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Intra-symbol Piecewise Equalization for OFDM Systems in Fast Fading Channels

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Abstract

Based on intra-symbol piecewise linear channel model, a novel piecewise equalizer is proposed for OFDM systems in high mobility environment. In this proposed scheme the complex ICI caused by fast fading channels has been resolved as new OFDM subsystems' simple ISI and lower ICI, respectively caused by the average channel and the time-varying channel over a piece-period. By further employing the characteristics of the channel matrix, a successive `ISI' and `ICI' mitigation based equalization with low complexity has been realized. Simulation results show that the proposed scheme has lower computation complexity than the comparative one, while perserving good performance.

1. Introduction

By expanding the symbol duration and adding the cyclic prefix, OFDM systems can easily overcome the multi-path dispersion by simple 1-tap equalization [1]. On the other hand, OFDM systems become more susceptible to time-variations as symbol length increases. Hence, the equalization over fast fading channels becomes a challenging and important issue for OFDM systems [2].

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The existing equalization schemes over fast fading channels can be roughly divided into 2 classes, namely, pure-equalization and turbo-equalization. The pure-equalization schemes usually assume the channel information completely known by the receiver, and focus on low complexity data symbol estimation by fully using the channel characteristics [1],[3]-[4]. While the turbo-equalization schemes estimate the channel and/or data symbol and equalize the channel alternatively and iteratively, and generally perform better than the former at the cost of higher complexity [5]-[6]. Despite the rigorous requirement of complete time varying channel information, pure-equalization schemes with low complexity still preserve their importance. First, because the tradeoff between the equalization performance and complexity is still a challenging problem, even with complete channel information [3]. Second, because turbo-equalization schemes usually employ pure-equalization as data symbol estimation component.

Inspired by the piecewise linear channel model in [2], a low complexity intra-symbol piecewise pure-equalization scheme over the fast fading channels is proposed in this paper. Based on the intra-symbol piecewise linear channel model, the complex ICI caused by the fast fading channel is resolved as the new OFDM subsystems' simple ISI and lower ICI, respectively caused by the average channel and time-varying channel over a piece-period. By fully exploiting the piecewise linearity of time varying channels, a successive ISI and ICI mitigation based equalization is realized with low complexity.

This paper is organized as follows. Section 2 introduces the intra-symbol piecewise OFDM system model and section 3 describes the proposed equalization scheme. Further complexity analysis and some simulation examples to demonstrate the performance of the proposed scheme are provided in section 4 and 5 respectively. And conclusions are drawn in section 5.

Notation: $(\bullet)^T$ and $(\bullet)^H$ represent transpose and complex conjugate transpose (Hermitian), respectively. $(\mathbf{A})^{UT}$ and $(\mathbf{A})^{LT}$ denotes the upper triangular part (excluding the diagonals) and the lower triangular part (including the diagonals) of matrix $\mathbf{A}$. $diag(\mathbf{a})$ is a diagonal matrix with the vector $\mathbf{a}$ on the diagonal and $blkdiag(\bullet)$ denotes block diagonal concatenation of the input arguments. Finally, $\mathbf{I}_N$ and $\mathbf{F}_N$ denote the $N \times N$ identity matrix and FFT matrix respectively.

2. System Model and Assumption

Consider an OFDM system with N subcarriers in a fast time-varying multipath channel. Let the fast fading channel be expressed as time varying tapped delay line model $h(n,l), n = 0,1,\cdots,N-1; l = 0,1,\cdots,L-1$. Assuming the channel's maximum delay length L less than cyclic prefix length and perfect synchronization, the time domain signal with CP removed at the receiver $\mathbf{r} = [r(0), r(1), \cdots, r(N-1)]^T$ can be written in matrix form [6] as

$$\mathbf{r} = \mathbf{H}\mathbf{F}_N^H \mathbf{X} + \mathbf{W} \tag{1}$$

where $\mathbf{X} = [X(0), X(1),..., X(N-1)]^T$ and $\mathbf{W} = [W(0), W(1),\cdots, W(N-1)]^T$ are 'frequency domain' data vector and additive white Gaussian noise (AWGN) vector respectively, and $N \times N$ time domain channel matrix $\mathbf{H}$ is the circular form of $N \times L$ channel impulse response (CIR) matrix $\mathcal{H}$ with $\mathcal{H}(p,q) = h(p-1, q-1)$. And define the circular shift operation on the CIR matrix as

$$\mathbf{H} = \mathbf{CSHIFT}(\mathcal{H}, N) \tag{2}$$

$$\mathbf{H}(k,m) = \begin{cases} \mathcal{H}(k, ((k-m))_N + 1) & 0 \le k - m \le L - 1 \\ or & -(N-1) \le \quad k - m \le -(N-L) \\ 0 & else \end{cases}$$

where $((\bullet))_N$ represents a cyclic shift in the base of N.

To match the fast varying channel, the channel is further approximated as intra-symbol piecewise linear model [7]. In this model the symbol period is divided into $M = 2^p$ pieces and p is a natural number, and the CIR varies linearly in each piece-period. Let the average CIR vector $\bar{\mathbf{h}}_m$ in the m-th piece-period be denoted as the middle instant CIR vector $\bar{\mathbf{h}}_m = \mathbf{h}(mN/M + N/(2M),:)$ as in [2], and the difference between the average CIR vector in the $(m+1)$-th and m-th piece-period be denoted as $\Delta\bar{\mathbf{h}}_m = \bar{\mathbf{h}}_m - \bar{\mathbf{h}}_{m-1}$. By using the CIR in the m-th piece-period, define the $(N/M) \times (N/M)$ CIR matrix $\mathbf{H}_m$, $\bar{\mathbf{H}}_m$ and $\overset{\Delta}{\mathbf{H}}_m$ as the circular form of $(N/M) \times L$ the CIR matrix $\mathcal{H}_m$ with $\mathcal{H}_m(p,q) = h(mN/M + p - 1, q - 1)$, average CIR matrix $\overline{\mathcal{H}}_m$ with $\overline{\mathcal{H}}_m(p,q) = h(mN/M + N/(2M), q - 1)$, and linear variation CIR matrix $\overset{\Delta}{\mathcal{H}}_m$ with $\overset{\Delta}{\mathcal{H}}_m(p,q) = \Delta\bar{\mathbf{h}}_m(q-1)$ respectively, which can be expressed as

$$\mathbf{H}_m = \mathbf{CSHIFT}(\mathcal{H}_m, N/M) \tag{3}$$

$$\bar{\mathbf{H}}_m = \mathbf{CSHIFT}(\overline{\mathcal{H}}_m, N/M) \tag{4}$$

$$\overset{\Delta}{\mathbf{H}}_m = \mathbf{H}_m - \bar{\mathbf{H}}_m = diag(\frac{2M*(0:(N/M-1))-N}{2N})\mathbf{CSHIFT}(\overset{\Delta}{\mathcal{H}}_m, N/M) \tag{5}$$

where the second equation in (5) is derived from the property of the piecewise linear channel model.

It can be derived that the channel matrices $\mathbf{H}$ and $\mathbf{H}_m$, $\bar{\mathbf{H}}_m$, $\overset{\Delta}{\mathbf{H}}_m$ satisfy the following matrix form if $2L \le N/M$.

$$\mathbf{H} = \underbrace{blkdiag(\bar{\mathbf{H}}_0, \bar{\mathbf{H}}_1, \cdots, \bar{\mathbf{H}}_{M-1})}_{\bar{\mathbf{H}}} + \underbrace{blkdiag(\overset{\Delta}{\mathbf{H}}_0, \overset{\Delta}{\mathbf{H}}_1, \cdots, \overset{\Delta}{\mathbf{H}}_{M-1})}_{\overset{\Delta}{\mathbf{H}}} + \underbrace{\begin{Bmatrix} -(\bar{\mathbf{H}}_0)^{UT} & \mathbf{0}\cdots & \mathbf{0} & (\bar{\mathbf{H}}_0)^{UT} \\ (\bar{\mathbf{H}}_1)^{UT} & -(\bar{\mathbf{H}}_1)^{UT} & \mathbf{0}\cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \vdots \\ \%\vdots & (\mathbf{H}_{M-2})^{UT} & (\mathbf{H}_{M-2})^{LT} & \mathbf{0} \\ \mathbf{0}\cdots & \mathbf{0} & (\bar{\mathbf{H}}_{M-1})^{UT} & -(\bar{\mathbf{H}}_{M-1})^{UT} \end{Bmatrix}}_{\overset{ISI}{\mathbf{H}}} + \begin{Bmatrix} -(\overset{\Delta}{\mathbf{H}}_0)^{UT} & \mathbf{0}\cdots & \mathbf{0} & (\overset{\Delta}{\mathbf{H}}_0)^{UT} \\ (\overset{\Delta}{\mathbf{H}}_1)^{UT} & -(\overset{\Delta}{\mathbf{H}}_1)^{UT} & \mathbf{0}\cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \vdots \\ \mathbf{0}\cdots & \mathbf{0} & (\overset{\Delta}{\mathbf{H}}_{M-1})^{UT} & -(\overset{\Delta}{\mathbf{H}}_{M-1})^{UT} \end{Bmatrix}$$

$$\tag{6}$$

Due to the sparsity of the matrix $(\overset{\Delta}{\mathbf{H}}_m)^{UT}$ and the small channel difference between the neighbouring piece-periods, the fourth term in the right hand is negligible. And let the first three terms be denoted by $\bar{\mathbf{H}}$, $\overset{\Delta}{\mathbf{H}}$ and $\overset{ISI}{\mathbf{H}}$ respectively.

Let the received signals in the m-th piece-period be denoted by $\mathbf{r}_m = [r(mM+0), r(mM+1), \cdots, r(mM-1+N/M)]^T$, then divide the 'frequency domain' data symbol $\mathbf{X}$ into M groups with the m-th data symbol group denoted as $\mathbf{X}_m = [X(m), X(m+1 \cdot M), X(m+2 \cdot M), \cdots X(m+(\frac{N}{M}-1) \cdot M)]^T$, the received signal $\mathbf{r}$ can be expressed by using the M symbol groups $\{\mathbf{X}_m\}_{m=0}^{M-1}$ as

$$\mathbf{r} = [(\mathbf{r}_0)^T (\mathbf{r}_1)^T \cdots (\mathbf{r}_{M-1})^T]^T = \mathbf{H} \cdot \underbrace{\begin{Bmatrix} \omega_{1,1}\mathbf{I}_{N/M} & \cdots & \omega_{1,M}\mathbf{I}_{N/M} \\ \omega_{2,1}\mathbf{I}_{N/M} & \cdots & \omega_{2,M}\mathbf{I}_{N/M} \\ \vdots & \ddots & \vdots \\ \omega_{M,1}\mathbf{I}_{N/M} & \cdots & \omega_{M,M}\mathbf{I}_{N/M} \end{Bmatrix}}_{\omega} \cdot \underbrace{\begin{Bmatrix} \mathbf{E}_0\mathbf{F}_{N/M}^H\mathbf{X}_0 \\ \mathbf{E}_1\mathbf{F}_{N/M}^H\mathbf{X}_1 \\ \cdots \\ \mathbf{E}_{M-1}\mathbf{F}_{N/M}^H\mathbf{X}_{M-1} \end{Bmatrix}}_{\mathbf{Y}} \approx \bar{\mathbf{H}} \cdot \omega \cdot \mathbf{Y} + \overset{\Delta}{\mathbf{H}} \cdot \omega \cdot \mathbf{Y} + \overset{ISI}{\mathbf{H}} \cdot \omega \cdot \mathbf{Y} + \mathbf{W} \tag{7}$$

where $\omega_{p,q} = e^{\frac{j2(p-1)(q-1)\pi}{M}}$, $\mathbf{E}_m = diag(1, e^{j2\frac{2m\pi}{N}}, \cdots, e^{j2\frac{(N/M-1)m\pi}{N}})$.

According to equation (7), the OFDM system can be equivalent as M subsystems, and the m-th subsystem is a new OFDM system with N/M subcarriers, data symbol $\mathbf{X}_m$ and subcarrier spacing $\Delta f' = M\Delta f$, and experiences frequency offset of $m\Delta f$. Thus the IFFT modulated signal of the original OFDM system in each piece-period can be viewed as the weighted-sum of M OFDM subsystems' symbols. Such intra-symbol piecewise OFDM system model for $M = 2$ has been shown in Fig. 1.

The first term and the second term in the right hand of equation (7) individually represents the *M* 'symbols' through the average channel and the linear time-varying channel in different piece-periods. The third term in the right hand of equation (7) represents the inter-symbol interference (ISI) of the OFDM subsystems owing to missing of cyclic-prefix between neighbouring symbols, which has also been shown in Fig. 1.

Based on such intra-symbol piecewise OFDM system model, the original OFDM system's complex ICI caused by the fast fading channel has been resolved as the subsystems' simple ISI and lower ICI, respectively caused by the time average channel and time-varying channel in multiple piece-periods. In the following section, a novel successive intra-symbol piecewise 'ISI' and 'ICI' mitigation method will be proposed to equalize the fast fading channel.
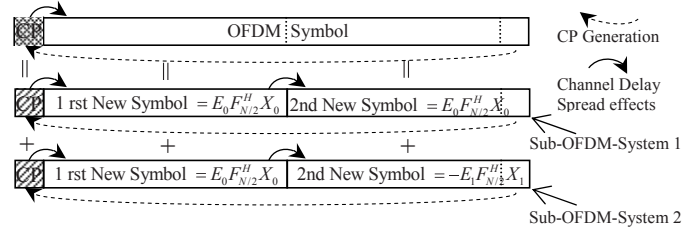

Fig. 1. Intra-symbol piecewise OFDM system model for $M = 2$

3. Intra-symbol Piecewise Equalization Method

Since we focus on the equalization of time-varying channel, AWGN will not be considered in the following description. For brevity, the intra-symbol piecewise equalization method for piece number $M = 2$ will be described in detail first in this section, then such proposed method is generalized for piece number $M = 2^p$.

The equalization for each data group $\mathbf{X}_m$ is similar and composed of three steps: symbol modification, 'ISI' mitigation and 'ICI' mitigation. And the equalization for the *m*-th data group is introduced as following.

3.1. symbol modification

First the received samples are modified as

$$\mathbf{r}' = \mathbf{F}_N^H \mathbf{B} \mathbf{F}_N \mathbf{G}_m \mathbf{r} \approx \mathbf{H} \mathbf{G}_m \mathbf{F}_N^H \mathbf{X} = \mathbf{H} \mathbf{G}_m \mathbf{\omega} \mathbf{Y} \tag{8}$$

where the frequency offset rotation matrix $\mathbf{G}_m$ and modifying matrix $\mathbf{B}$ are $N \times N$ diagonal matrix with the *n*-th diagonal element $\mathbf{G}_m(n,n) = e^{-j2\pi nm/N}$ and $\mathbf{B}(n,n) = [\mathbf{F}_N \mathbf{H} \mathbf{F}_N^H](n,n) / [\mathbf{F}_N \mathbf{H} \mathbf{F}_N^H](((n+m))_N, ((n+m))_N)$.

From (7) and (8), it can be derived that

$$\begin{bmatrix} \mathbf{F}_{N/2} \mathbf{r}_0' \\ \mathbf{F}_{N/2} \mathbf{r}_1' \end{bmatrix} = \underbrace{\begin{bmatrix} \overline{\mathbf{D}}_0 \tilde{\mathbf{E}}_{-m} \mathbf{X}_0 + \overline{\mathbf{D}}_0 \tilde{\mathbf{E}}_{-m+1} \mathbf{X}_1 \\ e^{jm\pi} (\overline{\mathbf{D}}_0 \tilde{\mathbf{E}}_{-m} \mathbf{X}_0 - \overline{\mathbf{D}}_0 \tilde{\mathbf{E}}_{-m+1} \mathbf{X}_1) \end{bmatrix}}_{\mathbf{Sig}} + \underbrace{\begin{bmatrix} \overset{\Delta}{\mathbf{D}}_0 \tilde{\mathbf{E}}_{-m} \mathbf{X}_0 + \overset{\Delta}{\mathbf{D}}_0 \tilde{\mathbf{E}}_{-m+1} \mathbf{X}_1 \\ e^{jm\pi} (\overset{\Delta}{\mathbf{D}}_0 \tilde{\mathbf{E}}_{-m} \mathbf{X}_0 - \overset{\Delta}{\mathbf{D}}_0 \tilde{\mathbf{E}}_{-m+1} \mathbf{X}_1) \end{bmatrix}}_{\overset{\Delta}{\mathbf{Sig}}} + \underbrace{\begin{bmatrix} \overline{\mathbf{D}}_0'' \tilde{\mathbf{E}}_{-m} \mathbf{X}_0 + \overline{\mathbf{D}}_0'' \tilde{\mathbf{E}}_{-m+1} \mathbf{X}_1 \\ e^{jm\pi} (\overline{\mathbf{D}}_0'' \tilde{\mathbf{E}}_{-m} \mathbf{X}_0 - \overline{\mathbf{D}}_0'' \tilde{\mathbf{E}}_{-m+1} \mathbf{X}_1) \end{bmatrix}}_{\overset{ISI}{\mathbf{Sig}}} \tag{9}$$

where $\overline{\mathbf{D}}_m = \mathbf{F}_{N/2} \overline{\mathbf{H}}_m \mathbf{F}_{N/2}^H$, $\tilde{\mathbf{E}}_m = \mathbf{F}_{N/2} \mathbf{E}_m \mathbf{F}_{N/2}^H$, $\overset{\Delta}{\mathbf{D}}_m = \mathbf{F}_{N/2} \overset{\Delta}{\mathbf{H}}_m \mathbf{F}_{N/2}^H$, $\overline{\mathbf{D}}'' = \mathbf{F}_{N/2} (\overline{\mathbf{H}}_m)^{UT} \mathbf{F}_{N/2}^H$. For simplification, define the three

terms in (9) as $\overline{\mathbf{Sig}}$, $\overset{\Delta}{\mathbf{Sig}}$ and $\overset{ISI}{\mathbf{Sig}}$ respectively.

3.2. 'ISI' mitigation

From (9), the m-th 'ISI' free data group estimate $\widehat{\mathbf{x}}_m^{'}$ satisfies the following formula

$$\left[\begin{smallmatrix} \overline{\mathbf{D}}_0\tilde{\mathbf{E}}_{-m}\widehat{\mathbf{X}}_0^{'}+\overline{\mathbf{D}}_0\tilde{\mathbf{E}}_{-m+1}\widehat{\mathbf{X}}_1^{'} \\ e^{jm\pi}(\overline{\mathbf{D}}_0\tilde{\mathbf{E}}_{-m}\widehat{\mathbf{X}}_0^{'}-\overline{\mathbf{D}}_0\tilde{\mathbf{E}}_{-m+1}\widehat{\mathbf{X}}_1^{'}) \end{smallmatrix} \right] = \overline{\mathbf{Sig}}+\overset{\Delta}{\mathbf{Sig}} \tag{10}$$

By subsitituting (10) into (9), we have

$$\left[\begin{smallmatrix} \overline{\mathbf{D}}_0\tilde{\mathbf{E}}_{-m}\widehat{\mathbf{X}}_0^{'}+\overline{\mathbf{D}}_0\tilde{\mathbf{E}}_{-m+1}\widehat{\mathbf{X}}_1^{'} \\ e^{jm\pi}(\overline{\mathbf{D}}_0\tilde{\mathbf{E}}_{-m}\widehat{\mathbf{X}}_0^{'}-\overline{\mathbf{D}}_0\tilde{\mathbf{E}}_{-m+1}\widehat{\mathbf{X}}_1^{'}) \end{smallmatrix} \right] = \left[\begin{smallmatrix} \mathbf{F}_{N/2}\mathbf{r}_0^{'} \\ \mathbf{F}_{N/2}\mathbf{r}_1^{'} \end{smallmatrix} \right] - \overset{ISI}{\mathbf{Sig}} \tag{11}$$

From (11), the 'ISI' free data symbol estimate $\widehat{\mathbf{x}}_m^{'}$ can be further approximated as

$$\widehat{\mathbf{X}}_m^{'} \approx \frac{1}{M}\sum_{m=0}^{M-1}\{(\overline{\mathbf{D}}_m)^{-1}\mathbf{F}_{N/M}\mathbf{r}_m^{'}\} \tag{12}$$

with estimate error $\widehat{\mathbf{Err}}_m = [(\overline{\mathbf{D}}_0)^{-1}\overline{\mathbf{D}}_0^{''}-(\overline{\mathbf{D}}_1)^{-1}\overline{\mathbf{D}}_1^{''}][(1-e^{jm\pi})\tilde{\mathbf{E}}_{-m}\mathbf{X}_0+(1+e^{jm\pi})\tilde{\mathbf{E}}_{-m+1}\mathbf{X}_1]$. Thanks to the sparsity of matrix $\overline{\mathbf{D}}_m^{''}$ and $(\overline{\mathbf{D}}_0)^{-1}\overline{\mathbf{D}}_0^{''}\approx(\overline{\mathbf{D}}_1)^{-1}\overline{\mathbf{D}}_1^{''}$, the estimate error $\widehat{\mathbf{Err}}_m$ of the m-th 'ISI' free data group is trivial and negligible.

3.3. ICI mitigation

Based on the obtained 'ISI' free data symbol estimate, the 'ICI' free data symbol estimate $\widehat{\mathbf{x}}_m$ can be further derived from (10).

$$\widehat{\mathbf{X}}_m = \frac{1}{2}[(\overline{\mathbf{D}}_0+\overset{\Delta}{\mathbf{D}}_0)^{-1}\overline{\mathbf{D}}_0+(\overline{\mathbf{D}}_1+\overset{\Delta}{\mathbf{D}}_1)^{-1}\overline{\mathbf{D}}_1]\widehat{\mathbf{X}}_m^{'} - \frac{1}{2}[(\overline{\mathbf{D}}_0+\overset{\Delta}{\mathbf{D}}_0)^{-1}\overline{\mathbf{D}}_0-(\overline{\mathbf{D}}_1+\overset{\Delta}{\mathbf{D}}_1)^{-1}\overline{\mathbf{D}}_1]\widehat{\mathbf{X}}_{mod(m+1,2)}^{'} \tag{13}$$

Considering that time variation matrix $\overset{\Delta}{\mathbf{D}}_m$ is much 'smaller' than $\overline{\mathbf{D}}_m$ with a sufficient large piece number M, the approximation $(\overline{\mathbf{D}}_m+\overset{\Delta}{\mathbf{D}}_m)^{-1}\approx(\overline{\mathbf{D}}_m)^{-1}-(\overline{\mathbf{D}}_m)^{-1}\overset{\Delta}{\mathbf{D}}_m(\overline{\mathbf{D}}_m)^{-1}$ can be obtained from matrix theory.

By substituting the approximation above into (13), the 'ICI' free data symbol estimate $\widehat{\mathbf{x}}_m$ is finally obtained.

$$\widehat{\mathbf{X}}_m = \widehat{\mathbf{X}}_m^{'} - \frac{1}{M}\left\{\sum_{m=0}^{M-1}\{(\overline{\mathbf{D}}_m)^{-1}\overset{\Delta}{\mathbf{D}}\}\right\}\widehat{\mathbf{X}}_m^{'} \tag{14}$$

Until all the $M=2$ data groups have been estimated through the steps above, the equalized data symbol can be finally obtained by permutating the data groups estimate $\{\widehat{\mathbf{X}}_m\}_{m=0}^{M-1}$.

The abovementioned intra-symbol piecewise equalization method for $M=2$ can be directly generalized for $M=2^p$, and the equalization steps: symbol modification, 'ISI' mitigation and 'ICI' mitigation for the m-th data group are described as (8), (12) and (14) respectively.

4. Complexity Analysis

The equalization complexity is evaluated in terms of complex additions (CA), complex multiplications (CM), and complex divisions (CD). According to (8), (12) and (14), the steps of the proposed method mainly includes FFT, IFFT and matrix-vector multiplications. Owing to the property of the cyclic toeplitz matrix, $\overline{\mathbf{D}}_m = \mathbf{F}_{N/M}\overline{\mathbf{H}}_m\mathbf{F}_{N/M}^H$ is a diagonal matrix with diagonal element vector $\mathbf{F}_{N/M}(\overline{\mathbf{H}}_m(1,:))^T$, and the matrix $\overset{\Delta}{\mathbf{D}}_m$ can be expressed in matrix form from (5)

$$\overset{\Delta}{\mathbf{D}}_m = \mathbf{F}_{N/M}\,diag\left(\frac{2M*(0:(N/M-1))-N}{2N}\right)\mathbf{F}_{N/M}^H \cdot \underbrace{\mathbf{F}_{N/M}\mathbf{CSHIFT}(\overset{\Delta}{\mathcal{H}}_m, N/M)\mathbf{F}_{N/M}^H}_{\overline{\Delta\mathbf{D}}_m}$$

where $\overline{\Delta\mathbf{D}}_m$ is also a diagonal matrix with diagonal element vector $\mathbf{F}_{N/M}(\Delta\overline{\mathbf{h}}_m)^T$. Hence the matrix-vector multiplication involved in (8), (12) and (14) can be simplified as diagonal matrix-vector multiplication and FFT/IFFT, which are both low complexity operations.

As the N-point IFFT/FFT operation require $N/2log_2N$ CM and $Nlog_2N$ CA, the three equalization steps on the m-th data symbol group $\mathbf{X}_m$ have computation complexity of $2Nlog_2N$ CA $+N(log_2N+2)$ CM $+N$ CD, $N[log_2(N/M)+1]$ CA $+0.5*Nlog_2(N/M)$ CM $+N$ CD and $N[2log_2(N/M)+1]$ CA $+N[log_2(N/M)+2]$ CM $+N$ CD respectively.

Table 1. Complexity comparison

	Proposed Equalization		Banded Equalization	
	M=2	M=4	Q=2	Q=4
N=1024	100N	230N	140N	564N
N=256	80N	184N	140N	564N
N=64	60N	138N	140N	564N

As a total, the proposed scheme requires $\{13Mlog_2N/2 - 9Mlog_2M/2 + 9M\}N$ complex operations for piece number M, while the classical simple equalization scheme in [1] with banded factor Q requires a total of $(8Q^3/3 + 20Q^2 + 52Q/3 + 4)N$ complex operation. Banded factor $Q=2,4$ and piece number $M=2,4$ is assumed in the following simulations that the two equalization schemes not only have comparative complexity, but also achieve better trade-off between performance and complexity in fast fading channels. For ease of comparison, the computation complexity of the two schemes for typical system parameters has been listed in Table 1. Among the listed typical parameter configuration cases the proposed method for $M=2$ can achieve the lowest complexity.

5. Simulation Results

In this section the uncoded BER performance is applied to evaluate the performance of different equalizers. An OFDM system with $N=256, L=8,32$, and QPSK modulation is supposed. Rayleigh fading channels with exponential power delay profile and maximum normalized Doppler frequency $f_d = 0.15, 0.30$ is assumed in the simulations. For fair comparison, the proposed equalizer and the banded equalizer [1] both have only the CSI at mid-time instant in each piece-period, and the complete CSI for the banded scheme is obtained by linear interpolation in each piece-period.

From Fig. 2, the proposed scheme achieved slightly worse performance than the banded scheme for $f_d = 0.15$ case, while achieved better performance for $f_d = 0.30$ case, since the ICI energy outside Q banded neighbouring subcarriers can not be negligible in fast fading channels, while the proposed scheme can

efficiently mitigate the much lower 'ICI' caused by time-varying channel in each piece-period. But the proposed scheme is more susceptible to the multipath length increase than the banded one, since the 'ISI' mitigation of the proposed scheme is dependent on the multipath length. The proposed scheme performs better for $M = 4$ than for $M = 2$ only in $L = 8, f_d = 0.30$ case, since the 'ISI' dominates the equalization performance for lower Doppler spread cases or big delay spread cases.

6. Conclusions

We have proposed an intra-symbol piecewise equalizer for OFDM systems over fast time-varying multipath channels. By using the intra-symbol piecewise OFDM system model and the characteristics of channel matrix, the equalization against fast fading channels can be realized through the successive `ISI' and `ICI' mitigation with complexity of $\{13Mlog_2N/2 - 9Mlog_2M/2 + 9M\}N$. We have shown the proposed scheme has lower computation complexity than the comparative one, while perserving good performance especially in fast fading channels with small delay spread.
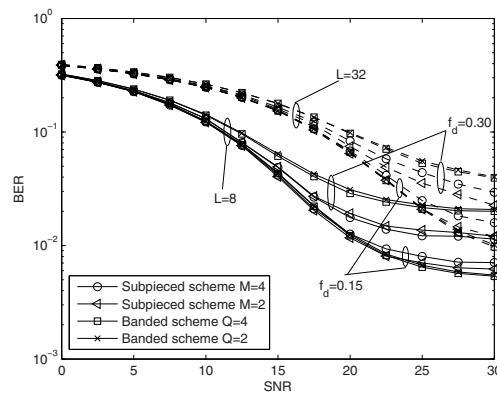

Fig. 2. BER comparison between the proposed scheme and banded scheme for $f_d = 0.15, 0.30$

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