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Thermosolutal convection in a viscoelastic dusty fluid with hall currents in porous medium

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ABSTRACT

An incompressible Oldroydian viscoelastic fluid layer heated and soluted from below in the presence of suspended (dust) particles and uniform vertical magnetic field to include the effect of Hall currents in porous medium is considered. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. For the case of stationary convection, Oldroydian viscoelastic fluid behaves like an ordinary Newtonian fluid. Dust particles and Hall currents are found to have a destabilizing effect on the thermosolutal convection, whereas magnetic field is found to have a stabilizing effect on the thermosolutal convection. Medium permeability has both stabilizing and destabilizing effect on the thermosolutal convection under certain conditions. Graphs have been plotted by giving numerical values to the parameters to depict the stability characteristics. The case of overstability is also considered.

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1. Introduction

The growing importance of the use of viscoelastic fluids in technology and industries has led various researchers to attempt diverse flow problems related to several non-Newtonian fluids. Recently an attention has been drawn by calculations of the rheological behavior of dilute suspensions and emulsions to the idealized incompressible viscoelastic liquids whose behavior at small variable shear stresses is characterized by three parameters coefficient of viscosity μ, a relaxation time λ, and a retardation time $\lambda_0(<\lambda)$. A theoretical model is proposed by Oldroyd [1] for a class of viscoelastic fluids. An experimental demonstration by Toms and Strawbridge [2] revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of the Oldroyd fluid. Sharma [3] studied the problem of the thermal instability in a viscoelastic fluid layer in hydromagnetics.

The problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of magnetic field is studied by Bhatia and Steiner [4]. The effect of magnetic field on thermosolutal instability of an Oldroydian viscoelastic fluid in porous medium is considered by Sharma and Bhardwaj [5]. They found that magnetic field has a stabilizing effect on the system while medium permeability has dual effect. In thermal

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Notations

a	dimensionless wave number
c	speed of light
d	depth of layer
D	derivative with respect to z $(= d/dz)e$ Charge of an electron
λ	relaxation time
λ_0	retardation time $(< \lambda)$
$\mathbf{g}(0, 0, -g)$	acceleration due to gravity field
$\mathbf{H}$	uniform magnetic field having components $(0, 0, H)$
(h_x, h_y, h_z)	perturbation in magnetic field having components
$k_x,\ k_y$	horizontal wave numbers
k	wave number
$k_T\vert$	thermal diffusivity
k_s	Solute diffusivity
M	Hall current parameter
N	electron number density
n	growth rate
p	fluid pressure
p_1	Prandtl number
p_2	magnetic Prandtl number
$\mathbf{q}$	velocity of fluid
(u, v, w)	perturbations in fluid velocity
Q	Chandrasekhar number
R	Rayleigh number
t	Time coordinate
T	temperature
C	concentration
$\mathbf{x}(x, y, z)$	space coordinates

Greek Symbols

η	electrical resistivity
α	coefficient of thermal expansion
β	uniform temperature gradient
θ	perturbation in temperature,
γ	perturbation in concentration,
δp	perturbation in pressure$p$,
ρ	fluid density
$\delta\rho$	perturbation in densityρ
μ_e	magnetic permeability
∇	del operator
∂	curly operator

and thermosolutal convection problems, the Boussinesq approximation is used, which is well justified in the case of incompressible fluids. Usually the magnetic field has a stabilizing effect on the instability. A numerical study of the hydromagnetic thermal convection in a viscoelastic dusty fluid in a porous medium is discussed by Goel and Agrawal [6].

The Hall Effect is likely to be important in many geophysical situations as well as in flow of laboratory plasma. There is growing importance of non-Newtonian fluids in chemical technology, industry and geophysical fluid dynamics. The Hall currents have relevance and importance in geophysics, MHD generator and industry. Hall effect on thermosolutal instability of Rivlin-Ericksen fluid with varying gravity field in porous medium is discussed by Sharma and Kishor [7]. Sunil et al. [8] investigated the Hall effects on thermosolutal instability of Walters' (model B') fluid in porous medium and found that magnetic field has a stabilizing effects, whereas the Hall currents have a destabilizing effect on the system. Kumar et al. [9] studied the Rayleigh–Taylor instability of rotating Oldroydian viscoelastic fluids in porous medium in the presence of a variable magnetic field.

The problem on a couple-stress fluid heated from below in hydromagnetics has been studied by Kumar and Kumar [10]. They found that magnetic field has both stabilizing and destabilizing effects on the thermal convection under certain conditions. Singh and Dixit [11] considered the stability of stratified Oldroydian fluid through porous medium in hydromagnetics in presence of suspended particles. Vikrant et al. [12] studied the problem of thermal convection in a compressible Walters' (model B') elastico-viscous dusty fluid with Hall currents and found that Hall currents have destabilizing effect on the system. The effect of Hall currents on thermal instability of compressible dusty viscoelastic fluid in porous medium is discussed by Kumar [13] and found that Hall currents have destabilizing effect on the thermal convection. The instability of the plane interface between two viscoelastic Kuvshiniski superposed fluids in porous in the presence of uniform rotation and variable magnetic field has been considered by Kumar [14].

Wang and Tan [15] considered the stability analysis of Soret-driven double-diffusive convection of Maxwell fluid in a porous medium. Bishnoi and Goyal [16] studied the problem of Soret-Dufour driven thermosolutal instability of Darcy-Maxwell fluid and found that the Dufour number enhances the stability of Darcy-Maxwell fluid for stationary convection as well as overstability. Kumar and Mohan [17] included the double-diffusive convection in an Oldroydian viscoelastic fluid under the simultaneous effects of magnetic field and suspended particles through porous medium.

In the past studies, instability in an Oldroydian viscoelastic fluid layer in porous medium heated and soluted from below has been investigated including the external constraints such as magnetic field and/or rotation. During the survey it was noticed that effect of Hall currents is completely neglected from the studies of Oldroydian viscoelastic dusty fluid in porous medium. Further, magnetic field and medium permeability have dual character. Therefore, an attempt has been made to study the effect of thermosolutal convection in an Oldroydian viscoelastic dusty fluid in presence of Hall currents in porous medium.

2. Formulation on the problem

Consider an infinite layer of an incompressible, finitely conducting (electrically and thermally both) Oldroydian viscoelastic dusty fluid, confined between two horizontal planes situated at $z = 0$ and $z = d$, acted upon by a uniform vertical magnetic field $\mathbf{H}(0, 0, H)$ and gravity field $\mathbf{g}(0, 0, -g)$. The fluid layer is heated and soluted from below leading to an adverse temperature gradient $\beta = \frac{T_0 - T_1}{d}$, where T_0 and T_1 are the constant temperatures of the lower and upper boundaries with $T_0 > T_1$ and$\beta' = \frac{C_0 - C_1}{d}$, where C_0 and C_1 are the constant concentrations of the lower and upper boundaries with $C_0 > C_1$. When the fluid permeated a porous material, the actual path of individual particles of fluid cannot be followed analytically. The gross effect is represented by Brinkman equation as the fluid slowly percolates through the pores of the rock. If ε is the porosity and k_1 is medium permeability then the hydromagnetic equations relevant to the physical model, following Boussinesq approximation, are

$$\frac{1}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\mathbf{q}\right] = -\frac{1}{\rho_0}\left(1+\lambda\frac{\partial}{\partial t}\right)\nabla p + \mathbf{g}\left(1+\lambda\frac{\partial}{\partial t}\right)\times\left(1+\frac{\delta\rho}{\rho_0}\right) + \frac{\upsilon}{\varepsilon}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\nabla^2-\frac{\varepsilon}{k_1}\right)\mathbf{q} + \left(1+\lambda\frac{\partial}{\partial t}\right)\frac{K'N_0}{\varepsilon\rho_0}(\mathbf{q}_d-\mathbf{q}) + \left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\mu_e}{4\pi\rho_0}[(\nabla\times\mathbf{H})\times\mathbf{H}], \tag{1}$$

$$\nabla.\mathbf{q} = 0, \tag{2}$$

$$\varepsilon\frac{d\mathbf{H}}{dt} = (\mathbf{H}.\nabla)\mathbf{q} + \varepsilon\eta\nabla^2\mathbf{H} - \frac{c\varepsilon}{4\pi Ne}\nabla\times[(\nabla\times\mathbf{H})\times\mathbf{H}], \tag{3}$$

$$\nabla.\mathbf{H} = 0. \tag{4}$$

$$mN_0\left[\frac{\partial \mathbf{q}_d}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}_d.\nabla)\mathbf{q}_d\right] = K'N_0(\mathbf{q}-\mathbf{q}_d), \tag{5}$$

$$\varepsilon\frac{\partial N_0}{\partial t} + \nabla.(N_0\mathbf{q}_d) = 0, \tag{6}$$

When the fluid flows through a porous medium, the equation of heat conduction is

$$(\rho c_f\varepsilon + \rho_s c_s(1-\varepsilon))\frac{\partial T}{\partial t} + \rho c_f(\mathbf{q}.\nabla)T + mN_0c_{pt}\left(\varepsilon\frac{\partial}{\partial t} + \mathbf{q}_d.\nabla\right)T = k_T\nabla^2 T. \tag{7}$$

An analogous solute concentration equation is

$$(\rho c_f\varepsilon + \rho_s c_s^{'}(1-\varepsilon))\frac{\partial C}{\partial t} + \rho c_f(\mathbf{q}.\nabla)C + mN_0c_{pt}\left(\varepsilon\frac{\partial}{\partial t} + \mathbf{q}_d.\nabla\right)C = k_s\nabla^2 C. \tag{8}$$

Since density variations are mainly due to variations in temperature and solute concentration, the equation of state for the fluid is given by

$$\rho = \rho_0[1-\alpha(T-T_0)+\alpha'(C-C_0)], \tag{9}$$

where $\mathbf{q}$, ρ, p, T and C denote, respectively the fluid velocity, density, pressure, temperature and concentration and k_T, k_s, α, α' μ_e, N, e, c and η stands for the thermal diffusivity, solute diffusivity, thermal coefficient of expansion, an analogous coefficient of expansion, magnetic permeability, electron number density, charge of an electron, speed of light and electrical resistivity. The suffix zero refers to the values at the reference level $z = 0$. Here, we assume that the distance between particles is quite large as compared with their diameter so that inter-particle reactions need not to be considered. The effect of pressure, gravity and magnetic field on the suspended particles, assuming large distance apart, is negligibly small and therefore ignored and mN_0 being mass of particles per unit volume. $\frac{d}{dt} = \frac{\partial}{\partial t} + q.\nabla$ stands for the convective derivative.

3. Basic state and perturbation equations

In the undisturbed state, let the fluid be at rest. Constants temperatures and concentrations are maintained in the fluid and a constant vertical magnetic field is applied, therefore, the steady state solution is

$$\mathbf{q} = (0,0,0),\ \mathbf{H} = (0,0,H),\ T = T(z),\ C = C(z),\ \rho = \rho(z),$$
$$\text{with } T(z) = T_0 - \beta z, C(z) = C_0 - \beta' z,\ \rho = \rho_0[1+\alpha\beta z - \alpha'\beta' z]$$
$$\text{and } N_0 = N_1 = \text{constant} \tag{10}$$

To use linearized stability theory and normal mode technique, here we assume small perturbations on the steady state solution. Let $\mathbf{q}(u, v, w)$, $\mathbf{h}(h_x, h_y, h_z)$, $\delta\rho$, δp, θ and γ denote, respectively the perturbations in the fluid velocity, magnetic field, density, pressure and temperature and concentration, then the linearized perturbation equations are

$$\frac{1}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial \mathbf{q}}{\partial t} = -\frac{1}{\rho_0}\left(1+\lambda\frac{\partial}{\partial t}\right)\nabla\delta p + \mathbf{g}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\delta\rho}{\rho_0} + \frac{\upsilon}{\varepsilon}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\nabla^2-\frac{\varepsilon}{k_1}\right)\mathbf{q} + \left(1+\lambda\frac{\partial}{\partial t}\right)\frac{K'N_1}{\varepsilon\rho_0}(\mathbf{q}_d-\mathbf{q}) + \left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\mu_e}{4\pi\rho_0}[(\nabla\times h)\times\mathbf{H}], \tag{11}$$

$$\nabla.\mathbf{q} = 0, \tag{12}$$

$$\varepsilon\frac{\partial h}{\partial t} = (\mathbf{H}.\nabla)\mathbf{q} + \varepsilon\eta\nabla^2 h - \frac{c\varepsilon}{4\pi Ne}\nabla\times[(\nabla\times h)\times\mathbf{H}], \tag{13}$$

$$\nabla.\mathbf{h} = 0, \tag{14}$$

$$(E + h_d\varepsilon)\frac{\partial\theta}{\partial t} = \beta(w + h_d s) + k_T\nabla^2\theta, \tag{15}$$

$$(E + h_d^{'}\varepsilon)\frac{\partial\gamma}{\partial t} = \beta'(w + h_d^{'} s) + k_s\nabla^2\gamma, \tag{16}$$

$$\left[\frac{m}{K'}\frac{\partial}{\partial t} + 1\right]\mathbf{q}_d = \mathbf{q}, \tag{17}$$

$$\varepsilon\frac{\partial N_1}{\partial t} + N_0(\nabla.\mathbf{q}_d) = 0, \tag{18}$$

Here $\frac{mN_1}{\rho_0} = f$ is the mass fraction, $h_d = f\frac{c_{pt}}{c_f}$, $h_d^{'} = f\frac{c_{pt}^{'}}{c_f}$, $E = \varepsilon - (1-\varepsilon)\frac{\rho_s c_s}{\rho_0 c_f}$ and ρ_0, c_f, ρ_s, c_s stand for density and heat capacity of fluid and solid matrix, respectively and E' is an analogous solute parameter. The change in density $\delta\rho$ caused by the perturbations θ,γ in temperature and solute concentration at the lower boundary $z = 0$ is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma). \tag{19}$$

Analyzing the perturbations into normal modes, we assume that the perturbation quantities are of the form

$$[w,\theta,\gamma,h_z,\zeta,\xi] = [W(z),\Theta(z),\Gamma(z),K(z),Z(z),X(z)]\exp\times\{ik_x x + ik_y y + nt\}, \tag{20}$$

where k_x and k_y are the wave numbers in x' and y' directions respectively,$k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number of propagation and n is the frequency of any arbitrary disturbance which is, in general, a complex constant. $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and$\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ are the z-components of the vorticity and current density respectively. Using equation (20), equations (11)–(18) in non-dimensional form become

$$\left[\frac{\sigma}{\varepsilon}(1+\sigma F)\left\{1+\frac{f}{1+\sigma\tau_1}\right\}-\frac{1}{\varepsilon}(1+\sigma F_0)(D^2-a^2)+\frac{1}{p_l}(1+\sigma F_0)\right](D^2-a^2)W + (1+\sigma F)\frac{ga^2d^2}{\nu}(\alpha\Theta-\alpha'\Gamma)-(1+\sigma F)\frac{\mu_e Hd}{4\pi\rho_0\nu}(D^2-a^2)DK=0, \tag{21}$$

$$\left[\frac{\sigma}{\varepsilon}(1+\sigma F)\left\{1+\frac{f}{1+\sigma\tau_1}\right\}-\frac{1}{\varepsilon}(1+\sigma F_0)(D^2-a^2)+\frac{1}{p_l}(1+\sigma F_0)\right]Z = (1+\sigma F)\frac{\mu_e Hd}{4\pi\rho_0\nu}DX, \tag{22}$$

$$(D^2-a^2-\sigma p_2)K = -\frac{Hd}{\varepsilon\eta}DW + \frac{cHd}{4\pi N e\eta}DX, \tag{23}$$

$$(D^2-a^2-\sigma p_2)X = -\frac{Hd}{\varepsilon\eta}DZ - \frac{cH}{4\pi N e\eta d}(D^2-a^2)DK, \tag{24}$$

$$(D^2-a^2-E_1\sigma p_1)\Theta = -\frac{\beta d^2}{k_T}\left(\frac{H_d+\sigma\tau_1}{1+\sigma\tau_1}\right)W \tag{25}$$

$$(D^2-a^2-E_1'\sigma q)\Gamma = -\frac{\beta' d^2}{k_s}\left(\frac{H_d'+\sigma\tau_1}{1+\sigma\tau_1}\right)W, \tag{26}$$

where we have put $a = kd$, $\sigma = \frac{nd^2}{\nu}$,$F = \frac{\lambda\nu}{d^2}$,$F_0 = \frac{\lambda_0\nu}{d^2}$, $p_1 = \frac{\nu}{k_T}$,$q = \frac{\nu}{k_s}$, $p_l = \frac{k_1}{d^2}$, $p_2 = \frac{\nu}{\eta}$,$\tau = \frac{\nu}{K}$,$\tau_1 = \frac{\tau\nu}{d^2}$,$H_d = h_d+1$, $H_d' = h_d'+1$,$E_1 = E + h_d\varepsilon$, $E_1' = E' + h_d'\varepsilon$ and $D^* = dD$[(*) is dropped for convenience]. On eliminating various physical parameters from equations (21)–(26), we obtain the final stability governing equation as

$$\begin{aligned}
&\Bigg\{\left\{\frac{\sigma(1+\sigma F)}{\varepsilon}\left\{1+\frac{f}{1+\sigma\tau_1}\right\}-\frac{(1+\sigma F_0)}{\varepsilon}(D^2-a^2)+\frac{(1+\sigma F_0)}{p_l}\right\}\left\{(D^2-a^2-\sigma p_2)^2+M(D^2-a^2)D^2\right\}\\
&+\frac{Q(1+\sigma F)}{\varepsilon}(D^2-a^2-\sigma p_2)D^2\Bigg\}\times\Bigg[\left\{\frac{\sigma(1+\sigma F)}{\varepsilon}\left\{1+\frac{f}{1+\sigma\tau_1}\right\}-\frac{(1+\sigma F_0)}{\varepsilon}(D^2-a^2)+\frac{(1+\sigma F_0)}{p_l}\right\}\\
&(D^2-a^2-E_1\sigma p_1)(D^2-a^2-E'\sigma q)(D^2-a^2)W-Ra^2(1+\sigma F)\left(\frac{H_d+\sigma\tau_1}{1+\sigma\tau_1}\right)(D^2-a^2-E'\sigma q)W\\
&+Sa^2(1+\sigma F)\left(\frac{H_d'+\sigma\tau_1}{1+\sigma\tau_1}\right)(D^2-a^2-E\sigma p_1)W\Bigg]+\frac{Q(1+\sigma F)}{\varepsilon}\Bigg[\Bigg\{\frac{\sigma(1+\sigma F)}{\varepsilon}\left\{1+\frac{f}{1+\sigma\tau_1}\right\}\\
&-\frac{(1+\sigma F_0)}{\varepsilon}(D^2-a^2)+\frac{(1+\sigma F_0)}{p_l}\Bigg\}(D^2-a^2-\sigma p_2)+\frac{Q(1+\sigma F)D^2}{\varepsilon}\Bigg](D^2-a^2-E\sigma p_1)\\
&(D^2-a^2-E'\sigma q)(D^2-a^2)D^2W=0.
\end{aligned} \tag{27}$$

where $R = \frac{g\alpha\beta d^4}{\nu k_T}$ is the Rayleigh number, $S = \frac{g\alpha'\beta' d^4}{\nu k_s}$ is the analogous solute Rayleigh number, $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ the Chandrasekhar number and $M = \left(\frac{cH}{4\pi N e\eta}\right)^2$ is the non-dimensional number accounting for Hall currents.

Here we consider the case of two free boundaries, and the medium adjoining the fluid is electrically non-conducting. The case of two free boundaries is slightly artificial, except in stellar atmospheres and in certain geophysical situations where it is most appropriate, but it allows for an analytical solution. Since both the boundaries are maintained at constant temperature and so the perturbations in the temperature are zero at the boundaries therefore, the appropriate boundary conditions are

$$\begin{aligned}&W=0,\ D^2W=0,\ DZ=0,\ \Theta=0,\ \Gamma=0,\ X=0\\ &\text{and } h_x, h_y, h_z \text{ and are continuous at } z=0,1\end{aligned} \tag{28}$$

The proper solution of equation (27) characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{29}$$

where W_0 is constant. Using equation (29), equation (27) gives

$$
\begin{aligned}
R_1(1+i\sigma_1\pi^2F) = {} & \left(\frac{1+x}{x}\right)\left(\frac{1+i\sigma_1\pi^2\tau_1}{H_d+i\sigma_1\pi^2\tau_1}\right)\left[\left\{\frac{i\sigma_1(1+i\sigma_1\pi^2F)}{\varepsilon}\left\{1+\frac{f}{1+i\sigma_1\pi^2\tau_1}\right\}+\frac{(1+i\sigma_1\pi^2F_0)(1+x)}{\varepsilon}\right.\right.\\
& \left.+\frac{(1+i\sigma_1\pi^2F_0)}{P}\right\}(1+x+i\sigma_1E_1p_1)+\frac{Q_1(1+i\sigma_1\pi^2F)}{\varepsilon}\left\{\left\{\frac{i\sigma_1(1+i\sigma_1\pi^2F)}{\varepsilon}\left\{1+\frac{f}{1+i\sigma_1\pi^2\tau_1}\right\}\right.\right.\\
& \left.\left.+\frac{(1+i\sigma_1\pi^2F_0)}{P}+\frac{(1+i\sigma_1\pi^2F_0)(1+x)}{\varepsilon}\right\}(1+x+i\sigma_1p_2)+\frac{Q_1(1+i\sigma_1\pi^2F)}{\varepsilon}\right\}(1+x+i\sigma_1E_1p_1)\times\\
& \left\{\left\{\frac{i\sigma_1(1+i\sigma_1\pi^2F)}{\varepsilon}\left\{1+\frac{f}{1+i\sigma_1\pi^2\tau_1}\right\}+\frac{(1+i\sigma_1\pi^2F_0)}{\varepsilon}(1+x)+\frac{(1+i\sigma_1\pi^2F_0)}{P}\right\}\left\{(1+x+i\sigma_1p_2)^2+M(1+x)\right\}\right.\\
& \left.\left.+\frac{Q_1(1+i\sigma_1\pi^2F)}{\varepsilon}(1+x+i\sigma_1p_2)\right\}^{-1}\right]+S_1(1+i\sigma_1\pi^2F)\left(\frac{H_d'+i\sigma_1\pi^2\tau_1}{H_d+i\sigma_1\pi^2\tau_1}\right)\left(\frac{1+x+i\sigma_1E_1p_1}{1+x+i\sigma_1E_1'q}\right)
\end{aligned}
\tag{30}
$$

where $R_1=\frac{R}{\pi^4}$, $S_1=\frac{S}{\pi^4}$, $i\sigma_1=\frac{\sigma}{\pi^2}$, $x=\frac{a^2}{\pi^2}$, $P=\pi^2p_l$ and $Q_1=\frac{Q}{\pi^2}$. Equation (30) is the required dispersion relation including the parameters characterizing the dust particles, solute gradient, Hall currents, magnetic field and medium permeability.

4. Stationary convection

When the instability sets in as stationary convection ($\sigma=0$), equation (30) reduces to

$$
R_1=\frac{(1+x)}{H_dx}\left[\left\{\frac{(1+x)}{\varepsilon}+\frac{1}{P}\right\}(1+x)+\frac{Q_1}{\varepsilon}\left\{\left\{\frac{(1+x)}{\varepsilon}+\frac{1}{P}\right\}(1+x)+\frac{Q_1}{\varepsilon}\right\}\left\{\left\{\frac{(1+x)}{\varepsilon}+\frac{1}{P}\right\}(1+x+M)+\frac{Q_1}{\varepsilon}\right\}^{-1}\right]+S_1\frac{H_d'}{H_d},
\tag{31}
$$

Thus, for the case of stationary convection, the relaxation time parameter F and the strain retardation time parameter F_0 vanishes with σ and Oldroydian viscoelastic fluid behaves like an ordinary Newtonian fluid. The above relation expresses the modified Rayleigh number R_1 as a function of the parameters H_d, S_1,M, Q_1, P and dimensionless wave number x. To study the effect of dust particles, solute gradient, Hall currents, magnetic field and medium permeability, we examine the nature of $\frac{dR_1}{dH_d}, \frac{dR_1}{dS_1}, \frac{dR_1}{dM}, \frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dP}$ analytically.

From equation (31), we have

$$
\begin{aligned}
\frac{dR_1}{dH_d}= {} & -\frac{(1+x)}{H_d^2x}\left[\left\{\left\{\left(\frac{(1+x)}{\varepsilon}+\frac{1}{P}\right)(1+x)+\frac{Q_1}{\varepsilon}\right\}^2\right.\right.\\
& \left.+M(1+x)\left(\frac{(1+x)}{\varepsilon}+\frac{1}{P}\right)^2\right\}\left\{\left(\frac{(1+x)}{\varepsilon}+\frac{1}{P}\right)(1+x+M)+\frac{Q_1}{\varepsilon}\right\}^{-1}\\
& \left.+S_1H_d'\right],
\end{aligned}
\tag{32}
$$

which is negative, therefore dust particles have a destabilizing effect on the thermosolutal convection in an Oldroydian viscoelastic fluid.

From equation (31), we have

$$
\frac{dR_1}{dS_1}=\frac{H_d'}{H_d},
\tag{33}
$$

which is positive, therefore solute gradient has a stabilizing effect on the thermosolutal convection in an Oldroydian viscoelastic fluid.

From equation (31), we have

$$
\frac{dR_1}{dM}=-\frac{\frac{(1+x)}{H_dx}\frac{Q_1}{\varepsilon}\left\{\frac{(1+x)}{\varepsilon}+\frac{1}{P}\right\}\left\{\frac{(1+x)^2}{\varepsilon}+\frac{(1+x)}{P}+\frac{Q_1}{\varepsilon}\right\}}{\left[\left\{\frac{1+x}{\varepsilon}+\frac{1}{P}\right\}(1+x+M)+\frac{Q_1}{\varepsilon}\right]^2},
\tag{34}
$$

which is negative, therefore Hall currents have a destabilizing effect on the thermosolutal convection in an Oldroydian viscoelastic fluid.

From equation (31), we have

$$
\begin{aligned}
\frac{dR_1}{dQ_1}= {} & \frac{(1+x)}{H_dx}\left[\left\{\frac{(1+x)}{\varepsilon}+\frac{1}{P}\right\}\left\{\frac{1+x}{\varepsilon}\left\{\frac{(1+x)}{\varepsilon}+\frac{1}{P}\right\}(1+x+M)\right.\right.\\
& +\frac{Q_1}{\varepsilon^2}(1+x+2M)\\
& \left.\left.+\frac{Q_1}{\varepsilon^2}(1+x)\right\}+\frac{Q_1^2}{\varepsilon^3}\right]\left[\left\{\frac{1+x}{\varepsilon}+\frac{1}{P}\right\}(1+x+M)+\frac{Q_1}{\varepsilon}\right]^{-2},
\end{aligned}
\tag{35}
$$

which shows that magnetic field has a stabilizing effect on the thermosolutal convection in an Oldroydian viscoelastic fluid.

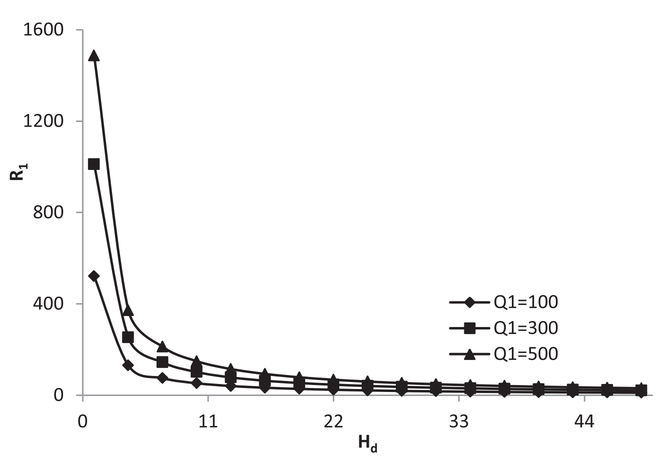

Fig. 1 – Variations of critical Rayleigh numberR_1 withH_d for fixed value of $\varepsilon=0.5$, $M=5$, $P=0.05$, $S_1=20$, $H_d'=10$ and $Q_1=100{,}200{,}300$.

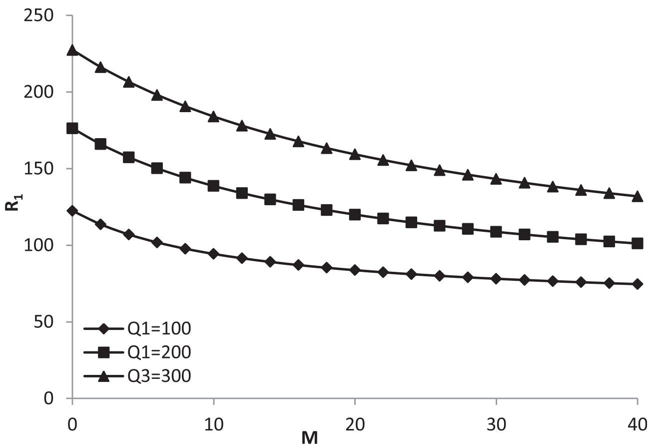

Fig. 2 – Variations of critical Rayleigh number R_1 with M for fixed value of $\varepsilon = 0.5, P = 0.05, S_1 = 20, H_d = 5, H_d' = 10$ and $Q_1 = 100{,}200{,}300$.

In the absence of Hall currents, equation (35) reduces to

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{H_d x \varepsilon}, \tag{36}$$

predicting that magnetic field has also stabilizing effect on thermosolutal convection in an Oldroydian viscoelastic fluid in the absence of Hall currents.

Further equation (31) yields

$$\frac{dR_1}{dP} = \frac{(1+x)}{H_d x P^2} \frac{\left[\frac{MQ_1^2}{\varepsilon^2} - \left\{\left\{\frac{(1+x)}{\varepsilon} + \frac{1}{P}\right\}(1+x+M) + \frac{Q_1}{\varepsilon}\right\}^2 (1+x)\right]}{\left\{\left\{\frac{1+x}{\varepsilon} + \frac{1}{P}\right\}(1+x+M) + \frac{Q_1}{\varepsilon}\right\}^2}, \tag{37}$$

which shows that medium permeability has stabilizing or destabilizing effect on the thermosolutal convection according as

$$\frac{MQ_1^2}{\varepsilon^2} > \text{or} < \left\{\left\{\frac{(1+x)}{\varepsilon} + \frac{1}{P}\right\}(1+x+M) + \frac{Q_1}{\varepsilon}\right\}^2 (1+x)$$

In the absence of Hall currents, equation (37) reduces to

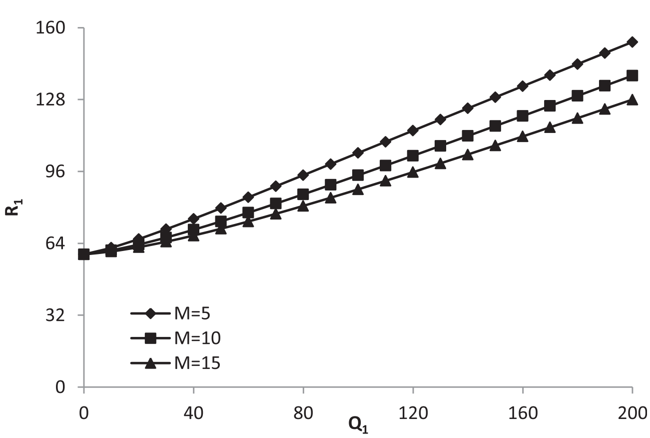

Fig. 3 – Variations of critical Rayleigh numberR_1 withQ_1 for fixed value of $\varepsilon = 0.5, S_1 = 20, P = 0.05, H_d = 5, H_d' = 10$ and $M = 5{,}10{,}15$.

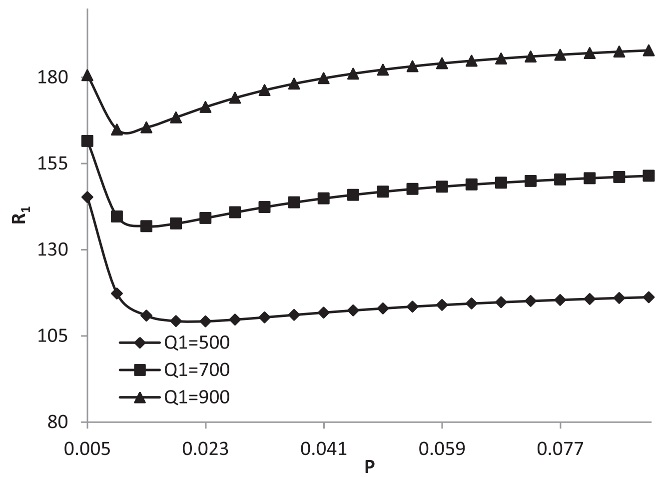

Fig. 4 – Variations of critical Rayleigh number R_1 with P for fixed value of $\varepsilon = 0.5, M = 10, S_1 = 20, H_d = 10, H_d' = 20$ and $Q_1 = 500{,}700{,}900$.

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{H_d x P^2}, \tag{38}$$

which clearly shows that medium permeability has a destabilizing effect on the thermosolutal convection. Thus medium permeability has a dual character, in the absence of Hall currents it has destabilizing effect while in the presence of Hall currents, it has both stabilizing and destabilizing effects on the system.

5. Numerical computation

For the stationary convection critical thermal Rayleigh number for the onset of instability is determined for critical wave

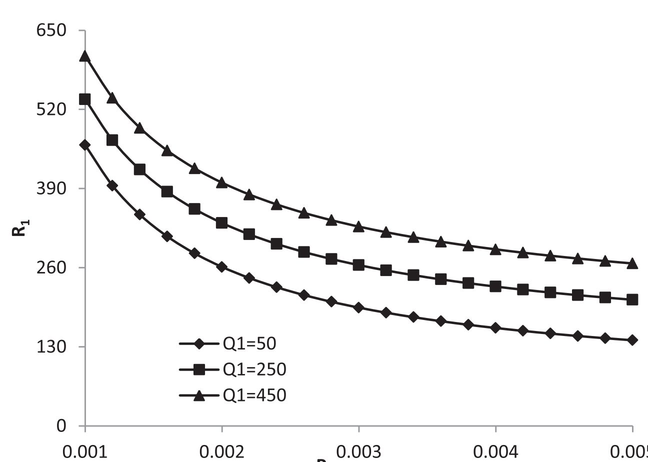

Fig. 5 – Variations of critical Rayleigh number R_1 with P for fixed value of $\varepsilon = 0.5, M = 0, S_1 = 20, H_d = 10, H_d' = 20$ and $Q_1 = 50{,}250{,}450$.

number obtained by the condition $\frac{dR_1}{dx} = 0$ and analyzed numerically using Newton–Raphson method.

In Fig. 1, critical Rayleigh number R_1 is plotted against dust particles parameterH_d for fixed values of $\varepsilon = 0.5,\ M = 5, P = 0.05, S_1 = 20, H_d^{'} = 10$ and $Q_1 = 100{,}200{,}300$ The critical Rayleigh number R_1 decreases with increase in dust particles parameter which shows that dust particles have destabilizing effect on the system.

In Fig. 2, critical Rayleigh number R_1 is plotted against Hall currents parameter M for fixed values of $\varepsilon = 0.5, P = 0.05, S_1 = 20, H_d = 5, H_d^{'} = 10$ and $Q_1 = 100{,}200{,}300$. The critical Rayleigh number R_1 decreases with increase in Hall currents parameter$M$ which shows that Hall current has a destabilizing effect on the system.

In Fig. 3, critical Rayleigh number R_1 is plotted against magnetic field parameter Q_1 for fixed value of $\varepsilon = 0.5, S_1 = 20,\ P = 0.05, H_d = 5, H_d^{'} = 10$ and $M = 5{,}10{,}15$. The critical Rayleigh number R_1 increases with increase in magnetic field parameter which shows that magnetic field has stabilizing effect on the system.

In Fig. 4, critical Rayleigh number R_1 is plotted against medium permeability P for fixed value of $\varepsilon = 0.5, M = 10, S_1 = 20,\ H_d = 10, H_d^{'} = 20$ and $Q_1 = 500{,}700{,}700$. The critical Rayleigh number R_1 decreases up to certain values of P and gradually increases there after which shows that medium permeability has both destabilizing and stabilizing effect on the system.

In Fig. 5, critical Rayleigh number R_1 is plotted against medium permeability P for fixed value of $\varepsilon = 0.5, M = 0, S_1 = 20,\ H_d = 10, H_d^{'} = 20$ and $Q_1 = 50{,}250{,}450$. The critical Rayleigh number R_1 decreases with increase in medium permeability which shows that medium permeability has destabilizing effect on the system.

6. Case of overstability

Here, we discuss the possibility as to whether instability may occur as overstability. Equating real and imaginary parts of equation (26) and eliminating R_1 between them, we obtain

$$A_9c_1^9+A_8c_1^8+A_6c_1^7+A_6c_1^6+A_5c_1^5+A_4c_1^4+A_3c_1^3+A_2c_1^2+A_1c_1+A_0=0, \tag{39}$$

where $c_1 = \sigma_1^2,\ b = 1 + x$ and

$$A_9 = \frac{\pi^{18}F^3\tau^5q^2p_2^4E_1^{'2}}{\varepsilon^2}\left\{\frac{bF\pi^2\tau}{\varepsilon}+\frac{E_1\pi^2\tau F_0p_1}{\varepsilon}\left(\frac{b}{\varepsilon}+\frac{1}{P}\right)\right. \\ \left.+\frac{FE_1p_1}{\varepsilon}(1+f-H_d)\right\}b(b-1), \tag{40}$$

$$\begin{aligned}A_0 = (b-1)\Bigg[&(H_d-1)\pi^2\tau\Bigg\{\left(\frac{2Mb^{10}}{\varepsilon^3}+\frac{2Mb^7}{P^3}+\frac{Q_1^3b^5}{\varepsilon^3}\right)+\left(\frac{6Mb^8}{P\varepsilon}+\frac{3MQ_1b^7}{\varepsilon^3}+\frac{3Q_1^2b^6}{\varepsilon^2}+\frac{MQ_1^2b^5}{\varepsilon^2}+\frac{3MQ_1b^6}{P\varepsilon}+\frac{b^{10}}{\varepsilon^2}+\frac{2b^9}{P\varepsilon}+\frac{b^8}{P^2}+\frac{M^2b^8}{\varepsilon^2}+\frac{2M^2b^7}{P\varepsilon}\right.\\
&\left.+\frac{M^2b^6}{P^2}+\frac{3Q_1b^7}{P\varepsilon}+\frac{3Q_1b^8}{\varepsilon^2}\right)\left(\frac{b}{\varepsilon}+\frac{1}{P}\right)\Bigg\}+(E_1p_1-p_2)\frac{H_dQ_1}{\varepsilon}\left(\frac{b^8}{\varepsilon^2}+\frac{2b^7}{P\varepsilon}+\frac{b^6}{P^2}+\frac{2Q_1b^6}{\varepsilon^2}+\frac{2Q_1b^5}{P\varepsilon}+\frac{Q_1^2b^4}{\varepsilon^2}\right)+\frac{H_dMQ_1^2\pi^2(F-F_0)}{\varepsilon^2}\left(\frac{b}{\varepsilon}+\frac{1}{P}\right)b^5\\
&+\frac{H_d(1+f)}{\varepsilon^3}b^{10}+\frac{2H_d}{\varepsilon^2}\left(\frac{M}{\varepsilon}+\frac{1}{P}\right)(1+f)b^9+\frac{H_d}{\varepsilon}\left(\frac{M^2}{\varepsilon^2}+\frac{2Q_1}{\varepsilon^2}+\frac{4M}{P\varepsilon}+\frac{1}{P^2}\right)(1+f)b^8+\frac{2MQ_1(E_1p_1+p_2)}{P\varepsilon}\Bigg\}b^6+\frac{H_dMQ_1}{\varepsilon}\left(\frac{(E_1p_1+p_2)}{P^2}\right.\\
&\left.+\frac{Q_1(E_1p_1-1)}{\varepsilon^2}\right)b^5+\frac{H_dMQ_1^2}{\varepsilon^2}\left(\frac{E_1p_1}{P}-\frac{fb}{\varepsilon}\right)b^4+\left(\frac{6H_dM}{P^2\varepsilon}+\frac{H_d}{P^3}+\frac{2H_dMQ_1}{\varepsilon^3}+\frac{3H_dM^2}{P\varepsilon^2}+\frac{4H_dQ_1}{P\varepsilon^2}\right)b^7+\left(\frac{2H_dM}{P^3}+\frac{H_dQ_1^2}{\varepsilon^3}+\frac{4H_dMQ_1}{P\varepsilon^2}+\frac{3H_dM^2}{P^2\varepsilon}\right.\\
&\left.+\frac{2H_dQ_1}{P^2\varepsilon}\right)b^6+\left(\frac{H_dM^2}{P^3}+\frac{H_dQ_1^2}{P\varepsilon^2}+\frac{2H_dMQ_1}{P^2\varepsilon}\right)b^5\Bigg\}+S_1(b-1)\Bigg\{(H_d-H_d^{'})\pi^2\tau\left(\frac{b^6}{P}+\frac{2Mb^5}{P}+\frac{M^2b^4}{P}+\frac{b^7}{P}+\frac{2Mb^6}{\varepsilon}+\frac{M^2b^5}{\varepsilon}+\frac{2Q_1b^5}{\varepsilon}\right.\\
&\left.+\frac{2MQ_1b^4}{\varepsilon}\right)\left(\frac{b}{\varepsilon}+\frac{1}{P}\right)+(H_d-H_d^{'})\frac{\pi^2\tau Q_1^2b^4}{\varepsilon^2}+H_dH_d^{'}(E_1p_1-E_1^{'}q)\left(\frac{b^5}{P}+\frac{b^6}{\varepsilon}+\frac{2Mb^5}{\varepsilon}+\frac{2Mb^4}{P}+\frac{M^2b^3}{P}+\frac{M^2b^4}{\varepsilon}+\frac{2Q_1b^4}{\varepsilon}+\frac{2MQ_1b^3}{\varepsilon}\right)\left(\frac{b}{\varepsilon}+\frac{1}{P}\right)\\
&+(E_1p_1-E_1^{'}q)\frac{H_dH_d^{'}Q_1^2b^3}{\varepsilon^2}\Bigg\}\Bigg]\end{aligned} \tag{41}$$

and the coefficients A_1–A_8 being quite lengthy and not needed in the discussion of stability, have not been written here.

Since σ_1 is real for overstability, the nine values of $c_1\,(=\sigma_1^2)$ are positive. The product of the roots $\left(=-\frac{A_0}{A_9}\right)$ is negative and this is to be positive.

It is clear from (40) and (41) that A_0 and A_9 are always positive if

$$1+f>H_d,\ \ H_d>1,\ \ H_d>H_d^{'},\ \ F>F_0,\ \ E_1p_1>b\pi^2(F-F_0),\ \ E_1p_1>1, \\ E_1p_1>p_2,\ \ E_1p_1>\frac{Pfb}{\varepsilon}\ \ \text{and}\ \ E_1p_1>E_1^{'}q \tag{42}$$

The inequalities (42) imply that the sufficient conditions for non-existence of overstability are

$$1+f>H_d,\ H_d>1,\ H_d>H_d^{'},\ F>F_0,\ E_1p_1>b\pi^2(F-F_0),\ E_1p_1>1, \\ E_1p_1>p_2,\ E_1p_1>\frac{Pfb}{\varepsilon}E_1p_1>E_1^{'}q$$

But $F > F_0$, as $\lambda > \lambda_0$, therefore, the sufficient conditions for non-existence of overstability becomes

$$1+f > H_d,\ H_d > 1,\ H_d > H_d',\ E_1p_1 > b\pi^2(F-F_0),\ E_1p_1 > 1,\ E_1p_1 > p_2,\ E_1p_1 > \frac{Pfb}{\varepsilon},\ E_1p_1 > E_1'q$$

i.e

$$c_f > c_{pt},\ \ c_{pt} > c_{pt}',\ \ E\frac{\nu}{k_T} > (1+x)\pi^2\left(\frac{\lambda\nu}{d^2}-\frac{\lambda_0\nu}{d^2}\right), E\frac{\nu}{k_T} > \frac{\nu}{\eta},\ E\frac{\nu}{k_T} > 1,$$

$$E_1\frac{\nu}{k_T} > E_1'\frac{\nu}{k_s},\ \text{and}\ E\frac{\nu}{k_T} > \frac{mN_1(1+x)\pi^2k_1}{\rho_0\varepsilon d^2}.\ \ i.e.\ c_f > c_{pt},\ c_{pt} > c_{pt}'$$

$$\text{and}\ \ E\frac{\nu}{k_T} > \max\left\{(1+x)\pi^2\left(\frac{\lambda\nu}{d^2}-\frac{\lambda_0\nu}{d^2}\right), \frac{\nu}{\eta}, 1, \frac{mN_1(1+x)\pi^2k_1}{\rho_0\varepsilon d^2}, E_1'\frac{\nu}{k_s}\right\}$$

These are, therefore, the sufficient conditions for the non-existence of overstability.

7. Conclusions

In the present paper, we have investigated the effect of Hall currents on an electrically conducting Oldroydian viscoelastic dusty fluid heated and soluted from below in porous medium. Dispersion relation governing the effects of dust particles, solute gradient, Hall currents, magnetic field and medium permeability is derived. The main results obtained from the analysis of this paper are as follows:

(1) For stationary convection, the relaxation time parameter F and the strain retardation time parameter F_0 vanishes with σ and thus an Oldroydian viscoelastic fluid behaves like an ordinary Newtonian fluid.

(2) For the case of stationary convection, suspended (dust) particles and Hall currents are found to have destabilizing effects whereas magnetic field has stabilizing effect on the system.

(3) It is also found, for stationary convection, that the medium permeability has both stabilizing and destabilizing effects on the system in contrast to its destabilizing effect in the absence of the Hall currents. Solute gradient has a stabilizing effect on the thermosolutal convection.

(4) It is also observed from Figs. 1–5 that suspended (dust) particles and Hall currents have destabilizing effects whereas the magnetic field has stabilizing effect on the system. The medium permeability, however, has both stabilizing and destabilizing effects in contrast to its destabilizing effect in the absence of Hall Currents.

(5) The conditions $E\frac{\nu}{k_T} > \max\left\{\left(1+x\right)\pi^2\left(\frac{\lambda\nu}{d^2}-\frac{\lambda_0\nu}{d^2}\right),\ \frac{\nu}{\eta},\ 1, \frac{mN_1(1+x)\pi^2k_1}{\rho_0\varepsilon d^2},\ E_1'\frac{\nu}{k_s}\right\}$, $c_f > c_{pt}$ and $c_{pt} > c_{pt}'$ are the sufficient conditions for the non-existence of overstability.

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