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Medical Image Enhancement With Brightness and Detail Preserving Using Multiscale Top-hat Transform by Reconstruction

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Abstract

Medical imaging help medical doctors provide faster and more efficient diagnoses to their patients. Medical image quality directly influences diagnosis. However, when medical images are acquired, they often present degradations such as poor detail or low contrast. This work presents an algorithm that improves contrast and detail, preserving the natural brightness of medical images. The proposed method is based on multiscale top-hat transform by reconstruction. It extracts multiple features from the image that are then used to enhance the medical image. To quantify the performance of the proposed method, 100 medical images from a public database were used. Experiments show that the proposal improves contrast, introducing less distortion and preserving the average brightness of medical images.

Keywords: Medical imaging, low contrast, natural brightness, multiscale top-hat transform by reconstruction

1 Introduction

Medical imaging has an important role in diagnosing and monitoring the effect of selected treatments for a disease [9]. Emergency situations, ambient noise, special conditions of photography patients, lighting conditions and technical limitations of imaging devices are some of the reasons why images may have low quality [29,7].

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In such cases, image enhancement techniques can be useful, especially when it is impossible to re-create images. These techniques are used to improve the visual quality of medical images.

At present, several contrast enhancement techniques have been proposed that enhance the visual quality of all types of images, including medical images. A very popular technique for improving medical imaging is Histogram Equalization (HE) [15] which improves image contrast by increasing the distribution of gray levels. The results obtained by HE are not necessarily good for all areas of the medical image, because it can damage the image as well as its border areas. For this reason, new approaches have been proposed to improve the performance of HE [20,2,5,14,16]. The mathematical morphology is another widely used method to process and analyze images [6,8,12,19,27,22]. Morphological operators can extract different types of features from the medical image, which in the end will be used to enhance the visual quality of the image. Also, multiscale approaches to the top-hat transform were proposed to improve its performance in image enhancement [24,17,4,23,1,13,25]. The multiscale approach of the top-hat transform has been used to improve images of retinal vessels [11], hand vein images [28], ultrasound images [21], mammography images [10] and other applications that use medical imaging.

This paper presents a technique for contrast enhancement, detail and brightness preservation in medical imaging. The proposal is inspired by the algorithm of Bai et al. [4] and is based on the multiscale top-hat transform by reconstruction. With the geodesic reconstruction, in the transformed top-hat, we seek to preserve some features that are connected in the image. Thus, by introducing this variation we obtain medical images with low distortion, contrast enhancement, detail improvement and preservation of their natural brightness.

This paper presents in Section 2 the morphological operators and the proposal for the improvement of the image, in Section 3 the numerical and visual experimental results are visualized; and finally, in Section 4 the conclusions of the work are presented.

2 Proposed method based on the multiscale top-hat transform by reconstruction

Mathematical morphology methods are based on the structural properties of objects. These methods use mathematical principles and relationships between categories to extract the components of an image, which are useful for describing the shape of areas. The morphological operators are non-linear and have as input two sets of data.The first set contains the original image and the second describes the structuring element [27].

Let f be an original image and B a structuring element. The two basic morphological operations are dilation ($\delta_B(f)$) and erosion ($\varepsilon_B(f)$), and are defined as follows [8]:

$$\delta_B(f)(u,v) = \underset{(x,y)\in B}{max}(f(u-x, v-y)), \tag{1}$$

$$\varepsilon_B(f)(u,v) = \underset{(x,y)\in B}{min}(f(u+x, v+y)), \tag{2}$$

where (u, v) and (x, y) are the pixel coordinates of f and B, respectively.

2.1 Morphological reconstruction

Morphological reconstruction is based on morphological operators with geodesic principles. The geodesic transformations use two input images [27]. However, classical morphological transformations use one image and one structuring element as input.

Geodesic dilation ($\delta_g^{(s)}$) and erosion ($\varepsilon_g^{(s)}$) are defined as [3]:

$$\delta_g^{(s)}(f) = \delta_g^{(1)}(\delta_g^{(s-1)}(f)), \tag{3}$$

$$\varepsilon_g^{(s)}(f) = \varepsilon_g^{(1)}(\varepsilon_g^{(s-1)}(f)), \tag{4}$$

where g is a mask image, $\delta_g^{(1)}(f)$ represents the minimum pixels between g and the dilation of f, and $\varepsilon_g^{(1)}(f)$ is the maximum pixels between g and the erosion of f; using in both cases a structuring element B, in an iterative way, as follows:

$$\delta_g^{(1)}(f) = min(g, \delta_B^{(1)}(f)), \tag{5}$$

$$\varepsilon_g^{(1)}(f) = max(g, \varepsilon_B^{(1)}(f)). \tag{6}$$

Reconstruction by dilation (R_g^{δ}) [3] is defined as the iterative geodesic dilation of f with respect to g until it reaches stability:

$$R_g^{\delta}(f) = \delta_g^{(s)}(f),\ where\ \delta_g^{(s)}(f) = \delta_g^{(s+1)}(f), \tag{7}$$

Reconstruction by erosion (R_g^{ε}) [3] is defined as the iterative geodesic erosion of f with respect to g until it reaches stability:

$$R_g^{\varepsilon}(f) = \varepsilon_g^{(s)}(f),\ where\ \varepsilon_g^{(s)}(f) = \varepsilon_g^{(s+1)}(f). \tag{8}$$

Opening by reconstruction (γ_B^R) is defined as the dilation by reconstruction of image f from the erosion of f with a structuring element B, as follows:

$$\gamma_B^R(f) = R_f^{\delta}(\varepsilon_B(f)), \tag{9}$$

Closing by reconstruction (ϕ_B^R) is defined as the erosion by reconstruction of image f from the dilation of f with a structuring element B, as follows:

$$\phi_B^R(f) = R_f^{\varepsilon}(\delta_B(f)). \tag{10}$$

2.2 Top-hat transform by reconstruction

From the opening and closing by reconstruction, the white top-hat transform by reconstruction ($RWTH$) and the black top-hat transform by reconstruction ($RBTH$) are defined as follows [3]:

$$RWTH(f) = f - \gamma_B^R(f), \tag{11}$$

$$RBTH(f) = \phi_B^R(f) - f. \tag{12}$$

$RWTH$ is used to extract bright features from the image and $RBTH$ is used to extract dark features from the image.

2.3 Proposed method

The proposed method for medical image contrast enhancement is a variation of the method described by Bai et al. [4] for grayscale images, called Multiscale Mophology for Contrast Enhancement (MMCE). This method extracts multiple bright and dark features from the image using the classic top-hat transform. The proposed variation consists of using the concept of geodesic reconstruction in the top-hat transform. This allows to enhance the visual quality of the image. The proposed method takes into account the operators connected in the images, making the improved images less distorted, preserving the details and the average brightness.

The following is a description of the proposed method. Given an image f, a disk shaped structuring element B, and the number of iterations n. Where the radius of B increases in each iteration in sizes of 1.

The first step is to obtain the multiple features of the bright areas, which are extracted by RWTH as follows:

$$RWTH_i(f) = f - \gamma_{B_i}^R(f), \tag{13}$$

where $RWTH_i$ are the i-scales of brightness that are extracted from the image. The multiple scales of the bright areas of the image can be expressed as follows:

$$\begin{gathered} RWTH_1(f) = f - \gamma_{B_1}^R(f) \\ RWTH_2(f) = f - \gamma_{B_2}^R(f) \\ RWTH_3(f) = f - \gamma_{B_3}^R(f) \\ \cdots \\ RWTH_n(f) = f - \gamma_{B_n}^R(f) \end{gathered}$$

Similarly, the multiple features of the dark areas of the image are obtained as follows:

$$RBTH_i(f) = \phi_{B_i}^R(f) - f, \tag{14}$$

where $RBTH_i$ are the i-scales of darkness that are extracted from the image. The multiple scales of the dark areas of the image can be expressed as follows:

$$RBTH_1(f) = \phi^R_{B_1}(f) - f$$
$$RBTH_2(f) = \phi^R_{B_2}(f) - f$$
$$RBTH_3(f) = \phi^R_{B_3}(f) - f$$
$$\cdots$$
$$RBTH_n(f) = \phi^R_{B_n}(f) - f$$

The second step will obtain the subtractions between the multiple scales of the bright regions of the image as follows:

$$RWTHS_{i-1}(f) = RWTH_i(f) - RWTH_{i-1}(f). \tag{15}$$

The multiple subtractions of the different scales of the bright areas of the image can be expressed as follows:

$$RWTHS_1(f) = RWTH_2(f) - RWTH_1(f)$$
$$RWTHS_2(f) = RWTH_3(f) - RWTH_2(f)$$
$$RWTHS_3(f) = RWTH_4(f) - RWTH_3(f)$$
$$\cdots$$
$$RWTHS_{n-1}(f) = RWTH_n(f) - RWTH_{n-1}(f)$$

Similarly, the subtractions between the multiple scales of the dark regions of the image will be obtained as follows:

$$RBTHS_{i-1}(f) = RBTH_i(f) - RBTH_{i-1}(f). \tag{16}$$

The multiple subtractions of the different scales of the dark areas of the image can be expressed as follows:

$$RBTHS_1(f) = RBTH_2(f) - RBTH_1(f)$$
$$RBTHS_2(f) = RBTH_3(f) - RBTH_2(f)$$
$$RBTHS_3(f) = RBTH_4(f) - RBTH_3(f)$$
$$\cdots$$
$$RBTHS_{n-1}(f) = RBTH_n(f) - RBTH_{n-1}(f)$$

The third step calculates the maximum values between all the multiple scales that were obtained in the different steps.

The maximum values of all bright and dark scales extracted from the image are defined as:

$$f_w^C = \max_{1 \leq i \leq n} \{RWTH_i(f)\}, \tag{17}$$

$$f_b^C = \max_{1 \leq i \leq n} \{RBTH_i(f)\}. \tag{18}$$

The maximum values of all bright and dark scales extracted from the image by subtraction are defined as:

$$f_w^D = \max_{1 \leq i \leq n-1} \{RWTHS_{i-1}(f)\}, \tag{19}$$

$$f_b^D = \max_{1 \leq i \leq n-1} \{RBTHS_{i-1}(f)\}. \tag{20}$$

The final step is to obtain contrast enhancement of the medical image as follows:

$$f_E = f + (f_w^C + f_w^D) - (f_b^C + f_b^D), \tag{21}$$

where f_E is the medical image with contrast enhancement.

The medical images contrast enhancement process called Geodesic Reconstruction MMCE (GR-MMCE) is described in the following Algorithm 1.

Algorithm 1 Geodesic Reconstruction MMCE.

Input: f, B, n

Output: f_E *(Enhanced image)*

Initialization : B

1: **for** $i = 1$ to n **do**

2: *Calculation of top-hat transform by reconstruction.*

$RWTH_i(f) = f - \gamma_{B_i}^R(f)$, (Equation (13))

$RBTH_i(f) = \phi_{B_i}^R(f) - f$. (Equation (14))

3: *Calculation of subtractions from neighboring scales, obtained through the top-hat transform by reconstruction.*

$RWTHS_{i-1}(f) = RWTH_i(f) - RWTH_{i-1}(f)$, (Equation (15))

$RBTHS_{i-1}(f) = RBTH_i(f) - RBTH_{i-1}(f)$. (Equation (16))

4: **end for**

5: *Maximum values of all the multiple scales obtained.*

$f_w^C = \max_{1 \leq i \leq n} \{RWTH_i(f)\}$, (Equation (17))

$f_b^C = \max_{1 \leq i \leq n} \{RBTH_i(f)\}$, (Equation (18))

$f_w^D = \max_{1 \leq i \leq n-1} \{RWTHS_{i-1}(f)\}$, (Equation (19))

$f_b^D = \max_{1 \leq i \leq n-1} \{RBTHS_{i-1}(f)\}$ (Equation (20))

6: *Medical images contrast enhancement calculation..*

$f_E = f + (f_w^C + f_w^D) - (f_b^C + f_b^D)$ (Equation (21))

7: **return** f_E

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3 Results and discussion

This section presents the comparative analysis between the proposed method and other methods of the literature. Contrast enhancement algorithms that modify the histogram are used for comparison: Histogram Equalization (HE), Contrast-Limited Adaptive Histogram Equalization (CLAHE) [30]. Also, it is compared with the MMCE algorithm which is a method based on multiscale mathematical morphology.

3.1 Assessment metrics

Four metrics are used to assess image enhancement:

Standard deviation (SD) [22,23], this metric assesses the global contrast of grayscale images, defined as:

$$SD(f) = \sqrt{\sum_{q=0}^{L-1} (q - E(f))^2 \times P(q)}, \tag{22}$$

where q is the pixel gray intensity level (u, v) of the image f, $L - 1$ represents the maximum gray intensity level for the image, $E(f)$ represents the average brightness of the image and $P(q)$ is the probability of occurrence of the intensity level q. In case the $SD(f_E)$ of the enhanced image is higher than the $SD(f)$ of the original image, then there is an enhancement in the image.

Peak Signal to Noise Ratio (PSNR), this metric is used to measure the amount of noise introduced in the resulting image f_E, is given by [4]:

$$PSNR(f, f_E) = 10 \log_{10} \left[\frac{(L-1)^2}{MSE(f, f_E)} \right], \tag{23}$$

where the Mean Squared Error (MSE) is defined as:

$$MSE(f, f_E) = \frac{\sum_{u=1}^{M} \sum_{v=1}^{N} [f(u, v) - f_E(u, v)]^2}{M \times N}, \tag{24}$$

where $M \times N$ is the image size f. The higher the PSNR value indicates that less noise is introduced into the enhanced image, and therefore the better the image quality will be.

Absolute Mean Brightness Error (AMBE) [2,20] is used to measure performance in preserving average brightness and is defined as:

$$AMBE(f, f_E) = |E(f) - E(f_E)|. \tag{25}$$

It should be noted that the lower the value of AMBE, the better the preservation of brightness.

The PL rate [4] is used to measure both the noise condition and the improved image clarity. This is defined as follows:

$$PL(f, f_E) = \frac{PSNR(f, f_E)}{\lambda(f_E)}, \tag{26}$$

where λ represents the linear blurring index, is given by:

$$\lambda(f_E) = \frac{2}{MN} \sum_{u=1}^{M} \sum_{v=1}^{N} min(r(u,v), 1 - r(u,v)), \tag{27}$$

$$r(u,v) = sin(\frac{\pi}{2} \times (1 - \frac{f_E(u,v)}{f_{max}})), \tag{28}$$

where f_{max} represents the maximum gray level of the f_E image. The higher PL value indicates that the image contains less noise and is clearer.

3.2 Experimental results

The experiment consists of comparing the proposed algorithm (GR-MMCE) with other techniques in the literature, taking into account SD, PSNR, AMBE and PL as assessment metrics. The aim is to show how good the proposed method is on enhancing contrast and preserving the brightness in medical imaging. Also, the advantages obtained against the other methods will be made known.

For the analysis of results, 100 medical images were selected from a public database [18]. Here we can find images of dental x-rays, hands, legs, backs, thorax, images of retinal vessels, ultrasound, among others. Tests were performed using the HE, CLAHE, MMCE algorithms and the proposed method. The HE and CLAHE algorithms were implemented using the MATLAB program (using the default parameters for CLAHE). The MMCE and the proposed algorithms were implemented in the ImageJ framework [26], the input parameters were the original medical image f, the number of iterations $n = 7$, and the initial disk structuring element B with radius $r = 1$.

Table 1 shows the average obtained by the algorithms in the SD, AMBE, PSNR and PL metrics for enhanced medical imaging.

Table 1
Averages of the assessments of the 100 enhanced medical images with the HE, CLAHE, MMCE algorithms and the proposed algorithm.

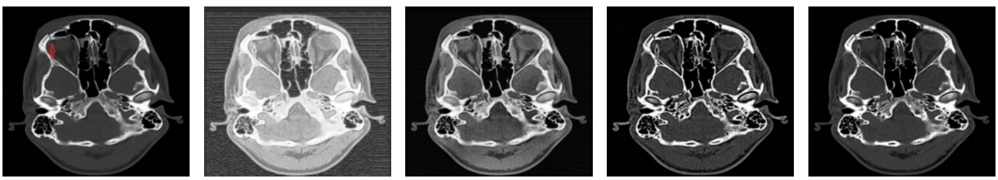
Methods	SD	AMBE	PSNR	PL
Original	61.05	-	-	-
HE	**70.65**	41.22	15.08	44.96
CLAHE	64.83	11.05	19.46	67.22
MMCE	65.90	1.03	24.25	82.81
Proposed algorithm	63.20	**0.95**	**29.83**	**100.37**

Results in Table 1 show that the HE algorithm improves the contrast of the medical image, but distorts it, loses the average brightness and the relationship between distortion and clarity is the lowest. CLAHE improves contrast, distorts to a lesser degree than HE, but loses the average brightness and in terms of the relationship between distortion and clarity it also has a low performance. MMCE improves the contrast, introduces distortions in the image in less proportion to HE and CLAHE, preserves the average brightness, and the relationship between distortion and clarity remains good. The proposed method enhances contrast in medical imaging, preserves the average brightness of medical imaging, distorts it less, and the relationship between distortion and clarity is higher than the other algorithms.

Figs. 1, 2, 3 and 4 it can be seen that all the algorithms improve medical images. In addition, we see that the HE method over-exposes the light details which causes high brightness noise to be introduced. The CLAHE and MMCE algorithms show a good brightness enhancement, but also a lot of noise is introduced. The proposal enhances the bright areas and further darkens the dark areas while preserving the details, being quite similar to the original image.

(a) (b) (c) (d) (e)

Fig. 1. (a) Original image 20.*png* with SD = 62.81; (b) Image enhanced with HE algorithm with SD=74.67, AMBE=68.12 and PSNR=10.76 ; (c) Image enhanced with CLAHE algorithm with SD=64.65, AMBE=15.01 and PSNR=19.76; (d) Image enhanced with MMCE algorithm with SD=65.48, AMBE=1.14 and PSNR=27.17; and (e) Image enhanced with the proposed algorithm with SD=63.49, AMBE=0.33 and PSNR=34.49.

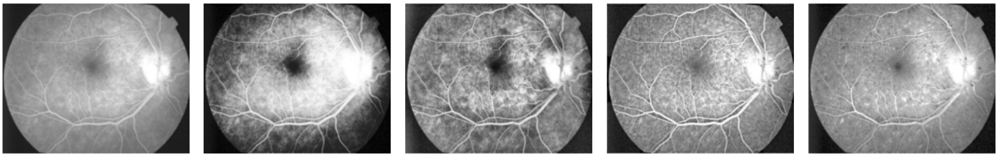


(a) (b) (c) (d) (e)

Fig. 2. (a) Original image 21.*png* with SD = 56.15; (b) Enhanced image with HE algorithm with SD=68.93, AMBE=87.28 and PSNR=8.68 ; (c) Enhanced image with CLAHE algorithm with SD=66.84, AMBE=20.30 and PSNR=18.88; (d) Image enhanced with MMCE algorithm with SD=68.53, AMBE=3.32 and PSNR=19.75; and (e) Image enhanced with the proposed algorithm with SD=61.08, AMBE=0.89 and PSNR=27.43.

(a) (b) (c) (d) (e)

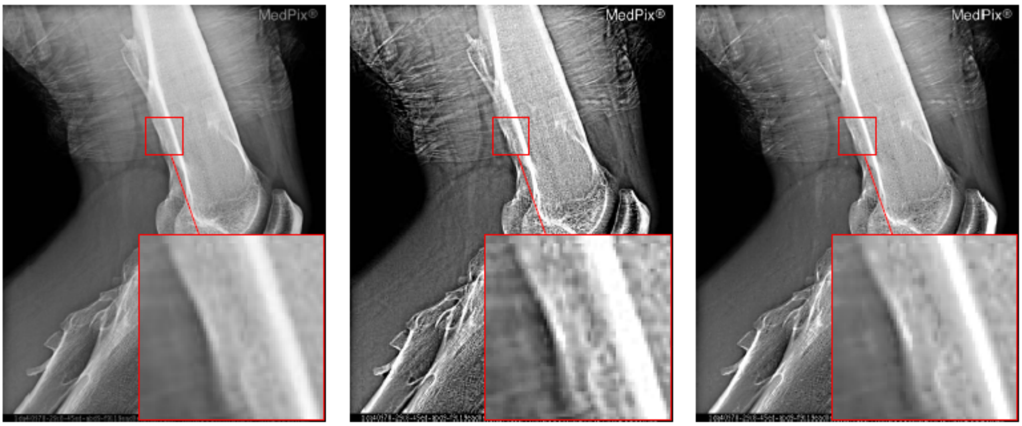
Fig. 3. (a) Original image 51.*png* with SD = 57.53; (b) Image enhanced with HE algorithm with SD=75.02, AMBE=21.89 and PSNR=18.98 ; (c) Image enhanced with CLAHE algorithm with SD=60.67, AMBE=8.10 and PSNR=19.28; (d) Image enhanced with MMCE algorithm with SD=61.49, AMBE=0.57 and PSNR=25.78; and (e) Image enhanced with the proposed algorithm with SD=59.25, AMBE=0.55 and PSNR=31.31.



(a) (b) (c) (d) (e)

Fig. 4. (a) Original image 98.*png* with SD = 49.79; (b) Image enhanced with HE algorithm with SD=74.84, AMBE=15.29 and PSNR=16.53 ; (c) Image enhanced with CLAHE algorithm with SD=50.07, AMBE=13.33 and PSNR=18.64; (d) Image enhanced with MMCE algorithm with SD=56.76, AMBE=0.78 and PSNR=21.99; and (e) Image enhanced with the proposed algorithm with SD=51.58, AMBE=0.35 and PSNR=33.93.

Fig. 5 shows this: (a) the zoom of the original medical image, (b) the zoom of the enhanced medical image with the MMCE algorithm and (c) the zoom of the enhanced medical image with the proposed algorithm. The image enhanced with the proposed method improves the contrast of the original image and introduces less distortion with respect to the MMCE algorithm.



(a) (b) (c)

Fig. 5. (a) Zoom of the original image 62.*png*, (b) Zoom of the image enhanced with the MMCE algorithm and (c) Zoom of the image enhanced with the proposed algorithm.

4 Conclusions

In this work we presented an algorithm to improve medical images using the multiscale top-hat transform by geodesic reconstruction. Morphological reconstruction allows us to preserve the details of the image. The performance of the proposed method was assessed with metrics measuring contrast enhancement, conservation of average brightness, signal-to-noise ratio of the image and the relationship between image distortion and clarity. The numerical results show that the proposed method performs well in PSNR, PL and AMBE metrics. It also improves the contrast of the original image according to SD. This indicates that the improved images have low distortion, preserve their natural brightness and have enhanced contrast.

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