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# The Invariant Set Hypothesis: A New Geometric Framework for the Foundations of Quantum Theory and the Role Played by Gravity

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**Abstract**

A new hypothesis is proposed about the nature of physical reality at its most primitive level. The hypothesis is framed in terms of invariance, a concept that forms the very bedrock of physics. Specifically, the Invariant Set Hypothesis proposes that states of physical reality are precisely those belonging to a non-computable fractal subset $I$ of state space, invariant under the action of some subordinate deterministic causal dynamics $D$. The Invariant Set Hypothesis provides a geometric framework for a new perspective on quantum physics.

*Keywords:* State Space Invariant Sets, Fractals, Quantum Foundations, Quantum Gravity.

## 1 Introduction

A new hypothesis is proposed about the nature of physical reality at its most primitive level [1]. The hypothesis is framed in terms of invariance, a concept that forms the very bedrock of physics. Specifically, the Invariant Set Hypothesis proposes that states of physical reality are precisely those belonging to a non-computable fractal subset $I$ of state space, invariant under the action of some subordinate deterministic causal dynamics $D$. As discussed below, the Invariant Set Hypothesis provides a geometric framework for a new perspective on quantum physics.

The Invariant Set Hypothesis is motivated by two concepts that would not have been known to the founding fathers of quantum theory: the generic existence of invariant fractal subsets of state space for certain nonlinear dynamical systems, and the notion that the irreversible laws of thermodynamics are fundamental rather than phenomenological in describing the physics of black holes.

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Why are nonlinear dynamics and black holes central to quantum theory, which is both linear and applicable to laboratory experiments far from black holes? To answer this it should first be noted that the Liouville equation, which describes conservation of probability, is always precisely linear even though the probabilities themselves are derived from profoundly nonlinear deterministic dynamics. As is well known, the Schrödinger equation especially in its Heisenberg form, is strongly reminiscent of a Liouville equation. Second, the notion that properties of black holes may be relevant to the formulation of quantum theory is not itself new. Penrose [2] motivates his Objective Reduction mechanism by positing some process where the flow in state space is divergent, to compensate for the convergence of phase-space flow associated with loss of information in black hole dynamics. Contrary to Penrose, we argue here there that there is no compelling argument for compensating divergence based on the Liouville theorem, and that the state space flow of a system (e.g., the Hawking Box) which might contain one or more black holes at some stage in its evolution, may asymptotically approach a zero-volume fractal invariant set in the presence of black-hole-induced convergence of state-space flow.

We outline here some aspects of the new perspective brought to quantum theory by the Invariant Set Hypothesis.

## 2 Contextuality

We know from the Bell-Kochen-Specker Theorem that any hidden-variable model which purports to underpin quantum theory, cannot be non-contextual. As emphasized by Penrose, one of the great mysteries of quantum theory is how to make sense of this in terms of ideas that we can comprehend.

The Invariant Set Hypothesis makes contextuality a readily understandable concept, by focusing on the sparseness (measure zero, nowhere dense property) of $I$. Imagine a world state on $I$, i.e., corresponding to a state of physical reality. Now perturb this state by changing one variable or parameter keeping all others fixed. This is the type of perturbation considered when one constructs counterfactual states (what Bob would have measured if, instead of measuring in the $x$ direction, he had measured in the $y$ direction). Because of the sparseness of $I$, such a perturbation almost certainly takes the world state off to a state of physical unreality.

Application of The Invariant Set Hypothesis in this way implies that the Bell-Kochen-Specker Theorem does not constrain $D$ to be non-local (i.e., to not be locally causal). The reason for this (a reason which Bell himself recognized as valid) is that from the perspective of the Invariant Set Hypothesis, experimental parameters, the orientation of polarisers and so on, cannot be considered as unconstrained free variables.

## 3 The Quantum State Vector and Corresponding Hilbert Space Formalism

With respect to the Invariant Set Hypothesis, the quantum state relative to some point $p \in I$ is to be considered as a coarse-grain probability mixture in some neighbourhood of $p$ (with dimension $\sqrt{\hbar}$) based on a partition $\Pi$ of $I$. The most elemental partition is a binary partition, e.g., imagine all points are labeled either red or blue. Dynamical evolution relative to some partition is the basis of symbolic dynamics and for generating partitions, the symbolic dynamics is homeomorphic to the exact dynamics. The "observables" of quantum theory correspond to different partitions of $I$. Let $D_i$ denote an $i$th forward iteration of $D$ from some initial state. Then $\Pi_i = D_i^{-1}(\Pi)$ denotes one of a countable infinity of dynamically coherent partitions of $I$.

In quantum theory the state of a quantum sub-system is defined as an element of a Hilbert space irrespective of whether a particular interaction between that sub-system and the rest of the universe (e.g., a measurement) is real or counterfactual. Consider the following analogy. We count and compare quantities of physical objects, like apples, using the integers, which can be represented by points on a regular grid on the real line. Using their algebraic properties (the sum, difference and product of two integers is a third), the integers can be "continued" onto a rectangular grid in the complex plane. Conversely, if we were told that one of the Gaussian integers corresponded to a particular quantity of apples, then we would know that its corresponding grid point lay on the real axis.

Similarly, we can "continue" to points off $I$, the mathematical object corresponding to a probability mixture. This is done by allowing this mathematical object to acquire the algebraic properties of the probability mixture, but dropping the requirement that the object describes a probability mixture. In this way, states off $I$ can be given a consistent mathematical structure, but one which cannot be interpreted in terms of elements of physical reality. The intricate structure of $I$ (i.e., which points are points of physical reality and which not) is hidden to quantum theory. As such it is possible to understand why quantum theory operates using mathematical objects (elements of a complex Hilbert Space) which cannot be unambiguously interpreted as probability mixtures. Since $I$ is non-computable, this ambiguity (and hence uncertainty) of interpretation is profound.

## 4 Wave-Particle Duality

A key element of quantum physics is the wave nature of coherent quantum objects. Again the fractal nature of the invariant set can explain this. Readers will be familiar with the periodicity found in animations which zoom into the Mandelbrot set. Here positive exponent Lyapunov vectors perform the function of the zoom, periodically revealing the self-similar structure of the partition of the invariant set. The periodicity revealed by the self-similarity of $I$ is manifest at a fixed point through the coherence associated with the family of dynamically defined partitions (i.e., observables) (see above).

## 5 Superposition and the Measurement Paradox

As discussed, with respect to the Invariant Set Hypothesis, the quantum state represents a probability distribution on the invariant set. Hence there is no fundamental ontological significance to the superposed state; quantum coherence is a consequence of the self-similar structure of the invariant set.

As such, there is also nothing fundamental about measurement. Rather, measurement is a process which on the one hand reveals to us humans the nature of the invariant set, but on the other hand is merely part of the many interactions which determines the invariant set. The invariant set is an atemporal concept; measurements performed to the future of some point p in state space help determine whether $p$ lies on $I$ or not. This simple fact can be used to understand the apparently paradoxical nature of delayed-choice experiments.

Decoherence theory can explain loss of coherence during measurement. However, decoherence theory does not account for the basis onto which the quantum state decoheres [2]. Effectively the Invariant Set Hypothesis provides a preferred basis for decoherence to operate.

## 6 Emergence of Classicality

A key notion underpinning the Invariant Set Hypothesis is that the invariant set is a primitive notion, whereas the dynamics $D$ is to be considered subordinate to $I$. This is quite unlike the situation in classical physics where $D$ is primitive and where there is no requirement for states to lie on an invariant set, even if one should exist. But how can the classical domain emerge from the Invariant Set Hypothesis? By the central limit theorem, the invariant measure for sufficiently time-averaged states of $I$ will be Gaussian, and hence not fractal. A smooth measure such as a Gaussian is neither sparse nor self-similar, and the arguments above for contextuality and periodicity fail. That is to say, by taking long enough time averages, classicality emerges from the Invariant Set Hypothesis.

## 7 Reconciling the Copenhagen Interpretation with Einstein Reality

On the one hand, consistent with Einstein's view, the Invariant Set Hypothesis indicates that quantum theory is incomplete in the sense that it is blind to the fractal structure of $I$. Quantum theory only sees the coarse-grain structure of $I$; it is theory saddled within spectacles. Moreover, with respect to $D$, physics is both deterministic (no dice) and locally causal (no spooky effects).

On the other hand, the Invariant Set Hypothesis implies that it is not meaningful to regard an individual quantum system as having any intrinsic properties independent of the invariant set on which the whole world state evolves. The invariant set is in part characterised by the properties of the experiments which inform us humans about the invariant set. Hence, the Invariant Set Hypothesis implies that it is not meaningful to regard a quantum particle as having any intrinsic properties independent of the instruments which measure the state of the quantum system. This is one of the key tenets of the Copenhagen Interpretation.

# 8 The Role of Gravity in Quantum Physics

Gravity has often been suggested as playing a role in quantum theory, e.g., as a mechanism that affects quantum state vector collapse [2]. However, as discussed above, the Invariant Set Hypothesis does not require superposed states at a deep ontological level, and hence does not require a collapse mechanism.

Rather we propose the following. General Relativity theory reveals that the role of gravity in classical physics can be understood in terms of its causal effect on space-time geometry. The Invariant Set Hypothesis conjectures that the role of gravity in quantum physics can be understood in terms of its atemporal effect on state-space geometry. If this idea is correct, then a challenge for the future will be to combine the pseudo-Riemannian geometry of space-time, and the fractal geometry of state space, into a single geometric entity, thus unifying gravity in the quantum and classical arenas. This is a very different perspective on "quantum gravity" than suggested in any existing approaches to the subject.

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