Cairo University

Egyptian Informatics Journal

www.elsevier.com/locate/eij
www.sciencedirect.com

ORIGINAL ARTICLE

Real parameter optimization by an effective differential evolution algorithm

Ali Wagdy Mohamed [a,b,*], Hegazy Zaher Sabry [c], Tareq Abd-Elaziz [b]

[a] *Statistics Department, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia*
[b] *Operations Research Department, Institute of Statistical Studies and Research, Cairo University, Giza, Egypt*
[c] *Mathematical Statistics Department, Institute of Statistical Studies and Research, Cairo University, Giza, Egypt*

Received 23 June 2012; revised 12 December 2012; accepted 1 January 2013
Available online 1 February 2013

KEYWORDS

Differential evolution;
Best–worst mutation;
Global optimization;
Modified BGA mutation

Abstract This paper introduces an Effective Differential Evolution (EDE) algorithm for solving real parameter optimization problems over continuous domain. The proposed algorithm proposes a new mutation rule based on the best and the worst individuals among the entire population of a particular generation. The mutation rule is combined with the basic mutation strategy through a linear decreasing probability rule. The proposed mutation rule is shown to promote local search capability of the basic DE and to make it faster. Furthermore, a random mutation scheme and a modified Breeder Genetic Algorithm (BGA) mutation scheme are merged to avoid stagnation and/or premature convergence. Additionally, the scaling factor and crossover of DE are introduced as uniform random numbers to enrich the search behavior and to enhance the diversity of the population. The effectiveness and benefits of the proposed modifications used in EDE has been experimentally investigated. Numerical experiments on a set of bound-constrained problems have shown that the new approach is efficient, effective and robust. The comparison results between the EDE and several classical differential evolution methods and state-of-the-art parameter adaptive differential evolution variants indicate that the proposed EDE algorithm is competitive with , and in some cases superior to, other algorithms in terms of final solution quality, efficiency, convergence rate, and robustness.

* Corresponding author at: Statistics Department, Faculty of Sciences, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia. Tel.: +966 556269723, +20 1005157657.
E-mail address: aliwagdy@gmail.com (A.W. Mohamed).
Peer review under responsibility of Faculty of Computers and Information, Cairo University.

Production and hosting by Elsevier

1. Introduction

Differential Evolution (DE) is a stochastic population-based search method, proposed by Storn and Price [1] for solving non-linear, high-dimensional and complex computational optimization problems. DE is considered the most recent EAs for solving real-parameter optimization problems [2]. DE has many advantages including simplicity of implementation, reliable, robust, and in general is considered as an effective global

1110-8665 © 2013 Faculty of Computers and Information, Cairo University. Production and hosting by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.eij.2013.01.001

optimization algorithm [3]. Therefore, it has been used in many real-world applications, such as flow shop scheduling [4], machine intelligence applications [5], financial markets dynamic modeling [6], pattern recognition studies [7], signal processing implementations [8], data mining [9], power systems [10], fuzzy logic systems [11], and many others. DE, nevertheless, also has shortcomings as all other intelligent techniques. Firstly, while the global exploration ability of DE is considered adequate, its local exploitation ability is regarded weak and its convergence velocity is too low [12]. Secondly, DE suffers from the problems of premature convergence and stagnation [13,14]. Finally, DE is sensitive to the choice of the control parameters and it is difficult to adjust them for different problems. Moreover, like other evolutionary algorithms, DE performance decreases as search space dimensionality increases [13]. Indeed, due to the above drawbacks, a lot of researchers have proposed to overcome these problems and to improve the overall performance of the DE algorithm. The choice of DE's control variables has been discussed by Storn and Price [1] who suggested a reasonable choice for *NP* (population size) between 5D and 10D (D being the dimensionality of the problem), and 0.5 as a good initial value of *F* (mutation scaling factor). The effective value of *F* usually lies in the range between 0.4 and 1. As for the CR (crossover rate), an initial good choice of $CR = 0.1$; however, since a large CR often speeds convergence, it is appropriate to first try CR as 0.9 or 1 in order to check if a quick solution is possible. After many experimental analysis, Gämperle et al. [15] recommended that a good choice for *NP* is between 3D and 8D, with $F = 0.6$ and CR lies in $[0.3, 0.9]$. Contrarily, Rönkkönen et al. [16] concluded that $F = 0.9$ is a good compromise between convergence speed and convergence probability. Additionally, CR depends on the nature of the problem, so CR with a value between 0.9 and 1 is suitable for non-separable and multimodal objective functions, while a value of CR between 0 and 0.2 when the objective function is separable. Due to the contradiction claims that can be seen from the literature, some techniques have been designed to adjust control parameters in adaptive or self-adaptive manner instead of trial-and-error procedure. A Fuzzy Adaptive Differential Evolution (FADE) algorithm was proposed by Liu and Lampinen [17]. They introduced fuzzy logic controllers to adjust crossover and mutation rates. Numerical experiments and comparisons on a set of well known benchmark functions showed that the FADE Algorithm outperformed basic DE algorithm. Likewise, Brest et al. [18] proposed an efficient technique, called jDE, for self-adapting control parameter settings by encoding the parameters into each individual and adapting them by means of evolution. The results showed that jDE is better than, or at least comparable to, the standard DE algorithm (FADE) algorithm and other evolutionary algorithms from the literature when considering the quality of the solutions obtained. In the same context, Omran et al [19] proposed a Self-adaptive Differential Evolution (SDE) algorithm. The scaling factor *F* is self-adapted using a mutation rule similar to the mutation operator in the basic DE. The experiments conducted showed that SDE generally outperformed DE algorithms and other evolutionary algorithms. Zaharie [20] introduced an adaptive DE (ADE) algorithm based on the idea of controlling the population diversity and implemented a multi-population approach Ali and Törn [21] proposed a new DE algorithm with two evolving populations. The crossover rate CR has been empirically studied and set equal to 0.5. Unlike CR, the value of the scaling factor *F* is adaptively calculated at each generation by using fitness-based adaptation scheme based on the maximum and minimum objective function values over the individuals of populations. In a similar way, the scale factor local search differential evolution (SFLSDE) is presented by Neri and Tirronen [22]. It is a DE-based memetic algorithm (MA) which employs within a self-adaptive scheme, two local search algorithms which are golden section search and hill climbing search. These local search algorithms aim at detecting a value of the scale factor corresponding to an offspring with a high performance, while the generation is executed. The local search algorithms thus assist in the global search and generate offspring with high performance which are subsequently supposed to promote the generation of enhanced solutions within the evolutionary framework. The efficiency of the proposed algorithm seems to be very high especially for large scale problems and complex fitness landscapes. SaDE (self-adaptive differential evolution) is proposed by Qin et al. [23]. The main idea of SaDE is to simultaneously implement two mutation schemes: "DE/rand/1/bin" and "DE/best/2/bin" and also to adapt mutation and crossover parameters. The Performance of SaDE evaluated over a suite of 26 several benchmark problems and it was compared with the conventional DE and three adaptive DE variants. The presented experimental results demonstrated that SaDE was more effective in obtaining better quality solutions and had higher success rate. Similarly, Zhang and Sanderson [24] introduced a new Differential Evolution (DE) algorithm, named JADE, to improve optimization performance by implementing a new mutation strategy "DE/current-to-*p*best" with optional external archive and updating control parameters in an adaptive manner. Simulation results show that JADE is better than, or at least comparable to, other classic or adaptive DE algorithms, Particle swarm and other evolutionary algorithms from the literature in terms of convergence performance for a set of 20 benchmark problems. Recently, motivated by the recent success of diverse self-adaptive DE approaches, Das et al. [25] developed a self-adaptive DE, called FiADE. In FiADE, an effective adaptation technique for tuning both *F* and Cr is proposed, on the run, without any user intervention. The adaptation strategy is based on the objective function value of individuals in the DE population. Comparison with the best-known and expensive variants of DE over fourteen well-known numerical benchmarks and one real-life engineering problem reflects the superiority of proposed parameter tuning scheme in terms of accuracy, convergence speed, and robustness. Practically, from the literature, it can be observed that the main modifications, improvements and developments on DE focus on adjusting control parameters in adaptive or self-adaptive manner. However, a few enhancements have been implemented to modify the standard mutation strategies or to propose new mutation rules so as to enhance the local search ability of DE or to overcome the problems of stagnation or premature convergence [13,26–29]. Consequently, proposing new mutations and adjusting control parameters are still an open challenge direction of research [30–36]. Therefore, in order to improve the global performance of basic DE, this research uses a new mutation rule to enhance the local exploitation tendency and to improve the convergence rate of the algorithm. The scaling factor and the crossover of DE are also introduced as uniform random number to enrich the whole search space and to enhance the diversity of the population. In order to avoid the stagnation and the premature convergence issues through generations, modified BGA mutation and a random mutation are embedded into the proposed EDE algorithm. Numerical experiments and comparisons conducted in this research effort on a set of well-known high dimensional benchmark functions indicate that the proposed Improved Differential Evolution (EDE) algorithm is superior and competitive with conventional DE and several state-of-the-art parameter adaptive DE variants particularly in the case of high dimensional complex optimization problems. The rest of the paper is organized as follows. In Section 2, the standard DE algorithm is introduced. Next, in Section 3, the new EDE algorithm is described in detail. Section 4 reports on the computational results of testing benchmark functions and on the comparison with other techniques is discussed. Section 5 discusses the effectiveness of the proposed modifications. Finally, conclusions and future works are drawn in Section 6.

2. The differential evolution algorithm

A bound constrained global optimization problem can be defined as follows [37]:

$$\min f(X), \quad X = [x_1, \dots, x_n]; \text{ S.t. } x_j \in [a_j, b_j]; \quad j = 1, 2, \dots n, \tag{1}$$

where f is the objective function, X is the decision vector consisting of n variables, and a_j and b_j are the lower and upper bounds for each decision variable, respectively. Virtually, there are several variants of DE [1]. In this paper, we use the scheme which can be classified using the notation as DE/rand/1/bin strategy [1,18]. This strategy is the most often used in practice. A set of D optimization parameters is called an individual, which is represented by a D-dimensional parameter vector. A population consists of NP parameter vectors x_i^G, $i = 1, 2, \dots, NP$. G denotes one generation. NP is the number of members in a population. It is not changed during the evolution process. The initial population is chosen randomly with uniform distribution in the search space. DE has three operators: mutation, crossover and selection. The crucial idea behind DE is a scheme for generating trial vectors. Mutation and crossover operators are used to generate trial vectors, and the selection operator then determines which of the vectors will survive into the next generation [18].

2.1. Initialization

In order to establish a starting point for the optimization process, an initial population must be created. Typically, each decision parameter in every vector of the initial population is assigned a randomly chosen value from the boundary constraints:

$$x_{ij}^0 = a_j + rand_j \cdot (b_j - a_j) \tag{2}$$

where $rand_j$ denotes a uniformly distributed number between $[0, 1]$, generating a new value for each decision parameter. a_j and b_j are the lower and upper bounds for the jth decision parameter, respectively.

2.2. Mutation

For each target vector x_i^G, a mutant vector v_i^{G+1} is generated according to the following:

$$v_i^{G+1} = x_{r_1}^G + F * (x_{r_2}^G - x_{r_3}^G), \quad r_1 \neq r_2 \neq r_3 \neq i \tag{3}$$

with randomly chosen indices and r_1, r_2, $r_3 \in \{1, 2, \dots, NP\}$.

Note that these indices must be different from each other and from the running index i so that NP must be at least four. F is a real number to control the amplification of the difference vector $(x_{r_2}^G - x_{r_3}^G)$. According to [30], the range of F is in [0, 2]. If a component of a mutant vector goes off the search space, then the value of this component is generated a new using (2).

2.3. Crossover

The target vector is mixed with the mutated vector, using the following scheme, to yield the trial vector u_i^{G+1}.

$$u_{ij}^{G+1} = \begin{cases} v_{ij}^G, rand(j) \leq CR & \text{or} \quad j = randn(i), \\ x_{ij}^G, rand(j) > CR & \text{and} \quad j \neq randn(i), \end{cases} \tag{4}$$

where $j = 1, 2, \dots, D, rand(j) \in [0, 1]$ is the jth evaluation of a uniform random generator number. $CR \in [0, 1]$ is the crossover probability constant, which has to be determined by the user. $randn(i) \in \{1, 2, \dots, D\}$ is a randomly chosen index which ensures that u_i^{G+1} gets at least one element from v_i^{G+1}; otherwise no new parent vector would be produced and the population would not alter.

2.4. Selection

DE adapts a greedy selection strategy. If and only if the trial vector u_i^{G+1} yields a better fitness function value than x_i^G, then u_i^{G+1} is set to x_i^{G+1}. Otherwise, the old vector x_i^G is retained. The selection scheme is as follows (for a minimization problem):

$$x_i^{G+1} = \begin{cases} u_i^{G+1}, f(u_i^{G+1}) < f(x_i^G), \\ x_i^G, f(u_i^{G+1}) \geqslant f(x_i^G). \end{cases} \tag{5}$$

3. Effective differential evolution algorithm

All evolutionary algorithms, including DE, are stochastic population-based search methods. Accordingly, there is no guarantee to reach the global optimal solution all the times. Nonetheless, adjusting control parameters such as the scaling factor, the crossover rate and the population size, alongside developing an appropriate mutation scheme, can considerably improve the search capability of DE algorithms and increase the possibility of achieving promising and successful results in complex and large scale optimization problems. Therefore, in this paper, four modifications are introduced in order to significantly enhance the overall performance of the standard DE algorithm.

3.1. Modification of mutations

Practical experience through many developed evolutionary algorithms and experimental investigation prove that a success of the population-based search algorithms is based on balancing two contradictory aspects: global exploration and local exploitation [13]. Moreover, the mutation scheme plays a vital role in the DE search capability and the convergence rate. However, even though the DE algorithm has good global exploration ability, it suffers from weak local exploitation ability as well as its convergence velocity is still too low as the region of the optimal solution is reached [26]. Obviously, from the mutation Eq. (3), it can be observed that three vectors are chosen at random for mutation and the base vector is then selected at random among the three. Consequently, the basic mutation strategy DE/rand/1/bin is able to maintain population diversity and global search capability, but it slows down the convergence of DE algorithms. Hence, in order to enhance the local search tendency and to accelerate the convergence of DE technique, a new mutation rule is proposed based on the best and the worst individuals among the entire population of a particular generation. The modified mutation scheme is as follows:

$$v_i^{G+1} = x_b^G + F \cdot \left(x_r^G - x_w^G\right) \tag{6}$$

where x_r^G is a random chosen vector and x_b^G and x_w^G are the best and worst vectors in the entire population, respectively. This modification is intended to replace the random base vector $x_{r_1}^G$ and $x_{r_3}^G$ in the mutation Eq. (3) by the best and worst individual vectors with the best and worst fitness, respectively (i.e. lowest and highest objective function value for minimization problem) in the population at generation G. This process explores the region around the best vector. Besides, it also favors exploitation ability since the mutant individuals are strongly attracted around the current best vector and at same time enhances the convergence speed. Obviously, from mutation Eq. (6), it can be observed that the new mutation scheme has two benefits. Firstly, for the difference vector, the perturbation part of the mutation, the global solution can be easily reached if all vectors follow the direction of the better individual vector besides they also follow the opposite direction of the worst individual vector. Therefore, the directed perturbation in the proposed mutation resembles the concept of gradient as the difference vector is oriented from the worst vector to the better vectors [38]. Secondly, indeed, the new mutation process exploits the nearby region around x_b^G in the direction of $(x_r^G - x_w^G)$ by different weights as will be discussed in the next subsection. As a result, the new mutation rule has better local search ability and faster convergence rate. The new mutation strategy is embedded into the DE algorithm and it is combined with the basic mutation strategy DE/rand/1/bin through a non-linear decreasing probability rule as follows:

$$\text{If}\left(u(0,1) \geq \left(1 - \frac{G}{GEN}\right)\right) \text{ Then} \tag{7}$$

$$v_i^{G+1} = x_b^G + F \cdot \left(x_r^G - x_w^G\right) \tag{8}$$

Else

$$v_i^{G+1} = x_{r1}^G + F \cdot \left(x_{r2}^G - x_{r3}^G\right) \tag{9}$$

where F is a uniform random variables, $u(0,1)$ returns a real number between 0 and 1 with uniform random probability distribution and G is the current generation number, and GEN is the maximum number of generations. From the above scheme, it can be realized that for each vector, only one of the two strategies is used for generating the current trial vector, depending on a uniformly distributed random value within the range $(0,1)$. For each vector, if the random value is smaller than $\left(1 - \frac{G}{GEN}\right)$, then the basic mutation is applied. Otherwise, the proposed one is performed. Of course, it can be seen that, from Eq. (7), the probability of using one of the two mutations is a function of the generation number, so $\left(1 - \frac{G}{GEN}\right)$ can be gradually changed from 1 to 0 in order to favor, balance, and combine the global search capability with local search tendency. The strength and efficiency of the above scheme is based on the fact that, at the beginning of the search, two mutation rules are applied but the probability of the basic mutation rule to be used is greater than the probability of the new strategy. So, it favors exploration. Then, in the middle of the search, through generations, the two rules are approximately used with the same probability. Accordingly, it balances the search direction. Later, two mutation rules are still applied but the probability of the proposed mutation to be performed is greater than the probability of using the basic one. Finally, it enhances exploitation. Therefore, at any particular generation, both exploration and exploitation aspects are done in parallel. On the other hand, although merging a local mutation scheme into a DE algorithm can enhance the local search ability and speed up the convergence velocity of the algorithm, it may lead to a premature convergence and/or to get stagnant at any point of the search space especially with high dimensional problems [13,27]. For this reason, random mutation and a modified BGA mutation are merged and incorporated into the DE algorithm to avoid both cases at early or late stages of the search process. Generally, in order to perform random mutation on a chosen vector x_i at a particular generation, a uniform random integer number j_{rand} between $[1, D]$ is first generated and than a real number between $(b_j–a_j)$ is calculated. Then, the j_{rand} value from the chosen vector is replaced by the new real number to form a new vector x'. The random mutation can be described as follows.

$$x'_j = \begin{cases} a_j + rand_j \cdot (b_j - a_j) & j = j_{rand} \\ x_j & \text{otherwise} \end{cases}, \quad j = 1, \ldots, D \tag{10}$$

Therefore, it can be deduced from the above equation that random mutation increases the diversity of the DE algorithm as well decreases the risk of plunging into local point or any other point in the search space. In order to perform BGA mutation, as discussed in [39], on a chosen vector x_i at a particular generation, a uniform random integer number j_{rand} between $[1, D]$ is first generated and then a real number between $0.1 \cdot (b_j–a_j) \cdot \alpha$ is calculated. Then, the j_{rand} value from the chosen vector is replaced by the new real number to form a new vector x'_i. The BGA mutation can be described as follows.

$$x'_j = \begin{cases} x_{j\pm}0.1 \cdot (b_j - a_j) \cdot \alpha & j = j_{rand} \\ x_j & \text{otherwise} \end{cases}, \quad j = 1, \ldots, D \tag{11}$$

The + or − sign is chosen with probability 0.5. α is computed from a distribution which prefers small values. This is realized as follows.

$$\alpha = \sum_{k=0}^{15} \alpha_k \cdot 2^{-k}, \quad \alpha_k \in \{0, 1\} \tag{12}$$

Before mutation, we set $\alpha_i = 0$. Afterwards, each α_i is mutated to 1 with probability $p_\alpha = 1/16$. Only α_k contributes to the sum as in Eq. (12). On average, there will be just one α_k with value 1, say α_m, then α is given by $\alpha = 2^{-m}$. In this paper, the modified BGA mutation is given as follows:

$$x'_j = \begin{cases} x_j \pm rand_j \cdot (b_j - a_j) \cdot \alpha & j = j_{rand} \\ x_j & \text{otherwise} \end{cases}, \quad j = 1, \ldots, D \tag{13}$$

where the factor of 0.1 in Eq. (11) is replaced by a uniform random number generator in the range [0, 1], because the constant setting of 0.1•(b_j–a_j) is not suitable. However, the probabilistic setting of $rand_j \cdot (b_j - a_j)$ enhances the local search capability with small random numbers besides it still has an ability to jump to another point in the search space with large random numbers so as to increase the diversity of the population. Practically, no vector is subject to both mutations in the same generation, and only one of the above two mutations can be applied with the probability of 0.5. However, both mutations can be performed in the same generation with two different vectors. Therefore, at any particular generation, the proposed algorithm has the chance to improve the exploration and exploitation abilities. Furthermore, in order to avoid stagnation as well as premature convergence and to maintain the convergence rate, a new mechanism for each solution vector is proposed that satisfies the following condition: if the difference between two successive objective function values for any vector except the best one at any generation is less than or equal a predetermined level δ for predetermined allowable number of generations K, then one of the two mutations is applied with equal probability of (0.5). This procedure can be expressed as follows:

$$\begin{aligned} &\text{If} \quad |f_c - f_p| \le \delta \text{ for } K \text{ generations, then} \\ &\qquad\qquad \text{If}(u(0,1) \ge 0.5), \text{then} \end{aligned} \tag{14}$$

$$x'_j = \begin{cases} a_j + rand_j \cdot (b_j - a_j) & j = j_{rand} \\ x_j & \text{otherwise} \end{cases}, \quad j = 1, \ldots, D, \quad \text{(Random Mutation)} \tag{15}$$

Else

$$x'_j = \begin{cases} x_j \pm rand_j \cdot (b_j - a_j) \cdot \alpha & j = j_{rand} \\ x_j & \text{otherwise} \end{cases}, \quad j = 1, \ldots, D, \quad \text{(Modified BGA mutation)} \tag{16}$$

where f_c and f_p indicate current and previous objective function values, respectively.

After many experiments, in order to make a comparison with other algorithms with all dimensions, we observed that δ = E−06 and K = 25 generations are the best settings for these two parameters over all benchmark problems and these values seem to maintain the convergence rate as well as avoid stagnation and/or premature convergence in case they occur. Indeed, these parameters were set to their mean values. In this paper, these settings were fixed for all dimensions without tuning them to their optimal values that may attain good solutions better than the current results and improve the performance of the algorithm over all the benchmark problems.

3.2. Modification of scaling factor

In the mutation Eq. (3), the constant of differentiation F is a scaling factor of the difference vector. It is an important parameter that controls the evolving rate of the population. In the original DE algorithm in [2], the constant of differentiation F was chosen to be a value in [0, 2]. The value of F has a considerable influence on exploration: small values of F lead to premature convergence, and high values slow down the search [38]. However, to the best of our knowledge, there is no optimal value of F that has been derived based on theoretical and/or systematic study using all complex benchmark problems. It can be clearly observed from mutation Eq. (6) that the difference vector is a directed difference vector from the worst to the better vectors. Hence, F must be a positive value in order to bias the search direction for the trial vectors in the same direction. In fact, if the value of F is kept constant value the diversity of the population is extremely decreased during the search process as the all the vectors are perturbed by the same difference vector component. Therefore, Instead of keeping F constant during the search process F is set as a random variable for each trial vector so as to perturb the best vector x_b^G by different directed weights. Therefore, F is introduced as a uniform random variable in [0.2, 0.8], where the range is determined empirically. Accordingly, this range ensures both exploitation tendency (with small F values) and exploration ability (with large F values). In order to reduce the number of parameters of the proposed algorithm F is also used with basic mutation.

3.3. Modification of the crossover rate

The crossover operator, as in Eq. (4), shows that the constant crossover (CR) reflects the probability with which the trial individual inherits the actual individual's genes [38]. The constant crossover (CR) practically controls the diversity of the population. If the CR value is relatively high, this will increase the population diversity and improve the convergence speed. Nevertheless, the convergence rate may decrease and/or the population may prematurely converge. On the other hand, small values of CR increase the possibility of stagnation and slow down the search process. Additionally, at the early stage of the search, the diversity of the population is large because the vectors in the population are completely different from each other and the variance of the whole population is large. Therefore, the CR must take a small value in order to avoid the exceeding level of diversity that may result in premature convergence and slow convergence rate. Then, through generations, the variance of the population will decrease as the vectors in the population become similar. Thus, in order to advance diversity and increase the convergence speed, the CR must be a large value. Based on the above analysis and discussion, and in order to balance between the diversity and the convergence rate, CR is introduced as a uniform random variable in [0.5, 0.9], where the range is determined empirically and as extensively used in the literature [2,17,15,18]. In order to reduce the number of parameters of the proposed algorithm CR is also used with basic mutation. The description of EDE is presented in Fig. 1.

4. Numerical experiments and comparisons

4.1. Benchmark functions

In order to evaluate the performance and show the efficiency and superiority of the proposed algorithm (EDE), 14 well-known benchmark test functions mentioned in [40,23] are used. All these functions are minimization problems. Among the functions, f_1–f_4 are unimodal and functions f_5–f_{14} are multimodal. However, the generalized Rosenbrock's function f_3 is a multimodal function when D > 3 [41]. These 14 test functions are dimension wise scalable. Definitions of the Benchmark Problems are as follows:

```
01. Begin
02.   G=0
03.   Create a random initial population $\vec{x}_i^G \ \forall i, i = 1, ..., NP$
04.   Evaluate $f(\vec{x}_i^G) \ \forall i, i = 1, ..., NP$
05.   Determine $x_{best}^G$ and $x_{worst}^G$ based on $f(\vec{x}_i^G), i = 1, ..., NP$
06.   For G=1 to GEN Do
07.       For i=1 to NP Do
08.           $CR$ =rand(0.5,0.9) (Crossover Rate)
09.           $F$ =rand(0.2,0.8) (Scaling Factor)
10.           If ( rand[0,1] >= ( 1-G/GEN)) Then (Use New Directed Mutation Scheme)
11.               Select randomly $r1 \neq best \neq worst \neq i \in [1, NP]$
12.               j_rand= randint(1,D)
13.             For j=1 to D Do
14.               If (rand_j[0,1]< CR or j= j_rand) Then
15.                   $u_{ij}^{G+1} = x_{best}^G + F \cdot (x_{r1}^G - x_{worst}^G)$
16.               Else
17.                   $u_{ij}^{G+1} = x_{ij}^G$
18.               End If
19.             End For
20.           Else (Use Basic Mutation Scheme)
21.               Select randomly $r1 \neq r2 \neq r3 \neq i \in [1, NP]$
23.               j_rand= randint(1,D)
24.             For j=1 to D Do
25.               If (rand_j[0,1]< CR or j= j_rand) Then
26.                   $u_{ij}^{G+1} = x_{r1}^G + F \cdot (x_{r2}^G - x_{r3}^G)$
27.               Else
28.                   $u_{ij}^{G+1} = x_{ij}^G$
29.               End If
30.             End For
31.           End If
32.             If( $f(\vec{u}_i^{G+1}) \le f(\vec{x}_i^G)$ )Then
33.                 $u_{ij}^{G+1} = x_{ij}^{G+1}$
34.             Else
35.                 $u_{ij}^{G+1} = x_{ij}^G$
36.             End If
37.           If $|f_{current} - f_{previous}| \le \delta = 10^{-6}$ for $K = 25$ generations Then
              (Use Random Mutation and Modified BGA Mutation rules)
38.               If $(rand(0,1) \ge 0.5)$ Then
39.                   $\vec{x}_{ij}^{G+1} = \begin{cases} a_j + rand_{ij} * (b_j - a_j) & j = j_{rand} \\ x_{ij} & \text{otherwise} \end{cases}, \ j = 1, ..., D$ (Random Mutation)
40.               Else
41.                   $\vec{x}_{ij}^{G+1} = \begin{cases} x_{ij} + rand_{ij} * (b_j - a_j) * \alpha & j = j_{rand} \\ x_{ij} & \text{otherwise} \end{cases}, \ j = 1, ..., D$ (Modified BGA mutation)
42.               End If
43.                   Determine $x_{best}^{G+1}$ and $x_{worst}^{G+1}$ based on $f(\vec{x}_i^{G+1}), i = 1, ..., NP$
44.             End If
45.       End For
46.     G=G+1
47.   End For
48. End
```

Figure 1 Description of **EDE** algorithm.

(1) Shifted sphere function

$$f_1(x) = \sum_{i=1}^{D} z_i^2, \quad \mathbf{z} = \mathbf{x} - \mathbf{o}, \boldsymbol{o} = [o_1, o_2, \ldots, o_D]$$

: the shifted global optimum

(2) Shifted Schwefel's Problem 1.2

$$f_2(x) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} z_j \right)^2, \ \mathbf{z} = \mathbf{x} - \mathbf{o}, \quad \mathbf{o} = [o_1, o_2, \ldots, o_D]$$

: the shifted global optimum

(3) Rosenbrock's function

$$f_3(x) = \sum_{i=1}^{D-1}(100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2)$$

(4) Shifted Schwefel's Problem 1.2 with noise in fitness

$$f_4(x) = \sum_{i=1}^{D}\left(\sum_{j=1}^{i} z_j\right)^2 (1 + 0.4|N(0,1)|), \quad \mathbf{z} = \mathbf{x} - \mathbf{o}, \quad \mathbf{o} = [o_1, o_2, \ldots, o_D] : \text{the shifted global optimum}$$

(5) Shifted Ackley's function

$$f_5(x) = -20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} z_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi z_i)\right) + 20 + e,$$

$\mathbf{z} = \mathbf{x} - \mathbf{o}$, $\mathbf{o} = [o_1, o_2, \ldots, o_D]$: the shifted global optimum

(6) Shifted rotated Ackley's function

$$f_6(x) = -20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} z_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi z_i)\right) + 20 + e,$$

$\mathbf{z} = \mathbf{M}(\mathbf{x} - \mathbf{o}), \mathrm{cond}(\mathbf{M}) = 1$,
$o = [o_1, o_2, \ldots, o_D]$: the shifted global optimum

(7) Shifted Griewank's function

$$f_7(x) = \sum_{i=1}^{D}\frac{z_i^2}{4000} - \prod_{i=1}^{D}\cos\left(\frac{z_i}{\sqrt{i}}\right) + 1, \quad \mathbf{z} = \mathbf{x} - \mathbf{o},$$

$\mathbf{o} = [o_1, o_2, \ldots, o_D]$: the shifted global optimum

(8) Shifted rotated Griewank's function

$$f_8(x) = \sum_{i=1}^{D}\frac{z_i^2}{4000} - \prod_{i=1}^{D}\cos\left(\frac{z_i}{\sqrt{i}}\right) + 1,$$

$\mathbf{z} = \mathbf{M}(\mathbf{x} - \mathbf{o}), \mathrm{cond}(\mathbf{M}) = 3$,
$\mathbf{o} = [o_1, o_2, \ldots, o_D]$: the shifted global optimum

(9) Shifted Rastrigin's function

$$f_9(x) = \sum_{i=1}^{D}(z_i^2 - 10\cos(2\pi z_i) + 10),$$

$\mathbf{z} = \mathbf{x} - \mathbf{o}$, $\mathbf{o} = [o_1, o_2, \ldots, o_D]$: the shifted global optimum

(10) Shifted rotated Rastrigin's function

$$f_{10}(x) = \sum_{i=1}^{D}(z_i^2 - 10\cos(2\pi z_i) + 10),$$

$\mathbf{z} = \mathbf{M}(\mathbf{x} - \mathbf{o}), \mathrm{cond}(\mathbf{M}) = 2$,
$\mathbf{o} = [o_1, o_2, \ldots, o_D]$: the shifted global optimum

(11) Shifted non-continuous Rastrigin's function

$$f_{11}(x) = \sum_{i=1}^{D}(z_i^2 - 10\cos(2\pi z_i) + 10),$$

$$y_i = \begin{cases} z_i & |z_i| < 1/2 \\ round(2z_i)/2 & |z_i| \geq 1/2 \end{cases} \quad \text{for} \quad i = 1, 2, \ldots, D,$$

$\mathbf{z} = (\mathbf{x} - \mathbf{o})$, $\mathbf{o} = [o_1, o_2, \ldots, o_D]$: the shifted global optimum

(12) Schwefel's function

$$f_{12}(x) = 418.9829 \times D - \sum_{i=1}^{D} x_i sin(|x_i|^{1/2})$$

(13) Composition function 1 (CF1) in [40].

The function $f_{13}(x)$ (CF1) is composed by using 10 sphere functions. The global optimum is easy to find once the global basin is found.

(14) Composition function 6 (CF6) in [40].

The function $f_{14}(x)$ (CF6) is composed by using 10 different benchmark functions, i.e. 2 rotated Rastrigin's functions, 2 rotated Weierstress functions, 2 rotated Griewank's functions, 2 rotated Ackley's functions and 2 rotated Sphere functions.

Note that the shifted and/or rotated features make the global optimum of the above functions are very difficult to be achieved. Where $\vec{o}$ indicates the position of the shifted optima, M is a rotation matrix, and cond (M) is the condition number of the matrix. The initialization ranges, the range of the search space, and the position of the global minimum for these 14 benchmark functions are presented in Table 1.

4.2. *Algorithms for comparisons*

In order to evaluate the benefits of the proposed modifications, a comparison of EDE with seven classical DE methods and six state-of-the-art self-adaptive DE algorithms is done. These approaches are DE/rand/1/bin with (F = 0.8, Cr = 0.9), DE/rand/1/bin with (F = 0.9,Cr = 0.1), DE/rand/1/bin with (F = 0.9,Cr = 0.9), DE/rand/1/bin with (F = 0.5,Cr = 0.3), DE/target-to-best/1/bin with (F = 0.5,Cr = 0.3), DE/target-to-best/2/bin with (F = 0.5,Cr = 0.3), DE/best/1/bin with (F = 0.9,Cr = 0.9), SaDE [23], jDE [18], SFLSDE [22], JADE [24], Modified DE2 [21] and FiADE [25]. The above benchmark functions f_1 to f_{14} be tested in 10-dimensions (10-D), and 30-dimensions (30-D). The maximum number of function evaluations is set to 100,000 for 10D problems and 300,000 for 30D problems. The population size is set to 50 for all dimensions with all functions. For each problem, 50 independent runs are performed and statistical results are provided including the mean and the standard deviation values. The performance of different algorithms is statistically compared with EDE by a non-parametric statistical test called Wilcoxon's rank-sum test for independent samples with significance level of 0.05 [42,43]. Numerical values −1, 0, 1 represent that the EDE is inferior to, equal to and superior to the algorithm with which it is compared, respectively.

4.3. *Experimental results and discussions*

The results (mean, standard deviation of the best-of-run errors and Wilcoxon's rank-sum-test results) of the comparisons between EDE and seven classical DE variants and six state-of-the-art self-adaptive DE algorithms are provided in Tables 2 and 3 for 10-dimensions problems, respectively. The results (mean, standard deviation of the best-of-run errors and t-test results) of the comparisons between EDE and seven classical DE variants and six state-of-the-art self-adaptive DE

Table 1 Global optimum, search ranges and initialization ranges of the test functions.

Functions	Dimension	Global optimum x^*	$f(x^*)$	Search range	Initialization range
f_1	10 and 30	$\mathbf{o}$	0	$[-100,100]^D$	$[-100,100]^D$
f_2		$\mathbf{o}$	0	$[-100,100]^D$	$[-100,100]^D$
f_3		$(1,1,\ldots,1)$	0	$[-100,100]^D$	$[-100,100]^D$
f_4		$\mathbf{o}$	0	$[-100,100]^D$	$[-100,100]^D$
f_5		$\mathbf{o}$	0	$[-32,32]^D$	$[-32,32]^D$
f_6		$\mathbf{o}$	0	$[-32,32]^D$	$[-32,32]^D$
f_7		$\mathbf{o}$	0	R	$[0600]^D$
f_8		$\mathbf{o}$	0	R	$[0600]^D$
f_9		$\mathbf{o}$	0	$[-5,5]^D$	$[-5,5]^D$
f_{10}		$\mathbf{o}$	0	$[-5,5]^D$	$[-5,5]^D$
f_{11}		$(420.96,\ldots,420.96)$	0	$[-500,500]^D$	$[-500,500]^D$
f_{12}		$(420.96,\ldots,420.96)$	0	$[-500,500]^D$	$[-500,500]^D$
f_{13}		$\mathbf{o_1}$	0	$[-5,5]^D$	$[-5,5]^D$
f_{14}		$\mathbf{o_1}$	0	$[-5,5]^D$	$[-5,5]^D$

$\mathbf{o}$ is the shifted vector. $\mathbf{o_1}$ is the shifted vector for the first basic function in the composition function.

algorithms are provided in Tables 4 and 5 for 30-dimensions problems, respectively. Note that the best-of-the-run error corresponds to absolute difference between the best-of-the-run value $f(\vec{x}_{best})$ and the actual optimum f_* of a particular objective function i.e. $|f(\vec{x}_{best}) - f_*|$. The results provided by these approaches were directly taken from reference [25]. The best results are marked in bold for all problems. From Table 2, it can be obviously seen that EDE outperformed all the contestant algorithms in statistically significant fashion over 13 problems. Meanwhile, it can be observed that the performance of the EDE and DE/rand/1/bin with (F = 0.8, Cr = 0.9) and DE/rand/1/bin with (F = 0.8, Cr = 0.9) algorithms are almost the same and they approximately achieved the same results on function f_{14}. However, the entire classical DE scheme with the exception of DE/rand/1/bin with (F = 0.9, Cr = 0.9) and DE/target-to-best/1/bin with (F = 0.5, Cr = 0.3) have performed better than EDE on problem f_7. From the t-test results, it can be observed that EDE is inferior to, equal to, superior to compared algorithms in 5, 2 and 91 cases, respectively out of the total 98 cases. Thus, the EDE is always either better or equal.

As can be seen from Table 3,out of 14, in 12 cases EDE could beat all its compared algorithms and from these 12 cases, in 10 cases, the results of EDE is statistically significantly better as compared to all other algorithms considered here. Obviously, it can be deduced that EDE is superior to all its contestant algorithms in all these 10 functions in terms of average and Standard deviation values. Contrarily, similar to results in table 2, EDE is surpassed by all other compared algorithms on function f_7. Furthermore, SaDE and jDE slightly perform better than EDE on problem f_{10}. All in all, from the t-test results, it can be observed that EDE is inferior to, equal to, superior to compared algorithms in 9, 9 and 66 cases, respectively out of the total 84 cases. Thus, the EDE is almost either better or equal.

According to Table 4, we can conclude that the performance of all other compared classical DE algorithm on the 30-dimensional benchmark function set is very similar to that on the 10-dimensional benchmark. The significant difference is that the performance of EDE is not affected in a worse way with the growth of the search-space dimensionality while the performance of all other compared algorithms declines significantly. Therefore, it can be deduced that EDE is superior to all classical DE algorithms with high quality final solution with lower mean and standard deviation values. Moreover, the results show that the proposed EDE algorithm outperforms other algorithm on the most functions by remarkable difference. From the t-test results, it is obvious that the EDE are inferior to, equal to, superior to compared algorithms in 6, 8 and 84 cases, respectively out of the total 98 cases. Thus, the EDE is almost either better or equal.

Table 5 indicates that the EDE algorithm produces 12, 8, 10,7 and 6 significantly better, and 2,2,3,3 and 1 slightly worse results than the Modified DE2, SFLSDE, JADE, SaDE and jDE algorithms, respectively. Furthermore, the EDE algorithm considerably performs best on the functions f_4 and f_9–f_{14} which become so difficult to solve as the dimension of variable increases. Finally, it can be observed that the performance of the EDE remained comparable to those of FiADE algorithm in most of the functions. All in all, from the t-test results, it can be observed that EDE is inferior to, equal to, superior to compared algorithms in 18, 19 and 47 cases, respectively out of the total 84 cases. Thus, the EDE is almost either better or equal.

A prolonged look of Tables 2–5 and based on the above analysis, results and comparisons, the proposed EDE algorithm is of better searching quality, efficiency and robustness for solving unconstrained global optimization problems. It is clear that the proposed EDE algorithm performs well and it has shown its outstanding superiority with separable, non-separable, unimodal and multimodal functions with shift in dimensionality, rotation, multiplicative noise in fitness and composition of functions. Consequently, its performance does not influenced by all these obstacles. Contrarily, it greatly balances the local optimization speed and the global optimization diversity in challenging optimization environment with invariant performance. Besides, its performance is superior and competitive with the performance of classical DE variants and the-state-of-the-art well-known self-adaptive DE algorithms. Factually, it can be obviously seen that the performance of the most of compared algorithm shows complete and/or significant deterioration with the growth of the search-space dimensionality while the performance of the EDE algorithm slightly diminishes and it is still more stable, efficient and robust

Table 2 Comparison between EDE and various classical DE methods on 10D problems.

Functions	DE/rand/1/bin, F = 0.8, Cr = 0.9		DE/rand/1/bin, F = 0.9, Cr = 0.1		DE/rand/1/bin, F = 0.9, Cr = 0.9		DE/rand/1/bin, F = 0.5, Cr = 0.3		DE/target-to-best/1/bin, F = 0.5, Cr = 0.3		DE/target-to-best/2/bin, F = 0.5, Cr = 0.3		DE/best/1/bin, F = 0.8, Cr = 0.9		EDE	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
f_1	3.72E−12	4.82E−12	1.18E−23	2.99E−20	4.95E−13	5.27E−13	1.88E−15	3.88E−15	3.99E−15	6.99E−14	2.66E−16	6.33E−17	4.95E−13	5.27E−13	**0 + 00E + 00**	**0 + 00E + 00**
	1		1		1		1		1		1		1		–	
f_2	8.89E−01	4.96E−01	8.89E−01	4.90E−01	1.44E−05	1.13E−05	9.63E−09	5.99E−09	3.49E−15	9.64E−15	9.45E−13	9.90E−13	1.44E−03	5.13E−02	**1.81E−28**	**2.57E−28**
	1		1		1		1		1		1		1		–	
f_3	1.23E + 00	7.92E−01	9.01E−01	7.94E−04	7.11E−03	2.74E−02	1.76E + 00	1.54E + 00	2.57E + 00	1.86E + 00	2.37E + 00	2.23E + 00	7.11E−01	2.74E−02	**1.62E−16**	**8.47E−16**
	1		1		1		1		1		1		1		–	
f_4	2.42E + 01	1.28E + 01	2.41E + 01	1.28E + 01	2.42E−04	1.38E−04	5.42E−06	4.44E−06	1.23E−17	6.92E−18	1.04E−08	1.20E−08	2.48E−04	1.38E−04	**7.01E−27**	**3.74E−26**
	1		1		1		1		1		1		1		–	
f_5	9.25E−01	6.11E−09	2.06E−12	1.29E−12	4.59E−07	2.41E−07	3.55E−15	1.28E−16	4.97E−15	1.77E−15	3.55E−15	1.87E−15	4.54E−15	2.58E−07	**0 + 00E + 00**	**0 + 00E + 00**
	1		1		1		1		1		1		1		–	
f_6	3.81E−05	1.33E−04	3.81E−15	1.33E−04	6.86E−07	3.89E−07	3.32E−15	9.01E−16	4.26E−15	1.45E−15	3.55E−15	2.98E−14	6.86E−07	3.84E−07	**0 + 00E + 00**	**0 + 00E + 00**
	1		1		1		1		1		1		1		–	
f_7	**4.86E−10**	**5.19E−10**	**5.32E−11**	**4.92E−15**	3.05E−01	2.02E−1	**1.11E−16**	**2.88E−15**	4.67E−03	8.13E−03	**3.66E−13**	**1.383−15**	**3.04E−12**	**2.02E−11**	1.74E−02	1.24E−02
	−1		−1		0		−1		0		−1		−1		–	
f_8	3.29E−01	4.77E−02	1.22E−01	2.77E−02	2.41E−01	2.00E−01	1.60E−01	3.75E−02	2.91E−01	3.14E−01	1.44E−01	3.97E−02	1.34E−01	2.03E−01	**3.51E−02**	**3.16E−02**
	1		1		1		1		1		1		1		–	
f_9	5.85E−01	1.78E−02	1.77E−15	3.45E−15	8.71E + 00	5.53E + 00	1.54E−15	1.47E−15	6.63E−02	2.52E−01	5.88E−16	1.88E−15	8.74E−04	5.52E−03	**0 + 00E + 00**	**0 + 00E + 00**
	1		1		1		1		1		1		1		–	
f_{10}	1.40E + 01	3.02E + 00	1.33E + 01	3.00E + 00	1.63E + 01	1.10E + 01	1.65E + 01	2.99E + 00	1.00E + 01	2.32E + 00	1.63E + 01	3.36E + 00	1.53E + 01	1.14E + 01	**7.56E + 00**	**3.35E + 00**
	1		1		1		1		1		1		1		–	
f_{11}	5.23E−02	2.95E−02	1.77E−15	8.28E−15	8.20E + 00	3.37E + 00	1.09E−15	4.92E−15	1.33E−01	3.46E−01	2.99E−15	1.28E−16	8.26E−03	3.49E−02	**0 + 00E + 00**	**0 + 00E + 00**
	1		1		1		1		1		1		1		–	
f_{12}	4.81E−02	8.02E−01	9.09E−13	3.99E−15	2.82E + 00	1.41E + 01	2.57E−12	9.66E−14	1.18E + 01	3.61E + 01	6.06E−14	2.31E−13	1.82E−02	1.41E−02	**0 + 00E + 00**	**0 + 00E + 00**
	1		1		1		1		1		1		1		–	
f_{13}	3.44E−02	5.41E−02	3.41E−02	5.98E−02	4.27E−13	3.74E−13	6.50E−01	3.56e + 00	1.83E + 01	4.62E + 01	1.02E−09	3.95E−09	4.27E−04	3.74E−05	**0 + 00E + 00**	**0 + 00E + 00**
	1		1		1		1		1		1		1		–	
f_{14}	**7.95E−02**	**6.91E + 00**	6.62E + 00	1.33E + 00	**2.17E−01**	**5.64E−01**	9.48E + 00	2.49E + 01	2.36E + 01	4.31E + 01	1.37E + 00	8.26E−01	1.17E + 01	1.82E + 00	**4.25E−01**	**7.32E−01**
	0		1		0		1		1		1		1		–	

Table 3 Comparison between EDE and various state-of-the-art DE methods on 10D problems.

Functions	Modified DE2		SFLSDE		JADE		FiADE		SaDE		jDE		EDE	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
f_1	3.82E−17	4.09E−11	1.07E−23	1.62E−22	2.56E−31	8.52E−30	3.62E−37	4.63E−36	2.72E−31	1.52E−31	1.12E−28	1.10E−28	**0+00E+00**	**0+00E+00**
	1		1		1		1		1		1		–	
f_2	9.63E−06	5.92E−06	3.71E−04	2.34E−03	9.43E−15	9.36E−16	7.36E−21	5.88E−21	6.82E−19	2.72E−18	3.1E−15	8.31E−15	**1.81E−28**	**2.57E−28**
	1		1		1		1		1		1		–	
f_3	1.71E+00	1.54E+00	2.57E−02	1.82E−02	2.37E−03	4.22E−03	**5.87E−17**	**4.28E−17**	**3.77E−16**	**2.76E−15**	1.34E−13	7.32E−13	**1.62E−16**	**8.47E−16**
	1		1		1		0		0		1		–	
f_4	7.42E−06	4.44E−06	5.24E−05	6.92E−04	3.04E−18	1.23E−18	5.63E−16	4.34E−16	1.92E−18	2.80E−19	1.88E−16	1.56E−17	**7.01E−27**	**3.74E−26**
	1		1		1		1		1		1		–	
f_5	1.94E−18	4.34E−16	4.97E−15	1.77E−15	3.55E−25	1.47E−23	6.90E−30	5.91E−28	5.09E−26	5.72E−27	7.72E−19	4.62E−20	**0+00E+00**	**0+00E+00**
	1		1		1		1		1		1		–	
f_6	3.32E−15	9.08E−16	2.24E−15	1.45E−15	3.53E−23	7.56E−21	3.43E−27	4.19E−27	1.55E−22	6.78E−21	5.92E−16	7.77E−17	**0+00E+00**	**0+00E+00**
	1		1		1		1		1		1		–	
f_7	**4.78E−16**	**5.42E−18**	**4.63E−15**	**8.13E−15**	**1.45E−34**	**2.89E−35**	**1.03E−30**	**4.54E−30**	**4.71E−27**	**8.18E−27**	**5.75E−04**	**2.21E−03**	1.74E−02	1.24E−02
	−1		−1		−1		−1		−1		−1		–	
f_8	1.60E−01	3.74E−02	2.94E−01	3.14E−01	1.44E−01	3.95E−02	**1.00E−02**	**2.34E−02**	**1.43E−02**	**4.14E−02**	**2.26E−02**	**1.77E−02**	**3.51E−02**	**3.16E−02**
	1		1		1		0		0		0		–	
f_9	3.87E−14	7.59E−15	6.65E−16	2.55E−17	7.45E−25	4.93E−24	4.11E−28	1.53E−27	5.90E−24	6.69E−24	1.96E−21	3.22E−20	**0+00E+00**	**0+00E+00**
	1		1		1		1		1		1		–	
f_{10}	1.65E+01	4.04E+00	1.00E+01	2.33E+00	1.63E+01	3.34E+00	**8.03E+00**	**2.78E+00**	**3.83E+00**	**1.35E+00**	**5.78E+00**	**2.10E+00**	7.56E+00	3.35E+00
	1		1		1		0		−1		−1		–	
f_{11}	7.92E−05	5.83E−04	1.34E−09	3.59E−03	5.98E−15	4.89E−06	3.02E−17	6.13E−17	4.84E−15	3.82E−15	6.34E−16	2.24E−17	**0+00E+00**	**0+00E+00**
	1		1		1		1		1		1		–	
f_{12}	4.86E−01	5.12E−02	4.13E−02	3.64E−02	5.81E−17	1.28E−15	8.97E−20	3.14E−19	1.04E−19	4.92E−19	9.23E−18	2.11E−18	**0+00E+00**	**0+00E+00**
	1		1		1		1		1		1		–	
f_{13}	6.50E−05	3.56E−06	1.83E−06	4.62E−06	1.02E−26	3.93E−25	4.81E−32	5.91E−31	5.12E−28	6.11E−29	1.33E+01	3.46E+01	**0+00E+00**	**0+00E+00**
	1		1		1		1		1		1		–	
f_{14}	2.90E+00	3.16E+00	7.92E+00	4.81E+00	**5.57E−01**	**4.46E−01**	**1.77E−01**	**4.81E−01**	**2.61E−01**	**3.98E−02**	1.27E+00	3.20E+00	4.25E−01	7.32E−01
	1		1		0		0		0		1		–	

Table 4 Comparison between EDE and various classical DE methods on 30D problems.

Functions	DE/rand/1/bin, $F = 0.8$, Cr = 0.9		DE/rand/1/bin, $F = 0.9$, Cr = 0.1		DE/rand/1/bin, $F = 0.9$, Cr = 0.9		DE/rand/1/bin, $F = 0.5$, Cr = 0.3		DE/target-to-best/1/bin, $F = 0.5$, Cr = 0.3		DE/target-to-best/2/bin, $F = 0.5$, Cr = 0.3		DE/best/1/bin, $F = 0.8$, Cr = 0.9		EDE	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
f_1	3.87E−08	4.76E−07	2.34E−20	5.23E−20	4.50E−02	6.15E−02	2.71E−05	8.33E−04	1.90E−07	1.04E−06	1.48E−07	1.84E−09	4.34E−14	6.15E−13	**1.60E−28**	**4.63E−28**
	1		1		1		1		1		1		1		–	
f_2	3.23E+03	7.26E+02	3.62E+03	6.22E+02	1.73E+03	1.40E+03	1.38E+03	2.53E+02	1.92E+01	2.27E+01	1.04E+02	8.25E+01	1.73E+03	3.42E+03	**2.29E−09**	**4.53E−09**
	1		1		1		1		1		1		1		–	
f_3	3.34E+01	5.51E+01	3.39E+01	1.51E+01	1.04E+02	6.25E+01	2.14E+01	1.98E+00	6.37E+01	4.01E+01	2.08E+01	1.19E+01	1.03E+01	6.23E+01	**1.85E−11**	**4.56E−11**
	1		1		1		1		1		1		1		–	
f_4	5.24E+04	2.34E+03	1.24E+04	2.15E+03	8.98E+03	5.74E+03	5.19E+03	1.24E+03	**2.36E+00**	**5.47E+00**	1.51E+03	2.07E+02	8.98E+03	5.74E+03	1.55E+00	2.97E+00
	1		1		1		1		0		1		1		–	
f_5	4.75E−04	7.75E−05	8.92E−13	3.72E−14	3.86E−02	2.18E−02	**4.04E−15**	**1.23E−15**	3.10E−02	1.70E−01	**7.58E−15**	**1.80E−15**	3.84E−08	2.15E−08	**3.55E−15**	**0.00E+00**
	1		1		1		0		1		0		1		–	
f_6	6.84E−05	4.32E−04	3.81E−05	1.34E−04	7.64E−02	5.11E−02	3.67E−05	6.49E−16	4.82E−03	2.64E−02	**7.34E−15**	**1.30E−15**	7.63E−02	5.13E−02	**3.55E−15**	**0.00E+00**
	1		1		1		1		1		0		1		–	
f_7	**5.87E−17**	**6.71E−17**	**4.78E−17**	**3.21E−18**	1.52E−01	1.15E−01	**1.24E−17**	**3.66E−17**	1.08E+01	1.00E+01	**1.94E−13**	**2.88E−14**	**1.52E−10**	**6.15E−10**	1.15E−03	3.07E−03
	−1		−1		1		−1		1		−1		−1		–	
f_8	3.15E−02	8.03E−02	9.12E−02	3.08E−02	9.01E−01	1.40E−01	2.24E−05	1.19E−04	1.82E+02	5.47E+01	3.97E−03	1.85E−02	**9.86E−05**	**4.40E−06**	4.10E−03	6.22E−03
	0		0		1		0		1		0		−1		–	
f_9	3.82E−07	4.73E−07	1.78E−15	1.65E−15	8.54E+01	3.30E+01	3.10E+01	3.24E+00	9.58E+00	3.85E+00	4.03E+01	3.73E+00	8.52E−12	5.53E−12	**0+00E+00**	**0+00E+00**
	1		1		1		1		1		1		1		–	
f_{10}	8.64E+02	1.04E+01	1.68E+02	1.43E+01	2.45E+02	2.20E+01	1.87E+02	1.09E+01	1.44E+02	1.09E+01	1.88E+02	7.15E+00	3.45E+02	2.22E+01	**3.88E+01**	**7.13E+00**
	1		1		1		1		1		1		1		–	
f_{11}	4.75E−07	5.08E−06	3.88E−15	3.09E−14	6.93E+01	2.37E+01	2.88E+01	1.94E+00	1.18E+01	2.70E+00	3.17E+01	2.25E+00	6.93E−04	2.37E−01	**0+00E+00**	**0+00E+00**
	1		1		1		1		1		1		1		–	
f_{12}	1.93e+02	2.78E+01	2.88E−14	8.34E−15	4.98E+03	1.75E+03	2.88E−15	3.16E−15	4.18E+02	1.75E+02	2.48E+03	4.01E+02	1.36E+02	1.09E+01	**0+00E+00**	**0+00E+00**
	1		1		1		1		1		1		1		–	
f_{13}	3.72E−03	2.12E−02	3.79E−03	1.15E−02	5.06E−03	5.54E−03	4.49E−06	4.62E−06	5.00E+01	8.20E+01	2.17E−09	1.19E−08	5.37E−03	2.53E−03	**4.16E−31**	**3.31E−31**
	1		1		1		1		1		1		1		–	
f_{14}	1.38E+01	3.32E+00	1.33E+01	3.08E+00	1.56E+01	5.46E+00	1.15E+01	1.54E+00	2.86E+01	7.69E+01	1.22E+01	2.99E+00	1.56E+01	5.46E+00	**1.98E+00**	**9.03E−01**
	1		1		1		1		1		1		1		–	

Table 5 Comparison between EDE and various state-of-the-art DE methods on 30D problems.

Functions	Modified DE2		SFLSDE		JADE		FiADE		SaDE		jDE		EDE	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
f_1	4.34E−10 1	6.15E−08	1.92E−13 1	1.25E−12	5.87E−28 0	4.98E−28	**4.82E−34** −1	**5.87E−33**	8.76E−18 1	2.76E−18	2.56E−25 0	3.54E−25	1.60E−28 –	4.63E−28
f_2	1.57E+03 1	1.95E+03	1.92E+00 1	2.32E+00	**1.03E−26** −1	**8.64E−26**	**3.89E−29** −1	**9.38E−18**	**4.34E−16** −1	**6.46E−17**	8.91E−11 0	1.27E−10	2.29E−09 –	4.53E−09
f_3	6.04E+02 1	6.25E+02	3.33E−10 0	4.01E−13	2.34E−04 1	1.18E−02	**8.63E−18** −1	**3.46E−20**	3.95E−01 1	1.22E+00	5.77E−01 1	1.38E+00	1.85E−11 –	4.56E−11
f_4	3.94E+03 1	1.74E+03	2.33E+02 1	5.43E+02	6.54E+01 1	2.36E+01	**1.56E+00** 0	**2.14E+00**	2.37E+00 1	4.37E+01	**2.15E+00** 0	**4.91E−01**	**1.55E+00** –	**2.97E+00**
f_5	3.83E−04 1	9.15E−05	2.34E−15 0	1.36E−15	7.54E−09 1	5.84E−09	**6.45E−18** −1	**4.85E−17**	4.82E−16 0	7.85E−16	4.82E−15 0	9.62−16	3.55E−15 –	0.00E+00
f_6	3.66E−05 1	6.49E−06	**4.35E−16** 0	**2.83E−16**	7.33E−05 1	1.54E−05	**4.77E−17** 0	**8.56E−17**	**8.93E−16** 0	**2.45E−16**	**9.76E−17** 0	**4.36E−16**	**3.55E−15** –	**0.00E+00**
f_7	**4.56E−06** −1	**6.92E−07**	3.65E−03 0	1.08E−01	**7.76E−34** −1	**3.87E−32**	**5.93E−30** −1	**3.98E−29**	**5.33E−33** −1	**5.04E−33**	**3.36E−33** −1	**4.87E−33**	1.15E−03 –	3.07E−03
f_8	**2.24E−05** −1	**3.73E−04**	**8.82E−06** −1	**4.54E−05**	**3.95E−05** −1	**1.85E−04**	**2.65E−06** −1	**4.53E−05**	8.54E−03 0	9.08E−03	5.17E−03 0	6.64E−02	4.10E−03 –	6.22E−03
f_9	4.10E−06 1	3.27E−04	9.53E−16 1	2.09E−17	4.04E−25 1	3.45E−24	6.85E−34 1	4.99E−30	4.22E−18 1	8.70E−17	5.23E−17 1	7.35E−17	**0+00E+00** –	**0+00E+00**
f_{10}	1.87E+02 1	1.49E+01	**1.89E+01** −1	**1.09E+01**	1.87E+02 1	7.54E+00	1.39E+02 1	9.34E+00	**1.86E+01** −1	**5.26E+00**	3.65E+01 0	8.29E+00	3.88E+01 –	7.13E+00
f_{11}	2.81E−01 1	1.94E−02	1.18E−04 1	2.73E−05	3.14E−12 1	2.25E−10	1.00E−21 1	2.13E−22	4.56E−19 1	2.76E−20	6.67E−02 1	2.54E−01	**0+00E+00** –	**0+00E+00**
f_{12}	7.90E−02 1	4.75E−03	4.18E+00 1	2.75E+00	2.79E+00 1	1.02E+00	4.82E−27 1	4.84E−26	3.17E−20 1	5.80E−18	6.32E+01 1	1.07E+02	**0+00E+00** –	**0+00E+00**
f_{13}	4.49E−06 1	4.34E−06	5.00E−05 1	2.27E−03	2.17E−16 1	1.19E−15	**3.92E−30** 0	**1.45E−28**	6.18E−17 1	1.45E−19	5.16E−16 1	2.36E−18	**4.16E−31** –	**3.31E−31**
f_{14}	1.15E+01 1	1.54E+00	2.86E+01 1	2.63E+01	1.22E+01 1	2.05E+00	**1.22E+00** −1	**4.43E−01**	2.17E+00 0	9.23E−01	1.24E+01 1	3.06E+01	1.98E+00 –	9.03E−01

Table 6 Comparison between EDE and EDE with different versions on 30D problems.

Functions	EDE1		EDE2		EDE3		EDE	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
f_1	3.28E−30	1.22E−29	3.36E−30	1.30E−29	1.06E−28	1.01E−28	**1.60E−28**	**4.63E−28**
	0		0		0		–	
f_2	9.58E−06	1.27E−05	1.19E−05	1.18E−05	3.85E−09	8.82E−09	**2.29E−09**	**4.53E−09**
	1		1		0		–	
f_3	1.31E+01	7.37E+00	2.07E+01	1.26E+01	5.31E−01	1.40E+00	**1.85E−11**	**4.56E−11**
	1		1		1		–	
f_4	**7.94E−02**	**1.46E−01**	**1.02E−02**	**1.15E−02**	**5.68E−01**	**7.93E−01**	1.55E+00	2.97E+00
	−1		−1		−1		–	
f_5	**3.55E−15**	**0.00E+00**	**3.55E−15**	**0.00E+00**	**3.55E−15**	**0.00E+00**	**3.55E−15**	**0.00E+00**
	0		0		0		–	
f_6	**3.55E−15**	**0.00E+00**	**3.55E−15**	**0.00E+00**	**3.55E−15**	**0.00E+00**	**3.55E−15**	**0.00E+00**
	0		0		0		–	
f_7	**0+00E+00**	**0+00E+00**	1.15E−03	3.07E−03	6.57E−04	2.54E−03	1.15E−03	3.07E−03
	−1		0		0		–	
f_8	2.79E−03	4.14E−03	**4.93E−04**	**1.91E−03**	2.62E−03	4.60E−03	4.10E−03	6.22E−03
	0		−1		0		–	
f_9	5.72E+01	1.34E+01	9.95E+01	1.72E+01	1.44E+01	5.59E+00	**0+00E+00**	**0+00E+00**
	1		1		1		–	
f_{10}	1.68E+02	7.49E+00	1.81E+02	6.67E+00	5.29E+01	1.53E+01	**3.88E+01**	**7.13E+00**
	1		1		1		–	
f_{11}	5.82E+01	1.17E+01	9.69E+01	1.22E+01	2.06E+1	5.30E+00	**0+00E+00**	**0+00E+00**
	1		1		1		–	
f_{12}	4.38E+00	1.69E+01	1.61E+03	2.09E+03	3.21E+02	2.12E+02	**0+00E+00**	**0+00E+00**
	1		1		1		–	
f_{13}	**3.50E−32**	**1.36E−31**	**1.75E−32**	**4.63E−32**	**3.94E−31**	**6.72E−31**	**4.16E−31**	**3.31E−31**
	0		0		0		–	
f_{14}	1.57E+00	8.03E−01	**8.97E−01**	**7.02E−01**	2.84E+00	1.13E+00	1.98E+00	9.03E−01
	0		−1		1		–	

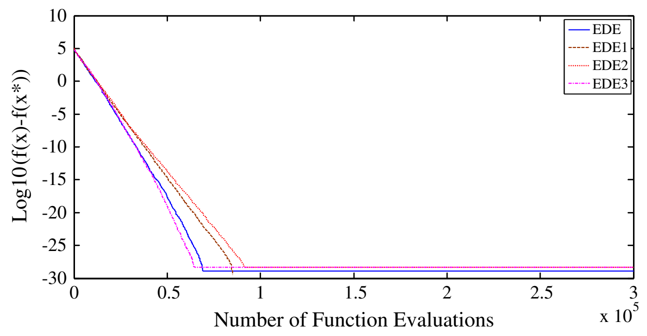
against the curse of dimensionality. Finally, it is easily implemented and a reliable approach for real parameter optimization.

5. A parametric study on EDE

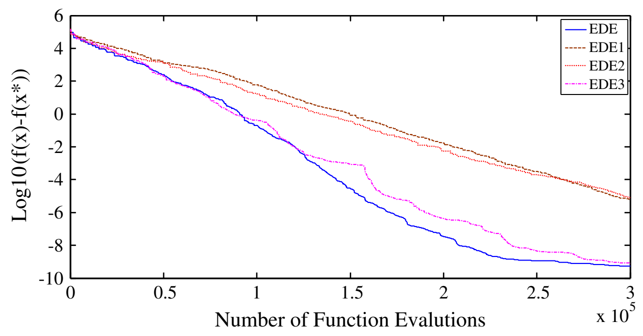
In this section, in order to investigate the impact of the proposed modifications, some experiments are conducted. Three different versions of EDE algorithm have been tested and compared against the proposed one.

1. *Version 1:* To study the effect of the proposed modifications for (CR) and (F) parameters with basic mutation strategy, DE/rand/1/bin strategy is combined with the proposed uniform cross over (CR) probability and uniform scaling factor (F). (Denoted as EDE1).
2. *Version 2:* To study the effect of the proposed modifications for (CR) and (F) parameters and random and modified (BGA) mutations with basic mutation strategy, DE/rand/1/bin strategy is combined with random and modified (BGA) mutations and the proposed uniform cross over (CR) probability and uniform scaling factor (F). (Denoted as EDE2).
3. *Version 3:* To study the effect of the proposed uniform (CR) and (F) parameters and basic mutation strategy with proposed mutation strategy, DE/rand/1/bin strategy is combined with the proposed mutation and the proposed uniform cross over (CR) probability and uniform scaling factor (F). (Denoted as EDE3).

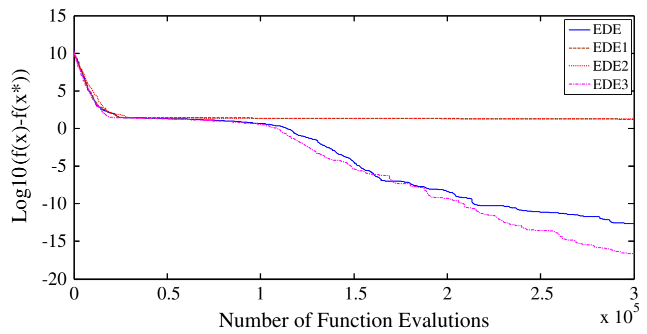
In order to evaluate the final solution quality, efficiency, convergence rate, and robustness produced by all algorithms, the performance of the three different versions of EDE algorithm are investigated based on the 30-dimensional functions. The parameters used are fixed as same as those in Section 4.2. The overall comparison results of the EDE algorithm against its versions and conventional DE algorithm are summarized in Table 6. Furthermore, in order to analyze the convergence behavior of each algorithm compared, the convergence characteristics in terms of the best fitness value of the median run of each algorithm for functions f_1–f_{14} with dimension 30 is illustrated in Fig. 2. Indeed, the presented results in Table 6 explain that EDE and its three different variants obtain better high quality results on unimodal problem f_1, multi-modal functions f_5, f_6 and composition function f_{13}. Therefore, it is clearly that the similar performance and common results exhibited by EDE and its versions on these functions is due to the effect of the proposed uniform cross over (CR) probability and uniform scaling factor (F). Furthermore, with respect to the remaining functions, for the EDE 2 algorithm, it is clearly observed that the incorporation of the random mutation and modified (BGA) mutation to EDE 1 deteriorates performance slightly on functions (f_3,f_7,f_9,f_{10} and f_{11}) while deteriorates performance significantly on function f_{12}. However, produces a significant improvement in composition function f_{14}. Finally, EDE 1 and EDE 2 exhibit the similar performance on functions (f_2, f_4 and f_8). Thus, the joining of random mutation and modified (BGA) mutation in EDE 2 has a slight negative influence on the final solution quality and the convergence speed on some



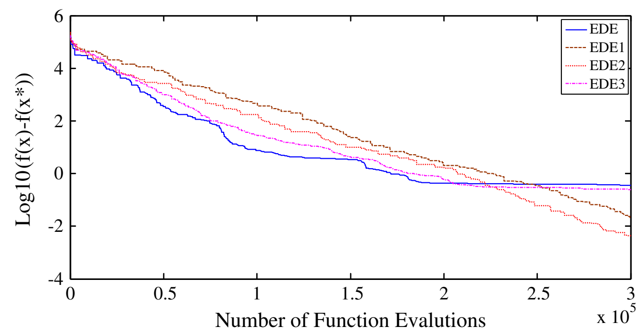
(a) F1



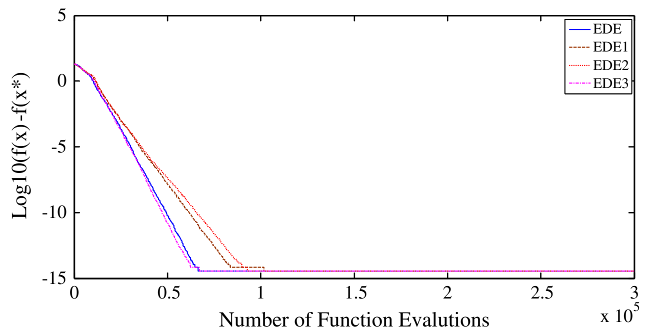
(b) F2



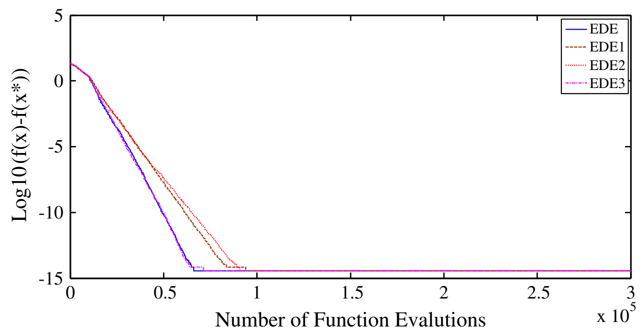
(c) F3



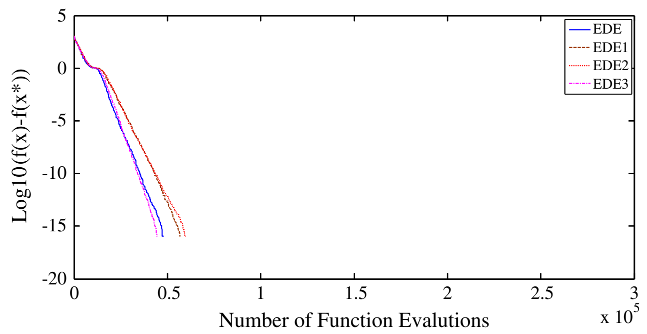
(d) F4



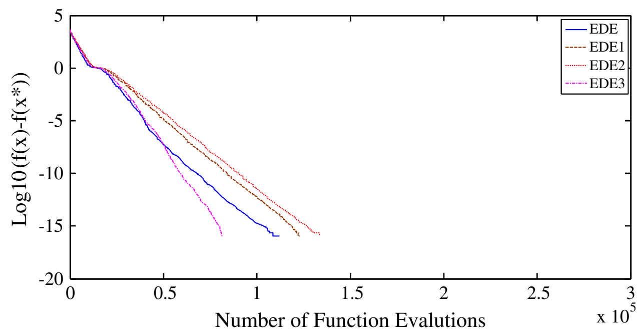
(e) F5



(f) F6



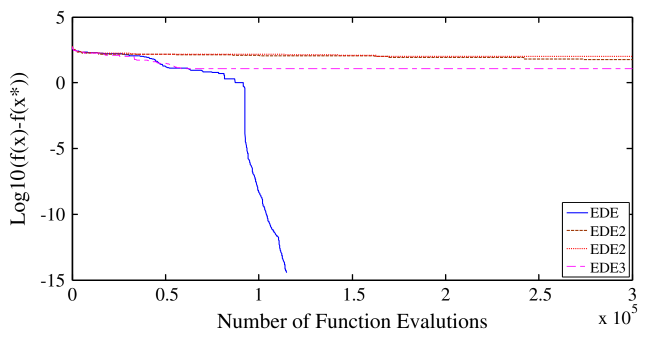
(g) F7



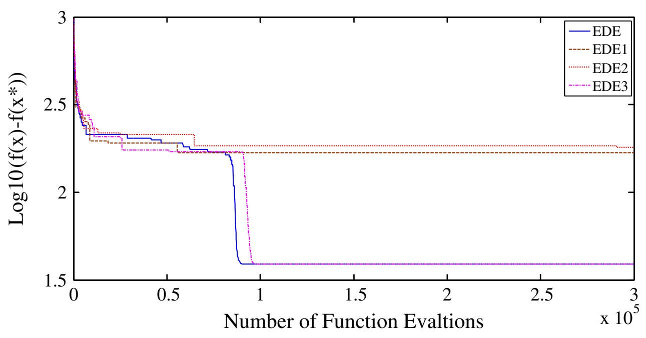
(h) F8

Figure 2 Convergence graph (median curves) of EDE, EDE1, EDE2 and EDE3 on 30-dimensional test functions f_1–f_{14}.

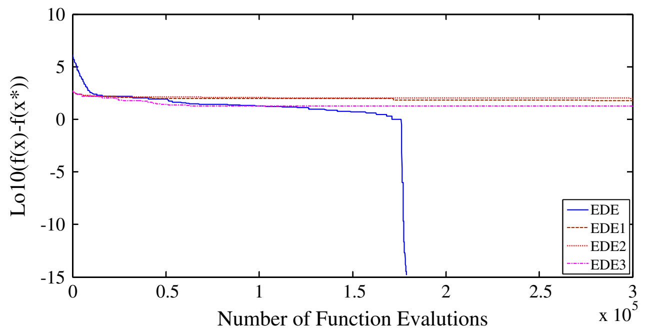
cases. On the other hand, for the EDE 3 algorithm, it can be seen that by embedding the proposed mutation in EDE 1 algorithm, a significant improvement in the performance of EDE 3 has been detected and achieved on functions (f_2–f_3 and f_9–f_{11}). On the contrary, EDE1 algorithm has performed better than EDE3 on problem (f_7, f_{12} and f_{14}). Meanwhile, EDE1 and IDE3 exhibit similar performance on functions f_4 and f_8. For the EDE algorithm, it exhibits substantial performance improvement on functions (f_2–f_3 and f_9–f_{12}). Therefore, it can be seen that by embedding the new mutation scheme , random mutation and modified (BGA) mutation together in EDE 1 algorithm, extreme and ultimate improvement in the performance of EDE 1



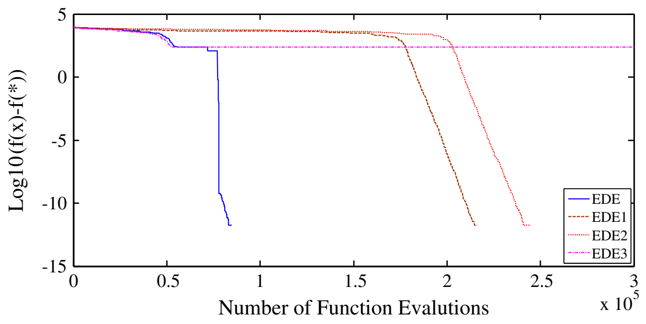
(i) F9



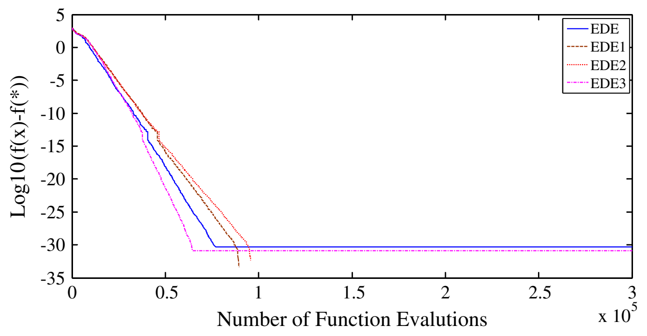
(j) F10



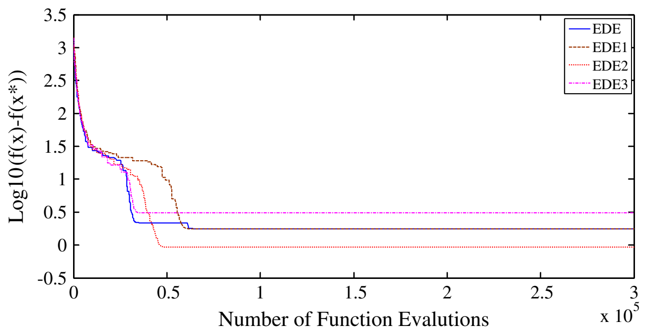
(k) F11



(l) F12



(m) F13



(n) F14

Figure 2 (*continued*)

has been detected and achieved on these functions. Moreover, from the *t*-test results, it can be observed that EDE is inferior to, equal to, superior to its compared versions in 6, 18 and 18 cases out of the total 42 cases, respectively. Consequently, EDE algorithm is always either better or equal. Overall, it can be concluded that the performance of the EDE is superior to and/or competitive with EDE 1, EDE 2 and EDE 3 algorithms in terms of final solution quality, stability and robustness. Additionally, as previously mentioned, the convergence graph in Fig. 2 illustrate that EDE 3 and EDE algorithms converge to better or global solution faster than EDE 1 and IDE 2 in all cases with exception to functions f_4 and f_{14} where EDE 2 converges faster than all compared algorithms. However, EDE algorithm converges faster than EDE 3 on functions (f_9, f_{11} and f_{12}) while EDE slightly slower than EDE1 and DE 2 on function f_4. It is clear that the proposed modifications play a vital role and has a significant impact in improving the convergence speed of EDE algorithm for most problems. The EDE algorithm has a considerable ability to maintain its convergence rate, improve its diversity as well as advance its local tendency through a search process. Thus, after the above analysis and discussion, the proposed algorithm EDE show competitive performance in terms of quality of solution, efficiency, convergence rate and robustness. It is superior to conventional DE methods, and it is also competitive with and, in some cases superior to the-state-of-the-art well-known self-adaptive DE algorithms and its three versions EDE 1, EDE 2 and EDE 3. Accordingly, the main benefits of the proposed modifications are the remarkable balance between the exploration capability and exploitation tendency through the optimization process that leads to superior performance with fast convergence speed and the extreme robustness over the entire range of benchmark functions which are the weak points of all evolutionary algorithms.

6. Conclusion and future works

In this paper, an Effective Differential Evolution (EDE) algorithm is presented for solving unconstrained global real-parameter optimization problems over continuous domain. In order to enhance the local search ability and advance the convergence rate, a new directed mutation rule was presented and it is combined with the basic mutation strategy through a linear decreasing probability rule. The proposed mutation rule is shown to enhance the local search capabilities of the basic DE and to increase the convergence speed. A new scaling factor is introduced as uniform random number to enrich the search behavior. Furthermore, a random mutation scheme and a modified Breeder Genetic Algorithm (BGA) mutation scheme are merged to avoid stagnation and/or premature convergence. Additionally, the scaling factor and crossover of DE are introduced as uniform random numbers to enrich the search behavior and to enhance the diversity of the population. The proposed EDE algorithm has been compared with seven classical DE methods and six recent state-of-the-art parameter adaptive differential evolution variants over a suite of 14 bound constrained numerical optimization problems. The experimental results and comparisons have shown that the EDE algorithm performs better in unconstrained optimization problems with different types, complexity and dimensionality; it performs better with regard to the search process efficiency, the final solution quality, the convergence rate, and robustness, when compared with other algorithms. Finally, the performance of the EDE algorithm is statistically superior to and conventional DE algorithms and it is competitive with other recent well-known self-adaptive DE algorithms especially with high dimensions problems. The effectiveness and benefits of the proposed modifications used in EDE have been experimentally investigated and compared. It is found that the proposed algorithm EDE shows competitive performance in terms of quality of solution, efficiency, convergence rate and robustness. It is statistically superior to and competitive with its three versions EDE1, EDE 2 and EDE 3 basically based on DE/rand/1/bin strategy. Several current and future works can be developed from this study. Firstly, Current research efforts focus on how to modify the EDE algorithm for handling constrained and multi-objective optimization problems as well as to solve practical engineering optimization problems and real world applications. Secondly, it would be very interesting to propose a self-adaptive EDE version. However, Future works may focus on applying the algorithm to solve standard benchmark functions and high dimensions or large scale global optimization problems and compare the results with the most recent algorithms.

References

[1] Storn R, Price K. Differential evolution-a simple and efficient adaptive scheme for global optimization over continuous spaces. Technical report TR-95-012, ICSI; 1995. < http://.icsi.berkeley.edu/~storn/litera.html >.

[2] Storn R, Price K. Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces. J Global Optim 1997;11(4):341–59.

[3] Price K, Storn R, Lampinen J. Differential evolution: a practical approach to global optimization. Heidelberg: Springer; 2005.

[4] Pan QK, Wang L, Gao L, Li WD. An effective hybrid discrete differential evolution algorithm for the flow shop scheduling with intermediate buffers. Inf Sci 2011;181(3):668–85.

[5] Omran M, Engelbrecht AP, Salman A. Differential evolution methods for unsupervised image classification. In: Proceedings of IEEE congress on evolutionary computation, vol. 2; 2005. p. 966–73.

[6] Hachicha N, Jarboui B, Siarry P. A fuzzy logic control using a differential evolution algorithm aimed at modeling the financial market dynamics. Inf Sci 2011;181(1):79–91.

[7] Das S, Abraham A, Konar A. Automatic clustering using an improved differential evolution algorithm. IEEE Trans Syst Man Cybern – Part A: Syst Hum 2008;38(1):218–37.

[8] Das S, Konar A. Two-dimensional IIR filter design with modern search heuristics: a comparative study. Int J Comput Intell Appl 2006;6(3):329–55.

[9] Das S, Sil S. Kernel-induced fuzzy clustering of image pixels with an improved differential evolution algorithm. Inf Sci 2010;180(8):1237–56.

[10] Wang Y, Li B, Weise T. Estimation of distribution and differential evolution cooperation for large scale economic load dispatch optimization of power systems. Inf Sci 2010;180(12):2405–20.

[11] Aliev AR, Pedrycz W, Guirimov BG, Aliev RR, Ilhan U, Babagil M, Mammadli S. Type-2 fuzzy neural networks with fuzzy clustering and differential evolution optimization. Inf Sci 2011;181(9):1591–608.

[12] Noman N, Iba H. Accelerating differential evolution using an adaptive local search. IEEE Trans Evol Comput 2008;12(1):107–25.

[13] Das S, Abraham A, Chakraborty UK, Konar A. Differential evolution using a neighborhood based mutation operator. IEEE Trans Evol Comput 2009;13(3):526–53.

[14] Lampinen J, Zelinka I. On stagnation of the differential evolution algorithm. In: Matoušek R, Ošmera P, editors. Proceedings of Mendel 2000, 6th international conference on soft computing; 2000. p. 76–83.

[15] Gämperle R, Müller SD, Koumoutsakos P. A parameter study for differential evolution. In: Grmela A, Mastorakis NE, editors. Advances in intelligent systems, fuzzy systems, evolutionary computation. Interlaken, Switzerland: WSEAS Press; 2002. p. 293–8.

[16] Rönkkönen J, Kukkonen S, Price KV. Real-parameter optimization with differential evolution. In: Proceedings of IEEE congress on evolutionary computation, IEEE Computer Society, Washington, DC, vol. 1; 2005. p. 506–13.

[17] Liu J, Lampinen J. A fuzzy adaptive differential evolution algorithm. Soft Comput 2005;9(6):448–62.

[18] Brest J, Greiner S, Bošković B, Mernik M, žumer V. Self-adapting control parameters in differential evolution: a comparative study on numerical benchmark problems. IEEE Trans Evol Comput 2006;10(6):646–57.

[19] Omran MGH, Salman A, Engelbrecht AP. Self-adaptive differential evolution. In: Computational intelligence and security, PT 1, proceedings, lecture notes in artificial intelligence; 2005. p. 192–99.

[20] Zaharie D. Control of population diversity and adaptation in differential evolution algorithms. In: Matousek R, Osmera P, editors. Proceedings of Mendel 2003, 9th international conference on soft computing; 2003. p. 41–6.

[21] Ali MM, Törn A. Population set based global optimization algorithms: some modifications and numerical studies. Comput Oper Res 2004;31:1703–25.

[22] Neri F, Tirronen V. Scale factor local search in differential evolution. Memetic Comput J 2009;1(2):153–71.

[23] Qin AK, Huang VL, Suganthan PN. Differential evolution algorithm with strategy adaptation for global numerical optimization. IEEE Trans Evol Comput 2009;13(2):398–417.

[24] Zhang JQ, Sanderson AC. JADE: adaptive differential evolution with optional external archive. IEEE Trans Evol Comput 2009;13(5):945–58.

[25] Ghosh A, Das S, Chowdhury A, Giri R. An improved differential evolution algorithm with fitness-based adaptation of the control parameters. Inf Sci 2011;181(18):3749–65.

[26] Fan HY, Lampinen J. A trigonometric mutation approach to differential evolution. J Global Optim 2003;27(1):105–29.
[27] Mohamed AW, Sabry HZ. Constrained optimization based on modified differential evolution algorithm. Inf Sci 2012;194:171–208.
[28] Mohamed AW, Sabry HZ, Khorshid M. An alternative differential evolution algorithm for global optimization. J Adv Res 2012;3(2):149–65.
[29] Kaelo P, Ali MM. A numerical study of some modified differential evolution. Eur J Oper Res 2006;196(3):1176–84.
[30] Liu G, Li Y, Nie X, Zheng H. A novel clustering-based differential evolution with 2 multi-parent crossovers for global optimization. Appl Soft Comput 2012;12(2):663–81.
[31] Ali M, Siarry P, Pant M. An efficient Differential Evolution based algorithm for solving multi-objective optimization problems. Eur J Oper Res 2012;217(2):404–16.
[32] Triguero I, Derrac J, Garci S, Herrera F. Integrating a differential evolution feature weighting scheme into prototype generation. Neurocomputing 2012;97:332–43.
[33] Ali M, Pant M, Abraham A. Improving differential evolution algorithm by synergizing different improvement mechanisms. ACM Trans Auton Adapt Syst 2012;7(2):1–32.
[34] Islam SM, Das S, Ghosh S, Roy S, Suganthan PN. An adaptive differential evolution algorithm with novel mutation and crossover strategies for global numerical optimization. IEEE Trans Syst Man Cybern Part B: Cybern 2012;42(2):482–500.
[35] Mallipeddi R, Suganthan PN, Pan QK, Tasgetiren MF. Differential evolution algorithm with ensemble of parameters and mutation strategies. Appl Soft Comput 2011;11(2):1679–96.
[36] Das S S, Suganthan PN. Differential evolution: a survey of the state-of-the-art. IEEE Trans Evol Comput 2011;15(1):4–31.
[37] Xu Y, Wang L, Li L. An effective hybrid algorithm based on simplex search and differential evolution for global optimization. Int Conf Intell Comput 2009:341–50.
[38] Feoktistov V. Differential evolution: in search of solutions. Berlin, Germany: Springer-Verlag; 2006.
[39] Mühlenbein H, Voosen DS. Predictive models for the breeder genetic algorithm – I. Continuous parameter optimization. Evol Comput 1993;1(1):25–49.
[40] Liang JJ, Suganthan PN, Deb K. Novel composition test functions for numerical global optimization. In: Proceedings of IEEE Swarm intelligence symposium, Pasadena, CA; 2005. p. 68–75.
[41] Shang YW, Qiu YH. A note on the extended rosenbrock function. Evol Comput 2006;14(1):119–26.
[42] Derrac J, García S, Molina D, Herrera F. A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms. Swarm Evol Comput 2011;1(1):3–18.
[43] Eiben AE, Smit SK. Parameter tuning for configuring and analyzing evolutionary algorithms. Swarm Evol Comput 2011;1(1):19–31.