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ORIGINAL ARTICLE

Generalized production planning problem under interval uncertainty

Samir A. Abass [a,*], Mohamed Ali Gomaa [b], Gaber A. Elsharawy [b], Marwa Sh. Elsaid [c]

[a] *Atomic Energy Authority, Cairo, Egypt*
[b] *Department of Mathematics, Faculty of Science, ElAzhar University (Girls), Cairo, Egypt*
[c] *Institute of Culture and Science, 6th October city, Cairo, Egypt*

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Abstract Data in many real life engineering and economical problems suffer from inexactness. Herein we assume that we are given some intervals in which the data can simultaneously and independently perturb. We consider the generalized production planning problem with interval data. The interval data are in both of the objective function and constraints. The existing results concerning the qualitative and quantitative analysis of basic notions in parametric production planning problem. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind.

* Corresponding author.
E-mail addresses: Samir.abdou@gmail.com (S.A. Abass), Gelsharawy@hotmail.com (M.A. Gomaa), Gomaa_sc1@yahoo.com (G.A. Elsharawy), Marwa_shehata2003@yahoo.com (M.Sh. Elsaid).

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1. Introduction

A production planning problem exists because there are limited production resources that cannot be stored from period to period. Choices must be made as to which resources to include and how to model their capacity and behavior, and their costs. Also, there may be uncertainty associated with the production function and the constraints. The production planning problem starts with a specification of customer demand that is to be met by the production plan. One might only include the most critical or limiting resource in the planning problem, e.g. a bottleneck. Alternatively, when there is not a dominant resource, then one must model the resources that could limit production.

The general references on production planning are Thomas and McClain [1], Shapiro [2], Silver et al. [3], Mula et al. [4]

and Jamalnia and Soukhakian [5]. Hax and Meal [6] introduced the notion of hierarchical production planning and provide a specific framework for this problem, where there is an optimization model with each level of the hierarchy. Each optimization model imposes a constraint on the model at the next level of the hierarchy.

The literature in production planning under uncertainty is vast. Different approaches have been proposed to cope with different forms of uncertainty (see, for example [7,8]). Galbraith [9] defines uncertainty as the difference between the amount of information required to perform a task and the amount of information already possessed. In the real world there are many forms of uncertainty that affect production processes.

Any planning problem starts with a specification of a customer demand but In most contexts, future demand is partially known. So one relies on a forecast for the future demand but the forecast is inaccurate. This leads to that demand that cannot be met in a period is lost, thus reducing revenue. In our work, we develop a new planning problem to minimize the lost demands and thus maximize revenues. First, we construct the production planning problem with interval numbers as uncertainty in both of the objective function and constraints. After that we will treat the uncertain of objective function and constraints. In Section 4, parametric study for the treatment problem is introduced. Finally, a numerical example is provided to clarify the proposed approach.

Parametric programming investigates the changes in the optimum linear programming solution due to predetermined continuous variations in the model's parameters. Parametric study of the mathematical programming problems is important and enhances the scope of application of the obtained solutions of those problems. There are three main different approaches to handle the parametric optimization problem, namely the sensitive analysis approach that concerns the minor changes in the parameters values and its effects on the obtained solutions, the parametric solution approach and finally the stability sets approach that deals with the stability of the optimal solutions in different cases. In our work we exhibit and apply the last approach [10]. The stability notion plays an important role in the mathematical programming field. It is important for solver or for the decision maker to preserve effort and time. Stability in mathematical programming has many types. one of these types depends on making perturbation to the decision space or to the objective space or to both by a parameter. This type is called stability in parametric programming problems [11–13]. Other types are called internal and external stability.

2. Problem formulation

First, let us exhibit the following production planning problem that in Ref. [14]:

$$\max \sum_{t=1}^{T} \sum_{i=1}^{I} [r_{it}(d_{it} - u_{it}) - cp_{it}p_{it} - cq_{it}q_{it} - cu_{it}u_{it}] \tag{1}$$

subject to

$$\sum_{i=1}^{I} a_{ik}p_{it} \leqslant b_{kt} \quad \forall k, t \tag{2}$$

$$q_{i,t-1} + p_{it} - q_{it} + u_{it} = d_{it} \quad \forall i, t \tag{3}$$

$$p_{it}, q_{it}, u_{it} \geqslant 0 \quad \forall i, t \tag{4}$$

where T is the number of time periods; I is the number of items (raw materials or finished products); K is the number of resources; a_{ik} is an amount of resource k required per unit of production of item i; b_{kt} is the amount of resource k available in time period t; d_{it} is the demand for item i in time period t; r_{it} is the unit revenue for item i in time period t; cp_{it} is the unit variable cost of production for item i in time period t; cu_{it} is the unit cost of unmet demand for item i in time period t; and cq_{it} is the unit inventory holding cost for item i in time period t.

The decision variables

p_{it}: an amount of production of item i during time period t;
q_{it}: an amount of inventory of item i at end of time period t;
u_{it}: an amount of unmet demand of item i during time period t.

For the above model, a linear relationship between the cost and time, and revenue and time is assumed. The objective function (1) maximizes revenues net of the production, inventory and lost sales costs. Eq. (2) is a set of resource constraints. Production in each period is limited by the availability of a set of shared resources. Production of one unit of item i requires q_{it} units of resource k, for $k = 1, 2, \ldots, K$. Typical resources are various types of labor, process and material handling equipment. Eq. (3) is a set of inventory balance constraints.

The optimization model of production planning problem that maximize revenues net of the production inventory and lost sales cost with interval numbers is formulated as follows:

$$\max \sum_{i=1}^{I} \sum_{t=1}^{T} [r_{it}([d_{it}^L, d_{it}^R] - u_{it}) - cp_{it}p_{it} - cq_{it}q_{it} - cu_{it}u_{it}] \tag{5}$$

subject to

$$\sum_{i=1}^{I} a_{ik}p_{it} \leqslant [b_{kt}^L, b_{kt}^R] \quad k = 1, \ldots, K, \ t = 1, \ldots, T \tag{6}$$

$$q_{i,t-1} + p_{it} - q_{it} + u_{it} = [d_{it}^L, d_{it}^R] \quad i = 1, \ldots, I, \ t = 1, \ldots, T \tag{7}$$

$$p_{it}, q_{it}, u_{it} \geqslant 0 \quad i = 1, \ldots, I, \ t = 1, \ldots, T \tag{8}$$

where $[b_{kt}^L, b_{kt}^R]$ is an interval number represents the amount of resource k available in time period t; and $[d_{it}^L, d_{it}^R]$ is an interval number represent the demand for item i in time period t.

The superscripts L and R denote lower and upper bounds of an interval number, respectively.

3. The optimization approach

Based on the proposed approach of Jiang [15] for treating interval number, we will treat the uncertainty of Eqs. (5)–(8) as follows:

3.1. Treatment of the uncertain objective function

Let $f(p_{it}, q_{it}, u_{it}, d_{it})$

$$= \sum_{i=1}^{I} \sum_{t=1}^{T} \left[r_{it}\left([d_{it}^L, d_{it}^R] - u_{it}\right) - cp_{it}p_{it} - cq_{it}q_{it} - cu_{it}u_{it}\right] \tag{9}$$

in interval mathematics, the uncertain objective function (5) can be transformed into two objective optimization problem as follows [16]:

$$m(f(p_{it}, q_{it}, u_{it}, d_{it})) = \frac{1}{2}(f^R(p_{it}, q_{it}, u_{it}, d_{it}) + f^L(p_{it}, q_{it}, u_{it}, d_{it})), \tag{10}$$

$$w(f(p_{it}, q_{it}, u_{it}, d_{it})) = \frac{1}{2}(f^R(p_{it}, q_{it}, u_{it}, d_{it}) - f^L(p_{it}, q_{it}, u_{it}, d_{it})). \tag{11}$$

where m is called the midpoint value, w is called the radius of interval number and the two functions f^L and f^R are given as follows:

$$f^L(p_{it}, q_{it}, u_{it}, d_{it}) = \min_{d\in D} f(p_{it}, q_{it}, u_{it}, d_{it}) \tag{12}$$

and

$$f^R(p_{it}, q_{it}, u_{it}, d_{it}) = \max_{d\in D} f(p_{it}, q_{it}, u_{it}, d_{it}) \tag{13}$$

where $d \in D = \{d | d^L < d < d^R\}$.

3.2. Treatment of uncertain constraints

The possibility degree of interval number represents certain degree that one interval number is larger or smaller than another [17]. The set of inequality constraints (6) can be written as

$$-\sum_{i=1}^{I} a_{ik}p_{it} \geqslant -[b_{kt}^L, b_{kt}^R] \quad \forall k, t \tag{14}$$

Let

$$x = -\sum_{i=1}^{I} a_{ik}p_{it} \tag{15}$$

as in interval linear programming [18], we can make an inequality constraint satisfied with a possibility degree level, and formulate the deterministic inequality by the following possibility degree $P_{x\geqslant -[b_{kt}^L, b_{kt}^R]}$:

$$P_{x\geqslant -[b_{kt}^L, b_{kt}^R]} = \begin{cases} 0 & x < -b_{kt}^L \\ (x + b_{kt}^L)/(-b_{kt}^R + b_{kt}^L) & -b_{kt}^L \leqslant x < -b_{kt}^R \\ 1 & x \geqslant -b^R \end{cases} \tag{16}$$

where $P_{x\geqslant -[b_{kt}^L, b_{kt}^R]} \geqslant \lambda_{kt}$ is the possibility degree of the ktth constraint and $0 \leqslant \lambda_{kt} \leqslant 1$ is a predetermined possibility degree level.

An equality constraint (7) can be transformed into the following form:

$$d_{it}^L \leqslant q_{i,t-1} + p_{it} - q_{it} + u_{it} \leqslant d_{it}^R \tag{17}$$

that can be written as:

$$q_{i,t-1} + p_{it} - q_{it} + u_{it} \geqslant d_{it}^L \quad \text{and} \quad q_{i,t-1} + p_{it} - q_{it} + u_{it} \leqslant d_{it}^R$$
$$\text{where } i = 1, \ldots I, \quad t = 1, \ldots, T \tag{18}$$

3.3. The deterministic form of Eqs. (5)–(8)

The linear combination method [19,20] is adopted with the multiobjective optimization. In multiobjective optimization, applying the linear combination method to integrate the objective function is relatively easy provided that the preferences of the objective functions are available. Through the above treatments of Eqs. (5)–(8), it is transformed into the following deterministic form:

$$\max\,[c_1 m(f(p_{it}, q_{it}, u_{it}, d_{it})) + c_2 w(f(p_{it}, q_{it}, u_{it}, d_{it}))] \tag{19}$$

subject to

$$b_{kt}^L - \sum_{i=1}^{I} a_{ik}p_{it} \geqslant \lambda_{kt}(b_{kt}^L - b_{kt}^R) \quad i = 1, \ldots I, \quad t = 1, \ldots, T \tag{20}$$

$$q_{i,t-1} + p_{it} - q_{it} + u_{it} \leqslant d_{it}^R \quad i = 1, \ldots I, \quad t = 1, \ldots, T \tag{21}$$

$$q_{i,t-1} + p_{it} - q_{it} + u_{it} \geqslant d_{it}^L \quad i = 1, \ldots I, \quad t = 1, \ldots, T \tag{22}$$

$$c_1, c_2 \geqslant 0, c_1 + c_2 = 1 \tag{23}$$

$$p_{it}, q_{it}, u_{it} \geqslant 0 \quad i = 1, \ldots, I, \quad t = 1, \ldots, T \tag{24}$$

4. The parametric study for Eqs. (19)–(24)

In this paper we assume that λ_{kt} are parameters rather than constants. Let $G(\lambda)$ denotes the decision space of Eqs. (19)–(24), $G(\lambda)$ is defined by:

$$G(\lambda) = \{(p_{it}, q_{it}, u_{it}) \in R^{3IT}, \forall i, t | \text{ satisfies the set of constraints (20)–(24)}\} \tag{25}$$

In what follows we give the definition of some basic notions for Eqs. (19)–(24). Such notions are the set of feasible parameters, the solvability set and the stability set of the first kind [13,21].

The set of feasible parameters

The set of feasible parameters of Eqs. (19)–(24) which is denoted by U, is defined by:

$$U = \{\lambda_{kt} \in R^{KT}, \forall k, t | G(\lambda) \text{ is not empty set}\} \tag{26}$$

The solvability set

The solvability set of Eqs. (19)–(24) which is denoted by V is defined by:

$$V = \{\lambda_{kt} \in U, \forall k, t | \text{ Eqs. (19)–(24) has optimal solution}\} \tag{27}$$

The stability set of the first kind

Suppose that $\lambda^* \in V$ with corresponding optimal solution (p^*, q^*, u^*) for Eqs. (19)–(24). The stability set of the first kind of Eqs. (19)–(24) that is denoted by $S(p^*, q^*, u^*)$ is defined by:

$$S(p^*, q^*, u^*) = \{\lambda^* \in V | (p^*, q^*, u^*) \text{ is an optimal solution of Eqs. (19)–(24)}\} \tag{28}$$

4.1. Determination of the stability set of the first kind

Going back to Eqs. (19)–(24), the Lagrange function is

$$\begin{aligned} LF = Z &- \theta_{kt}\left(b_{kt}^L - \sum_{i=1}^{I} a_{ik}p_{it} - \lambda_{kt}(b_{kt}^L - b_{kt}^R)\right) \\ &- \alpha_{it}(-q_{i,t-1} - p_{it} + q_{it} - u_{it} + d_{it}^R) \\ &- \beta_{it}(q_{i,t-1} + p_{it} - q_{it} + u_{it} - d_{it}^L) \\ &- \phi_{it}p_{it} - \psi_{it}q_{it} - \eta_{it}u_{it} \end{aligned} \tag{29}$$

where

$$Z = \frac{1}{2}c_1 \sum_{i=1}^{I}\sum_{t=1}^{T} \times \begin{pmatrix} r_{it}(d_{it}^R - u_{it}) - cp_{it}p_{it} - cq_{it}q_{it} - cu_{it}u_{it} + r_{it}(d_{it}^L - u_{it}) - cp_{it}p_{it} \\ -cq_{it}q_{it} - cu_{it}u_{it} \end{pmatrix} + \frac{1}{2}c_2 \sum_{i=1}^{I}\sum_{t=1}^{T} \times \begin{pmatrix} r_{it}(d_{it}^R - u_{it}) - cp_{it}p_{it} - cq_{it}q_{it} - cu_{it}u_{it} - r_{it}(d_{it}^L - u_{it}) + cp_{it}p_{it} \\ +cq_{it}q_{it} + cu_{it}u_{it} \end{pmatrix} \quad (30)$$

The Kuhn–Tucker necessary optimality conditions corresponding to this problem will take the following form:

$$\partial LF/\partial p_{it} = 0, \quad \partial LF/\partial u_{it} = 0, \quad \partial LF/\partial q_{it} = 0 \quad i = 1,\ldots,I, \ t = 1,\ldots,T \quad (31)$$

$$\alpha_{it}(-q_{i,t-1} - p_{it} + q_{it} - u_{it} + d_{it}^L) = 0 \quad i = 1,\ldots,I, \ t = 1,\ldots,T \quad (32)$$

$$\beta_{it}(q_{i,t-1} + p_{it} - q_{it} + u_{it} - d_{it}^R) = 0 \quad i = 1,\ldots I, \ t = 1,\ldots,T \quad (33)$$

$$\theta_{kt}\left(b_{kt}^L - \sum_{i=1}^{I} a_{ik}p_{it} - \lambda_{kt}(b_{kt}^L - b_{kt}^R)\right) = 0 \quad k = 1,\ldots,K, \ t = 1,\ldots,T \quad (34)$$

$$\phi_{it}p_{it} = 0 \quad i = 1,\ldots I, \ t = 1,\ldots,T \quad (35)$$

$$\psi_{it}q_{it} = 0 \quad i = 1,\ldots,I, \ t = 1,\ldots,T \quad (36)$$

$$\eta_{it}u_{it} = 0 \quad i = 1,\ldots,I, \ t = 1,\ldots,T \quad (37)$$

$$q_{i,t-1} + p_{it} - q_{it} + u_{it} - d_{it}^L \geqslant 0 \quad i = 1,\ldots,I, \ t = 1,\ldots,T \quad (38)$$

$$q_{i,t-1} + p_{it} - q_{it} + u_{it} - d_{it}^R \geqslant 0 \quad i = 1,\ldots,I, \ t = 1,\ldots,T \quad (39)$$

$$\alpha_{it}, \beta_{it}, \theta_{kt}, \phi_{\mathrm{it}}, \psi_{\mathrm{it}}, \eta_{\mathrm{it}} \geqslant 0 \quad (40)$$

where α_{it}, β_{it}, θ_{kt}, ϕ_{it}, ψ_{it}, η_{it} $\forall i,k,t$ are the Lagrange multipliers. All the relations of the above system are evaluated at the optimal solution of Eqs. (19)–(24).

5. Numerical example

Consider the instance of production planning problem to maximize net revenues given by

$$I = 3, \quad K = 3, \quad T = 2, \quad (b_{it}^L, b_{it}^R) = \begin{Bmatrix} (4,13)(1,12) \\ (3,10)(8,17) \\ (4,12)(5,15) \end{Bmatrix},$$

$$(d_{it}^L, d_{it}^R) = \begin{Bmatrix} (1,6)(2,4) \\ (2,6)(1,6) \\ (3,6)(4,9) \end{Bmatrix}, \quad a_{ik} = \begin{Bmatrix} 344 \\ 165 \\ 256 \end{Bmatrix}, \quad cp_{it} = \begin{Bmatrix} 12 \\ 31 \\ 21 \end{Bmatrix},$$

$$cu_{it} = \begin{Bmatrix} 23 \\ 12 \\ 21 \end{Bmatrix}, \quad r_{it} = \begin{Bmatrix} 43 \\ 24 \\ 44 \end{Bmatrix}, \quad \lambda_{kt} = \begin{Bmatrix} 0.90.8 \\ 0.60.7 \\ 0.50.9 \end{Bmatrix}, \quad cq_{it} = \begin{Bmatrix} 12 \\ 31 \\ 12 \end{Bmatrix}.$$

By using the optimization approach which is described in Section 3, the deterministic form for this example is obtained. Then we get the following optimal solution $u_{21}^* = 2$, $u_{31}^* = 3$, $u_{32}^* = 3.3$, $p_{11}^* = 1.8$, $p_{12}^* = 1.2$, $p_{22}^* = 1$, $p_{32}^* = 0.7$, $q_{11}^* = 0.8$ and all other variables are equal to zero. Objective function value is equal to 41.8.

The set of feasible parameters, solvability set and stability set of the first kind are calculated where Set of feasible parameters is $U = \{\lambda_{kt} \in R^{kt} | 0 < \lambda_{kt} \leqslant 1, k = 1,2,3$ and $t = 1,2\}$ and the solvability set is $V = \{\lambda_{kt} \in U | \lambda_{kt} = 1, \ k = 1,2,3$ and $t = 1,2\}$ and the stability set of the first kind is $S(p^*, q^*, u^*) = \{\lambda^* \in V | \lambda_{11} = 0.1, 0 < \lambda_{12} \leqslant 1, \lambda_{21} = 0.2, 0 < \lambda_{22} \leqslant 1, 0 < \lambda_{31} \leqslant 1, \lambda_{32} = 0.6.\}$.

Table 1 The comparison results between the two approaches.

Results in the paper	Results obtained from the model of Stephen C. Graves
Objective function value = 41.8	Objective function value = 25
Variables	Variables
$u_{21}^* = 2$, $u_{31}^* = 3$,	$u_{21}^* = 2$, $u_{31}^* = 2$,
$u_{32}^* = 3.3$, $p_{11}^* = 1.8$,	$p_{11}^* = 1$, $p_{12}^* = 2$,
$p_{12}^* = 1.2$, $p_{22}^* = 1$,	$p_{22}^* = 1$, $p_{31}^* = 1$
$p_{32}^* = 0.7$, $q_{11}^* = 0.8$	and other variables equal zero
and other variables equal zero	

Table 1 shows the comparison between the results of our approach which is based on uncertainty case and one's of Stephen C. Graves approach which is based on the deterministic case.

It is clear that the result of the paper is better than the result obtained by Stephen C. Graves especially for the objective function value.

6. Conclusions

In this paper, we have discussed the concepts of stability of generalized production planning problem under interval data environment. We have defined and characterized some basic notions for the problem under consideration. These notions are the set of feasible parameters the solvability set and stability set of the first kind. However, as a point for future research, a comparison study is needed between the interval and fuzzy programming to tackle the production planning problem, where each of fuzzy programming and interval programming are two forms of uncertainty. This point for future research is to determine which of interval and fuzzy programming is more suitable for problem of concern.

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