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Teaching-learning-based optimization algorithm to minimize cross entropy for Selecting multilevel threshold values

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ABSTRACT

Image thresholding is one of the most important approaches for image segmentation. Among multilevel thresholding techniques, cross entropy has been widely used by researchers to find multilevel threshold values. In multilevel cross entropy thresholding techniques, main target is to find an optimal combination of threshold values at different levels for minimizing the cross entropy. In this paper, Teaching-Learning-based Optimization (TLBO) algorithm is used to find an optimal combination of threshold values at different levels for minimizing the cross entropy. TLBO algorithm is inspired by passing on knowledge within a classroom environment where students first gain knowledge from a teacher and then through mutual interaction. This new proposed approach is called the TLBO-based minimum cross entropy thresholding (TLBO-based MCET) algorithm. The performance of the proposed algorithm is tested on a number of standard test images. For comparative analysis, the results of TLBO-based MCET algorithm are compared with the results of Firefly-based minimum cross entropy thresholding (FF-based MCET), Honey Bee Mating Optimization-based minimum cross entropy thresholding (HBMO-based MCET) and Quantum Particle Swarm Optimization-based minimum cross entropy thresholding (Quantam PSO-based MCET). Comparative analysis is done based on two most popular objective performance measures, Peak Signal to Noise Ratio (PSNR) and Uniformity. From experimental results, it is observed that the proposed method is an efficient and feasible method to search an optimal combination of threshold values at 2nd, 3rd, 4th and 5th levels.

1. Introduction

Image segmentation is an active field in medical imaging, machine vision and satellite imagery. Main goal of image segmentation is to partitioning an image into regions which are meaningful for a particular task. Segmentation is generally the essential component of pattern recognition systems in which objects of interest are found and isolated from the rest of the scene. After image segmentation, some features are extracted from objects and at the end, objects are classified in particular groups or classes based on extracted features. For medical applications, segmentation is used for the detection of organs such as the brain, heart, lungs or liver in CT or MR images [1]. It is also used to distinguish pathological tissue such as a tumor from normal tissue. Depending on the particular application, different techniques for image segmentation have been used, such as image thresholding, edge detection, region growing, stochastic models, ANN and clustering techniques [2]. Thresholding is one of the most frequently used methods in image segmentation because it is computationally simple and never fails to define disjoint regions with closed, connected boundaries [3]. Its basic idea is to divide the image into target and background regions by the threshold value. Intensity value of each

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pixel of the image is compared to the threshold value. If its value is greater than the threshold value, then the pixel is considered as target region pixel and set to white; otherwise the pixel is considered as background region pixel and set to black. The success of thresholding depends upon the selection of an optimal threshold value. Over the years, various thresholding techniques have been proposed by researchers to select a suitable threshold value [4]. In the past years, many thresholding techniques using entropy measures have been proposed by researchers to select an appropriate threshold value for segmenting images [5–9]. In these techniques, first, two probability distributions, one for the target and the other for the background are obtained from gray level values of input image. After this, the procedure is adopted for maximizing the total entropy of two probability distributions to obtain the threshold. Different definitions of entropies differentiated these techniques from each other. Liu et al. [10] proposed a fuzzy classification approach for image thresholding. Cheng et al. [11] used fuzzy c-partition entropy approach to select threshold. In this approach, parameterized fuzzy membership functions are used to classify pixels of image into target and background. This approach is based on the selection of an optimal combination of parameters of fuzzy membership functions for maximizing the entropy of fuzzy 2-partition. After this, the selected optimal parameters are used to find optimal threshold. Benabdelkader & Boulemden [12] proposed a recursive approach to search a suitable combination of parameters of fuzzy membership functions. In this approach, trapezium fuzzy membership function with two parameters is used. In 2011, Tang et al. [13] also proposed a recursive programming approach to find a suitable combination of fuzzy membership function parameters. This recursive approach is applied on two fuzzy membership functions (s-function and z-function) with three parameters. Tao et al. [14] applied ant colony optimization to search a combination of parameters of fuzzy membership functions for maximizing the entropy of fuzzy 2-partition. Li and Lee [15] proposed minimum cross entropy approach for image segmentation. In 2011, Nie et al. [16] proposed a thresholding method that is based on two-dimensional cross entropy. In this method, gray level co-occurrence matrix is used to obtain two-dimensional cross entropy. Horng [17] proposed honey bee mating optimization algorithm for calculating the minimum cross entropy objective function. Tang et al. [18] used genetic algorithm to reduce computation burden for computing the minimum cross entropy objective function. Horng and Liou [19] proposed firefly algorithm to search multilevel thresholds by using cross entropy principal. Recently, fuzzy entropy based techniques have been proposed by numerous researchers to find optimal threshold. These techniques are frequently used for thresholding because it is the general belief that fuzziness and uncertainty exist in images. Although the cross entropy has been applied by many researchers to search multilevel thresholds for image segmentation, it is worth noting that selection of an optimal combination of thresholds for minimizing the cross entropy of fuzzy 2-partition in reasonable amount of time is a challenging task. Thus, selection of an optimal combination of thresholds for minimizing the cross entropy can be formulated as a combinatorial optimization problem. Over the last decade, various metaheuristic algorithms have been used by researchers to solve combinatorial optimization problems. Such algorithms are Genetic Algorithm (GA) [18,20–22], Ant Colony Optimization (ACO) algorithm [14,23], Biogeography based Optimization (BBO) approach [24,25], bacterial foraging optimization algorithm [26,27], gravitational search algorithm [28], cuckoo optimization algorithm [29], hybrid approaches [30,31], etc. Teaching-Learning-Based Optimization (TLBO) algorithm is a newly introduced member in the optimal algorithm family [32,33]. It is inspired by the philosophy of teaching and learning. The search mechanism of TLBO algorithm is a population-based. Initially, a set of some feasible solution candidates of the given problem is randomly generated called the population. After this, feasible solutions are modified to achieve optimal solution by the simulation of a classical school learning process. This process consists of two phases: teaching phase and student phase [34]. Teacher phase simulates the learning of the students through the teacher. During this phase, the best feasible solution acts as teacher. Other feasible solutions are improved by moving their positions towards the position of the teacher by taking into account the current mean value of the feasible solutions. Student phase simulates the learning of the students through their mutual interaction. During this phase, two feasible solutions are randomly selected. If the first one is better than second one, then the first one is moved towards the second one. Otherwise, the first one is moved away from the second one. Main advantage of TLBO algorithm over other optimization algorithms is that it used only common controlling parameters while it is free from algorithm-specific parameters [35]. Common controlling parameters are common in running any population based optimization algorithms like population size and number of generations while algorithm-specific parameters are specific to that algorithm and different algorithms have different algorithm-specific parameters to control. For example, GA's algorithm-specific parameters are mutation rate and crossover rate. Similarly, BBO's algorithm-specific parameters are maximum immigration rate, maximum emigration rate and mutation rate. The optimal selection of algorithm-specific parameters is also a problem. The improper selection of algorithm-specific parameters decreases the performance of optimization algorithms. Due to the improper selection of algorithm-specific parameters, either the computational cost of the algorithm will increase or yield the local optimal solution [36]. Recently, TLBO algorithm has been widely applied to obtain global optimal solutions for a variety of optimization problems [37–41] with less computational cost and high consistency. From this motivation, the feasibility of TLBO algorithm is investigated to search an optimal combination of thresholds for minimizing the cross entropy.

The objective of this paper is to search an optimal combination of thresholds for minimizing the cross entropy. The selected optimal combination of thresholds is used to segment the input image. The proposed thresholding approach is called the TLBO-based minimum cross entropy thresholding (TLBO-based MCET) algorithm. Here, the fitness function for TLBO algorithm is cross entropy of the input image and segmented image.

In this study, a set of five standard test images is used to evaluate the performance of the proposed algorithm. The proposed approach is compared with three different approaches included Firefly-based minimum cross entropy thresholding (FF-based MCET) [19], Honey Bee Mating Optimization-based minimum cross entropy thresholding (HBMO-based MCET) [17] and Quantum Particle Swarm Optimization-based minimum cross entropy thresholding (Quantam PSO-based MCET) [42].

The rest of the paper is organized as follows. Section 2 introduces the TLBO algorithm. In Section 3, the proposed TLBO-based minimum cross entropy thresholding (TLBO-based MCET) algorithm is described in details. Performance evaluation is discussed in detail in Section 4. Finally, Section 5 concludes the paper and suggests future research directions.

2. Teacher-Learning-based Optimization algorithm

Teacher-Learning-based Optimization (TLBO) algorithm is a new kind of metaheuristic algorithm that is based on a teaching–learning process. It is firstly introduced by Rao et al. [32] to solve constrained mechanical design optimization problems. It is inspired by passing on knowledge within a classroom environment where students first gain knowledge from a teacher and then through mutual interaction [34]. TLBO algorithm is a population-based optimization algorithm in which a group or class of students considered as population. Thus, a student of class represents a feasible solution of the problem. Different subjects offered to the class are considered as different design variables of the optimization problem and student's result is treated as the fitness value of feasible solution of the optimization problem.

TLBO algorithm consists of two phases: Teacher phase and student phase. Working of these phases is described below [32–34,36].

2.1. Teacher phase

The teacher phase means learning of the students from the teacher. On the basis of teaching-learning philosophy, the most experienced, knowledgeable and highly learned person in the society is considered as teacher. The teacher tries to improve the knowledge level of students and helps students to obtain good marks. But, students gain knowledge and obtain marks according to the quality of teaching delivered by the teacher and the quality of students present in the class. For simulation, suppose there are *'n'* number of subjects (design variables, $j = 1, 2,\ldots,n$) offered to *'N_p'* number of students (population size, $i = 1, 2,\ldots,N_p$). At any teaching-learning cycle (iteration, $k = 0, 1, 2,\ldots I_n$), M_j^k is the mean result of students in a particular subject *'j'*. Teacher is the most experienced, knowledgeable and highly learned person in the society. To simulate this concept, the best student (feasible solution) in the entire population is considered as teacher. Let X_T^k be the most feasible solution of the population at kth teaching-learning cycle and $X_{T,j}^k$ denotes the jth design variable in the best feasible solution of the population at kth teaching-learning cycle i.e. the result of the teacher in subject *'j'*. The difference between the result of the teacher and the mean result of the students in subject *'j'* is given by [32]

$$D_j^k = r(X_{T,j}^k - T_F M_j^k) \tag{1}$$

where T_F is a teaching factor that decides the value of the mean to be changed and r is a random number in the range [0 1]. T_F is not a parameter of the TLBO algorithm and its value can either be 1 or 2 [36].

Feasible solutions (students) are improved by moving their positions towards the position of the best feasible solution (teacher) by taking into account the current mean value of the feasible solutions. To simulate this fact, the ith feasible solution in the population at kth teaching-learning cycle is updated according to the following expression:

$$X_{new,i,j}^k = X_{old,i,j}^k + D_j^k \tag{2}$$

If $X_{new,i}^k$ is better than $X_{old,i}^k$, then $X_{new,i}^k$ is accepted; Otherwise it is rejected. All the accepted feasible solutions are maintained and these become the input to the student phase.

2.2. Student phase

In this phase, students gain knowledge through mutual interaction. A student interacts randomly with other students of the class to improve knowledge. A student (u) learns something new from another student (v) of the class if student (v) has more knowledge than student (u). Thus, if student (v) is better than student (u), then student (u) is moved towards student (v). Otherwise, student (u) is moved away from student (v). The learning philosophy of this phase is simulated as below:

Two students (feasible solutions, X_u^k, X_v^k) are randomly selected from the class (population), where, u, v are two integer random numbers belong to $[1, N_p]$ and $u \neq v$.

If $\quad F(X_u^k) > F(X_v^k)$
$\quad X_{new\_SP,u,j}^k = X_{u,j}^k + r(X_{u,j}^k - X_{v,j}^k)$
Else
$\quad X_{new\_SP,u,j}^k = X_{u,j}^k + r(X_{v,j}^k - X_{u,j}^k)$
Endif

where $F(X)$ is a fitness function that is used to find the fitness value of feasible solution, $X_{new\_SP,u,j}^k$ denotes the jth design variable of the modified feasible solution in student phase at kth teaching-learning cycle.

After this, the fitness value of $X_{new\_SP,u}^k$ is evaluated

If $\quad F(X_{new\_SP,u}^k) > F(X_{new,u}^k)$
$\quad X_{new,u}^k = X_{new\_SP,u}^k$
Else
$\quad X_{new,u}^k = X_{new,u}^k$
Endif

2.3. Basic TLBO algorithm

Based upon the above discussion, TLBO algorithm can be re-written in the following steps [32,43]:

Step1: [Initialization] Initialize the optimization parameters
- Population size (the number of students or learners): N_p
- Number of iterations: I_n
- Number of design variables or parameters (the subjects or courses offered)
- Limits of design variables

Step2: [Initialize the population]
Generate random population according to the population size and the number of design variables.
Step3: [Fitness Evaluation]
Evaluate fitness of feasible solutions in the population and arrange these solutions according to their fitness values
Step4: [Teacher Phase]
Modify solution by simulating the concept: the learning of the students through the teacher
Step5: [Student Phase]
Modify solution by simulating the concept: the learning of the students through their mutual interaction
Step6: [Repeat] Go to Step 3 until the stopping criteria (maximum iteration: In) is not met
Step7: Stop

3. Proposed TLBO-based MCET algorithm

In this section, first the objective function based on cross entropy is generated. After this, the generated objective function is minimized using Teaching-Learning-based Optimization (TLBO) algorithm to obtain multilevel threshold values for segmenting images.

3.1. Cross entropy based objective function

The cross entropy was developed by Kullback [44] in 1968. The cross entropy is a measure of closeness between two sampling distributions. Let $P = \{p_1, p_2, \ldots, p_n\}$ and $Q = \{q_1, q_2, \ldots, q_n\}$ be two probability distributions defined on the same set of values. The cross entropy between P and Q is defined as

$$E_C(P,Q) = \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i} \quad (3)$$

Let $f(x, y)$ be a mathematical function that defines a digital image having $L_{\max} + 1$ gray levels ranging from 0 to $L_{\max}$ and $M \times N$ be the size of the image, then $f(x,y) \in \{0,1,2,\ldots,r,\ldots,L_{\max}\}$ is a gray level value of the pixel that has coordinate position (x, y), $x \in \{1,2,\ldots,M\}$ and $y \in \{1,2,\ldots,N\}$. Let h_r be the frequency of occurrence of a particular gray level value r in the digital image f, then $p_r = h_r/(M \times N)$ is the probability of occurrence of gray level value r in the image.

For bi-level image segmentation, a threshold value $T_1 \in [0, L_{\max}]$ is selected that segments the image into two regions: one is Background (BG) and other is Target (TG). Gray level values of all pixels of BG region is less than or equal to T_1 while gray level values of all pixels of TG region is greater than T_1. In segmented image $g_{T_1}(x,y)$, belongingness of pixel (x,y) to BG and TG regions is described as

$$g_{T_1}(x,y) = \begin{cases} S(0,T_1) & f(x,y) \leqslant T_1 \\ S(T_1+1,L_{\max}) & f(x,y) > T_1 \end{cases} \quad (4)$$

where

$$S(a,b) = \frac{\sum_{r=a}^{b} rp_r}{\sum_{r=a}^{b} p_r} \quad (5)$$

Now, probability distribution of the gray level values of the input image region that belongs to BG region is described as

rp_r where $r = 0,1,\ldots,T_1$

Probability distribution of the gray level values of BG region is described as

$S(0,T_1)p_r$ where $r = 0,1,2,\ldots,T_1$

Then, the cross entropy of the input image region that belongs to BG region and BG region is defined as

$$E_{C,BG}(T_1) = \sum_{r=0}^{T_1} rp_r \log \frac{rp_r}{S(0,T_1)p_r} \quad (6)$$

Now, probability distribution of the gray level values of the input image region that belongs to TG region is described as rp_r, where $r = T_1 + 1, T_1 + 2, \ldots, L_{\max}$

Probability distribution of the gray level values of TG region is described as

$S(T_1+1,L_{\max})p_r$, where $r = T_1 + 1, T_1 + 2, \ldots, L_{\max}$

Then, the cross entropy of the input image region that belongs to TG region and TG region is defined as

$$E_{C,TG}(T_1) = \sum_{r=T_1+1}^{L_{\max}} rp_r \log \frac{rp_r}{S(T_1+1,L_{\max})p_r} \quad (7)$$

The total cross entropy of the input image and segmented image is defined as

$$E_C(T_1) = E_{C,BG}(T_1) + E_{C,TG}(T_1) \quad (8)$$

$$E_C(T_1) = \sum_{r=0}^{T_1} rp_r \log \frac{rp_r}{S(0,T_1)p_r} + \sum_{r=T_1+1}^{L_{\max}} rp_r \log \frac{rp_r}{S(T_1+1,L_{\max})p_r} \quad (9)$$

$$E_C(T_1) = \sum_{r=0}^{T_1} rp_r \log \frac{r}{S(0,T_1)} + \sum_{r=T_1+1}^{L_{\max}} rp_r \log \frac{r}{S(T_1+1,L_{\max})} \quad (10)$$

$$E_c(T_1) = \sum_{r=0}^{T_1} rp_r \log r - \sum_{r=0}^{T_1} rp_r \log[S(0,T_1)] + \sum_{r=T_1+1}^{L_{\max}} rp_r \log r - \sum_{r=T_1+1}^{L_{\max}} rp_r \log[S(T_1+1,L_{\max})] \quad (11)$$

$$E_C(T_1) = \sum_{r=0}^{T_1} rp_r \log r + \sum_{r=T_1+1}^{L_{\max}} rp_r \log r - \sum_{r=0}^{T_1} rp_r \log[S(0,T_1)] - \sum_{r=T_1+1}^{L_{\max}} rp_r \log[S(T_1+1,L_{\max}) \quad (12)$$

$$E_C(T_1) = \sum_{r=0}^{L_{\max}} rp_r \log r - \sum_{r=0}^{T_1} rp_r \log[S(0,T_1)] - \sum_{r=T_1+1}^{L_{\max}} rp_r \log[S(T_1+1,L_{\max})] \quad (13)$$

In segmentation, the main goal is to minimize the variance between input image and segmented image. So here, the main objective is to find an optimal value of threshold value (T_1) such that the total cross entropy (E_C) of the input image and segmented image is minimal.

$$T_1^* = \arg\min_{T_1}^{L_{\max}}[E_C(T_1)] \quad (14)$$

First term in Eq. (12) is a constant term. So the objective function can be rewritten as

$$O(T_1) = -\sum_{r=0}^{T_1} rp_r \log[S(0,T_1)] - \sum_{r=T_1+1}^{L_{\max}} rp_r \log[S(T_1+1,L_{\max})] \quad (15)$$

$$O(T_1) = -\sum_{r=0}^{T_1} rp_r \log\left[\frac{\sum_{r=0}^{T_1} rp_r}{\sum_{r=0}^{T_1} p_r}\right] - \sum_{r=T_1+1}^{L_{\max}} rp_r \log\left[\frac{\sum_{r=T_1+1}^{L_{\max}} rp_r}{\sum_{r=T+1_1}^{L_{\max}} p_r}\right] \quad (16)$$

Let $m^0(a,b) = \sum_{r=a}^{b} p_r$ and $m^1(a,b) = \sum_{r=a}^{b} rp_r$, then

$$O(T_1) = -m^1(0,T_1)\log\left[\frac{m^1(0,T_1)}{m^0(0,T_1)}\right] - m^1(T_1+1,L_{\max}) \times \log\left[\frac{m^1(T_1+1,L_{\max})}{m^0(T_1+1,L_{\max})}\right] \quad (17)$$

For three-level image segmentation, two threshold values $T_1, T_2 \in [0, L_{\max}]$ are selected that segments the image into three regions.

$$O(T_1,T_2) = -m^1(0,T_1)\log\left[\frac{m^1(0,T_1)}{m^0(0,T_1)}\right] - m^1(T_1+1,T_2) \times \log\left[\frac{m^1(T_1+1,T_2)}{m^0(T_1+1,T_2)}\right] - m^1(T_2+1,L_{\max}) \times \log\left[\frac{m^1(T_2+1,L_{\max})}{m^0(T_2+1,L_{\max})}\right] \quad (18)$$

For k-level image segmentation, $k - 1$ threshold values $T_1, T_2, \ldots, T_{k-2}, T_{k-1} \in [0, L_{\max}]$ are selected that segments the image into k regions.

$$O(T_1,T_2,\ldots T_{k-1}) = -m^1(0,T_1)\log\left[\frac{m^1(0,T_1)}{m^0(0,T_1)}\right] - m^1(T_1+1,T_2)\log\left[\frac{m^1(T_1+1,T_2)}{m^0(T_1+1,T_2)}\right] - m^1(T_2+1,T_3)\log\left[\frac{m^1(T_2+1,T_3)}{m^0(T_2+1,T_3)}\right] - \ldots - m^1(T_{k-1}+1,L_{\max})\log\left[\frac{m^1(T_{k-1}+1,L_{\max})}{m^0(T_{k-1}+1,L_{\max})}\right] \quad (19)$$

$$O(T_1,T_2,\ldots T_{k-1}) = -\sum_{i=1}^{k} m^1(T_{i-1},T_i)\log\left[\frac{m^1(T_{i-1},T_i)}{m^0(T_{i-1},T_i)}\right] \quad (20)$$

where $T_0 = 0$ and $T_k = L_{\max}$

3.2. TLBO algorithm to minimize cross entropy for Selecting Multiple threshold values

The formulation of optimization problem is as follows:
Minimize $O(T)$

Where *O(T)* is the objectives function and *T* is a vector for design variables
(T1, T2, ...,T_{k-1})

Subject to constraints

(i) $0 \leqslant T_1, T_{2,\ldots,} \quad T_{k-1} \leqslant L_{\max}$
(ii) $0 \leqslant T_1 \leqslant T_2 \leqslant \ldots \leqslant T_{k-1} \leqslant L_{\max}$

In case of gray level images whose pixel values are 8 bits, $L_{\max} = 255$

In the other words, the objective is to find an optimal combination of $T_1, T_2, \ldots, T_{k-1}$ such that $O(T_1, T_2, \ldots T_{k-1})$ is minimized. Here, objective function is

$$O(T_1, T_2, \ldots T_{k-1}) = -\sum_{i=1}^{k} m^1(T_{i-1}, T_i) \log \left[\frac{m^1(T_{i-1}, T_i)}{m^0(T_{i-1}, T_i)}\right] \tag{21}$$

In case of gray level images that have 256 gray levels, the search space of parameters is as follow:

$(T_1, T_2, \ldots T_{k-1}) \in [0, 255]$

In this paper, Teaching-Learning-based Optimization (TLBO) algorithm is used to find an optimal combination of threshold values for minimizing the cross entropy. The block diagram of the proposed algorithm is shown in Fig. 1 and the detail is introduced as follows:

Initialization
Step 1: Initialize the parameters

- Population size (the number of students or learners): N_p
- Number of iterations: $I_{\max}$
- Number of design variables or parameters (the subjects or courses offered: $k-1$): $T_1, T_2, \ldots, T_{k-1}$
- Limits of design variables: $0 \leqslant T_1, T_2, \ldots T_{k-1} \leqslant 255$

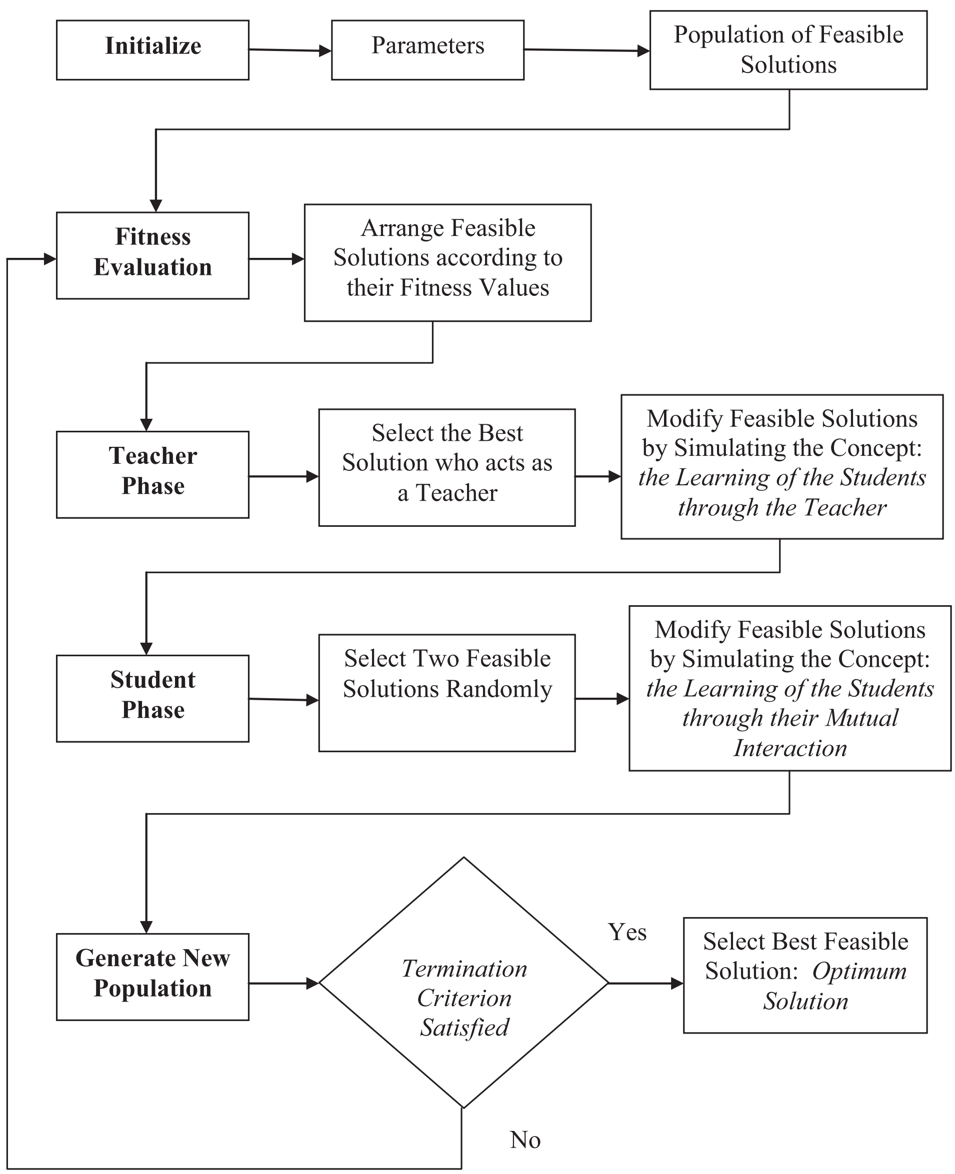

Fig. 1. Block Diagram of Proposed Algorithm.

Step 2: Initialize the population

Generate random population according to the population size and the number of design variables.

$$Pop = X[N_p, k-1]$$

Population is represented by a matrix of size $N_p \times k-1$

$$\{X_{i1}\} \in T_1, \{X_{i2}\} \in T_2, \ldots \{X_{ik-1}\} \in T_{k-1}, \quad i = 1,2,3,\ldots,N_p$$

Population is a random set of individuals. It is not necessary that randomly generated population satisfies the constrain: $0 \leqslant T_1 \leqslant T_2 \leqslant \ldots \leqslant T_{k-1} \leqslant L_{\max}$. To solve this problem, the following mathematical processing is done:

For bi-level segmentation

$$X'_{i1} = X_{i1}$$

$$\{X'_{i1}\} \in T_1$$

For three-level segmentation

$$X'_{i1} = X_{i1}$$

$$X'_{i2} = X'_{i1} + (255 - X'_{i1}) * (X_{i2}/255)$$

$$\{X'_{i1}\} \in T_1, \{X'_{i2}\} \in T_2$$

For four-level segmentation

$$X'_{i2} = X_{i2}$$

$$X'_{i3} = X'_{i2} + (255 - X'_{i2}) * (X_{i3}/255)$$

$$X'_{i1} = X'_{i12} * (X_{i1}/255)$$

$$\{X'_{i1}\} \in T_1, \{X'_{i2}\} \in T_2, \{X'_{i3}\} \in T_3$$

For more than four level segmentation (fifth-level segmentation to k-level segmentation)

$$X'_{i2} = X_{i2}$$

$$X'_{i3} = X'_{i2} + (255 - X'_{i2}) * (X_{i3}/255)$$

$$X'_{i1} = X'_{i2} * (X_{i1}/255)$$

$$X'_{i4} = X'_{i3} + (255 - X'_{i3}) * (X_{i4}/255)$$

$$X'_{ik-1} = X'_{ik-2} + (255 - X'_{ik-2}) * (X_{ik-1}/255)$$

$$\{X'_{i1}\} \in T_1, \{X'_{i2}\} \in T_2, \{X'_{i3}\} \in T_3, \ldots, \{X'_{ik-1}\} \in T_{k-1}$$

Thus, after the above said modification, population is described as follows

$$Pop = X'[N_p, k-1]$$

Hence, initial generated population is as follows:

$$X^I = X'$$

where $I = 0, 1, 2, \ldots, I_{\max}$
Initially, $I = 0$

Fitness Evaluation

Step 3: Evaluate fitness of feasible solutions in the population and arrange these solutions according to their fitness values

Evaluate the value of the objective function for each feasible solution candidate as its fitness (if the numeric value of objective function for a feasible solution is minimum, then it is called the most fit feasible solution)

$$F_i^I = O(X_i^I), I = 1, 2, \ldots, I_{\max} and \quad i = 1, 2, 3, \ldots, N_p$$

Arrange feasible solution candidates of the population in ascending order according to their respective objective function numeric values X_i^I and the corresponding fitness function value F_i^I

$$\text{i.e.} X^I = \begin{bmatrix} X_1^I \\ X_2^I \\ \cdot \\ \cdot \\ X_{N_p}^I \end{bmatrix} \text{ and } F^I = \begin{bmatrix} F_1^I \\ F_2^I \\ \cdot \\ \cdot \\ F_{N_p}^I \end{bmatrix}$$

$$\{X_{i1}^I\} \in T_1, \{X_{i2}^I\} \in T_2, \{X_{i3}^I\} \in T_3, \ldots, \{X_{ik-1}^I\} \in T_{k-1}$$

Teacher Phase

Step 4: Modify solution by simulating the concept: the learning of the students through the teacher

This phase simulates the learning of the students through the teacher. During the teacher phase, the best feasible solution candidate acts as teacher. Other feasible solution candidates are improved by moving their positions towards the position of the teacher by taking into account the current mean value of the feasible solution candidates.

For this, **s**elect the best solution ($X^I_{O(X_i^I)=\min}$) who acts as a teacher for that iteration

$$X^I_{Teacher} = X_1^I = X^I_{O(X_i^I)=\min}$$

Calculate the mean of the population column-wise, which will give the mean for the particular course (design variable or parameter) as

$$M^I = [m_1^I, m_2^I, m_3^I, \ldots, m_{k-1}^I]$$

$$m_1^I = \frac{X_{11}^I + X_{21}^I + \cdots . + X_{N_p1}^I}{N_p}$$

$$m_2^I = \frac{X_{12}^I + X_{22}^I + \cdots + X_{N_p2}^I}{N_p}$$

$$m_3^I = \frac{X_{13}^I + X_{23}^I + \cdots + X_{N_p3}^I}{N_p}$$

$$m_{k-1}^I = \frac{X_{1k-1}^I + X_{2k-1}^I + \cdots + X_{N_pk-1}^I}{N_p}$$

The teacher will try to shift the mean from M^I towards $X^I_{Teacher}$ which will act as a new mean for the iteration. So,

$$M^I_{new} = X^I_{Teacher}$$

The difference between two means is expressed as

$$D^I = r(M^I_{new} - T_F M^I)$$

where T_F is a teaching factor that decides the value of the mean to be changed, and r is a random number in the range [0 1]

Now, the above difference between two means modifies the existing solution according to the following expression

$$X^I_{new\_TP,i} = X^I_{old,i} + D^I$$

where $X^I_{old,i} = X_i^I$

Find the fitness function value $F^I_{new\_TP,i}$ of each $X^I_{new\_TP,i}$, $i = 1, 2, \ldots, N_p$

$If F^I_{new\_TP,i} < F^I_{old,i} \&\& \quad 0 \leqslant X^I_{new\_TP,i1} \leqslant X^I_{new\_TP,i2} \leqslant \ldots \leqslant X^I_{new\_TP,ik-1} \leqslant 255$
  $X^I_{new,i} = X^I_{new\_TP,i}$
Else
  $X^I_{new,i} = X^I_{old,i}$
Endif

where $F^I_{old,i} = F^I_i i = 1, 2, \ldots N_p$

Student Phase

Step 5: Modify solution by simulating the concept: the learning of the students through their mutual interaction

In this phase, students gain knowledge through mutual interaction. A student (X^I_u) tries to improve knowledge by peer learning from an arbitrary student (X^I_v), where, u, v are two integer random numbers belong to $[1, N_p]$ and $u \neq v$. If X^I_v is better than X^I_u, then X^I_u is moved towards X^I_v. Otherwise, X^I_u is moved away from X^I_v. This concept is simulated as follows:

Select two feasible solution candidates X^I_u and X^I_v from X^I_{new}

$If F^I_u < F^I_v$
  $X^I_{new\_SP,u} = X^I_u + r(X^I_u - X^I_v)$
Else
  $X^I_{new\_SP,u} = X^I_u + r(X^I_v - X^I_u)$
Endif

where F^I_u, F^I_v are fitness values of X^I_u, X^I_v respectively.

After this, evaluate the fitness value ($F^I_{new\_SP,u}$) of $X^I_{new\_SP,u}$

If $\quad F^I_{new\_SP,u} < F^I_{new,u} \quad \&\& \quad 0 \leqslant X^I_{new\_SP,u1} \leqslant X^I_{new\_SP,u2} \leqslant \ldots \leqslant X^I_{new\_SP,uk-1} \leqslant 255$
  $X^I_{new,u} = X^I_{new\_SP,u}$
Else
  $X^I_{new,u} = X^I_{new,u}$
Else

Thus, new population is generated as follows:

$X^{I+1} = X^I_{new}$

Step 6: Go to Step 3 until the stopping criteria (maximum iteration: I_{max}) is not met.

Step 7: Stop

Pseudo- Code of TLBO Algorithm to Minimize Cross Entropy for Selecting Multiple Threshold Values

Input:

Population Size: N_p
Number of Iterations: Imax
Number of Variables $(k-1) : T_1, T_2, \ldots, T_{k-1}$
/*generate a random set of N_p individuals (population) that satisfy the constraint: $0 \leqslant T_1, T_2, \ldots T_{k-1} \leqslant 255$*/
Pop = *X* [N_p, $k-1$]

(*continued*)

Input:

$\{X_{i1}\} \in T_1, \{X_{i2}\} \in T_2, \ldots \{X_{ik-1}\} \in T_{k-1}$, i = 1, 2, 3, …,N_p
/*modify population to satisfy the constraint:
$0 \leqslant T_1 \leqslant T_2 \leqslant \ldots \leqslant T_{k-1} \leqslant L_{max}$*/
/*for bi-level segmentation*/
$X'_{i1} = X_{i1}$
$\{X'_{i1}\} \in T_1$
/*for three-level segmentation*/
$X'_{i1} = X_{i1}$
$X'_{i2} = X'_{i1} + (255 - X'_{i1}) * (X_{i2}/255)$
$\{X'_{i1}\} \in T_1, \{X'_{i2}\} \in T_2$
/*for four-level segmentation*/
$X'_{i2} = X_{i2}$
$X'_{i3} = X'_{i2} + (255 - X'_{i2}) * (X_{i3}/255)$
$X'_{i1} = X'_{i12} * (X_{i1}/255)$
$\{X'_{i1}\} \in T_1, \{X'_{i2}\} \in T_2, \{X'_{i3}\} \in T_3$
/*for more than four level segmentation
(fifth-level segmentation to k-level segmentation)*/
$X'_{i2} = X_{i2}$
$X'_{i3} = X'_{i2} + (255 - X'_{i2}) * (X_{i3}/255)$
$X'_{i1} = X'_{i2} * (X_{i1}/255)$
$X'_{i4} = X'_{i3} + (255 - X'_{i3}) * (X_{i4}/255)$
. . ..**......................................
$X'_{ik-1} = X'_{ik-2} + (255 - X'_{ik-2}) * (X_{ik-1}/255)$
$\{X'_{i1}\} \in T_1, \{X'_{i2}\} \in T_2, \{X'_{i3}\} \in T_3, \ldots, \{X'_{ik-1}\} \in T_{k-1}$
/*representation of ith feasible solution candidate of the population*/
$X'_i \leftarrow [X'_{i1} \quad X'_{i2} \quad X'_{i3} \ldots X'_{ik-1}]$
$X'_i = [T_{i1} \quad T_{i2} \quad T_{i3} \ldots T_{ik-1}]$
/*modified population*/
$Pop = X'$
/*initially generated population*/
$X^I = X'$
$I = 0, 1, 2, \ldots, I_{max}$
/*initial*/
I = 0
/*representation of ith feasible solution candidate of the initially generated population*/
$X^I_i \leftarrow [X^I_{i1} \quad X^I_{i2} \quad X^I_{i3} \ldots X^I_{ik-1}]$
$\{X^I_{i1}\} \in T_{i1}, \{X^I_{i2}\} \in T_{i2}, \{X^I_{i3}\} \in T_{i3}, \ldots, \{X^I_{ik-1}\} \in T_{ik-1}$
$X^I_i = [T^I_{i1} \quad T^I_{i2} \quad T^I_{i3} \ldots T^I_{ik-1}]$, i = 1, 2, 3, …, N_p

Output:

/*representation of the best feasible solution candidate with the largest fitness value*/
$X^{I_{max}}_{Best} = X^{I_{max}}_1 = X^{I_{max}}_{O(X^{I_{max}}_i)=\min} = [X^{I_{max}}_{11} \quad X^{I_{max}}_{12} \quad X^{I_{max}}_{13} \ldots X^{I_{max}}_{ik-1}] = [T^*_1 \quad T^*_2 \quad T^*_3 \ldots T^*_{k-1}]$

Begin

I = 0
  for $\quad i = \quad 1 \quad$ *to* $\quad N_p \quad$ *do*
    $X^I_i = X^I[i, k-1]$
    $F^I_i = O(X^I_i)$
  endfor
/*arrange feasible solution candidates of the population in ascending order according to their respective fitness values and find best fit feasible solution candidate along with its fitness value*/
$X^I_{Best} = X^I_1$
$F^I_{Best} = F^I_1$

(*continued on next page*)

(continued)

Input:

```
While(I < I_max)do
Teacher = X^I_Best
for  j = 1tok − 1do
  sum_m_j = 0
endfor
for  i = 1toN_pdo
  sum_m_1 = sum_m_1 + X^I[i, 1]
  sum_m_2 = sum_m_2 + X^I[i, 2]
  .
  .
  sum_m_{k−1} = sum_m_{k−1} + X^I[i, k − 1]
endfor
  m_1 = sum_m_1/N_p
  m_2 = sum_m_2/N_p
  .
  .
  m_{k−1} = sum_m_{k−1}/N_p
  M = [m_1  m_2  m_3.  .m_{k−1}]
  r = rand(0, 1)
  TF = rand(0, 1)
  D = r(Teacher − TF * M)
for  i = 1toN_pdo
  X^I_new_TP,i = X^I_i + D
  F^I_new_TP,i = O(X^I_new_TP,i)
If  F^I_new_TP,i < F^I_i&&  0 ⩽ X^I_new_TP,i1 ⩽ X^I_new_TP,i2
⩽ ... ⩽ X^I_new_TP,ik−1 ⩽ 255
  X^I_new,i = X^I_new_TP,i
else
  X^I_new,i = X^I_i
endif
endfor
do
  u = rand(1, N_p)
  v = rand(1, N_p)
}while(u≠v)
X_u = X^I_new,u
X_v = X^I_new,v
X_u = O(X_u)
X_v = O(X_v)
If  F_u < F_v
  X_SP,u = X_u + r(X_u − X_v)
else
  X_SP,u = X_u + r(X_v − X_u)
endif
F_SP,u = O(X_SP,u)
If  F_SP,u < F^I_u&&0 ⩽ X_SP,u1 ⩽ X_SP,u2 ⩽ ... ⩽ X_SP,uk−1 ⩽ 255
  X^I_new,u = X_SP,u
else
  X^I_new,u = X^I_new,u
endif
/*generation of new population*/
  for  i = 1toN_pdo
  X^{I+1}_i = X^I_new,i
  endfor
/*arrange feasible solution candidates of the new
 population in ascending order according to their
```

(continued)

Input:

```
  respective fitness values and find best fit feasible
  solution candidate*/
   X^{I+1}_Best = X^{I+1}_1
   I = I + 1
  endwhile
End
```

Pseudo-Code of Fitness Value

Input:

```
  /*input image*/
  f = imread ('Input Image')
  /*feasible solution candidates of the population in an
   arbitrary iteration*/
  X_i i = 1, 2, ..., N_p
  /*number of segments required: k*/
  2 ⩽ k < L_max
Output:
  /*fitness values of feasible solutions of the population in an
   arbitrary iteration*/
  F_i,  i = 1, 2, ..., N_p
Begin
  /*find size of input image*/
  [M, N] = size (f)
  /*find the number of pixels in the image whose gray level r,
   r = 0, 1, 2, ...,L_max*/
  h_r,  r = 0, 1, 2, ..., L_max
  /*find the probability distribution of the gray level values of
   the image*/
  p(r) = h_r/(M × N)
  for i = 1 to N_p
    T_1 = X [i, 1]
    T_2 = X [i, 2]
    .
    .
  T_{k−1} = X [i, k − 1]
  /*find cross entropy*/
  S = 0
  for j = 0 to k
    S1_j = 0
    S2_j = 0
    S3_j = 0
    if j = 1
      a = 0
      b = T_j
    else
      if j = k
        a = T_{j−1} + 1
        b = L_max
      else
        a = T_{j−1} + 1
        b = T_j
      endif
    endif
  for r_1 = a to b
    S1_j = S1_j + p(r_1)
    S2_j = S2_j + r_1*p(r_1)
  endfor
  L_j = log(S2_j/S1_j)
```

(*continued*)

Input:

for $r_2 = a$ to b

   $S3_j = S3_j + r_2 * p(r_2) * L_j$

endfor

   $S = S + S3_j$

endfor

$F_i = -S$

endfor

End

4. Experimental results &Analysis

In this section, the proposed TLBO-based minimum cross entropy thresholding (TLBO-based MCET) algorithm is implemented in MATLAB 7.7.0 (R2008b) with 2.2 GHz Intel(R) Core (TM) 2 Duo CPU T7500 machine of 1.99 GB RAM. The fitness function for the proposed work is cross entropy of the input image and segmented image as shown in the following equation:

$$O(T_1, T_2, \ldots T_{k-1}) = -\sum_{i=1}^{k} m^1(T_{i-1}, T_i) \log \left[\frac{m^1(T_{i-1}, T_i)}{m^0(T_{i-1}, T_i)} \right] \tag{22}$$

Here, Teaching-Learning-based Optimization (TLBO) algorithm is used to find an optimal combination of threshold values $(T_1, T_2, \ldots T_{k-1})$ for minimizing the cross entropy.

Five standard test images, named "Lena", "Pepper", "Bird", "Cameraman" and "Goldhill" with size 512×512, 512×512, 204×204, 256×256 and 512×512, respectively, are used for conducting experiments. These original standard images are shown in Fig. 2(a) - 6(a) and their histograms are shown in Fig. 2 (b) - 6(b). The segmented images with different thresholds using the proposed approach are illustrated in Fig. 2(c, e, g, i) – Fig. 6(c, e, g, i) and the performance characteristics of the proposed approach to segment five standard test images with different thresholds are displayed in Fig. 2(d, f, h, j) – Fig. 6(d, f, h, j). As we increase the number of thresholds, the segmented image rapidly tends the original image form the visual point of view. Fig. 2(c, e, g, i) – Fig. 6(c, e, g, i) show that segmented images obtained from the proposed approach at different levels are visually acceptable. Thus, from subjective evaluation point of view, it is observed that the proposed approach (TLBO-based MCET algorithm) has ability to segment images. For comparative analysis, three other algorithms namely Quantum PSO-based MCET, FF-based MCET and HBMO-based MCET are considered. (See Figs. 3–5)

In order to obtain the objective evaluation of the proposed algorithm and the consistent comparison analysis with other methods, two most popular objective elevation parameters, peak signal to noise ratio (PSNR) [17,19] and uniformity [16,18], are used. PSNR is defined as

$$PSNR = 20\log_{10}\left(\frac{255}{RMSE}\right) \tag{23}$$

where RMSE is the root mean-squared error that is defined as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}[f(i,j) - g(i,j)]^2}{M * N}} \tag{24}$$

Here f and g are input and segmented images of size $M \times N$, respectively.

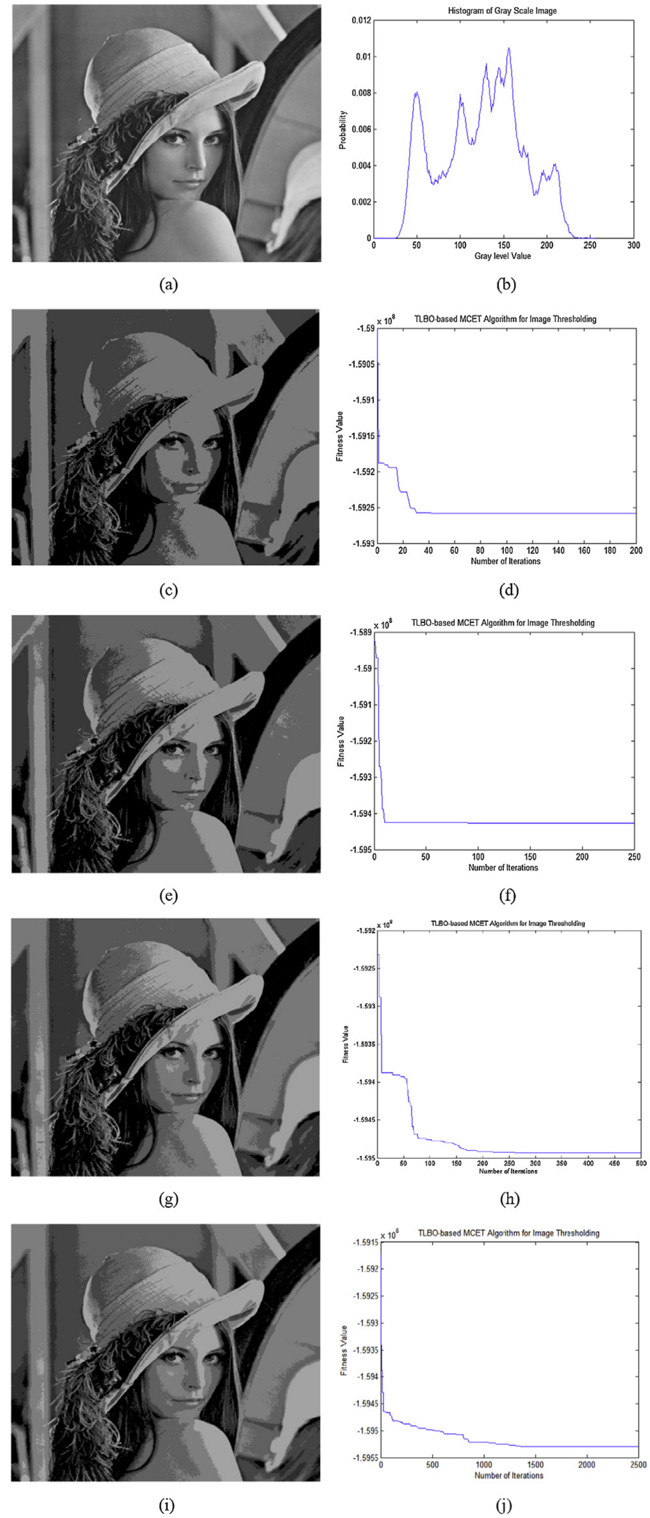

Fig. 2. (a) The test image "Lena", (b) its histogram, (c) 2-level thresholding image by TLBO-based MCET algorithm, (d) the performance characteristics of TLBO-based MCET algorithm at 2-level thresholding, (e) 3-level thresholding image by TLBO-based MCET algorithm, (f) the performance characteristics of TLBO-based MCET algorithm at 3-level thresholding, (g) 4-level thresholding image by TLBO-based MCET algorithm, (h) the performance characteristics of TLBO - based MCET algorithm at 4-level thresholding, (i) 5-level thresholding image by TLBO-based MCET algorithm and (j) the performance characteristics of TLBO - based MCET algorithm at 5-level thresholding.

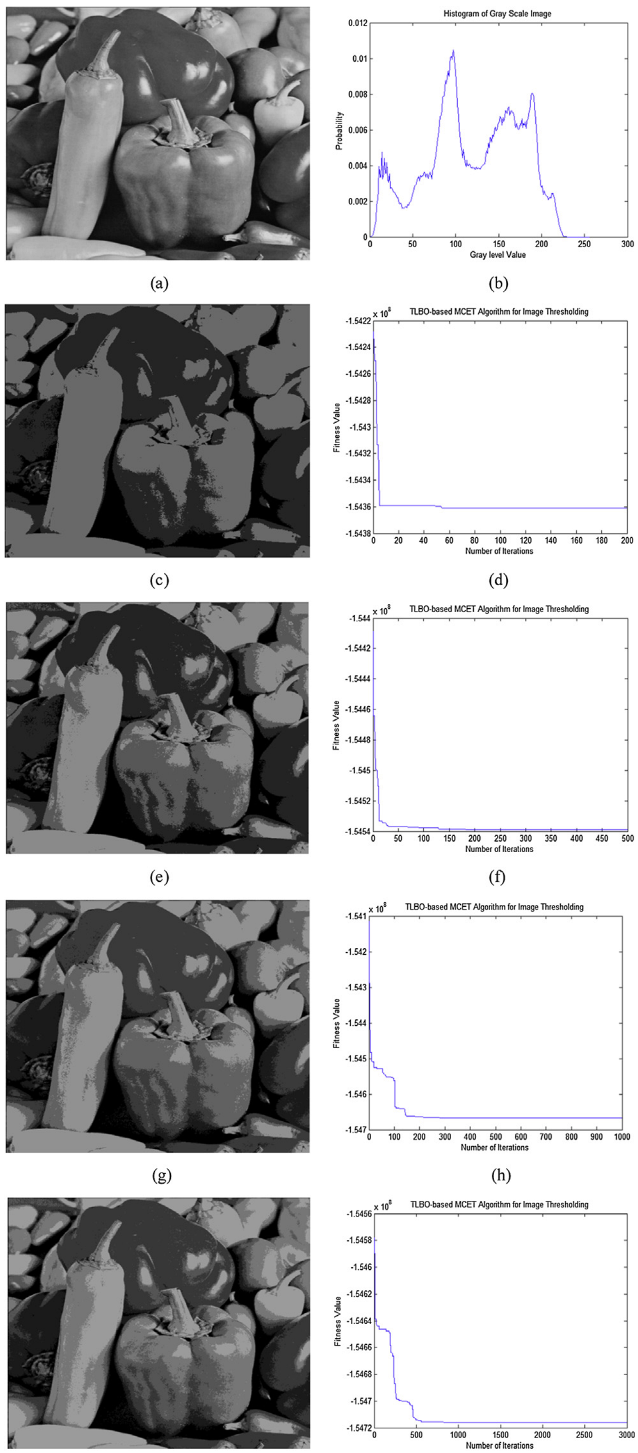

Fig. 3. (a) The test image "Pepper", (b) its histogram, (c) 2-level thresholding image by TLBO-based MCET algorithm, (d) the performance characteristics of TLBO-based MCET algorithm at 2-level thresholding, (e) 3-level thresholding image by TLBO-based MCET algorithm, (f) the performance characteristics of TLBO-based MCET algorithm at 3-level thresholding, (g) 4-level thresholding image by TLBO-based MCET algorithm, (h) the performance characteristics of TLBO - based MCET algorithm at 4-level thresholding, (i) 5-level thresholding image by TLBO-based MCET algorithm and (j) the performance characteristics of TLBO - based MCET algorithm at 5-level thresholding.

PSNR is measured in decibel (dB) and is used to determine the quality of the segmented images. Higher PSNR indicates that the quality of segmented image quality is better.

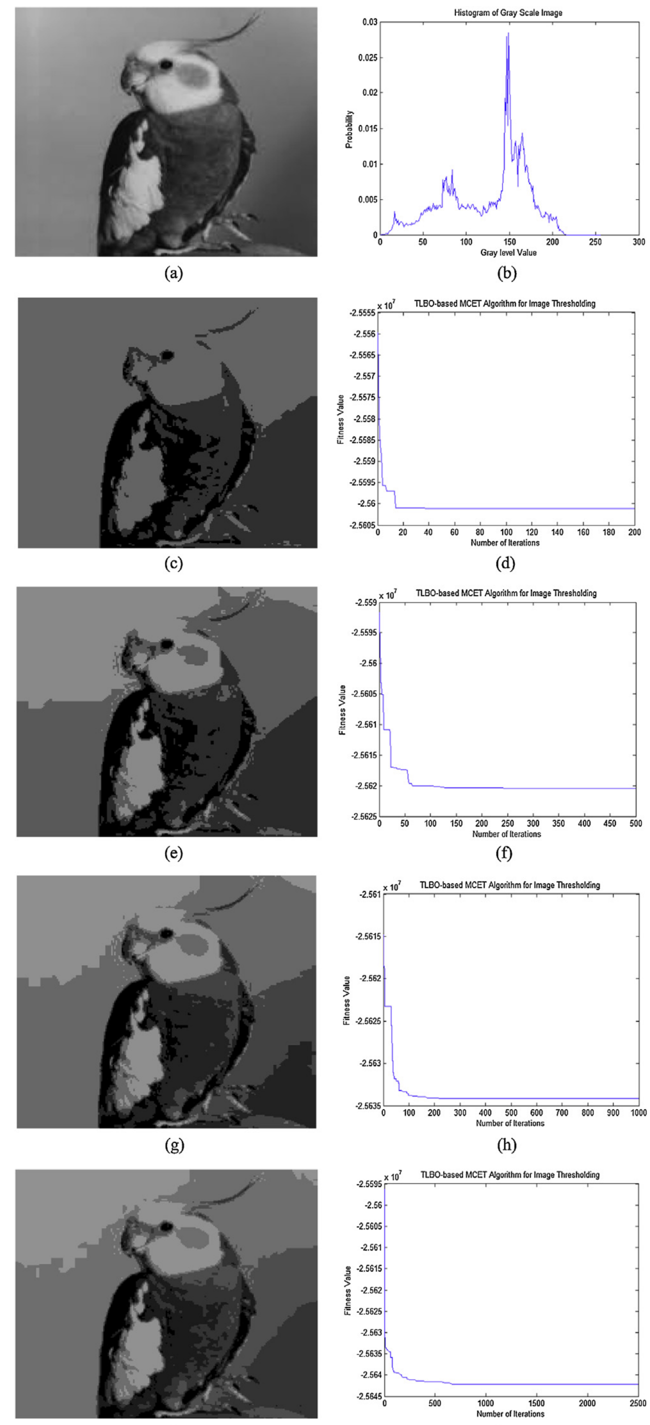

Fig. 4. (a) The test image "Bird", (b) its histogram, (c) 2-level thresholding image by TLBO-based MCET algorithm, (d) the performance characteristics of TLBO-based MCET algorithm at 2-level thresholding, (e) 3-level thresholding image by TLBO-based MCET algorithm, (f) the performance characteristics of TLBO-based MCET algorithm at 3-level thresholding, (g) 4-level thresholding image by TLBO-based MCET algorithm, (h) the performance characteristics of TLBO - based MCET algorithm at 4-level thresholding, (i) 5-level thresholding image by TLBO-based MCET algorithm and (j) the performance characteristics of TLBO - based MCET algorithm at 5-level thresholding.

Uniformity parameter measures region homogeneity in image and it is defined as

$$U = 1 - \frac{\left[2\sum_{k=0}^{1}\sum_{(i,j)\in R_k}\{f(i,j) - \mu_k\}^2\right]}{M * N * (f_{\max} - f_{\min})^2} \tag{25}$$

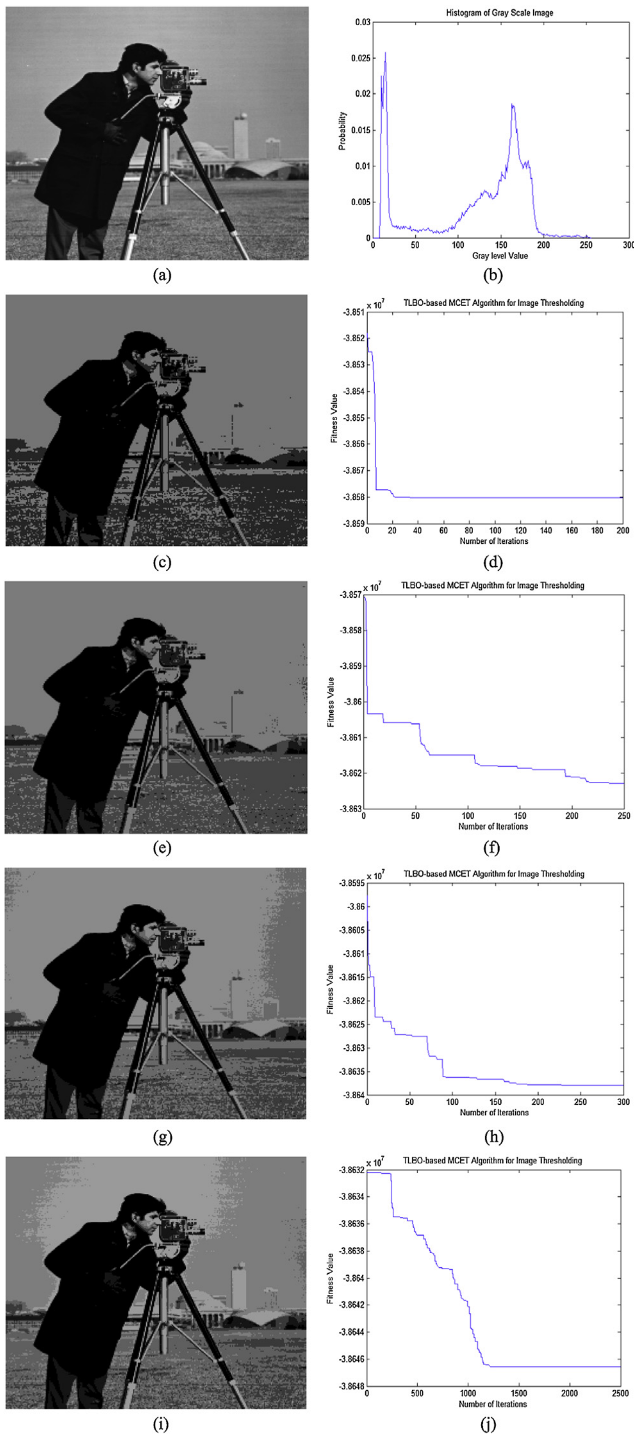

Fig. 5. (a) The test image "Cameraman", (b) its histogram, (c) 2-level thresholding image by TLBO-based MCET algorithm, (d) the performance characteristics of TLBO-based MCET algorithm at 2-level thresholding, (e) 3-level thresholding image by TLBO-based MCET algorithm, (f) the performance characteristics of TLBO-based MCET algorithm at 3-level thresholding, (g) 4-level thresholding image by TLBO-based MCET algorithm, (h) the performance characteristics of TLBO - based MCET algorithm at 4-level thresholding, (i) 5-level thresholding image by TLBO-based MCET algorithm and (j) the performance characteristics of TLBO - based MCET algorithm at 5-level thresholding.

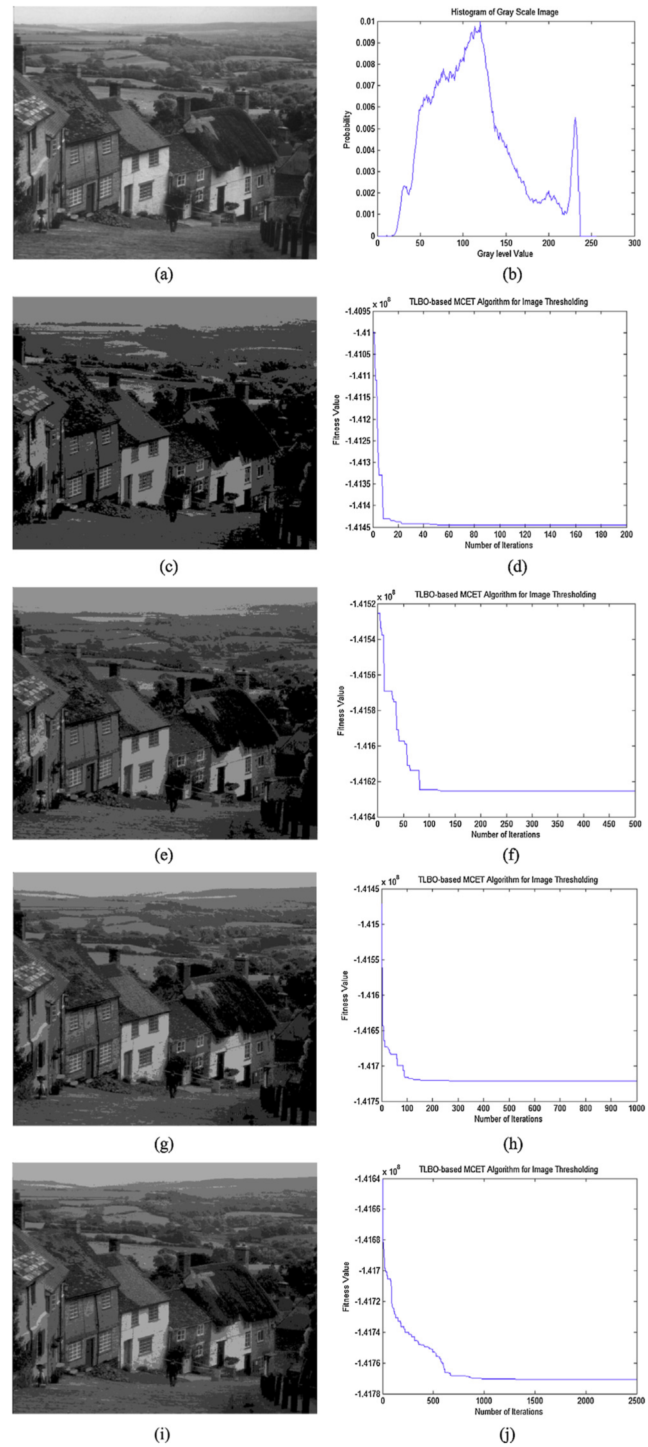

Fig. 6. (a) The test image "Goldhill", (b) its histogram, (c) 2-level thresholding image by TLBO-based MCET algorithm, (d) the performance characteristics of TLBO-based MCET algorithm at 2-level thresholding, (e) 3-level thresholding image by TLBO-based MCET algorithm, (f) the performance characteristics of TLBO-based MCET algorithm at 3-level thresholding, (g) 4-level thresholding image by TLBO-based MCET algorithm, (h) the performance characteristics of TLBO - based MCET algorithm at 4-level thresholding, (i) 5-level thresholding image by TLBO-based MCET algorithm and (j) the performance characteristics of TLBO - based MCET algorithm at 5-level thresholding.

where R_k is the kth segmented region; $f(i,j)$ is the gray level value of pixel (i,j); $f_{\max}$ and f_{min} are the maximum and minimum gray level in the input image, respectively and μ_k is the mean gray level of pixels in kth region that is defined as

$$\mu_k = \frac{\sum_{(i,j)\in R_k} f(i,j)}{n_k} \tag{26}$$

Here n_k is the total number of pixels in the segmented region R_k.The value of the uniformity lies between 0 and 1. For the better segmented image quality, the value of uniformity should be higher.

For evaluating the performance of the proposed TLBO-based MCET algorithm, five standard test images are segmented by this algorithm at 2-level, 3-level, 4-level and 5-level. The performance metrics for checking the effectiveness of the proposed approach are PSNR and uniformity which are used to determine the quality of the segmented images. For comparison, the results of Quantum PSO-based MCET, FF-based MCET and HBMO-based MCET algorithms are also considered for the same five standard test images. Table 1 shows the selected thresholds (2, 3, 4 and 5 threshold values) of the five test images using TLBO-based MCET algorithm, Quantum PSO-based MCET, FF-based MCET and HBMO-based MCET algorithms. PSNR and uniformity values of five segmented images obtained by TLBO-based MCET, Quantum PSO-based MCET, FF-based MCET and HBMO-based MCET algorithms are tabulated in Table 2 and Table 3 respectively. Table 4 shows the fitness values of objective function obtained by TLBO-based MCET, Quantum PSO-based MCET, FF-based MCET and HBMO-based MCET algorithms to segment five standard test images at various levels. Table 2, Table3 and Table 4 provide quantitative standard for evaluating. Such tables show that the number of thresholds increase, the PSNR, the uniformity and the fitness value are enlarged. From Table 1, it is found there is no significance difference in the selected thresholds at various levels for four test images, named "Pepper", "Bird", "Cameraman" and "Goldhill" by TLBO-based MCET algorithm, Quantum PSO-based MCET, FF-based MCET and HBMO-based MCET algorithms. However, there is significance difference in the selected thresholds at various levels for "Lena" standard test image by the proposed approach as compared to other algorithms.

Table 1
Multilevel threshold values obtained through TLBO-based MCET algorithm, GA-based MCET algorithm, FF-based MCET algorithm and HBMO-based MCET algorithm for five standard test images.

Image (Size)	k	TLBO-based MCET Algorithm	Quantum PSO-based MCET Algorithm	FF-based MCET Algorithm	HBMO-based MCET Algorithm
Lena	2	**81, 140**	53, 117	53, 117	53, 117
(512 × 512)	3	**72, 119, 165**	45, 91, 143	46, 95, 150	46, 95, 150
	4	**68, 107, 139, 175**	43, 76, 121, 157	40, 77, 114, 160	40, 77, 114, 160
	5	**59, 86, 115, 144, 178**	30, 55, 92, 107, 157	29, 53, 84, 117, 161	28, 52, 83, 117, 161
Pepper	2	**52, 125**	52, 126	52, 125	52, 125
(512 × 512)	3	47, 107, 157	45, 106, 158	48, 107, 157	48, 107, 157
	4	35, 74, 116, 162	30, 78, 117, 158	35, 75, 117, 163	35, 75, 117, 163
	5	34, 71, 104, 137, 171	34, 72, 111, 140, 173	34, 72, 104, 137, 172	34, 71, 104, 136, 171
Bird	2	60, 118	61, 119	61, 118	61, 118
(204 × 204)	3	56, 109, 155	59, 114, 165	59, 111, 157	59, 111, 157
	4	42, 79, 118, 158	53, 82, 114, 159	45, 83, 122, 160	45, 83, 122, 160
	5	36, 66, 95, 129, 162	35, 73, 103, 135, 168	37, 66, 97, 132, 164	37, 66, 98, 132, 164
Cameraman	2	**50, 135**	50, 138	50, 136	50, 136
(256 × 256)	3	**31, 86, 142**	30, 86, 142	29, 82, 143	29, 82, 143
	4	**27, 75, 123, 156**	32, 80, 117, 155	28, 75, 124, 157	28, 75, 124, 157
	5	**28, 73, 114, 144, 170**	26, 63, 112, 141, 171	27, 70, 112, 144, 171	27, 70, 114, 144, 171
Goldhill	2	84, 148	85, 149	85, 149	85, 149
(512 × 512)	3	68, 107, 162	68, 106, 163	70, 109, 163	70, 109, 163
	4	61, 93, 129, 178	62, 93, 138, 184	62, 94, 130, 179	62, 94, 130, 179
	5	54, 80, 106, 137, 183	53, 82, 105, 133, 187	55, 81, 107, 138, 184	55, 81, 107, 138, 184

Bold values indicate optimal threshold values, searched by proposed algorithm, through which either PSNR or Uniformity is obtained maximum.

Table 2
PSNR values of TLBO-based MCET algorithm, GA-based MCET algorithm, FF-based MCET algorithm and HBMO-based MCET algorithm for five standard test images.

Image (Size)	k	TLBO-based MCET Algorithm	Quantum PSO-based MCET Algorithm	FF-based MCET Algorithm	HBMO-based MCET Algorithm
Lena	2	**15.5306**	14.8638	14.8638	14.8638
(512 × 512)	3	17.3866	**17.7146**	17.6864	17.6864
	4	18.7021	**19.6546**	19.5150	19.5150
	5	20.0644	19.2888	**20.1972**	20.1558
Pepper	2	15.1896	**15.2337**	15.1896	15.1896
(512 × 512)	3	17.6338	17.4696	**17.7093**	**17.7093**
	4	19.9153	19.8414	**19.9644**	**19.9644**
	5	21.6366	21.6358	**21.6751**	21.6136
Bird	2	**16.0486**	16.1355	16.0431	16.0431
(204 × 204)	3	**18.5123**	17.9528	18.4755	18.4755
	4	20.3177	19.7368	**20.5288**	**20.5288**
	5	22.1644	22.2094	**22.2883**	**22.2964**
Cameraman	2	**15.9935**	15.8946	15.9713	15.9713
(256 × 256)	3	**18.7556**	18.7461	18.4850	18.4850
	4	21.1499	**21.3946**	21.1308	21.1308
	5	**22.7206**	22.1507	22.6106	22.5393
Goldhill	2	**14.4587**	14.4082	14.4082	14.4082
(512 × 512)	3	17.0286	**17.0313**	16.8969	16.8969
	4	**18.8143**	18.6798	18.7570	18.7570
	5	**20.5004**	**20.5888**	20.4292	20.4292

Bold value indicates maximum PNSR value.

Table 3
Uniformity of TLBO-based MCET algorithm, GA-based MCET algorithm, FF-based MCET algorithm and HBMO-based MCET algorithm for five standard test images.

Image (Size)	k	TLBO-based MCET Algorithm	Quantum PSO-based MCET Algorithm	FF-based MCET Algorithm	HBMO-based MCET Algorithm
Lena	2	**0.9714**	0.9555	0.9555	0.9555
(512 × 512)	3	**0.9792**	0.9613	0.9632	0.9632
	4	**0.9831**	0.9714	0.9715	0.9715
	5	**0.9843**	0.9609	0.9686	0.9684
Pepper	2	0.9692	**0.9693**	0.9692	0.9692
512 × 512)	3	0.9744	0.9740	**0.9746**	**0.9746**
	4	0.9776	0.9762	**0.9778**	**0.9778**
	5	0.9813	0.9814	**0.9814**	**0.9812**
Bird	2	0.9733	0.9736	**0.9734**	**0.9734**
(204 × 204)	3	0.9774	0.9774	**0.9781**	**0.9781**
	4	0.9788	0.9784	**0.9797**	**0.9797**
	5	0.9812	0.9812	0.9815	**0.9816**
Cameraman	2	0.9824	**0.9826**	0.9825	0.9825
(256 × 256)	3	**0.9827**	0.9826	0.9825	0.9825
	4	0.9844	0.9840	**0.9846**	**0.9846**
	5	**0.9872**	0.9862	0.9870	0.9871
Goldhill	2	0.9703	**0.9706**	**0.9706**	**0.9706**
(512 × 512)	3	0.9731	0.9731	**0.9737**	**0.9737**
	4	0.9787	0.9784	**0.9789**	**0.9789**
	5	0.9806	0.9797	**0.9809**	**0.9809**

Bold value indicates maximum uniformity.

Table 4
Fitness values of TLBO-based MCET algorithm, GA-based MCET algorithm, FF-based MCET algorithm and HBMO-based MCET algorithm for five standard test images.

Image (Size)	k	TLBO-based MCET Algorithm	Quantum PSO-based MCET Algorithm	FF-based MCET Algorithm	HBMO-based MCET Algorithm
Lena	2	−**1.5926e+008**	−1.5909e+008	−1.5909e+008	−1.5909e+008
(512 × 512)	3	−**1.5943e+008**	−1.5930e+008	−1.5930e+008	−1.5930e+008
	4	−**1.5949e+008**	−1.5944e+008	−1.5944e+008	−1.5944e+008
	5	−**1.5953e+008**	−1.5943e+008	−1.5947e+008	−1.5947e+008
Pepper	2	−**1.5436e+008**	−**1.5436e+008**	−**1.5436e+008**	−**1.5436e+008**
(512 × 512)	3	−**1.5454e+008**	−**1.5454e+008**	−**1.5454e+008**	−**1.5454e+008**
	4	−**1.5467e+008**	−1.5466e+008	−**1.5467e+008**	−**1.5467e+008**
	5	−**1.5472e+008**	−1.5471e+008	−**1.5472e+008**	−**1.5472e+008**
Bird	2	−**2.5601e+007**	−**2.5601e+007**	−**2.5601e+007**	−**2.5601e+007**
(204 × 204)	3	−**2.5620e+007**	−2.5619e+007	−**2.5620e+007**	−**2.5620e+007**
	4	−**2.5634e+007**	−2.5632e+007	−**2.5634e+007**	−**2.5634e+007**
	5	−**2.5642e+007**	−2.5641e+007	−**2.5642e+007**	−**2.5642e+007**
Cameraman	2	−**3.8580e+007**	−**3.8580e+007**	−**3.8580e+007**	−**3.8580e+007**
(256 × 256)	3	−**3.8623e+007**	−**3.8623e+007**	−**3.8623e+007**	−**3.8623e+007**
	4	−**3.8638e+007**	−3.8636e+007	−**3.8638e+007**	−**3.8638e+007**
	5	−**3.8647e+007**	−3.8646e+007	−**3.8647e+007**	−**3.8647e+007**
Goldhill	2	−**1.4144e+008**	−**1.4144e+008**	−**1.4144e+008**	−**1.4144e+008**
(512 × 512)	3	−**1.4163e+008**	−**1.4163e+008**	−**1.4163e+008**	−**1.4163e+008**
	4	−**1.4172e+008**	−1.4171e+008	−**1.4172e+008**	−**1.4172e+008**
	5	−**1.4177e+008**	−**1.4177e+008**	−**1.4177e+008**	−**1.4177e+008**

Bold value indicates minimum cross entropy value.

In this case, the proposed approach is superior to the other methods in terms of uniformity and fitness value. Form experimental results, it is possible to appear the fact that the selected thresholds of the TLBO-based MCET algorithm can effectively find the adequate solutions based on the minimum cross entropy criterion. Thus, we can say that the proposed method is an efficient and feasible method to search an optimal combination of threshold values at 2nd, 3rd, 4th and 5th levels.

Table 5 shows the computation time and the number of iterations required to find an optimal combination of threshold values at 2nd, 3rd, 4th and 5th levels for the segmentation of five standard test images. From computational complexity point of view, table values indicate that the proposed algorithm has an ability to find an optimal combination of threshold values at 2nd, 3rd, 4th and 5th levels for the segmentation of standard test images in the reasonable amount of time and iterations.

5. Conclusion and future scope

In this paper, a new multilevel image thresholding approach based on Teacher-Learning-based Optimization, which is named TLBO-based minimum cross entropy thresholding (TLBO-based MCET) algorithm, has been presented to find multilevel optimal threshold values for segmenting gray-scale digital images. Optimal combinations of threshold values at 2nd, 3rd, 4th and 5th levels are searched by TLBO for five standard test images, named "Lena", "Pepper", "Bird", "Cameraman" and "Goldhill".

Table 5
Number of iterations and computation time of TLBO-based MCET algorithm for test images.

Image (Size)	k	Number of Iterations	Computation Time (s)
Lena	2	**47**	**0.9237**
(512 × 512)	3	**92**	**1.8032**
	4	**261**	**5.1156**
	5	**1391**	**27.2636**
Pepper	2	**57**	**1.2027**
(512 × 512)	3	**184**	**3.8824**
	4	**292**	**6.1612**
	5	**859**	**18.1249**
Bird	2	**38**	**0.3717**
(204 × 204)	3	**244**	**2.3668.**
	4	**222**	**2.1534**
	5	**668**	**6.4796**
Cameraman	2	**37**	**0.1624**
(256 × 256)	3	**237**	**1.0402**
	4	**232**	**0.9976**
	5	**1205**	**5.1815**
Goldhill	2	**52**	**0.8892**
(512 × 512)	3	**123**	**2.1033**
	4	**263**	**4.4973**
	5	**1363**	**23.3073**

The major contribution of the proposed approach is the application of the cross entropy based TLBO for gray-scale digital image segmentation. The proposed approach is a new variant of multilevel thresholding algorithm to segment gray-scale digital images by employing the concept of cross entropy. The proposed algorithm selects multilevel optimal threshold values to segment gray-scale digital images. This is done by the concept of cross entropy and framing the problem of threshold selection as an optimization problem. Here, the optimization problem is to minimize the cross entropy between the segmented image and the original image. One of recent optimization techniques named Teaching-Learning-based Optimization (TLBO) has been used to solve the optimization problem. The proposed approach is novel, as the concept of multilevel threshold selection has not been explored using cross entropy and TLBO. To examine the performance of the proposed approach, five different standard digital test images have been segmented through selected threshold values at 2nd, 3rd, 4th and 5th levels by the proposed approach. The simulation and experimental results show the proposed algorithm has an ability to search multilevel optimal threshold values to segment digital images. For evaluating the effectiveness of the proposed algorithm, two measures, namely PSNR, uniformity, are used. PSNR and uniformity are used to measure the quality of the thresholded images. From the experimental results on various types of images, it is observed that the proposed method produces the better quality thresholded images than the compared methods. Thus, the proposed method is an efficient method to search multilevel optimal threshold values for segmenting digital images.

The proposed approach has great potential future in the field of image segmentation. The work is under further progress to segment medical images like mammograms, CT or MR images.

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