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An appropriate discrete-transformation technique for order reduction methodology

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ABSTRACT

Various transformation techniques are known for transforming continuous-time to discrete-time domains and vice-versa, among which only two techniques are used in application frequently. This paper attempts to access both the techniques presenting a suitable consequence of their sole importance. The assessment of these two techniques is made on the basis of computation of reduced order model of discrete-time interval systems. Approximate models obtained after the transformations lead to the same response, and the outcome in favor of one technique. As a conclusion, the appropriate transformation is mathematically preferred over another based on its simplicity and ease of computation. Examples illustrating the comparisons are accompanied. Eventually, the assessment demonstrated in this article would be helpful to the mathematicians, mathematics educators and researchers working in the area of model order reduction.

1. Introduction

Discretization is an important data processing task and includes many advantages as; it is less prone to variance in estimation from small fragmented data; amount of data under consideration is reduced as redundant data can be recognized and neglected; provides better performance for the rule extraction. There are numerous ways of transformation from continuous-time domain to discrete-time domain and vice-versa and can be accessed in Refs. [1–4]. But well-known and easily accessible are *i*) Eulers Forward differentiation method, *ii*) Eulers Backward differentiation method, *iii*) Zero Order Hold (*ZOH*) method, *iv*) Tustins method with frequency prewarping, or Bilinear transformation, and *v*) Matched Pole-Zero mapping. All these transformations fall under frequency-domain. In addition, time-domain transformations are impulse-invariance and step-invariance methods. Each of the transformations has their own practical and theoretical importance with differences among each other, which when studied would be lengthy and exhaustive. At par, their individual study is out of scope for this article. However, an attempt to present an appropriate transformation techniques for its wide application to order reduction methodologies in discrete-time domain is made here.

Till date, to the authors' knowledge, no discussion is available that specifies, which of the either transformation is more accurate or preferable for obtaining reduced order models. This sets the motive of the paper to attempt for a convincing difference between the two transformation methods based on their simplicity and ease of computation via order reduction of discrete-time interval systems. Moreover, this assessment of the frequently used transformation techniques would be helpful to the researchers who work on a higher order system for improvement of the system performance. Also, this paper does not provide any new outcome, but attempts to offer a firm justification, in a manner to directly aim which of the two transformation techniques is to be used.

In this exploration, focus is on the two frequently used discretization techniques, which are different from the computation point of view. A brief about the outcome of the discussion made in this submission is presented in section 1 followed by a literature survey section 2, for the methodologies available for order reduction using different transformation techniques. In section 3, the two transformation techniques chosen for realization or assessment in the presented illustration is reported. Section 4 comprises the preliminaries required throughout the discussion to validate the statement of proof for the conclusion derived. This section is divided in two subsections *a*) reduction methodology used and *b*) the performance measure of the obtained results via two transformations. Section 5 aim to present some numerical examples to solidify the findings via varied transformations. Assessment and

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validation of the obtained results through step responses of the higher-order and obtained reduced-order systems is in section 6 along with the computed errors. The limitation found during the findings of the outcome from this article is discussed in section 7. Finally, the key difference between the two methods is offered in section 8 as conclusions with a possible future scope.

2. Literature survey

Systems with huge parameters are represented through finite differential equations, resulting in an overall higher order system transfer function. It then becomes anticipated for researchers to represent such models by an approximate model of lower order. Here, outburst the idea of order reduction, to estimate the original large-scale system by a smaller model, which recovers the former information numerically by integrating the model of reduced size. The reduced models also preserve fewer of the dynamic characteristic like time moments, markov parameters, and stability. Literature in Refs. [5–11] extensively provide numerous techniques for order reduction of non-interval, which now have advanced to interval systems.

With time span and easy access of digital signals and systems, discrete-time systems gained their consideration over analogue systems for control and analysis. Since, the study and analysis of such systems of higher order in their raw form is uneasy; they also demand their order reduction for easy access. Order reduction techniques available in literature showcase the advancement of fewer order reduction techniques in continuous-time domain directly to the discrete-time domain, both for non-interval and interval systems. Such type of algorithms includes Pade approximation, balanced truncation, direct truncation, aggregation. Apart from these, an algorithm like Routh approximation in the continuous-time domain is not applicable to discrete-time domain directly. A vast range of algorithms based on Routh approximation is consisted in Ref. [12]. This call for an appropriate discretization technique of the discrete-time systems to continuous-time and vice-versa with an ease of computation. Precisely, the transformation modifies the discrete-time system to continuous-time system; order reduction methodology applied, and finally, an appropriate inverse transformation results in the desired reduced model in discrete-time domain.

From the literature available, transformation techniques used widely for either discrete-time non-interval or interval systems are namely Tustin or bilinear or trapezoidal method [13–21] and Eulers Forward differentiation method [22–25]. In recent literature, Ruchira [26] employed bilinear transformation and Potturu and Prasad [27], used linear transformation. In 2020, Deveerasetty and Nagar [28] proposed reduction of discrete-time interval systems using both linear and bilinear transformations.

Available library of literature represent the possible dilemma of the researchers in terms of which transformation technique to be employed for better outcome. The present article might be a possible solution to such researchers.

3. Transformation techniques

Conversion from discrete-time to continuous-time domain is important in a manner to apply the continuous-time algorithms to discrete-time systems. As stated in introduction, there are many transformation techniques but only two of them, namely, Tustin and Forward difference transformation techniques are considered in this paper, as these are applied widely for order reduction of discrete-time systems as survey in section 2. Here, they have been chosen to establish their grandness in the area of discrete-time interval system, stating a significant difference among each other. Short discussion about both the techniques is offered in this section for their better understanding [29]. Their integral approximations can be seen in Fig. 1 for bilinear or Tustin or trapezoidal transformation (w-domain) and Fig. 2 for forward difference respectively.

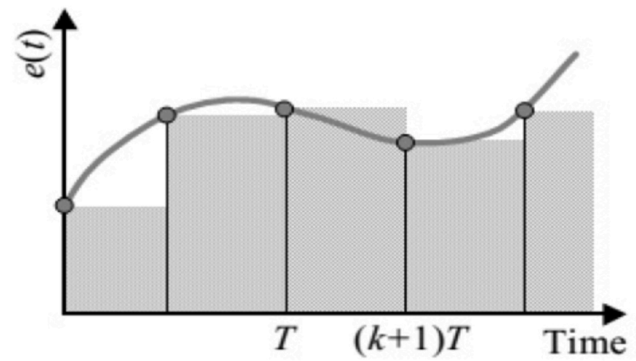

Fig. 1. Bilinear or Tustin or trapezoidal transformation (w-domain).

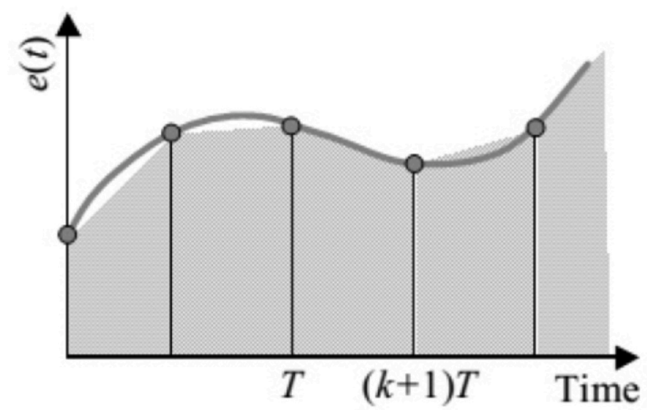

Fig. 2. Forward difference or linear transformation (p-domain).

3.1. Bilinear or tustin or trapezoidal transformation (w-domain)

In z-plane, the frequency appears as $z = e^{j\omega t}$, and its response loses the simplicity of logarithmic plots. It is to be noted that the z-transformation maps the primary and complementary strips of the left of the s-plane into the unit circle in the z-plane. Thus conventional frequency response methods, do not apply to the z-plane. To overcome this difficulty, the pulse transfer function in the z-plane is transformed to w-plane. The w-transformation states $z = \left(\frac{1+(T/2)w}{1-(T/2)w}\right)$, where T is the sampling period. The inverse transformation is $w = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$.

Through the z-transformation and the w-transformation, the primary strip of the left half of the s-plane is first mapped inside the unit circle in the z-plane and then mapped to the entire left half of the w-plane. The origin in the z-plane maps to the point $w = -\frac{2}{T}$ in the w-plane.

As s varies from $0 \to j\frac{\omega_s}{2}$ along $j\omega$ axis, z varies from 1 to -1 along the unit circle in z-plane, and w varies from 0 to 1 along the imaginary axis in the w plane. The difference between the s-plane and w-plane is that the frequency range $-\frac{1}{2}\omega_s \leq \omega \leq \frac{1}{2}\omega_s$ in the s-plane maps to the range in the w-plane, where ν is the fictitious frequency. Thus, there is a compression of the frequency scale. Although, the w-plane resembles the s-plane geometrically, the frequency axis in the w-plane is distorted.

3.2. Forward difference or linear transformation (p-domain)

Tustin transformation had fever difficulties in its application to filter design, thus calling for matched z-transformation where $z = e^{pt}$. This was successfully implemented and used in digital control systems. It's a simpler version where $z = 1 + p$ and is prevailed by employing forward Euler rule to the matched z-transform equality and retaining the first two terms in the resultant expansion. Such transformations are of special significance in the design of audio and telephone networks.

3.3. Comparison between the techniques based on stability

In terms of stability transformation from z-to-w or p-domain can be better understood by the figures below. Fig. 3 and Fig. 4, illustrates stability region Re*(s)* < *0* mapped on the complex z-plane for the forward difference and trapezoidal approximation respectively. By using forward difference approximation, the stability region (*LHP*) is mapped to the half-plane to the left of 1 on the complex z-plane. Thus, with forward difference approximation, it is possible that a stable discrete-time controller will be approximated by an unstable continuous-time controller.

The bilinear transformation (trapezoidal or Tustin approximation) maps the left half s-plane into the unit disc. Hence, stable discrete (continuous) controllers are approximated by stable continuous (discrete) controllers and unstable continuous (discrete) controllers are mapped to unstable discrete (continuous) controllers. In practice, the Tustins approximation (bilinear transformation) is the approximation of choice for converting continuous-time (discrete) controllers to discrete-time (continuous) controllers.

4. Preliminaries

This section is divided under two subheadings to understand the discovery of the paper through *a*) Reduction methodology applied for the approximation and *b*) the Performance Analysis of the obtained systems on the basis of error computation and step response.

4.1. Reduction methodology

Till date, there are numerous order reduction methodologies available both in continuous-time as well as discrete-time domain ranging from non-interval systems to interval systems. Any of the prevailing reduction algorithms can be chosen for deriving the reduced models.

The methodology considered here is available from the literature as *Gamma-Delta approximation* [13]. The desires of the reductions methodologies to be depicted below;

Consider a higher order system (1) whose equivalent model of reduced dimension (2) is to be derived, where $k < n$.

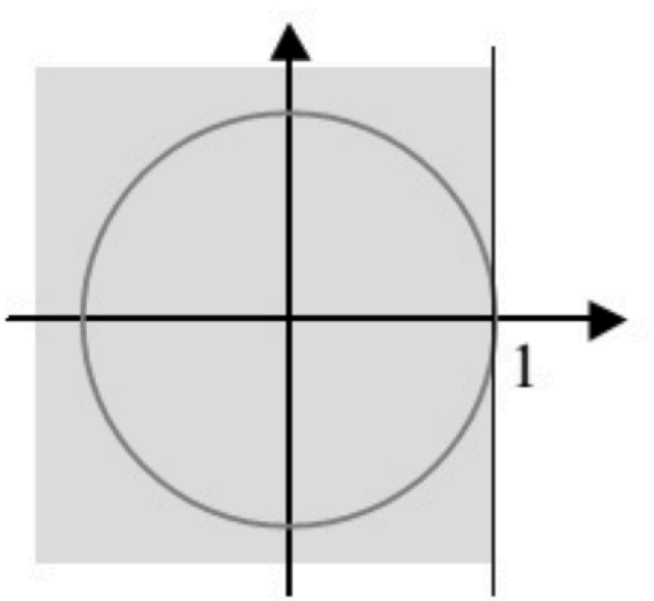

Fig. 3. Forward difference or linear transformation (p-domain).

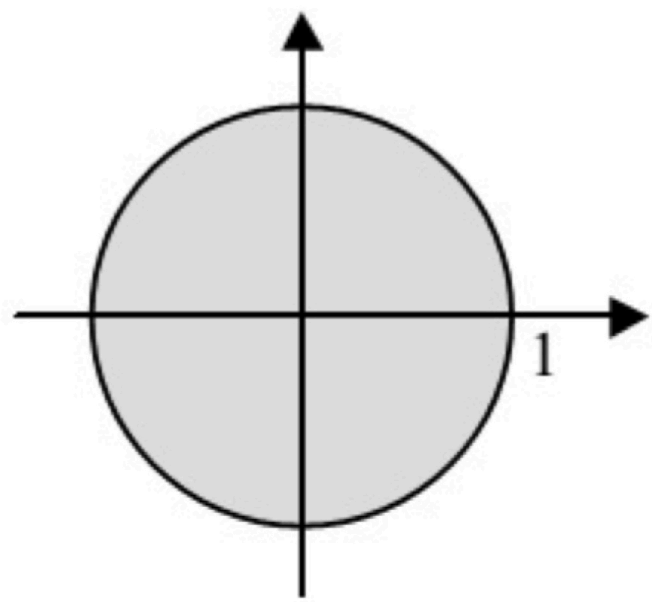

Fig. 4. Bilinear or Tustin or trapezoidal transformation (w-domain).

$$T_n(z) = \frac{C_n(z)}{D_n(z)} \tag{1}$$

where $C_n(z) = [C_1^-, C_1^+]z^{n-1} + [C_2^-, C_2^+]z^{n-2} + \dots + [C_n^-, C_n^+]$

$D_n(z) = [D_0^-, D_0^+]z^n + [D_1^-, D_1^+]z^{n-1} + \dots + [D_n^-, D_n^+]$

$$R_k(z) = \frac{C_k(z)}{D_k(z)} \tag{2}$$

where $C_k(z) = [c_1^-, c_1^+]z^{k-1} + [c_2^-, c_2^+]z^{k-2} + \dots + [c_k^-, c_k^+]$

$D_k(z) = [d_0^-, d_0^+]z^k + [d_1^-, d_1^+]z^{k-1} + \dots + [d_k^-, d_k^+]$

Furthermore, the algorithmic rules illustrated in the flowchart in Fig. 5, pose all the similarities step by step, except the transformations techniques, $z = \left(\frac{1+w}{1-w}\right)$ in w-domain and $z = 1 + p$ in p-domain and their respective inversions.

As depicted, the essentials of the higher-order systems and reduced-order models are understood, an illustrative explanation of the employed approximation methodology is performed here for both the transformation techniques.

4.1.1. Bilinear transformation (w-domain)

The mentioned transformation on (1) results the higher-order interval system as

$$T_n(w) = \frac{N_n(w)}{D_n(w)} \tag{3}$$

where $N_n(w) = [n_0^-, n_0^+]w^n + [n_1^-, n_1^+]w^{n-1} + \dots + [n_n^-, n_n^+]$

$D_n(w) = [d_0^-, d_0^+]w^n + [d_1^-, d_1^+]w^{n-1} + \dots + [d_n^-, d_n^+]$

Now, tabulate the first two rows of Tables 1 and 2 using the $D_n(w)$ and $N_n(w)$ respectively of $T_n(w)$.

The coefficients from third rows of Tables 1 and 2 are computed by the Routh algorithm with $i = 2; 3$ and $j = 0; 1; 2$.

$$[d_{i,j}^-, d_{i,j}^+] = [d_{i-2,j+1}^-, d_{i-2,j+1}^+] - \frac{[d_{i-2,0}^-, d_{i-2,0}^+][d_{i-1,j+1}^-, d_{i-1,j+1}^+]}{[d_{i-1,0}^-, d_{i-1,0}^+]} \tag{4}$$

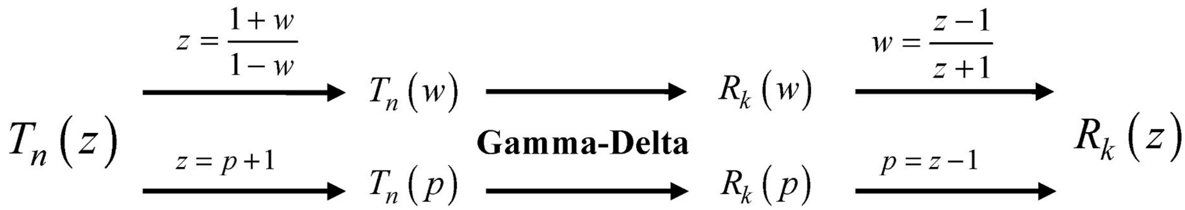

Fig. 5. Flowchart of the algorithmic rules of Gamma-Delta Approximation.

Table 1
Routh table for Denominator.

$[d_n^-, d_n^+] = [d_{0,0}^-, d_{0,0}^+]$	$[d_{n-2}^-, d_{n-2}^+] = [d_{0,1}^-, d_{0,1}^+]$	$[d_{n-4}^-, d_{n-4}^+] = [d_{0,2}^-, d_{0,2}^+]$
$[d_{n-1}^-, d_{n-1}^+] = [d_{1,0}^-, d_{1,0}^+]$	$[d_{n-3}^-, d_{n-3}^+] = [d_{1,1}^-, d_{1,1}^+]$	$[d_{n-5}^-, d_{n-5}^+] = [d_{1,2}^-, d_{1,2}^+]$
…		
$[d_{n-1,0}^-, d_{n-1,0}^+]$		
$[d_{n,0}^-, d_{n,0}^+]$		

Table 2
Routh table for Numerator.

$[n_n^-, n_n^+] = [n_{1,0}^-, n_{1,0}^+]$	$[n_{n-2}^-, n_{n-2}^+] = [n_{1,1}^-, n_{1,1}^+]$	$[n_{n-4}^-, n_{n-4}^+] = [n_{1,2}^-, n_{1,2}^+]$
$[n_{n-1}^-, n_{n-1}^+] = [n_{2,0}^-, n_{2,0}^+]$	$[n_{n-3}^-, n_{n-3}^+] = [n_{2,1}^-, n_{2,1}^+]$	$[n_{n-5}^-, n_{n-5}^+] = [n_{2,2}^-, n_{2,2}^+]$
…		
$[n_{n-1,0}^-, n_{n-1,0}^+]$		
$[n_{n,0}^-, n_{n,0}^+]$		

$$\left[n_{i,j}^-, n_{i,j}^+\right] = \left[n_{i-2,j+1}^-, n_{i-2,j+1}^+\right] - \frac{\left[n_{i-2,0}^-, n_{i-2,0}^+\right]\left[d_{i-2,j+1}^-, d_{i-2,j+1}^+\right]}{\left[d_{i-2,0}^-, d_{i-2,0}^+\right]} \tag{5}$$

The completion of the two tables formulate towards the desired kth order reduced model which is obtained by retaining the $\gamma-\delta$ parameters of first k rows of Tables 1 and 2

The desired interval parameters $\gamma' s$ and $\delta' s$ are obtained from Tables 1 and 2 respectively as

$$\left[\gamma_k^-, \gamma_k^+\right] = \frac{\left[d_{k-1,0}^-, d_{k-1,0}^+\right]}{\left[d_{k,0}^-, d_{k,0}^+\right]} \text{ and } \left[\delta_k^-, \delta_k^+\right] = \frac{\left[n_{k,0}^-, n_{k,0}^+\right]}{\left[d_{k,0}^-, d_{k,0}^+\right]} \tag{6}$$

with $k = 1, 2, 3, \ldots$

On appropriate substitution of the computed $\gamma - \delta$ parameters, the reduced model is observed as

$$R_k(w) = \frac{N_k(w)}{D_k(w)} \tag{7}$$

with

$$D_k(w) = w^2 D_{k-2}(w) + \left[\gamma_k^-, \gamma_k^+\right] D_{k-1}(w) \tag{8}$$

$$N_k(w) = \left[\delta_k^-, \delta_k^+\right] w^{k-1} + w^2 N_{k-2}(w) + \left[\gamma_k^-, \gamma_k^+\right] N_{k-1}(w) \tag{9}$$

where $D_{-1}(w) = \frac{1}{w}$, $D_0(w) = 1$, $N_{-1}(w) = 0$, $N_0(w) = 0$.

Considering equations (7)–(9), the first and second order reduced models obtained are

$$R_1(w) = \frac{\left[\delta_1^-, \delta_1^+\right]}{w + \left[\gamma_1^-, \gamma_1^+\right]} \tag{10}$$

$$R_2(w) = \frac{\left[\delta_2^-, \delta_2^+\right] w + \left[\gamma_2^-, \gamma_2^+\right]\left[\delta_1^-, \delta_1^+\right]}{w^2 + \left[\gamma_2^-, \gamma_2^+\right] w + \left[\gamma_2^-, \gamma_2^+\right]\left[\gamma_1^-, \gamma_1^+\right]} \tag{11}$$

The computed reduced model in w-domain, is reversed back to its equivalent z-domain by enforcing inverse Tustin transformation $w = \left(\frac{z-1}{z+1}\right)$.

4.1.2. Euler forward difference transformation (p-domain)

For analysis through this transformation, replace z in the higher order transfer function (1) by $z = 1 + p$ and proceed through algorithmic steps from (3) to (11) and apply inverse transformation $p = z - 1$ to get the system in z-domain.

4.2. Performance Analysis

Gamma-Delta approximation, is used to retain the dynamic characteristic of the reduced model comparable to higher order system. Step response, a common analysis tool and Integral Square Error, most practiced performance measure (both available in literature) is used to validate the obtained reduced models after the two transformations.

Since, the paper deals with discrete-time interval systems, performance measure J is modified as weighted error sum over a fixed interval of time, determined by the error between the transient responses of the higher order system, and the lower order system, expressed as;

$$J = \sum_{k=0}^{\infty} \left[y_n(k) - y_k(k)\right]^2 \tag{12}$$

where $y_n(k)$ and $y_k(k)$ are the step responses of the higher order $T_n(z)$ and reduced order system $R_k(z)$ respectively.

The obtained reduced system is guaranteed to be approximate when J is minimum. For this analysis, the higher and reduced systems are considered as, *1*) Transfer function with only lower limits and *2*) Transfer function with only upper limits. Thereafter, J is computed independently for the two transfer functions for observation under the error column for lower limit and upper limit as shown in the tables in section 6. The stated section also offers the step responses of the higher and reduced order interval systems.

5. Experimental results

Two examples with the same reduction methodology, but varied in the realization of transformation are provided here to solidify the observations made in the previous sections. This would help in understanding the difference between the two techniques.

Example 1: Consider the third order interval system as

$$T_3(z) = \frac{[3.25, 3.35]z^2 + [3.5, 3.65]z + [2.8, 3]}{[5.4, 5.5]z^3 + [1, 1.1]z^2 + [1.5, 1.6]z + [2.1, 2.15]} \tag{13}$$

(a) The Tustin transformation (w-domain) $z = \left(\frac{1+w}{1-w}\right)$ leads to

$$T_3(w) = \frac{[-2.85, -2.4]w^3 + [1.3, 2.35]w^2 + [-9.05, -8.9]w + [9.55, 10]}{[3.65, 4]w^3 + [19.8, 20.45]w^2 + [9.15, 9.8]w + [10, 10.35]} \tag{14}$$

Interval parameters $\gamma' s$ and$\delta' s$, are computed as

$$\left[\gamma_1^-, \gamma_1^+\right] = [1.02, 1.13]\ ;\ \left[\gamma_2^-, \gamma_2^+\right] = [0.51, 0.69]$$

$$\left[\delta_1^-, \delta_1^+\right] = [0.97, 1.09]\ ;\ \left[\delta_2^-, \delta_2^+\right] = [-0.63, -0.49]$$

(b) The forward difference or linear transformation (p-domain) $z = 1 + p$ transforms (13) to (15) and the respective $\gamma's$ and$\delta's$, parameters as

$$T_3(p) = \frac{[3.25, 3.35]p^2 + [10, 10.35]p + [9.55, 10]}{[5.4, 5.5]p^3 + [17.2, 17.6]p^2 + [19.7, 20.3]p + [10, 10.35]} \quad (15)$$

$$[\gamma_1^-, \gamma_1^+] = [0.49, 0.53] \; ; \; [\gamma_2^-, \gamma_2^+] = [1.12, 1.18]$$

$$[\delta_1^-, \delta_1^+] = [0.47, 0.51] \; ; \; [\delta_2^-, \delta_2^+] = [0.57, 0.60]$$

On substitution of the above obtained parameters in (11), results in simplified z-domain model with varied transformations as

$$R_{2w}(z) = \frac{[-0.137, 0.244]z^2 + [0.85, 1.107]z + [0.995, 1.376]}{[2.032, 2.464]z^2 + [-0.958, -0.447]z + [0.834, 1.266]} \quad (16)$$

$$R_{2p}(z) = \frac{[0.568, 0.601]z + [-0.475, -0.402]}{z^2 + [-0.881, -0.820]z + [0.550, 0.619]} \quad (17)$$

Example 2: Consider an eighth order interval real-time system as

$$T_8(z) = \frac{C_7(z)}{D_8(z)} \quad (18)$$

where

$$C_7(z) = [1.6484, 1.7156]z^7 + [1.0937, 1.1383]z^6 + [-0.2142, -0.2058]z^5 + [0.1490, 0.1550]z^4 + [-0.5263, -0.5057]z^3 + [-0.2672, -0.2568]z^2 + [0.0431, 0.0449]z + [-0.0061, -0.0059]$$

$$D_8(z) = [23.52, 24.48]z^8 + [-1.7156, -1.6484]z^7 + [-1.1383, -1.0937]z^6 + [0.2058, 0.2142]z^5 + [-0.1550, -0.1490]z^4 + [0.5057, 0.5263]z^3 + [0.2568, 0.2672]z^2 + [-0.0449, -0.0431,]z + [0.0059, 0.0061]$$

Upon respective transformation, the $\gamma's$ and$\delta's$, parameters are in Table 3. Further, using the computed parameters, the reduced order models in respective domains after inverse transformation into z-domain are obtained as

$$R_{2w}(z) = \frac{[0.0332, 0.0400]z^2 + [0.0070, 0.0102]z + [-0.0345, -0.0246]}{[1.3645, 1.4733]z^2 + [-1.9224, -1.8902]z + [0.6204, 0.7292]} \quad (19)$$

$$R_{2p}(z) = \frac{[0.0302, 0.0394]z + [-0.0358, -0.0250]}{z^2 + [-1.6665, -1.5683]z + [0.6080, 0.7232]} \quad (20)$$

6. Assessment and validation

This section attempts to assess the transformation techniques based on the computation of errors and step responses obtained between the higher-order and reduced lower-order models obtained through the different transformation techniques. Errors obtained for the Examples 1 and 2 from section 5 are shown in Tables 4 and 5 correspondingly.

Table 3
w-Domain and p-Domain $\gamma - \delta$ Parameters For Example 2.

parameters	w-domain	p-domain
$[\gamma_1^-, \gamma_1^+]$	[0.1190, 0.1313]	[0.1189, 0.1314]
$[\gamma_2^-, \gamma_2^+]$	[0.3257, 0.4184]	[0.3335, 0.4317]
$[\delta_1^-, \delta_1^+]$	[0.0107, 0.0121]	[0.0107, 0.0121]
$[\delta_2^-, \delta_2^+]$	[0.0297, 0.0380]	[0.0302, 0.0394]

Table 4
Error for 2nd order for example 1.

Transformation	Error	
	Lower Limit	Upper Limit
w-domain	0.0822	0.0116
p-domain	0.0011	0.0441

Table 5
Error for 2nd order for example 2.

Transformation	Error	
	Lower Limit	Upper Limit
w-domain	0.1158	0.1352
p-domain	0.0016	0.1724

The step responses of the reduced models and higher order systems for Example 1 and 2 are shown in Figs. 6 and 7 respectively. The heavy solid line represents the response of the higher-order system and the other dotted lines depict the responses of the reduced models through both transformation techniques respectively.

From Figs. 6 and 7, it can observed that the step responses of the higher order and reduced order systems for both the examples via two transformations are almost identical. Similarly, the errors in Tables 4 and 5 respectively are comparatively minimum and can be considered accordingly. Thus, the above results and thorough observations of the methodologies, evoke that p-domain discretization technique is quite simple and easy to transform, making it preferable over the w-domain techniques to produce similar reduced models.

Briefly, the validation of the obtained reduced models is performed on the basis of minimum error computation and approximate tracking of the step responses.

7. Limitation

A feasible query developed or the limitation about the offered assessment is its relevance for comparison of the performance based on the model reduction as the result may depend on the reduction techniques as well as the numerical examples. The answer is; yes, they are a major factor to be considered but on a large scale of simplification, the linear transformation can be applied directly to obtain an acceptable result.

8. Conclusions

The analysis of the two transformation techniques having their own advantages and disadvantages is outlined individually in this paper. The main goal to examine which of them supplies more convenient discretization with ease is performed successfully. It is found in general that, forward difference achieves the advantages of being easy and simple at every step. The main reason is its form of linearity *i.e.* $z = 1 + p$ instead of using rational form as $z = \left(\frac{1+w}{1-w}\right)$. These observations indicate that any of the two techniques can be used; as both of them result in almost equivalent reduced forms but for convenience and ease of computation, p-domain proves to be superior. The achieved results are found a basis for further work in the area of discrete-time to continuous-time transformation and vice-versa. Overall, the conclusion is that the

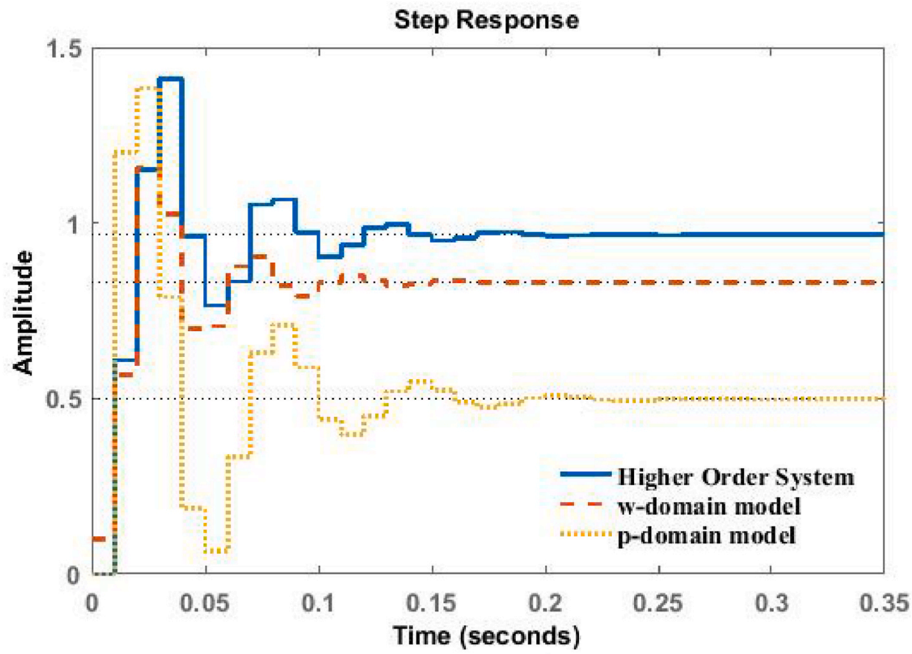

Fig. 6. Step response for 2nd system for Example 1.

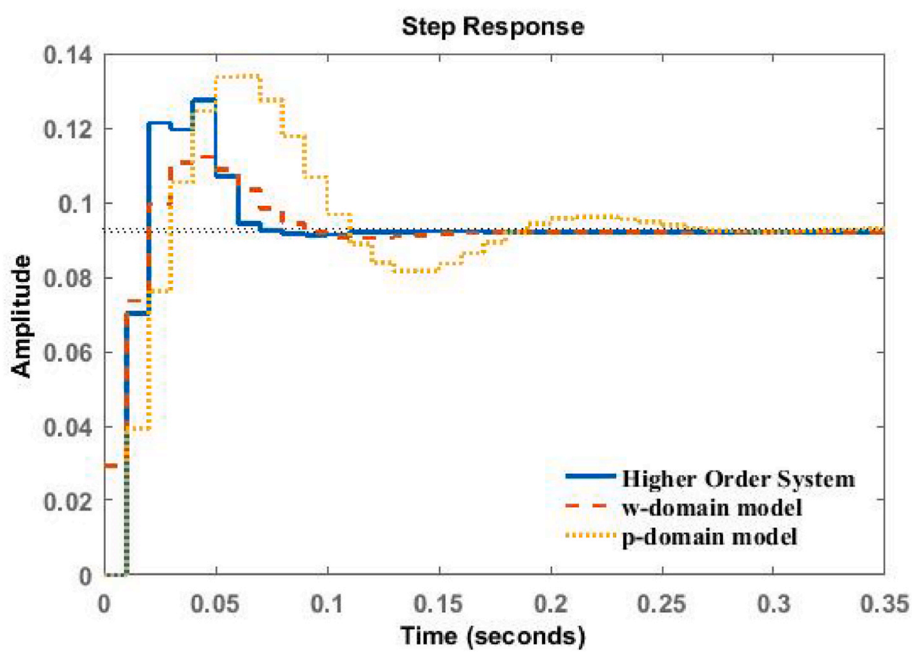

Fig. 7. Step response for 2nd system for Example 2.

latter technique is much simpler, matches as many moments as the former one does and hence appears to be better than the former technique. Lastly, the attempt made in this paper is believed to be helpful to the researchers working on higher order systems. A future work developed from this presented discussion is its practical approach on physical system models.

Declaration of interest statement

Authors have no conflicts of interest to declare.

Appendix

Intervals $a = [a^-, a^+] = \{a \in : a^- \leq a \leq a^+\}$

$b = [b^-, b^+] = \{b \in : b^- \leq b \leq b^+\}$

Lower Limits of interval systems a^-, b^-
Upper Limits of interval systems a^+, b^+
Arithmetic operations where $\odot \in \{+, -, \times, \div\}$ $[a] \odot [b] = \{a \odot b / a \in [a], b \in [b]\}$
End point formulas for arithmetic operations

$a + b = [a^- + b^-, a^+ + b^+]$

$a - b = [a^- - b^+, a^+ - b^-]$

$a \times b = [\min C, \max C], C = [a^- b^-, a^- b^+, a^+ b^-, a^+ b^+]$

$a / b = a \times (1 / b); 1 / b = [1 / b^+, 1 / b^-], 0 \notin b$

Order of higher order systems n
Higher-order interval transfer function $T_n(z)$
Higher-order numerator interval polynomial $C_n(z)$
Higher-order denominator interval polynomial $D_n(z)$
Higher order interval transfer function in w-domain $T_n(w)$
Higher order interval transfer function in p-domain $T_n(p)$
Order of reduced order systems k
Reduced - order interval transfer function $R_k(z)$
Reduced -order numerator interval polynomial $C_k(z)$
Reduced -order denominator interval polynomial $D_k(z)$
Reduced order interval transfer function in w-domain $R_k(w)$
Higher order interval transfer function in p-domain $R_k(p)$
Modified weighted error sum J
Step response of higher order interval transfer function $T_n(z)$ $y_n(k)$
Step response of reduced order interval transfer function $R_k(z)$ $y_k(k)$
Second order reduced model via w-domain $R_{2w}(z)$
Second order reduced model via p-domain $R_{2p}(z)$

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