~p),S(y~u,c)
17 R(y4~p),S(y~p,c) -> R(y4~p!1)S(y~p,c!1)
18 R(y4~p!1)S(y~p,c!1) -> R(y4~p),S(y~p,c)
19 G(a!1,b)S(y~p!1,c),R(y4~p) <-> G(a!2,b)S(y~p!2,c!1)R(y4~p!1)
20 G(a!2,b!1)S(y~p!2,c)O(d!1),R(y4~p) <-> G(a!3,b!2)S(y~p!3,c!1)R(y4~p!1)O(d!2)
21 G(a,b),S(y~p,c!1)R(y4~p!1) <-> G(a!2,b)S(y~p!2,c!1)R(y4~p!1)
22 G(a,b),S(y~p,c) <-> G(a!1,b)S(y~p!1,c)
23 G(a,b!),S(y~p,c) <-> G(a!1,b!)S(y~p!1,c)
24 G(a,b!2)O(d!2),R(y4~p!1)S(y~p,c!1) <-> G(a!3,b!2)O(d!2)R(y4~p!1)S(y~p!3,c!1)
```

Species

number and κ-expression:

```
0   E(r)
1   E(r!1)R(l!1,r,y4~p,y6~p)
2   R(l,r,y4~p,y6~p)
3   G(a,b)
```

```
4   S(c,y~p)
5   O(d)
6   E(r!1)E(r!2)R(l!1,r!3,y4~p,y6~p)R(l!2,r!3,y4~p,y6~p)
7   R(l,r,y4~p,y6~u)
8   E(r!1)R(l!1,r,y4~p,y6~u)
9   R(l,r,y4~u,y6~p)
10  E(r!1)R(l!1,r,y4~u,y6~p)
11  S(c,y~u)
12  G(a!1,b)R(l,r,y4~p,y6~p!1)
13  E(r!1)G(a!2,b)R(l!1,r,y4~p,y6~p!2)
14  G(a,b!1)O(d!1)
15  R(l,r,y4~p!1,y6~p)S(c!1,y~p)
16  E(r!1)R(l!1,r,y4~p!2,y6~p)S(c!2,y~p)
17  G(a!1,b)S(c,y~p!1)
18  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p,y6~p!3)R(l!1,r!4,y4~p,y6~p)
19  E(r!1)E(r!2)R(l!1,r!3,y4~p,y6~p)R(l!2,r!3,y4~u,y6~p)
20  E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~p)R(l!1,r!3,y4~p,y6~p)S(c!4,y~p)
21  E(r!1)E(r!2)R(l!1,r!3,y4~p,y6~p)R(l!2,r!3,y4~p,y6~u)
22  E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p,y6~p!3)R(l!2,r!5,y4~p,y6~p!4)
23  E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p,y6~p!3)R(l!2,r!4,y4~u,y6~p)
24  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p)R(l!1,r!4,y4~p,y6~p!3)S(c!5,y~p)
25  E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p,y6~p!3)R(l!2,r!4,y4~p,y6~u)
26  E(r!1)E(r!2)R(l!1,r!3,y4~u,y6~p)R(l!2,r!3,y4~u,y6~p)
27  E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~p)R(l!1,r!3,y4~u,y6~p)S(c!4,y~p)
28  E(r!1)E(r!2)R(l!2,r!3,y4~p,y6~u)R(l!1,r!3,y4~u,y6~p)
29  E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~p)R(l!2,r!3,y4~p!5,y6~p)S(c!4,y~p)S(c!5,y~p)
30  E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~p)R(l!2,r!3,y4~p,y6~u)S(c!4,y~p)
31  E(r!1)E(r!2)R(l!1,r!3,y4~p,y6~u)R(l!2,r!3,y4~p,y6~u)
32  R(l,r,y4~u,y6~u)
33  E(r!1)R(l!1,r,y4~u,y6~u)
34  R(l,r,y4~p!1,y6~u)S(c!1,y~p)
35  E(r!1)R(l!1,r,y4~p!2,y6~u)S(c!2,y~p)
36  E(r!1)G(a!2,b)R(l!1,r,y4~u,y6~p!2)
37  G(a!1,b)R(l,r,y4~u,y6~p!1)
38  R(l,r,y4~p!1,y6~p)S(c!1,y~u)
39  E(r!1)R(l!1,r,y4~p!2,y6~p)S(c!2,y~u)
40  G(a!1,b)R(l,r,y4~p!2,y6~p!1)S(c!2,y~p)
41  E(r!1)G(a!2,b)R(l!1,r,y4~p!3,y6~p!2)S(c!3,y~p)
42  G(a!1,b!2)O(d!2)R(l,r,y4~p,y6~p!1)
43  E(r!1)G(a!2,b!3)O(d!3)R(l!1,r,y4~p,y6~p!2)
44  G(a!1,b!2)O(d!2)R(l,r,y4~u,y6~p!1)
45  E(r!1)G(a!2,b!3)O(d!3)R(l!1,r,y4~u,y6~p!2)
46  G(a!1,b!2)O(d!2)R(l,r,y4~p!3,y6~p!1)S(c!3,y~p)
47  E(r!1)E(r!2)G(a!3,b!4) O(d!4)R(l!1,r!5,y4~p,y6~p!3)R(l!2,r!5,y4~p,y6~p)
48  E(r!1)G(a!2,b!3)O(d!3)R(l!1,r,y4~p!4,y6~p!2)S(c!4,y~p)
49  G(a!1,b!2)O(d!2)S(c,y~p!1)
50  E(r!1)G(a!2,b)R(l!1,r,y4~p!3,y6~p!2)S(c!3,y~u)
51  G(a!1,b)R(l,r,y4~p!2,y6~p!1)S(c!2,y~u)
52  E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~p)R(l!2,r!3,y4~p,y6~p)S(c!4,y~u)
53  R(l,r,y4~p!1,y6~u)S(c!1,y~u)
54  E(r!1)R(l!1,r,y4~p!2,y6~u)S(c!2,y~u)
55  G(a!1,b)R(l,r,y4~p!2,y6~p)S(c!2,y~p!1)
56  E(r!1)G(a!2,b)R(l!1,r,y4~p!3,y6~p)S(c!3,y~p!2)
57  E(r!1)G(a!2,b)G(a!3,b)R(l!1,r,y4~p!4,y6~p!3)S(c!4,y~p!2)
58  G(a!1,b)G(a!2,b)R(l,r,y4~p!3,y6~p!2)S(c!3,y~p!1)
59  E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p)R(l!2,r!4,y4~p,y6~p)S(c!5,y~p!3)
60  G(a!1,b)R(l,r,y4~p!2,y6~u)S(c!2,y~p!1)
61  E(r!1)G(a!2,b)R(l!1,r,y4~p!3,y6~u)S(c!3,y~p!2)
62  G(a!1,b!2)O(d!2)R(l,r,y4~p!3,y6~p)S(c!3,y~p!1)
63  E(r!1)G(a!2,b!3)O(d!3)R(l!1,r,y4~p!4,y6~p)S(c!4,y~p!2)
64  E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~u)R(l!1,r!3,y4~p,y6~p)S(c!4,y~u)
65  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~p,y6~p)S(c!5,y~u)
66  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~p,y6~p)S(c!5,y~p)
67  E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p,y6~p)R(l!2,r!4,y4~u,y6~p!3)
68  E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p,y6~p)R(l!2,r!5,y4~u,y6~p!3)
69  E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~p,y6~p)S(c!6,y~p)
70  E(r!1)E(r!2)R(l!1,r!3,y4~p,y6~p)R(l!2,r!3,y4~u,y6~u)
71  E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p!4)R(l!1,r!5,y4~p,y6~p)S(c!6,y~p!3)
72  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~u)R(l!1,r!4,y4~p,y6~p)S(c!5,y~p!3)
73  E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p)R(l!1,r!5,y4~p,y6~p)S(c!6,y~p!3)
74  E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~u)R(l!1,r!3,y4~p,y6~p)S(c!4,y~p)
75  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~u)R(l!1,r!4,y4~p,y6~p!3)S(c!5,y~u)
76  E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p!4)R(l!1,r!5,y4~p,y6~p!3)S(c!6,y~u)
77  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p)R(l!1,r!4,y4~p,y6~p!3)S(c!5,y~u)
78  E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p!4)R(l!1,r!5,y4~p,y6~p!3)S(c!6,y~p)
79  E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p,y6~p!3)R(l!2,r!5,y4~u,y6~p!4)
80  E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p,y6~p!5)R(l!2,r!6,y4~u,y6~p!3)
81  E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~p,y6~p!5)S(c!7,y~p)
82  E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p,y6~p!3)R(l!2,r!4,y4~u,y6~u)
83  E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p)R(l!1,r!5,y4~p,y6~p!3)S(c!6,y~p!4)
84  E(r!1)E(r!2)G(a!3,b)G(a!4,b)G(a!5,b)R(l!2,r!6,y4~p!7,y6~p!5)R(l!1,r!6,y4~p,y6~p!3)S(c!7,y~p!4)
85  E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~u)R(l!1,r!5,y4~p,y6~p!3)S(c!6,y~p!4)
86  E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p,y6~p!5)R(l!2,r!6,y4~p,y6~p!3)
87  E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p)R(l!1,r!6,y4~p,y6~p!5)S(c!7,y~p!3)
88  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~u)R(l!1,r!4,y4~p,y6~p!3)S(c!5,y~p)
89  E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~u)R(l!1,r!3,y4~u,y6~p)S(c!4,y~u)
90  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~u,y6~p)S(c!5,y~u)
91  E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~p)R(l!1,r!3,y4~u,y6~p)S(c!4,y~u)
92  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~u,y6~p)S(c!5,y~p)
93  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~u,y6~p!3)R(l!1,r!4,y4~u,y6~p)
94  E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~u,y6~p!3)R(l!1,r!5,y4~u,y6~p)
95  E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~u,y6~p)S(c!6,y~p)
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96  E(r!1)E(r!2)R(l!1,r!3,y4~u,y6~p)R(l!2,r!3,y4~u,y6~u)
97  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p)R(l!1,r!4,y4~u,y6~p)S(c!5,y~p!3)
98  E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p!4)R(l!1,r!5,y4~u,y6~p)S(c!6,y~p!3)
99  E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~u)R(l!1,r!4,y4~u,y6~p)S(c!5,y~p!3)
100 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p,y6~p!3)R(l!1,r!5,y4~u,y6~p)
101 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p)R(l!1,r!5,y4~u,y6~p)S(c!6,y~p!3)
102 E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~u)R(l!1,r!3,y4~u,y6~p)S(c!4,y~p)
103 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~p)R(l!2,r!3,y4~p!5,y6~u)S(c!4,y~p)S(c!5,y~u)
104 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~p!6,y6~p)S(c!6,y~p)S(c!5,y~u)
105 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~p)R(l!2,r!3,y4~p!5,y6~p)S(c!4,y~p)S(c!5,y~u)
106 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~p!6,y6~p)S(c!6,y~p)S(c!5,y~p)
107 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p)R(l!2,r!4,y4~u,y6~p!3)S(c!5,y~p)
108 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~u,y6~p!3)S(c!6,y~p)
109 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~p!7,y6~p)S(c!7,y~p)S(c!6,y~p)
110 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~p)R(l!2,r!3,y4~u,y6~u)S(c!4,y~p)
111 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p)R(l!2,r!4,y4~p!6,y6~p)S(c!6,y~p!3)S(c!5,y~p)
112 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p!4)R(l!1,r!5,y4~p!7,y6~p)S(c!6,y~p!3)S(c!7,y~p)
113 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p)R(l!2,r!4,y4~p!6,y6~u)S(c!6,y~p!3)S(c!5,y~p)
114 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~p,y6~p!3)S(c!6,y~p)
115 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~p!7,y6~p)S(c!7,y~p!3)S(c!6,y~p)
116 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~p)R(l!2,r!3,y4~p!5,y6~u)S(c!4,y~p)S(c!5,y~p)
117 E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~u)R(l!1,r!3,y4~p,y6~u)S(c!4,y~u)
118 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~p,y6~u)S(c!5,y~u)
119 E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~p)R(l!1,r!3,y4~p,y6~u)S(c!4,y~u)
120 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~p,y6~u)S(c!5,y~p)
121 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p,y6~u)R(l!2,r!4,y4~u,y6~p!3)
122 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p,y6~u)R(l!2,r!5,y4~u,y6~p!3)
123 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~p,y6~u)S(c!6,y~p)
124 E(r!1)E(r!2)R(l!1,r!3,y4~p,y6~u)R(l!2,r!3,y4~u,y6~u)
125 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p)R(l!1,r!4,y4~p,y6~u)S(c!5,y~p!3)
126 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p!4)R(l!1,r!5,y4~p,y6~u)S(c!6,y~p!3)
127 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~u)R(l!1,r!4,y4~p,y6~u)S(c!5,y~p!3)
128 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p,y6~p!3)R(l!1,r!5,y4~p,y6~u)
129 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p)R(l!1,r!5,y4~p,y6~u)S(c!6,y~p!3)
130 E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~u)R(l!1,r!3,y4~p,y6~u)S(c!4,y~p)
131 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~u)R(l!2,r!3,y4~p!5,y6~u)S(c!4,y~u)S(c!5,y~u)
132 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~p!6,y6~u)S(c!6,y~u)S(c!5,y~u)
133 E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~p)R(l!1,r!3,y4~p!5,y6~u)S(c!5,y~u)S(c!4,y~u)
134 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~p!6,y6~u)S(c!5,y~p)S(c!6,y~u)
135 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~u)R(l!2,r!4,y4~u,y6~p!3)S(c!5,y~u)
136 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~u)R(l!2,r!5,y4~u,y6~p!3)S(c!6,y~u)
137 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~p!7,y6~u)S(c!6,y~p)S(c!7,y~u)
138 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~u)R(l!2,r!3,y4~u,y6~u)S(c!4,y~u)
139 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p)R(l!1,r!4,y4~p!6,y6~u)S(c!5,y~p!3)S(c!6,y~u)
140 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p!4)R(l!1,r!5,y4~p!7,y6~u)S(c!6,y~p!3)S(c!7,y~u)
141 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~u)R(l!2,r!4,y4~p!6,y6~u)S(c!6,y~p!3)S(c!5,y~u)
142 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~u)R(l!2,r!5,y4~p,y6~p!3)S(c!6,y~u)
143 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p)R(l!1,r!5,y4~p!7,y6~u)S(c!6,y~p!3)S(c!7,y~u)
144 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~u)R(l!2,r!3,y4~p!5,y6~u)S(c!5,y~p)S(c!4,y~u)
145 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~p!7,y6~p!4)S(c!6,y~u)S(c!7,y~u)
146 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p!3)R(l!2,r!4,y4~p!6,y6~p)S(c!5,y~u)S(c!6,y~u)
147 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~p!7,y6~p!4)S(c!7,y~p)S(c!6,y~u)
148 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~u,y6~p!4)S(c!6,y~u)
149 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~u,y6~p!3)S(c!7,y~u)
150 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p!8,y6~p!3)S(c!8,y~p)S(c!7,y~u)
151 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p!3)R(l!2,r!4,y4~u,y6~u)S(c!5,y~u)
152 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~p!7,y6~p)S(c!7,y~p!4)S(c!6,y~u)
153 E(r!1)E(r!2)G(a!3,b)G(a!4,b)G(a!5,b)R(l!1,r!6,y4~p!7,y6~p!3)R(l!2,r!6,y4~p!8,y6~p!5)S(c!8,y~p!4)S(c!7,y~u)
154 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~p!7,y6~u)S(c!7,y~p!4)S(c!6,y~u)
155 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p,y6~p!3)S(c!7,y~u)
156 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p!8,y6~p)S(c!8,y~p!3)S(c!7,y~u)
157 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p!3)R(l!2,r!4,y4~p!6,y6~u)S(c!6,y~p)S(c!5,y~u)
158 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~p)R(l!2,r!3,y4~p!5,y6~p)S(c!4,y~u)S(c!5,y~u)
159 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p!3)R(l!1,r!4,y4~p!6,y6~p)S(c!5,y~p)S(c!6,y~u)
160 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p)R(l!2,r!4,y4~u,y6~p!3)S(c!5,y~u)
161 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~u,y6~p!3)S(c!6,y~u)
162 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~p!7,y6~p)S(c!6,y~p)S(c!7,y~u)
163 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~p)R(l!2,r!3,y4~u,y6~u)S(c!4,y~u)
164 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p)R(l!2,r!4,y4~p!6,y6~p)S(c!6,y~p!3)S(c!5,y~u)
165 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p!4)R(l!1,r!5,y4~p!7,y6~p)S(c!6,y~p!3)S(c!7,y~u)
166 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p)R(l!2,r!4,y4~p!6,y6~u)S(c!6,y~p!3)S(c!5,y~u)
167 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~p,y6~p!3)S(c!6,y~u)
168 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~p!7,y6~p)S(c!7,y~p!3)S(c!6,y~u)
169 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~p)R(l!2,r!3,y4~p!5,y6~u)S(c!5,y~p)S(c!4,y~u)
170 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~p!7,y6~p!4)S(c!6,y~p)S(c!7,y~p)
171 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~u,y6~p!4)S(c!6,y~p)
172 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~u,y6~p!3)S(c!7,y~p)
173 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p!8,y6~p!3)S(c!7,y~p)S(c!8,y~p)
174 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p!3)R(l!2,r!4,y4~u,y6~u)S(c!5,y~p)
175 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~p!7,y6~p)S(c!7,y~p!4)S(c!6,y~p)
176 E(r!1)E(r!2)G(a!3,b)G(a!4,b)G(a!5,b)R(l!1,r!6,y4~p!7,y6~p!3)R(l!2,r!6,y4~p!8,y6~p!5)S(c!8,y~p!4)S(c!7,y~p)
177 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~p!7,y6~u)S(c!7,y~p!4)S(c!6,y~p)
178 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p,y6~p!3)S(c!7,y~p)
179 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p!8,y6~p)S(c!8,y~p!3)S(c!7,y~p)
180 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p!3)R(l!2,r!4,y4~p!6,y6~u)S(c!5,y~p)S(c!6,y~p)
181 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~u,y6~p!3)R(l!2,r!5,y4~u,y6~p!4)
182 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~u,y6~p!5)R(l!2,r!6,y4~u,y6~p!3)
183 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~u,y6~p!5)S(c!7,y~p)
184 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~u,y6~p!3)R(l!2,r!4,y4~u,y6~u)
185 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p)R(l!1,r!5,y4~u,y6~p!3)S(c!6,y~p!4)
186 E(r!1)E(r!2)G(a!3,b)G(a!4,b)G(a!5,b)R(l!2,r!6,y4~p!7,y6~p!5)R(l!1,r!6,y4~u,y6~p!3)S(c!7,y~p!4)
187 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~u)R(l!1,r!5,y4~u,y6~p!3)S(c!6,y~p!4)
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188 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p,y6~p!3)R(l!1,r!6,y4~u,y6~p!5)
189 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p)R(l!1,r!6,y4~u,y6~p!5)S(c!7,y~p!3)
190 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~u)R(l!1,r!4,y4~u,y6~p!3)S(c!5,y~p)
191 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~u,y6~p!3)R(l!2,r!7,y4~u,y6~p!5)
192 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!5)R(l!1,r!7,y4~u,y6~p!3)S(c!8,y~p)
193 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~u,y6~p!3)R(l!2,r!5,y4~u,y6~u)
194 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p)R(l!1,r!6,y4~u,y6~p!3)S(c!7,y~p!5)
195 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!2,r!7,y4~p!8,y6~p!6)R(l!1,r!7,y4~u,y6~p!3)S(c!8,y~p!5)
196 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~u)R(l!1,r!6,y4~u,y6~p!3)S(c!7,y~p!5)
197 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p,y6~p!5)R(l!1,r!7,y4~u,y6~p!3)
198 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p)R(l!1,r!7,y4~u,y6~p!3)S(c!8,y~p!5)
199 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~u)R(l!1,r!5,y4~u,y6~p!3)S(c!6,y~p)
200 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~p!3)R(l!2,r!7,y4~p!9,y6~p!5)S(c!8,y~p)S(c!9,y~p)
201 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~u,y6~u)S(c!6,y~p)
202 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!3)R(l!2,r!6,y4~p!8,y6~p)S(c!8,y~p!5)S(c!7,y~p)
203 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!1,r!7,y4~p!8,y6~p!3)R(l!2,r!7,y4~p!9,y6~p!6)S(c!9,y~p!5)S(c!8,y~p)
204 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!3)R(l!2,r!6,y4~p!8,y6~u)S(c!8,y~p!5)S(c!7,y~p)
205 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~p!3)R(l!2,r!7,y4~p,y6~p!5)S(c!8,y~p)
206 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~p!3)R(l!2,r!7,y4~p!9,y6~p)S(c!9,y~p!5)S(c!8,y~p)
207 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~p!7,y6~u)S(c!6,y~p)S(c!7,y~p)
208 E(r!1)E(r!2)R(l!1,r!3,y4~u,y6~u)R(l!2,r!3,y4~u,y6~u)
209 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~p)R(l!1,r!4,y4~u,y6~u)S(c!5,y~p!3)
210 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!2,r!5,y4~p!6,y6~p!4)R(l!1,r!5,y4~u,y6~u)S(c!6,y~p!3)
211 E(r!1)E(r!2)G(a!3,b)R(l!2,r!4,y4~p!5,y6~u)R(l!1,r!4,y4~u,y6~u)S(c!5,y~p!3)
212 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p,y6~p!3)R(l!1,r!5,y4~u,y6~u)
213 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p)R(l!1,r!5,y4~u,y6~u)S(c!6,y~p!3)
214 E(r!1)E(r!2)R(l!2,r!3,y4~p!4,y6~u)R(l!1,r!3,y4~u,y6~u)S(c!4,y~p)
215 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~p!7,y6~p)S(c!6,y~p!3)S(c!7,y~p!4)
216 E(r!1)E(r!2)G(a!3,b)G(a!4,b)G(a!5,b)R(l!2,r!6,y4~p!7,y6~p!5)R(l!1,r!6,y4~p!8,y6~p)S(c!8,y~p!3)S(c!7,y~p!4)
217 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~p!7,y6~u)S(c!6,y~p!3)S(c!7,y~p!4)
218 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p)R(l!2,r!6,y4~p,y6~p!3)S(c!7,y~p!5)
219 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p)R(l!2,r!6,y4~p!8,y6~p)S(c!7,y~p!5)S(c!8,y~p!3)
220 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~p)R(l!2,r!4,y4~p!6,y6~u)S(c!5,y~p!3)S(c!6,y~p)
221 E(r!1)E(r!2)G(a!3,b)G(a!4,b)G(a!5,b)G(a!6,b)R(l!1,r!7,y4~p!8,y6~p!4)R(l!2,r!7,y4~p!9,y6~p!6)S(c!8,y~p!3)S(c!9,y~p!5)
222 E(r!1)E(r!2)G(a!3,b)G(a!4,b)G(a!5,b)R(l!1,r!6,y4~p!7,y6~p!4)R(l!2,r!6,y4~p!8,y6~u)S(c!7,y~p!3)S(c!8,y~p!5)
223 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!1,r!7,y4~p!8,y6~p!6)R(l!2,r!7,y4~p,y6~p!3)S(c!8,y~p!5)
224 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!1,r!7,y4~p!8,y6~p!6)R(l!2,r!7,y4~p!9,y6~p)S(c!8,y~p!5)S(c!9,y~p!3)
225 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~p!4)R(l!2,r!5,y4~p!7,y6~u)S(c!6,y~p!3)S(c!7,y~p)
226 E(r!1)E(r!2)G(a!3,b)G(a!4,b)R(l!1,r!5,y4~p!6,y6~u)R(l!2,r!5,y4~p!7,y6~u)S(c!6,y~p!3)S(c!7,y~p!4)
227 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~u)R(l!2,r!6,y4~p,y6~p!3)S(c!7,y~p!5)
228 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p)R(l!1,r!6,y4~p!8,y6~u)S(c!8,y~p!5)S(c!7,y~p!3)
229 E(r!1)E(r!2)G(a!3,b)R(l!1,r!4,y4~p!5,y6~u)R(l!2,r!4,y4~p!6,y6~u)S(c!5,y~p!3)S(c!6,y~p)
230 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p,y6~p!3)R(l!2,r!7,y4~p,y6~p!5)
231 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p)R(l!1,r!7,y4~p,y6~p!3)S(c!8,y~p!5)
232 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~u)R(l!1,r!5,y4~p,y6~p!3)S(c!6,y~p)
233 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~p)R(l!2,r!7,y4~p!9,y6~p)S(c!8,y~p!3)S(c!9,y~p!5)
234 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~p!7,y6~u)S(c!6,y~p!3)S(c!7,y~p)
235 E(r!1)E(r!2)R(l!1,r!3,y4~p!4,y6~u)R(l!2,r!3,y4~p!5,y6~u)S(c!4,y~p)S(c!5,y~p)
236 G(a!1,b!2)O(d!2)R(l,r,y4~p!3,y6~u)S(c!3,y~p!1)
237 E(r!1)G(a!2,b!3)O(d!3)R(l!1,r,y4~p!4,y6~u)S(c!4,y~p!2)
238 G(a!1,b!2)O(d!2)R(l,r,y4~p!3,y6~p!1)S(c!3,y~u)
239 E(r!1)G(a!2,b!3)O(d!3)R(l!1,r,y4~p!4,y6~p!2)S(c!4,y~u)
240 G(a!1,b!2)G(a!3,b)O(d!2)R(l,r,y4~p!4,y6~p!3)S(c!4,y~p!1)
241 E(r!1)G(a!2,b!3)G(a!4,b)O(d!3)R(l!1,r,y4~p!5,y6~p!4)S(c!5,y~p!2)
242 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p!3)R(l!2,r!5,y4~p,y6~p)S(c!6,y~u)
243 E(r!1)G(a!2,b!3)G(a!4,b)O(d!3)R(l!1,r,y4~p!5,y6~p!2)S(c!5,y~p!4)
244 G(a!1,b!2)G(a!3,b)O(d!2)R(l,r,y4~p!4,y6~p!1)S(c!4,y~p!3)
245 G(a!1,b!2)G(a!3,b!4)O(d!2)O(d!4)R(l,r,y4~p!5,y6~p!1)S(c!5,y~p!3)
246 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!3)R(l!2,r!6,y4~p,y6~p)S(c!7,y~p!5)
247 E(r!1)G(a!2,b!3)G(a!4,b!5)O(d!3)O(d!5)R(l!1,r,y4~p!6,y6~p!2)S(c!6,y~p!4)
248 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~u)R(l!1,r!5,y4~p,y6~p)S(c!6,y~p!3)
249 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~u)R(l!1,r!5,y4~u,y6~p)S(c!6,y~p!3)
250 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~u,y6~p)S(c!7,y~p!3)
251 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~p!7,y6~u)S(c!7,y~p!3)S(c!6,y~p)
252 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~p!5)R(l!2,r!7,y4~p,y6~p)S(c!8,y~p!3)
253 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~u)R(l!2,r!5,y4~p,y6~u)S(c!6,y~p!3)
254 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p,y6~u)S(c!7,y~p!3)
255 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~u)R(l!1,r!6,y4~p,y6~p!5)S(c!7,y~p!3)
256 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p!8,y6~p)S(c!7,y~p!3)S(c!8,y~p)
257 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!1,r!7,y4~p!8,y6~p!5)R(l!2,r!7,y4~p,y6~p!6)S(c!8,y~p!3)
258 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!5)R(l!1,r!6,y4~p,y6~p)S(c!7,y~p!3)
259 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!2,r!8,y4~p!9,y6~p!3)R(l!1,r!8,y4~p,y6~p!7)S(c!9,y~p!5)
260 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~p,y6~p!5)S(c!8,y~p!6)
261 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~p,y6~p!5)S(c!7,y~u)
262 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~u,y6~p)S(c!8,y~p!5)
263 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~u,y6~p)S(c!7,y~p!5)
264 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~u,y6~p)S(c!6,y~u)
265 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~p!9,y6~p)S(c!8,y~p!5)S(c!9,y~p)
266 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~p!8,y6~p)S(c!7,y~p!5)S(c!8,y~p)
267 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~p!7,y6~p)S(c!7,y~p)S(c!6,y~u)
268 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~p,y6~u)S(c!8,y~p!5)
269 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~p,y6~u)S(c!7,y~p!5)
270 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~p,y6~u)S(c!6,y~u)
271 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~p!9,y6~u)S(c!8,y~p!5)S(c!9,y~u)
272 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~p!8,y6~u)S(c!7,y~p!5)S(c!8,y~u)
273 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~p!7,y6~u)S(c!7,y~u)S(c!6,y~u)
274 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!5)R(l!1,r!6,y4~p!8,y6~u)S(c!7,y~p!3)S(c!8,y~u)
275 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~u)R(l!2,r!5,y4~p!7,y6~u)S(c!7,y~p!3)S(c!6,y~u)
276 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!1,r!8,y4~p!9,y6~p!7)R(l!2,r!8,y4~p!10,y6~p!3)S(c!10,y~p!5)S(c!9,y~u)
277 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!1,r!7,y4~p!8,y6~p!5)R(l!2,r!7,y4~p!9,y6~p!3)S(c!9,y~p!6)S(c!8,y~u)
278 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p!8,y6~p!3)S(c!7,y~u)S(c!8,y~u)
279 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!1,r!7,y4~p!8,y6~p!5)R(l!2,r!7,y4~p!9,y6~p!6)S(c!9,y~p!3)S(c!8,y~u)
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280 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p!8,y6~u)S(c!8,y~p!3)S(c!7,y~u)
281 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~p!9,y6~p)S(c!8,y~p!5)S(c!9,y~u)
282 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~p!8,y6~p)S(c!7,y~p!5)S(c!8,y~u)
283 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~p!7,y6~p)S(c!7,y~u)S(c!6,y~u)
284 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!5)R(l!1,r!6,y4~p!8,y6~p)S(c!7,y~p!3)S(c!8,y~u)
285 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~p)R(l!2,r!5,y4~p!7,y6~u)S(c!7,y~p!3)S(c!6,y~u)
286 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!1,r!8,y4~p!9,y6~p!7)R(l!2,r!8,y4~p!10,y6~p!3)S(c!10,y~p!5)S(c!9,y~p)
287 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!1,r!7,y4~p!8,y6~p!5)R(l!2,r!7,y4~p!9,y6~p!3)S(c!9,y~p!6)S(c!8,y~p)
288 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p!8,y6~p!3)S(c!7,y~p)S(c!8,y~u)
289 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!1,r!7,y4~p!8,y6~p!5)R(l!2,r!7,y4~p!9,y6~p!6)S(c!9,y~p!3)S(c!8,y~p)
290 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p!5)R(l!2,r!6,y4~p!8,y6~u)S(c!8,y~p!3)S(c!7,y~p)
291 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!2,r!8,y4~p!9,y6~p!3)R(l!1,r!8,y4~u,y6~p!7)S(c!9,y~p!5)
292 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~u,y6~p!5)S(c!8,y~p!6)
293 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~u,y6~p!5)S(c!7,y~u)
294 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!2,r!7,y4~p!8,y6~p!6)R(l!1,r!7,y4~u,y6~p!5)S(c!8,y~p!3)
295 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~u)R(l!1,r!6,y4~u,y6~p!5)S(c!7,y~p!3)
296 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b!8)O(d!4)O(d!6)O(d!8)R(l!2,r!9,y4~p!10,y6~p!5)R(l!1,r!9,y4~u,y6~p!3)S(c!10,y~p!7)
297 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!2,r!8,y4~p!9,y6~p!5)R(l!1,r!8,y4~u,y6~p!3)S(c!9,y~p!7)
298 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!5)R(l!1,r!7,y4~u,y6~p!3)S(c!8,y~u)
299 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!2,r!8,y4~p!9,y6~p!7)R(l!1,r!8,y4~u,y6~p!3)S(c!9,y~p!5)
300 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~u)R(l!1,r!7,y4~u,y6~p!3)S(c!8,y~p!5)
301 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b!8)O(d!4)O(d!6)O(d!8)R(l!1,r!9,y4~p!10,y6~p!3)R(l!2,r!9,y4~p!11,y6~p!5)S(c!11,y~p!7)S(c!10,y~p)
302 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!1,r!8,y4~p!9,y6~p!3)R(l!2,r!8,y4~p!10,y6~p!5)S(c!10,y~p!7)S(c!9,y~p)
303 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~p!3)R(l!2,r!7,y4~p!9,y6~p!5)S(c!8,y~p)S(c!9,y~u)
304 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!1,r!8,y4~p!9,y6~p!3)R(l!2,r!8,y4~p!10,y6~p!7)S(c!10,y~p!5)S(c!9,y~p)
305 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~p!3)R(l!2,r!7,y4~p!9,y6~u)S(c!9,y~p!5)S(c!8,y~p)
306 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~u,y6~u)S(c!8,y~p!5)
307 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~u,y6~u)S(c!7,y~p!5)
308 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~u,y6~u)S(c!6,y~u)
309 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!5)R(l!1,r!6,y4~u,y6~u)S(c!7,y~p!3)
310 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~u)R(l!1,r!5,y4~u,y6~u)S(c!6,y~p!3)
311 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!2,r!8,y4~p!9,y6~p!3)R(l!1,r!8,y4~p!10,y6~p)S(c!10,y~p!7)S(c!9,y~p!5)
312 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~p!9,y6~p)S(c!9,y~p!5)S(c!8,y~p!6)
313 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~p!8,y6~p)S(c!8,y~p!5)S(c!7,y~u)
314 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!2,r!7,y4~p!8,y6~p!6)R(l!1,r!7,y4~p!9,y6~p)S(c!9,y~p!5)S(c!8,y~p!3)
315 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~p)R(l!2,r!6,y4~p!8,y6~u)S(c!7,y~p!5)S(c!8,y~p!3)
316 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)G(a!8,b)O(d!4)O(d!6)R(l!1,r!9,y4~p!10,y6~p!8)R(l!2,r!9,y4~p!11,y6~p!3)S(c!10,y~p!7)S(c!11,y~p!5)
317 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)G(a!7,b)O(d!4)R(l!1,r!8,y4~p!9,y6~p!6)R(l!2,r!8,y4~p!10,y6~p!3)S(c!9,y~p!5)S(c!10,y~p!7)
318 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!1,r!7,y4~p!8,y6~p!6)R(l!2,r!7,y4~p!9,y6~p!3)S(c!8,y~p!5)S(c!9,y~u)
319 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)G(a!7,b)O(d!4)R(l!1,r!8,y4~p!9,y6~p!6)R(l!2,r!8,y4~p!10,y6~p!7)S(c!9,y~p!5)S(c!10,y~p!3)
320 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!1,r!7,y4~p!8,y6~p!6)R(l!2,r!7,y4~p!9,y6~u)S(c!8,y~p!5)S(c!9,y~p!3)
321 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!2,r!8,y4~p!9,y6~p!3)R(l!1,r!8,y4~p!10,y6~u)S(c!10,y~p!7)S(c!9,y~p!5)
322 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~p!9,y6~u)S(c!9,y~p!5)S(c!8,y~p!6)
323 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~p!8,y6~u)S(c!8,y~p!5)S(c!7,y~u)
324 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)G(a!6,b)O(d!4)R(l!2,r!7,y4~p!8,y6~p!6)R(l!1,r!7,y4~p!9,y6~u)S(c!9,y~p!5)S(c!8,y~p!3)
325 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!1,r!6,y4~p!7,y6~u)R(l!2,r!6,y4~p!8,y6~u)S(c!7,y~p!5)S(c!8,y~p!3)
326 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b!8)O(d!4)O(d!6)O(d!8)R(l!2,r!9,y4~p!10,y6~p!5)R(l!1,r!9,y4~p,y6~p!3)S(c!10,y~p!7)
327 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!2,r!8,y4~p!9,y6~p!5)R(l!1,r!8,y4~p,y6~p!3)S(c!9,y~p!7)
328 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!5)R(l!1,r!7,y4~p,y6~p!3)S(c!8,y~u)
329 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!2,r!8,y4~p!9,y6~p!7)R(l!1,r!8,y4~p,y6~p!3)S(c!9,y~p!5)
330 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~u)R(l!1,r!7,y4~p,y6~p!3)S(c!8,y~p!5)
331 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b!8)O(d!4)O(d!6)O(d!8)R(l!2,r!9,y4~p!10,y6~p!5)R(l!1,r!9,y4~p!11,y6~p)S(c!11,y~p!3)S(c!10,y~p!7)
332 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!2,r!8,y4~p!9,y6~p!5)R(l!1,r!8,y4~p!10,y6~p)S(c!10,y~p!3)S(c!9,y~p!7)
333 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!5)R(l!1,r!7,y4~p!9,y6~p)S(c!9,y~p!3)S(c!8,y~u)
334 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!2,r!8,y4~p!9,y6~p!7)R(l!1,r!8,y4~p!10,y6~p)S(c!10,y~p!3)S(c!9,y~p!5)
335 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~p)R(l!2,r!7,y4~p!9,y6~u)S(c!8,y~p!3)S(c!9,y~p!5)
336 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!2,r!7,y4~p!8,y6~p!3)R(l!1,r!7,y4~p!9,y6~u)S(c!8,y~p!5)S(c!9,y~p)
337 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!3)R(l!1,r!6,y4~p!8,y6~u)S(c!7,y~p!5)S(c!8,y~p)
338 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!2,r!5,y4~p!6,y6~p!3)R(l!1,r!5,y4~p!7,y6~u)S(c!7,y~p)S(c!6,y~u)
339 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b)O(d!4)R(l!2,r!6,y4~p!7,y6~p!5)R(l!1,r!6,y4~p!8,y6~u)S(c!7,y~p!3)S(c!8,y~p)
340 E(r!1)E(r!2)G(a!3,b!4)O(d!4)R(l!1,r!5,y4~p!6,y6~u)R(l!2,r!5,y4~p!7,y6~u)S(c!7,y~p!3)S(c!6,y~p)
341 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b!8)G(a!9,b!10)O(d!4)O(d!6)O(d!8)O(d!10)
    R(l!1,r!11,y4~p!12,y6~p!3)R(l!2,r!11,y4~p!13,y6~p!7)S(c!12,y~p!5)S(c!13,y~p!9)
342 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b!8)G(a!9,b)O(d!4)O(d!6)O(d!8)R(l!1,r!10,y4~p!11,y6~p!3)R(l!2,r!10,y4~p!12,y6~p!7)S(c!11,y~p!5)S(c!12,y~p!9)
343 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b!8)O(d!4)O(d!6)O(d!8)R(l!1,r!9,y4~p!10,y6~p!3)R(l!2,r!9,y4~p!11,y6~p!7)S(c!10,y~p!5)S(c!11,y~u)
344 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b!8)G(a!9,b)O(d!4)O(d!6)O(d!8)R(l!1,r!10,y4~p!11,y6~p!3)R(l!2,r!10,y4~p!12,y6~p!9)S(c!11,y~p!5)S(c!12,y~p!7)
345 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b!8)O(d!4)O(d!6)O(d!8)R(l!1,r!9,y4~p!10,y6~p!3)R(l!2,r!9,y4~p!11,y6~u)S(c!10,y~p!5)S(c!11,y~p!7)
346 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)G(a!8,b)O(d!4)O(d!6)R(l!1,r!9,y4~p!10,y6~p!3)R(l!2,r!9,y4~p!11,y6~p!5)S(c!10,y~p!7)S(c!11,y~p!8)
347 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!1,r!8,y4~p!9,y6~p!3)R(l!2,r!8,y4~p!10,y6~p!5)S(c!9,y~p!7)S(c!10,y~u)
348 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)G(a!8,b)O(d!4)O(d!6)R(l!1,r!9,y4~p!10,y6~p!3)R(l!2,r!9,y4~p!11,y6~p!8)S(c!10,y~p!7)S(c!11,y~p!5)
349 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!1,r!8,y4~p!9,y6~p!3)R(l!2,r!8,y4~p!10,y6~u)S(c!9,y~p!7)S(c!10,y~p!5)
350 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~p!3)R(l!2,r!7,y4~p!9,y6~p!5)S(c!8,y~u)S(c!9,y~u)
351 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!1,r!8,y4~p!9,y6~p!3)R(l!2,r!8,y4~p!10,y6~p!7)S(c!10,y~p!5)S(c!9,y~u)
352 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~p!3)R(l!2,r!7,y4~p!9,y6~u)S(c!9,y~p!5)S(c!8,y~u)
353 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)G(a!8,b)O(d!4)O(d!6)R(l!1,r!9,y4~p!10,y6~p!7)R(l!2,r!9,y4~p!11,y6~p!8)S(c!10,y~p!3)S(c!11,y~p!5)
354 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)G(a!7,b)O(d!4)O(d!6)R(l!1,r!8,y4~p!9,y6~p!7)R(l!2,r!8,y4~p!10,y6~u)S(c!9,y~p!3)S(c!10,y~p!5)
355 E(r!1)E(r!2)G(a!3,b!4)G(a!5,b!6)O(d!4)O(d!6)R(l!1,r!7,y4~p!8,y6~u)R(l!2,r!7,y4~p!9,y6~u)S(c!8,y~p!3)S(c!9,y~p!5)
```

Organisations

number and set:

```
0  {  }
1  { 32 }
2  { 11 }
3  { 5 }
4  { 3 }
5  { 0 }
```

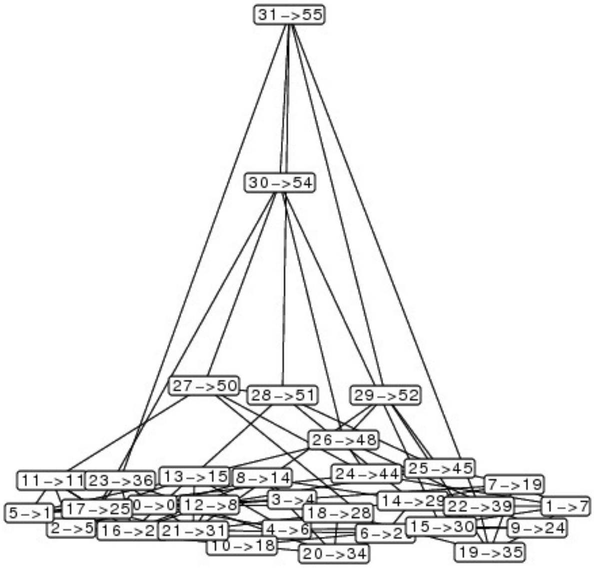

Fig. A.3. Hasse diagram of organisations of the EGF example, $n \to m$ means organisation with number n is mapped to f–organisation number m by Θ. The unlabelled terminal node stands for the empty set.

```
6  { 11 32 }
7  { 5 32 }
8  { 5 11 }
9  { 3 32 }
10 { 3 11 }
11 { 0 11 }
12 { 0 5 }
13 { 0 3 }
14 { 5 11 32 }
15 { 3 11 32 }
16 { 0 5 11 }
17 { 0 3 11 }
18 { 3 5 14 }
19 { 3 5 14 32 }
20 { 3 5 11 14 }
21 { 0 3 5 14 }
22 { 3 5 11 14 32 }
23 { 0 3 5 11 14 }
24 { 0 1 2 6 7 8 9 10 19 21 26 28 31 32 33 70 96 124 208 }
25 { 0 1 2 5 6 7 8 9 10 19 21 26 28 31 32 33 70 96 124 208 }
26 { 0 1 2 3 6 7 8 9 10 12 13 18 19 21 22 23 25 26 28 31 32 33 36 37 67 70
     79 82 93 96 121 124 181 184 208 }
27 { 0 1 2 4 6 7 8 9 10 11 15 16 19 20 21 26 27 28 29 30 31 32 33 34 35 38
     39 52 53 54 64 70 74 89 91 96 102 103 105 110 116 117 119 124 130 131
     133 138 144 158 163 169 208 214 235 }
28 { 0 1 2 4 5 6 7 8 9 10 11 15 16 19 20 21 26 27 28 29 30 31 32 33 34 35
     38 39 52 53 54 64 70 74 89 91 96 102 103 105 110 116 117 119 124 130
     131 133 138 144 158 163 169 208 214 235 }
29 { 0 1 2 3 5 6 7 8 9 10 12 13 14 18 19 21 22 23 25 26 28 31 32 33 36 37
     42 43 44 45 47 67 68 70 79 80 82 86 93 94 96 100 121 122 124 128 181
     182 184 188 191 193 197 208 212 230 }
305 { 0 1 2 3 4 6 7 8 9 10 11 12 13 15 16 17 18 19 20 21 22 23 24 25 26 27
      28 29 30 31 32 33 34 35 36 37 38 39 40 41 50 51 52 53 54 55 56 57 58
      59 60 61 64 65 66 67 70 71 72 74 75 76 77 78 79 82 83 84 85 88 89 90
      91 92 93 96 97 98 99 102 103 104 105 106 107 110 111 112 113 116 117
      118 119 120 121 124 125 126 127 130 131 132 133 134 135 138 139 140
      141 144 145 146 147 148 151 152 153 154 157 158 159 160 163 164 165
      166 169 170 171 174 175 176 177 180 181 184 185 186 187 190 208 209
      210 211 214 215 216 217 220 221 222 225 226 229 235 }
316 { 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
      26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48
      49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71
```

```
72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94
95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112
113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129
130 131 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146
147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163
164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180
181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197
198 199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214
215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231
232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248
249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265
266 267 268 269 270 271 272 273 274 275 276 277 278 279 280 281 282
283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299
300 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316
317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333
334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350
351 352 353 354 355 }
```

Fragments

number and κ-expression:

```
0  E(r!1)R(y4~p!2,l!1,r!)S(y7~p!3,c!2)G(a!3,b!4)O(d!4)
1  E(r!1)R(y4~p!2,l!1,r)S(y7~p!3,c!2)G(a!3,b!4)O(d!4)
2  G(a!2,b!1)S(y7~p!2,c!3)R(y4~p!3,l,r)O(d!1)
3  R(y4~p!1,l,r),S(y7~p,c!1)
4  G(a,b!1)O(d!1)
5  E(r!1)R(y4~p!2,l!1,r)S(y7~p,c!2)
6  E(r!1)R(y4~p!2,l!1,r!)S(y7~p,c!2)
7  G(a!2,b!1)S(y7~p!2,c)O(d!1)
8  S(y7~p,c)
9  G(a!1,b)S(y7~p!1,c)
10 G(a,b)
11 E(r!1)R(y4~p!2,l!1,r!)S(y7~p!3,c!2)G(a!3,b)
12 E(r!1)R(y4~p!2,l!1,r)S(y7~p!3,c!2)G(a!3,b)
13 G(a!1,b)S(y7~p!1,c!2)R(y4~p!2,l,r)
14 R(y4~p,l,r)
15 E(r!1)R(y4~p,l!1,r)
16 E(r!1)R(y4~p,l!1,r!)
17 E(r!1)R(y4~p!2,l!1,r!)S(y7~u,c!2)
18 E(r!1)R(y4~p!2,l!1,r)S(y7~u,c!2)
19 R(y4~p!1,l,r)S(y7~u,c!1)
20 S(y7~u,c)
21 O(d)
22 E(r!1)R(y6~p!2,l!1,r!)G(a!2,b!!3)O(d!3)
23 E(r!1)R(y6~p!2,l!1,r)G(a!2,b!3)O(d!3)
24 G(a!2,b!1)R(y6~p!2,l,r)O(d!1)
25 G(a!1,b)R(y6~p!1,l,r)
26 E(r!1)R(y6~p!2,l!1,r)G(a!2,b)
27 E(r!1)R(y6~p!2,l!1,r!)G(a!2,b)
28 R(y6~p,l,r)
29 E(r!1)R(y6~p,l!1,r)
30 E(r!1)R(y6~p,l!1,r!)
31 E(r!1)R(y4~u,l!1,r!)
32 E(r!1)R(y4~u,l!1,r)
33 R(y4~u,l,r)
34 E(r!1)R(y6~u,l!1,r!)
35 E(r!1)R(y6~u,l!1,r)
36 R(y6~u,l,r)
37 E(r)
```

F–organisations

number and set:

```
0  {  }
1  { 37 }
2  { 36 }
3  { 33 }
4  { 21 }
5  { 20 }
6  { 10 }
7  { 33 36 }
8  { 21 37 }
9  { 21 36 }
10 { 21 33 }
11 { 20 37 }
```

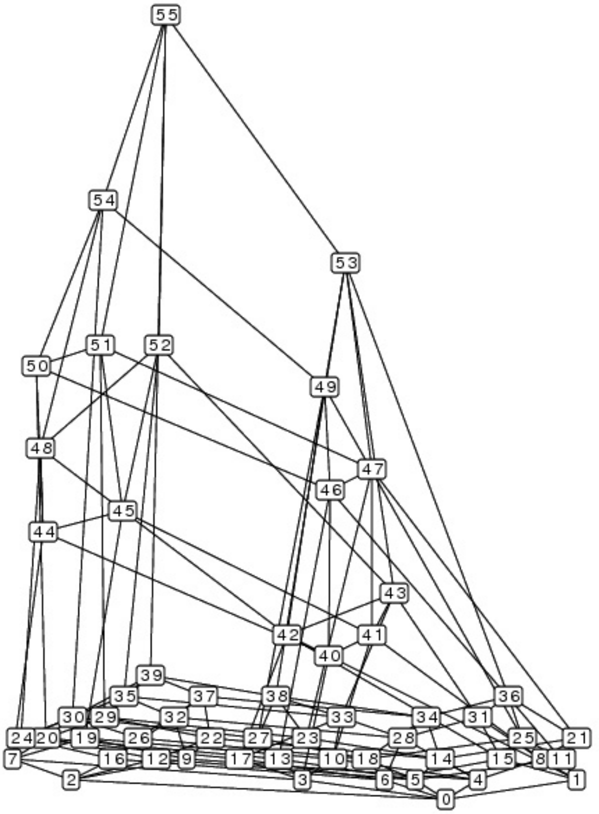

Fig. A.4. Hasse diagram of f–organisations of the EGF example. The unlabelled terminal node stands for the empty set.

```
12 { 20 36 }
13 { 20 33 }
14 { 20 21 }
15 { 10 37 }
16 { 10 36 }
17 { 10 33 }
18 { 10 20 }
19 { 21 33 36 }
20 { 20 33 36 }
21 { 20 21 37 }
22 { 20 21 36 }
23 { 20 21 33 }
24 { 10 33 36 }
25 { 10 20 37 }
26 { 10 20 36 }
27 { 10 20 33 }
28 { 4 10 21 }
29 { 20 21 33 36 }
30 { 10 20 33 36 }
31 { 4 10 21 37 }
32 { 4 10 21 36 }
33 { 4 10 21 33 }
34 { 4 10 20 21 }
35 { 4 10 21 33 36 }
36 { 4 10 20 21 37 }
37 { 4 10 20 21 36 }
38 { 4 10 20 21 33 }
39 { 4 10 20 21 33 36 }
40 { 14 15 16 31 32 33 37 }
41 { 14 15 16 21 31 32 33 37 }
```

```
42 { 10 14 15 16 31 32 33 37 }
43 { 4 10 14 15 16 21 31 32 33 37 }
44 { 14 15 16 28 29 30 31 32 33 34 35 36 37 }
45 { 14 15 16 21 28 29 30 31 32 33 34 35 36 37 }
46 { 3 5 6 8 14 15 16 17 18 19 20 31 32 33 37 }
47 { 3 5 6 8 14 15 16 17 18 19 20 21 31 32 33 37 }
48 { 10 14 15 16 25 26 27 28 29 30 31 32 33 34 35 36 37 }
49 { 3 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 31 32 33 37 }
50 { 3 5 6 8 14 15 16 17 18 19 20 28 29 30 31 32 33 34 35 36 37 }
51 { 3 5 6 8 14 15 16 17 18 19 20 21 28 29 30 31 32 33 34 35 36 37 }
52 { 4 10 14 15 16 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 }
53 { 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 31 32 33 37 }
54 { 3 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 25 26 27 28 29 30 31 32
     33 34 35 36 37 }
55 { 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
     26 27 28 29 30 31 32 33 34 35 36 37 }
```