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ScienceDirect
Electronic Notes in Theoretical Computer Science 261 (2010) 131–145
Electronic Notes in Theoretical Computer Science
www.elsevier.com/locate/entcs

Flexible adjustment of the short-term correlation of LRD M/G/∞-based processes

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Abstract

Video represents a larger and larger portion of the traffic in Internet. This traffic is characterized by a high burstiness and a strong short- and long-range correlation, that is very important from the performance point of view. Particularly, the efficient generation of synthetic sample paths is fundamental because real traces are usually of limited length and lack the necessary diversity required to perform such analysis. In this paper, we focus on the M/G/∞ process due to its theoretical simplicity, its flexibility to exhibit both Short- and Long-Range Dependence and its advantages for simulation studies when compared to other types of processes, and we improve the adjustment of the short-term correlation of LRD M/G/∞-based processes adding autoregressive filters.

Keywords: Video traffic modeling; M/G/∞ process; Correlation; Synthetic efficient on-line generation

1 Introduction

With the increasing popularity of multimedia applications, video data represents a larger and larger portion of the traffic in Internet. Consequently, adequate models of video traffic, characterized by a high burstiness and a strong positive correlation, are very important for the performance evaluation of network architectures and protocols.

In the last decade several traffic studies have convincingly shown the existence of persistent correlations in several kinds of traffic as VBR video [14,3,23,4,7,32]. These experimental findings stimulated the opening of a new branch in the stochastic modeling of traffic, since the impact of the correlation on the performance metrics may be drastic [17,15,22,16,8]. So, working with classes of stochastic processes that

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This work was supported by the Ministerio de Educación y Ciencia, under grant TSI2006-12507-C03-02, and is partially funded by FEDER.

doi:10.1016/j.entcs.2010.01.009

can display diverse forms of correlation by making use of few parameters (parsimonious modeling) is essential for traffic modeling purposes. Some of these processes are Fractional Gaussian Noise (FGN), Fractional AutoRegressive Integrated Moving Average (F-ARIMA) and M/G/∞.

The use of these processes in performance evaluation has opened new problems and research issues in simulation studies, where the efficient generation of synthetic sample paths is fundamental because real traces are usually of limited length and lack the necessary diversity required to perform such analysis.

Several works have been conducted in modeling VBR video traffic, based on different stochastic methods [10,13,18,20,21,9,25]. We focus on the M/G/∞ process [5], due to its theoretical simplicity, its flexibility to exhibit both Short-Range Dependence (SRD) and Long-Range Dependence (LRD) in a parsimonious way and its advantages for simulation studies [13,24], such as the possibility of on-line generation (the main drawback of FGN and F-ARIMA processes is that only off-line methods for trace generation are efficient enough to be of practical use [19]) and the lower computational cost [27].

In this paper, we improve the adjustment of the short-term correlation of LRD M/G/∞-based processes adding autoregressive filters.

In order to apply a model to the synthetic generation of traces with a correlation structure similar to that of real sequences, a fundamental problem is the estimation of the parameters of the model. Among the methods proposed in the literature [31,1,30], those based on the Whittle estimator are especially interesting because they permit both fitting the whole spectral density and obtaining confidence intervals of the estimated parameters. Moreover, in [26] we have presented a method based on the prediction error of the Whittle estimator to choose, among several models for compressed VBR video traffic based on the M/G/∞ process, the one that gives rise to a better adjustment of the spectral density, and therefore of the correlation structure, of the traffic to model. In this paper we extend this method in order to deal with the new models. Moreover, we check if the numerically better adjustment is significant or not.

The remainder of the paper is organized as follows. We begin reviewing the main concepts related to the M/G/∞ process in Section 2 and those related to the Whittle estimator in Section 3. In Section 4 we explain the models for VBR video traffic that we propose and compare in this work. In Section 5 we explain how to use the Whittle estimator to check that the new models are more flexible in order to fit the autocorrelation function measured from empirical VBR traces. Finally, concluding remarks and guidelines for further work are given in Section 6.

2 M/G/∞ process

The M/G/∞ process [5], X, is a stationary version of the occupancy process of an M/G/∞ queueing system. Let λ be the arrival rate to the system, and denote by S the service time distribution, with finite mean value $\mathsf{E}[S]$.

Considering a discrete time analysis [29], if the initial number of users is a

Poisson random variable of mean value $\lambda\mathsf{E}[S]$, and their service times are mutually independent and have the same distribution as the residual life of S, $\widehat{S}$:

$$\Pr\left[\widehat{S}=k\right]=\frac{\Pr[S\geq k]}{\mathsf{E}[S]}$$

then the stochastic process X is strict-sense stationary, ergodic, and enjoys the following properties:

- it has Poissonian marginal distribution, with mean value:

$$\mathsf{E}[X]=\lambda\mathsf{E}[S],$$

- the autocorrelation function is:

$$r[k]=\Pr\left[\widehat{S}>k\right]\quad\forall k.$$

So, the autocorrelation structure of X is completely determined by the distribution of S.

If the autocorrelation function is summable, $\sum_{k=0}^{\infty}r[k]<\infty$, the process exhibits SRD and conversely, if the autocorrelation function is not summable, $\sum_{k=0}^{\infty}r[k]=\infty$ the process exhibits LRD. In particular, the M/G/∞ process exhibits LRD when S has infinite variance, as it happens in heavy-tailed distributions.

In [13] the authors show that an $\mathbb{R}^{+}$-valued sequence $r[k]$ can be the autocorrelation function of the stationary M/G/∞ process, with integrable S, if and only if it is decreasing and integer-convex, with $r[0]=1$ and $\lim_{k\to\infty}r[k]=0$. In such a case the probability mass function of S is given by:

$$\Pr[S=k]=\frac{r[k-1]-2r[k]+r[k+1]}{1-r[1]}\quad\forall k>0,\tag{1}$$

and its mean value is:

$$\mathsf{E}[S]=\frac{1}{1-r[1]}.$$

3 Whittle estimator

Let $f_\theta(\lambda)$ be the spectral density function of a zero-mean Gaussian stochastic process, $X=\{X_n;n=1,2,\dots\}$ and let $I_{X^{\mathsf{N}}}(\lambda)=\frac{1}{2\pi\mathsf{N}}\left|\sum_{i=0}^{\mathsf{N}-1}X_{i+1}\,\mathrm{e}^{-\jmath\lambda i}\right|^2$ be the periodogram of a sample of size N of X. $\theta=\{\theta_k;k=1,\dots,\mathsf{M}\}$ is the vector of parameters to be estimated.

The approximate Whittle MLE [31] is the vector $\widehat{\theta}=\left\{\widehat{\theta}_k;k=1,\dots,\mathsf{M}\right\}$ that minimizes, for a given sample x^{N} of size N of X, the statistic:

$$Q_{X^{\mathsf{N}}}(\theta)\triangleq\frac{1}{2\pi}\left[\int_{-\pi}^{\pi}\frac{I_{X^{\mathsf{N}}}(\lambda)}{f_\theta(\lambda)}\,\mathrm{d}\lambda+\int_{-\pi}^{\pi}\log f_\theta(\lambda)\,\mathrm{d}\lambda\right].\tag{2}$$

Moreover, if θ^{o} is the real value of θ:

$$\Pr\left[\left|\hat{\theta}-\theta^{\mathrm{o}}\right|<\epsilon\right] \xrightarrow[\mathsf{N}\to\infty]{} 1 \quad \forall\epsilon>0,$$

then $\sqrt{\mathsf{N}}(\hat{\theta}-\theta^{\mathrm{o}})$ converges in distribution to ζ, as $\mathsf{N}\to\infty$, where ζ is a zero-mean Gaussian vector with matrix of covariances $\mathrm{C}\left(\theta^{\mathrm{o}}\right)=2\,\mathrm{D}^{-1}\left(\theta^{\mathrm{o}}\right)$, being:

$$\mathrm{D}_{ij}\left(\theta^{\mathrm{o}}\right)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\partial}{\partial\theta_i}\log f_{\theta}\left(\lambda\right)\frac{\partial}{\partial\theta_j}\log f_{\theta}\left(\lambda\right)\,\mathrm{d}\lambda\bigg|_{\theta=\theta^{\mathrm{o}}}. \tag{3}$$

So, confidence intervals of the estimated values can be obtained.

A simplification of (2) can be achieved by choosing a special scale parameter θ_1, such that:

$$f_{\theta}\left(\lambda\right)=\theta_1 f_{\theta^*}\left(\lambda\right)=\theta_1 f^*_{\eta}\left(\lambda\right),$$

and:

$$\int_{-\pi}^{\pi}\log f_{\theta^*}\left(\lambda\right)\,\mathrm{d}\lambda=\int_{-\pi}^{\pi}\log f^*_{\eta}\left(\lambda\right)\,\mathrm{d}\lambda=0,$$

where $\eta=\{\theta_i;i=1,2,...,\mathsf{M}\}$ and $\theta^*=(1,\eta)$.

Thus:

$$\theta_1=\exp\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\log f_{\theta}\left(\lambda\right)\,\mathrm{d}\lambda\right)=\frac{\sigma^2_{\epsilon}}{2\pi},$$

where σ^2_{ϵ} is the optimal one-step-ahead prediction error, that is equal to the variance of the innovations of the AR(∞) representation of the process [2]:

$$X_i=\sum_{j=1}^{\infty}\beta_j X_{i-j}+\epsilon_i.$$

Equation (2) therefore simplifies to:

$$Q_{X^{\mathsf{N}}}\left(\theta^*\right)=Q^*_{X^{\mathsf{N}}}\left(\eta\right)=\int_{-\pi}^{\pi}\frac{I_{X^{\mathsf{N}}}\left(\lambda\right)}{f_{\theta^*}\left(\lambda\right)}\,\mathrm{d}\lambda=\int_{-\pi}^{\pi}\frac{I_{X^{\mathsf{N}}}\left(\lambda\right)}{f^*_{\eta}\left(\lambda\right)}\,\mathrm{d}\lambda.$$

Additionally [2]:

$$\widehat{\sigma^2_{\epsilon}}=Q_{X^{\mathsf{N}}}\left(\widehat{\eta}\right).$$

We use $\widehat{\sigma^2_{\epsilon}}$ as a measure of the suitability of a model, since smaller values of $\widehat{\sigma^2_{\epsilon}}$ (or $\widetilde{Q}^*_{X^{\mathsf{N}}}\left(\widehat{\eta}\right)$) mean better adjustment to the actual correlation of the sample.

4 M/G/∞-based LRD models

First, we remember the main features of M/G/∞-based processes able to exhibit LRD that we have proposed in previous works.

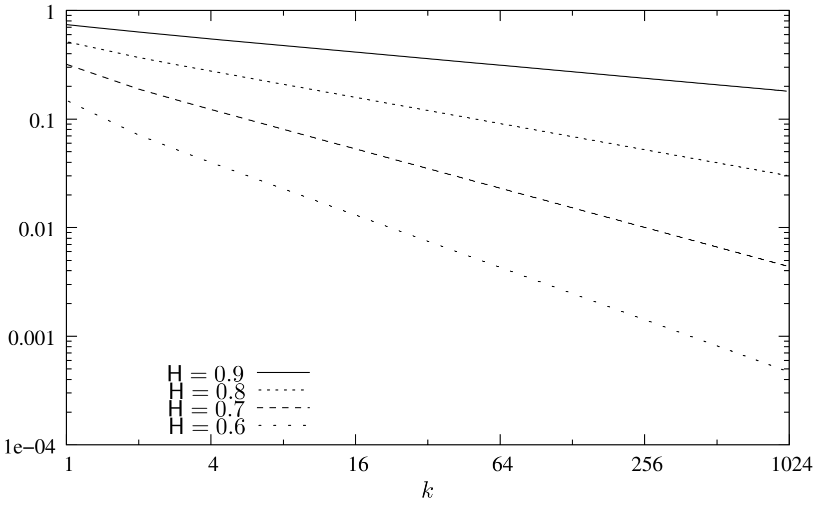

Fig. 1. Autocorrelation function of the M/F/∞ process.

4.1 LRD-1 model: M/F/∞

This model fixes the autocorrelation function of the resulting M/G/∞ process within the class of exactly second order self-similar processes:

$$r[k] = r_{\mathsf{H}}[k] \triangleq \frac{1}{2}\left[(k+1)^{2\mathsf{H}} - 2k^{2\mathsf{H}} + (k-1)^{2\mathsf{H}}\right] \quad \forall k,$$

i.e., the same one that the autocorrelation function of the FGN process.

H is the Hurst parameter [12]. For $0.5 < \mathsf{H} < 1$ the process is LRD.

From (1) the probability mass function of the service time in a M/G/∞ system generating an occupancy process with such correlation function is obtained. We denote this random variable by F.

In Fig. 1 we show the autocorrelation function of the resulting M/F/∞ process for several values of the parameter H.

4.2 LRD-2 model: M/S/∞

Now, the model considers the heavy-tailed distribution, S, proposed in [29] as the service time of the M/G/∞ queueing system.

The resulting autocorrelation function is:

$$r[k] = r_{\{\mathsf{m},\alpha\}}[k] \triangleq \begin{cases} 1 - \dfrac{\alpha-1}{\mathsf{m}\alpha}k & \forall k \in (0, \mathsf{m}] \\ \dfrac{1}{\alpha}\left(\dfrac{\mathsf{m}}{k}\right)^{\alpha-1} & \forall k \geq \mathsf{m}, \end{cases}$$

with:

$$\alpha = 3 - 2\mathsf{H},$$
$$\mathsf{m} = \begin{cases} (\alpha r[1])^{\frac{1}{\alpha-1}} & \forall r[1] \in \left(0, \frac{1}{\alpha}\right] \\ \dfrac{\alpha-1}{\alpha - \alpha r[1]} & \forall r[1] \in \left[\frac{1}{\alpha}, 1\right). \end{cases}$$

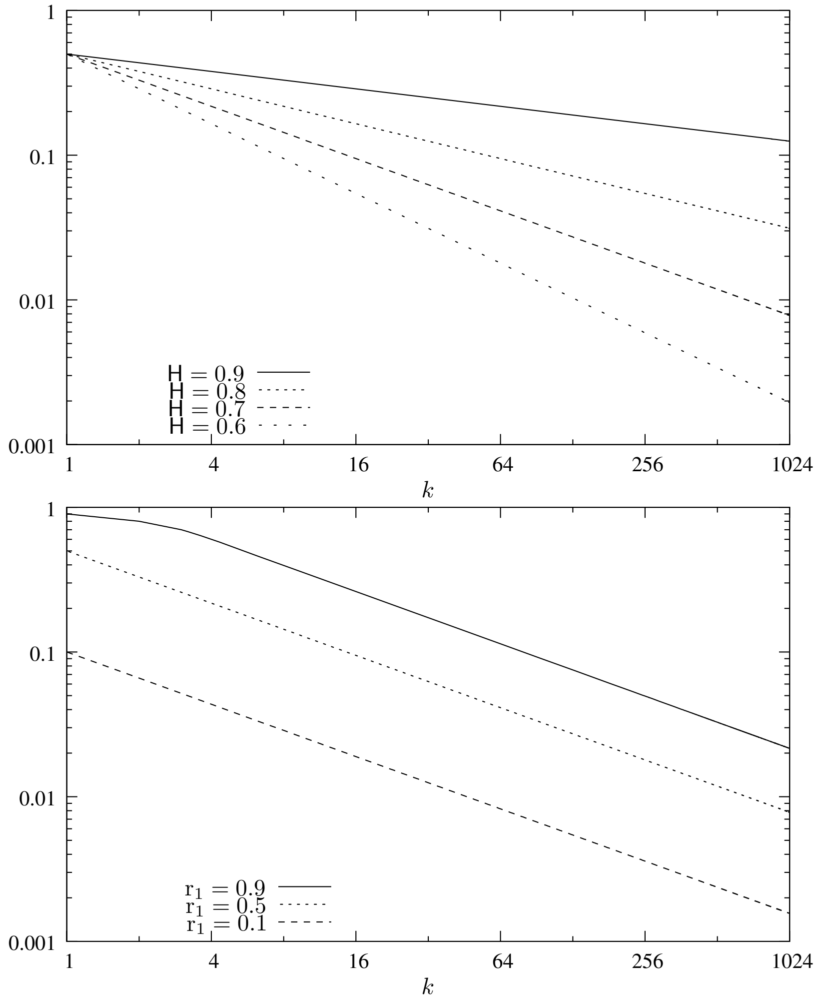

Fig. 2. Autocorrelation function of the M/S/∞ process for $\mathrm{r}_1 = 0.5$ (top) and $\mathsf{H} = 0.7$ (bottom).

If $\alpha \in (1, 2)$, then $\mathsf{H} \in (0.5, 1)$ and $\sum_{k=0}^{\infty} r_{\{\mathsf{m},\alpha\}}[k] = \infty$. Hence, in this case, this correlation structure gives rise to an LRD process.

In Fig. 2 we show the autocorrelation function of the resulting M/S/∞ process for several values of the parameters H and $\mathrm{r}_1 = r_{\{\mathsf{m},\alpha\}}[1]$.

In Fig. 3, where we represent the queue of the autocorrelation function of different LRD processes with the same value of the Hurst parameter, $\mathsf{H} = 0.7$, we can see that only the M/G/∞ process scales the tail of the queue.

4.3 On improving the adjustment of the short-term correlation

In order to improve the adjustment of the short-term correlation of the previous processes, we propose to add an autoregressive filter. Specifically, we focus on the particular case of an AR(1) filter.

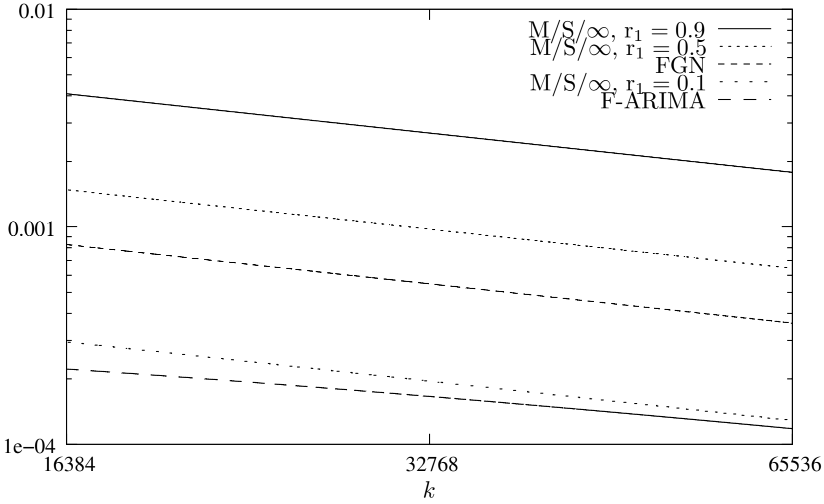

Fig. 3. Tail of the autocorrelation function of several types of LRD processes.

If Y is the M/F/∞ or M/S/∞ original process, the new one is obtained as:

$$X_n = \alpha_1 X_{n-1} + Y_n,$$

with its mean values and covariances related by:

$$\mathsf{E}[X] = \frac{\mathsf{E}[Y]}{1-\alpha_1},$$

$$\gamma_k^X = \frac{1}{1-\alpha_1^2}\Big(\gamma_k^Y + \sum_{i=1}^{\infty}\gamma_{k+i}^Y\alpha_1^i + \sum_{i=1}^{\infty}\gamma_{k-i}^Y\alpha_1^i\Big). \tag{4}$$

It is important to stand out that this will not modify the behavior of the tail of the autocorrelation function, because its influence decays exponentially, as we observe in equation (4).

We denote the resulting processes as M/F/∞-AR and M/S/∞-AR (and the models based on them as LRD-1-AR and LRD-2-AR respectively).

If the resulting autocorrelation function is decreasing and convex, we can obtain the service time distribution of the process X by means of (1). In other case, we will have to use an autoregressive filter to generate X from Y, discarding as many samples as necessary to initialize the generator approximately in steady state.

4.4 Application of the Whittle estimator to the proposed models

For the LRD-1 model the spectral density:

$$f(\lambda) = f_{\mathsf{H}}(\lambda) \triangleq \mathsf{c}_f \left|e^{\jmath\lambda} - 1\right|^2 \sum_{i=-\infty}^{+\infty} |2\pi i + \lambda|^{-2\mathsf{H}-1}$$

$$\forall \lambda \in [-\pi, \pi],$$

can be computed efficiently with the Euler's formula.

For the LRD-2 model, the spectral density is given by [28]:

$$f(\lambda) = f_{\{\mathsf{m},\alpha\}}(\lambda) \triangleq \mathsf{Var}[X] \left\{ \frac{\mathsf{m}^{\alpha-1}}{\alpha\cos(\lambda) - \alpha} f_{\mathsf{h}}(\lambda) + \frac{1}{2\pi} \right.$$

$$\left. + \frac{1}{\pi} \sum_{k=1}^{\lfloor \mathsf{m} \rfloor} \cos(\lambda k) \left[\frac{\alpha(m-k)+k}{\mathsf{m}\alpha} - \frac{1}{\alpha} \left(\frac{\mathsf{m}}{k} \right)^{\alpha-1} \right] \right\}$$

$$\forall \lambda \in [-\pi, \pi],$$

where f_{h} is the spectral density of the FGN process with $\mathsf{h} = \frac{(1-\alpha)}{2}$, scaled by the variance.

For LRD-1-AR and LRD-2-AR models, if $f_Y(\lambda)$ is the spectral density of the process Y, the spectral density of X can be obtained as:

$$f_X(\lambda) = \frac{f_Y(\lambda)}{|1 - \alpha_1 \, \mathrm{e}^{j\lambda}|^2} \quad \forall \lambda \in [-\pi, \pi].$$

5 Using the Whittle estimator for the selection of the best adjustment of the correlation

In this section we explain, by means of two examples, how to use the Whittle estimator to check that the new models are more flexible in order to fit the autocorrelation function obtained from empirical VBR traces.

We consider the following empirical traces of the Group of Pictures (GoP) sizes of the MPEG encoded videos: *"Robin Hood"* (T-1) and *"Ice Age I"* (T-2), both of length $\mathsf{N} = 9000$ and available in [11].

In order to adjust simultaneously the marginal distribution and the autocorrelation, as the marginal distribution in all cases is approximately Lognormal, we apply a change of distribution.

In each case, A denotes the process we want to generate and C the M/G/∞ process from which we start off, that should have a high enough mean value so as the Poissonian marginal distribution can be considered approximately Gaussian (we select $\sigma_C^2 = \mu_C = 10^6$). Moreover, we consider the intermediate process $B = \log(A)$, from which we estimate the parameters.

If A has Lognormal marginal distribution, then $B = \log(A)$ has Gaussian marginal distribution, with mean, variance and autocorrelation given by [6]:

$$\mu_B = \log \frac{\mu_A^2}{\sqrt{\mu_A^2 + \sigma_A^2}},$$

$$\sigma_B^2 = \log\left(1 + \frac{\sigma_A^2}{\mu_A^2}\right),$$

Model	T-1	T-2
LRD-1	$4.5173 \cdot 10^{-2}$	$7.6194 \cdot 10^{-2}$
LRD-1-AR	$4.1202 \cdot 10^{-2}$	$7.6157 \cdot 10^{-2}$
LRD-2	$4.1296 \cdot 10^{-2}$	$7.6135 \cdot 10^{-2}$
LRD-2-AR	$4.1287 \cdot 10^{-2}$	$7.6128 \cdot 10^{-2}$

Table 1
$\widehat{\sigma_\epsilon^2}$ with each model.

$$\mathrm{r}_B[k] = \frac{1}{\sigma_B^2} \log\left(1 + \frac{\mathrm{r}_A[k]\sigma_A^2}{\mu_A^2}\right). \tag{5}$$

The estimations of the parameters of each model, computed via the Whittle estimator, are as follows:

- T-1: *"Robin Hood"*
  - · Model LRD-1: $\widehat{\mathsf{H}} \simeq 1$.
  - · Model LRD-1-AR: $\widehat{\alpha_1} = 0.541$ and $\widehat{\mathsf{H}} = 0.799$.
  - · Model LRD-2: $\widehat{\mathrm{r}_1} = 0.892$ and $\widehat{\mathsf{H}} = 0.869$.
  - · Model LRD-2-AR: $\widehat{\alpha_1} = 0.016$, $\widehat{\mathrm{r}_1} = 0.887$ and $\widehat{\mathsf{H}} = 0.861$.
- T-2: *"Ice Age I"*
  - · Model LRD-1: $\widehat{\mathsf{H}} = 0.879$.
  - · Model LRD-1-AR: $\widehat{\alpha_1} = -0.042$ and $\widehat{\mathsf{H}} = 0.894$.
  - · Model LRD-2: $\widehat{\mathrm{r}_1} = 0.696$ and $\widehat{\mathsf{H}} = 0.873$.
  - · Model LRD-2-AR: $\widehat{\alpha_1} = -0.131$, $\widehat{\mathrm{r}_1} = 0.749$ and $\widehat{\mathsf{H}} = 0.849$.

The variance (or confidence intervals) of the estimations can be computed from (3).

We use $\widehat{\sigma_\epsilon^2}$ as a measure of the suitability of a model, since smaller values of $\widehat{\sigma_\epsilon^2}$ mean numerically better adjustment to the empirical correlation of the sample.

In Table 1 we show the estimations of the prediction error.

The results show that increasing the number of parameters leads to smaller prediction errors (better fit of the spectral density) within each family of models.

In Figs. 4 and 5 we show the adjustment of the autocorrelation function for each empirical trace after the transformation. The case when $\widehat{\mathsf{H}} \simeq 1$ in the LRD-1 model is not represented. In this case the strong short term correlation, that we also observe in the high value of α_1 in the LRD-1-AR model, makes the H parameter tend to be one.

Once we have generated a sample of the process C, to obtain Lognormal marginal distribution with the mean value and variance of the empirical traces, we apply the inverse transformation:

$$B = T(C) = \left(\frac{C - \mu_C}{\sigma_C}\right)\widehat{\sigma_B} + \widehat{\mu_B},$$

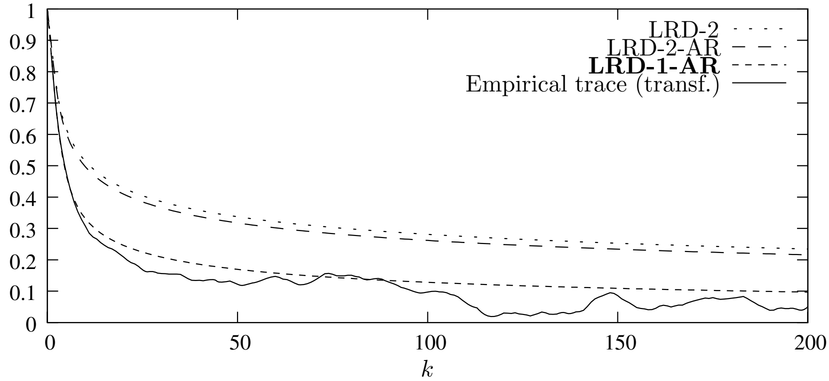

Fig. 4. Adjustment of the autocorrelation function of the empirical trace "*Robin Hood*" (T-1) with each model, after the transformation.

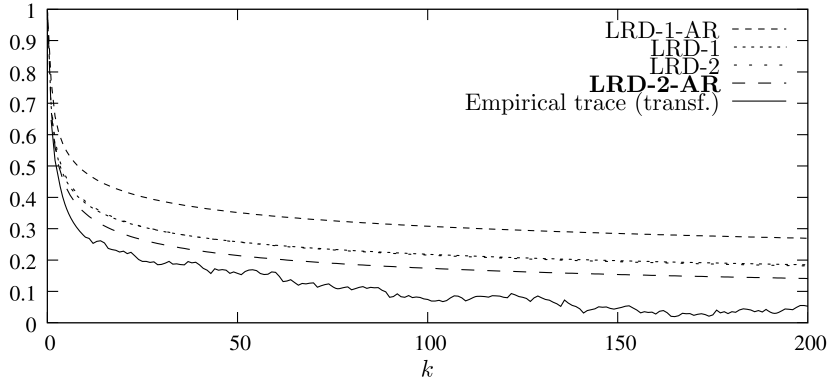

Fig. 5. Adjustment of the autocorrelation function of the empirical trace "*Ice Age I*" (T-2) with each model, after the transformation.

$$A = \exp(B),$$

being $\widehat{\sigma_B^2}$ the estimation of the variance of B computed with the Whittle estimator, that is, considering the autocorrelation structure.

From (5) we obtain the relationship between the autocorrelation of A and B:

$$\mathrm{r}_A[k] = \frac{\widehat{\mu_A^2}\left(\exp\left(\widehat{\sigma_B^2}\mathrm{r}_B[k]\right) - 1\right)}{\widehat{\sigma_A^2}},$$

with:

$$\widehat{\mu_A} = \exp\left(\widehat{\mu_B} + \frac{\widehat{\sigma_B^2}}{2}\right),$$

$$\widehat{\sigma_A^2} = \exp\left(2\widehat{\mu_B} + 2\widehat{\sigma_B^2}\right) - \exp\left(2\widehat{\mu_B} + \widehat{\sigma_B^2}\right).$$

In Figs. 6 and 7 we represent the adjustment of the autocorrelation function of each empirical trace, that is, GoP sizes. Again, the case when $\widehat{\mathsf{H}} \simeq 1$ is not represented.

An open issue to use this criterion for the selection of the best model is the following:

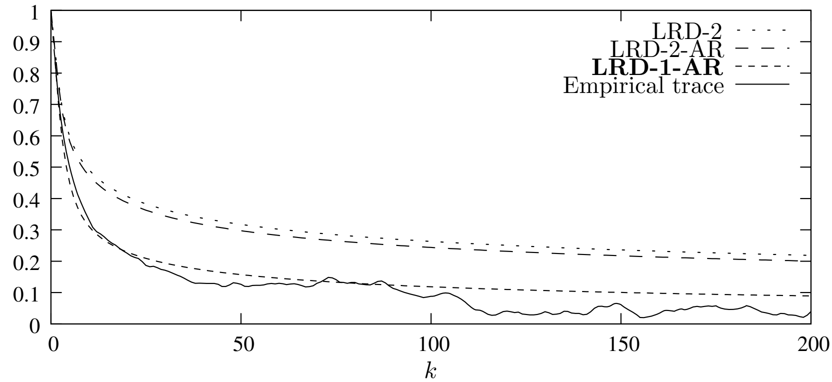

Fig. 6. Ajustment of the autocorrelation function of the empirical trace with each model. "*Robin Hood*" (T-1).

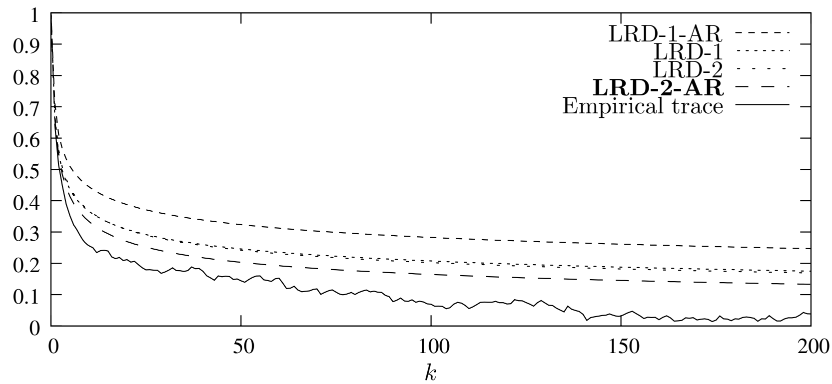

Fig. 7. Ajustment of the autocorrelation function of the empirical trace with each model. "*Ice Age I*" (T-2).

- Increasing the number of parameters supposes a major flexibility in the adjustment of the autocorrelation function, and therefore a reduction of the estimation of the prediction error but, is this improvement significant in order to compensate the increase of complexity of the model?

5.1 Hypothesis test over the spectral density

To solve this question we apply an hypothesis test over the spectral density that we have proposed in a previous work.

Considering two models, 0 and 1, we set out the following null hypothesis:

$$H_0 : f(\lambda) = f_0(\lambda),$$

being $f_0(\lambda)$ the spectral density of the process 0, base of the model 0, and the alternative one:

$$H_1 : f(\lambda) = f_1(\lambda),$$

being $f_1(\lambda)$ the spectral density of the process 1, base of the model 1.

We define the test statistic as the difference between the estimation of the prediction error when the null hypothesis is supposed and the estimation of the prediction error when the alternative hypothesis is supposed divided by the sample variance:

$$E \triangleq \frac{\widehat{\sigma^2_\epsilon}(H_0) - \widehat{\sigma^2_\epsilon}(H_1)}{\widehat{\sigma^2}}.$$

We apply the hypothesis test in the classical way, choosing as process 0 the one with fewer parameters, and we select a critical region with a fixed degree of significance ϵ. In this way, if we can not reject the null hypothesis we should consider that the apparently better adjustment of the model with more parameters is not significant, and consequently we should consider the model with fewer parameters.

As an example, we apply it to the decision between the models LRD-1 and LRD-1-AR.

So, we consider as null hypothesis:

$$H_0 : f(\lambda) = f_{\widehat{\mathsf{H}_0}}(\lambda),$$

and as alternative hypothesis:

$$H_1 : f(\lambda) = f_{\widehat{\alpha_1},\widehat{\mathsf{H}_1}}(\lambda),$$

being $\widehat{\mathsf{H}_0}$ the value obtained for the parameter H using the Whittle estimator and considering the M/F/∞ process as the base process, and $\widehat{\alpha_1}$ and $\widehat{\mathsf{H}_1}$ the values obtained for the parameters α_1 and H using the Whittle estimator and considering the M/F/∞-AR process as base process.

We generate 500 synthetic traces of different sizes:

$$\mathsf{N} \in \{4096, 8192, 16384, 32768, 65536, 131072\}$$

of the M/F/∞ process for three different values of the Hurst parameter $\mathsf{H} \in \{0.6, 0.75, 0.9\}$and we calculate the value of the statistic.

The estimated critical region, W, for a degree of significance $\epsilon = 5\%$ is shown in Table 2.

To verify the power of the test in several concrete cases, we generate 500 synthetic traces of different sizes of the M/F/∞-AR process, and for different combinations of H and α_1. Specifically, $\mathsf{N} \in \{4096, 8192, 16384, 32768, 65536, 131072\}$, $\mathsf{H} \in \{0.6, 0.75, 0.9\}$ and $\alpha_1 \in \{-0.3, 0.3\}$; we check if for the estimated value of the H parameter supposing the null hypothesis is rejected, interpolating the border of the critical region. We show the results in Table 3.

For each one of the empirical traces we estimate the H parameter assuming the null hypothesis and interpolate the border of the critical region in order to check if the null hypothesis is rejected. The results are shown in Table 4.

For T-1 we can observe that the null hypothesis is rejected, so the improvement is significant.

For T-2 the test fails to reject the null hypothesis, so for this trace the numerically better adjustment does not compensate the increase of complexity of the model.

N	H = 0.6	H = 0.75	H = 0.9
4096	$(9.82 \cdot 10^{-4},\infty)$	$(9.4 \cdot 10^{-4},\infty)$	$(1.5 \cdot 10^{-4},\infty)$
8192	$(4.531 \cdot 10^{-4},\infty)$	$(4.302 \cdot 10^{-4},\infty)$	$(2.881 \cdot 10^{-4},\infty)$
16384	$(2.27 \cdot 10^{-4},\infty)$	$(2.12 \cdot 10^{-4},\infty)$	$(1.4 \cdot 10^{-4},\infty)$
32768	$(1.251 \cdot 10^{-4},\infty)$	$(1.008 \cdot 10^{-4},\infty)$	$(5.6 \cdot 10^{-5},\infty)$
65536	$(5.39 \cdot 10^{-5},\infty)$	$(4.35 \cdot 10^{-5},\infty)$	$(3.8 \cdot 10^{-5},\infty)$
131072	$(2.911 \cdot 10^{-5},\infty)$	$(2.231 \cdot 10^{-5},\infty)$	$(1.211 \cdot 10^{-5},\infty)$

Table 2
Estimated critical region in the test to decide between LRD-1 and LRD-1-AR.

N	H = 0.6	H = 0.75	H = 0.9
	$\alpha_1 = -0.3$	$\alpha_1 = -0.3$	$\alpha_1 = -0.3$
4096	500	500	500
8192	500	500	500
16384	500	500	500
32768	500	500	500
65536	500	500	500
131072	500	500	500

N	H = 0.6	H = 0.75	H = 0.9
	$\alpha_1 = 0.3$	$\alpha_1 = 0.3$	$\alpha_1 = 0.3$
4096	498	500	500
8192	499	500	500
16384	499	500	498
32768	500	500	500
65536	500	500	500
131072	500	500	500

Table 3
Power of the test to decide between LRD-1 and LRD-1-AR.

TRACE	$\widehat{E}$	$\widehat{\mathsf{H}_0}$	$W_{\widehat{\mathsf{H}_0}}$
T-1	$2.45 \cdot 10^{-2}$	~ 1	$(5.21 \cdot 10^{-4},\infty)$
T-2	$2.64 \cdot 10^{-4}$	0.879	$(4.48 \cdot 10^{-4},\infty)$

Table 4
Application of the test to decide between LRD-1 and LRD-1-AR to the empirical traces.

It is important to highlight that in order to select the best model for each trace the test must be extended to other combinations of models.

6 Conclusions and further work

In this paper we have improved the adjustment of the short-term correlation of LRD $M/G/\infty$-based processes that we have proposed in previous works adding autoregressive filters. Moreover, we have checked if the numerically better adjustment is significative or not.

The resulting processes enjoy several interesting features as the original ones: highly efficient, on-line generation and the possibility of capturing the whole correlation structure in a parsimonious way.

In the same way as we have done with an AR(1) filter, the order can be increased, and so the number of parameters of the models, until the improvement is not significant.

References

[1] Abry, P. and D. Veitch, *Wavelet analysis of Long-Range Dependent traffic*, IEEE Transactions on Information Theory **42** (1998), pp. 2–15.

[2] Beran, J., "Statistics for Long-Memory Processes," Champan and Hall, 1994.

[3] Beran, J., R. Sherman, M. S. Taqqu and W. Willinger, *Long-Range Dependence in Variable-Bit-Rate video traffic*, IEEE Transactions on Communications **43** (1995), pp. 1566–1579.

[4] Conti, M., E. Gregori and A. Larsson, *Study of the impact of MPEG-1 correlations on video sources statistical multiplexing*, IEEE Journal on Selected Areas in Communications **14** (1996), pp. 1455–1471.

[5] Cox, D. R. and V. Isham, "Point Processes," Chapman and Hall, 1980.

[6] Crouse, M. S. and R. G. Baraniuk, *Fast, exact synthesis of Gaussian and non-Gaussian Long-Range Dependent processes* (1999), enviado a IEEE Transactions on Information Theory.

[7] Crovella, M. E. and A. Bestavros, *Self-similarity in World Wide Web traffic: Evidence and possible causes*, IEEE/ACM Transactions on Networking **5** (1997), pp. 835–846.

[8] Erramilli, A., O. Narayan and W. Willinger, *Experimental queueing analysis with Long-Range Dependent packet traffic*, IEEE/ACM Transactions on Networking **4** (1996), pp. 209–223.

[9] Frey, M. and S. Nguyen-Quang, *A Gamma-based framework for modeling Variable-Bit-Rate MPEG video sources: The GoP GAP model*, IEEE/ACM Transactions on Networking **8** (2000), pp. 710–719.

[10] Garrett, M. W. and W. Willinger, *Analysis, modeling and generation of self-similar VBR video traffic*, in: *Proc. ACM SIGCOMM '94*, London, UK, 1994, pp. 269–280.

[11] http://trace.eas.asu.edu/tracemain.html, *Video traces for network performance evaluation*.

[12] Hurst, H. E., *Long-term storage capacity of reservoirs*, Transactions of the American Society of Civil Engineers **116** (1951), pp. 770–799.

[13] Krunz, M. and A. Makowski, *Modeling video traffic using M/G/∞ input processes: A compromise between Markovian and LRD models*, IEEE Journal on Selected Areas in Communications **16** (1998), pp. 733–748.

[14] Leland, W. E., M. S. Taqqu, W. Willinger and D. V. Wilson, *On the self-similar nature of Ethernet traffic (extended version)*, IEEE/ACM Transactions on Networking **2** (1994), pp. 1–15.

[15] Li, S. Q. and C. L. Hwang, *Queue response to input correlation functions: Discrete spectral analysis*, IEEE/ACM Transactions on Networking **1** (1993), pp. 317–329.

[16] Likhanov, N., B. Tsybakov and N. Georganas, *Analysis of an ATM buffer with self-similar ("fractal") input traffic*, in: *Proc. IEEE INFOCOM '95*, Boston, MA, USA, 1995, pp. 985–992.

[17] Livny, M., B. Melamed and A. K. Tsiolis, *The impact of autocorrelation on queueing systems*, Management Science **39** (1993), pp. 322–339.

[18] Lombardo, A., G. Morabito and G. Schembra, *An accurate and treatable Markov model of MPEG video traffic*, in: *Proc. IEEE INFOCOM '98*, San Francisco, CA, USA, 1998, pp. 217–224.

[19] López, J. C., C. López, A. Suárez, M. Fernández and R. F. Rodríguez, *On the use of self-similar processes in network simulation*, ACM Transactions on Modeling and Computer Simulation **10** (2000), pp. 125–151.

[20] Ma, S. and C. Ji, *Modeling video traffic using wavelets*, IEEE Communications Letters **2** (1998), pp. 100–103.

[21] Melamed, B. and D. E. Pendarakis, *Modelinf full-length VBR video using Markov-renewal-modulated TES models*, IEEE Journal on Selected Areas in Communications **16** (1998), pp. 600–611.

[22] Norros, I., *A storage model with self-similar input*, Queueing Systems **16** (1994), pp. 387–396.

[23] Paxson, V. and S. Floyd, *Wide-area traffic: The failure of Poisson modeling*, IEEE/ACM Transactions on Networking **3** (1995), pp. 226–244.

[24] Poon, W. and K. Lo, *A refined version of M/G/∞ processes for modeling VBR video traffic*, Computer Communications **24** (2001), pp. 1105–1114.

[25] Sarkar, U. K., S. Ramakrishnan and D. Sarkar, *Modeling full-length video using Markov-modulated Gamma-based framework*, IEEE/ACM Transactions on Networking **11** (2003), pp. 638–649.

[26] Sousa, M. E. and A. Suárez, *Using the Whittle estimator for the selection of an autocorrelation function family*, Lecture Notes in Computer Science **5055** (2008), pp. 16–30.

[27] Sousa, M. E., A. Suárez, M. Fernández, C. López and R. F. Rodríguez, *A highly efficient M/G/∞ generator of self-similar traces*, in: *2006 Winter Simulation Conference*, Monterey, CA, 2006, pp. 2146–2153.

[28] Sousa, M. E., A. Suárez, J. C. López, C. López, M. Fernández and R. F. Rodríguez, *Application of the Whittle estimator to the modeling of traffic based on the M/G/∞ process*, IEEE Communications Letters **11** (2007), pp. 817–819.

[29] Suárez, A., J. C. López, C. López, M. Fernández, R. F. Rodríguez and M. E. Sousa, *A new heavy-tailed discrete distribution for LRD M/G/∞ sample generation*, Performance Evaluation **47** (2002), pp. 197–219.

[30] Taqqu, M. S. and V. Teverovsky, *On estimating the intensity of Long-Range Dependence in finite and infinite variance time series*, in: R. Adler, R. Feldman and M. S. Taqqu, editors, *A Practical Guide to Heavy Tails*, Birkhauser, 1998 pp. 177–217.

[31] Whittle, P., *Estimation and information in stationary time series*, Arkiv Matematick **2** (1953), pp. 423–434.

[32] Willinger, W., M. S. Taqqu, R. Sherman and D. V. Wilson, *Self-similarity through high variability: Statistical analysis of Ethernet LAN traffic*, IEEE/ACM Transactions on Networking **5** (1997), pp. 71–86.