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A characterization of monotonicity with collective quantifiers

Gilad Ben-Avi and Yoad Winter

Computer Science Department
Technion – Israel Institute of Technology
Haifa 32000 Israel

Abstract

This paper studies the monotonicity behavior of plural determiners that quantify over collections. Following previous work, we describe the collective interpretation of determiners such as *all, some* and *most* using generalized quantifiers of a higher type that are obtained systematically by applying a type shifting operator to the standard meanings of determiners in Generalized Quantifier Theory. Unlike previous proposals, one unified *determiner fitting* operator both captures existential quantification with plural determiners and respects their monotonicity properties. However, some previously unnoticed facts indicate that monotonicity of plural determiners is not always preserved when they apply to collective predicates. We show that the proposed operator describes this behavior correctly, and characterize the monotonicity of the collective determiners it derives. It is proved that determiner fitting always preserves monotonicity properties of determiners in their second argument, but monotonicity in the first argument of a determiner is preserved if and only if it is monotonic in the same direction in the second argument.

1 Introduction

Monotonicity and collectivity phenomena are pivotal in logical theories of natural language semantics. However, despite the many advances in the formal investigation of these phenomena, relatively little attention has been paid to the effects of collective interpretations of quantifiers on their monotonicity properties. In this paper we aim to formally study these relations. Using a novel collectivity operator on determiners that combines insights from previous works in generalized quantifier theory, we show that the *conservativity* of quantification in natural language is responsible for two curious asymmetries in the monotonicity properties of collective determiners. First, natural language determiners change their monotonicity behavior when they quantify over collections only if their standard monotonicity properties are different in their two arguments. Second, only the first argument of the determiner may change its monotonicity properties when the determiner quantifies over collections. These two asymmetries follow from the conservativity of collective quantification as implemented in the proposed operator, together with standard assumptions about their "logical constancy" and "non-triviality".

Generalized Quantifier Theory (GQT), as was applied to natural language semantics in the influential works of [1], [7] and [3], concentrated on the behavior of determiners in sentences such as the following.

(1) *All the students are happy. Some girls arrived. Most teachers are Republican*

In these sentences, the denotations of both the nominal (e.g. *students, girls*, etc.) and the predicate (e.g. *happy, arrived*, etc.) are traditionally treated as *distributive predicates*, which correspond to subsets of a domain of (arbitrary) atomic entities. Standard GQT assigns determiners such as *all, some* and *most* denotations that are relations between such sets of atomic entities.

However, this treatment does not account for the interactions between quantifiers and *collective* predicates as in the following sentences.

(2) *All the colleagues cooperated. Some girls gathered. Most of the sisters saw each other.*

According to most theories of plurals, nominals such as *colleagues* and *sisters* and verb phrases such as *cooperated, gathered* and *saw each other* denote sets of *collections* of atomic entities. For our purposes in this paper it is sufficient to assume that collections are *sets* of atomic entities. Thus, we assume that collective predicates denote *sets of sets* of atomic entities. Consequently, the standard denotation of determiners in GQT as relations between sets of atoms is not directly applicable to sentences with collective predicates.

Early contributions to the study of collective quantification in natural language, most notably [5], propose that meanings of "collective statements" as in (2) are systematically obtained from standard "distributive" denotations of determiners in GQT. More recent works, including (among others) [9,10] and [11,12], use *type shifting operators* that apply to a standard determiner over a domain E, and derive a determiner of a higher type, which ranges over collections of elements in E. In this paper we follow [12] and adopt one general type shifting operator that is referred to as *determiner fitting* (*dfit*). This operator maps standard determiners over E to "collective" determiners over $\wp(E)$ (relations between subsets of $\wp(E)$).

A natural question that arises in this context is: what are the relations between the semantic properties of a standard determiner D and the properties of its "collectivized" version $dfit(D)$? Especially, we are interested here in the relations between the *monotonicity* properties of standard determiners and the monotonicity of their collectivized counterparts. We first observe that monotonicity entailments are not always preserved when a determiner quantifies over collections. Consider for instance the contrast between the sound entailment in (3a) and the lack of entailment in (3b) below.

(3)a. All the students are happy $\Rightarrow$ All the rich students are happy.
b. All the students drank a whole glass of beer together $\not\Rightarrow$ All the rich students drank a whole glass of beer together.

In a situation where the students are s_1, s_2 and s_3 and the rich students are s_1 and s_2, assume that the group $\{s_1, s_2, s_3\}$ drank a whole glass of beer together, but no other group did. In this situation, the antecedent in (3b) is obviously true, but the consequent is false. This observation shows that when the determiner *all* quantifies over collections it "loses" its downward left monotonicity property.

It should be noted that the use of the collective predicate *drank a whole glass of beer together* is crucial in the observation that *all* loses its left monotonicity with collective predicates. However, this observation is obscured with many other collective predicates. For instance, the predicate *be similar* is reasonably "downward monotone" in the sense that if a set A is in its extension (i.e. the members of A are similar to each other), so is any subset of A with at least two members. Consequently, the entailment in (4) below holds, in contrast to (3b).

(4) All the students are similar $\Rightarrow$ All the rich students are similar.

However, in order to conclude that *all* is not left downward monotone with collective predicates, the existence of cases like (3b) is sufficient. The predicate *drink a whole glass of beer together* is general enough to establish non-left-monotonicity of *all*, since the existence of a set A whose members drank a whole glass of beer together does not entail the existence of any other set with the same property.

Unlike *all*, many other determiners do not lose their monotonicity properties when they quantify over collections. For example, consider the determiner *some*, which is upward monotone in both its arguments. That *some* remains upward left monotone also with collective predicates can be observed in the following entailment.

(5) Some rich students drank a whole glass of beer together $\Rightarrow$ Some students drank a whole glass of beer together.

In distinction to this contrast between determiners concerning their left monotonicity, we claim that *right* monotonicity of determiners is always preserved when they quantify over collections. This claim can be exemplified by entailments such as the following.

(6) All the/more than five/some students drank a whole glass of *dark* beer together $\Rightarrow$ All the/more than five/some students drank a whole glass of

beer together.

We will prove that using the *dfit* operator, these two general contrasts – between different determiners and between the left and right arguments – follow from the asymmetric *conservativity* principle of quantification in natural language that this operator presupposes.

2 Notions from Generalized Quantifier Theory

This section reviews some familiar notions from standard GQT that are important for the developments in subsequent sections. For an exhaustive survey of standard GQT see [4].

A *generalized quantifier* over a domain E is a function from $\wp(E)$ to $\{0,1\}$. A *determiner* D_E over a domain E is a function that maps each subset of E to a generalized quantifier over E. Thus, a determiner is a function from $\wp(E) \times \wp(E)$ to $\{0,1\}$. When the E domain is understood from the context, we omit it and sloppily refer to the determiner D_E by 'D'. The main property of determiners that is studied in this paper is *monotonicity*.

Definition 2.1 (monotonicity) *A determiner* D_E *is* left upward monotone *iff for all* $A, A', B \subseteq E$ *if* $A \subseteq A'$ *then* $D_E(A)(B) = 1 \Rightarrow D_E(A')(B) = 1$.
D_E *is* left downward monotone *iff for all* $A, A', B \subseteq E$ *if* $A' \subseteq A$ *then* $D_E(A)(B) = 1 \Rightarrow D_E(A')(B) = 1$.
D_E *is* right upward monotone *iff for all* $A, B, B' \subseteq E$ *if* $B \subseteq B'$ *then* $D_E(A)(B) = 1 \Rightarrow D_E(A)(B') = 1$.
D_E *is* right downward monotone *iff for all* $A, B, B' \subseteq E$ *if* $B' \subseteq B$ *then* $D_E(A)(B) = 1 \Rightarrow D_E(A)(B') = 1$.
We say that D_E *is left (right)* monotone *iff it is either left (right) upward monotone or left (right) downward monotone.*

We use the notations $\uparrow$MON, $\downarrow$MON and $\sim$MON for the classes of determiners that are upward-, downward- and non-monotone in their left argument. Similarly for the right argument.

The denotation of determiner expressions varies with the choice of the domain E. A *global determiner* is a functional that maps a domain E to a (local) determiner D_E. We say that a determiner expression and the global determiner functional D that it denotes are upward (downward) monotonic in their left (right) argument if D_E has this property for every choice of E.

The following property characterizes the class of global determiners that remains "constant" across different isomorphic domains.

Definition 2.2 (isomorphism invariance) *A global determiner* D *is* isomorphism invariant *(ISOM) iff for all bijections* $\pi : E \rightarrow E'$*, for all* $A, B \subseteq E$*:* $D_{E'}(\{\pi(x) : x \in A\})(\{\pi(y) : y \in B\}) = D_E(A)(B)$.

We say that a global determiner D satisfies *extension* if the truth value that D_E assigns to two subsets of E does not change when E is replaced by a superset of E. Formally,

Definition 2.3 (extension) *A global determiner D satisfies* extension *(EXT) iff for all $A, B \subseteq E \subseteq E'$: $D_E(A)(B) = D_{E'}(A)(B)$.*

The well-known *conservativity* property of natural language determiners says that the truth value that they assign to any pair of sets does not depend on entities that are not members of the first argument. Formally,

Definition 2.4 (conservativity) *A (local) determiner D_E is* conservative *(CONS) over E iff for all $A, B \subseteq E$: $D_E(A)(B) = D_E(A)(A \cap B)$.*

We say that a global determiner D is conservative if D_E is conservative for every domain E.

The ISOM and EXT properties characterize the "logical" behavior of most natural language determiners. Conservativity is a restriction on the class of logical determiners, which reflects the special role of the first argument of natural language determiners in restricting the domain of quantification. The following definition characterizes two trivial classes of conservative logical determiners.

Definition 2.5 (left and right triviality) *A (local) determiner D_E is called* left trivial *(LTRIV) iff for all $A, A', B \subseteq E$: $D(A)(B) = D(A')(B)$. D_E is called* right trivial *(RTRIV) iff for all $A, B, B' \subseteq E$: $D(A)(B) = D(A)(B')$.*

We say that a global determiner D is left (right) trivial if the local determiner D_E is left (right) trivial for every domain E. Intuitively, a left (right) trivial determiner is insensitive to the identity of its left (right) argument. We occasionally restrict our attention to determiners that are not right trivial, because every such a conservative determiner is also not left trivial. [1]

In this paper we study the monotonicity properties of non-RTRIV global determiners that satisfy ISOM, EXT and CONS. These determiners, which are the main focus of GQT, are referred to as *quantifying non-trivial* (**QNT**) *determiners.*

3 Determiner fitting and the witness condition

The *dfit* operator is defined as a conjunction of two operators. One operator, called *count*, is a reformulation of the "neutral" operator of [5] and

[1] **Proof:** Assume that D is conservative and LTRIV. For any domain E, by left triviality of D, we have for all $A, B \subseteq E$: $D_E(A)(B) = 1$ iff $D_E(\emptyset)(B) = 1$. Conservativity of D entails that for every $B \subseteq E$: $D_E(\emptyset)(B) = 1$ iff $D_E(\emptyset)(\emptyset) = 1$. We conclude that for all $A, B \subseteq E$: $D_E(A)(B) = 1$ iff $D_E(\emptyset)(\emptyset) = 1$. Especially: D is RTRIV. On the other hand, the determiner D s.t. $D_E(A)(B) = 1$ iff $A \neq \emptyset$ is an example for a conservative determiner that is RTRIV but not LTRIV.

[9,10].

Definition 3.1 (counting operator) *Let D be a (local) determiner over E. The corresponding determiner $count(D)$ over $\wp(E)$ is defined for all $\mathcal{A}, \mathcal{B} \subseteq \wp(E)$ by:*

$$(count(D))(\mathcal{A})(\mathcal{B}) \stackrel{def}{=} D(\cup\mathcal{A})(\cup(\mathcal{A} \cap \mathcal{B})).$$

This process of counting members of collections consists of two separate steps: (i) a *conservativity* step, in which the second argument is modified by intersecting it with the first argument; (ii) a *participation* step, in which the first argument and the result of step (i) are both unioned, and serve as the first and second arguments of the determiner respectively. To illustrate the process consider sentence (7) and its analysis in (8) below. Note that, as in most other theories of plurals, distributive predicates, which range over atoms, can be mapped to predicates that range over sets of atoms using a *distributivity operator*. The powerset operator $\wp$ is sufficient as a distributivity operator for our purposes in this paper. For instance, the plural common noun *students* in (8) is treated as denoting the powerset of the singular noun *student*.

(7) Exactly 5 students drank a whole glass of beer together.

(8) $count(\mathbf{exactly\_5'})(\wp(\mathbf{student'}))(\mathbf{drink\_beer'})$
$\Leftrightarrow\ \mathbf{exactly\_5'}(\cup\wp(\mathbf{student'}))(\cup(\wp(\mathbf{student'}) \cap \mathbf{drink\_beer'}))$
$\Leftrightarrow\ |\{x \in A : A \subseteq \mathbf{student'} \wedge \mathbf{drink\_beer'}(A)\}| = 5$

The analysis in (8) guarantees that *exactly* five students participated in sets of students drinking beer.

It is easy to verify that the *count* operator respects the semantics of conservative determiners on distributive predicates in the following sense.

Fact 3.2 *For every conservative determiner D over E, for all $A, B \subseteq E$: $(count(D))(\wp(A))(\wp(B)) = D(A)(B)$.*

The major problem with the *count* operator is that its outcome is not always the intuitively correct one due to the lack of an existential requirement. Reconsider sentence (7) and its analysis in (8) above. Proposition (8) can be true in a situation where there is no set of five students who drank a whole glass of beer together. For instance, this may happen when there are exactly two sets of students in the extension of *drank a whole glass of beer together*: $\{s_1, s_2, s_3\}$ and $\{s_4, s_5\}$. Despite the truth of (8) in this situation, sentence (7) is intuitively false in these conditions. To solve this problem, we add to the *count* operator an "existential" condition that is formalized using a *witness operator*.

Definition 3.3 (witness operator) *Let D be a local determiner over E. The corresponding determiner $wit(D)$ over $\wp(E)$ is defined for all $\mathcal{A}, \mathcal{B} \subseteq \wp(E)$ by:*

$$(wit(D))(\mathcal{A})(\mathcal{B}) = 1 \Leftrightarrow \mathcal{A} \cap \mathcal{B} = \emptyset \vee \exists W \in \mathcal{A} \cap \mathcal{B}[D(\cup\mathcal{A})(W) = 1].$$

In words: the witness operator maps a determiner D over E to a determiner over $\wp(E)$ that holds of any two sets of sets $\mathcal{A}, \mathcal{B}$ iff their intersection $\mathcal{A} \cap \mathcal{B}$ is empty or contains a *witness set* of D and $\cup\mathcal{A}$. [2]

To exemplify the operation of the witness operator, consider the analysis in (9) below of sentence (7).

(9) $wit(\mathbf{exactly\_5'})(\wp(\mathbf{student'}))(\mathbf{drink\_beer'})$
$\Leftrightarrow [\wp(\mathbf{student'}) \cap \mathbf{drink\_beer'} \neq \emptyset \rightarrow$
$\exists W \in \wp(\mathbf{student'}) \cap \mathbf{drink\_beer'}[\mathbf{exactly\_5'}(\cup\wp(\mathbf{student'}))(W)]]$
$\Leftrightarrow \exists W \subseteq \mathbf{student'}[\mathbf{drink\_beer'}(W) \wedge |W| = 5]$

The analysis in (9) verifies that there exists at least one set constituted by exactly five students who drank a whole glass of beer together. By conjoining (9) with (8) we get the intuitively correct interpretation of sentence (7). Accordingly, the general *determiner fitting* operator is a conjunction of the counting operator and the witness operator.

Definition 3.4 (determiner fitting operator) *Let D be a local determiner over E. The corresponding determiner $dfit(D)$ over $\wp(E)$ is defined for all $\mathcal{A}, \mathcal{B} \subseteq \wp(E)$ by:*

$$(dfit(D))(\mathcal{A})(\mathcal{B}) = 1 \Leftrightarrow (count(D))(\mathcal{A})(\mathcal{B}) = 1 \wedge (wit(D))(\mathcal{A})(\mathcal{B}) = 1.$$

This operator, similarly to the *bounded composition* operator of Dalrymple et al. in [2], [3] imposes an existential requirement on non-$\mathrm{MON}\!\downarrow$ determiners, but does not lead to undesired existential analyses with $\mathrm{MON}\!\downarrow$ determiners (cf. the warning in [8, p. 52-53]). The disjunct $\mathcal{A} \cap \mathcal{B} = \emptyset$ within the witness operator prevents such undesired analyses.

It can be shown that fact 3.2 about the *count* operator is generalized to the case of the *dfit* operator. Formally,

Fact 3.5 *For every conservative determiner D over E, for all $A, B \subseteq E$: $(dfit(D))(\wp(A))(\wp(B)) = D(A)(B)$.*

Thus, when both arguments of a conservative determiner are distributive predicates, the witness operator is redundant in *dfit*.

[2] Following [1], a witness set of a determiner D and a set A is any subset W of A that satisfies $D(A)(W) = 1$. [1] define witness sets on generalized quantifiers explicitly, but they reach the argument A indirectly by defining what they call a *live on set* of the quantifier. This complication is unnecessary for our purposes.
A similar strategy for quantification over witness sets is proposed by Szabolcsi in [6]. While Szabolcsi's witness operation is used only for $\mathrm{MON}\!\uparrow$ determiners, the witness operator that is defined above is designed to be used for all determiners.

[3] The *dfit* operator and Dalrymple et al's **BC** operator differ in their counting process. For instance, for sentence (7) to be true, the **BC** operator does not require that the total number of students who participated in groups of students drinking beer is exactly five, but only requires that each such group of maximal cardinality includes exactly five students. We leave a detailed comparison between Dalrymple et al's **BC** operator and the proposed *dfit* operator to another occasion

4 Monotonicity properties of collective determiners

The main result in this paper is that determiners like *all* and *not all*, which are monotone in their left argument but have a different monotonicity property in their right argument, lose their left monotonicity under *count*. In contrast, determiners like *some* and *no* that have similar monotonicities in both arguments do not lose their left monotonicity under *count*. Right monotonicity is always preserved under *count*, as well as (right and left) non-monotonicity. Although we concentrate here on the *count* operator, it is not hard to show that all these results are naturally extended to the case of the general operator, *dfit*. Thus, it is not the existential requirement within the *dfit* operator that is responsible for the exceptional monotonicity properties of collective determiners, but the *count*ing process itself that collective quantification involves.

The final results are summarized in table 1 for each of the nine classes of (non-)monotone determiners. The exclamation marks emphasize the cases in which left monotonicity is not preserved.

Monotonicity of D	Monotonicity of $dfit(D)$	Example
↑MON↑	↑MON↑	*some*
↓MON↓	↓MON↓	*less than five*
↓MON↑	∼MON↑ (!)	*all*
↑MON↓	∼MON↓ (!)	*not all*
∼MON∼	∼MON∼	*exactly five*
∼MON↓	∼MON↓	*not all and (in fact) less than five (of the)*
∼MON↑	∼MON↑	*most*
↓MON∼	∼MON∼ (!)	*all or less than five (of the)*
↑MON∼	∼MON∼ (!)	*some but not all (of the)*

Table 1
(non-)monotonicity under *dfit*

The following simple fact characterizes all the cases in which monotonicity of determiners is preserved under *count*.

Fact 4.1 *Let D be a determiner over E. If D is* ↑MON↑ *(*↓MON↓*), then the determiner $count(D)$ over $\wp(E)$ is also* ↑MON↑ *(*↓MON↓*). If D is* MON↓ *(*MON↑*), then $count(D)$ is* MON↓ *(*MON↑*).*

The proof of this fact follows directly from the definition of the *count* operator.

Non-monotonicity of QNT determiners is preserved under *count*, as stated in fact 4.2 below. The proof of this fact follows directly from fact 3.2.

Fact 4.2 *Let D be a QNT determiner that is* $\sim$MON *(*MON$\sim$*). Then the determiner $count(D)$ over $\wp(E)$ is also* $\sim$MON *(*MON$\sim$*).*

Proof. We only prove this fact for the case of $\sim$MON determiners. The proof for MON$\sim$ determiners is similar.
Let E be a domain in which D_E is $\sim$MON. Such a domain exists since there exist E' and E'' such that $D_{E'}$ is not $\downarrow$MON and $D_{E''}$ is not $\uparrow$MON. Since D is EXT, we can choose $E = E' \cup E''$.
Thus, there exist $A_1, A_1', B_1, A_2, A_2', B_2 \subseteq E$ such that $A_1 \subseteq A_1'$ and $A_2 \subseteq A_2'$, and the following hold:

$D_E(A_1)(B_1) = 1$ and $D_E(A_1')(B_1) = 0$ (D is not $\uparrow$MON);
$D_E(A_2')(B_2) = 1$ and $D_E(A_2)(B_2) = 0$ (D is not $\downarrow$MON).

By corollary 3.2 (page 6) the following hold:

$(count(D_E))(\wp(A_1))(\wp(B_1)) = 1$ and $(count(D_E))(\wp(A_1'))(\wp(B_1)) = 0$;
$(count(D_E))(\wp(A_2'))(\wp(B_2)) = 1$ and $(count(D_E))(\wp(A_2))(\wp(B_2)) = 0$.

Since $\wp(A_1) \subseteq \wp(A_1')$ and $\wp(A_2') \subseteq \wp(A_2)$, we get that $count(D_E)$ is $\sim$MON, which implies that $count(D)$ is $\sim$MON.

Results pertaining to *non*-preservation of monotonicity under *count* are less straightforward, and they are therefore in the focus of the rest of this paper.

As mentioned above, the contrast between determiners with "mixed" monotonicity and determiners with similar monotonicity in both arguments, and the difference between the left argument and the right argument, both stem from the "conservativity process" within *count* that intersects the right argument with the left argument. Recall that by definition, $(count(D))(\mathcal{A})(\mathcal{B}) = D(\cup\mathcal{A})(\cup(\mathcal{A} \cap \mathcal{B}))$. Intuitively, when D is a $\downarrow$MON$\uparrow$ ($\uparrow$MON$\downarrow$) determiner and $\mathcal{A}$ and $\mathcal{B}$ are two sets of sets, replacing $\mathcal{A}$ in the left argument of $count(D)$ by a subset (superset) $\mathcal{A}'$ of $\mathcal{A}$, does not guarantee that the set $\cup(\mathcal{A}' \cap \mathcal{B})$ remains the same as $\cup(\mathcal{A} \cap \mathcal{B})$, or a superset (subset) of this set. Hence, there is no guarantee that $count(D)$ is a $\downarrow$MON ($\uparrow$MON) determiner over $\wp(E)$. By contrast, replacing $\mathcal{A}$ by a superset (subset) $\mathcal{A}'$ of $\mathcal{A}$ does guarantee that $\cup(\mathcal{A}' \cap \mathcal{B})$ is a superset (subset) of $\cup(\mathcal{A} \cap \mathcal{B})$ or equal to this set, which is the reason why for $\uparrow$MON$\uparrow$ ($\downarrow$MON$\downarrow$) determiners left monotonicity is preserved.

Here we prove a "non-preservation" theorem for $\downarrow$MON$\uparrow$ determiners. The "non-preservation" proof for the case of $\uparrow$MON$\downarrow$ determiners follows directly, using the fact that any $\uparrow$MON$\downarrow$ determiner is the negation of a $\downarrow$MON$\uparrow$ determiner. Similarly, the non-preservation proof for $\uparrow$MON$\sim$ determiners follows directly from the non-preservation theorem for $\downarrow$MON$\sim$ determiners that is proven below.

Before we turn to the non-preservation theorems, it should be mentioned that in each of the two classes, $\downarrow$MON$\uparrow$ and $\uparrow$MON$\downarrow$, there is a single QNT determiner that does not lose its left monotonicity under *count*.

These two determiners have a property to which we refer as *triviality for plurals* (PTRIV). Formally:

Definition 4.3 (triviality for plurals) *A (local) determiner D_E is called* trivial for plurals *(PTRIV) iff for all $A, B, B' \subseteq E$: if $|A| > 1$ then $D(A)(B) = D(A)(B')$.*

Informally, a PTRIV determiner is indifferent to the identity of its right argument, whenever its left argument is a set with two or more entities. However, plural nouns presuppose semantic plurality (e.g. the noun *students* presupposes that there are at least two students). Consequently, no plural noun could appear with such a determiner without leading to a trivial statement. Therefore, PTRIV determiners are not expected to be members of the class of *plural* determiners in natural language.

For the sake of presentation of our non-preservation results, we define the following condition.

Condition 1 *We say that a global determiner D satisfies condition 1 iff there exist a domain E and three subsets of E: X, Y and Y' with $|X| > 1$ and $Y' \subseteq Y \subseteq X$ such that:*

(∗) $D_E(X)(Y) = 1$ and $D_E(X)(Y') = 0$.

The next lemma claims that $\mathrm{MON}\!\uparrow$ determiners that are not PTRIV satisfy condition 1.

Lemma 4.4 *Let D be a conservative global determiner that is $\mathrm{MON}\!\uparrow$ and not PTRIV. Then D satisfies condition 1.*

Proof. Let E be a domain in which D_E is not PTRIV. Then there exist $A, B, B' \subseteq E$ with $|A| > 1$ such that $D_E(A)(B) = 1$ and $D_E(A)(B') = 0$. From conservativity of D we can assume, without loss of generality, that both B and B' are subsets of A. Since D is $\mathrm{MON}\!\uparrow$, it follows from $D_E(A)(B) = 1$ that $D_E(A)(B \cup B') = 1$. Therefore, the result holds for $X = A$, $Y = B \cup B'$ and $Y' = B'$.

Determiners that are $\downarrow\!\mathrm{MON}\!\sim$ satisfy condition 1 as well. This is proven in the next lemma. Note that this lemma also implies that a $\downarrow\!\mathrm{MON}\!\sim$ determiner is necessarily not PTRIV, and hence so is any $\uparrow\!\mathrm{MON}\!\sim$ determiner. This is the reason why there is no PTRIV exception for the case of $\downarrow\!\mathrm{MON}\!\sim$ and $\uparrow\!\mathrm{MON}\!\sim$ determiners.

Lemma 4.5 *Let D be a QNT determiner that is $\downarrow\!\mathrm{MON}\!\sim$. Then D satisfies condition 1.*

Proof. Let E be a domain in which D_E is $\downarrow\!\mathrm{MON}\!\sim$ (cf. the proof of fact 4.2). Thus, there exist six subsets of E: A, A', B, B', C and C' with $B' \subseteq B$ and $C \subseteq C'$ such that $D_E(A)(B) = 1$, $D_E(A)(B') = 0$, $D_E(A')(C) = 1$ and $D_E(A')(C') = 0$. ¿From conservativity, we can assume that $B' \subseteq B \subseteq A$ and $C \subseteq C' \subseteq A'$, and it follows that both A and A' are not the empty set.

If $|A| > 1$ then we simply choose $X = A$, $Y = B$ and $Y' = B'$. Otherwise, $A = B = \{x\}$ for some x in E, and $B' = \emptyset$. Since D is ISOM, we can assume that $A \subset A'$ (otherwise choose any permutation on E that maps x to an element in A'), and it follows that $|A'| > 1$. By $\downarrow\mathrm{MON}$, it follows from $D_E(A)(B') = 0$ that $D_E(A')(B') = 0$. Therefore, we can choose $X = A'$, $Y = C$ and $Y' = B' = \emptyset$.

Non-preservation of monotonicity in the left argument will follow immediately from the following lemma.

Lemma 4.6 *Let D be a QNT determiner that is $\downarrow\mathrm{MON}$. If D satisfies condition 1, then the determiner $count(D)$ over $\wp(E)$ is $\sim\mathrm{MON}$.*

Proof. Using X, Y and Y' in condition 1, we define the following three subsets of $\wp(E)$: $\mathcal{A} = \wp(X)$; $\mathcal{A}' = \wp(X) \setminus \{Y\}$ and $\mathcal{B} = \{Y, Y'\}$. With these definitions, the following relations hold: $\mathcal{A}' \subseteq \mathcal{A}$; $\cup\mathcal{A} = X$; $\cup\mathcal{A}' = X$; $\cup(\mathcal{A} \cap \mathcal{B}) = Y$ and $\cup(\mathcal{A}' \cap \mathcal{B}) = Y'$. By substitution in $(*)$ in condition 1 we get that: $D_E(\cup\mathcal{A})(\cup(\mathcal{A} \cap \mathcal{B})) = (count(D_E))(\mathcal{A})(\mathcal{B}) = 1$ and $D_E(\cup\mathcal{A}')(\cup(\mathcal{A}' \cap \mathcal{B})) = (count(D_E))(\mathcal{A}')(\mathcal{B}) = 0$. Since $\mathcal{A}' \subseteq \mathcal{A}$, it follows that $count(D_E) \notin \downarrow\mathrm{MON}$.
To prove that $count(D) \notin \uparrow\mathrm{MON}$, note that since D is not LTRIV, it follows from $D \in \downarrow\mathrm{MON}$ that $D \notin \uparrow\mathrm{MON}$. Thus, there exist a domain E and three subsets of E: A, A' and B such that $A \subseteq A'$, $D_E(A)(B) = 1$ and $D_E(A')(B) = 0$. The rest of the proof directly follows from fact 3.2.

It is now straightforward to show that determiners that are $\downarrow\mathrm{MON}\uparrow$ or $\downarrow\mathrm{MON}\sim$ lose their left monotonicity property under *count*.

Theorem 4.7 *Let D be a QNT determiner that is $\downarrow\mathrm{MON}\uparrow$. Then the determiner $count(D)$ over $\wp(E)$ is $\sim\mathrm{MON}\uparrow$ iff D is not PTRIV.*

Proof. $(\Leftarrow)$ Assume, first, that D is not PTRIV. It follows directly from lemma 4.4 and lemma 4.6 that $count(D)$ is $\sim\mathrm{MON}$. By fact 4.1, $count(D)$ is $\mathrm{MON}\uparrow$.
$(\Rightarrow)$ Assume that D is PTRIV. By fact 4.1, $count(D)$ is $\mathrm{MON}\uparrow$. Let E be a domain and let $\mathcal{A}, \mathcal{A}'$ and $\mathcal{B}$ be three subsets of $\wp(E)$ such that $\mathcal{A}' \subseteq \mathcal{A}$ and $(count(D_E))(\mathcal{A})(\mathcal{B}) = D_E(\cup\mathcal{A})(\cup(\mathcal{A}\cap\mathcal{B})) = 1$. We Show that $(count(D_E))(\mathcal{A}')(\mathcal{B}) = D_E(\cup\mathcal{A}')(\cup(\mathcal{A}' \cap \mathcal{B})) = 1$. There are three possibilities: $|\cup\mathcal{A}| > 1$, $\cup\mathcal{A} = \emptyset$ and $\cup\mathcal{A} = \{x\}$. In the first two cases the proof is trivial. For the third case, note that from upward right monotonicity of D, $D(\{x\})(\{x\}) = 1$ and by CONS and $\downarrow\mathrm{MON}$, $D(\emptyset)(\emptyset) = 1$. If $\cup\mathcal{A}' = \cup\mathcal{A}$, then the lemma trivially holds. Otherwise, $\cup\mathcal{A}' = \emptyset$, which implies that $D_E(\cup\mathcal{A}')(\cup(\mathcal{A}' \cap \mathcal{B})) = D(\emptyset)(\emptyset) = 1$.

Theorem 4.8 *Let D be a QNT determiner that is $\downarrow\mathrm{MON}\sim$. Then the determiner $count(D)$ over $\wp(E)$ is $\sim\mathrm{MON}\sim$.*

Proof. Directly by lemmas 4.5 and 4.6, $count(D)$ is $\sim\mathrm{MON}$. By fact 4.2, $count(D)$ is $\mathrm{MON}\sim$.

Examples for the result in theorem 4.8 with natural language determiners are more complex, since we are not familiar with any lexical determiner that is monotone in its left argument but non-monotone in its right argument. However, such a compound determiner can be obtained by conjoining a determiner that is $\uparrow\mathrm{MON}\uparrow$ (e.g. *some*) with a determiner that is $\uparrow\mathrm{MON}\downarrow$ (e.g. *not all*). The result is a determiner such as *some but not all*, which is $\uparrow\mathrm{MON}\sim$ with distributive predicates. However, with collective predicates, *some but not all* is *not* $\uparrow\mathrm{MON}$. Consider the lack of entailment in (10) below.

(10) Some but not all of the rich students drank a whole glass of beer together
$\not\Rightarrow$
Some but not all of the students drank a whole glass of beer together.

In a situation where the students are s_1, s_2, s_3 and s_4, the rich students are s_1, s_2 and s_3, and there are only two groups that drank a whole glass of beer together: $\{s_1, s_2\}$ and $\{s_1, s_2, s_3, s_4\}$, most speakers we consulted consider the antecedent of (10) to be true, whereas the consequent is false, in agreement with the analysis using the *dfit* operator.

5 Conclusion

The formal study of the interactions between quantifiers and collective predicates has to deal with many seemingly conflicting pieces of evidence that threaten to blur the interesting logical questions that these phenomena raise. In this paper we have studied the monotonicity properties of collective quantification, which is a central aspect of the problem of collectivity. We showed that to a large extent, the principles that underly monotonicity of collective quantification follow from standard assumptions on quantification in natural language in general. The *count* operator, which is a straightforward extension of Scha's 'neutral' analysis of collective determiners, involves a simple 'conservativity element' – intersection of the right argument with the left argument, and a 'participation element' – union of both set of sets arguments. The conservativity element within the *count* operator is responsible for the two *a priori* unexpected asymmetries in the monotonicity behavior of collective determiners:

(i) Only determiners with 'mixed' monotonicity properties change their behavior when they quantify over collections.

(ii) Only the *left* monotonicity properties of such determiners may change in these cases.

We believe that the reduction of certain asymmetries in the domain of collective quantification to the well-known asymmetric conservativity principle is a desirable result that reveals another aspect of the central role that this principle plays in natural language semantics.

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