Available online at www.sciencedirect.com

ScienceDirect

AASRI Procedia 5 (2013) 114 – 119

AASRI Procedia

www.elsevier.com/locate/procedia

2013 AASRI Conference on Parallel and Distributed Computing and Systems

Intuitionistic Fuzzy Real Time Multigraphs for Communication Networks: A Theoretical Model

Siddhartha Sankar Biswas[a*], Bashir Alam[a] and M. N. Doja[a]

[a]*Department of Computer Engineering, Faculty of Engineering & Technology, Jamia Millia Islamia, New Delhi – 110025, INDIA*

Abstract

Many problems of computer science, communication network, transportation systems, etc. can not be modeled into graphs, but into multigraphs only, and then can be easily solved. Nowadays, the networks are expanding very fast in huge volumes in terms of their nodes and links/arcs. For a given alive network, in many situations, its complete topology may not be always available to the communication systems at a given point of time because of the reason that few or many of its links/arcs may be temporarily disable owing to damage or external attack or blockage upon them, and of course they are under repair at that point of time. Besides that, in most of the cases the cost parameters corresponding to its links are not crisp numbers, rather intuitionistic fuzzy numbers (or fuzzy numbers). Thus at any real time instant, the complete multigraph is not available but a submutigraph of it is available to the system for executing its communication or packets transfer. There is no mathematical model available in the existing literature to represent such type of real time network. In this paper the authors propose a mathematical model for such types of multigraphs be called by 'Real Time Multigraphs' (RT-multigraphs) in which all real time information (being updated every q quantum of time) are incorporated so that the communication/transportation system can be made very efficiently with optimal results. It is a theoretical work, a kind of intuitionistic fuzzy mathematical model being the most generalized form of the crisp multigraphs.

© 2013 The Authors. Published by Elsevier B.V. Open access under CC BY-NC-ND license.
Selection and/or peer review under responsibility of American Applied Science Research Institute

Keywords : IFS; IFN; Multigraphs; RT-multigraphs; RT-graphs; Tbl; link status; LSV; LSC; Tbn; rn; Communicable node.

* Corresponding author. Tel.: +91 9910115368 ; fax: +91(11)2698 0229.
E-mail address: ssbiswas1984@gmail.com.

2212-6716 © 2013 The Authors. Published by Elsevier B.V. Open access under CC BY-NC-ND license.
Selection and/or peer review under responsibility of American Applied Science Research Institute
doi:10.1016/j.aasri.2013.10.066

1. Introduction

Graph theory [4, 5, 7, 9] has wide applications in several branches of Engineering, in particular in Computer Science, Communication systems, Civil Engineering, etc. and also in, Science, Social Science, Optimization, Management Science, Medical Science, Economics, etc. to list a few only out of many. The mathematical model 'multigraph' [4, 11, 13, 14] is a generalized concept of graph as multiple links (edges/arcs) may exist between nodes. For example, in a communication model in a Adhoc Network or MANET, multipath features between two adjacent nodes are very common. Two neighboring routers in a network topology might share more than one multiple direct connections between them (instead of just one), so as to reduce the bandwidth as compared to the case of single connection. Many real life important problems of communication network, transportation network, etc. cannot be modeled into graphs, but can be well modeled into multigraphs. Besides that, in most of the cases of such directed multigraphs, the real data about the weights of the arcs are not always crisp but intuitionistic fuzzy (or fuzzy) numbers. In this paper we propose a very generalized notion of 'multigraphs' which is a highly flexible and appropriate model as it incorporates the real time information of the network problem to facilitate the decision maker to search for an efficient and optimized results/solutions. We call such multigraphs by 'Real Time Multigraphs' or 'RT-multigraphs'. Clearly a RT-multigraph is a variable representation of a network with respect to time. In this network modeling we make an application of Atanassov's intuitionistic fuzzy numbers (IFN).

2. Preliminaries

In this section we present basic preliminaries on the IFS theory of Atanassov [1-3] and also on the existing notion of multigraphs [4, 11, 13, 14].

2.1. Intuitionistic Fuzzy Set (IFS)

The Intuitionistic fuzzy set (IFS) theory of Atanassov [1-3] is now a well known powerful soft computing tool to the world. If X be a universe of discourse, an intuitionistic fuzzy set A in X is a set of ordered triplets $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A, \nu_A : X \rightarrow [0, 1]$ are functions such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1 \ \forall x \in X$. For each $x \in X$ the values $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of nonmembership of the element x to $A \subset X$, respectively, and the amount $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the hesitation part. Of course, a fuzzy set is a particular case of the intuitionistic fuzzy set if $\pi_A(x) = 0$, $\forall x \in X$. For details of the classical notion of intuitionistic fuzzy set (IFS) theory, one could see the book [2] authored by Atanassov.

2.2. Multigraph

A multigraph G is an ordered pair (V, E) which consists of two sets V and E, where V or V(G) is the set of vertices (or, nodes), and E or E(G) is the set of edges (links or arcs).

Here, although multiple edges or arcs might exist between pair of vertices but in our discussion in this paper we consider that no loop exists.

Multigraphs may be of two types: undirected multigraphs and directed multigraphs. In an undirected multigraph the edge (i, j) and the edge (j, i), if exist, are obviously identical unlike in the case of directed multigraph. For a latest algebraic study on the theory of multigraphs, the work [14] and also [4, 11, 13] may be seen.

Fig.1.(a) shows below a directed multigraph G = (V, E), where V = {A, B, C, D} and E = {AB_1, AB_2, BA, AD, AC, CB, BD, DB}.

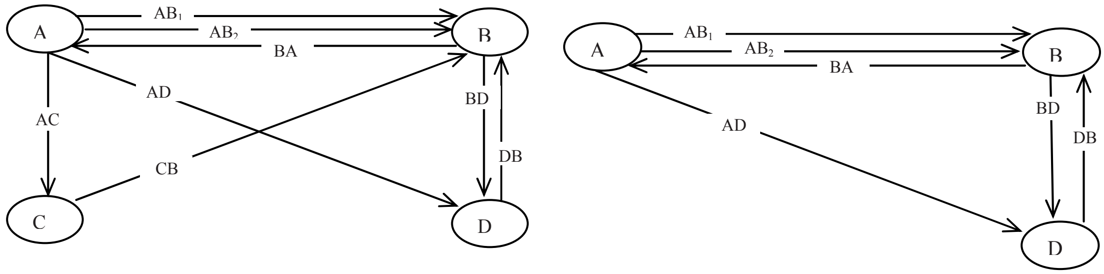

Fig. 1. (a)A multigraph G; (b) A submultigraph H of G

A multigraph H = (W, F) is called a submultigraph of the multigraph G = (V, E) if $W \subseteq V$ and $F \subseteq E$. The Fig.1.(b) shows a submultigraph H of the multigraph G. We now consider the modeling of a very real situation of networks and define a generalized notion of multigraphs/graphs.

3. Real Time Multigraph (RT-Multigraph) : An Intuitionistic Fuzzy Model

In most of the real life problems of networks, be it in a communication model or a transportation model, the weights of the arcs are not always crisp but intuitionistic fuzzy numbers (IFNs) or at best fuzzy numbers. For example, the Fig.2 below shows a public road transportation model for a traveller where the cost parameters for travelling each arc have been considered as IFN which are the more generalized form of fuzzy numbers involving two independently estimated degrees: degree of acceptance and a degree of rejection.

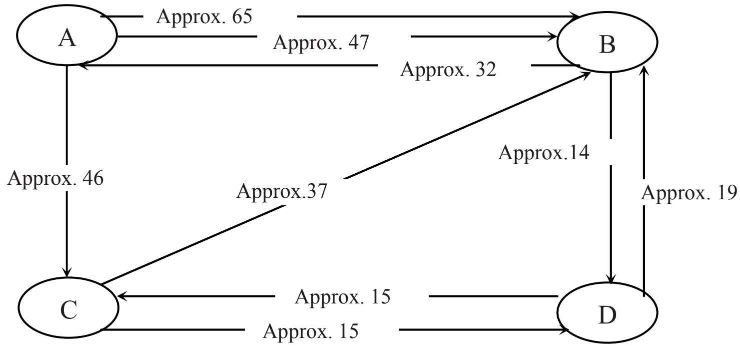

Fig.2. A multigraph G with IF weights (IFNs) of arcs.

3.1. 'Neighbour' node

For a given node u the node v will be designated as a **'neighbour'** node of u if u has at least one link from u to v. In our work here we consider more real situations which are actually and frequently faced by the present communication systems. For example, consider an Adhoc Network or a MANET where there may exist multiple paths between two neighbour nodes, but because of some reasons one or more number of paths may be temporarily damaged and hence temporarily unavailable for transmission of packets by a node u to its neighbour node v. This is a very useful information to the communication system if available to the sender node in advance.

Consider the following directed RT-Multigraph G where the IF weights (IFNs) are shown against each link. We want to solve the single-source IF shortest paths problem taking the vertex A as the source vertex and the vertex D as the destination vertex, where and the LSCs of all the nodes of the RT-multigraph G are $\mathbf{I_A}$, $\mathbf{I_B}$, $\mathbf{I_C}$ and $\mathbf{I_D}$ given by :

$\mathbf{I_A} = \{ I_{AB} , I_{AC} \}$ where $I_{AB} = (0, 1)$, $I_{AC} = (1)$; $\mathbf{I_B} = \{ I_{BC} , I_{BD} \}$ where $I_{BC} = (1)$, $I_{BD} = (1)$; and
$\mathbf{I_C} = \{ I_{CB} , I_{CD} \}$ where $I_{CB} = (1)$, $I_{CD} = (1, 0)$;

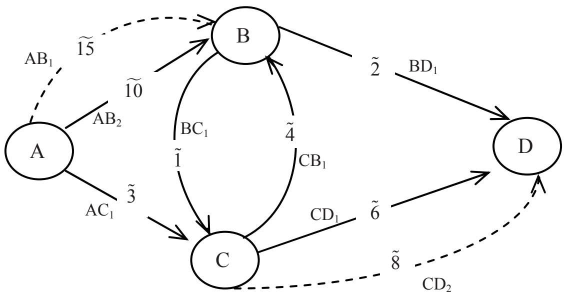

Fig.3. A Multigraph G having few links damaged temporarily.

In our proposed mathematical model of multigraphs, we incorporate the real time data from the network to make the multigraphs more dynamic, more useful, and hence more efficient to the users. Every node of the multigraph carries an information vector corresponding to each of its neighbour nodes. If the node u has the node v as a neighbour node then u carries the following information handy with it :-

(i) Suppose that there are n ($\geq$1) number of links from u to v outward which are $uv_1, uv_2, \ldots\ldots, uv_n$. Let u designate them as $1^{st}, 2^{nd}, 3^{rd}, \ldots\ldots, n^{th}$.

(ii) In real life situation, because of natural phenomenon (flood, earthquake, thunderstorm, solar storm, etc. etc.) or because of some kind of external attack or technical failure or because of an predictable/unpredictable damage of the link, etc. (to list a few only out of many such type of real life problems), it may happen in reality that during a period of time the r^{th} link uv_r of the node u to its neighbour v is non-functional ($r = 1, 2, 3, \ldots, n$). In our proposed model, this is a precious information and is available with the node u here in advance.

3.2. Link Status Vector (LSV)

Corresponding to every neighbour node v, there exist a **Link Status Vector (LSV)** $I_{uv} = (i_1, i_2, i_3, \ldots\ldots, i_n)$ of u, where at any given point of time i_r takes any of the two values from $\{0,1\}$ for $r = 1, 2, 3, \ldots, n$ with the following significance :-

$i_r = 0$, if the link uv_r is non-functional.
$= 1$, if the link uv_r is functional.

3.3. Temporarily Blocked Link (tbl) and Link Status

If at a given time i_r happens to be 0, i.e. if the link uv_r is non-functional then we say that the link uv_r is a **temporarily blocked link (tbl)** from u. The value i_r is called the **'link status'** of the link uv_r.

In real situation the complete multigraph thus may not be available due to existence of non-functional status for few links, i.e. due to existence of few tbls and consequently a sub-multigraph of it be available for communication (Example: for communication of packets in an Adhoc Network/ MANET, or for a salesman to travel many cities, or for a buss/truck carrying goods/passengers in a transportation network, etc.).

(iii) If a node u has k (≥0) number of neighbour nodes v1, v2, v3,…,vk, then u carries k number of LSV: I_{uv1}, I_{uv2} , I_{uv3} ,……,I_{uvk}. In our mathematical model, we propose that there is a system S for the multigraph which updates all the information vectors of all the nodes after every quantum time τ. This quantum τ is fixed (can be reset) for the system S a multigraph, but different for different multigraphs, in general depending upon the various properties of the physical problem for which a multigraph is modelled.

3.4. Link Status Class (LSC)

For a given node u, the collection of all LSV are called 'Link Status Class' (LSC) of u denoted by $\mathbf{I_u}$. If a node u has k (≥0) number of neighbour nodes x1, x2, x3,..…, xk, then

$$\mathbf{I_u} = \{I_{ux1}, I_{ux2}, I_{ux3}, \ldots\ldots, I_{uxk}\}.$$

3.5. Temporarily Blocked Neighbour & Reachable Neighbour

If v is a neighbour node of a given node u, and if I_{uv} is a null vector at a given instant of time then v is called a **temporarily blocked neighbour (tbn)** of u for that instant.

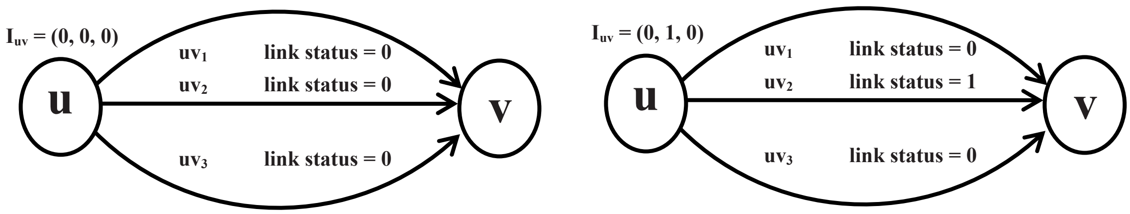

Fig.4. (a) A tbn v of the node u; (b) A rn v from the node u.

However, since it is a temporary phenomenon, and if any of the links be repaired in due time, then obviously a 'blocked neighbour' may regain its 'neighbour' status at some later stage. If a neighbour v is not a tbn, then it is called a **reachable neighbour** (rn) of u.

3.6. Communicable Node

For a given node u, if $I_u \neq \varphi$ and at least one member of I_u is non-null at a given time, then the node u is called a communicable node for that instant of time. If u does not have any neighbor node then $I_u = \varphi$, and in that case it is trivial that further communication is never possible. However, if $I_u \neq \varphi$ and all the members of I_u are null vectors at a point of time, then it signifies that further communication is not possible temporarily.

(iv) All the real time information mentioned/defined above will get automatically updated at every node of the multigraph at every q quantum of time (for a quantum q to be pre-fixed depending upon the properties of the network, on what kind of communication/transportation it is performing).

We call such type of multigraphs by **'Real Time Multigraphs'** or **'RT-multigraphs'** as they contain real time information of the networks. Consequently, for a given network the RT-multigraph is not a static multigraph but changes with time. As a special case, if a network can be modelled into a graph then we call our proposed model as **'Real Time Graph'** or **'RT-graph'**.

4. Conclusion & Future Work

Multigraph is a mathematical model which is a generalization of graph. There are many real life problems of network, transportation, communication, etc. which cannot be modeled into graphs but into multigraphs only. In real life situations, in many of these directed multigraphs, the weights of the arcs are not always crisp but intuitionistic fuzzy or fuzzy. Besides that, due to some reason few links may be temporarily unavailable to the communication system. Consequently, the complete topology of the multigraph of a network may not be available to the system but a submultigraph of it. In this paper we make a mathematical modeling of such real time status of a network by introducing a generalized structure called by 'RT-multigraph' or 'RT-graph'. The real time data/information is updated at each node of the RT-multigraph at every quantum time q. This proposed theoretical model "Intuitionistic Fuzzy RT-multigraph' is a new concept on the generalization of multigraphs or graphs considering its huge potential for real time applications in communication or transportation systems. There is no attempt made so far in the literature for searching an IF shortest path in a multigraph. Our next research work will be to develop a method to find IF shortest path from a source vertex to a destination vertex of a directed RT-multigraph.

Corresponding Author:

Siddhartha Sankar Biswas , Email : ssbiswas1984@gmail.com , Mobile: (+91) 9910115368

References

[1] Atanassov, K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, Vol. 20, 1986, 87–96.
[2] Atanassov, K., Intuitionistic Fuzzy Sets: Theory and Applications, Physica-Verlag. 1999.
[3] Atanassov, K., "More on intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 33, pp.37-46, 1989.
[4] Balakrishnan, V. K., Graph Theory, McGraw-Hill; 1997.
[5] Bollobas, Bela, Modern Graph Theory, Springer; 2002.
[6] D.F. Li, A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to MADM problem, Computer and Mathematics with Applications 60 (2010) 1557-1570.
[7] Diestel, Reinhard., Graph Theory, Springer 2000
[8] Dubois,D. and Prade,H. , Fuzzy Sets and Systems, Academic Press, New York, 1980.
[9] Harary, Frank., Graph Theory, Addison Wesley Publishing Company, 1995
[10] Jenson P, Barnes J., Network Flow Programming, John Wiley and Sons, New York. 1980
[11] J. Ivančo, Decompositions of multigraphs into parts with the same size, Discussiones Mathematicae Graph Theory 30(2)(2010), 335-347.
[12] M. G. Karunambigai, Parvathi Rangasamy, K.Atanassov, N. Palaniappan, An Intuitionistic Fuzzy Graph Method for Finding the Shortest Paths in Networks, Advances in Soft Computing Volume 42, 2007, pp 3-10.
[13] Mariusz Meszka and Zdzislaw Skupien, Decomposition of a Complete Multigraph into Almost Arbitrary Paths, Discussiones Mathematicae Graph Theory32(2)(2012) 357–372.
[14] Siddhartha Sankar Biswas, Basir Alam and M.N. Doja, A Theoretical Characterization of the Data Structure 'Multigraphs', Journal of Contemporary Applied Mathematics, Vol2(2) December'2012, pp 88-106.
[15] Zadeh, L.A., Fuzzy Sets, Inform. And Control, Vol.8(1965) 338-353.