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# Estimates Dynamic Material Properties of Ligaments under the Fast Strain Rate

Jiuguang Zhou[a,b], Zuwen Qian[c], Zhiwen Cui[a,*], Ruilan Zhang[b], Kexie Wang[a]

[a]*Department of Acoustics and Microwave Physics,College of Physics, Jilin University, Qianjin Street 2699, Changchun, 130012 ,china*
[b]*College of Basic medical, Beihua University, huashan Road 2699, jilin, 132001, China*
[c]*Institute of Acoustics, Chinese Academy of Sciences, Beijing,100190,china*

**Abstract**

The cyclic mechanical property of ligament plays an important role in biomechanics. If the microstructural volume fractions, orientations, and interactions among components for different ligament types are understood, it is conceivable to develop a general ligament model that is based on microstructural properties. But the cycle loading experiment is very difficult to operate, for convenience we can use the relationship between the complex modulus and relaxation modulus. Thus, in order to characterize the comprehensive viscoelastic behavior of cervical spine ligaments within their physiological range, the mechanical response of three types of human cervical spines: the anterior longitudinal ligament, the posterior longitudinal ligament and the ligamentum flavum and a porcine posterior longitudinal ligament collagen fascicles were simulated using quasi-linear viscoelastic theory model using the data from stress relaxation experiments at fast strain rate. A seven parameter model was investigated, which is sufficient to characterize the ligament dynamic material properties. Such model used the instantaneous elastic functions approximated as linear under fast strain rate. Dynamic material properties including storage modulus, loss modulus, the dynamic modulus of elasticity and the internal damping were investigated by transforming the stress relaxation data into frequency domain by Laplace transformation.



*Keywords*: Ligament; Fascicle; Viscoelasticity; Modulus; Damping.

* Corresponding author. Tel.: +86-431-86781598; fax: +86-431-85166112.
*E-mail address:* cuizw@jlu.edu.cn; zhoujg10@mail.jlu.edu.cn


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## 1. Introduction

Measurement and evaluation of dynamic material properties (storage modulus, loss modulus, the dynamic modulus of elasticity $|M|$ and the internal damping $\tan\delta$), especially, the internal damping for biological tissue, is not only beneficial to the research on Physiology, Kinematics, Pathology and propagation characteristics of ultrasonic beam in tissues, but also is conducive to explain the problem that the heated focus region in high intensity focused ultrasound (HIFU) diagnosis is shift.

So far, the quasi-linear viscoelastic(QLV) theory proposed by Fung[1] has been well developed and widely adopted for modeling the viscoelastic behavior of ligaments by many laboratories[3-11]. Also some researchers have reported that QLV is only appropriate to certain strain levels[3], and some have stated that QLV is appropriate at fast strain rates [6-8]. However, a limitation exists that the QLV theory is based on a step change in strain which is not possible to perform experimentally. Accounting for this limitation，it may result in regression algorithms that converge poorly and yield nonunique solutions with highly variable constants[9]. In order to obtain better approximate solutions, one of the common methods is to use five constants to describe the instantaneous elastic response (constants A and B) and reduced relaxation function (constants $C$, $\tau_1$ , and $\tau_2$) on experiments with finite ramp times followed by stress relaxation to equilibrium. Abramowitch SD et al.[10] introduced this approach in goat femur-medial collateral ligament-tibia tests with slow strains. Then the approach was well developed by Lucas et al.[7,8,11] using seven constants to describe the instantaneous elastic response (constants A and B) and reduced relaxation function (constants $G_1$, $G_2$, $G_3$, $G_4$, $G_\infty$) on experiments with finite ramp times under both the slow strain rates and the fast strain rates.

As far as the author knows, there is a lack of published data concerning the cyclic viscoelastic properties of ligaments in the current literature. Yoganandan et al.[12] determined that the tensile strength, stiffness, and energy absorbing capacity of cervical spine anterior longitudinal ligament(ALL) and ligamentum flavum(LF) were dependent on the rate of applied loading. The stress-relaxation and hysteresis behavior of the posterior longitudinal ligament(PLL) and ALL were reported by Hukins et al.[13]. Yahia et al.[14] investigated the hysteresis, stress relaxation and stepwise load relaxation behavior of human lumbar ISL+SSL ligament complexes. Recently Troyer et al.[15] studied the comprehensive viscoelastic characterization of the human lower cervical spine ligament using seven constants, including stress relaxation, dynamic mechanical analysis (the storage modulus, the loss modulus, the complex modulus, and the $\tan\delta$). However, the dynamic mechanical analysis for human cervical spine ligaments using the modified QLV theory (seven constants) under fast strain rate has not been investigated. The objective of the presented study is to estimate dynamic material properties of ligaments in frequency domain using modified QLV approach. We focus on the investigation of three types of ligaments and compare them with porcine collagen fascicles under fast strain rates.

## 2. Structure of Ligament

Ligaments play an important role in spine biomechanics. Therefore, in order to understand and model spine biomechanics accurately, it is requisite to gain a precise understanding of the internal structures of ligaments, such as connective tissues, of the body region that is modeled. Furthermore, as connective tissues are viscoelastic, their properties must be valid for the range of simulated loading conditions, including strain magnitude and strain rate. Ligaments are composed of varied amounts of the same microstructural components. Thus, it is conceivable to develop a material model based on the microstructure that can be used to predict the force response of an arbitrary ligament-type based on its microstructural composition and the microstructural material behavior. In as early as the nineteenth century, anatomical investigations[16] identified human connective tissue to be organized in a hierarchical manner from macroscopic organized structures to microscopic molecules. Kastelic[18] recognized the collagen fascicle and deemed the fascicle as the fundamental structural unit of tendons. The passive tensile tissues, ligament, are also composed largely of water (60-80%) and collagen, elastin, but contain very little of the ground substance (arranged in parallel with a Maxwell element which accounts for the viscoelasticity of the proteoglycans) that give ligament its unique mechanical properties. To keep with the functional role of these tissues, the collagen fibrils are organized primarily in long strands parallel with the axis of loading. As the Ligament is loaded, the bending angle of the crimp structure of the collagen fibers can be seen to reversibly decrease, which indicates that deformation of this structure is one source of elasticity. Individual collagen fibrils also display some inherent elasticity, and these two features are believed to determine the bulk properties of passive tensile tissues[17].

## 3. Cyclic mechanical properties

If a linear viscoelastic material is subjected to harmonic oscillations, the strain will "lag" the stress with a lag phase $\delta$ due to internal damping within the material, which is a consequence of the viscous component of the material[19]. The tangent of the phase lag (i.e., $\tan\delta$) is called the loss tangent and is a measure of a material's internal damping. However, for viscoelastic body, in the stable vibration, stress and strain meet plural type constitutive relation is $\sigma(t)=E(i\omega)\varepsilon(t)$, where $E(i\omega)$ is the dynamic complex modulus of a material which can be expressed as:

$$E(i\omega)= E_1(i\omega)+iE_2(i\omega)=|M|e^{i\delta} \tag{1}$$

where $E_1(i\omega)$ is storage modulus and $E_2(i\omega)$ is loss modulus, $|M|$ is elastic modulus, and

$$\tan\delta=E_2(i\omega)/ E_1(i\omega) \tag{2}$$

When a viscoelastic body was exerted a loading (stress), the deformation (strain) is after. Then we keep deformation (strain) unchanged, the stress-strain relationship can be expressed as $\sigma(t) = \varepsilon_0 G(t)$, where $G(t)$ is the stress relaxation function. In the constitutive equation $\sigma(t)=E(i\omega)\varepsilon(t)$, if we make $\varepsilon(t) =H(t)$(for $t<0$, $H(t)=0$;for $t=0$, $H(t)=1/2$;and for $t>0$, $H(t)=1$), according to the Laplace transform, we can get the relationship between the complex modulus and relaxation modulus

$$E(i\omega)=i\omega G(i\omega) \tag{3}$$

So we can estimated the dynamic material properties of soft tissue indirectly if we get the complex modulus by measuring the relaxation function.

## 4. Modified QLV theory

For each ligament, the stress response, $\sigma(t)$, was analyzed in terms of the fascicle elongation using a hereditary integral for nonlinear displacement-time separable viscoelastic responses[2]:

$$\sigma(t) = \int_{-\infty}^{t} G(t-\xi)\frac{\partial\sigma^e(\varepsilon)}{\partial\varepsilon}\frac{\partial\varepsilon}{\partial\xi}d\xi \tag{4}$$

where $G(t)$ is the reduced relaxation function, $\sigma^e(\varepsilon)$ is the instantaneous elastic stress, $\varepsilon$ is the elongation, $t$ is the time, and $\xi$ is a dummy variable for integration. The stress response during the strain ramp onset was included in this analysis.

The instantaneous elastic stress[2], $\sigma^e(\varepsilon)$, was approximated as:

$$\sigma^e(\varepsilon)=A(e^{B\varepsilon}-1) \tag{5}$$

where parameters $A$ and $B$ are material constants[2].

The reduced relaxation function was expressed as a summed exponential function as follows:

$$G(t)=G_\infty+\sum_{n=1}^{4}G_n e^{-t/\tau_n} \tag{6}$$

where $G_\infty$ is the long-term relaxation parameter (i.e., $G_\infty=\lim_{t\to\infty}G(t)$), and $G_n$ is the relaxation parameter corresponding to each of the time constant $\tau_n$. The time constants, $\tau_n$, were constrained to decade values and Equation *(6)* was subjected to the constraint $G_1+G_2+G_3+G_4+G_\infty=1$[6].

It is possible for stress relaxation to occur during the ramping phase because of a finite ramping time exists in the relaxation experiments (versus an instantaneous Heaviside step function). Therefore, we can separate equation *(5)* into ramping and relaxation region, and the parameters were simultaneously fitted to both regions using a previously reported procedure [10]. During the ramping region, the ligament was subjected to a constant strain rate $\gamma$ over the times $0<t\leq t_0$. The time $t_0$ was determined by finding the time associated with the maximum stress magnitude of the stress versus time. After inputting the relations $\varepsilon=\gamma t$ and $\partial\varepsilon/\partial\xi=\gamma$ (for $0<t\leq t_0$), finding $\partial\sigma^e(\varepsilon)/\partial\xi$, the ramping region of equation *(4)* was written as:

$$\sigma(t:0<t\leq t_0,\theta)=AB\gamma\int_0^t(G_\infty+\sum_{n=1}^{4}G_n e^{-(t-\xi)/\tau_n}e^{B\gamma\xi})d\xi$$

$$=AB\gamma\left\{G_\infty(e^{Bt\gamma}-1)/(B\gamma)+\sum_{n=1}^{4}G_n e^{-t/\tau_n}\tau_n/(1+B\gamma\tau_n)\left[e^{(B\gamma\tau_n+1)t/\tau_n}-1\right]\right\} \tag{7}$$

where $\theta=\{A, B, G_1, G_2, G_3, G_4, G_\infty\}$

The stress in the relaxation region of the curve is equal to the stress from the previous ramping region plus the stress history during the relaxation region. Thus, equation *(7)* can be described as:

$$\sigma(t:t>t_0,\theta)=AB\gamma\int_0^{t_0}G(t-\xi)e^{B\gamma\xi}+AB\gamma\int_0 G(1-\xi)e^{B\gamma\xi}d\xi$$

$$=AB\gamma\left\{G_\infty(e^{B\gamma t_0}-1)/(B\gamma)+\sum_{n=1}^{4}G_n e^{-t/\tau_n}\tau_n/(1+B\gamma\tau_n)\left[e^{(B\gamma\tau_n+1)t_0/\tau_1}-1\right]\right\} \tag{8}$$

An optimal solution for $A$, $B$, $G_1$, $G_2$, $G_3$, $G_4$ and $G_\infty$ can be found by a generalized reduced gradient technique or nonlinear least-squares method.

## 5. Numerical simulation

According to equation *(2)* and *(6)*, and Fourier transform, in the frequency domain we can get

$$E(i\omega) = G_{\infty}\frac{1}{\omega} + G_1\frac{\omega^2\tau_1^2}{1+\omega^2\tau_1^2}...G_4\frac{\omega^2\tau_4^2}{1+\omega^2\tau_4^2} + i(G_1\frac{\omega\tau_1}{1+\omega^2\tau_1^2} + ...G_4\frac{\omega\tau_4}{1+\omega^2\tau_4^2}) \tag{10}$$

$$\tan\delta = (G_1\frac{\omega\tau_1}{1+\omega^2\tau_1^2} + ...G_4\frac{\omega\tau_4}{1+\omega^2\tau_4^2})/(G_{\infty}\frac{1}{\omega} + G_1\frac{\omega^2\tau_1^2}{1+\omega^2\tau_1^2}...G_4\frac{\omega^2\tau_4^2}{1+\omega^2\tau_4^2}) \tag{11}$$

Firstly we investigate relaxation function under the fast strain rate. The isolated ALL, PLL and LF bone–ligament–bone complexes from male and female human cervical spines under fast-rate deformations were harvested for mechanical testing in Ref [7]. And isolated collagen fascicle segments were removed from seven porcine lumbar posterior longitudinal ligament (PLL) bone–ligament–bone complexes and prepared for mechanical testing in Ref [11]. The results of the Ref [7] and [11] indicate that QLV is appropriate for both spinal ligaments and isolated collagen fascicles under fast rate deformations. In this study, the time constants, $\tau_n$, were the same between the fascicle model and the spinal ligament mode and constrained to decade values ($\tau_1$ = 1s, $\tau_2$ = 100 ms, $\tau_3$ = 10 ms, $\tau_4$ = 1ms)[7].

Table 1. The relaxation parameters.

| Type | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_\infty$ |
|---|---|---|---|---|---|
| Collagen fascicle | 0.03 | 0.02 | 0.05 | 0.59 | 0..31 |
| ALL | 0.03 | 0.03 | 0.04 | 0.65 | 0.25 |
| PLL | 0.03 | 0.03 | 0.04 | 0.69 | 0.21 |
| LF | 0.06 | 0.06 | 0.08 | 0.61 | 0.19 |

The average five relaxation coefficients in Ref [7] and Ref [11] are shown in Table 1. The parameters of the instantaneous elastic stress in Ref [7] and Ref [11]are as follow**:** *A*=6,89N; *B*=8.15; the fast strain rates were 10.9–84.6 $s^{-1}$.

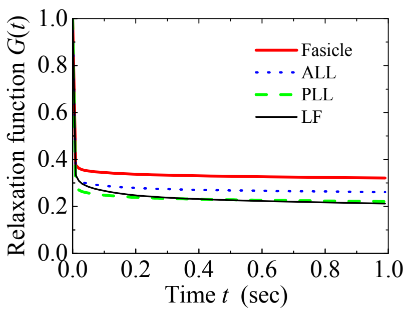


Fig. 1. Reduced relaxation function in comparison the fascicle mode with the spinal ligament model. The ALL, PLL and LF data are from Ref [7], the fascicle mode date are from Ref [11].

In order to compare the reduced relaxation functions of the collagen fascicle, ALL, PLL, and LF, we display curves in Fig. 1. As is shown in Fig. 1, the fascicle and ligament relaxation functions are similar in shape and magnitude, although there is a slight divergence in early relaxation behavior. The fascicle fast rate coefficient in this paper ($G_4$ = 0.59) is smaller in magnitude than the average ALL, PLL, and LF fast rate relaxation coefficients. Thus, the ligament relaxation functions exhibit a larger decrease in magnitude early in the relaxation time history.

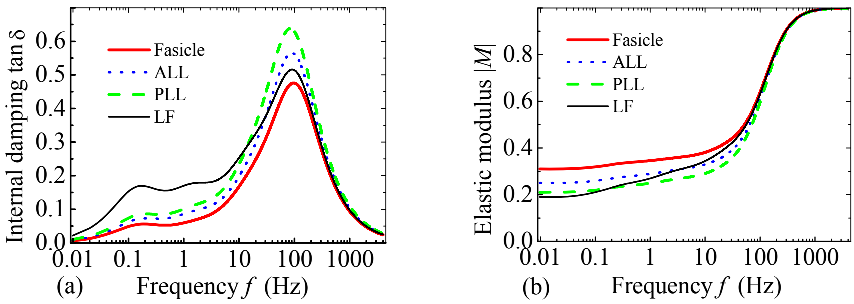


Fig. 2. The internal damping tan$\delta$ and the dynamic modulus of elasticity |*M*| plotted as a function of logarithm of frequency in comparison the fascicle mode with the spinal ligament model. The data is same as Fig. 1. (a) tan$\delta$; (b) |*M*|.

When the internal damping tan$\delta$ and the elastic modulus |*M*| are plotted against the frequency *f*, curves as shown in Fig. 2 are obtained. At slower frequencies, damping tan$\delta$ increase slowly with the frequency *f*, and reaches a peak when the frequency *f* is near 100 Hz. Then decrease sharply and in final the damping is approximately equal to zero when the frequency is greater than 4500 Hz. Correspondingly, the elastic modulus |*M*| increase all the time with the frequency *f* and rises dramatically in the neighborhood of 100Hz. And for comparison, the storage modulus and loss modulus are shown in Fig. 3. It is obvious that the storage modulus and the elastic modulus |*M*| have the similar characteristics with the increase of frequency. Also internal damping tan$\delta$ and the loss modulus have the same characteristics with the increase of the frequency.

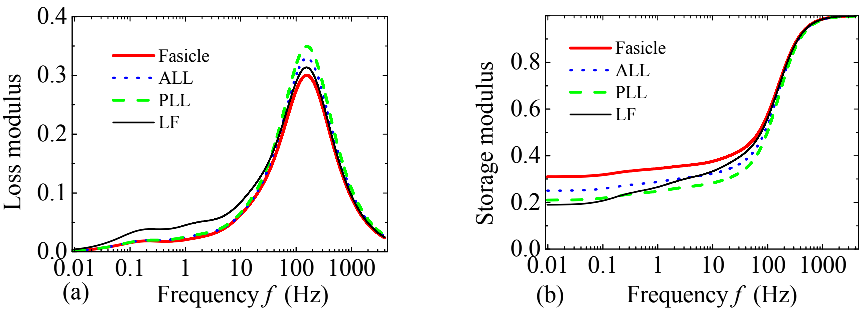


Fig. 3. The loss modulus and the storage modulus plotted as a function of logarithm of frequency in comparison the fascicle mode with the spinal ligament model. The data is same as Fig. 1. (a) the loss modulus; (b) the storage modulus.

There is a histological difference between the LF and the ALL and PLL. Water makes up about two-thirds of the weight of normal ligaments, and the remaining weight is primarily made up of collagen and elastin. Elastin is mostly known as “linearly” elastic materials, however, collagen is a basic structural element for soft and hard tissues in animals[2].The elastin to collagen ratio for the LF is 2:1[20]. The dry weights of elastin and collagen in the cervical spine PLL are only approximately 5.9% and 67.1% respectively (giving a 1:11 elastin to collagen ratio)[21], which is similar to the ALL. The cyclic mechanical property of the ligaments is likely to be affected by the differences in elastin and collagen composition. Fig. 2 shows that, for LF, the amplitude of internal damping tan$\delta$ is lower than the cervical spine PLL and ALL; the similar shape of frequency spectrum with respect to internal damping tan$\delta$ for cervical spine PLL and ALL is different from LF. Moreover, the amplitude of internal damping for the fascicle is lower than that of ALL and PLL although the shape of frequency spectrum curves is same for the fascicle, ALL and PLL. One of the important reasons is that the dry weights of ground substance (Proteoglycans) and collagen in the cervical spine PLL are only approximately 20%~30% and 67.1% respectively, and viscous damping of ground substance is stronger. One factor for these difference may be the large (orders of magnitude) difference in ramp times and relaxation holding times – both of which were extremely short for this study, emphasizing fast strain-rate behavior compared with the non-injurious time-frames examined previously[22].

In these fast-rate deformations, ligament viscoelasticity which is partly due to fluid flow and interactions among collagen fascicles and ground substance[23-25] is ignored. Because the comparable collagen fascicle and spinal ligament relaxation functions suggest that, at fast rate deformations, ligament viscoelasticity may be attributed more to the intrinsic viscoelastic properties of the collagen fascicle.

## 6. Conclusions

All ligament types and collagen fascicle exhibited nonlinear viscoelastic cyclic behavior. Our study supports the assertion that QLV is valid for a ligament under fast stain rate. The storage modulus, loss modulus, the dynamic modulus of elasticity $|M|$ and the internal damping tan$\delta$ for all ligament types and collagen fascicle are dependent on frequency. All ligament types and collagen fascicle exhibited an increased cyclic viscoelastic with response frequencies at slower frequencies; when the frequency $f$ is near 100 Hz, the damping tan$\delta$ and loss modulus reach a peak, while the dynamic modulus of elasticity $|M|$ and the storage modulus show the fastest rise; then all of them diminished viscoelasticity as frequency increased. The ALL, PLL and the collagen fascicle exhibited similar damping properties, but the damping of the collagen fascicle is lower. However, LF exhibited a different damping characteristic and lower than the ALL and PLL. Overall, the LF exhibited a more elastic response than the ALL and PLL. It is interesting work to compare the dynamic material properties in the present paper with that of a ligament model under slow strain rate. This is the subject of our ongoing research.

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