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Epistemic Logics with Structured Knowledge

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Abstract

Multi-agent Dynamic Epistemic Logic, as a suitable modal logic to reason about knowledge evolving systems, has emerged in a number of contexts and scenarios. The agents knowledge in this logic is simply characterised by valuations of propositions. This paper discusses the adoption of other richer structures to make these representations, as graphs, algebras or even epistemic models. This method of building epistemic logics over richer structures is called "Epistemisation". On this view a parametric method to build such Epistemic Logics with Public Announcements is introduced. Moreover, a parametric notion of bisimulation is presented, and the modal invariance of the proposed logics, with respect to this relation, are proved. Some interesting application horizons opened with this construction are stated.

Keywords: Dynamic Epistemic Logic; Structured States; Parametric construction of Logics

1 Introduction

Multi-agent epistemic logic has been investigated in Computer Science [17,10,28] to represent and reason about agents, or groups of agents, knowledge and beliefs.

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Models for these logics are Epistemic Kripke Structures - multi-modal Kripke structures whose states formalise informations and, an equivalence relation of edges for each agent, relating indistinguishable states from their perception. Typically, the states information is characterised by a set of propositions. This paper introduces a method to build dynamic epistemic logics to deal with scenarios where the representation of knowledge needs richer presentations than a simply set of propositions. The method to derive these logics (called 'epistemisations'), in a demand driven fashion, is parametric to the (logic) formalism that better fits the nature of the knowledge involved. We illustrate the approach representing knowledge by means of three core structures of computer science: propositions, graphs and abstract data types. This is figured out in three instances of the method, namely the 'epistemisations' of propositional logic (that captures Dynamic Epistemic Logic (DEL)), of Equational Logic (as the classic formalism for abstract data types specification) and of Hybrid Logic with Binders (as the standard logic for graphs representation).

There are works that use epistemic models in which states have a structure [1,16]. In [1] it is introduced a multi-agent epistemic logic in which states are positions in $\mathbb{R}^n$ and the accessibility relations represent the possible states (positions) compatible with the current one. Also in [16,26,27], (dynamic) epistemic logics based on the notion of visibility or observability of propositional variables are presented, i.e, some propositional variable are observable and others are not.

Other works deal with values in an epistemic setting [29,30,31]. Here, states are equipped with a register that can store values ([29]) or with constants that can have their values updated ([30,31]).

In the case of the epistemisation of equational logic it reminds first order modal logic [14], where epistemic states are relational structures.

The combination of logics is an active research topic in modern logic. In addition of the well known symmetric combination of logics with methods as fibring, product and fusion of logics [7], combinations where the particular features of a logic are combined 'on top' of another one have been considered in several contexts. Other example of this kind of combination can be found in the works on 'modalisation' of logics, which endows systematically logics with Kripke semantics (w.r.t. the standard $\Box$ and $\Diamond$ modalities) presented in [11,9]. These works were extended by the authors with the 'hybridisation' construction [8], by enriching these logics with the hybrid logic machinery. The 'temporalisation' of logics [13] and the 'probabilisation' of logics [2] are other remarkable examples on this kind of combinations. Other proposals in the literature abstract the combination pattern by considering the 'top logic' itself arbitrary. Such is the case of what is called *parametrisation of logics* in [6]. In brief, a logic is parametrized by another one if an atomic part of the first is replaced by the second. Therefore, the method distinguishes a parameter to fill (the atomic part), a parametrised logic (the 'top' logic) and a parameter logic (the logic inserted within). The method of *importing logics* [25] aims at formalising this kind of asymmetric combinations resorting to a graph-theoretic approach. A more general account of this combinations can be found in [21]. The Epistemisation process proposed in this paper can be regarded as an asymmetric combination.

The remaining of this paper is organised as follows: we start by introducing the notion of *knowledge representation framework*, i.e. a generic notion of logic that will be used to specify/support structured states of information. Based on this parameter, we introduce the building *'epistemisation' method*, with three paradigmatic illustrations. These logics are then enriched with *public announcements*. Finally, we trace a research agenda to follow up from this work, discussing a set of potential applications and research lines.

2 Representation of Structured Knowledge

In order to represent knowledge of structured states a generic notion of logic is used. This formalism will be the parameter for the construction introduced in the next section, and is defined as follows:

Definition 2.1 A *knowledge representation framework* consists of a tuple $\mathcal{L} = (\mathrm{Fm}_{\mathcal{L}}, \mathrm{Mod}_{\mathcal{L}}, \models_{\mathcal{L}})$, where

- $\mathrm{Fm}_{\mathcal{L}}$ is a countable set of formulas,
- $\mathrm{Mod}_{\mathcal{L}}$ is the set of models for $\mathcal{L}$ and
- relation $\models_{\mathcal{L}} \subseteq \mathrm{Mod}_{\mathcal{L}} \times \mathrm{Fm}_{\mathcal{L}}$ is a relation called *satisfaction relation.*

The usual notion of elementary equivalence is also used in this work. Let us recall it:

Definition 2.2 The *elementary equivalence in* $\mathcal{L}$ is the relation $\equiv_{\mathcal{L}} \subseteq \mathrm{Mod}_{\mathcal{L}} \times \mathrm{Mod}_{\mathcal{L}}$ defined by

$$\equiv_{\mathcal{L}} = \{(M, M') \mid \text{ for any } \varphi_0 \in \mathrm{Fm}_{\mathcal{L}},\, M \models_{\mathcal{L}} \varphi_0 \text{ iff } M' \models_{\mathcal{L}} \varphi_0\}$$

As examples of Knowledge representation frameworks we can enumerate all the logics with a complete calculus. Relevant for the present work we have the standard Classic Propositional and Equational Logic, and Hybrid Logic with Binders [4].

3 Parametric construction of Epistemic Logics with structured States

Let us fix a knowledge representation framework $\mathcal{L}$. We introduce in the following a generic construction to building the logic $\mathcal{E}(\mathcal{L})$, the *epistemisation of* $\mathcal{L}$.

Definition 3.1 The formulas for the $\mathcal{L}$-epistemic logic for a finite set of agents $\mathcal{A}$, in symbols $\mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$, is defined by the following grammar:

$$\varphi ::= \varphi_0 \mid \mathbf{tt} \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid K_a\varphi \mid C_G\varphi$$

where $\varphi_0 \in \mathrm{Fm}_{\mathcal{L}}$, $a \in \mathcal{A}$ and $G \subseteq \mathcal{A}$.

The standard connectives can be presented as abbreviations, namely $\mathbf{ff} \equiv \neg\mathbf{tt}$, $\varphi \vee \phi \equiv \neg(\neg\varphi \wedge \neg\phi)$, $\varphi \to \phi \equiv \neg(\varphi \wedge \neg\phi)$, $B_a\varphi \equiv \neg K_a\neg\varphi$ and $E_G\varphi \equiv \bigwedge_{a\in G} K_a\varphi$.

The intuitive meaning of the modal formulas are:

- φ_0 - is an assertion about the (structured) epistemic states, expressed in the knowledge representation framework $\mathcal{L}$;
- $K_a\varphi$ - agent a knows φ;
- $E_G\varphi$ - every agent $a \in G$ knows φ;
- $C_G\varphi$ - it is common knowledge among all members of group G that it is the case that φ.

Definition 3.2 An *$\mathcal{L}$-epistemic model* for a finite set of agents $\mathcal{A}$, $\mathcal{A}$-model for short, is a tuple $\mathcal{M} = (W, \sim, M)$ where

- W is a non-empty set of states;
- $\sim$ is an $\mathcal{A}$-family of equivalence relations $(\sim_a \subseteq W \times W)_{a \in \mathcal{A}}$; and
- $M : W \to \mathrm{Mod}_{\mathcal{L}}$ is a function, that assigns the knowledge structure of each state.

We also consider the relations $\sim_G = \bigcup_{a \in G} \sim_a$ and $\sim_G^* = (\sim_G)^*$, where $(\sim_G)^*$ is the reflexive, transitive closure of $\sim_G$. The set of $\mathcal{L}$-epistemic models for a set of agents $\mathcal{A}$ is denoted by $\mathrm{Mod}_{\mathcal{E}(\mathcal{L})}$.

Definition 3.3 For any $\mathcal{A}$-model $\mathcal{M} = (W, \sim, M)$, for any $w \in W$, and $\varphi \in \mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$, the satisfaction relation

$$\models \subseteq \mathrm{Mod}_{\mathcal{E}(\mathcal{L})} \times \mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$$

is recursively defined as follows:

- $\mathcal{M}, w \models \varphi_0$ iff $M(w) \models_{\mathcal{L}} \varphi_0$
- $\mathcal{M}, w \models \neg\phi$ iff $\mathcal{M}, w \not\models \phi$
- $\mathcal{M}, w \models \phi \wedge \psi$ iff $\mathcal{M}, w \models \phi$ and $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models K_a\phi$ iff for all $w' \in W : w \sim_a w' \Rightarrow \mathcal{M}, w' \models \phi$
- $\mathcal{M}, w \models C_G\phi$ iff for all $w' \in W : w \sim_G^* w' \Rightarrow \mathcal{M}, w' \models \phi$

We write $\mathcal{M} \models \varphi$ whenever, for any $w \in W$, $\mathcal{M}, w \models \varphi$.

It is easy to see that

- $\mathcal{M}, w \models E_G\phi$ iff for all $w' \in W$ we have $w \sim_G w' \Rightarrow \mathcal{M}, w' \models \phi$, and
- $\mathcal{M}, w \models B_a\phi$ iff there is a $w' \in W$ such that $w \sim_a w'$ and $\mathcal{M}, w' \models \phi$.

Now we are in condition to introduce the first illustration of the paper that suggests that the standard propositional epistemic logic can be captured with our epistemisation method:

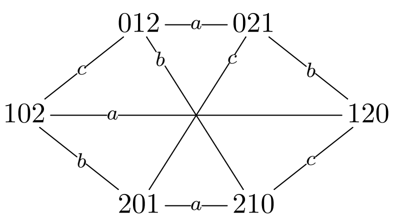
Example 3.4 This example is adapted from [28]. Suppose that a father has three envelopes, each containing: **0**, **1** and **2** dollars inside respectively. The father has three children: **a**nna, **b**ob and **c**lara. Each child receives one envelope and do not know the content of the envelopes of the other children.

In order to represent this scenario, we adopt $\mathcal{E}(\mathcal{PL})$, for $\mathcal{PL}$ the classic propositional logic. Note that this logic corresponds exactly to the standard *Multi-Agent Epistemic Logic* (e.g. [28]).

Hence, for the construction of the base formulas (in $\mathrm{Fm}_{\mathcal{PL}}$) we use propositional sentences over the set of variables $\mathrm{Var} = \{0_n, 1_n, 2_n \mid n \in \{a, b, c\}\}$ meaning "child n has envelope **0**, **1** or **2**. Moreover, the knowledge in each epistemic state will be simply structured as valuations $\mathbf{2}^{\mathrm{Var}}$. We represent each state by the envelope that each child has in that state, for instance 012 is the state where child **a** has **0**, child **b** has **1** and child **c** has **2**; hence $W = \{012, 021, 102, 120, 201, 210\}$. This notation also supports the information of the local models Knowledge representation. For instance, the knowledge structure in state 012 is a function $M(012)$ defined, for each $C_n \in \mathrm{Var}$, as:

$$M(012)(C_n) = \begin{cases} \mathbf{tt} & \text{if } C = 0 \text{ and } n = a \\ \mathbf{tt} & \text{if } C = 1 \text{ and } n = b \\ \mathbf{tt} & \text{if } C = 2 \text{ and } n = c \\ \mathbf{ff} & \text{otherwise} \end{cases}$$

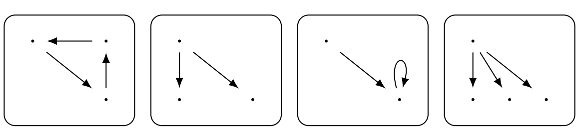
Finally, we take the $\{a, b, c\}$-epistemic relation $\sim$ as expected. For instance, we have, for Anna, the relation $\sim_a = \{(012, 012), (012, 021), (021, 021), \dots\}$. The graphical representation of $\mathcal{M} = (W, \sim, M)$ is given as follows:



Observe that $\mathcal{M}, 012 \models B_b 0_a$ since, $012 \sim_b 012$ and $\mathcal{M}, 012 \models 0_a$ (because $M(012)(0_a) = \mathbf{tt}$, i.e. $M(012) \models_{\mathcal{PL}} 0_a$). Moreover, it is not difficult to see that $021 \models E_{ac} 2_b$ does not hold.

The next two examples introduce two new logics by means of two instantiations of our epistemisation method:

Example 3.5 Let us now consider the following game: from the universe of structures

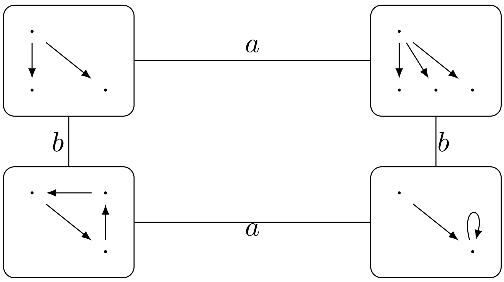


one graph is chosen. Anna and Bob will play with this hidden graph with the following additional information:

- Anna knows that the graph is deterministic
- Bob knows that the graph has exactly 3 nodes

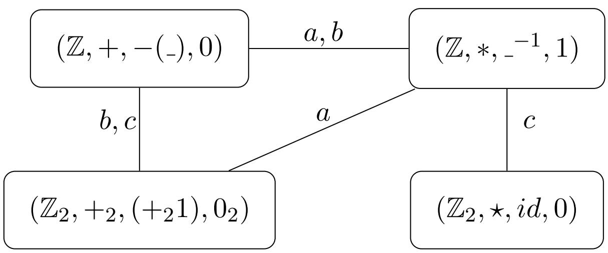
This knowledge perception can be represented directly by means of an epistemic structured model, whose epistemic states consist precisely in these four possible graphs. As it is well known, *hybrid logic with binders* is a very good candidate to express (local) properties about these structures (e.g. [12]). We can build the set of hybrid formulas over a signature $(Prop, Nom)$, where $Prop$ is the set of propositions and Nom is the set of nominals. It is a standard modal language over $Prop$ enriched with the nominals i, which are special propositions that hold at a unique state named by i, $@_i\rho$, which asserts that formula ρ holds in the state named by the nominal i and the formulas $\downarrow x.\rho$ that are true in a given state w, if ρ is true in w whenever all the occurrences of x in φ refers to w.

Hence, let us build the logic $\mathcal{E}(\mathcal{H}(\downarrow, @))$ in order to deal with this scenario. Hence, the intended $H(\downarrow, @)$-epistemic structure $\mathcal{M}$ is represented as follows:



Using the features of hybrid logic, we are able to express the players information with the sentences
$K_a\big(\downarrow x.(\diamond \mathbf{tt} \longrightarrow \diamond \downarrow y.@_x\Box y)\big)$ and $K_b\big((i \vee j \vee k) \wedge (\neg @_j k \wedge \neg @_i k \wedge \neg @_i j)\big)$. Other formulas valid in this model are *Anna knows, that Bob does not knows, that the graph is deterministic* expressed in the sentence $K_a\big(\neg K_b(\downarrow x.(\diamond \mathbf{tt} \longrightarrow \diamond \downarrow y.@_x\Box y)\big)$.

Example 3.6 In order to illustrate the 'epistemisation' of equational logic, the logic $\mathcal{E}(\mathrm{EQ})$, let us consider a game as the one used in the previous example. From the four algebras (in a signature with a binary symbol $\odot$, an unary operation symbol inv and a constant symbol e) depicted in the model $\mathcal{N}$ represented bellow, one algebra is chosen.



The operations of the algebras of the top line are the usual integers sum and product and the respective inverses. The operations of the bottom left algebra are the standard sum and inverse on the cyclic field $\mathbb{Z}_2$. The binary operation $\star$ of the bottom right algebra is defined by

$$\star(x,y) = \begin{cases} 1 & \text{if } x = 0, y = 1 \\ 0 & \text{otherwise} \end{cases}.$$

The following properties are valid in this model: *Anna knows that* $\odot$ *is associative*, expressed by the sentence $K_a\big((x \odot y) \odot z = x \odot (y \odot z)\big)$; *Bob knows that* e *is a neutral element* expressed by $K_b\big(x \odot e = x \wedge e \odot x = x\big)$; and finally, *Clara knows that every element has an inverse*, expressed by the sentence $K_c\big(x \odot x^{-1} = e \wedge x^{-1} \odot x = e\big)$.

4 Public Announcements

The present section enriches 'epistemisations' with the public announcements. These are announcements that are made publicly to all agents and their content is common knowledge among all agents.

Definition 4.1 The formulas for the $\mathcal{L}$-*epistemic logic with public announcement* (for a set of agents $\mathcal{A}$) , in symbols $\mathrm{Fm}^{\mathrm{pub}}_{\mathcal{E}(\mathcal{L})}$, is defined by the following grammar:

$$\varphi ::= \varphi_0 \mid \top \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_a\varphi \mid C_G\varphi \mid [\varphi]\varphi$$

where $\varphi_0 \in \mathrm{Fm}_{\mathcal{L}}$, $a \in \mathcal{A}$ and $G \subseteq \mathcal{A}$.

This grammar extends the one presented in Definition 3.1 with the formulas $[\varphi]\varphi'$ expressing the situation 'after the public announcement of φ, formula φ' is true'.

Definition 4.2 Given $\mathcal{L}$-epistemic *model* for a set of agents $\mathcal{A}$, $w \in W$, and $\varphi \in \mathrm{Fm}^{\mathrm{pub}}_{\mathcal{E}(\mathcal{L})}$, the satisfaction relation

$$\models^{\mathrm{pub}} \subseteq \mathrm{Mod}_{\mathcal{E}(\mathcal{L})} \times \mathrm{Fm}^{\mathrm{pub}}_{\mathcal{E}(\mathcal{L})}$$

extends the definition of the relation $\models$ with:

- $\mathcal{M}, w \models^{\mathrm{pub}} [\chi]\varphi$ iff $\mathcal{M}, w \models^{\mathrm{pub}} \chi$ implies $\mathcal{M}|_\chi, w \models^{\mathrm{pub}} \varphi$

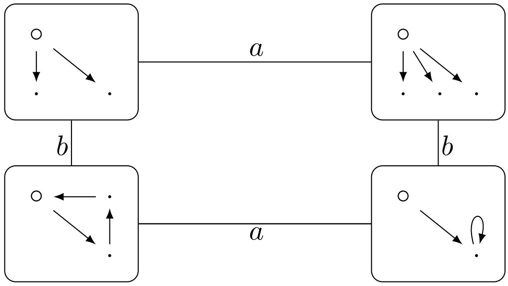
where $\mathcal{M}|_\chi = (W|_\chi, \sim|_\chi, M|_\chi)$ is the the $\mathcal{L}$-epistemic *model* for a set of agents $\mathcal{A}$ where:

- $W_\chi = \{w \mid \mathcal{M}, w \models^{\mathrm{pub}} \chi\}$;
- for any $a \in \mathcal{A}$, $(\sim_a)|_\chi = \sim_a \cap (W_\chi \times W_\chi)$;
- $M|_\chi$ is the restriction of M to W_χ, i.e. the function defined for any $w \in W_\chi$ by $M|_\chi(w) = M(w)$.

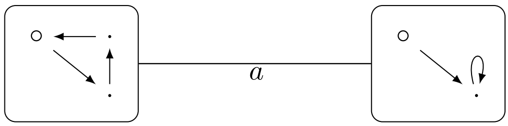
As usual we use $\langle\chi\rangle\varphi$ to denote its dual $\neg[\chi]\neg\varphi$. Semantically, we have

$$\mathcal{M}, w \models^{\mathrm{pub}} \langle\chi\rangle\varphi \text{ iff } \mathcal{M}, w \models^{\mathrm{pub}} \chi \text{ and } \mathcal{M}|_\chi, w \models^{\mathrm{pub}} \varphi$$

Example 4.3 Let us suppose, in the context of Example 3.5, that it is announced that 'We can execute two consecutive transitions' $\diamond\diamond\,\mathbf{tt}$. This entails that the updated model $\mathcal{M}|_{\diamond\diamond\mathbf{tt}}$ depicted as follows:

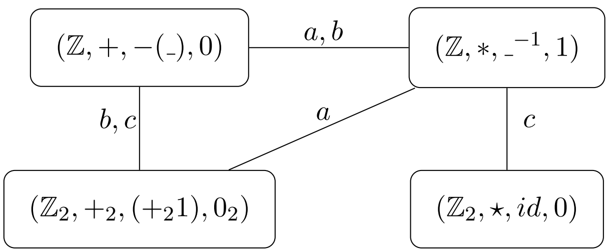


$\Downarrow_{\diamond\diamond\mathbf{tt}}$

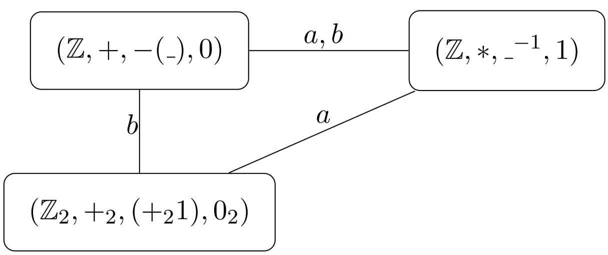


Now, we have, for instance that $\mathcal{M} \models \neg K_b(\Diamond(\downarrow x.\Diamond\Diamond\Diamond x))$ but $\mathcal{M} \models [\diamond\diamond\mathbf{tt}]K_b(\Diamond(\downarrow x.\Diamond\Diamond\Diamond x))$.

Example 4.4 In the situation of Example 3.6, let us suppose that it is announced that the algebra chosen is a group (let us assume Φ as the conjunction of the equational axiomatisation of the (variety) of Groups e.g.[5]). The model $\mathcal{N}|_\Phi$ is represented in the following diagram:



$\Downarrow_\Phi$



Now, we have for instance that $\mathcal{N} \models [\Phi]K_c(a \odot (b \odot c) = (a \odot b) \odot c$ but $\mathcal{N} \not\models K_c(a \odot (b \odot c) = (a \odot b) \odot c$.

5 Bisimulation Invariance

In this section we introduce a parametric notion of bisimulation that preserves properties in 'epistemisations'.

Definition 5.1 Let $\mathcal{L}$ be a Knowledge representation framework and $\mathcal{M} = (W, \sim, M)$ and $\mathcal{M}' = (W', \sim', M')$ two $\mathcal{L}$-epistemic models for the set of agents $\mathcal{A}$. A *bisimulation* between $\mathcal{M}$ and $\mathcal{M}'$ is a relation $R \subseteq W \times W'$ such that, for each $a \in \mathcal{A}$, and for any $w, v \in W$ with $(w, w') \in R$:

(atom) $M(v) \equiv_{\mathcal{L}} M(v')$;

(zig) for any $v \in W$, if $w \sim_a v$, then there is a $v' \in W'$ such that $w' \sim'_a v'$ and $(v, v') \in R$;

(zag) for any $v' \in W'$, if $w' \sim'_a v'$, then there is a $v \in W$ such that $w \sim_a v$ and $(v, v') \in R$.

Whenever a bisimulation between $w \in W$ and $w' \in W'$ exists, we say that w and w' are bisimilar and we write $(M, w) \leftrightarroweq (M', w')$.

Note that, as it is well known, the *bisimilarity* relation $\leftrightarroweq$ is itself a bisimulation. The modal invariance result for the Epistemic Logic is extended for the generic $\mathcal{L}$-Epistemic logics:

Theorem 5.2 (Invariance) *Let $\mathcal{M} = (W, \sim, M)$ and $\mathcal{M}' = (W', \sim', M')$ two $\mathcal{L}$-epistemic models for the set of agents $\mathcal{A}$ and $w \in W$ and $w' \in W'$. Then, if $(\mathcal{M}, w) \leftrightarroweq (\mathcal{M}', w')$, we have that, for any formula $\varphi \in \mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$,*

$$\mathcal{M}, w \models \varphi \textit{ iff } \mathcal{M}', w' \models \varphi$$

Proof. We prove this result by induction over the structure of $\mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$. For the atomic sentences $\varphi_0 \in \mathrm{Fm}_{\mathcal{L}}$, we just have to observe that

$$
\begin{array}{ll}
 & \mathcal{M}, w \models \varphi_0 \\
\Leftrightarrow & \quad \{ \models \text{defn} \} \\
 & M(w) \models_{\mathcal{L}} \varphi_0 \\
\Leftrightarrow & \quad \{ \text{By Defn 5.1 } M(w) \equiv_{\mathcal{L}} M'(w') \} \\
 & M'(w') \models_{\mathcal{L}} \varphi_0 \\
\Leftrightarrow & \quad \{ \models \text{defn} \} \\
 & \mathcal{M}', w' \models \varphi_0
\end{array}
$$

The proof for other cases is done as for the Epistemic Logic (eg. [28]). □

Let us consider the following auxiliary result:

Lemma 5.3 *Let $\mathcal{M} = (W, \sim, M)$ and $\mathcal{M}' = (W', \sim', M')$ two $\mathcal{L}$-epistemic models for the set of agents $\mathcal{A}$, $w \in W, w' \in W'$ and $\varphi \in \mathrm{Fm}_{\mathcal{E}(\mathcal{L})}$. Then, if $(\mathcal{M}, w) \leftrightarroweq$*

$(\mathcal{M}', w')$ and $\mathcal{M}, w \models \varphi$, we have $(\mathcal{M}|_\varphi, w) \leftrightarrows (\mathcal{M}'|_\varphi, w')$.

Proof. Let us assume a bisimulation $B \subseteq W \times W'$ with $(w, w') \in B$. Let us suppose that $\mathcal{M}, w \models \varphi$. Hence, in order to prove $(\mathcal{M}|_\varphi, w) \leftrightarrows (\mathcal{M}'|_\varphi, w')$, it is enough to show that $B|_\varphi = B \cap (W|_\varphi \times W'|_\varphi)$ is a bisimulation. Let us start to see (**zig**). For any $a \in \mathcal{A}$ and $v \in W|_\varphi$,

$$
\begin{array}{ll}
& w\,(\sim_a)|_\varphi v \\
\Rightarrow & \quad \{\ |_\varphi \text{ defn}\} \\
& w\ \sim_a\ v \\
\Rightarrow & \quad \{\ \text{Since } B \text{ is bisimulation}\ \} \\
& \exists v' \in W' \text{ such that } w' \sim'_a v' \text{ and } (v, v') \in B \\
\Rightarrow & \quad \{\ \text{By Thm 5.2, } w' \in W'|_\varphi + |_\varphi \text{ dfn.}\} \\
& w'\,(\sim'_a)|_\varphi v' \text{ and } (v, v') \in B
\end{array}
$$

The proof for (**zag**) is analogous and for (**atom**) is trivial. □

Theorem 5.4 *Let $\mathcal{M} = (W, \sim, M)$ and $\mathcal{M}' = (W', \sim', M')$ two $\mathcal{L}$-epistemic models for the set of agents $\mathcal{A}$ and $w \in W$ and $w' \in W'$. Then, if $(\mathcal{M}, w) \leftrightarrows (\mathcal{M}', w')$, we have that, for any formula $\varphi \in \mathrm{Fm}^{\mathrm{pub}}_{\mathcal{E}(\mathcal{L})}$,*

$$\mathcal{M}, w \models^{\mathrm{pub}} \varphi \textit{ iff } \mathcal{M}', w' \models^{\mathrm{pub}} \varphi$$

Proof.

This proof extends the one of Theorem 5.2 with the proof of invariance of the sentences $[\varphi]\rho$. On this view, we just have to observe that

$$
\begin{array}{ll}
& \mathcal{M}, w \models^{\mathrm{pub}} [\varphi]\rho \\
\Leftrightarrow & \quad \{\ \models^{\mathrm{pub}} \text{ defn}\} \\
& \mathcal{M}, w \models^{\mathrm{pub}} \varphi \Rightarrow \mathcal{M}|_\varphi, w \models^{\mathrm{pub}} \rho \\
\Leftrightarrow & \quad \{\ \text{Lemma 5.3} + \text{H.I.}\} \\
& \mathcal{M}', w' \models^{\mathrm{pub}} \varphi \Rightarrow \mathcal{M}'|_\varphi, w' \models^{\mathrm{pub}} \rho \\
\Leftrightarrow & \quad \{\ \models^{\mathrm{pub}} \text{ defn}\} \\
& \mathcal{M}', w' \models^{\mathrm{pub}} [\varphi]\rho
\end{array}
$$

□

6 Research agenda

This paper introduced a method for building suitable epistemic logics to deal with scenarios where the representation of knowledge needs richer presentations than sets of propositions. The illustrations provided shown the generality of the method by presenting epistemic logics which states are represented with three paradigmatic structures of computer science - sets of propositions, graphs and algebras. Moreover, a parametric notion of bisimulations, invariant to the 'epistemisations' was also introduced. As happens in other modal logics, this is a very important result, not only to identify equivalent epistemic models, but also to base the development of algorithms for the minimisation of models (a crucial aspect to scale up this approach to the analysis of real practical scenarios).

Next step would be to provide a method to axiomatise our "epistemisations" and to establish (under some conditions) their completeness, soundness, finite model property, decidability and complexity.

It is important to notice that this approach can be potentially useful in a wide range of application domains. For instance, in order to model a given autonomous hybrid system, we can understand sensors as agents that partially know the state of the system (e.g. some of them know the vertical acceleration, some other the current position etc.). The 'epistemisation' of differential dynamic logic [24], is a good candidate to perform this analysis. By specifying, as common knowledge, the classic Newtonian laws of mechanics (on means of ordinary differential equations), we are able to reason about the whole system behaviour, to formalise cooperative strategies, etc.

Another natural application domain for this research would be the analysis of cryptographic protocols, in particular asymmetrical cryptography protocols, by assuming public keys as common knowledge and private keys as specific agent knowledge. The structured representation of epistemic states, for instance as algebras, endows the method with expressibility to analyse more complex cryptographic schemes dealing with partial shared information.

Other interesting, less conventional, logics can emerge for this method. For instance, it would be interesting to derive the 'epistemisation' of the probabilistic [23] or of the fuzzy logic (e.g. [15]) and analyse their relation with the probabilistic and multi-valued epistemic logics in the literature (e.g. [18,3]).

Finally, we note that the parametric nature of the method can also pave the way for the development of their own computational supporting tools. Our previous experience in demand driven generation of specification logics, parametric to the specificities of some classes of complex systems (e.g. [19,20,3]) will certainly hint this research. For instance, in the line of what we have done on the parametric construction of hybrid logics [19], we expect to introduce a method to derive logic calculus for 'epistemisation', parametric to the calculus of the base logic (cf. [22]). This would provide the proof support necessary to apply this research to the analysis of real case scenarios.

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