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The Non-fragile Controller Design Based on Quadratic Performance Optimization

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Abstract

This paper presents a non-fragile controller design method based on system quadratic performance optimization. For the additive controller gain variations, the necessary and sufficient conditions for the existence of non-fragile state feedback controller are given and transformed to the LMI problems, which simplifies the solutions to obtain non-fragile state feedback controllers. The flight control simulation results prove the reliability and validity of the method.

Keywords: non-fragile control, robust control, quadratic performance optimization, Linear Matrix Inequality (LMI);

1. Introduction

System modelling can never describes the engineering plant exactly, and lots of scholars showed interesting on the system optimal problems with system and external uncertainty [ZHOU et al., 1996, YU et al., 2002, WU et al., 2006]. The Guaranteed Cost Control presented by Chang [Sheldon et al., 1972] tackled this problem, and can be seemed as the linear quadratic optimal problem's extension to system uncertainty. But such feedback controller which tackles the uncertainty of controlled plant and the closed-loop performance

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optimization needs to be implemented exactly. This demand seems validity, but actually, because of the inevitable uncertainty in advanced controller operation, it's impossible in the actual engineering. The controller uncertainty is composed by environment temperature variation, the digital device's word capacity, data transform error, etc.

As the analysis and synthesis of closed system which is composed of plant and controller, the robust control approach has the consideration of the control fragile problem. And the fruitful results of robust control theory are helpful to tackle the non-fragile problem. Theoretical research and engineering practice show that, if we divide the uncertainty of plant and controller, the robust and non-fragile are interactive. When the robust from the closed system to the uncertainty is stronger, then the closed system is fragile or sensitive to the uncertainty of controller. Keel et al., 1997 concludes that the fragile of controller will be worse as the order of controller become higher. So it's important to consider the controller's uncertainty during the controller design and synthesis, e.g. non-fragile control.

Over recent years, many remarkable achievements have been achieved in developing advanced non-fragile controller [Famularo et al., 1998, Yang et al., 2000], but the common approaches with operator theory or Riccati inequality are complex. According to the additive controller gain uncertainty, this paper presents the non-fragile controller synthesis problem based on the optimal performance control, which controller will be tackled using the LMI toolbox in Matlab® software. A mathematical simulation of flight controller design is presented to show the validity of the method above.

2. Problem statements

Lemma 1 [WU et al., 2006] For given proper dimension matrix Y, D and E, in which the Y is symmetric,

$$Y + DFE + E^T F^T D^T < 0,$$

is existed for all the matrix F which satisfy $F^T F < I$, if and only if there exists the constant $\varepsilon > 0$ and

$$Y + \varepsilon DD^T + \varepsilon^{-1} E^T E < 0.$$

Lemma 2 [WU et al., 2006] With the given symmetric matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix},$$

in this matrix, S_{11}, S_{22} is nonsingular invertible matrix, then the three conditions below is equivalent:

(1) $S < 0$;

(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0$, $S_{11} - S_{12}^T S_{22}^{-1} S_{12} < 0$;

Consider linear continuous time-invariable equation

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, A, and B is the matrices with proper dimension. The matrices uncertainty satisfy

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) \end{bmatrix} = DF(t)\begin{bmatrix} E_1 & E_2 \end{bmatrix}. \tag{2}$$

The optimal controller $u(t)=Kx(t)$ makes the performance index

$$J = \int_0^{\infty} \left[x^T(t)Qx(t) + u^T(t)Ru(t) \right] dt, \tag{3}$$

reach the minimum.

For the given quadratic performance index and system uncertainty, the closed system with state feedback controller $u(t)=Kx(t)$ is optimal, if the symmetric matrix P and K exist which satisfy

$$P\left[A+BK+DF(E_1+E_2K)\right]+\left[A+BK+DF(E_1+E_2K)\right]^T P+K^T RK+Q<0, \quad (4)$$

and then system performance upper bound is $J_* = x_0^T P x_0$.

Consider the controller has parameter perturbation

$$K = K_0+\Delta K=K_0+D_k F_k(t)E_k, \quad (5)$$

the non-fragile controller design approach with optimal performance can be given as follows.

3. Main results

3.1. Non-fragile controller design with robust performance optimal

Theorem 1 For system (1) with uncertainty (2), the necessary and sufficient condition to the optimal performance index (3) with non-fragile feedback control strategy is that, if there exists positive define matrix X, W, and real constant $\lambda > 0$ that satisfy the LMIs below

$$\begin{bmatrix} (\mathrm{AX}+\mathrm{BW})+(AX+BW)^T+\varepsilon DD^T & (E_1X+E_2W)^T & X & W^T & \lambda BD_k & XE_k^T \\ (E_1X+E_2W)^T & -\varepsilon I & 0 & 0 & \lambda E_2 D_k & 0 \\ X & 0 & -Q^{-1} & 0 & 0 & 0 \\ W & 0 & 0 & -R^{-1} & \lambda D_k & 0 \\ (\lambda BD_k)^T & (\lambda E_2 D_k)^T & 0 & \lambda D_k^T & -\lambda I & 0 \\ E_k X & 0 & 0 & 0 & 0 & -\lambda I \end{bmatrix} < 0 \quad (6)$$

Moreover, if the feasible solution for the inequality (6) above exists, then the control strategy $u(t)=WX^{-1}x(t)$ is the controller for the system (1), and the performance upper bound is $J_* = x_0^T P x_0$.

Proof. According to the Eq. (4), the necessary and sufficient condition for minimizing the performance index (3) with system uncertainty is

$$Y+PDF(E_1+E_2K)+(E_1+E_2K)^T F^T (PD)^T = Y+\varepsilon PD(PD)^T+\varepsilon(E_1+E_2K)^T(E_1+E_2K)<0,$$

where $Y=Q+K^T RK+P(A+BK)+(A+BK)^T P$.

$$M=\begin{bmatrix} P(A+BK)+(A+BK)^T P+\varepsilon PDD^T P & (E_1+E_2K)^T & I & K^T \\ (E_1+E_2K) & -\varepsilon I & 0 & 0 \\ I & 0 & -Q^{-1} & 0 \\ K & 0 & 0 & -R^{-1} \end{bmatrix}<0$$

Substitute the Eq. (3) into the equation above, then we can get the results by **Lemma 1**

$$M=\begin{bmatrix} P(A+BK)+(A+BK)^T P+\varepsilon PDD^T P & (E_1+E_2K_0)^T & I & K_0^T \\ (E_1+E_2K_0) & -\varepsilon I & 0 & 0 \\ I & 0 & -Q^{-1} & 0 \\ K_0 & 0 & 0 & -R^{-1} \end{bmatrix}$$

$$+\begin{bmatrix} PBD_k \\ E_2D_k \\ 0 \\ D_k \end{bmatrix} F_k \begin{bmatrix} E_k & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} E_k \\ 0 \\ 0 \\ D_k \end{bmatrix} F_k^T \begin{bmatrix} (PBD_k)^T & (E_2D_k)^T & 0 & D_k^T \end{bmatrix}$$

$$\leq \begin{bmatrix} P(A+BK_0)+(A+BK_0)^T P+\varepsilon PDD^T P & (E_1+E_2K_0)^T & I & K_0^T \\ (E_1+E_2K_0) & -\varepsilon I & 0 & 0 \\ I & 0 & -Q^{-1} & 0 \\ K_0 & 0 & 0 & -R^{-1} \end{bmatrix}$$

$$+\lambda \begin{bmatrix} PBD_k \\ E_2D_k \\ 0 \\ D_k \end{bmatrix} \begin{bmatrix} PBD_k \\ E_2D_k \\ 0 \\ D_k \end{bmatrix}^T + \lambda^{-1} \begin{bmatrix} E_k^T \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_k & 0 & 0 & 0 \end{bmatrix}$$

By the **Lemma 2**, $M < 0$ is equal to

$$\begin{bmatrix} P(A+BK_0)+(A+BK_0)^T P+\varepsilon PDD^T P & (E_1+E_2K_0)^T & I & K_0^T & \lambda PBD_k & E_k^T \\ (E_1+E_2K_0) & -\varepsilon I & 0 & 0 & \lambda E_2D_k & 0 \\ I & 0 & -Q^{-1} & 0 & 0 & 0 \\ K_0 & 0 & 0 & -R^{-1} & \lambda D_k & 0 \\ (\lambda PBD_k)^T & (\lambda E_2D_k)^T & 0 & \lambda D_k^T & -\lambda I & 0 \\ E_k & 0 & 0 & 0 & 0 & -\lambda I \end{bmatrix} < 0.$$

Multiply the inequality above right and left with $diag\{P^{-1}, I, I, I, I, I\}$ respectively.

$$\begin{bmatrix} (A+BK_0)X+X(A+BK_0)^T+\varepsilon DD^T & X(E_1+E_2K_0)^T & X & XK_0^T & \lambda BD_k & XE_k^T \\ (E_1+E_2K_0)X & -\varepsilon I & 0 & 0 & \lambda E_2D_k & 0 \\ X & 0 & -Q^{-1} & 0 & 0 & 0 \\ K_0X & 0 & 0 & -R^{-1} & \lambda D_k & 0 \\ (\lambda BD_k)^T & (\lambda E_2D_k)^T & 0 & \lambda D_k^T & -\lambda I & 0 \\ E_kX & 0 & 0 & 0 & 0 & -\lambda I \end{bmatrix} < 0.$$

Let $X = P^{-1}$, $K_0X = W$, then the Eq. (6) can be got. If the feasible solution to the LMIs (6) is existed, the control strategy $u(t) = W^*(X^*)^{-1}x(t)$ is the non-fragile state feedback controller. □

3.2. Non-fragile controller design based on Linear Quadratic Regulator (LQR)

If the system uncertainty doesn't exist, then this problem regresses to the non-fragile control with conventional linear quadratic performance index, e.g.

$$\Delta A = 0,\ \Delta B = 0$$

Theorem 2 For system (1), the necessary and sufficient condition to the optimal performance index (3) with non-fragile feedback control strategy is that, if the positive define matrix X, W exist, and real constant $\lambda > 0$ that satisfy the following LMIs

$$\begin{bmatrix} (AX+BW)+(AX+BW)^T & X & W^T & \lambda BD_k & XE_k^T \\ X^T & -Q^{-1} & 0 & 0 & 0 \\ W & 0 & -R^{-1} & \lambda D_k & 0 \\ (\lambda BD_k)^T & 0 & \lambda D_k^T & -\lambda I & 0 \\ E_kX & 0 & 0 & 0 & -\lambda I \end{bmatrix} < 0, \tag{7}$$

then $K = KW^{-1}$ is the feedback control solution for control system (1).

Proof: According to the Eq. (4), the necessary and sufficient condition for minimizing the performance index (3) with system uncertainty is

$$L=\begin{bmatrix} (A+BK_0)^T P+P(A+BK_0) & I & K_0^T \\ I & -Q^{-1} & 0 \\ K_0 & 0 & -R^{-1} \end{bmatrix}$$

$$+\begin{bmatrix} PBD_k \\ 0 \\ D_k \end{bmatrix} F_k(t)\begin{bmatrix} E_k & 0 & 0 \end{bmatrix}+\begin{bmatrix} (E_k)^T \\ 0 \\ 0 \end{bmatrix} F_k^T(t)\begin{bmatrix} (PBD_k)^T & 0 & (D_k)^T \end{bmatrix}$$

$$\le \begin{bmatrix} (A+BK_0)^T P+P(A+BK_0) & I & K_0^T & \lambda PBD_k & E_k^T \\ I & -Q^{-1} & 0 & 0 & 0 \\ K_0 & 0 & -R^{-1} & \lambda D_k & 0 \\ \lambda(PBD_k)^T & 0 & \lambda D_k^T & -\lambda I & 0 \\ E_k & 0 & 0 & 0 & -\lambda I \end{bmatrix} < 0 .$$

Multiply the inequality above right and left with $diag\{P^{-1}, I, I, I, I\}$ respectively.

$$\begin{bmatrix} X(A+BK_0)^T+(A+BK_0)X & X & XK_0^T & \lambda BD_k & XE_k^T \\ X & -Q^{-1} & 0 & 0 & 0 \\ K_0 X & 0 & -R^{-1} & \lambda D_k & 0 \\ \lambda(BD_k)^T & 0 & \lambda D_k^T & -\lambda I & 0 \\ E_k X & 0 & 0 & 0 & -\lambda I \end{bmatrix} < 0 .$$

Let $X = P^{-1}$, $K_0 X = W$, then the Eq. (7) can be got. □

4. Simulation examples

Consider the following flight model

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{8}$$

$$A=\begin{bmatrix} -0.0316 & -6.9739 & -1.0087 & -9.8036 \\ -0.002 & -0.6256 & 0.9128 & 0 \\ -0.0027 & -0.4 & -0.8845 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B=\begin{bmatrix} 0 \\ -0.0873 \\ -4.1063 \\ 0 \end{bmatrix},$$

the corresponding system states are: flight velocity V, attack angle α, pitch rate q, and pitch angle θ, aircraft system input is elevator angle δ_e.

System controller addictive gain perturbation can be written as the Eq. (5)

$$E_k = [1],\ F_k = [0.1 \quad 0.1 \quad 0.1 \quad 0.1],\ |\Delta_k(t)| \le 1$$

Using the inequality (7) above, the controller can be solved as

$$K = [-2.3685 \quad 83.6093 \quad 3.536 \quad 45.6491]$$

For examining the control strategy, the flight system response with controller is given. Fig. 1 to 4 show the state curves with perturbation.

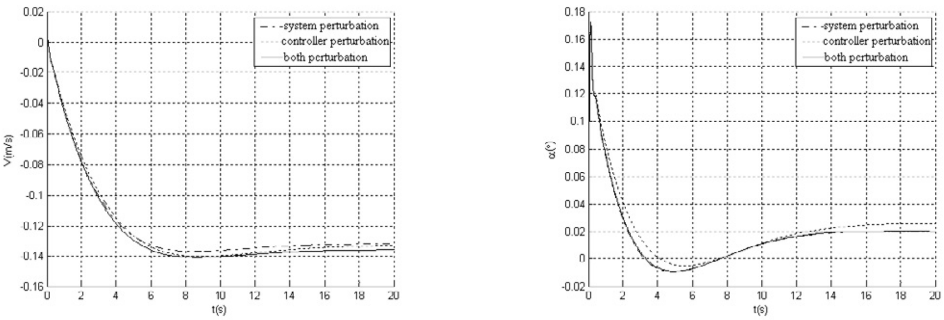

Fig. 1. (a) flight velocity ΔV response curves; (b) angle of attack $\Delta\alpha$ response curves

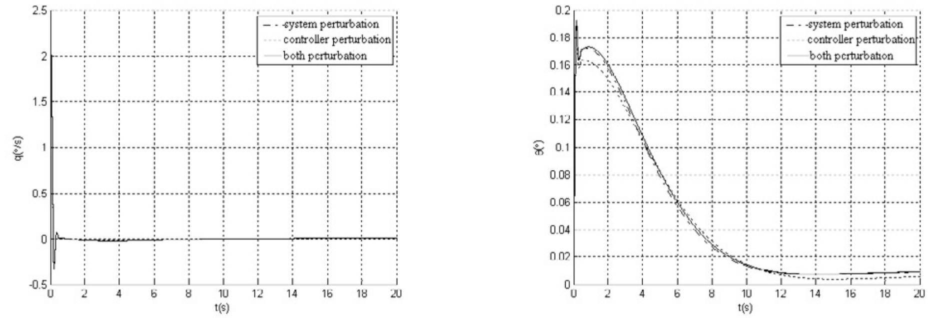

Fig. 2. (a) pitch rate Δq response curves; (b) pitch angle $\Delta\theta$ response curves

5. Conclusions

Using the Quadratic performance optimization method, the control stability problem with controller additive gain perturbation is discussed, and then the stability theorem and state feedback control approach are derived in this paper. In Section 4, the flight system performance of aircraft has been verified by mathematical simulation to validate the controller design approach.

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