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Hausdorff compactifications of topological function spaces via the theory of continuous lattices

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Abstract

It is known from the theory of continuous lattices that if X is a locally compact Hausdorff space then the set *LSC(X)* of lowersemicontinuous functions defined on X with values on the extended real lineadmits a unique compact Hausdorff topology making the functional *(f, g)to min(f, g)* continuous, namely the Lawson topology of the continuouslattice *LSC(X)*. It is natural to wonder whether the relative topologyon the subset *C(X)* of continuous functions is the compact-opentopology. Unfortunately, it turns out to be strictly weaker. But a relatedconstruction does produce a Hausdorff compactification of *C(X)*. Weshow that if X is a locally compact Hausdorff space and Y is aHausdorff topological space which is perfectly embedded into a continuouslattice L endowed with the Scott topology, then the Lawson topologyon the continuous lattice *LSC(X,L)* of Scott continuous maps from$X$ to L induces the compact-open topology on the space*C(X,Y)* of continuous maps from X to Y. Thus, by takingthe closure of the image of *C(X,Y)* in *LSC(X,L)*, one gets aHausdorff compactification of *C(X,Y)*. Three particular cases are ofinterest. (1) If Y is the Euclidean real line one can take L as the lattice of compact connected subsets of the two-pointcompactification of Y ordered by reverse inclusion. In this case,*C(X,Y)* is already dense in *LSC(X,L)*. (2) If Y is alocally compact Hausdorff space, one can take L as the compactsubsets of the one-point compactification of Y. (3) As a furtherparticular case of (2), if X and Y are compact Hausdorff, oneconcludes that the Vietoris topology on the closed subsets of the cartesianproduct of X and Y induces the compact-open topology on*C(X,Y)*, by identifying continuous functions with their closed graphs,using the fact that the Lawson topology coincides with the Vietoristopology.

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