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Parallel Construction of Variable Precision Concept Lattice in Fuzzy Formal Context

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Abstract

Based on the fuzzy cut method, the fuzzy concept lattice was analyzed and researched and a new kind of concept lattice which called variable precision concept lattice was proposed. This kind of concept lattice can be constructed with every different confidence through selecting different level. Two corresponding construction methods, context cut method and concept cut method, were discussed in the two different angles, and some results were given. Furthermore, a case study was carried on.

Keywords: Fuzzy parallel construction; variable pricision concept lattice; fuzzy cut; fuzzy concept lattice.

1. Introduction

In the real world, the relationship between the extent and intent is not certain at all the time. Much more often, the fuzzy relations exist in the concept, and they can not described by the accurate concept lattice. The usual formal concept [1] is a two-tuple which consists of extent and intent. The extent of a concept is the element set and the intent is the set of attributes which all the elements have. Burusco[2] First, the fuzzy logic Applied to the concept of grid, followed by Belohlavek [3-6], Georgescu[7] studied the fuzzy concept

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lattices; Fan [8] such as the establishment of the four kinds of fuzzy concept lattice, with Concept lattice on the background of the four kinds of classic form one-to-one correspondence, and research-based fuzzy Concept of the grid fuzzy inference problem. Many scholars introduces fuzzy concept lattice [9-16] successively to express the fuzzy relationship between extent and intent. It was more close to reality, and accord with the fuzzy concept ideas of human. In the other hand, it plays a crucial role in the practical problems processing.

The construction process of fuzzy concept lattice shows every node contains objects, attributes and the relationship which expressed by between the two. However, in many cases, membership is similar to the probability, and not able to clearly and accurately describe the next decision activity we must carry on.

Concept lattice can be used for the discovery, sort and display of the concept. The binary definition made that concept lattice is very suitable for association rule. In many cases, people just need a series of fuzzy rules, but not the completely determined rules. And people's demand for the degree of rules is also varying according to the different times. Therefore, this paper tries to use the method of fuzzy cut set to construct a classic concept lattice under the fuzzy formal background. This kind of concept can be adjusted with the different valuing of λ.

2. Basic Concepts

2.1. Fuzzy Cut Set

Fuzzy set is an important concept which was proposed by American computer and cybernetics expert L. A. Zadeh in 1965, and then the mathematic basic is put forward for the quantitative description of fuzzy concept and the objective regularity of fuzzy object. The basic idea is expanding the membership relation in the classic set theory, and the membership degree is expanded from the two numbers (0 and 1) to the interval $[0,1]$.

Suppose U is the domain, then a fuzzy set A is a real function from U : $\mu_A : U \to [0,1], u \mapsto \mu_A(u)$. For $u \in U$, $\mu_A(u)$ call the membership degree for A, and μ_A call the membership function of A.

Fuzzy set is an abstract concept and its elements are uncertain. We can only know and master it through the membership function μ_A. The size of $\mu_A(u)$ reflects the membership degree of u to the fuzzy set A. The closer $\mu_A(u)$ to 1, the membership degree is higher.

The membership function is the most basic concept to describe the fuzzy set and the most important tool for the research on fuzzy set theory and its application. In the fuzzy set theory, another important concept is fuzzy cut which plays a connection role in the translation between fuzzy set and classic set.

Suppose $A \in F(U)$, for $\forall \lambda \in [0,1]$, $A_\lambda = \{u \mid u \in U, A(u) \geq \lambda\}$, call the λ-cut of A, λ is the confidence level. And $A_{\underset{\cdot}{\lambda}} = \{u \mid u \in U, A(u) > \lambda\}$, call $A_{\underset{\cdot}{\lambda}}$ the λ power cut of A.

2.2. Fuzzy Concept Latticet

Suppose $L(U, A, f, g)$ are all the fuzzy concepts on the object space U and attribute space A, i.e. $L(U, A, f, g) = \{(A, B) \mid \forall A \in F(U), \forall B \in F(A), F(X) = B, g(B) = X\}$.

Fuzzy concept lattice and weighted concept lattice, the extent contains the objects which meet all the attributes in the intent. To find the concepts which have partial attributes, we must scan the concept lattice and combine the concepts. The time cost is so larger especially for large concept lattice. While, in the rough concept lattice, although the concepts which have partial attributes can be searched, but there may be a lot of objects which only have an attribute of the intent, thus the support and confidence degree of constructing association rules will be greatly reduced. In practical applications, we often care the object set which have a certain number or percent of attributes in intent.

3. Parallel Construction of Variable Precision Concept Lattice

3.1. Definition

The construction methods can be divided into two kinds: Batch and Gradualism. Batch method is simple, visualized and easy for parallelization. And some specific methods can generate the Hasse figure in the constructing process.

Suppose the formal context is $K=(O,D,R)$ and an accurate concept $C_i(O_i,D_i)$ ($O_i \in O, D_i \in D$).In the generating process of its child node, operators are almost carried on the attribute in D_i^c , i.e. the complementary set of D_i . For every attribute y in the set D_i^c , we should calculate relevant $g(y)=\{x \mid xRy, \forall x \in O_i\}$, and then judge the largest subset according to $g(y^{'})$. Here, $y^{'}$ is the attribute beside y in D_i^c . Finally, the child node is decided to generate or not

Definition 1 For fuzzy formal context $K=(O,D,I)$, a advisable λ （ $0<\lambda<1$ ） is choose, and the attribute value which smaller than λ will be changed to 0 and others will be changed to 0, then a classic context K_λ is obtained from the fuzzy formal context $K=(O,D,I)$. Here, K_λ is called the λ-cut of K .

The intent of fuzzy concept contains all the attributes in the context and the membership degree with the extent. According to the ideas of fuzzy cut set, we can carry on the following operators:

(1) Giving a suitable value of λ ;
(2) Deleting the attributes whose value are smaller than λ ;
(3) Changing the membership degree of the remaining attribute to 1.

Then a classic concept was obtained. This concept is called λ-cut of fuzzy concept. We will verify the λ-concept lattice is a λ-cut of fuzzy concept lattice.

3.2. Construction Methods

Context Cut Method supports the process from up to down and the steps are listed in the following:

Step1: Choose the value of λ ;
Step2: Change fuzzy formal context to accurate context;
Step3: Deicide the top node;
Step4: Generate all the child nodes of the top node;
Step5: Choose a child node, and generate all the children;
Step6: Carry out the Step5 on every child node.

The construction of fuzzy concept lattice contains two operators: decision of the fuzzy top node and child nodes. The fuzzy top node is determined by fuzzy formal context, and the fuzzy child nodes are determined by the following three chooses:

(l) Choose the father node. Selecting a current node to be the father node, and carry on the generating process of child nodes.
(2) Decide the child nodes. If a current node has child nodes, then we should carry on the operators.
(3) Decide the bother nodes. If a current node has bother nodes, then we should carry on the operators.

After the construction of fuzzy concept lattice, the following three steps should be carried on.

(l) Chose a suitable value of λ , and calculate the λ-cut of every node concept;
(2) Sort the λ-cuts according to the intent.
(3) Retain the extent which has the most elements, for one intent set.

Then, the variable precision concept lattice was constructed.

4. Example

The Table 1 is a fuzzy formal context whose element set is $\{e_1, e_2, e_3, e_4\}$ and attribute set is $\{a_1, a_2, a_3, a_4, a_5\}$. Here, the above two methods of λ-concept lattice are carried out and some results are discussed.

Table 1. An example of a context

	a_1	a_2	a_3	a_4	a_5
e_1	0.2	0.4	0.6	0.8	0.5
e_2	0.1	0.2	0.7	0.9	0.4
e_3	0.7	0.6	0.3	0.4	0.3
e_4	0.5	0.8	0.2	0.6	0.1

Here, we select two values of λ: $\lambda = 0.5$, $\lambda = 0.4$. ***For** $\lambda = 0.5$ **, following operators are carried out.***

4.1. Context Cut Method

(1) Cut the context and Change the fuzzy formal context to classic context; See in Table 2.

Table 2. The accurate context of table1 for $\lambda = 0.5$

	a_1	a_2	a_3	a_4	a_5
e_1	0	0	1	1	1
e_2	0	0	1	1	0
e_3	1	1	0	0	0
e_4	1	1	0	1	0

(2) Determine the top node; #1 $(\{e_1, e_2, e_3, e_4\}, \phi)$
(3) Determine the child nodes; #2 $(\{e_1, e_2, e_4\}, a_4)$, #3 $(\{e_3, e_4\}, a_1, a_2)$, #4 $(\{e_1, e_2\}, a_3, a_4)$
(4) Find the child nodes of the nodes in the step (3); #5 $(\{e_4\}, a_1, a_2, a_4)$, #6 $(\{e_1\}, a_3, a_4, a_5)$,
(5) Determine the root node. #7 $(\phi, a_1, a_2, a_3, a_4, a_5)$

For** $\lambda = 0.4$ **, the same operators are carried out.

Determine the top node and all the child nodes. Then we can get the λ-concept lattice when $\lambda = 0.4$.
#1 $(\{e_1, e_2, e_3, e_4\}, a_4)$, #2 $(\{e_1, e_3, e_4\}, a_2)$, #3 $(\{e_1, e_2\}, a_3, a_4, a_5)$, #4 $(\{e_3, e_4\}, a_1, a_2)$, #5 $(\{e_1, e_4\}, a_2, a_4)$, #6 $(\{e_1\}, a_2, a_3, a_4, a_5)$, #7 $(\{e_3\}, a_1, a_2, a_4)$, #8 $(\phi, a_1, a_2, a_3, a_4, a_5)$

The Hasse for $\lambda = 0.5$ and $\lambda = 0.4$ are see in Figure. 1.

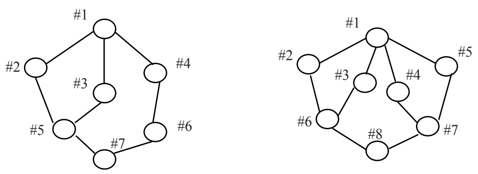

Fig. 1. The Hasse for $\lambda = 0.5$ and $\lambda = 0.4$

4.2. Concept Cut Method

Finally, repeat the above process for the three fuzzy child nodes and their child nodes. Then, the thirteen fuzzy formal concepts are obtained. According to the concept cut method, the cut operators are carried out for the above thirteen fuzzy formal concepts for $\lambda = 0.5$.

#1 $(\{1,2,3,4\},\frac{0.1}{a}+\frac{0.2}{b}+\frac{0.2}{c}+\frac{0.4}{d}+\frac{0.1}{e}) \to (\{1,2,3,4\},\phi)$ ***Remain Marked as #1***

#2 $(\{1,3,4\},\frac{0.2}{a}+\frac{0.4}{b}+\frac{0.2}{c}+\frac{0.4}{d}+\frac{0.1}{e}) \to (\{1,3,4\},\phi)$

#3 $(\{1,2,3\},\frac{0.1}{a}+\frac{0.2}{b}+\frac{0.3}{c}+\frac{0.4}{d}+\frac{0.3}{e}) \to (\{1,2,3\},\phi)$

#4 $(\{1,2,4\},\frac{0.1}{a}+\frac{0.2}{b}+\frac{0.2}{c}+\frac{0.6}{d}+\frac{0.1}{e}) \to (\{1,2,4\},d)$ ***Remain Marked as #2***

#5 $(\{3,4\},\frac{0.5}{a}+\frac{0.6}{b}+\frac{0.2}{c}+\frac{0.4}{d}+\frac{0.1}{e}) \to (\{3,4\},a,b)$ ***Remain Marked as #3***

#6 $(\{1,3\},\frac{0.2}{a}+\frac{0.4}{b}+\frac{0.3}{c}+\frac{0.4}{d}+\frac{0.3}{e}) \to (\{1,3\},\phi)$

#7 $(\{1,4\},\frac{0.2}{a}+\frac{0.4}{b}+\frac{0.2}{c}+\frac{0.6}{d}+\frac{0.1}{e}) \to (\{1,4\},d)$

#8 $(\{3\},\frac{0.7}{a}+\frac{0.6}{b}+\frac{0.3}{c}+\frac{0.4}{d}+\frac{0.3}{e}) \to (\{3\},a,b)$

#9 $(\{4\},\frac{0.5}{a}+\frac{0.8}{b}+\frac{0.2}{c}+\frac{0.6}{d}+\frac{0.1}{e}) \to (\{4\},a,b,d)$ ***Remain Marked as #5***

#10 $(\{\phi\},\frac{1.0}{a}+\frac{1.0}{b}+\frac{1.0}{c}+\frac{1.0}{d}+\frac{1.0}{e}) \to (\{\phi\},a,b,c,d,e)$ ***Remain Marked as #7***

#11 $(\{1\},\frac{0.2}{a}+\frac{0.4}{b}+\frac{0.6}{c}+\frac{0.7}{d}+\frac{0.5}{e}) \to (\{1\},c,d,e)$ ***Remain Marked as #6***

#12 $(\{1,2\},\frac{0.1}{a}+\frac{0.2}{b}+\frac{0.6}{c}+\frac{0.8}{d}+\frac{0.4}{e}) \to (\{1,2\},c,d)$ ***Remain Marked as #4***

#13 $(\{2\},\frac{0.1}{a}+\frac{0.2}{b}+\frac{0.7}{c}+\frac{0.9}{d}+\frac{0.4}{e}) \to (\{2\},c,d)$

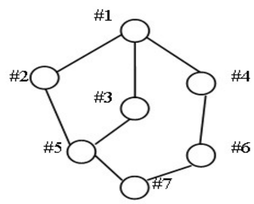

Fig. 2. The Hasse for $\lambda = 0.5$

The result is same to $\lambda = 0.5$, so the two methods are equivalent in some sense. In fact, the construction of concept is a clustering process of concept. The construction methods can be divided.

5. Conclusion

The relationships between fuzzy concept lattice and classic concept were analyzed in this paper, and then the ideas of fuzzy cut set were introduced into the construction of concept lattice. A new kind of concept lattice was presented to describe the different confidence of rules. A case study proved that this kind of concept is valuable. The next study is to apply this tool into the engineering practice, such as rules extracting, fuzzy decision, and intelligent control, etc.

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