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Numerical Simulation of the Airflow over Complex Terrains at Low Altitude

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Abstract

In order to avoid the problems of existing methods, a numerical simulation method for two-dimensional airflow over complex terrains is developed in this paper for the engineering use of flight dynamics. Based on the potential flow theories, the effects of terrains on the wind field are considered by a serial of two-dimensional vortexes, whose strengths are solved by combining with the ground boundary conditions. Numerical examples are studied by the proposed method, and the method is also evaluated by comparing the results with ones from the existing method. The result shows that the two-dimensional profile of complex terrains could be described by a cubic spline curve precisely. The computation procedure proposed in this paper is very simple and efficient, and it could provide a result of wind field with considerable accuracy. Therefore, this method could be used for flight principle evaluation and flight simulators.

Keywords: Complex terrains, Airflow over mountain, Cubic spline, Potential flow theory;

1. Introduction

During take-off and landing, aircrafts are always affected by the atmospheric disturbances at low-altitude. The disturbances will also have a great influence on the flight characteristics of aircrafts that are executing the missions like low-altitude manned penetration. The techniques of wind field modeling and simulation are very important for both of the flight safety and flight dynamic response analysis. As the complex terrain is the main

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reason that induces disturbances at low-altitude, and it is also an important element to establish the low-altitude flight environment in a simulator, so it is necessary to develop a numerical method for the low-altitude wind field simulation which could consider the influence of complex terrains.

There are mainly three different types of low-altitude wind field simulation methods at present. The first one is developed by the atmospheric dynamics theory, which could consider the variation of atmospheric parameters, such as the temperature, energy and pressure. Terry L. Clark[1] and Peng Hu[2] simulate the three-dimensional airflow over mountains by using this method, then compare the results with ones from observation and wind tunnel test and get a good match. However, its computation progress is very complex, which is not suitable for the engineering application of flight dynamics analysis. The second one is the engineering simulation method based on the potential flow theory of fluid dynamics. The assumption of the ideal gas has been adopted, and the temperature variation has been neglected. Guanxin Hong[3] and Gang Liu[4] simulate the two-dimensional airflow over the mountains by the airflow over a cylinder, which could be represented by a fundamental solution of potential flow, one of the streamlines has been chosen as the shape of the terrains. It has a simple process, and could describe the airflow over mountains clearly. But there are only limited kinds of mountain shapes, and need a lot of manual parameter adjust works. The last one is the table lookup method, which means all the data is measured and stored in the computer. It is close to practice, but requires a huge amount of data storage space, and costs a lot of labor and resources.

Based on the potential flow theory of aerodynamics and the boundary conditions of terrain shapes, a numerical simulation method has been proposed according to the engineering application requirements. This method could simulate the wind field at low-altitude that influenced by arbitrary terrain profiles. Moreover, numerical examples have been studied, and the results are compared with the ones mentioned in reference [3] and [4].

2. Theory

2.1. Complex terrain modeling

The Cartesian coordinate system Oxy is used to describe the terrain shape and wind field, where, the origin O is an arbitrary point, the x axis represents the horizontal distance, and the y axis represents the altitude. For the two-dimensional cases, the complex terrains could be represented by a complex curve in plane Oxy (as shown in Fig. 1). In this paper, the terrain curve is simulated by a cubic spline curve. If the cubic spline curve passes through a number of predefined points (x_i, y_i), where, j=1, 2, …, m, and m is the number of the points. Then the cubic spline curve could be described by a serial of third order polynomials

$$y = a_j x^3 + b_j x^2 + c_j x + d_j \qquad (x \in [x_j, x_{j+1}],\ j = 1, 2, \cdots, m-1) \tag{1}$$

where, a_j, b_j, c_j, d_j are the fitting coefficients of the jth section. All of the coefficients could be solved by using the coordinates of the predefined points and the continuity conditions of cubic spline curve.

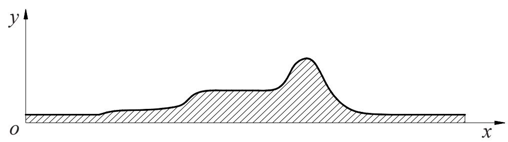

Fig.1 Complex terrains in the reference coordinate system

2.2. Wind field modeling

Similarly to the solving approach of vortex lattice method in aerodynamics, the curve that describes the terrain is divided into a number of straight line elements, and the number is n. A two-dimensional vortex is arranged at the quarter point of each element. Choose the three-quarter point of the element as the colocation point. At this point the actual boundary condition is implemented. Therefore, the effect of the terrains on the wind field could be represented by the sum of induce velocities of all the vortexes at any arbitrary point in the wind field. Use the uniform free stream to represent the wind field without terrains, the stream function is,

$$\psi_0 = -v_0 x + u_0 y \tag{2}$$

where, u_0 and v_0 are two components of the uniform free stream velocity $\boldsymbol{V}_\infty$, and $\boldsymbol{V}_\infty = [u_0\ v_0]^{\mathrm{T}}$.

For the ith element, assume that the vortex strength is Γ_i. The coordinates of vortex is (x_{vj}, y_{vj}), which could be obtained by cubic spline function eq.(1), also does the coordinates (x_{cj}, y_{cj}) and normal vector $\boldsymbol{n}_i$ of the collocation point. Based on potential flow theory, the stream function of ith vortex is

$$\psi_i = -\frac{\Gamma_i}{2\pi} \ln \sqrt{(x - x_{\mathrm{v}i})^2 + (y - y_{\mathrm{v}i})^2} \tag{3}$$

where, $P(x, y)$ is an arbitrary point in the wind field. Therefore the velocity that induced by the ith vortex at point P could be expressed as

$$\begin{aligned} u_{\mathrm{P}i} &= -\frac{(y - y_{\mathrm{v}i})}{2\pi r^2}\Gamma_i \\ v_{\mathrm{P}i} &= -\frac{(x - x_{\mathrm{v}i})}{2\pi r^2}\Gamma_i \end{aligned} \tag{4}$$

where, $r = [(x-x_{vi})^2+(y-y_{vi})^2]^{½}$ is the distance between P and the ith vortex. So the induced velocities of all the collocation points could be represented as

$$\begin{aligned} \boldsymbol{u} &= \boldsymbol{W}_x \boldsymbol{\Gamma} \\ \boldsymbol{v} &= \boldsymbol{W}_y \boldsymbol{\Gamma} \end{aligned} \tag{5}$$

where $\boldsymbol{u}$, $\boldsymbol{v}$ are the vectors that represent the induced velocity components of all the collocation points along the x axis and y axis, respectively; $\boldsymbol{W}_x$, $\boldsymbol{W}_y$ are their influence coefficients matrixes; and $\boldsymbol{\Gamma}$ is vortex strength vector of all the vortex elements.

To search for a singularity distribution that creates enclosed streamlines, the Neumann boundary condition [5] is used. For the collocation point of the ith vortex element, there is

$$(\boldsymbol{V}_\infty + \boldsymbol{V}_i)\boldsymbol{n}_i = 0 \tag{6}$$

where, $\boldsymbol{V}_i$ is the induced velocity at the ith colocation point, $\boldsymbol{V}_i = [u_i\ v_i]^{\mathrm{T}}$. $\boldsymbol{n}_i$ is the normal vector at ith element, $\boldsymbol{n}_i = [n_{xi}\ n_{yi}]^{\mathrm{T}}$. Then expanding eq. (6) yields

$$u_i n_{xi} + v_i n_{yi} = -u_0 n_{xi} - v_0 n_{yi} \tag{7}$$

Express the boundary condition equations of all the collocation points by matrix,

$$\boldsymbol{A}_{\mathrm{AIC}} \boldsymbol{\Gamma} = \boldsymbol{A}_0 \tag{8}$$

where, $\boldsymbol{A}_{AIC}$ is influence coefficient matrix of the normal induced velocity; $\boldsymbol{A}_0$ is constant coefficients vector that relative to the normal direction of the elements and the free stream velocity. Then the strength of the vortexes could be solved as

$$\boldsymbol{\Gamma} = \boldsymbol{A}_{\mathrm{AIC}}^{-1} \boldsymbol{A}_0 \tag{9}$$

Superposition of the effects of uniform free stream and two-dimensional vortexes, the stream function of the wind field with complex terrains could be represented as

$$\psi = \psi_0 + \sum_{i=1}^{n} \psi_i \tag{10}$$

The wind velocity at any arbitrary position could be solved by eq. (10).

3. Numerical examples

Several numerical examples have been studied in this paper, including the examples that proposed by reference [3] and [4].

3.1. Mountain shape simulation

In reference [3], the airflow over a single mountain is simulated by the ideal flow around a cylinder. One of the streamline has been chosen as the shape of the terrains, a factor S is used to get different mountain shapes. And reference [4] simulates the airflow over multiple mountains by superposition of the ideal flow around multiple cylinders. Because of the inherent property of the method, the kind of terrain shapes that could be modelled is limited. Moreover, all the parameters need manual adjustment, in order to get a much more approximate result of the mountain shape.

The flow around a cylinder could be solved by superposing two fundamental solutions of potential flow, which are the doublet and the uniform free stream. Write the speed of uniform free stream as V_∞, which is along the x axis. The number of doublets is l, coordinate of ith doublet is (x_i, y_i), and the radius of the relative cylinder is R_i. Therefore, the stream function of the wind field is,

$$\psi = V_\infty \left[y - \sum_{i=1}^{l} R_i^2 \frac{(y - y_i)}{(x - x_i)^2 + (y - y_i)^2} \right] \tag{11}$$

3.2. Results comparison

- The results of airflow around a single cylinder

Set the position of the cylinder as (0, -300), and R=200m. Select the streamline passes through the point (-∞, 0) as the terrain shape. It could be expressed as

$$\frac{\psi_0}{V_\infty} = y - R^2 \frac{y+300}{x^2 + (y+300)^2} \tag{12}$$

where, ψ_0 is stream function value, and the attitude of the mountain is 100m.

Table 1 Numerical results of the wind velocity that affected by the single mountain

x / m	y /m	V / m/s	θ / °	ΔV	$\Delta\theta$
-2000	300	4.97	0.29	0.24%	0.32%
-1000	300	4.94	1.47	0.20%	-2.71%
0	300	5.56	0.26	0.16%	-
1000	300	4.94	-1.55	0.22%	3.12%
2000	300	4.97	-0.30	0.22%	4.28%

The wind field is defined as $x\in[-3000,3000]$, $y\in[0,500]$. In this area, the terrain is divided into 100 elements. Set V_∞=5 m/s. Fig.2 shows the distribution of the wind velocity over the terrains, and Fig.3 shows streamlines calculated by the proposed method and the analytical solution of the flow around a cylinder. Viscous effects are both not considered. Table 1 lists the magnitude and direction of the wind velocities at several positions; it also lists the difference between the numerical results and the analytical results. Where, (x, y) is coordinates of the position has been analysed, and V is magnitude of the wind velocity, and θ is the angle between the x axis and the wind velocity.

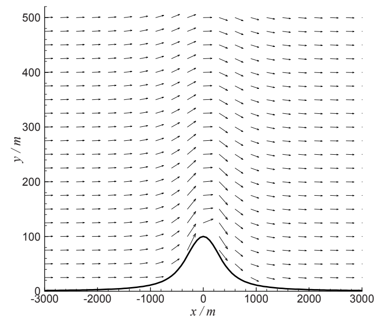

Fig. 2 Wind velocity over the single mountain;

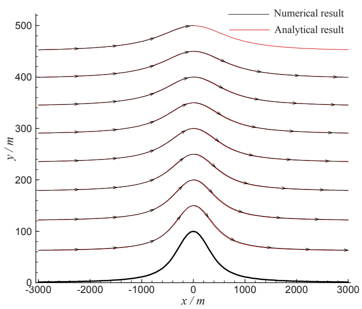

Fig. 3 Streamline over the single mountain

- The results of airflow around multiple cylinders

The position coordinates and geometric parameters of three cylinders are listed in table 2. Also choose the streamline that pass through the point (-∞, 0) as the terrain shape, then the streamline can be expressed as

$$\frac{\psi_0}{V_\infty} = y - \sum_{i=1}^{3} R_i^2 \frac{y - y_i}{(x - x_i)^2 + (y - y_i)^2} \tag{13}$$

Similarly, ψ_0 is the stream function value. It could be solved that the altitude of the three mountain are h_1=110 m， h_2=170 m， h_3=210 m, respectively.

Table 2 Position and geometric parameters of the cylinders

	1	2	3
(x_i, y_i) / m	(-1500, -300)	(0, -300)	(1500, -300)
R_i/ m	200	260	320

The wind field is defined as $x\in[-5000,6000]$， $y\in[0,1000]$. Velocity of uniform free stream is 5 m/s. In this area, the curve of terrain is divided into 200 elements. Fig.4 and Fig.5 shows the results of wind velocity and streamlines over the multiple mountains. As same as table 1， table 3 also lists the magnitude and direction of the wind velocities at several positions, it also lists the difference between the numerical results and the analytical results.

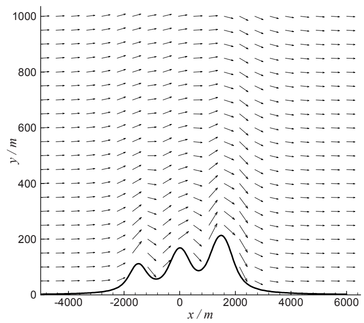

Fig. 4 Wind velocity over the multiple mountains;

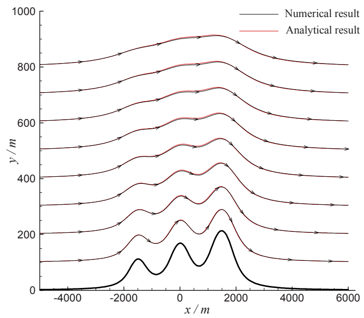

Fig. 5 Streamline over multiple mountains

Table 3 Numerical results of the wind velocity that affected by multiple mountain

x / m	*y /m*	*V / m/s*	*θ / º*	*ΔV*	*Δθ*
-3000	600	4.94	0.97	0.41%	-3.23%
-1500	600	5.17	1.39	0.35%	-1.10%
0	600	5.32	1.02	0.27%	4.81%
1500	600	5.58	-0.85	0.30%	-24.05%

Fig. 2 ~ Fig. 5 indicates that, the methods mentioned in reference [3] and [4] could only get the symmetrical terrain models. Compared with the analytical results, the numerical results obtained by the proposed method match it very well, the errors of the velocity magnitudes are all less than 1%. And the absolute values of the error of the velocity direction are about 5%. And the error could be decreased by increasing the elements number.

3.3. Wind field over arbitrary terrains

Because the ideal flow over a cylinder has a fixed type, the terrain shapes that could be simulated by this method are still limited, even the shape factor is used. Therefore, a lot of parameter adjusting work will be needed for the multiple cylinders superposition method to simulate an arbitrary terrain as shown in fig.1, and it has a pretty low efficiency.

For the terrains shown in fig.1, the sea level range is $x\in[-\infty, 0]$, with a gentle slope at $x\in[0, 1000]$, it transfers to the complex terrains at the range of $x\in[1000, 4000]$. The plain range is $x\in[4000, +\infty]$. The wind field is defined as $x\in[-2000,6000]$， $y\in[0,2000]$. Velocity of uniform free stream is still 5 m/s. Using the straight line and cubic spline curve to describe this complex terrain shape, and the curve of terrain is divided into 400 elements. Fig. 6 and Fig.7 shows the results of wind velocity and streamlines over the complex terrain.

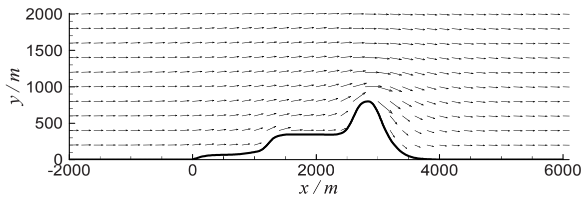

Fig. 6 Wind field affected by the arbitrary terrain

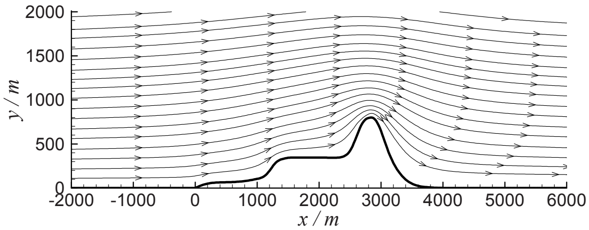

Fig. 7 Streamline over the arbitrary terrain

4. Conclusions

Based on the aerodynamic potential flow theories, a numerical simulation method for two-dimensional airflow over complex terrains is developed in this paper. The effects of terrains on the wind field are considered by a serial of two-dimensional vortexes, and the vortex strengths are solved by combining with the ground boundary conditions. Numerical examples are studied by the proposed method, and the method is also evaluated by comparing the results with ones from the existing method. Conclusions are summarized below.

1) Complex terrain is represented by the combination of the straight lines and cubic spline curves, which could be defined by several predefined pass through points. The two-dimensional profile of complex terrains could be described by this method precisely.

2) Compared with the analytical results of the airflow around a single cylinder and multiple cylinders, the numerical results obtained by the proposed method match the analytical results very well.

3) As a lot of parameter adjusting work will be needed for the multiple cylinders superposition method to simulate an arbitrary terrain, the computation procedure proposed in this paper is very simple and efficient, and could provide a wind field with considerable accuracy, which fits the requirements of engineering application very well. It could be used for flight principle evaluation and flight simulators.

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