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On Edge-magic Labelings of Forests

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Abstract

Given an n-vertex graph $G = (V, E)$ with m edges, a labeling f of $V \cup E$ that uses all the labels in the set $\{1, 2, \ldots, n + m\}$ is *edge-magic* if there is an integer k such that $f(u) + f(v) + f(uv) = k$ for every edge $uv \in E$. Furthermore, if the labels in $\{1, 2, \ldots, n\}$ are given to the vertices, then f is called *super edge-magic*. Kotzig [On magic valuations of trichromatic graphs, Reports of the CRM, 1971] started the investigation of super edge-magic labelings of forests. Following this line of research, we prove that some forests of stars admit a super edge-magic labeling and that some forests of caterpillars admit an edge-magic labeling.

Keywords: Edge-magic labelings, Stars, Caterpillars, Forests.

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1 Introduction

Let $G = (V, E)$ be an n-vertex graph with m edges. A function $f : V \cup E \to \{1, 2, \ldots, n + m\}$ is an *edge-magic labeling* of G if f is bijective and, for some integer k, we have $f(u) + f(v) + f(uv) = k$ for every edge $uv \in E$. The number k is a *magic constant* of G. Furthermore, if $f(V) = \{1, 2, \ldots, n\}$, then the labeling f is called *super edge-magic*.

Magic labelings have applications in communication networks. Such networks consist of devices and communication lines connecting these devices [1,2]. To avoid collision, each communication line has a unique identifier. For security reasons, one must be able to deduce the identifier of a line from the devices it connects. This goal is achieved if the sum of the identifier of the line with the identifiers of the two connecting devices is constant in the whole network. Another application of magic labelings is in the use of radar impulses to measure the distance between objects [1,3].

A *star* with n vertices is a tree isomorphic to the complete bipartite graph $K_{1,n-1}$. We denote by $|S|$ the size of a star S, which is its number of edges. A *caterpillar* is a tree composed by a central path and vertices directly connected to this path. Note that a star is also a caterpillar.

Edge-magic labelings were introduced by Kotzig and Rosa [9], who proved that the following graphs are edge-magic: bipartite complete graphs $K_{p,q}$ for all p, $q \geq 1$, cycles C_n for all $n \geq 3$, paths P_n for all $n \geq 2$, stars, caterpillars, and 1-regular graphs with an odd number of edges. Later [10] they proved that the complete graph K_n is edge-magic if and only if $n = 2, 3, 5, 6$. They also asked if any tree is edge-magic, a question that is open to this day. The concept of super edge-magic labelings was introduced by Enomoto, Lladó, Nakamigawa, and Ringel [4], who proved that any n-vertex super edge-magic graph with m edges must satisfy $m \leq 2n - 3$. Furthermore, they showed that C_n is super edge-magic if and only if n is odd, K_n is super edge-magic if and only if $n = 1, 2, 3$, and $K_{p,q}$ is super edge-magic if and only if $p = 1$ or $q = 1$. Note that this last result includes the stars. They also verified that all trees with up to 15 vertices are super edge-magic, and conjectured that any tree is super edge-magic.

In this paper we are interested in particular types of forests. Kotzig [8] showed that if G is a 3-colorable edge-magic graph, then any graph composed by the union of an odd number of copies of G is also edge-magic. This directly implies that if T is a path, a caterpillar, or a star, then a forest composed by a union of an odd number of copies of T is edge-magic. Figueroa-Centeno, Ichishima and Muntaner-Batle [6] showed that a forest with k copies of $K_{1,n-1}$ is super edge-magic if k is odd. They also proved that, if a forest consisting only of paths is super edge-magic, then a graph composed by k copies of such forest is also super edge-magic for k odd. This implies that forests composed by k copies of the same star (or path) are super edge-magic for k odd. For more results about edge-magic labelings, we refer the reader to the books [12,14] and to the survey [7].

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Our contribution is twofold: we investigate the problem of describing edge-magic labelings in some forests of stars and some forests of caterpillars. A *star forest* is a forest whose components are stars. We say that a forest of stars is *odd* if it consists of an odd number of stars. It is easy to see that a 1-regular graph with k edges, which is a star forest, is not super edge-magic if k is even [6, Lemma 1.2]. The following conjecture was posed in 2002.

Conjecture 1.1 (Lee–Kong [11]) *Every odd star forest is super edge-magic.*

Lee and Kong [11] proved that some star forests with less than five stars are super edge-magic. In what follows, let $(S_1, S_2, \ldots, S_k)$ be a star forest with $|V(S_i)| = n_i$ for $1 \leq i \leq k$. Zhenbin and Chongjin [15] proved that, if $k = 2q + 1$, $n_i = a$ for $1 \leq i \leq q+1$, and $n_i = b$ for $q+2 \leq i \leq 2q+1$, then $(S_1, S_2, \ldots, S_k)$ is super edge-magic. Recently, Manickam, Marudai, and Kala [13] showed that, for $r \geq 3$ odd, if $n_1, n_2, \ldots, n_k$ is an increasing sequence of positive integers with $n_i = 1 + (i-1)d$ for $1 \leq i \leq k$ and any d, then $(S_1, S_2, \ldots, S_k)$ is super edge-magic. For instance, for $n_1 = 1, \ldots, n_k = k$, the star forest $(S_1, S_2, \ldots, S_k)$ is super edge-magic.

We are interested in *symmetric* star forests, which are those such that the number of stars of each size is even except for at most one size. In our first main result (Theorem 2.2), we prove that every odd symmetric star forest has a super edge-magic labeling, providing a positive result regarding Conjecture 1.1. Furthermore, the labeling we show in this result is also *special super edge-magic*, which is a super edge-magic labeling f of a bipartite graph $G[X, Y]$ where $f(X) = \{1, 2, \ldots, |X|\}$. An open problem regarding such labelings is to characterize the star forests which admit special super edge-magic labelings [12, Problem 2.9].

Our second result concerns *caterpillar forests*, which are forests whose components are caterpillars. Being a tree, a caterpillar is also a bipartite graph, so we say it is of *type* (r, s) if r and s are the sizes of the parts of its unique bipartition, where $r \leq s$. A caterpillar forest is *odd* if it has an odd number of caterpillars and it is *uniform* if all of its caterpillars are of the same type. In Theorem 3.1, we prove that every odd uniform caterpillar forest has an edge-magic labeling.

This paper is organized as follows. In Section 2 we prove Theorem 2.2, which deals with star forests. The result concerning caterpillar forests (Theorem 3.1) is proved in Section 3.

2 Star forests

When dealing with super edge-magic labelings, the following result given by Figueroa-Centeno, Ichishima, and Muntaner-Batle [5] turns out to be very useful and it will be used in this section.

Lemma 2.1 ([5]) *An n-vertex graph $G = (V, E)$ with m edges is super edge-magic if and only if there exists a bijective function $f \colon V \to \{1, \ldots, n\}$ such that the set $L = \{f(u) + f(v) \colon uv \in E\}$ consists of m consecutive integers. In such a case, f extends to a super edge-magic labeling of G.*

Let $\mathcal{C}$ be an odd symmetric star forest on n vertices, and let S be the only star that appears in $\mathcal{C}$ an odd number of times. Let $p = |\mathcal{C}|$ and r be such that $p = 2r-1$. Let $S_1, \ldots, S_p$ be the stars in $\mathcal{C}$ so that $S_r = S$, $|S_i| = |S_{p-i+1}|$ for every $i = 1, \ldots, r-1$, and $|S_i| \leq |S_{i+1}|$ for every $i = 1, \ldots, r-2$. Note that such an order exists because $\mathcal{C}$ is odd and symmetric. Let c_i denote the central vertex of the star S_i. From now on, we consider that any odd symmetric star forest is a sequence of stars described as in such order.

A super edge-magic labeling f of $\mathcal{C}$ is *standard* if $f(c_i) = i$ for $i = 1, \ldots, p$ and, whenever $n > p$, the smallest sum in $L = \{f(u) + f(v) \mid uv \in E(\mathcal{C})\}$ is $r + p + 1$.

The next theorem shows that every odd symmetric star forest has a standard super edge-magic labeling. Figure 1 shows an example of such labeling for a star forest with seven stars.

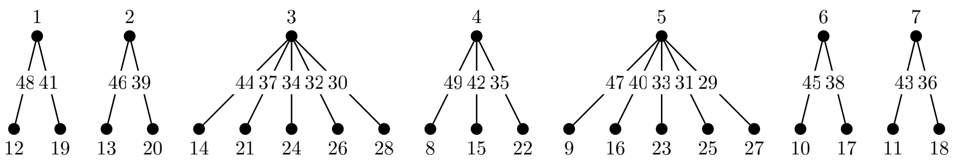

Fig. 1. Example of a standard super edge-magic labeling with magic constant 61 on an odd symmetric star forest.

Theorem 2.2 *Every odd symmetric star forest has a standard super edge-magic labeling.*

Proof. Let $\mathcal{C}$ be an odd symmetric star forest and $p = |\mathcal{C}|$. The proof is by induction on the number n of vertices of $\mathcal{C}$. If $n = p$, then $\mathcal{C}$ consists of only the centers of the stars. Set $f(c_i) = i$ for $i = 1, \ldots, p$. Trivially this is a standard super edge-magic labeling of $\mathcal{C}$.

Suppose $n > p$, that is, $\mathcal{C}$ contains at least one non-trivial star. Let $p' > 0$ be the number of non-trivial stars in $\mathcal{C}$. Note that p' is odd if the central star S is nontrivial; otherwise p' is even. Let $\mathcal{C}'$ be the star forest obtained from $\mathcal{C}$ by removing a leaf from each non-trivial star in $\mathcal{C}$. Call $u_1, \ldots, u_{p'}$ these removed leaves, in the order of their stars in $\mathcal{C}$, and let $U = \{u_1, \ldots, u_{p'}\}$. Note that $\mathcal{C}'$ is odd and symmetric, and the order of the stars in $\mathcal{C}$ has the same properties in $\mathcal{C}'$. Let n' be the number of vertices in $\mathcal{C}'$. Then $n' = n - p'$, and we can apply induction on $\mathcal{C}'$ to obtain a standard super edge-magic labeling f' for $\mathcal{C}'$.

We define the standard labeling f on $\mathcal{C}$ as follows. First, let $f(c_i) = f'(c_i) = i$ for $i = 1, \ldots, p$. Second, for every vertex v in $\mathcal{C}'$ which is not a star center, let $f(v) = f'(v) + p'$. See Figure 2. Now we describe the labels for vertices in U. Let $\ell = \lceil (p'+1)/2 \rceil$. The idea is to label the vertices in U cyclically starting at u_ℓ, with labels $p+1, \ldots, p+p'$. Set $f(u_j) = p + j - \ell + 1$ for $j = \ell, \ldots, p'$ and $f(u_j) = (p + p' - \ell + 1) + j = f(u_{p'}) + j$ for $j = 1, \ldots, \ell - 1$. Let us show that f can be extended to a standard super edge-magic labeling. That is, let us show that the set $L = \{f(u) + f(v) \mid uv \in E(\mathcal{C})\}$ consists of consecutive integers starting from $r + p + 1$.

Because f' is a standard super edge-magic labeling by induction, by Lemma 2.1,

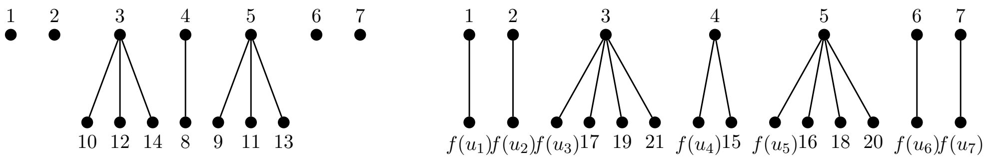

Fig. 2. Labeling for a star forest $\mathcal{C}$ on the right obtained from the labeling of the star forest $\mathcal{C}'$ on the left.

we know that $L' = \{f'(u) + f'(v) \mid uv \in E(\mathcal{C}')\}$ consists of consecutive integers starting from $r+p+1$. By the way we defined f on $V(\mathcal{C}')$, the set $L_1 = \{f(u)+f(v) \mid uv \in E(\mathcal{C}')\}$ consists of the consecutive integers starting from $r+p+1+p'$ (the same consecutive integers plus p'). Let us argue that $L_2 = \{f(u) + f(v) \mid uv \in E(\mathcal{C}) \setminus E(\mathcal{C}')\}$ consists of the p' consecutive integers starting from $r+p+1$.

If S_r is non-trivial, then u_ℓ is in S_r and thus $f(c_r) + f(u_\ell) = r + p + 1$. For $j = \ell+1, \ldots, p'$, vertex u_j is in $S_{r+j-\ell}$ (which is non-trivial by the order of the stars in $\mathcal{C}$), and therefore $f(c_{r+j-\ell}) + f(u_j) = (r+j-\ell)+(p+j-\ell+1) = r+p+1+2(j-\ell)$. Now, u_1 is in S_q for $q = p-(r+p'-\ell)+1 = r-p'+\ell$ and hence $f(c_q)+f(u_1) = q+(p+p'-\ell+2) = p+r+2$. For $j = 2, \ldots, \ell-1$, vertex u_j is in S_{q+j-1} (which is also non-trivial), and so $f(c_{q+j-1}) + f(u_j) = (q+j-1)+(p+p'-\ell+j+1) = p+r+2j$.

If S_r is trivial, then u_ℓ is in S_{r+1} and thus $f(c_{r+1}) + f(u_\ell) = (r+1)+p+1 = p+r+2$. For $j = \ell+1, \ldots, p'$, vertex u_j is in $S_{r+1+j-\ell}$ (which is non-trivial by the order of the stars in $\mathcal{C}$), hence $f(c_{r+1+j-\ell})+f(u_j) = (r+1+j-\ell)+(p+j-\ell+1) = r+p+2+2(j-\ell)$. Now, let $q = p-(r+1+p'-\ell)+1 = r-1-p'+\ell$. For $j = 1, \ldots, \ell-1$, vertex u_j is in the non-trivial star S_{q+j-1}, and so $f(c_{q+j-1})+f(u_j) = (q+j-1)+(p+p'-\ell+j+1) = p+r-1+2j$.

This concludes the proof that $L_1 \cup L_2$ is a set of $n-p$ consecutive integers and therefore, by Lemma 2.1, f can be extended to a standard super edge-magic labeling for $\mathcal{C}$. $\square$

3 Caterpillar forests

Let $\mathcal{A}$ be an odd uniform caterpillar forest of type (r,s). Let $p = |\mathcal{A}|$ and let $C_1, \ldots, C_p$ be the caterpillars in $\mathcal{A}$. We will consider the following notation, which is depicted in Figure 3. Let $u_{i1}, \ldots, u_{ir}$ denote the vertices of one part of the caterpillar C_i and $v_{i1}, \ldots, v_{is}$ denote the vertices of the other part, so that if $u_{ij}v_{ik} \in E(C_i)$, then $u_{ij'}v_{ik'} \notin E(C_i)$ for any j' and k' such that (i) $j' > j$ and $k' < k$ or (ii) $j' < j$ and $k' > k$. Note that since caterpillars are planar graphs, there is always such an order of its vertices. Furthermore, let $e_{i(r+s-1)} = u_{i1}v_{i1}$, let $e_{i(r+s-2)}$ be the next edge in the considered order of vertices, and so on, until $e_{i1} = u_{ir}v_{is}$. Thus, the edges of C_i are, in order, $e_{i(r+s-1)}, e_{i(r+s-2)}, \ldots, e_{i1}$.

The next theorem shows that every odd uniform caterpillar forest has an edge-magic labeling. Figure 4 shows an example of such a labeling for a forest with five caterpillars.

Theorem 3.1 *Every odd uniform caterpillar forest has an edge-magic labeling.*

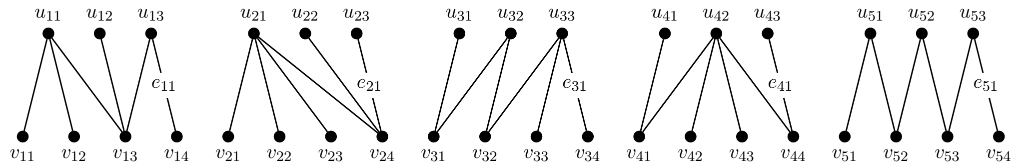

Fig. 3. An odd uniform caterpillar forest of type $(3, 4)$.

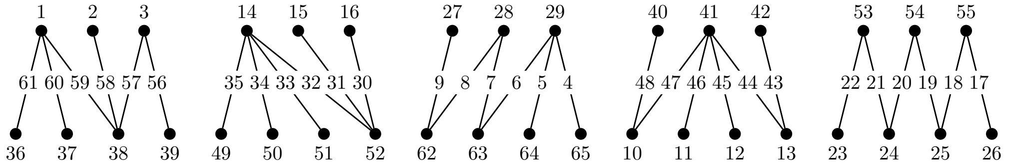

Fig. 4. Example of an edge-magic labeling with magic constant 98 on an odd uniform forest with five caterpillars of type $(3, 4)$.

Proof. Let $\mathcal{A}$ be an odd uniform caterpillar forest of type (r, s) and let $C_1, \ldots, C_p$ be the caterpillars in $\mathcal{A}$. For every $1 \le i \le p$, denote the vertices of C_i by $u_{i1}, \ldots, u_{ir}$, $v_{i1}, \ldots, v_{is}$ and the edges of C_i by $e_{i1}, \ldots, e_{i(r+s-1)}$ as described in the beginning of this section.

For simplicity, we use x for the sum of the number of vertices and the number of edges in a caterpillar C_i, i.e., $x = 2(r+s) - 1$. We define an edge-magic labeling f of $\mathcal{A}$ as follows:

(i) $f(u_{ij}) = j + (i-1)x$ for any $1 \le i \le p$ and $1 \le j \le r$;

(ii) $f(v_{ij}) = (2r + s - 1) + j + \big((\frac{p-1}{2} + i - 1) \bmod p\big)x$ for any $1 \le i \le p$ and $1 \le j \le s$;

(iii) $f(e_{ij}) = r + j + \big((2p - 2i + 1) \bmod p\big)x$ for any $1 \le i \le p$ and $1 \le j \le r + s - 1$.

The intuition behind the above labeling can be better seen in the next diagram, which contains the labeling for the caterpillar forest depicted in Figure 3:

$f(u_{11}) = 1$	$f(u_{12}) = 2$	$f(u_{13}) = 3$	$f(e_{31}) = 4$	$\ldots$	$f(e_{36}) = 9$	$f(v_{41}) = 10$	$\ldots$	$f(v_{44}) = 13$
$f(u_{21}) = 14$	$f(u_{22}) = 15$	$f(u_{23}) = 16$	$f(e_{51}) = 17$	$\ldots$	$f(e_{56}) = 22$	$f(v_{51}) = 23$	$\ldots$	$f(v_{54}) = 26$
$f(u_{31}) = 27$	$f(u_{32}) = 28$	$f(u_{33}) = 29$	$f(e_{21}) = 30$	$\ldots$	$f(e_{26}) = 35$	$f(v_{11}) = 36$	$\ldots$	$f(v_{14}) = 39$
$f(u_{41}) = 40$	$f(u_{42}) = 41$	$f(u_{43}) = 42$	$f(e_{41}) = 43$	$\ldots$	$f(e_{46}) = 48$	$f(v_{21}) = 49$	$\ldots$	$f(v_{24}) = 52$
$f(u_{51}) = 53$	$f(u_{52}) = 54$	$f(u_{53}) = 55$	$f(e_{11}) = 56$	$\ldots$	$f(e_{16}) = 61$	$f(v_{31}) = 62$	$\ldots$	$f(v_{34}) = 65$

First we prove in Claim 3.2 that, for every edge uv, we have $f(u) + f(v) + f(uv) = k$ for some constant $k = k(r, s, p)$ that depends only on r, s and p. Then, in order to finish the proof, we show that all labels given by f are different and lie between 1 and xp.

Claim 3.2 *For every $1 \le i \le p$ and every edge uv in C_i,*

$$f(u) + f(v) + f(uv) = 4r + 2s + \left(\frac{3p-3}{2}\right)x \ .$$

Proof of Claim 3.2. We start by analyzing some particular labeling of a caterpillar. We say that a labeling of C_i is *well-behaved* if all the labels are different and it uses consecutive integers $a+1\dots,a+r$ respectively for the vertices $u_{i1},\dots,u_{ir}$, consecutive integers $b+1,\dots,b+s$ respectively for the vertices $v_{i1},\dots,v_{is}$, and consecutive integers $c+1,\dots,c+(r+s-1)$ for the edges $e_{i1},\dots,e_{i(r+s-1)}$. For any well-behaved labeling f of a caterpillar C_i, a moment of thought shows that $f(u)+f(v)+f(uv)$ is the same for every edge uv of C_i.

Clearly, function f defined above restricted to C_i, which we denote by $f|_{C_i}$, is well-behaved. Thus, as $f(u)+f(uv)+f(v)$ is the same constant for every edge uv of C_i, it is enough to show that, for any $1\leq i\leq p$,

$$\begin{aligned}
f(u_{i1})&+f(v_{i1})+f(u_{i1}v_{i1})\\
&=f(u_{i1})+f(v_{i1})+f(e_{i(r+s-1)})\\
&=1+(i-1)x+(2r+s-1)+1+\left(\left(\frac{p-1}{2}+i-1\right)(\operatorname{mod}p)\right)x\\
&\quad+r+(r+s-1)+((2p-2i+1)(\operatorname{mod}p))x\\
&=4r+2s+\left(\frac{3p-3}{2}\right)x.
\end{aligned}$$

Hence the claim is proved. □

We now proceed with the proof of the theorem. As discussed before, it remains to prove that all labels given by f are different and lie between 1 and xp.

Clearly, all $f(u_{ij})$, $f(v_{ij})$ and $f(e_{ij})$ are positive. It is also clear that $f(u_{ij})\leq xp$, for $1\leq i\leq p$ and $1\leq j\leq r$. Since $\big((\frac{p-1}{2}+i-1)\bmod p\big)x\leq(p-1)x$ and $(2r+s-1)+j\leq x$, in view of (ii) in the definition of f, we have $f(v_{ij})\leq xp$, for $1\leq i\leq p$ and $1\leq j\leq s$. Also, since $\big((2i-1)\bmod p\big)x\leq(p-1)x$ and $r+j\leq 2r+s-1<x$, in view of (iii) in the definition of f, we have $f(e_{ij})\leq xp$, for $1\leq i\leq p$ and $1\leq j\leq r+s-1$.

Since the numbers multiplying x in (i), (ii), and (iii) are always between 0 and $p-1$, we have that all $f(u_{ij})$ are different, and the same happens to all $f(v_{ij})$ and to all $f(e_{ij})$, in their respective ranges. Thus, we only need to prove the following claim.

Claim 3.3 *The following statements hold for every $1\leq i,\,k,\,q\leq p$, every $1\leq j\leq r$, every $1\leq\ell\leq s$, and every $1\leq t\leq r+s-1$:*

(a) $f(v_{k\ell})\neq f(e_{qt})$;

(b) $f(e_{qt})\neq f(u_{ij})$;

(c) $f(u_{ij})\neq f(v_{k\ell})$.

Proof of Claim 3.3. For simplicity, we let $\alpha=\left(\left(\frac{p-1}{2}+k-1\right)\bmod p\right)$ and $\beta=\big((2p-2q+1)\bmod p\big)$. To see that (a) holds, suppose for a contradiction that

$$f(v_{k\ell})=(2r+s-1)+\ell+\alpha x=r+t+\beta x=f(e_{qt})\ .$$

Then, recalling that $x=2r+2s-1$, we have

$$(\alpha+1-\beta)x=r+t+s-\ell\ .$$

But since $1 \leq \ell \leq s$ and $1 \leq t \leq r+s-1$, we have $0 < r+t+s-\ell < x$, and thus the above equality does not hold.

We proceed similarly to prove that (b) holds. Suppose for a contradiction that

$$f(e_{qt}) = r + t + \beta x = j + (i-1)x = f(u_{ij}) \ .$$

Then,

$$\big((i-1) - \beta\big)x = r + t - j \ .$$

Since $1 \leq j \leq r$ and $1 \leq t \leq r+s-1$, we have $0 < r+t-j < x$, and thus the above equality does not hold.

Finally, to see that (c) holds, suppose for a contradiction that

$$f(u_{ij}) = j + (i-1)x = (2r+s-1) + \ell + \alpha x = f(v_{k\ell}) \ .$$

Then,

$$\big((i-1) - \alpha\big)x = 2r + s - 1 + \ell - j \ .$$

But since $1 \leq j \leq r$ and $1 \leq \ell \leq s$, we have $0 < 2r+s-1+\ell-j < x$, and thus the above equality does not hold. □

Therefore, since we proved Claim 3.2, the proof of Theorem 3.1 is complete. □

4 Final remarks

Edge-magic labelings have been studied since they were introduced in 1970, by Kotzig and Rosa. One of the main questions related to this topic, raised by them, is whether any tree is edge-magic. Despite its simplicity, this question is open for almost 50 years. Even the more recent and restrictive question of whether any tree is super edge-magic, posed in 2002 by Enomoto, Lladó, Nakamigawa, and Ringel, is still open. When attacking these questions, it is natural to consider their variants for forests. Indeed, also in 2002, Lee and Kong conjectured that any odd star forest is super edge-magic. In this abstract, we presented a proof for this conjecture for odd symmetric star forests. Searching results for classes of forests less restricted than star forests, we managed to prove that odd uniform forests of catterpillars are edge-magic. We hope that our results can be used somehow to answer Kotzig and Rosa's question for some classes of trees.

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