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Intuitionistic fuzzy entropy and distance measure based TOPSIS method for multi-criteria decision making

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Abstract In this paper, an intuitionistic fuzzy TOPSIS method for multi-criteria decision making (MCDM) problem to rank the alternatives is proposed. The proposed method is based on distance measure and intuitionistic fuzzy entropy. The proposed method also uses conversion theorem to convert fuzzy set to intuitionistic fuzzy set given by Jurio et al. (2010). A real case study is taken as an example to find the ranking of four organizations: Bajaj steel, H.D.F.C. bank, Tata steel and Infotech enterprises using real data. In order to compare the different rankingS, they are applied in a portfolio selection problem. Different portfolios are constructed and are analyzed for their risk and return. It is observed that if the portfolios are constructed using the ranking obtained with proposed method, the return is increased with slight increment in risk.

1. Introduction

In multi-criteria (or attribute) decision making (MCDM/MADM) problem, a decision maker selects or ranks alternatives after qualitative or quantitative assessment of a finite set of interdependent or independent criteria. Desirable alternative can be chosen by providing preference information in terms of exact numerical value or interval. However, preference information in real life situation can be assessed in a qualitative way with vague or imprecise knowledge. In such cases, ambiguity caused by vague or imprecise preference information is a big challenge for decision makers. This fact was a great motivation for researchers to extend MCDM techniques in fuzzy environment. Technique for order preference by similarity to an ideal solution (TOPSIS), one of the most known classical MCDM methods was developed by Hwang and Yoon [1] is based upon the concept that the chosen alternative should be the closest from the positive ideal solution and the farthest from the negative ideal solution.

Chen [2] developed a methodology for solving MCDM problems using TOPSIS in fuzzy environment. Jahanshahloo et al. [3] extended the concept of TOPSIS to develop a methodology for solving MCDM problems with fuzzy data. Wang and Chang [4] developed a fuzzy TOPSIS model to choose optimal initial training aircraft for Taiwan Airforce Academy. Various researchers [5–7] proposed fuzzy multi-criteria decision analysis methods and their applications to portfolio selection problems.

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In 1986, Atansasov [8] introduced intuitionistic fuzzy set (IFS) as the generalization of fuzzy set proposed by Zadeh [9]. Because of prominent characteristic of IFS to assign each element a membership degree and non-membership degree, this proved to be an ideal tool to handle the non-determinacy in the system. Later on various operators on IFSs were proposed by De et al. [10], Grzegorzewski and Mrowka [11]. The preferences in TOPSIS are essentially human judgments based on human perceptions, so intuitionistic fuzzy approach allow for a more accurate description of MCDM problems. Many researchers introduced IFS with TOPSIS to give a hybrid method for MCDM problems. Some methods for MCDM problems based on IFS using entropy weights and linear programming were also given by [12–15].

Some intuitionistic fuzzy aggregation operators were proposed by Xu [16] to aggregate intuitionistic fuzzy information in MCDM problems. Boran et al. [17] used weighted average operator to aggregate opinions of decision makers and proposed an intuitionistic fuzzy TOPSIS method for supplier selection problem. Recently many researchers [18–21] extended TOPSIS for MCDM and MADM for interval-valued intuitionistic fuzzy and hesitant fuzzy information.

In the present study, TOPSIS method proposed by [1] is extended for intuitionistic fuzzy environment by using construction theorem given by [22] and distance measure given by [23]. In order to see the relative importance of criteria, weights to each criterion have been given by using intuitionistic fuzzy entropy measure proposed by [24]. The proposed intuitionistic fuzzy TOPSIS method is applied to rank the four organizations based on some important criteria. Further, the obtained ranking is implemented in portfolio selection problem to analyze the risk and return in different portfolios.

The rest of the paper is organized as follows: Section 2 briefly introduces the basic concepts of fuzzy sets, intuitionistic fuzzy set, fuzzy and intuitionistic fuzzy entropy, and construction theorem given by [22]. The different steps in the proposed intuitionistic fuzzy TOPSIS method are presented in section 3. In section 4, a real case study taken as an example to illustrate the proposed method. In section 5, the proposed method is compared with conventional TOPSIS and fuzzy TOPSIS by taking a portfolio selection problem along with some basic definitions related to portfolio analysis. Finally in section 6, findings and conclusions are presented.

2. Preliminaries

In this section, basic definitions of fuzzy set, intuitionistic fuzzy set, fuzzy entropy and intuitionistic fuzzy entropy are presented. Conversion theorem for IFS given by [22] is also discussed in this section.

2.1. Fuzzy set, intuitionistic fuzzy set (IFS)

In 1965, Zadeh [9] first introduced the concept of fuzzy set for modeling the vagueness type of uncertainty. In 1986, Atanasov [8] gave the concept of IFS to model the uncertainty because of degree of hesitation in system. Following are some basic definitions of fuzzy and IFSs found in the literature:

Definition 1. A fuzzy set A in the universe of discourse X is characterized by membership function $\mu_A{:}X \to [0,1]$. A fuzzy set A is represented by the following order pair:

$$A = \{(x, \mu_A(x)) : \forall x \in X\} \tag{1}$$

where μ_A is the grade of membership of element x in the set A. The greater the amount of μ_A the greater is the truth of the statement that 'the element x belongs to the set A.

Definition 2. An IFS I on a universe X is defined as an object of the following form:

$$I = \{< x, \mu_I(x), v_I(x) >: \forall x \in X\} \tag{2}$$

where the functions $\mu_I(x){:}X \to [0,1]$ and $v_I(x){:}X \to [0,1]$ represent the degree of membership and the degree of non-membership of an element $x \in I \subset X$ respectively.

Definition 3. For every $x \in X$, $\pi_I(x) = 1 - \mu_I(x) - v_I\ (x)$ is called degree of uncertainty or non-determinism hesitation of IFS set A in X, with the condition $0 \leqslant \mu_I(x) + v_I\ (x) \leqslant 1$.

2.2. Entropy, fuzzy entropy and intuitionistic fuzzy entropy

Traditional entropy is derived from the concept of probability and measures the discrimination of criteria when applied in MCDM. De Luca and Termini [25] first approximated non-probabilistic entropy and introduced some requirements to capture intuitive comprehension of the degree of fuzziness. Many researchers [26–28] proposed many other ways to view the degree of fuzziness. Scmidt and Kacprzyk [29,30] also introduced few axioms for distance between the IFSs and non-probabilistic entropy measure for them. Following are the definitions of Shanon entropy, fuzzy entropy and intuitionistic fuzzy entropy.

Definition 4. Let $\Delta_n = \{P = (p_1, \ldots\ldots, p_n) : p_i \geqslant 0, \sum_{i=1}^{n} p_i = 1\}$ be a set of n-complete probability distributions. For any probability distribution $P = (p_1, \ldots, p_n) \in \Delta_n$, Shanon's entropy [31] is defined as follows:

$$H(P) = -\sum_{i=1}^{n} p(x_i) \log p(x_i) \tag{3}$$

Definition 5. Let A be a fuzzy set defined in the universe of discourse X. Fuzzy entropy for A as follows [24]:

$$H(A) = -\frac{1}{n}\sum_{i=1}^{n}[\mu_A(x_i) \log \mu_A(x_i) + (1 - \mu_A(x_i) \log(1 - \mu_A(x_i))] \tag{4}$$

Definition 6. Let I be a fuzzy set defined in the universe of discourse X. Intuitionistic fuzzy entropy is given as follows [30]:

$$E(I) = \frac{1}{n}\sum_{i=1}^{n} \frac{\min(\mu_I(x_i), v_I(x_i)) + \pi_I(x_i)}{\max(\mu_I(x_i), v_I(x_i)) + \pi_I(x_i)} \tag{5}$$

2.3. Construction of IFS from fuzzy set

In this subsection, method of construction of IFS from fuzzy sets given by [22] is presented. Importance of the method lays on the fact that IFSs are built in such a way indeterminacy index for each element is fixed beforehand. The method is as follows:

Let $A_F \in FS_s(U)$, where $FS_s(U)$ denotes the set of all fuzzy sets in the universal set U and let π, δ:$U \to [0,1]$ be two mapping. Then

$$I = \{< u_i, f(\mu_{AF}(u_i), \pi(u_i), \delta(u_i)), \ \forall u_i \in U \tag{6}$$

is an Atanassov [8] IFS, where the mapping
f: $[0,1]^2 \times [0,1] \to L^*$ given by
$f(x, y, \delta) = (f_\mu(x, y, \delta), f_v(x, y, \delta))$ where

$$f_\mu(x, y, \delta) = x(1 - \delta y), \quad f_v(x, y, \delta) = 1 - x(1 - \delta y) - \delta y \tag{7}$$

and

$$L^* = \{(x, y) : (x, y) \in [0,1] \times [0,1] \text{ and } x + y \leqslant 1\} \tag{8}$$

$$D = \begin{bmatrix} (\mu_{A1}(x_1), \ v_{A1}(x_1), \ \pi_{A1}(x_1)) & (\mu_{A1}(x_2), \ v_{A1}(x_2), \ \pi_{A1}(x_2)) \ldots & (\mu_{A1}(x_n), \ v_{A1}(x_n), \ \pi_{A1}(x_n)) \\ (\mu_{A2}(x_1), \ v_{A2}(x_1), \ \pi_{A2}(x_1)) & (\mu_{A2}(x_2), \ v_{A2}(x_2), \ \pi_{A2}(x_2)) \ldots & (\mu_{A2}(x_n), \ v_{A2}(x_n), \ \pi_{A2}(x_n)) \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ (\mu_{Am}(x_1), \ v_{Am}(x_1), \ \pi_{Am}(x_1)) & (\mu_{Am}(x_2), \ v_{Am}(x_2), \ \pi_{Am}(x_2)) \ldots & (\mu_{Am}(x_n), \ v_{Am}(x_n), \ \pi_{Am}(x_n)) \end{bmatrix}$$

satisfies the following conditions:

(i) If $y_1 \leqslant y_2$ then $\pi(f(x, y_1, \delta)) \leqslant \pi(f(x, y_2, \delta))$ for all x, $\delta \in [0,1]$.
(ii) $f_\mu(x, y, \delta) \leqslant x \leqslant 1 - f_v(x, y, \delta)$ for all $x \in [0,1]$.
(iii) $f(x, 0, \delta) = (x, 1 - x)$.
(iv) $f(0, y, \delta) = (0, 1 - \delta y)$.
(v) $f(x, y, 0) = (x, 1 - x)$.
(vi) $\pi(f(x, y, \delta)) = \delta y$.

3. An algorithm for intuitionistic fuzzy TOPSIS

In this section, step wise algorithm for proposed intuitionistic fuzzy TOPSIS method is presented. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of m alternatives and decision maker will choose the best one from A according to a criterion set $C = \{C_1, C_2, C_3, \ldots, C_n\}$ which include n criteria. Various steps in the proposed intuitionistic fuzzy TOPSIS are as follows:

Step 1: Construction of intuitionistic fuzzy decision matrix.

In this step, intuitionistic fuzzy decision matrix $D = [d_{ij}]_{n \times m}$ of intuitionistic fuzzy value $d_{ij} = (\mu_{ij}, v_{ij})$ is constructed. Here μ_{ij} and v_{ij} are degrees of membership and non-membership of the alternative A_j satisfying the criterion C_i. The intuitionistic fuzzy index $\pi_{ij} = 1 - \mu_{ij} - v_{ij}$ shows the decision maker's hesitation of the alternative A_j with respect to criterion C_i. This step includes the following sub steps.

(i) Define the universe of discourse for each alternative against each criterion by taking their actual numerical values.
(ii) Construct the appropriate fuzzy sets for each criterion.
(iii) Apply the method given by [22] to construct IFSs from fuzzy sets obtained in sub step (ii).

Intuitionistic fuzzy decision matrix can be presented as follows:

Step 2: Determination the weights of criteria.

In this step following intuitionistic fuzzy entropy measure given by [24] is used to obtain the weight vector $w = (w_1, w_2, \ldots, w_n)$, where $w_i \geqslant 0$ and $\sum_{i=1}^{n} w_i = 1$.

$$w_j = \frac{1}{(n - T)} \times (1 - a_j), \quad j = 1, 2, \ldots, n \tag{9}$$

$$a_j = \sum_{i=1}^{m} h_{ij}, \quad T = \sum_{j=1}^{n} a_j \tag{10}$$

Normalized entropy is given by following expression:

$$h_{ij} = \frac{E_{ij}}{\max(E_{ij})} \tag{11}$$

where E is intuitionistic fuzzy entropy.

Step 3: Construction of weighted intuitionistic fuzzy decision matrix.

After the construction of intuitionistic fuzzy decision matrix and determining the weights of each criterion, the weighted intuitionistic fuzzy decision matrix is constructed according to the following expression given by Atanassov [32]:

$$\lambda I = (1 - (1 - \mu_I)^\lambda, (v_I)^\lambda), \quad 0 < \lambda < 1 \tag{12}$$

The weighted intuitionistic fuzzy decision matrix can be defined as follows:

$$D^* = \begin{bmatrix} (\mu_{A1W}(x_1), \ v_{A1w}(x_1), \ \pi_{A1w}(x_1)) & \ldots & (\mu_{A1w}(x_n), \ v_{A1w}(x_n), \ \pi_{A1w}(x_n)) \\ (\mu_{A2w}(x_1), \ v_{A2w}(x_1), \ \pi_{A2w}(x_1)) & \ldots & (\mu_{A2w}(x_n), \ v_{A2w}(x_n), \ \pi_{A2w}(x_n)) \\ \vdots & & \\ \vdots & & \\ (\mu_{Amw}(x_1), \ v_{Amw}(x_1), \ \pi_{Amw}(x_1)) & \ldots & (\mu_{Amw}(x_n), \ v_{Amw}(x_n), \ \pi_{Amw}(x_n)) \end{bmatrix}$$

Step 4. Intuitionistic fuzzy positive-ideal solution (IFPIS) and intuitionistic fuzzy negative ideal solution (IFNIS).

In TOPSIS method, the evaluation criteria can be categorized into two categories, benefit and cost. Let G be a collection of benefit criteria and B be a collection of cost criteria. According to intuitionistic fuzzy theory and the principle of classical TOPSIS method, IFPIS and IFNIS can be defined as follows:

$$A^+ = \left[\left\{C_j, \langle(\max_i \mu_{ij}(C_j)/j \in G), (\min_i \mu_{ij}(C_j)/j \in B)\rangle, \left\langle(\min_i v_{ij}(C_j)/j \in G), (\max_i v_{ij}(C_j)/j \in B)\right\rangle\right\} \middle| i \in m\right] \quad (13)$$

$$A^- = \left[\left\{C_j, \langle(\min_i \mu_{ij}(C_j)/j \in G), (\max_i \mu_{ij}(C_j)/j \in B)\rangle, \left\langle(\max_i v_{ij}(C_j)/j \in G), (\max_i v_{ij}(C_j)/j \in B)\right\rangle\right\} \middle| i \in m\right] \quad (14)$$

Step 5: Calculation of distance measures from IFPIS and IFNIS.

To measure distance of each alternative A_i from IFPIS and IFNIS, the intuitionistic separation measure given by [23] is used and expressed by following expression:

$$S_i^+ = \sum_{i=1}^{n} \max\left(|\mu_{A^+}(x_i) - \mu_B(x_i)|, |v_{A^+}(x_i) - v_B(x_i)|\right) \quad (15)$$

$$S_i^- = \sum_{i=1}^{n} \max\left(|\mu_{A^-}(x_i) - \mu_B(x_i)|, |v_{A^-}(x_i) - v_B(x_i)|\right) \quad (16)$$

Step 6. Calculation of relative closeness coefficient (CC)

Finally, relative closeness coefficient of each alternative with respect to intuitionistic fuzzy ideal solutions is computed by using following expression and rank the preference order of all alternatives.

$$C_j = \frac{S^-}{S^+ + S^-}, \quad j = 1, 2, 3, \ldots, m \quad (17)$$

The larger value of relative closeness coefficient *i*ndicates that an alternative is closer to IFPIS and farther from IFNIS simultaneously. Therefore, the ranking order of all the alternatives can be determined according to the descending order of relative coefficient values. The most preferred alternative is the one with the highest value.

4. A real case study

Assessment and evaluations of the organizations based on some financial criteria is very important in financial system. In this section, proposed intuitionistic fuzzy TOPSIS method is applied in the ranking of four organizations, Bajaj Steel (A_1), H.D.F.C. Bank (A_2), Tata Steel (A_3) and Infotech Enterprises (A_4) using real data. Further, the obtained ranking is also implemented in portfolio selection problem.

4.1. Implementation of proposed intuitionistic fuzzy TOPSIS

Four alternatives (A_1, A_2, A_3, A_4) are assessed for their performance on the basis of following five inter-independent criteria (c_1, c_2, c_3, c_4, c_5).

(i) c_1: Earnings per share(EPS).
(ii) c_2: Face value.
(iii) c_3: P/C (Put–Call) Ratio.
(iv) c_4: Dividend.
(v) c_5: P/E (Price-to-earnings) ratio.

First two criteria belong to benefit criteria, i.e., high value indicates good growth prospects and last three belong to non-beneficial criteria, i.e., low value indicates good growth prospects. Therefore, c_1, $c_2 \in G$ and c_3, c_4, $c_5 \in B$. Actual numerical values of these five criteria of the four alternatives are retrieved from http://www.moneycontrol.com from date 21.9.2012 to 27.9.2012 and average of this information is placed in following table:

Step 1: Crisp numerical values given in table 1 are fuzzified by defining the following fuzzy sets for each criteria and fuzzy decision matrix (table 2) is obtained.

$$A_1 = 0.2846/20.5 + 0.75/10 + 0.234/2.21 + 0.23/2 + 0.24/4.8$$
$$A_2 = 0.318/23.31 + 0.248/2 + 0.766/24.7 + 0.097/0.67 + 0.76/27$$
$$A_3 = 0.759/60.06 + 0.75/10 + 0.315/5.65 + 0.32/2.92 + 0.278/6.5$$
$$A_4 = 0.241/16.86 + 0.437/5 + 0.41/9.7 + 0.155/1.25 + 0.394/11.4$$

Following intuitionistic fuzzy decision matrix (Table 3) is constructed the fuzzy decision matrix given by table 2 and construction theorem given by Eqs. 6 and 7.

Table 1 Average of actual numerical value of criteria.

Alternatives	c_1	c_2	c_3	c_4	c_5
a_1	20.50	10.00	2.21	2.00	4.80
a_2	23.31	2.00	24.7	0.67	27.00
a_3	60.06	10.00	5.65	2.92	6.50
a_4	16.86	5.00	9.70	1.25	11.40

Table 2 Fuzzy decision matrix.

	c_1	c_2	c_3	c_4	c_5
a_1	0.2846	0.75	0.234	0.23	0.24
a_2	0.318	0.248	0.766	0.097	0.76
a_3	0.759	0.75	0.315	0.32	0.278
a_4	0.241	0.437	0.41	0.155	0.394

Table 3 Intuitionistic fuzzy decision matrix.

Alternatives	c_1	c_2	c_3	c_4	c_5
$\boldsymbol{a}_1$	(0.23, 0.587)	(0.61, 0.2)	(0.192, 0.63)	(0.22, 0.75)	(0.196, 0.62)
$\boldsymbol{a}_2$	(0.26, 0.554)	(0.2, 0.61)	(0.63, 0.192)	(0.094, 0.875)	(0.62, 0.196)
$\boldsymbol{a}_3$	(0.62, 0.197)	(0.61, 0.2)	(0.259, 0.56)	(0.31, 0.66)	(0.227, 0.59)
$\boldsymbol{a}_4$	(0.197, 0.62)	(0.36, 0.454)	(0.337, 0.484)	(0.15, 0.82)	(0.322, 0.50)

Table 4 Weighted intuitionistic fuzzy decision matrix.

Alternatives	c_1	c_2	c_3	c_4	c_5
a_1	(0.058, 0.885)	(0.172, 0.725)	(0.042, 0.87)	(0.31, 0.965)	(0.047, 0.90)
a_2	(0.067, 0.873)	(0.044, 0.91)	(0.18, 0.72)	(0.0123, 0.98)	(0.192, 0.70)
a_3	(0.20, 0.69)	(0.172, 0.725)	(0.058, 0.89)	(0.045, 0.95)	(0.055, 0.89)
a_4	(0.049, 0.896)	(0.085, 0.854)	(0.079, 0.865)	(0.020, 0.975)	(0.082, 0.86)

Step 2: The following weights of each criterion are calculated using entropy measure for IFS:

$$w_1 = 0.230, \quad w_2 = 0.200, \quad w_3 = 0.200, \quad w_4 = 0.125, \quad w_5 = 0.220$$

Step 3: Weighted intuitionistic fuzzy decision matrix (Table 4) is obtained by multiplying intuitionistic fuzzy values with weight of the each criterion.

Step 4: Intuitionistic fuzzy positive-ideal solution and intuitionistic fuzzy negative-ideal solution are calculated by using Eqs. (13) and (14) and are as follows:

$$\mathbf{A}^+ = \{(0.20, 0.69)(0.172, 0.725)(0.042, 0.89)(0.0123, 0.98)(0.047, 0.90)\}$$
$$\mathbf{A}^- = \{(0.049, 0.896)(0.044, 0.91)(0.18, 0.72)(0.045, 0.95)(0.192, 0.90)\}$$

Step 5: Intuitionistic separation measure of each alternative from the positive-ideal solution and negative ideal solution are calculated using Eqs. (16) and (17) and are given by Table 5.

Step 6: Relative closeness coefficients are calculated using Eq. (17) and are given in Table 6.

According to descending order of relative closeness coefficients values four alternatives are ranked as $a_3 > a_1 > a_4 > a_2$. The ranking of these four alternatives is also obtained by TOPSIS and fuzzy TOPSIS method proposed by [1,3] and shown in following table (Table 7):

Table 5 Intuitionistic separation measures.

Alternatives	S_i^+	S_i^-
a_1	0.2377	0.5060
a_2	0.7380	0.2557
a_3	0.0587	0.698
a_4	0.4170	0.336

Table 6 Relative closeness coefficients.

Alternatives	a_1	a_2	a_3	a_4
C_i	0.680	0.257	0.922	0.446

Table 7 Ranking order of alternatives for different methods.

Method	Ranking	Best alternative
TOPSIS proposed by [11]	$a_3 > a_4 > a_1 > a_2$	a_3
Fuzzy TOPSIS proposed by [12]	$a_3 > a_1 > a_4 > a_2$	a_3
Proposed intuitionistic fuzzy TOPSIS	$a_3 > a_1 > a_4 > a_2$	a_3

It is found that there is no conflict in the preference ordering of all the alternatives by fuzzy TOPSIS method and proposed intuitionistic fuzzy TOPSIS method. There is only one conflict is found in deciding the preference ordering of a_1 and a_4.

5. An application of intuitionistic fuzzy TOPSIS in portfolio selection

To compare the performance of proposed intuitionistic fuzzy TOPSIS with TOPSIS [1] and fuzzy TOPSIS method [3], a portfolio selection problem is taken. Basic definitions related to portfolio and portfolio analysis are also given in this section.

5.1. Basic definitions related to portfolio analysis

Some basic definitions related to portfolio analysis given by Markowitz [33,34] are presented in this subsection.

Definition 7. An asset is a valuable economic entity from which the future economic benefits are expected to flow to the owner of the asset.

Definition 8. Return on an asset is an indicator of gain/loss in the investment of an asset in financial market. If current price of an asset is $A(0)$ and after T time period, the asset is sold off at amount $A(T)$, then return r, on asset for T time period is given by following:

$$r = \frac{A(T) - A(0)}{A(0)} \tag{18}$$

Definition 9. Risk is defined as the degree of uncertainty of return on an asset. It signifies the possibility of loss in the investment. The risk can be either zero (risk free asset) or positive (risky asset). There are two kinds of risk associated with risky asset **(i)** systematic risk which is concerned with money, capital market, credit and fiscal policy and economic policy which govern the market economy; **(ii)** unsystematic risk is concerned with outcome of unfavorable litigation, sudden discovery of deficiencies in a product of a company and natural catastrophe.

Definition 10. A portfolio is a collection of two or more assets, say $a_1, a_2, a_3, \ldots, a_n$, represented by an ordered n-tuple $(x_1, x_2, x_3, \ldots, x_n)$, where $x_i \in R$ is the number of units of the asset $a_i (i = 1, 2, 3, \ldots, n)$ owned by the investor.

Definition 11. Let $Vi(0)$ and $Vi(t)$ be the values of the ith asset at time $T = 0$ and $T = t$, respectively. Value of portfolio $(x_1, x_2, x_3, \ldots, x_n)$ at $T = 0$ and $T = t$, is denoted by $V(0)$ and $V(t)$ and defined as follows:

Table 8 Assets prices (in Rs.) at different equal intervals.

Date of observation	Bajaj Steel (a_1)	H.D.F.C. Bank (a_2)	Tata Steel (a_3)	Infotech Enterprise (a_4)
22/09/2012	92.50	607	396	196.50
23/09/2012	92.50	607	396	196.50
24/09/2012	93.60	626	410	195
25/09/2012	103.30	634	406.05	194
26/09/2012	101.05	639.25	398.55	193
27/09/2012	101	634.60	396.90	193
28/09/2012	100	631	400.05	191

$$V(0) = \sum_{i=1}^{n} x_i V_i(0) \quad \text{and} \quad V(t) = \sum_{i=1}^{n} x_i V_i(t) \tag{19}$$

Definition 12. The weight w_i of the asset a_i is the percentage of the value of the asset in the portfolio $(x_1, x_2, x_3, \ldots, x_n)$ at $t = 0$, i.e.,

$$w_i = \frac{x_i V_i(0)}{\sum_{i=1}^{n} x_i V_i(0)} \quad (i = 1, 2, 3, \ldots, n) \tag{20}$$

It can be observed that

$$w_1 + w_2 + w_3 + \ldots w_n = 1 \tag{21}$$

5.2. Portfolio analysis

We construct different portfolios with the assets of companies (a_1, a_2, a_3, a_4). Further the constructed portfolios are analyzed for their risk and return.

For analyze the portfolio the assets prices of four companies (a_1, a_3, a_4, a_2) are taken form www.moneycontrol.com at different days and given in following table (Table 8) :

Let us assume that total number of units of assets cannot exceed by 100 in the portfolio (x_1, x_2, x_3, x_4). According to our ranking $a_3 > a_1 > a_4 > a_2$, let us take $x_1 = 30$, $x_2 = 10$, $x_3 = 40$, $x_4 = 20$, where x_i represents the number of units of assets of a company a_i.

By using the Eq. (20), we find weight of each company corresponding to each observation and are placed in Table 9 as follows:

In order to follow ranking proposed by TOPSIS method [1], i.e.,

$a_3 > a_4 > a_1 > a_2$, we take $x_1 = 20$, $x_2 = 10$, $x_3 = 40$, $x_4 = 30$ and using Eq. (20), the weight of the asset of each company on different observations are calculated and are placed in Table 10.

Table 9 Weight matrix of the assets by proposed method.

Date of observation	a_1	a_2	a_3	a_4
22/09/2012	0.0969	0.212	0.554	0.137
23/09/2012	0.0969	0.212	0.554	0.137
24/09/2012	0.0956	0.213	0.558	0.133
25/09/2012	0.105	0.214	0.549	0.131
26/09/2012	0.103	0.219	0.545	0.132
27/09/2012	0.104	0.218	0.545	0.133
28/09/2012	0.103	0.217	0.549	0.131

Table 10 Weight matrix of the assets by TOPSIS method.

Date of observation	a_1	a_2	a_3	a_4
22/09/2012	0.0969	0.212	0.554	0.137
23/09/2012	0.0969	0.212	0.554	0.137
24/09/2012	0.0665	0.206	0.5397	0.1925
25/09/2012	0.0678	0.208	0.5331	0.191
26/09/2012	0.06704	0.212	0.545	0.192
27/09/2012	0.06726	0.2113	0.5286	0.1927
28/09/2012	0.103	0.217	0.549	0.131

We also determine ranking by fuzzy TOPSIS method, which is same as given by our proposed method. So the weight matrix of the assets by fuzzy TOPSIS method will be same as given in Table 10. Return series is formed with the help of weight matrix of companies. Return series is the matrix of order $n \times m$, where n is number of observations and m is number of assets with equally spaced incremental return observations. The first row is the oldest observation, and the last row is the most recent. The return series can be formed by multiplying the assets prices with corresponding weights. Matrices A, B and C represent the return series for proposed, TOPSIS and fuzzy TOPSIS method respectively:

$$\boldsymbol{A} = \begin{bmatrix} 8.963 & 128.7 & 219.4 & 26.92 \\ 8.963 & 128.7 & 219.4 & 26.92 \\ 8.95 & 133.3 & 228.78 & 25.93 \\ 10.84 & 135.68 & 222.92 & 25.414 \\ 10.4 & 139.99 & 217.21 & 25.47 \\ 10.5 & 138.34 & 216.3 & 25.7 \\ 10.3 & 136.937 & 219.63 & 25.02 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 5.70 & 124.40 & 211.46 & 39.10 \\ 5.70 & 128.40 & 211.46 & 39.10 \\ 5.767 & 128.956 & 221.315 & 37.546 \\ 7.0047 & 131.870 & 216.460 & 37.050 \\ 6.7700 & 135.500 & 210.700 & 37.056 \\ 6.7900 & 134.09 & 209.815 & 37.190 \\ 6.6600 & 132.50 & 214.02 & 36.480 \end{bmatrix} \quad \boldsymbol{C} = \begin{bmatrix} 8.963 & 128.7 & 219.4 & 26.92 \\ 8.963 & 128.7 & 219.4 & 26.92 \\ 8.95 & 133.3 & 228.78 & 25.93 \\ 10.84 & 135.68 & 222.92 & 25.414 \\ 10.4 & 139.99 & 217.21 & 25.47 \\ 10.5 & 138.34 & 216.3 & 25.7 \\ 10.3 & 136.937 & 219.63 & 25.02 \end{bmatrix}$$

Table 11 Portfolio risk and return.

Portfolio	By proposed method & fuzzy TOPSIS method [3]		By TOPSIS method [1]	
	Portfolio risk	Portfolio return	Portfolio risk	Portfolio return
Portfolio 1	0.1384	35.894	0.1336	34.4186
Portfolio 2	0.3614	64.8725	0.2157	64.2829
Portfolio 3	0.8725	95.9662	0.7087	94.1472
Portfolio 4	1.4197	217.1047	1.3688	214.5882
Portfolio 5	1.9731	158.2431	2.0403	153.87587
Portfolio 6	2.5315	189.3816	2.7148	183.7400
Portfolio 7	3.8924	220.5200	3.7753	213.6043
	Coefficient of correlation (r) = 0.843016		Coefficient of correlation (r) = 0.835655	

Using above return series, we construct four portfolios. Risk and return associated with each portfolio are placed in Table 11.

6. Conclusion

In this paper, an intuitionistic fuzzy TOPSIS method for MCDM problems has been proposed. The proposed method uses the construction theorem given by [22] and the weight of each criterion is calculated using intuitionistic fuzzy entropy measure proposed by [24]. A real case study is taken to rank the four organizations based on five criteria using the proposed method. Further, in order to compare the ranking obtained by proposed method with the methods given by [1,3], a portfolio selection problem is considered. By retrieving the stock prices of four organizations taken in the study from www.moneycontrol.com and considering the different rankings, different portfolios are constructed and analyzed for their risk and returns. Table 11 shows the return of different portfolios with associated risk and coefficient of correlation between them. As there is no conflict in the ranking the alternatives with the fuzzy TOPSIS method given by [3] and proposed intuitionistic fuzzy TOPSIS method, return of each portfolio and involved risk is same. But, when we compare the proposed method with the TOPSIS method given by [1], we find the return of each portfolio is higher with slightly high risk involved in each portfolio. Coefficient of correlation (r = 0.843016) also confirms the better relationship between portfolio risk and return in different portfolios constructed using the ranking obtained by proposed method.

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