Encoding First Order Proofs in SMT

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Abstract

We present a method for encoding first order proofs in SMT. Our implementation, called
ChewTPTP-SMT, transforms a set of first order clauses into a propositional encoding (modulo
theories) of the existence of a rigid first order connection tableau and the satisfiability of unifica-
tion constraints, which is then fed to Yices. For the unification constraints, terms are represented
as recursive datatypes, and unification constraints are equations on terms. The finiteness of the
tableau is encoded by linear real arithmetic inequalities.
We compare our implementation with our previous implementation ChewTPTP-SAT, encoding
rigid connection tableau in SAT, and show that for Horn clauses many fewer propositional clauses
are generated by ChewTPTP-SMT, and ChewTPTP-SMT is much faster than ChewTPTP-SAT.
This is not the case for our non-Horn clause encoding. We explain this, and we conjecture a rule
of thumb on when to use theories in encoding a problem.

Keywords: SMT, first-order, tableau, Yices

1   Introduction

Recent techniques in SAT solving have resulted in extremely fast proce-
dures for solving propositional satisfiability problems[8], based on the DPLL
method[4]. As an application of these techniques, we have developed an au-
tomated theorem prover called ChewTPTP-SAT[6], which encodes rigid first

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order theorem proving problems as SAT problems, and solves those SAT problems using Minisat[8].

Rigid unsatisfiability has been studied as early as [3,1]. A set of first order clauses is rigidly unsatisfiable if and only if there exists a closed rigid connection tableau for that set of clauses[10]. Our encoding uses this fact and solves the satisfiability of a set of rigid clauses by encoding the existence of a rigid connection tableau in SAT.

A set of Horn clauses is encoded by creating propositional clauses representing the following requirements of a tableau T: (1) The root of the tableau must be a clause with only negative literals. (2) If a clause is in the tableau, then all its negative literals are in the tableau. (3) If a negative literal is in the tableau, then it must be extended by some clause. (4) If a negative literal $\neg A$ is extended by a clause C, then A must unify with the positive literal in C. (5) All unifications must be consistent with each other. (6) The tableau must be finite, i.e., there is no cycle.

For connection tableaux for non-Horn clauses, literals are either extended or complementary to an ancestor literal in its branch. For efficiency reasons, we choose to encode a clause tableau as a DAG. So there may be many branches from the root to a node. Therefore, we encode the fact that each literal L in the tableau must either be extended or all paths from the root to that node must contain a literal complementary to L. A tableau may have the same clause on different branches, and those branches may be closed with different literals. Therefore, we may have to add more instances of clauses to find a closed tableau. This cannot be avoided, since rigid Horn clause satisfiability is NP-complete, but rigid non-Horn clause satisfiability is Σ_2^p-complete[9]. However, because of the DAG structure, we can often encode many instances of a clause with just one instance.

Since we encode rigid proofs, the proof of unsatisfiability of a set of clauses may require repeating the encoding with fresh variants of each clause. However, there are also applications which really require rigid proofs[5].

Our original ChewTPTP-SAT implementation[6] performed well on some problems, but some of the encodings created huge sets of clauses. Some parts of our encoding represented choices made, such as which clause to extend each literal with. But other parts of our encoding represented deterministic procedures, such as deciding the consistency of unification constraints and deciding the acyclicity of the DAG, which verifies that a particular property holds of the DAG. Furthermore, in experimental results with Horn clauses, approximately 99% of the clauses generated were encoding the deterministic procedures, and only about 1% represented the choices. We had an eager encoding of unification and acyclicity. We decided the implementation would be more efficient if

unification and acyclicity were encoded lazily and implemented these changes in ChewTPTP-SMT. It makes sense to expresses choices involved in building the tableau using SAT, and verifcation of unification and acyclicity using underlying theories. Therefore, we chose to encode our problem as Satisfiability modulo Theories[12], and we replaced Minisat[8] with Yices[7].

Yices has a theory for recursive datatypes, which can be used to represent terms. A term can be defined by using function symbols as constructors. Each function symbol of arity n is a constructor with n arguments. Constants are constructors with no arguments. Predicate symbols are viewed the same as function symbols. Variables are instances of terms. Then unification is represented as equality of terms. We represent acyclicity using linear arithmetic. Consider a graph $G = (V, E)$. If an edge (u, v) exists in E, then we assert an inequality $x_u < x_v$ for some real numbers x_u and x_v. Then G is acyclic if and only if the set of inequalities is consistent.

In this paper, we describe our implementation of ChewTPTP-SMT, and compare our results with ChewTPTP-SAT. We show that in the Horn encoding, ChewTPTP-SMT produces far fewer clauses than ChewTPTP-SAT. The time needed to decide the satisfiability is also drastically reduced. This is not the case for non-Horn clauses. We explain why this is the case and give a rule of thumb for when theories should be used for encoding.

2   Clausal Tableau

See [2] for a detailed description of first order logic and a background discussion on the validity of a first order logic formula.

We use the following definition of tableau [10].

Definition 2.1 *Clausal tableaux are trees with nodes labeled with literals and branches labeled either open or closed. Clausal tableaux are inductively defined as follows. Let $S = \{C_1...C_n\}$ be a set of clauses. If T is a tree consisting of a single unlabeled node N then T is a clausal tableau for S. The branch consisting of only the root node N is open. If N is a leaf node on an open branch B in the tableaux T for S and one of the following inference rules are applied to T then the resulting tree is a clausal tableaux for S.*

(Expansion rule) Let C_k be a clause in S. Replace each variable in C_k with a new variable not appearing in T. Suppose $L_{k1} \lor ... L_{ki}$ is the resulting clause. Construct a new tableaux T' by adding i nodes as children of N and labeling them L_{k1} through L_{ki}. Label each of the i branches open. T' is a clausal tableaux for S.

(Closure rule) Suppose L_{ij} is the literal at N and for some predecessor node with literal L_{pq} such that L_{ij} and $\neg L_{pq}$ are unifiable. Construct T' from

T by applying the unifier to T and labeling the branch containing L_{ij} as closed. T' is a clausal tableaux for S.

A clause which is added to the root node is called the *start clause* and we say that a clause is *in* a tableaux if the clause was used in an application of the expansion rule.

Definition 2.2 *A clausal tableaux is* tightly connected *if each clause (except the start clause) in the tableaux contains some literal which is unifiable with the negation of its predecessor.*

Connected clausal tableaux use an additional rule called *extension rule.*

Definition 2.3 (Extension Rule) *Let N be a node in the tableau T and let C_k be a clause in S such that there exists a literal L_{ik} in C_k which is unifiable with the negation of N. Apply the expansion rule with C_k and immediately apply the closure rule with L_{ik}.*

Definition 2.4 *The calculus for connection tableaux consists of the expansion rule (for the start clause only), the closure rule, and the extension rule.*

We call a tableau closed if each leaf node has been closed by an application of the closure rule. By [11] we can require that the start clause is a negative clause since there exists a negative clause in any minimally unsatisfiable set.

2.1 Rigid Unsatisfiability

The main problem in Automated Theorem Proving is to determine if a set of hypotheses implies a conclusion, or equivalently that a formula F is unsatisfiable. We will assume that F is in CNF. The problem of rigid unsatisfiability of F is to determine whether there exists a ground instance of F which is unsatisfiable. A rigid tableau is a tableau in which multiple instances of a clause appearing in the tableau are identical copies of the clause appearing in F. One result of Tableaux Theory is the completeness and soundness of closed connection tableaux.

Theorem 2.5 *There exists a closed connection (rigid) tableau for F iff F is (rigidly) unsatisfiable[10].*

3   Tableau Encoding

Our method to determine the rigid unsatisfiability of F generates a set S of propositional logic clauses modulo the theories of unification and arithmetic for F which encodes a rigid closed connection tableau for F and tests the satisfiability of S with a SMT solver.

We provide two encodings, the first for problems containing only Horn clauses and the second for those containing non-Horn clauses. Given F we enumerate each of the clauses in F and each of the literals in each clause. We denote clause i by C_i and denote the jth literal in clause i by L_{ij}. We denote A_{ij} to be the atom of L_{ij}. Therefore L_{ij} is either of the form A_{ij} or $\neg A_{ij}$.

3.1 Encoding for Horn Clauses

Let F be a set of first order logic formulas.

We define a set of propositional variables c_m, l_{mn}, e_{mnq}, disjoint from the symbols in F, as follows: Define $c_m = T$ iff C_m appears in the tableau. Define $l_{mn} = T$ iff L_{mn} is an internal node in the tableau. Define $e_{mnq} = T$ iff C_q is an extension of L_{mn}. For each pair of clauses C_i and C_j we define $x_i < x_j = T$ (where x_i and x_j do not exist in F) iff there exists a path from C_i to C_j. For each pair of atoms A_i and A_j in F, we define $(A_i = A_j) = T$ iff A_i and A_j are the two atoms involved in an application of the closure rule.

Below we list the set of clauses that we generate and provide their meaning.

At least one clause containing only negative literals appears in the tableau:

$$(1) \qquad \bigvee_{C_m \text{ is a negative clause}} c_m$$

If C_m appears in the tableau and L_{mn} is a negative literal then L_{mn} is an internal node in the tableau:

$$(2) \qquad c_m \Rightarrow l_{mn}$$

If L_{mn} is an internal node in the tableau then for some q_j, C_{q_j} is an extension of L_{mn}:

$$(3) \qquad l_{mn} \Rightarrow (e_{mnq_1} \vee ... \vee e_{mnq_k})$$

where $\{C_{q_1}...C_{q_k}\}$ represent the set of all clauses whose positive literals are unifiable with L_{mn}

If C_q is an extension of L_{mn} then C_q exists in the tableau:

$$(4) \qquad e_{mnq} \Rightarrow c_q$$

If C_q is an extension of L_{mn} and L_{qr} is the positive literal in C_q then A_{mn} and A_{qr} are unifiable:

$$(5) \qquad e_{mnq} \Rightarrow (A_{mn} = A_{qr})$$

If C_q is an extension of L_{mn} then there is a path from C_m to C_q:

$$(6) \qquad e_{mnq} \Rightarrow (x_m < x_q)$$

The encoding is satisfiable if and only if the original set of first order Horn clauses is rigidly unsatisfiable. We encode non-rigid unsatisfiability by continually adding new instances of each clause, renamed apart.

3.2 Encoding for Non-Horn Clauses

For non-Horn problems we use a different set of variables and generate a different set of clauses. Note: we say that two literals are *complementary* if they have opposite signs and their atoms are unifiable.

We define the variables, disjoint from the symbols in F, s_m, c_{mn}, l_{mn}, e_{mnqj}, o_{ijkl} and q_{mnij} as follows: Define $s_m = T$ iff C_m is the start clause. Define $c_{mn} = T$ iff C_m appears in the tableau and L_{mn} is complementary to its parent. Define $l_{mn} = T$ iff L_{mn} is a node in the tableau and is not a leaf node created by an application of the extension rule. Define $e_{mnqj} = T$ iff C_q is an extension of L_{mn} and L_{qj} is the complement of L_{mn}. Define $o_{ijkl} = T$ iff L_{ij} and L_{kl} are a pair of literals used in a closure but not by the extension rule. If a path to a node N contains the complement of N, then we say that the path is closed. Define $q_{mnij} = T$ iff L_{mn} is a leaf and L_{ij} is a node on a path from the root node to L_{mn} and every path from the root to L_{ij} contains a complement of L_{mn}. For each pair of clauses C_i and C_j we define $x_i < x_j = T$ (where x_i and x_j do not exist in F) iff there exists a path from C_i to C_j. For each pair of atoms A_i and A_j in F, we define $(A_i = A_j) = T$ iff A_i and A_j are the two atoms involved in an application of the closure rule.

The clauses are as follows.

There exists a start clause in the tableau which only contains negative literals:

$$(7) \qquad \bigvee_{s_m \text{ is a negative clause}} s_m$$

If C_m is the start clause in the tableau then each literal L_{mn} of C_m is in the tableau:

$$(8) \qquad s_m \Rightarrow l_{mn}$$

If C_i appears in the tableau and L_{ij} is the complement of a literal in its parent then all other literals of C_i are in the tableau:

$$(9) \qquad c_{ij} \Rightarrow l_{ik} \text{ where } j \neq k$$

If L_{ij} exists in the tableau and is not a leaf node created by an application of the closure rule then either every branch ending at L_{ij} is closed or there is an extension of L_{ij}:

$$(10) \qquad l_{ij} \Rightarrow (q_{ijij} \vee (\bigvee_{k,l} e_{ijkl}))$$

If L_{ij} is extended with C_k then C_k is in the tableau and some L_{kl} of C_k is the complement of L_{ij}:

$$(11) \qquad e_{ijkl} \Rightarrow c_{kl}$$

If clause C_m is an extension of L_{ij} and literals L_{ij} and L_{ml} are complements then A_{ij} and A_{ml} are unifiable.

$$(12) \qquad\qquad e_{ijml} \Rightarrow (A_{ij} = A_{ml})$$

If L_{ij} and L_{kl} are a pair used in a closure then they must be unifiable:

$$(13) \qquad\qquad o_{ijkl} \Rightarrow (A_{ij} = A_{kl})$$

If L_{ij} has the same sign as L_{kl} or their respective atoms are not unifiable then they are not complements:

$$(14) \qquad\qquad \neg o_{ijkl} \text{ where } L_{ij} \text{ and } L_{kl} \text{ are not unifiable}$$

If every path through L_{kl} to leaf L_{ij} is closed and C_k is an extension of L_{mn} then either L_{ij} is a complement of L_{mn} or every path through L_{mn} to L_{ij} is closed:

$$(15) \qquad\qquad q_{ijkl} \Rightarrow (e_{mnkp} \Rightarrow (o_{ijmn} \vee q_{ijmn}))$$

If C_k is an extension of L_{ij} then there is a path from clause C_i to clause C_k:

$$(16) \qquad\qquad e_{ijkl} \Rightarrow (x_i < x_k)$$

If C_i is the start clause then there are no inferences into any of the literals in C_i:

$$(17) \qquad\qquad s_i \Rightarrow \neg e_{klij}$$

If C_i is the start clause, L_{mn} is a leaf, and all paths that traverse L_{ij} to L_{mn} are closed, then L_{ij} and L_{mn} are complementary:

$$(18) \qquad\qquad s_i \Rightarrow (q_{mnij} \Rightarrow o_{mnij})$$

We represent our tableau as a DAG, so there is some structure sharing. But even with the structure sharing, a non-Horn clause tableau may need more than one instance of the same clause. Rigid unsatisfiability could be determined by continually adding identical instances of a clause. Non-Horn encoding could also be extended to the non-rigid case in the same way as the Horn encoding.

4 Implementation and Experimental Results

We have implemented our tableau encoding in our theorem prover ChewTPTP-SMT, which is an extension of ChewTPTP-SAT[6]. In ChewTPTP-SAT, instead of using theories, we encoded the consistency of the unifiers and the acyclicity of the tableau with additional propositional clauses. To encode the consistency of the unifiers, we encoded the equations that would be created if a unification algorithm was run. We do not know

ahead of time which unifiers we will have to create, so we encode everything that can possibly occur when the unification algorithm is run. To encode the absence of a cycle, we encode the existence of a path from one clause to another and the fact that there is no path from a clause to itself. This requires encoding all possible transitivity and irreflexivity axioms that may occur.

Our implementation allows the user to decide whether ChewTPTP encodes the problem as a SAT problem or an SMT problem. If the user chooses SMT, our implementation uses Yices to test the satisfiability of the encoding. If the user chooses SAT, then the user can also choose whether to test the satisfiability using Yices or Minisat, with a DIMACS encoding of SAT.

We tested our prover in all three settings on a subset of TPTP[13] problems. Tables 1-4 provide empirical data from these tests.

SMT-Y denotes our prover run in SMT mode, SAT-Y is SAT mode using Yices, and SAT-M is SAT mode using Minisat. For Horn clauses, we ran ChewTPTP on all the Horn problems in the TPTP database, but for non-Horn we only had time to run it through the GRP problems. We report all problems that both provers solved within five minutes but SAT-M took greater than one second. We believe the problems in these tables are representative of the overall results. Columns in the table show the running time of each method, the clause generation time rounded off to the nearest second, the number of clauses generated, and the number of variables generated for each method. We also show whether or not the problem is rigidly satisfiable. For these experiments, we only tested rigid satisfiability with one instance of each clause.

We wanted to see if working modulo theories would improve the performance of ChewTPTP. In the Horn case the running time was reduced significantly, except for a small percentage of exceptions. In the non-Horn case, working modulo theories often increased the running time. Generally, Yices was faster than Minisat on SAT problems without theories.

We believe we have an explanation for our results. In the Horn problems the number of clauses is reduced by an order of magnitude, whereas in the non-Horn problems the number of clauses is not reduced by much. This implies that working modulo theories is only useful when the clauses size is reduced significantly.

In the Horn encoding, everything can be encoded in $O(n^2)$ except for the encoding of unification and acyclicity, which require $O(n^3)$ space. When we remove the clauses used to represent unification and acyclicity, the number of clauses is now $O(n^2)$. However, for the encoding of non-Horn clauses, we must encode the fact of a leaf node having a complementary literal as an ancestor. This encoding is $O(n^3)$. We do not know how to encode this using

Table 1
ChewTPTP Times For Horn Problems

Name	SAT-M/Y Clause Gen	SMT-Y Clause Gen	SAT-M Total	SAT-Y Total	SMT-Y Total
PUZ008-1.p	1	0	1.06	0.89	0.11
NLP106-1.p	2	0	1.8	1.9	0.06
NLP104-1.p	2	0	1.82	1.9	0.05
NLP105-1.p	2	0	1.83	1.89	0.06
NLP107-1.p	2	0	2.47	1.99	0.06
GRP033-3.p	1	0	2.48	1.8	0.28
NLP109-1.p	1	0	2.49	1.99	0.05
NLP113-1.p	2	0	2.51	2.01	0.06
NLP110-1.p	2	0	2.74	1.84	0.07
NLP112-1.p	2	0	2.92	1.92	0.07
NLP111-1.p	1	0	2.94	1.93	0.06
NLP108-1.p	2	0	2.94	1.94	0.07
PUZ036-1.005.p	3	0	4.33	2.92	0.03
RNG037-2.p	4	0	5.33	5.35	6.2
RNG038-2.p	4	0	5.34	3.89	19.94
RNG001-5.p	4	0	6.93	5.32	0.84
SWV015-1.p	9	0	9.64	10.08	0.08
SWV017-1.p	11	0	10.82	11.27	0.1
RNG006-2.p	7	0	11.19	7.53	6.03

the theories of Yices, so we have kept the propositional encoding. Therefore, when we remove the encoding of unification and acyclicity, the entire coding of the problem is still $O(n^3)$. We conjecture a good rule of thumb for deciding when it is useful to encode properties using theories. We conjecture that if the number of clause can be reduced by a factor of n, then the coding is useful, but if the asymptotic complexity remains the same, then it is not a good idea.

Table 2
ChewTPTP Clause and Variable Count For Horn Problems

Name	SAT-M/Y Cls Ct	SMT-Y Cls Ct	SAT-M/Y Var Ct	SMT-Y Var Ct	Result
PUZ008-1.p	52957	323	207608	216	sat
NLP106-1.p	130174	338	513774	392	unsat
NLP104-1.p	130724	344	515712	398	unsat
NLP105-1.p	130724	344	515712	398	unsat
NLP107-1.p	137380	315	542996	370	unsat
GRP033-3.p	115013	737	445065	383	sat
NLP109-1.p	137380	315	542996	370	unsat
NLP113-1.p	137897	319	544836	374	unsat
NLP110-1.p	128150	296	506951	350	unsat
NLP112-1.p	135667	287	537099	342	unsat
NLP111-1.p	135667	287	537099	342	unsat
NLP108-1.p	135667	287	537099	342	unsat
PUZ036-1.005.p	185292	45	729464	91	unsat
RNG037-2.p	221760	1524	876393	714	sat
RNG038-2.p	230063	1522	910786	718	sat
RNG001-5.p	258888	1527	1026821	725	sat
SWV015-1.p	559284	1047	2105121	532	unsat
SWV017-1.p	625119	1137	2354882	578	unsat
RNG006-2.p	432194	2058	1702459	925	sat

5   Conclusion

We have given an application of SMT to theorem proving in first order logic
by encoding the existence of a rigid connection tableau in SMT. We have im-
plemented the SMT encoding in our theorem prover ChewTPTP-SMT. We
compared it with our initial version of ChewTPTP-SAT, where a rigid con-
nection tableau was encoded in SAT.

Table 3
ChewTPTP Times For Non-Horn Problems

Name	SAT-M/Y Clause Gen	SMT-Y Clause Gen	SAT-M Total	SAT-Y Total	SMT-Y Total
ANA025-2.p	1	0	1.02	1.04	2.43
COL121-2.p	0	1	1.02	0.92	1.41
ANA004-4.p	1	0	1.33	1.87	2.77
GRA001-1.p	2	2	1.92	1.74	4.08
ANA029-2.p	2	2	2.05	2.08	4.68
ANA005-2.p	2	1	2.38	2.31	4.72
ANA004-2.p	2	1	2.39	2.3	5.06
ANA003-2.p	3	1	2.96	2.81	5.53
GRP123-1.003.p	3	2	3.41	3.76	18.11
ANA001-1.p	4	2	4	3.84	7.94
GRP123-2.003.p	4	3	5.55	5.37	17.66
ANA002-2.p	5	3	5.73	5.34	10.56
ANA002-1.p	5	3	6.17	5.67	11.84
GRP124-2.004.p	9	6	10.51	11.4	43.91
GRP033-3.p	15	6	20.11	15.69	23.18
GRP123-3.003.p	28	20	30.63	30.73	80.84
ALG002-1.p	1	1	43.51	64.92	75.33
ANA004-5.p	2	1	47.25	21.5	83.54
GRP124-3.004.p	46	31	88.23	83.83	171
COM003-2.p	82	49	88.72	84.54	168.1

Compared to our encoding in SAT, the encoding in SMT is more natural and more efficient. As part of our encoding, we need to encode the solving of unification problems and the acyclicity of the tableau. In SAT, it was necessary to add cubically many clauses to encode the solving of unification. In addition, it was necessary to add cubically many clauses to encode the acyclicity of the

Table 4
ChewTPTP Clause and Variable Count For Non-Horn Problems

Name	SAT-M/Y Cls Ct	SMT-Y Cls Ct	SAT-M/Y Var Ct	SMT-Y Var Ct	Result
ANA025-2.p	41129	36020	2655	2286	sat
COL121-2.p	47725	20335	2322	1538	sat
ANA004-4.p	44142	36844	3160	2631	sat
GRA001-1.p	64222	60849	3292	3161	sat
ANA029-2.p	79860	66884	4107	3388	sat
ANA005-2.p	93806	68206	4907	3802	unsat
ANA004-2.p	93806	68206	4907	3802	unsat
ANA003-2.p	114945	78930	5654	4243	unsat
GRP123-1.003.p	111866	94335	4589	3596	unsat
ANA001-1.p	154246	113596	6680	5185	unsat
GRP123-2.003.p	180783	154243	6723	5450	unsat
ANA002-2.p	226149	151313	7457	5436	unsat
ANA002-1.p	229871	151313	7544	5437	unsat
GRP124-2.004.p	339070	283967	10854	8953	unsat
GRP033-3.p	699160	301901	15989	8961	sat
GRP123-3.003.p	1003831	934044	17763	15377	unsat
ALG002-1.p	54559	32731	3524	2460	unsat
ANA004-5.p	101166	44953	4981	3196	unsat
GRP124-3.004.p	1596801	1468732	25314	21981	unsat
COM003-2.p	2920669	2365922	46818	36051	sat

tableau. However, when encoding this information in SMT, there was no need to encode the solving of unification, since this was accomplished directly with the Yices recursive datatype theory. The number of unification clauses was reduced from a cubic to a quadratic number. Similarly for acyclicity of tableau, we did not need to encode the transitivity and irreflexivity of the path

relation. We only needed to express edges in the tableau as inequalities. The number of clauses to represent acyclicity also dropped from a cubic number to a quadratic number.

In the Horn encoding, all the other information in the tableau can also be encoded with a quadratic number of clauses. Therefore the entire encoding of the existence of a tableau dropped from a cubic number of clauses in SAT to a quadratic number in SMT. This drastically reduced the number of clauses, and simultaneously decreased the time needed to decide the satisfiability of the clauses. There was only a small reduction in number of clauses for non-Horn clauses, because we still need to encode the fact that all paths in the tableau can be closed. Therefore the entire encoding is still cubic, and the running time was actually worse. We conjecture a rule of thumb saying that it is worthwhile to use theories if the number of clauses is reduced by a factor of n, but not worthwhile if the asymptotic number remains the same.

For future work, we hope to be able to use SMT to further reduce the representation for non-Horn clauses, ideally cutting it down to a quadratic number of clauses. It would be possible to define a theory to do this directly, but we have not yet figured out how to do it with the existing theories in Yices. In addition, in order to prove the general first order problem we also need to find a good way to decide exactly which clauses should be copied. We would like a method to decide satisfiability from rigid satisfiability. It would be useful to have an encoding of rigid clauses modulo a non-rigid theory, as discussed in [5]. This way, we could immediately identify some clauses as non-rigid, and work modulo those clauses.

This paper shows the usefulness of SMT to theorem proving in first order logic. We suspect there are other logics which could also be solved efficiently using SMT.

Acknowledgement

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