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Composition Theorems for Generalized Sum and Recursively Defined Types

Alexander Rabinovich

School of Computer Science
Tel Aviv University
Tel Aviv 69978, Israel
e-mail: rabinoa@post.tau.ac.il

Composition theorems are tools which reduce reasoning about compound data structures to reasoning about their parts. For example, the truth value of a sentence about the Cartesian product of two structures can be reduced to the truth values of sentences on the components of the product.

A seminal example of a compositional theorem is the Feferman-Vaught Theorem [2]. Feferman and Vaught introduced a generalized product construct which encompasses many algebraic constructions on mathematical structures. Their main theorem reduces the first-order theory of generalized products to the first order theory of its factors and the monadic second-order theory of its index structure.

Shelah [24] defined the notion of a generalized sum and provided the composition theorem which reduces the monadic second-order theory of the generalized sum to the monadic second-order theories of the summands and of its index structure. An important example of generalized sums is the ordinal sum of linearly ordered sets. In [24] the composition theorem for linear orders was one of the main tools for obtaining remarkable decidability results for the monadic theory of linear orders.

In [6] several composition theorems for monadic-second order logic over trees were given.

Two important applications of the compositional methods to algebra and logics are related to

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Decidability. Here the method serves as a very powerful tool to obtain new results as well as to simplify other decidability proofs [2,24,3,6,8,1,12].

Expressibility. Analyze the expressive power of the first-order and monadic second-order languages. The compositional method is useful both for the proofs of the limitation of the expressive power [7,8,9,15,16] and for the proofs of positive results on the expressive power [10,14,15,16,5,18,19].

The composition theorem for linear orders was described in survey expositions by Gurevich [4] and Thomas [26]; it was illustrated there for the decidability of monadic logic of order, as an alternative to the automata-theoretic approach advanced by Büchi. The composition method, despite of its success, is still largely ignored by the theoretical computer science community. W. Thomas surveys in [26] Shelah's composition theorem for linear orders and writes: "Although the subject was exposed in Gurevich's concise survey [4], it did not attract much attention among theoretical computer scientists. Preference was (and is still) given to automata theoretic method . . . because it does not involve frightening logical technicalities as one finds them in [24]. Thus, there is a tendency that the merits of model theoretic approach are overlooked. This is unfortunate, because it excludes some interesting applications."

In this talk in addition to surveying classical compositional results, my main technical contributions will be as follows:

(i) A composition theorem for first-order logic over the generalized sum [20]

(ii) A composition theorem for recursively defined types [21].

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