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Object-Oriented Structure Refinement – A Graph Transformational Approach

Xiaojian Liu[†,‡], Zhiming Liu[‡] and Liang Zhao[‡]

[†] *Computer School, Northwest Polytechnical University, Xi'an, China*
[‡] *UNU-IIST, Macao SAR, China*
Emails: liuxiaojian@nwpu.edu.cn, {z.liu, liang}@iist.unu.edu

Abstract

In UML, the general structure of objects, their attributes and relations are modeled as a *class graph*, and an instance of a class graph is defined as an *object graph*. The class graph of a system determines the general properties of objects and how objects collaborate in realizing a use case. In this paper, we define class graphs and their object graphs as *directed labelled graphs*, and investigate in a graph theoretical approach what changes in the object structure maintain the capability of providing services. We define the general notion of *structure refinements*. A structure refinement is a transformation from one graph to another that preserves the *capability* of providing services, that is the resulting class graph should be able to provide at least as well as the original graph. We give a small set of structure refinement rules that is proved to be sound and complete for a kind of structure refinement.

Keywords: Object systems, class graphs, object graphs, labelled graphs, graph transformations, refinement

1 Introduction

An object program can be represented in the form of $Cdecls \bullet Main$, where the *class declaration section* $Cdecls$ declares a sequences of classes with their attributes, methods, and inheritance relations; and $Main$ declares a *main class* and *main method* [5]. The main class declares, as its attributes, the global variables whose types are either primitive built-in data types or classes declared in $Cdecls$. The main method implements an application by calling some *public* methods of *public classes* in the declaration section. A class declaration section can be depicted by a UML class diagram [6]. Such a class diagram also contains the methods and their bodies. Otherwise, sequence diagrams and state diagrams are needed.

Different class declaration sections $Cdecls_i$, $i = 1, 2$, may support the same main class. Formally, if for any $Main$, $Cdecls_2 \bullet Main$ behaves "at least as well as" (or *refines*) $Cdecls_1 \bullet Main$, we call $Cdecls_2$ a *structure refinement* of $Cdecls_1$ [5]. Here, we are only concerned with functional correctness.

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Structure refinement is important for an object oriented design to be maintainable, reusable and cohesive. In this paper, we propose a calculus of structure refinement by using graph transformations. We define a class declaration section as a directed labelled graph, called a *class graph*. The nodes are labelled with names of classes or primitive data types, such as *Int*, *Char* and *String*, and edges are labelled with attribute names (also represents UML associations) or symbol $\triangleright$ denoting the direct *inheritance relation*.

Given a class graph and a set of *public variables* declared in a main class, we define a *system state* as an *object graph*, a directed labelled graph with a *root node* ε. The root represents the reference of the instance of the main class [1]. Any node different from the root is labelled by a value which is a pair (v, T), where v is either an *object reference* if T is a class, or an element of T otherwise. For each public variable $x : T$, there is an edge labelled by x from the root to a node whose type is T. There is an edge from (v_1, T_1) to (v_2, T_2) labelled with a if and only if T_1 is a class and a is an edge from T_1 to T_2 in the class graph. Therefore, a class graph CG defines a set of object graphs for each set of public variables. An execution of a command c defined under CG from a given object graph (system state) OG will change OG to another object graph OG'.

Then for a structure refinement ρ from CG to CG_1, we can derive

(i) a transformation ρ_o from an object graph OG of CG to an object graph OG_1 of CG_1, and

(ii) a transformation ρ_c from commands defined under CG to commands defined under CG_1, such that

(iii) the diagram in Figure 1 commutes.

We will prove a small set of structure refinement rules, that is complete for a restricted definition of structure refinement.

$$\begin{tikzcd}
\mathrm{CG} \arrow[r, "i"] \arrow[d, "\rho"'] & \mathrm{OG} \arrow[r, "c"] \arrow[d, "\rho_o"'] & \mathrm{OG}' \arrow[d, "\rho_o"'] \\
\mathrm{CG}_1 \arrow[r, "i"] & \mathrm{OG}_1 \arrow[r, "\rho_c(c)"] & \mathrm{OG}_1'
\end{tikzcd}$$

Fig. 1. Structure Refinement

There exists a big body of research on formal semantics of object-oriented programs, e.g. [3,7,10,4,5]. There is a common feeling, though rarely stated in literatures, that *one does not (at least feel very difficult to) understand semantics of object-oriented programs defined by another*. Except for a restricted class of static properties, the different semantic definitions do not seem to be effective for analysis and verification of object-oriented programs. Verification of refinement of object-oriented specifications and designs are even harder. The operational semantics (and its denotational counterpart) proposed in this paper based on class graphs and object graphs is promising to improve this situation.

[1] An object system only has one instance of the main class.

An algebraic calculus of class structures is given in [7] based on predicate transformers. Structure refinement is studied in [5] in a denotational semantic model. The structure refinement rules in this paper agree all the rules in rCOS, except for those that allow the removal of redundant classes and attributes and for compressing attributes. However, purpose of the graph calculus presented in this paper is to improve the understanding of structure refinement and for future development of tool support to graph transformations.

The work [9,12] handles class and interface refinements. However, there the focus is substitutability of individual classes in a class structure. Our work, however, investigates the refinement of class models as a whole and supports structure design at different stages of the system development. In [8], a notion of equivalence between class graphs is proposed. There, the notion is defined according to properties of objects, instead of functionalities and object behavior. Thus, it does not address functional refinement.

The use of object graphs is influenced by notation of graphs for pointer structures in [1], and the idea of using paths of a graph comes from the trace model of pointers and objects with pointers [2].

Section 2 shows how a class declaration section can be defined as a directed labelled graph. In Section 3 we define object graphs for class graphs to represent system states. We also propose an informal, yet precise and obviously formalizable, operational semantics of programming commands based on class graphs and object graphs. In Section 4, we define structure refinements between class graphs and their derived relations between object graphs. Section 5 establishes a set of class graph refinement rules and prove that they are sound refinements. We also show that this set of rules are complete with respect to a restricted notion of structure refinement.

2 Class Graphs

A class declaration section can be represented as a directed and labelled graph. We use names of *data types* and *classes* to label the nodes and names of attributes and an annotation of inheritance to label the edges. For this, we assume an infinite set $\mathcal{CN}$ of class names, an infinite set $\mathcal{T}$ of names of primitive data types, an infinite set $\mathcal{A}$ of attribute names, and a single name $\rhd$ to annotate the inheritance relation. Let $\mathcal{N}$ be union of *types* in $\mathcal{CN}$ and $\mathcal{T}$.

Definition 2.1 A **class graph** is a directed labelled graph $\Gamma = \langle N, A, E\rangle$, where

- $N \subseteq \mathcal{N}$: is the set of *nodes* representing *types*, including both classes and data types
- $A \subseteq \mathcal{A}$: a set of *attributes* names
- $E \subseteq N \times (A \cup \{\rhd\}) \times N$: are the *edges* of the graph. An edge $(C, a, D) \in E$ for $a \in A$ means that class C has an attribute a of type D, and $(C, \rhd, D) \in E$ means that C is a direct subclass of D.

We use $\preceq$ to denote the reflexive and transitive closure of the direct subclass relation $\rhd$, and call D a superclass of C and C a subclass of D if $C \preceq D$ holds. Obviously, not all analysis class graphs as defined above correspond *well-formed* class declarations.

Definition 2.2 A class graph $\Gamma = \langle N, A, E\rangle$ is **well-formed** if it satisfies the following conditions:

(i) Data types are leaves of the graph: if $(C, a, D) \in E$, then $C \notin \mathcal{T}$

(ii) The inheritance relation is only defined among classes: if $(C, \rhd, D)$, then $C, D \in \mathcal{CN}$.

(iii) The inheritance relation is required to satisfy the following conditions
   (a) There is at most one $\rhd$ edge from each class, that is we assume no multiple inheritance.
   (b) There is no cycle formed by $\rhd$ edges.
   (c) No attributes of the superclass can be redeclared in the subclasses: if $C_1 \preceq C$, $C_1 \neq C$ and $(C, a, D) \in E$ then $(C_1, a, C_2) \notin E$ for any a and D and C_2.

We simply call a well-formed class graph a class graph if there is no confusion. Notice that a class graph has three disjoint sets of edges.

- *data attributes*: are those edges (C, x, T) such that T is a primitive type.
- *relational attributes*: are those edges (C, a, D) such that D is a class. We can also call a an association between class C and D.
- *inheritances*: are the edges $(C, \rhd, D)$ for all C and D in the graph.

We do not consider multiplicities of an association, as that will only introduce multi-objects (container objects) in the object graphs. Neither do we distinguish aggregation from general associations.

For a class node C of Γ, we define the following two sets.

- $attr(C) \stackrel{def}{=} \{a \in A \mid \exists D \in N \cdot (C, a, D) \in E\}$ denotes all labels of the outgoing edges from C, i.e. the set of the attributes directly defined in class C.
- $Attr(C) \stackrel{def}{=} \{a \mid \exists D \cdot C \preceq D \wedge a \in attr(D)\}$ is the set of labels of the outgoing edges from C and all its superclasses.

For a labelled graph Γ, we abuse the OO notation $v_0.a_0.\cdots.a_{k-1}$ to denote a *path* $[(v_0, a_0, v_1),\ (v_1, a_1, v_2), \cdots, (v_{k-1}, a_{k-1}, v_k)]$; and use $dest(v_0.a_0.\cdots.a_{k-1})$ to denote the *destination* v_k of the path. And for two $p_1 = C.\alpha$ and $p_2 = D.\beta$ such that $D = dest(C.\alpha)$, the concatenation $p_1.p_2$ of p_1 and p_2 is $C.\alpha.\beta$.

Only attributes determine the navigation paths in an object graph. However, the inheritance relation defines the attributes that one class inherits from the others. A sequence $\alpha = \{(v_i, a_i, v_{i+1}) \mid v_i, v_{i+1} \in N, a_i \in A, i = 0, \ldots, k\}$ of "edges" is called a *navigation path* of class graph $\Gamma = \langle N, A, E\rangle$ if for all $i = 0, \ldots, k$

$$\exists u_i, u_{i+1} \cdot (v_i \preceq u_i \wedge v_{i+1} \preceq u_{i+1} \wedge (u_i, a_i, u_{i+1}) \in E)$$

Notice that the above path is a navigation path iff for all $i = 0, \ldots, k$, $a_i \in Attr(v_i)$ and v_{i+1} is a subtype of the type declared for a_i in Γ.

Example 2.3 The left part of Figure 2 is an analysis class graph which represents the UML class diagram illustrated in the right part of Figure 2.

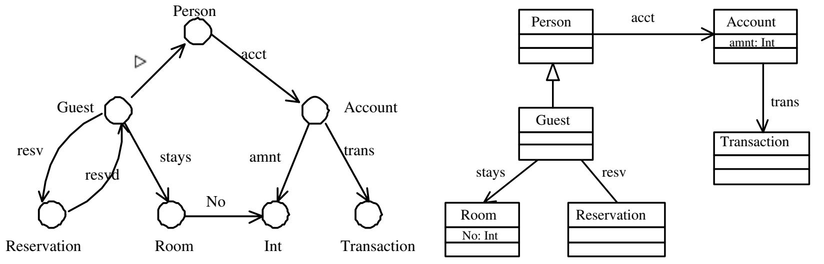


Fig. 2. An example

3 Object Graphs and Execution of Commands

A class graph declares a family of types and itself can be understood as a "complex" type whose elements are object graphs. For a class graph Γ, we use N_Γ to denote its nodes, E_Γ the edges and A_Γ the attribute names.

In general, we represent a *value* v of a type T as a pair (r, T), where r is an element of T if T is a primitive type and a *reference* otherwise. We assume an infinite set REF of references, including a special value $null$. For a class graph, we use V_Γ to denote the set of all values of types declared in Γ.

3.1 Object graphs as program states

An object graph is defined for a class graph Γ and a given finite set X of *global variables* $x_1 : T_1, \ldots, x_n : T_n$ such that all the types T_i are elements of the class graph. The variables are assumed to be used in the main method.

Definition 3.1 Let Γ be a class graph and X a set of global variable declarations. An **object graph** of Γ with variables X, is a rooted, directed and labelled graph $\Sigma = \langle L, N, E, \varepsilon \rangle$, where

- $L = X \cup A_\Gamma$ is the set of names that will be used to label the edges.
- A node in N is either the **root node** ε or an element in V_Γ, that represents a data value or a reference with its type.
- $E \subseteq N \times L \times N$ are the edges of Σ.
- The root node ε has no coming-in edges, and other node must have at least one coming-in edge.
- For each node v in N, there is at least one path p from the root with $dest(p) = v$.

An object graph Σ of Γ is *complete and correctly typed* if every attributes of a class in Γ is assigned a value with its correct type. The type system is defined by the navigation paths of the class graph.

Definition 3.2 An object graph Σ of a class graph Γ is **complete and correctly typed** (CCT) with respect to Γ if the following conditions hold

(i) **Type correctness of nodes:** if $(r, C) \in N$, then C must be node in Γ.

(ii) **Type correctness of attributes:** for any edge $e \in E$

 (a) if $e = (\varepsilon, x, (r_2, D))$ for $x : T \in X$, then $D \preceq T$,

 (b) if $e = ((r_1, C), a, (r_2, D))$ then (C, a, D) is a navigation path of Γ.

(iii) **Completeness:** For each node $v \in N$,

 (a) if $v = \varepsilon$, it has one and only one outgoing edge for each $x : T \in X$,

 (b) otherwise if $v = (r, T)$, then there exists an edge $((r, T), a, (r_1, T_1))$ in Σ iff T is a class name in Γ, $r \neq null$ and $a \in Attr(T)$.

We use $\mathcal{M}_X(\Gamma)$ to denote all the CCT object graphs of Γ for variables X, and simply call a CCT object graph an object graph and omit the subscript X, when there is no confusion. Figure 3 is an example of an object graph of the class graph of Figure 2, with $X = \{y_1 : Room,\ y_2 : Guest,\ y_3 : Reservation\}$ as its global variables.

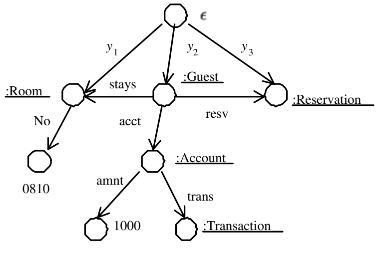


Fig. 3. An object graph

For an edge (C, a, D) of the class graph Γ, $dtype(C.a)$ (or simply $dtype(a)$ when there is no confusion) denotes the type D, called the *declared type* of a in Γ. For an edge $((r_1, C), a, (r_2, D_1))$ in the object graph Σ, $type(r_1.a)$ denotes D_1, called the *current type* of attribute a of object r_1 in the object graph Σ. Also, $type(r, C)$ denotes the *current type* C of the node (r, C) in the object graph. Definition 3.2 ensures that each object node in the object graph represents an object of a class declared in the class graph, and the current type of each attribute is a subtype of its declared type in the class graph.

The root object, representing the instance of the main class, can access an object or a property of an object via different navigation paths. We can thus use the set of all paths to a node to represent the object that the node intends to model. In Figure 3, for example, $\{\epsilon.y_1, \epsilon.y_2.stays\}$ represents the `:Room` instance, and $\epsilon.y_2.acct.amnt$ the value 1000.

3.2 Execution of a Command

Object graphs of a class graph can be seen as *states* of the object system. An execution of the object program is an execution of the main method command from an initial state to a final state, if terminates. The execution first calls a method $o.m(x; y)\{c\}$ of an object o of a class in the class graph Γ, where x is input parameter,

y the output parameter, and c the body command. This object o is a node in the initial state Σ_0, and x is also a node of Σ_0 though it can be a data value.

The execution of $o.m(x; y)$ changes the object graph Σ_0 according to the semantics of the body c of the method $m()$. A syntax and a formal denotational semantics of a OO language is defined in rCOS[5]. However, here we give it an informal operational interpretation:

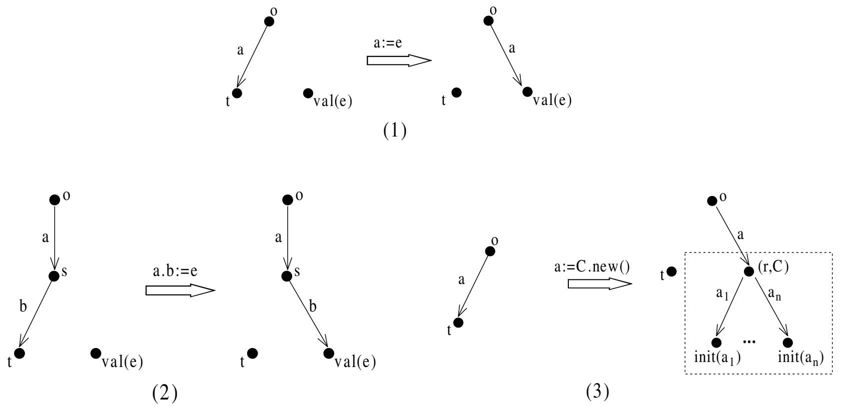


Fig. 4. The change of object graphs

(i) If c is a simple assignment $a := e$, where a is an attribute of o, then the execution changes the edge (o, a, t) in Σ_0 to the edge $(o, a, val(e))$, where $val(e)$ is the value of expression e, which can be an object or a data value. This is shown in Figure 4(1).

(ii) If c is a complex assignment $a.b := e$, where a is an attribute of o of a class type and b is an attribute of $o.a$, then the execution changes the path $(o, a.b, t)$ to the path $(o, a.b, val(e))$ as illustrated in Figure 4(2). The general attribute assignment $a.b_1.\cdots.b_k := e$ is performed by induction.

(iii) If c is an object creation $a := C.new()$, where a is an attribute of o and C is a class node of Γ and a subclass of $dtype(a)$, the execution changes the edge (o, a, t) in Σ_0 to a newly created rooted graph with (r, C) as the root and the initial values of the attributes of C (that we would like to ignore here) as nodes. This is shown in Figure 4(3).

(iv) The meaning of compositions of commands can be defined inductively.

If we want to consider type casting, we just need to extend the node from a pair to a triple (r, C, D) to represent the reference, the current type and the casted type.

Notice that the execution of a command may cause an object in the object graph unreachable from the root. In this case, the node of object will be deleted from the object graph, just like what garbage collection does.

For an object graph Σ_0 of Γ and an object node o in it, the set of all possible object graphs caused by the execution of $o.m()$ for a method $m()\{c\}$ defined in the type of o is denoted by $[\![o.m()]\!]_\Gamma(\Sigma_0)$.

We believe that it is easy to prove that the above operational semantics is consistent with the rCOS semantics defined in [5], though we have not done the proof. We allow nondeterministic choice and specification commands too. Then the execution of a method maps an object graph to a set of object graphs. Thus, commands refinement, as well as equivalence, can be defined for methods of class.

Definition 3.3 [Method refinement] Let

(i) Γ be a class graph,

(ii) X be a set of variables with types in Γ,

(iii) $m(u:T_1;v:T_2)\{c_1\}$ and $m(u:T_1;v:T_2)\{c_2\}$ be two methods defined in a class C for some $x:C \in X$.

We say method $m(u:T_1;v:T_2)\{c_2\}$ is a **refinement** of method $m(u:T_1;v:T_2)\{c_1\}$, denoted by $m(u:T_1;v:T_2)\{c_1\} \sqsubseteq m(u:T_1;v:T_2)\{c_2\}$, if for any object diagram Σ of Γ, any node u of Σ such that $type(u) \preceq T_1$, the set $[\![x.m()\{c_2\}]\!]_\Gamma(\Sigma)$ is a subset of $[\![x.m()\{c_1\}]\!]_\Gamma(\Sigma)$, where x is the object that x points to in the object graph Σ.

However, to support design by stepwised refinements, we must be able to change the structure of the class graph, though the public class names are not changed.

Definition 3.4 Let $\mathcal{PC}$ be a set of class names, Γ_1 and Γ_2 be two class graphs both containing $\mathcal{PC}$ as some of their nodes. Γ_2 is a **structure refinement** of Γ_1, denoted as $\Gamma_1 \sqsubseteq \Gamma_2$, if for any $C \in \mathcal{PC}$ and a variable $x:C$, there exists a mapping ρ_o from $\mathcal{M}(\Gamma_1)_{\{x:C\}}$ to $\mathcal{M}(\Gamma_2)_{\{x:C\}}$ such that for any method $m(u:T_1;v:T_2)\{c_1\}$ defined in class C of Γ_1, we can define a correspondence method $m(u:T_1;v:T_2)\{c_2\}$ in class C of Γ_2 and

$$[\![x.m()\{c_2\}]\!]_{\Gamma_2}(\rho_o(\Sigma)) \subseteq \rho_o([\![x.m()\{c_1\}]\!]_{\Gamma_1}(\Sigma))$$

4 Structure Transformation

4.1 Structure transformation

We now show that some refinement can be realized by certain class graph transformations.

Definition 4.1 Let Γ_1 and Γ_2 be class graphs, f a subset of the class nodes of Γ_1. A mapping ρ from Γ_1 to Γ_2 is a f**-framed transformation**, denoted by $\rho_{[f]}$, if the following conditions hold

(i) the restriction $\rho|_{N_1}$ of ρ to the nodes N_1 of Γ_1 maps each class name in the frame f to itself, that is $\rho(C) = C \in N_2$ for each $C \in f$.

(ii) the restriction $\rho|_{E_1}$ to the edges E_1 of Γ_1 maps each relational or inheritance edge $(C,a,D) \in E_1$ to a nonempty path from $\rho|_{N_1}(C)$ to $\rho|_{N_1}(D)$ in Γ_2; and maps each data attribute (C,a,T) to a nonempty set of paths of Γ_2 starting from $\rho|_{N_1}(C)$ with destinations that are data types in $\mathcal{T}$.

We decompose the restriction $\rho|_{E_1}$ into two restrictions $\rho|_r$ and $\rho|_d$ to the relational (including inheritance) edges and data attributes, respectively.

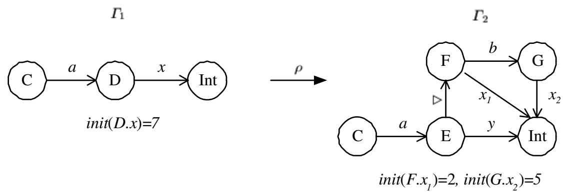


Fig. 5. An example of structure transformation

Obviously, not all framed structure transformations defined above are structure refinements. In what follows, we give a number of sufficient conditions for such a transformation to be a structure refinement.

Proposition 4.2 *A framed transformation* $\rho_{[f]}$ *from* Γ_1 *to* Γ_2 *is a* structure refinement *if it satisfies the following properties:*

(i) *If* $(C, \rhd, D)$ *is an inheritance edge, then* $\rho|_r(C, \rhd, D)$ *is a path containing only inherence edges.*

(ii) *If* (C, a, D) *is a relational edge, then the path* $\rho|_r(C, a, D)$ *contains at least one relational attribute.*

(iii) *For two different relational or inheritance edges* (C_1, a_1, D_1) *and* (C_2, a_2, D_2)*,* $\rho|_r(C_1, a_1, D_1)$ *is not a suffix of* $\rho|_r(C_2, a_2, D_2)$*.*

(iv) *For any data edges* (C_1, a_1, T_1) *and* (C_2, a_2, T_2)*, a path* p_1 *in* $\rho|_d(C_1, a_1, T_1)$ *and a path* p_2 *in* $\rho|_d(C_2, a_2, T_2)$*,* p_1 *is not a suffix of* p_2 *unless* $C_1 = C_2$*,* $a_1 = a_2$ *and* $p_1 = p_2$ *(obviously* $T_1 = T_2$ *too).*

(v) *For a data edge* (C, a, T)*, let* $\rho|_d(C, a, T) = \{C_1.\beta_1.a_1, \ldots, C_1.\beta_n.a_n\}$ *such that* $dest(C_1.\beta_i.a_i) = T_i$ *for* $1 \le i \le n$*, there exists a surjective operation* g *of the type* $T_1 \times \ldots \times T_n \to T$ *such that the initial value of* $C.a$ *can be calculated from those of the target attributes:* $init(C.a) = g(init(D_1.a_1), \ldots, init(D_n.a_n))$*, where* $D_i = dest(C_1.\beta_i)$*.*

A structure transformation from Γ_1 to Γ_2 in fact defines an *implementation* of the classes, their attributes and associations in Γ_1 by those of Γ_2. A single relational attribute (edge) in Γ_1 can be realized by a path, and an data attribute can be a set of paths in Γ_2. These are captured by conditions (ii)-(v).

The validity of the proposition is to be established in the next section in two steps:

- Soundness: prove a small set of rules that are structure refinements.
- Completeness: prove that any structure transformation that satisfies the conditions in Proposition 4.2 can be obtained by sequentially applying the proposed refinement rules.

Example 4.3 Figure 5 illustrates a structure transformation $\rho_{[f]}$, where $f = \{C\}$, $\rho|_{N_1}(C) = C$, $\rho|_{N_1}(D) = F$, $\rho|_r(C, a, D) = C.a.\rhd$, $\rho|_d(D, x, Int) = \{F.x_1, F.b.x_2\}$, and the addition operation on integers preserves the initial values of attributes:
$init(D.x) = init(F.x_1) + init(G.x_2)$.

4.2 Structure relation between object graphs

Given variables X, a structure transformation ρ from class graph Γ_1 to class graph Γ_2 determines a derived relation ρ_o between $\mathcal{M}_X(\Gamma_1)$ and $\mathcal{M}_X(\Gamma_2)$.

Definition 4.4 Let ρ be a structure transformation from class graph Γ_1 to class graph Γ_2 with a frame $\{C_1, \ldots, C_n\}$ satisfying the conditions in **Proposition** 4.2, and $X = \{x_1 : C_1, \ldots, x_n : C_n\}$ be variables. Σ_1 and Σ_2 be object graphs with variables X for Γ_1 and Γ_2 respectively. The **derived structure relation** of ρ, denoted by $\rho_o(\Sigma_1, \Sigma_2)$, is a relation between object graphs $\mathcal{M}(\Gamma_1)$ and $\mathcal{M}(\Gamma_2)$ such that the following conditions hold

(i) for each edge $e = ((r_1, C), a, (r_2, D))$ in Σ_1 such that a is a relational attribute, there is a path $\rho_o(e) = ((r_1', C'), \alpha, (r_2', D'))$ in Σ_2 such that
  (a) $C' = \rho|_{N_1}(C)$, $D' = \rho|_{N_1}(D)$, and
  (b) $(C', \alpha, D') = \rho|_r(C, a, D)\backslash\rhd$ that is the path obtained from $\rho|_r(C, a, D)$ after removing the inheritance edges.

(ii) for each data attribute edge $e = ((r, C), a, (v, T))$ in Σ_1, there is a set of paths
$$\rho_o(e) = \{((r', C'), \beta_1.a_1, (v_1, T_1)), \ldots, ((r', C'), \beta_n.a_n, (v_n, T_n))\}$$
such that $v = g(v_1, \ldots, v_n)$, $C' = \rho|_{N_1}(C)$ and $\rho|_d(C, a, T) = \{C'.\gamma_i.a_i\}$, where $\beta_i = \gamma_i\backslash\rhd$ for $1 \le i \le n$ and g is the primitive operation corresponding to (C, a, T) in Γ_1.

5 Structure Refinement Rules

We give a set of rules in Figure 6 which transform a class graph $\Gamma_1 = \langle A_1, N_1, E_1 \rangle$ to another $\Gamma_2 = \langle A_2, N_2, E_2 \rangle$. Notice that each rule has a frame representing the unchanged class names before and after the transformation. The purpose of introducing precondition for each rule is to ensure that the class graph after transforming is a well-formed one. Here, we use $\mathcal{C}$ to denote the set of class names declared in Γ_1.

5.1 Soundness

It is straightforward to prove that each rule defines a structure transformation on class graphs, which satisfies the conditions of **Proposition** 4.2. Thus each rule R determines a structure relation R_o between the object graphs of the corresponding class graphs.

Also, each rule R derives a transformation R_c. R_c transforms a command c that is *syntactically well-formed* under Γ_1 to a command $R_c(c)$ that is *syntactically well-formed* under Γ_2. This means that the variables and types in the command are all defined in the graph. Statements and expressions are correctly typed [5]. The command transformations are given in Figure 7, where notation $[D/C]$ denotes a substitution for each class name C by D, and notation $[C.b/C.a]$ denotes a substitution for each expression of the form $e.a$ by another expression $e.b$ if the declared type of e is C. The meaning of notations $[C.b.a/C.a]$ and $[g(C.x_1, \cdots, C.x_n)/C.x]$ is defined inductively.

Theorem 5.1 (Soundness of Rules) *If rule $R_{[f]}$ transforms Γ_1 to Γ_2, then $\Gamma_1 \sqsubseteq \Gamma_2$.*

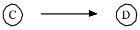
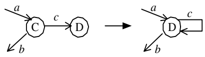
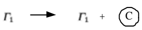
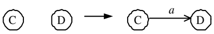
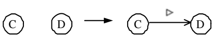
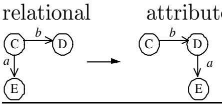
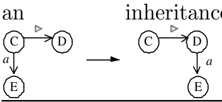
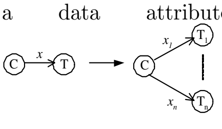
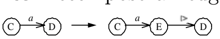
Rules	Description	Precondition	Frame
R1 Rename a class 	class name C is changed to a different name D	$C \in N_1, D \notin N_1$	$\mathcal{C} \setminus \{C\}$
R2 Merge classes 	merge class C to another class D such that all the outgoing (incoming) edges from (to) C become those from (to) D, and the edges between C and D become self loop edges	$C, D \in N_1$, $(C, a, E), (D, a, F) \in E_1$ implies $E = F$ and $init(C.a) = init(D.a)$	$\mathcal{C} \setminus \{C\}$
R3 Rename an attribute 	the name of an attribute (C, a, D) is changed to (C, b, D)	$(C, a, D) \in E_1$, $a \neq b$ and $(E, b, F) \notin E_1$ for any $F \in N_1$ if E is a superclass or subclass of C	$\mathcal{C}$
R4 Add a new class 	add a class C	$C \notin N_1$	$\mathcal{C}$
R5.1 Add an attribute 	add an attribute (C, a, D)	$C, D \in N_1$, $(E, a, F) \notin E_1$ for any $F \in N_1$ if E is a superclass or subclass of C	$\mathcal{C}$
R5.2 Add an inheritance 	add an inheritance edge $(C, \rhd, D)$	$C, D \in N_1$, for each $C' \in N_1$ $(C, \rhd, C') \notin E_1$, $D \npreceq C$, and for each pair of edges $(E, a, E'), (F, b, F') \in E_1$ where $E \preceq C$ and $F \succeq D$, we have $a \neq b$	$\mathcal{C}$
R6.1 Forward an attribute through a relational attribute 	an attribute (C, a, E) is forwarded through a relational attribute (C, b, D) to form another attribute (D, a, E)	$(C, a, E), (C, b, D) \in E_1$, $(F, a, F') \notin E_1$ for any $F' \in N_1$ if F is a superclass or subclass of D	$\mathcal{C}$
R6.2 Forward an attribute through an inheritance 	an attribute (C, a, E) is forwarded through an inheritance edge $(C, \rhd, D)$ to form another attribute (D, a, E)	$(C, a, E), (C, \rhd, D) \in E_1$, $(F, a, F') \notin E_1$ for any $F' \in N_1$ if F is a superclass or subclass of D	$\mathcal{C}$
R7 Decompose a data attribute 	a data attribute (C, x, T) is decomposed to a set of data attributes $(C, x_1, T_1), \ldots, (C, x_n, T_n)$	$(C, x, T) \in E_1$, there exists a surjective primitive operation $g : T_1 \times \ldots \times T_n \to T$ preserving the initial values of attributes, and for each $1 \leq i \leq n$, $(D, x_i, D') \notin E_1$ for any $D' \in N_1$ if D is a superclass or subclass of C	$\mathcal{C}$
R8 Decompose an edge 	a relational attribute or inheritance edge (C, a, D) is decomposed to two edges (C, a, E) and $(E, \rhd, D)$	$(C, a, D) \in E_1$, $E \in N_1$	$\mathcal{C}$

Fig. 6. Basic Rules

Rule R	Command Transformation R_c
R1 Rename a class	$R_c(c) = c[D/C]$
R2 Merge classes	$R_c(c) = c[D/C]$
R3 Rename an attribute	$R_c(c) = c[C.b/C.a]$
R4 Add a new class	$R_c(c) = c$
R5.1 Add an attribute	$R_c(c) = c$
R5.2 Add an inheritance	$R_c(c) = c$
R6.1 Forward an attribute through a relational attribute	$R_c(c) = c[C.b.a/C.a]$
R6.2 Forward an attribute through an inheritance	$R_c(c) = c$
R7 Decompose a data attribute	$R_c(c) = c[g(C.x_1, \cdots, C.x_n)/C.x]$
R8 Decompose an edge	$R_c(c) = c$

Fig. 7. Command transformation

Proof (Outline) If a method $m(u : T_1; v : T_2)\{c\}$ is defined in class C $(C \in f)$ of Γ_1, then we can also define a corresponding method $m(u : T_1; v : T_2)\{R_c(c)\}$ in class C of Γ_2 such that the condition given in Definition 3.4 holds. $\square$

This theorem implies that all rules given in Figure 6 are structure refinements. It also shows the commutativity of the diagram of Figure 1 in the Section 1. Obviously, the structure refinement relation defined in Definition 3.4 is transitive.

Corollary 5.2 *If Γ_1 is transformed to Γ_2 by a sequential applications of rules $R_{1[f_1]}, \ldots, R_{k[f_k]}$, then $\Gamma_1 \sqsubseteq \Gamma_2$, provided $f_1 \cap \cdots \cap f_k \neq \emptyset$.*

5.2 Validity of Proposition 4.2

We now establish the validity of **Proposition** 4.2 as the following completeness theorem.

Theorem 5.3 (Completeness of Rules) *If $\rho_{[f]}$ is a structure transformation from Γ_1 to Γ_2 that satisfies the conditions of* **Proposition** *4.2, then there exist a finite number of sequential applications of rules $R_{1[f_1]}, \ldots, R_{k[f_k]}$ that transforms Γ_1 to Γ_2 and $f \subseteq f_i$ for $1 \leq i \leq k$.*

Proof (Outline) Given a structure transformation $\rho_{[f]}$, we can identify a sequence of applications of refinement rules as follows.

(i) change each class C to $\rho|_{N_1}(C)$, by applications of R1 and R2. When mapping $\rho|_{N_1}$ is injective, R2 is not needed.

(ii) decompose each data attribute (C, x, T) to a set of data attributes:

$$\{(C, x_1, T_1), \ldots, (C, x_n, T_n)\},$$

provided $\rho|_d(C, x, T) = \{(C.\beta_1.x_1), \ldots, (C.\beta_n.x_n)\}$ and $dest(C.\beta_i.x_i) = T_i$, using rule R7.

(iii) for edges there are two cases

(a) change each data edge (C, x, T) to a path $C.\beta.x$ in $\rho|_d(C, x, T)$, using R4, R5 and R6.

(b) change each relational or inheritance edge (C, a, D) to a path $\rho|_r(C, a, D)$, by applications of R3, R4, R5, R6 and R8.

(iv) add additional nodes and edges by using rules R4 and R5 if necessary.

□

Example 5.4 For the structure transformation illustrated in Example 4.3, Figure 8 shows the applications of the rules that transform Γ_1 to Γ_2.

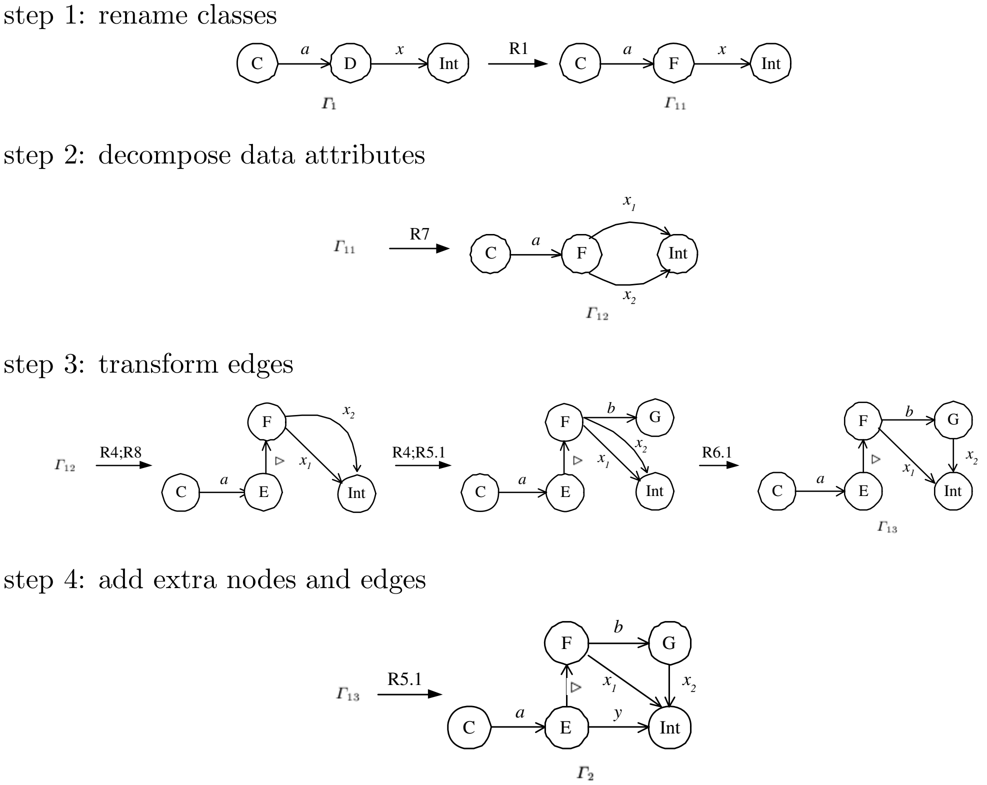


Fig. 8. An example

Now we have established the validity of **Proposition** 4.2.

Corollary 5.5 Proposition *4.2 holds.*

6 Conclusion

We have proposed a graph theoretical approach to studying the relation between changes in class declarations and changes in method definitions. The main purpose is to make the semantics and refinement of object oriented programs easier to understand and more operational. We believe this is important for development of tool support to object system development by transformations [11].

Another contribution of this paper is the proposal of an operational semantics for object oriented programs in the graph theoretical notation. This allows us to understand the execution of an object program in the same as an imperative program by taking graphs as the states. In our future work, we will study this operational semantics together with the study of operations and properties of graphs. This will lead to the development of a Hoare-logic for object-oriented programs with predicates of graphs.

The approach presented here suggests a design method of object oriented systems that allows the automatic derivation of methods definitions from their specifications and structure transformation.

This work is still at its early stage in that the structure refinements are restricted to only expanding the graph. No rules are provided for removing classes and attributes or compressing long paths to shorter paths. Therefore, some refinement laws proved in rCOS [5] have not been established. In other words, only "true" refinements are treated, but not the "abstractions" that preserve functionality. The difficulty in establishing this kind of abstraction rules is due to the fact that we consider arbitrary methods definable in a class graph. In further work, we will consider rules of class refinement for fixed methods in the public methods.

References

[1] Y. Chen and J. Sanders. Compositional reasoning for pointer structures. *Proc. of 8th International Conference on Mathematics of Program Construction (MPC06)*. Springer. 2006.

[2] C.A.R. Hoare and J. He. A trace model for pointers and objects. In *ECOOP'99, LNCS1628*, pages 1-17. Springer, 1999.

[3] M.Abadi and L. Cardelli. *A Theory of Objects*. Springer-Verlag, 1996.

[4] P. America and F.de Boer. Reasoning about dynamically eveloving process structures. *Formal Aspects of Computing*, 6(3):269-316,1994.

[5] J. He, X. Li and Z. Liu. rCOS: A Refinement Calculus for Object-oriented Systems. Accepted for publication in *Theoretical Computer Science*. Also available as Technical Report 322. UNU/IIST, P.O.Box 3058, Macao SAR China (http://www.iist.unu.edu).

[6] Z.Liu, J.He, X.Li and Y.Chen. A relational model for formal requirements analysis in UML. Proc. of ICFEM03, LNCS 2885, 641-664, 2003.

[7] P. Borba, A. Sampaio, and M. Cornelio. A Refinement Algebra for Object-Oriented Programming. In *ECOOP 2003, LNCS 2743*, pages 457-482. Springer, 2003.

[8] R. Gheyi, T. Massoni and P. Borba. An abstract equivalence notation for object models. *Proc. SBMF2004*, pages 1-15. 2004.

[9] A. Mikhajlova and E. Sekerinski. Class refinement and interface refinement in object-oriented programs. In *Proc of FME'97, LNCS 1313*. Springer, 1997.

[10] EB Johnsen and O. Owe. Object-Oriented Specification and Open Distributed Systems. In *LNCS 2635, pages 137-164*. Springer, 2004.

[11] Tata Consultancy Services. MasterCraft.(http://www.tata-mastercraft.com/)

[12] R. Back, A.Mikhajlova, and J. von Wright. Class refinement as semantics of correct object substitutability. *Formal Aspects of Computing*, 2:18-40, 2000.