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Optimal inventory system for deteriorated goods with time-varying demand rate function and advertisement cost

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ABSTRACT

This research work presents a depleted demand inventory model with constant deterioration. The rate of change of demand is assumed to be a time-dependent function. Initial non-zero demand occurs due to advertisements. Advertisement cost is assumed to be constant. Two types of models are considered for two replenishment strategies viz., without shortages and with shortages. This study aims to obtain a suitable policy for replenishment for minimizing the total inventory cost. Four examples about the alterations made in the optimal solutions due to different values of independent parameters used in the models are considered and discussed. Sensitivity analysis is done and numerical illustrations are provided for validating the approach presented.

1. Introduction

To achieve organisational goals, it is desirable to know the norms and specifics of resources such as labour, goods, finances, etc. These aspects enable decision-makers to frame their objectives for getting maximum profit via utilising limited services and resources. One of their objectives is inventory. Literature on inventory theory has been continuously modified to reflect the most practical characteristics of existing inventory systems.

Two important factors play a major role in inventory. Firstly, the supply chain of almost every company faces the problem of holding stocks of obsolete or deteriorating goods. For, some goods gradually lose their potential usefulness over time; some deteriorate directly as they are stored; some experience physical depletion over time through desertion; and some deteriorate through direct spoilage. Therefore, the best inventory management strategy for this class of products is to minimise the loss caused by deterioration. Secondly, the change in demand rates affects their inventory policy.

In this paper, the demand function has two distinct phases: The demand rate is a function of time that initially increases, and after some time, begins to stabilise or becomes constant. This demand rate function is referred to as the ramp-type demand function. When a new consumer product brand enters the market, retailers initially invest heavily in transmitters. And when customers are happy with the item's price and quality and service rendered by the company, sales volume gradually increases over a duration, and then, becomes constant. Therefore, the ramp type-demand function must have a minimum of one broken point between the two-time intervals in which it is non-differentiable [1].

Most of the literature available today is on an inventory model with a ramped demand rate, accounting for backorders, stockouts, and deterioration. A negative, exponentially deteriorating inventory model was presented by Ghare and Schrader [2]. Covert and Philip [3] replaced the negative exponential with two-parameter Weibull distributions for deterioration rate. Misra [4] and Chakrabarty [5] came up with economic order quantity models that concentrated on the above type of commodity. Mandal and Pal [6] introduced ramped demand rate for items with constant expiration rate inventory. Wu and Ouyang [7] gave the most suitable solution for the inventory system introduced by Mandal and Pal [6]. Also, they discussed two possible shortage models with ramp-type demand rate functions [1].

Manna and Chaudhuri [8] developed a model in which the deterioration rate of the ramped demand goods was time-dependent and the finite production rate is proportional to the demand rate. Panda, Senapati, and Basu [9] discussed a single-level inventory model for a seasonal item since the demand rate of seasonal products corresponds to the characteristics of a ramped demand rate. Skouri, Konstantaras, Papachristos, and Ganas [10] developed the model introduced by Covert and Philip, with the ramped demand rate and the Weibull distribution deterioration rate, partial backlog. Skouri, Manna, Konstantaras, and Chaudhuri [11] supplemented the work discussed in

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Ref. [8] by taking into account the demand rate without bottlenecks, which stabilizes after the production stop time; and the demand rate with bottlenecks, which stabilizes after the production stop time. Moreover, they further modified their model by considering the demand rate as a time function [1].

Recently, in contrast to the work above, Ahmed, Al-Khamis, and Benkherouf [12], for the first time, considered a general deterioration rate. Sanni and Chukwu [13] developed an EOQ inventory model for items with a three-parameter Weibull distribution of deterioration [1]. Amutha and Chandrasekaran [14] advanced the EOQ model with quadratic demand and time-dependent carrying cost of inventory. Srivastava and Singh [15] built on the inventory model with linear demand by varying the deterioration rates and partial backlogs.

Singh, Mishra, and Pattanayak [16] presented a model with a linear function of deterioration and a "ramp-type demand function. Uthayakumar and Karruppasamy [17] provided a model with time-varying demand for healthcare industries. Shaikh, A.A., and others [18] described an inventory model with addition facility for deteriorating goods with a ramp-type demand and a trade credit policy. Sharma and Kaushik [19] studied the inventory with ramp-type demand and offers with delayed payments. Palanivelu and Chandrasekaran [20] delivered the replenishment strategy for Giffen goods with time-dependent demand. Supakar, P., and Mahato, S. K. [21], developed the deteriorating inventory models with and without an advanced payment scheme, and they considered the ramp-type function for the demand rate.

It can be observed that until now, most of the researchers in this ramp-type demand rate have completely ignored the initial demand or considered a zero demand for starting the inventory cycle. In some particular situations or products, non-zero initial demand will exist. In a real-life situation, there is an initial demand for newly launched items such as mobile phones, computers, software, automobiles, etc., before they enter into the market because of the advertisement or canvassing. It moves upward with time when these items are launched, and after some time, it begins to stabilise.

In the healthcare industries too, newly introduced medicines have an initial demand before they are available in the market. Nonzero initial demand is also possible due to pre-booking or reserving of that product [22–24]. This paper contributes to that type of demand and takes into consideration the constant deterioration rate and constant holding cost. A ramp-type demand rate function is assumed here. The model is first discussed under the assumption that there is no inventory shortage, and then, extended to account for the shortage [25,26].

Here is how the rest of the article is structured: The assumptions and symbols as well as the description and formulation of Model-I, both without the shortage and with the shortage are presented in Section 2. It also includes some numerical examples to provide further insights. Section 3 presents a sensitivity analysis for the model parameters. The results thereof are presented in Section 4. Section 5 finally draws a conclusion of the study.

2. Materials and methods

This part of the paper presents the assumptions made, symbols used, and the formulation of the model used in this study.

2.1. Assumptions

- Inventory system considers only one item.
- Demand rate is characterized by a time-dependent function of the ramp type.
- Time horizon is infinite.
- There is a constant holding cost.
- There is an unbounded replacement rate.
- There has been no lead time.
- There is a constant deterioration rate, and no replacement is required during the cycle.
- Model I does not allow stock outs.
- Stock outs are allowed in Model II.

2.2. Notations

- A_1- The ordering cost/order
- AC-Advertisement cost/cycle
- $\boldsymbol{D(t)}$-Deterministic demand,
- γ – the constant rate of deterioration
- q_1- The economic order quantity (EOQ)
- h - holding cost per unit item per time
- $I_1(t)$,$I_2(t)$- the inventory at the time duration t;
- OC_1, OC_2 - Ordering cost per cycle
- AC_1, AC_2 – Advertisement cost per cycle
- HC_1, HC_2-carrying cost per cycle
- DC_1, DC_2-Total deterioration cost/cycle;
- TIC_1, TIC_2-Total Inventory Cost/unit time
- T_1, T_2 - Replenishment period of cycle time.

At the beginning of each period, it is supposed that I is the total amount of inventory, and market demand and deterioration induce the inventory level to slowly decrease during the duration $(0, T_1)$ and eventually move to zero at $t = T_1$. Due to the backlog, shortages occur during the time interval $T_1 \le t \le T$. Therefore, the differential equation governing the inventory $I(t)$ has two phases of the cycle time T, which are as follows:

$$\frac{dI(t)}{dt} = \begin{cases} -\gamma I(t) - D(t) 0 \le t < T_1 \\ -D(t) T_1 \le t \le T \end{cases} \tag{1}$$

2.3. Mathematical model

2.3.1. Model 1 - deterministic model without shortage

As mentioned earlier, demand for these types follows a ramp-type demand rate function. Let us consider this function as-

$$\boldsymbol{D(t) = a + bt + b(\mu - t)H(t-\mu)}, \text{where } \mathbf{a} \text{ and b are non} - \text{negative} \tag{2}$$

Where $\boldsymbol{H(t-\mu)}$ is a piecewise continuous 'Unit Heaviside function',

$$H(t-\mu) = \begin{cases} 1, \mu \le t \\ 0, \mu > t \end{cases}$$

The shortages can be avoided if the stock is replenished instantly when stock levels become sufficiently low or reach zero (see Fig. 1). There are no shortages in Model I, but this assumption is relaxed in

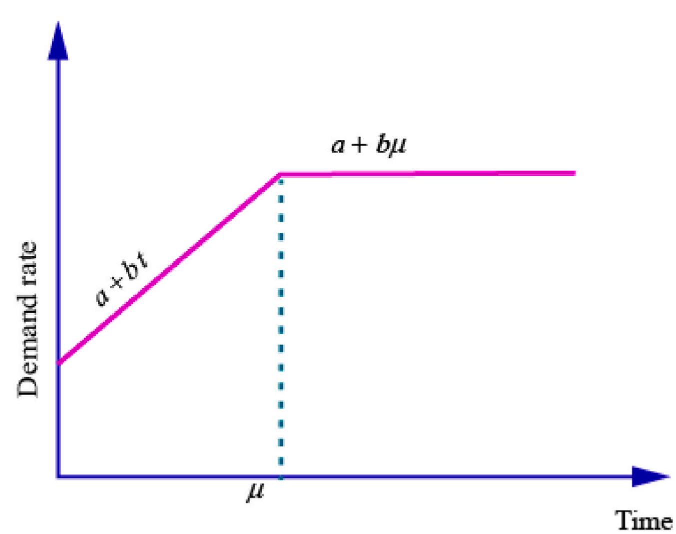

Fig. 1. Ramp Type Demand rate curve.

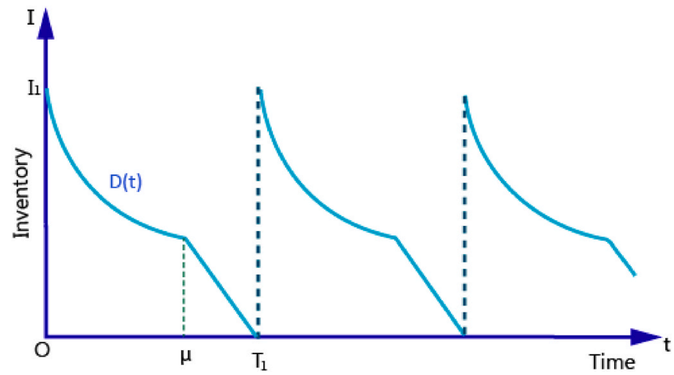

Fig. 2. The inventory model that excludes shortage.

Model- II. A batch of I_1 units is ordered at time 0 to raise the initial inventory level from 0 to I_1. This process is repeated each time the inventory level falls back to 0. This can be seen in Fig. 2.

The governing differential equation (1) when the shortage is not allowed is given by-

$$\frac{dI_1(t)}{dt} + \gamma I_1(t) + D(t) = 0 \tag{3}$$

Substituting the ramp-type rate demand function (2) in (3), the following equations are obtained.

$$\frac{dI_1(t)}{dt} + \gamma I_1(t) + (a + bt) = 0, 0 \le t \le \mu \tag{4}$$

$$\frac{dI_1(t)}{dt} + \gamma I_1(t) + (a + b\mu) = 0, \mu \le t \le T_1 \tag{5}$$

with initial and boundary conditions,

$I_1(0) = q_1, I_1(T_1) = 0$

Total Inventory Cost function TIC_1 is given by-

$$TIC_1 = \frac{(OC_1 + AC_1 + HC_1 + DC_1)}{T_1} \tag{6}$$

on solving equations (2) and (3),

$$I_1(t) = e^{-\gamma t}\left(q_1 + \frac{a}{\gamma} - \frac{b}{\gamma^2}\right) - \frac{a}{\gamma} - \frac{bt}{\gamma} + \frac{b}{\gamma^2}, 0 \le t \le \mu, \tag{7}$$

$$I_1(t) = \left(q_1 + \frac{a}{\gamma} + \frac{b\mu}{\gamma}\right)e^{-\gamma t} - \frac{a}{\gamma} - \frac{b\mu}{\gamma}, \mu \le t \le T_1, \tag{8}$$

By using the boundary conditions, one can obtain the initial order quantity by using the following equation:

$$q_1 = \left(\frac{a}{\gamma} - \frac{b}{\gamma^2}\right)\left(e^{-\gamma T_1} - 1\right) - \frac{be^{\gamma T_1}}{\gamma^2}, 0 \le t \le \mu, \tag{9}$$

$$q_1 = \left(\frac{a + b\mu}{\gamma}\right)\left(e^{\gamma T_1} - 1\right), \mu \le t \le T_1 \tag{10}$$

Ordering cost

$$OC_1 = A_1 \tag{11}$$

Carrying cost

$$HC_1 = \int_0^{T_1} hI_1(t)dt$$

$$= h\left[\int_0^{\mu} e^{-\gamma t}\left(q_1 + \frac{a}{\gamma} - \frac{b}{\gamma^2}\right) - \frac{a}{\gamma} - \frac{bt}{\gamma} + \frac{b}{\gamma^2}dt + \int_{\mu}^{T_1}\left(q_1 + \frac{a}{\gamma} + \frac{b\mu}{\gamma}\right)e^{-\gamma t} - \frac{a}{\gamma} - \frac{b\mu}{\gamma}dt\right]$$

$$\begin{aligned} &= h\left[\left(\frac{b}{\gamma^2} - \frac{a}{\gamma}\right)\left(\mu + \frac{e^{-\gamma\mu}}{\gamma}\right) - \frac{b\mu^2}{\gamma} - \frac{qe^{-\gamma\mu}}{\gamma}\right] \\ &\quad - qh\left[\left(\frac{e^{-\gamma T_1} - e^{-\gamma\mu}}{\gamma}\right) - \left(\frac{a + b\mu}{\gamma}\right)\left(\frac{e^{-\gamma T} - e^{-\gamma\mu}}{\gamma} + T_1 - \mu\right)\right] \end{aligned} \tag{12}$$

Deterioration cost

$$DC_1 = d\left[q_1 - \int_0^{T_1} D(t)dt\right]$$

$$= d\left[q_1 - \int_0^{\mu}(a + bt)dt - \int_{\mu}^{T_1}(a + b\mu)dt\right]$$

$$= d\left\{q_1 - \left(a\mu + \frac{b\mu^2}{2}\right) - (a + b\mu)(T_1 - \mu)\right\} \tag{13}$$

Total inventory cost

$$TIC_1 = \frac{(OC_1 + HC_1 + DC_1)}{T_1}$$

$$\begin{aligned} &= \frac{1}{T_1}\left[A_1 + A_3 + h\left[\left\{\left(\frac{b}{\gamma^2} - \frac{a}{\gamma}\right)\left(\mu + \frac{e^{-\gamma\mu}}{\gamma}\right) - \frac{b\mu^2}{\gamma} - \frac{q_1 e^{-\gamma\mu}}{\gamma}\right\}\right.\right. \\ &\quad \left. - q_1\left(\frac{e^{-\gamma T_1} - e^{-\gamma\mu}}{\gamma}\right) - \left(\frac{a + b\mu}{\gamma}\right)\left(\frac{e^{-\gamma T_1} - e^{-\gamma\mu}}{\gamma} + T_1 - \mu\right)\right] \\ &\quad \left. + d\left\{q_1 - \left(a\mu + \frac{b\mu^2}{2}\right) + (a + b\mu)(\mu - T_1)\right\}\right] \end{aligned} \tag{14}$$

2.3.2. Model II – inventory model with shortage

A company policy restricts the shortages of any of their products. Shortage costs occur when the required quantity of the good (demand) exceeds the available inventory. The company that has a supply shortage with its customers has to bear the shortage cost. This can be interpreted as the loss of goodwill with customers and the resulting unwillingness to do business with the company. That is, it includes the cost of belated revenue, and the additional administrative expenditure involved. A producer experiencing a shortage of materials needed to meet the demand for it has to spend an additional amount to delay the completion of the production process. When there is additional demand over available inventory, the company won't wait for the next shipment to meet this additional demand. This is shown in Fig. 3. The governing equation corresponding to shortages is given below-

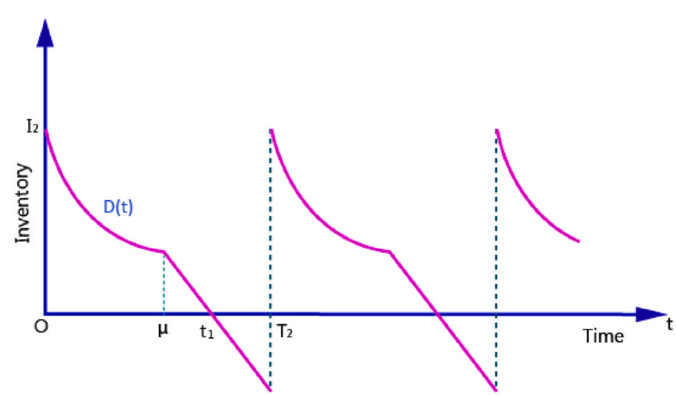

Fig. 3. The inventory model that includes shortage.

$$\frac{dI_2(t)}{dt} = \begin{cases} -(\gamma I(t) + D(t)) 0 \le t \le t_1 \\ -D(t) t_1 < t \le T_2 \end{cases} \tag{15}$$

with the same ramped demand is

$$\frac{dI_2(t)}{dt} + \gamma I_2(t) + (a + bt) = 0, 0 \le t \le \mu \tag{16}$$

$$\frac{dI_2(t)}{dt} + \gamma I_2(t) + (a + b\mu) = 0, \mu \le t \le t_1 \tag{17}$$

$$\frac{dI_2(t)}{dt} + (a + b\mu) = 0, t_1 \le t \le T_2 \tag{18}$$

with initial and boundary conditions

$$I_2(0) = q_2, I_2(t_1) = 0 \tag{19}$$

By solving equations (16)–(18), the following equations are obtained.

$$I_2(t) = q_2 e^{-\gamma t} + \left(\frac{a\gamma - b^2}{\gamma^2}\right) e^{-\gamma t} - \left(\frac{a + bt}{\gamma} - \frac{b}{\gamma^2}\right), 0 \le t \le \mu, \tag{20}$$

$$I_2(t) = q_2 e^{-\gamma t} + \left(\frac{a + b\mu}{\gamma}\right)(e^{-\gamma t} - 1), \mu \le t \le t_1, \tag{21}$$

$$I_2(t) = (t - t_1)(a + b\mu), t_1 \le t \le T_2. \tag{22}$$

And by using equation (19)

$$q_2 = \left(\frac{a + bt_1}{\gamma} - \frac{b}{\gamma^2}\right) e^{\gamma t_1} - \frac{a}{\gamma} + \frac{b}{\gamma^2}, 0 \le t \le \mu, \tag{23}$$

$$q_2 = \frac{1}{\gamma}(a + b\mu)(e^{\gamma t_1} - 1), \mu \le t \le t_1 \tag{24}$$

Carrying cost

$$HC_2 = h \int_0^{t_1} I_2(t) dt$$

$$= h\left\{ \int_0^{\mu} q_2 e^{-\gamma t} + \left(\frac{a + b\mu}{\gamma}\right)(e^{-\gamma t} - 1) dt + \int_{\mu}^{t_1} (t - t_1)(a + b\mu) dt \right\} \tag{25}$$

Deterioration cost

$$DC_2 = d\left\{ q_2 - \int_0^{t_1} D(t) dt \right\}$$

$$= d\left\{ q_2 - \int_0^{\mu} (a + bt) dt - \int_{\mu}^{t_1} (a + b\mu) dt \right\} \tag{26}$$

Shortage Cost

$$SC = s(a + b\mu) \int_{t_1}^{T_2} (t - t_1) dt$$

$$= s\left((a + b\mu) \frac{(T_2 - t_1)^2}{2} \right)$$

Substituting all the above information in the following total inventory cost equation

$$TIC_2 = \frac{1}{T_2}(OC_2 + AC_2 + HC_2 + DC_2 + SC) \tag{27}$$

the following is obtained

$$TIC_2 = \frac{1}{T_2}\left[A_2 + A_3 + h\left\{ \left(q_2 + \frac{a\gamma - b}{\gamma^2}\right)(e^{-\gamma\mu} - 1) - \frac{1}{\gamma}\left(a\mu + \frac{b\mu^2}{2} - \frac{b\mu}{\gamma}\right) - (e^{-\gamma t_1} - e^{-\gamma\mu})\left(q_2 + \frac{a + b\mu}{\gamma^2}\right) - \left(\frac{a + b\mu}{\gamma}\right)(t_1 - \mu) \right\} + d\left\{ q_2 - a\mu - \frac{b\mu^2}{2} - (a + b\mu)(t_1 - \mu) \right\} + s\left((a + b\mu) \frac{(T_2 - t_1)^2}{2} \right) \right] \tag{28}$$

2.4. Methodology

To attain the minimum total cost C_1 , the function TIC_1 given by (12) is differentiated with respect to T_1 and the equation given by the necessary condition $\frac{dTIC_1}{dT_1} = 0$ is solved. This helps in obtaining a transcendental equation in T_1. MATLAB software is used to perform the sensitivity for the inventory function on the minimum total cost TIC_1, and optimal ordering quantity $(q_1{}^*)$ caused by any incremental changes in the parameter.

The following necessary requirement for TIC_2 is applied.

$$\frac{\partial TIC_2}{\partial T_2} = 0, \frac{\partial TIC_2}{\partial t_1} = 0 \tag{29}$$

By differentiating equation (28) with respect to $t_1$andT_2 and applying the necessary condition (29), the simultaneous equations in transcendental function of $t_1$andT_2 are obtained. MATLAB software is used to calculate the effects of the decision variables by variations given to the system parameters of the inventory model function such as *a, b, , h* and *s.* Tables 1 and 2 present the results thereof (see Table 3).

2.4.1. Numerical example

To utilize and to check the above models, four cases are discussed with various parameter values.

2.4.2. Model I- without shortage

Example 1.

Setting the values of parameters in (13) as $a = 150$, $b = 2$, $A_1 = 500/$order, $A_3 = 500$/cycle, $d = 50$/unit, $\mu = 1.5$, $h = 2$ and $\gamma = 0.01$, the optimal point $T_1^* = 2.1257$ is obtained. This optimal point T_1^* is substituted in equations (12) and (7) to obtain the optimal cost $TIC_1^* = 887.41$ and $q_1^* = 327$.

Table 1
The corresponding alterations in other parameters for model I.

Parameter	Values	T_1^*	TIC_1^*	q_1^*
a	120	2.3052	804	285
	135	2.2101	846	307
	150	2.1257	887.41	327
	165	2.0501	927	346
	180	1.9820	965	364
b	1	2.1987	879	336
	2	2.1257	887.41	327
	3	2.0666	894.73	320
	4	2.0178	901.26	314
	5	1.9767	907.16	309
γ	0.005	2.2199	844.23	340
	0.01	2.1257	887.41	327
	0.015	2.0420	929.02	315
	0.02	1.9670	969.21	305
	0.025	1.8993	1008	296
h	1.0	2.5618	708.90	396
	1.5	2.3125	802.41	356
	2.0	2.1257	887.41	327
	2.5	1.9787	965.92	304
	3	1.8592	1039	285

Table 2
The corresponding changes in other parameters for model II.

Parameter	Values	t_1*	T_2^*	TIC_2^*	q_2^*
a	140	1.4613	3.8745	517.63	208
	165	1.3876	3.6491	552.94	226
	180	1.3235	3.4594	586.30	242
	200	1.2671	3.2967	618.01	257
	220	1.2170	3.1551	648.30	271
b	1	1.3059	3.4505	583.89	238
	2	1.3235	3.4594	586.30	242
	3	1.3379	3.4649	588.66	245
	4	1.3498	3.4680	590.98	248
	5	1.3598	3.4693	593.28	251
γ	0.005	1.4198	3.5100	573.73	259
	0.01	1.3235	3.4594	586.30	242
	0.015	1.2395	3.4159	597.44	227
	0.02	1.1656	3.3782	607.38	214
	0.025	1.1000	3.3453	616.31	202
h	1.0	1.7939	3.7091	525.73	329
	1.5	1.5215	3.5618	560.06	278
	2.0	1.3235	3.4594	586.30	242
	2.5	1.1724	3.3840	607.06	214
	3	1.0530	3.3260	623.95	192
s	1.0	1.1941	3.9965	512.84	218
	1.25	1.2653	3.6837	553.21	231
	1.5	1.3235	3.4594	586.30	242
	1.75	1.3723	3.2897	614.06	251
	2.0	1.4139	3.1564	637.77	258

Table 3
The change of direction in T_1^*, TIC_1^*, q_1^* through the parameter.

Change of direction in Parameter	Effects on T_1^*, $\boldsymbol{TIC_1^*}$, q_1^*		
	T_1^*	$\boldsymbol{TIC_1^*}$	q_1^*
↑ in a	↓	↑	↑
↑ in b	↓	↑	↓
↑ in γ	↓	↑	↓
↑ in h	↓	↑	↓

Example 2.

Considering the change in demand at the breakthrough point μ as 2.5, $a = 150$, $b = 2$, $A = 1000$ per order, d = 50 per unit, $h = 2$, and $\gamma = 0.01$, the values $T_1^* = 2.3259$, $\boldsymbol{TIC_1^*} = 880$ and $q_1^* = 359$ are obtained.

2.4.3. Model II- without shortage

Example 3.

With the parameter values $a = 180$, $A_2 = 500$ per order, $A_4 = 500/$ cycle, b = 2, $\gamma = 0.01$,$d = 50$, $s = 1.5$, $h = 2$, and $\mu\ = 1.5$, the optimum values $t_1^* = 1.3235, T_2^* = 3.4594$, $TIC_2^* = 586.30$ and $q_2^* =$ 242 are obtained.

Example 4.

For identical parameter values like that in example 3, except μ, which is considered as 2.5, the model gives the optimum values as t_1^* = 1.5463, T_2^* = 3.7348, $TIC_2^* = 607.31$ and $q_2^* = 283$.

It is observed from examples 1, 2, 3, and 4 that when the breakthrough point μ is greater than the cycle length, a stock-out condition occurs. The inventory cost and the initial order quantity of Model-I are greater than those of Model II to avoid a stock-out situation. The shortage in Model II ushers in the chance of losing the customer's goodwill.

3. Sensitivity analysis

3.1. Sensitivity analysis for model I

A sensitivity analysis is taken for the values as given in Example 1. It is observed that the changes in the values of T_1^*, TIC_1^* andq_1^* bring about corresponding changes in other parameters. This is shown in Table 1.

3.2. Sensitivity analysis for model II

Similarly, a sensitivity analysis is conducted for the values as given in Example 3. It is evident that alterations in the values of $t_1^*, T_2^*, TIC_2^*, q_2^*$, bring corresponding changes in other parameters. This is shown in Table 2.

4. Results and discussion

4.1. Model-I's effect on T_1^*, TIC_1^* and q_1^* caused by a, b, γ, and hare

- An increase in a, b, γ, and h in Model-1 will cause $\boldsymbol{TIC_1^*}$ to be increased.
- It is clear that the variations of $\boldsymbol{TIC_1^*}$ and q_1^* are in the same direction as 'a'.
- T_1^*, $\boldsymbol{TIC_1^*}$ and q_1^* are perceptive towards the alterations in the scale parameter 'b'.
- T_1^*, $\boldsymbol{TIC_1^*}$ and q_1^* are discreetly penetrative with an increase in the parameter γ.
- T_1^* and q_1^* sensibly change with alterations in the parameter h. T_1^* and q_1^* drop when the carrying cost per unit h increases, but $\boldsymbol{TIC_1^*}$ upturns when 'h' rises.

The sensitivity analysis for Model-I is depicted in the following Figs. 4–7.

From Fig. 4, it clear that the cycle length is inversely proportional to the parameter 'a' while the total cost and the economic order quantities are directly proportional.

From Fig. 5, it is seen that the cycle length and economic order quantities decrease as the parameter 'b' increases. However, there is an increase in the total cost.

From Fig. 6, it can be seen that with an increase in the deterioration rate, the cycle length and the economic order quantities decrease. However, the total cost increases.

From Fig. 7, it is evident that an increase in the holding cost will lead to the increase of total cost. The cycle length and the economic order quantities decrease.

The vital role of the shortages on the impacts of the change in parameters a, b, c, and h on t_1^*, T_2^*, TIC_2^*, q_2^*.are analyzed for Model II. This is shown in Table 4.

- The changes in TIC_2^* and q_2^* are in the same direction as changes in all parameters, excluding shape parameter 'a'
- As b increases, there is an increase in the values of t_1^*, T_2^*, TIC_2^*, q_2^*.
- T_2^*, TIC_2^*, and q_2^* are reasonably sensitive with $\boldsymbol{\Upsilon}$.
- t_1^*, T_2^* and q_2^* move in the opposite direction as the carrying cost per unit h rises. TIC_2^* alone does not abide by this condition.
- t_1^*, T_2^*, TIC_2^* and q_2^* are highly perceptive with the changes in the parameters.

In model II, the sensitivity analysis shows that the shortage cost plays a vital role on order quantity and total inventory cost.

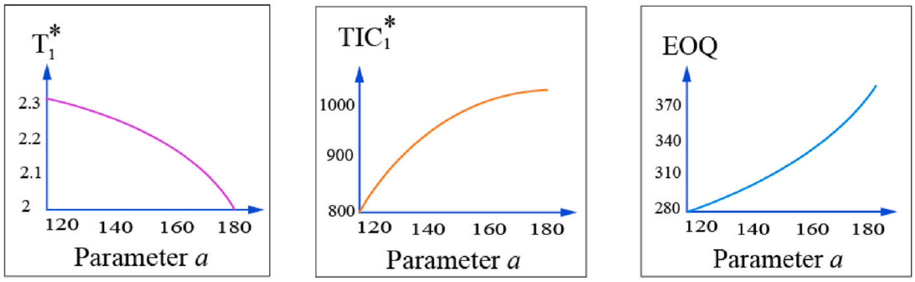

Fig. 4. The influence of parameter ‘a’ on T_1^*, TIC_1^* and q_1^*.

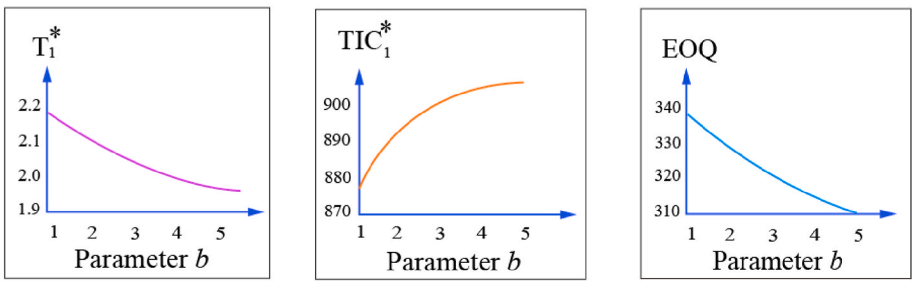

Fig. 5. The influence of parameter ‘b’ with T_1^*, TIC_1^* and q_1^*.

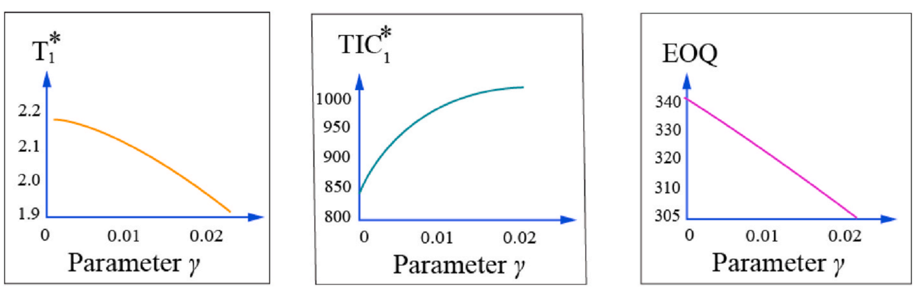

Fig. 6. Influence of parameter ‘Υ’ with T_1^*, TIC_1^* and q_1^*.

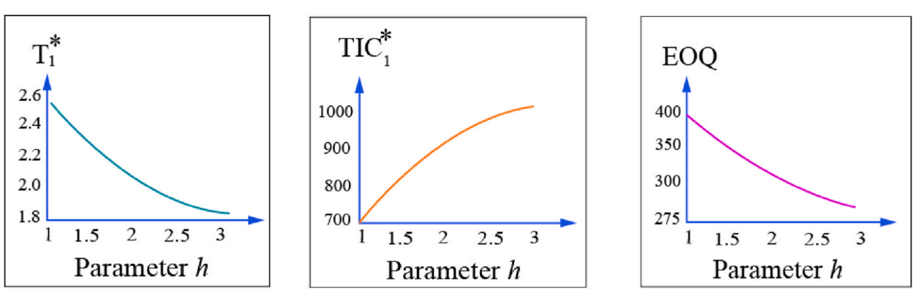

Fig. 7. The influence of parameter ‘h’ with T_1^*, TIC_1^* and q_1^*.

Table 4
Change in the direction of t_1^*, T_2^*, TIC_2^*, q_2^* for different parameters.

Change of direction in Parameter	Effect on t_1^*, T_2^*, TIC_2^*, q_2^*			
	t_1^*	T_2^*	TIC_2^*	q_2^*
↑ in a	↓	↓	↑	↑
↑ in b	↑	↑	↑	↑
↑ in Υ	↓	↓	↑	↓
↑ in h	↓	↓	↑	↓
↑ in s	↑	↓	↑	↑

The graphs in Figs. 8–12 clarify the significance of the sensitivity analysis for.

Model II.

Fig. 8 clarifies that the change of direction is the same for the total cost and the economic order quantity as it is for the direction of parameter ‘a’. But the cycle length reflects the opposite direction.

From Fig. 9, it is observed that the change of the parameter ‘b’ is directly proportional to the economic order quantity, total cost, and cycle length.

From Fig. 12, it is clear that to avoid the stock out situation, the economic order quantities should be increased. When stock-out situation occurs, the increase in the total inventory cost cannot be avoided due to the shortage cost.

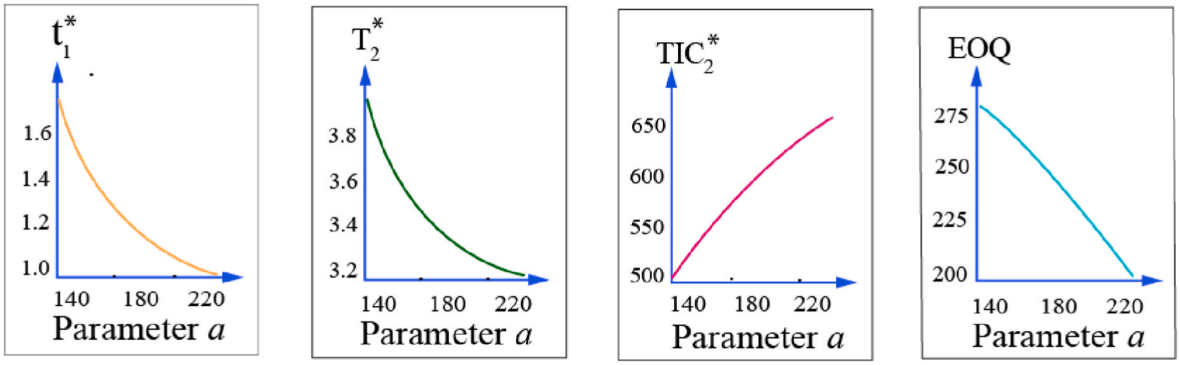

Fig. 8. The influence of parameter ‘a’ on t_1^*, T_2^*, TIC_2^* and q_2^*.

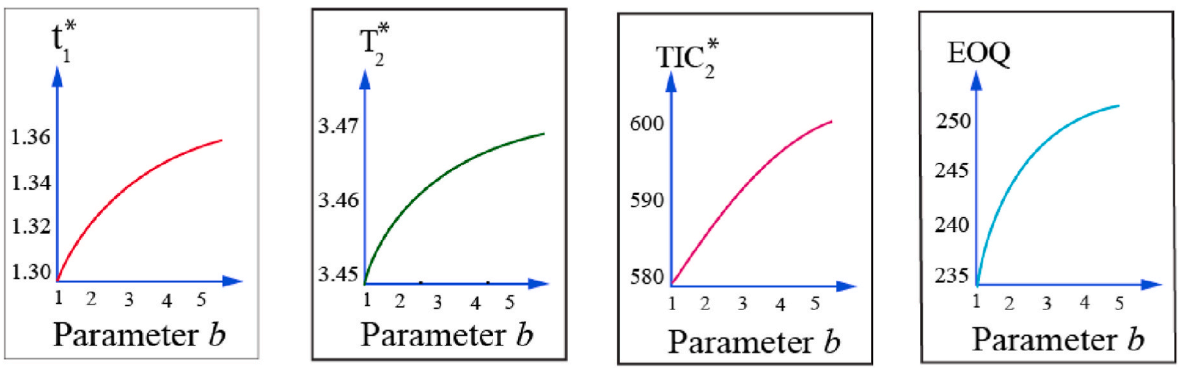

Fig. 9. The influence of parameter ‘b’ on t_1^*, T_2^*, TIC_2^* and q_2^*.

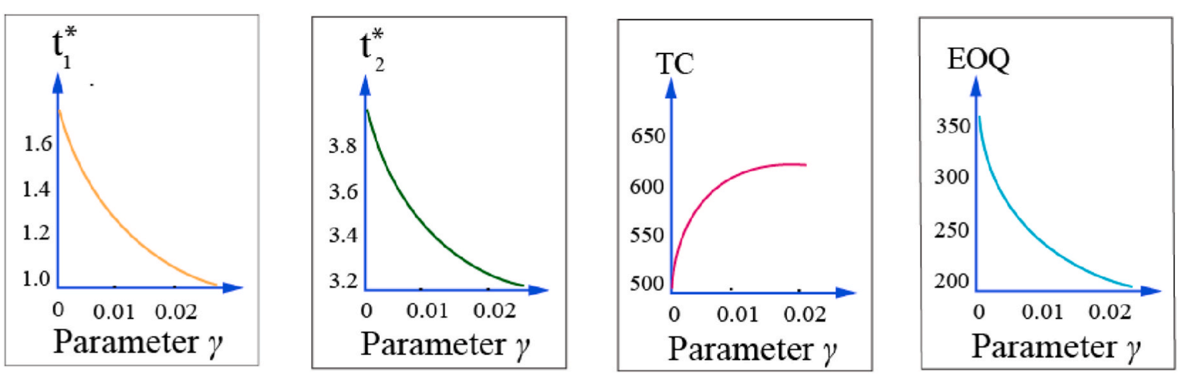

Fig. 10. The Influence of Parameter ‘ϒ’ on t_1^*, T_2^*, TIC_2^* and q_2^* It is clear from Fig. 10 that the deterioration parameter plays a vital role in the inventory cost and the economic order quantities. The cycle length cannot be longer if the deterioration rate increases.

5. Conclusion

This paper has proposed an inflated model for a time-dependent demand of the ramp type with a constant holding cost, constant advertisement cost, and expiration rate, taking into consideration two conditions viz., with and without shortage. The model provides an overall solution for reducing inventory holding costs is provided. So, it is also quite feasible for the latest technologies used for goods inventory and healthcare industry products under time-dependent demand. This approach’s sensitivity was verified by considering the different parameters of the system. This replenishment policy can be used for commodities such as fashion items, healthcare products, and milk products whose expiry rates increase with time. This can help policymakers in successfully and effectively managing practical problems. There are numerous opportunities to expand the future model. Partial backlogs, lost sales, shortage substitution, allowable withholding of payments, nonlinear ramped function rates of demand, fuzzy demand rates, etc. are some factors to be considered.

Credit author statement

Palanivelu Saranya: Conceptualization, Methodology / Study design, Software, Validation, Formal analysis, Resources, Data curation, Writing – original draft, Visualization, Ekambaram Chandrasekaran: Validation, Formal analysis, Investigation, Data curation, Writing – review and editing, Visualization, Supervision, Project administration

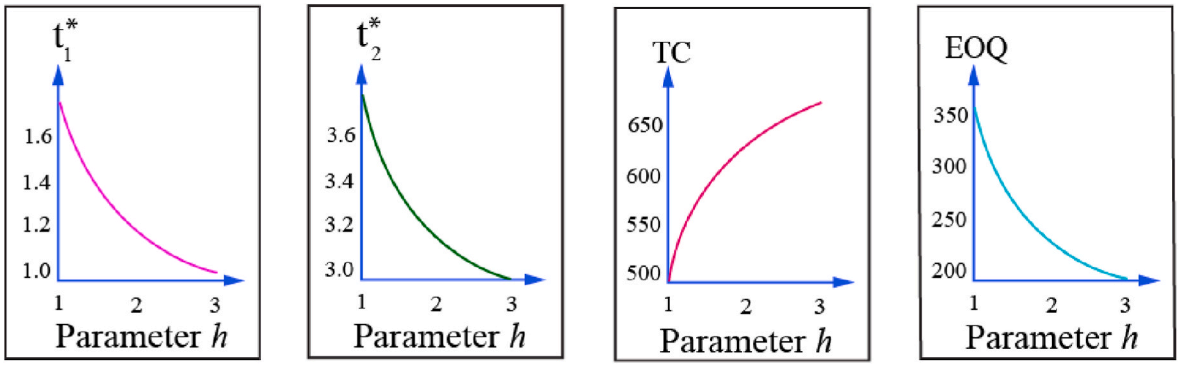

Fig. 11. The Influence of Parameter ‘h’ on t_1^*, T_2^*, TIC_2^* and q_2^* It is observed from Fig. 11, with an increase in the holding cost of the inventory, the total inventory cost increases. So, the goods cannot be held for a long time.

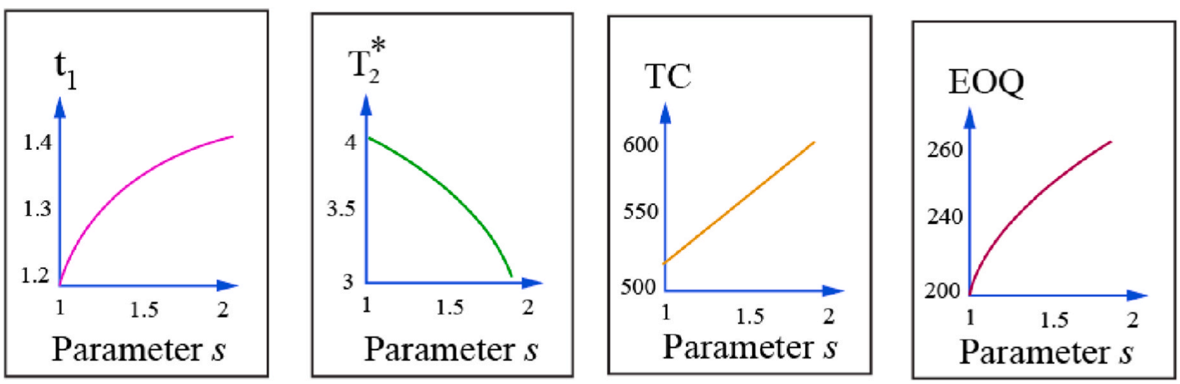

Fig. 12. The Influence of parameter’s’ with t_1^*, T_2^*, TIC_2^* and q_2^*.

Author contributions statement

Palanivelu Saranya and **Ekambaram Chandrasekaran** have contributed equally to the manuscript, **Palanivelu Saranya** and **Ekambaram Chandrasekaran** conducted the experiment(s), **Palanivelu Saranya** and **Ekambaram Chandrasekaran** analyzed the results. All authors have written and reviewed the manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

References

[1] Shi Y, Zhang Z, Zhou F, Shi Y. Optimal ordering policies for a single deteriorating item with ramp-type demand rate under permissible delay in payments. J Oper Res Soc 2019;70(10):1848–68. https://doi.org/10.1080/01605682.2018.1468865.

[2] Ghare PM, Schrader GF. An inventory model for exponentially deteriorating items. J Ind Eng 1963;14(2):238–43.

[3] Covert RP, Philip GC. An EOQ model for items with Weibull distribution deterioration. AIIE Trans 1973;5(4):323–6. https://doi.org/10.1080/05695557308974918.

[4] Misra RB. Optimum production lot size model for a system with deteriorating inventory. Int J Prod Res 1975;13:495-505. https://doi.org/10.1080/00207547508943019.

[5] Chakrabarty T, Giri BC, Chaudhuri KS. An EOQ model for items Weibull distribution deterioration shortages and trended demand—an extension of Philips model. Comput Oper Res 1998;25(7/8):649–57. https://doi.org/10.1016/S0305-0548(97)00081-6.

[6] Mandal B, Pal AK. Order level inventory system with ramp type demand rate for deteriorating items. J Interdiscipl Math 1998;1:49–66. https://doi.org/10.1080/09720502.1998.10700243.

[7] Wu KS, Ouyang LY. A replenishment policy for deteriorating items with ramp type demand rate. Proc. Nat. Sci. Council ROC(A) 2000;24:279–86.

[8] Manna SK, Chaudhuri KS. An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages. Eur J Oper Res 2006;171:557–66. https://doi.org/10.1016/j.ejor.2004.08.041.

[9] Panda S, Senapati S, Basu M. Optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand. Comput Ind Eng 2008;54:301–14. https://doi.org/10.1016/j.cie.2007.07.011.

[10] Skouri K, Konstantaras I, Papachristos S, Ganas I. Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. Eur J Oper Res 2009;192:79–92. https://doi.org/10.1016/j.ejor.2007.09.003.

[11] Skouri K, Konstantaras I, Manna SK, Chaudhuri KS. Inventory models with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages. Ann Oper Res 2011;191:73–95. https://doi.org/10.1007/s10479-011-0984-2.

[12] Ahmed MA, Al-Khamis TA, Benkherouf L. Inventory models with ramp type demand rate, partial backlogging and general deterioration rate. Appl Math Comput 2013;219(9):4288–307. https://doi.org/10.1016/j.amc.2012.09.068.

[13] Sanni SS, Chukwu WIE. An economic order quantity model for Items with three-parameter Weibull distribution deterioration, ramp-type demand and shortages. Appl Math Model 2013;37:9698–706. https://doi.org/10.1016/j.apm.2013.05.017.

[14] Amutha R, Chandrasekaran E. An EOQ model for deteriorating items with quadratic demand and tie dependent holding cost. Int J Emerg Sci Eng 2013;1(5):5–6.

[15] Srivastava S, Singh H. Deterministic inventory model for items with linear demand, variable deterioration and partial backlogging. Int J Inventory Res 2017;4(4):333–49. https://doi.org/10.1504/IJIR.2017.090378.

[16] Singh T, Mishra PJ, Pattanayak H. An EOQ inventory model for deteriorating items with time-dependent deterioration rate, ramp-type demand rate and shortages. Int J Math Oper Res 2018;12(4):423–37. https://doi.org/10.1504/IJMOR.2018.092103.

[17] Uthayakumar R, Karuppasamy SK. An EOQ model for deteriorating items with different types of time-varying demand in healthcare industries. J Anal 2019;27(1):3–18. https://doi.org/10.1007/s41478-018-0100-y.

[18] Shaikh AA, Panda GC, Khan MAA, Mashud AHM, Biswas A. An inventory model for deteriorating items with preservation facility of ramp type demand and trade

credit. Int J Math Oper Res 2020;17(4):514–51. https://doi.org/10.1504/IJMOR.2020.110895.

[19] Sharma A, Kaushik J. Inventory model for deteriorating items with ramp type demand under permissible delay in payment. Int J Procure Manag 2021;14(5): 578–95. https://doi.org/10.1504/IJPM.2021.117292.

[20] Palanivelu S, Chandrasekaran E. A deterministic replenishment policy for constant deteriorating giffen goods with time-dependent demand. In: Proceedings of 2nd international conference on mathematical modeling and computational science. Singapore: Springer; 2021. p. 29–30. https://doi.org/10.1007/978-981-19-0182-9_28.

[21] Supakar P, Mahato SK. An EPQ model with time proportion deterioration and ramp type demand under different payment schemes with fuzzy uncertainties. Int J Syst Sci: Operations & Logistics 2022;9(1):96–110. https://doi.org/10.1080/23302674.2020.1826594.

[22] Fattahi Parviz, Hajipour Vahid, Hajiloo Sara. A multi-objective parameter-tuned soft computing based algorithm to optimize competitive congested location-pricing problem within multi-type service. Array 2021;10:100062. https://doi.org/10.1016/j.array.2021.100062. ISSN 2590-0056.

[23] Zhao Jing, Wang Lisha. Pricing and retail service decisions in fuzzy uncertainty environments. Appl Math Comput 2015;250. https://doi.org/10.1016/j.amc.2014.11.005. ISSN 0096-3003.

[24] Pinto Rui, Gil Gonçalves. Application of artificial immune systems in advanced manufacturing. Array 2022;15:100238. https://doi.org/10.1016/j.array.2022.100238. ISSN 2590-0056.

[25] Abosaif Aghabi N, Hamza Haitham S. Quality of service-aware service selection algorithms for the internet of things environment: a review paper. Array 2020;8. https://doi.org/10.1016/j.array.2020.100041. ISSN 2590-0056.

[26] Zambrano Valentina, Mueller-Roemer Johannes, Sandberg Michael, Prasad Talasila, Zanin Davide, , Peter Gorm Larsen, Loeschner Elke, Thronicke Wolfgang, Pietraroia Dario, Landolfi Giuseppe, Fontana Alessandro, Laspalas Manuel, Antony Jibinraj, Poser Valerie, Kiss Tamas, Bergweiler Simon, Pena Serna Sebastian, Izquierdo Salvador, Viejo Ismael, Juan Asier, Serrano Francisco, Stork André. Industrial digitalization in the industry 4.0 era: classification, reuse and authoring of digital models on Digital Twin platforms. Array 2022;14:100176. https://doi.org/10.1016/j.array.2022.100176. ISSN 2590-0056.