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Shifted Chebyshev wavelet-quasilinearization technique for MHD squeezing flow between two infinite plates and Jeffery–Hamel flows

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ABSTRACT

In this article, shifted Chebyshev wavelets method is merged with quasilinearization technique to tackle with the nonlinearity of physical problems. The accuracy of the proposed method is verified by the help of two nonlinear physical models, one MHD squeezing flow between two infinite plates and other Jeffery–Hamel flow that is obtained using proper similarity transforms. Numerical solution is also sought using Runge–Kutta order 4 method. Results obtained for different iterations and different values of degree of polynomial are described in tables and graphs which verify the accuracy and stability of the proposed method.

1. Introduction

After the pioneering works of Stefan [1], squeezing flows have been of much interest to the researchers due to their many practical and industrial applications. Many mechanical equipment work under the principle of moving pistons where two plates exhibit squeezing moment normal to their own surfaces. Electric motors, engines and hydraulic lifters also have this squeezing flow in some of their parts. Its biological applications are also of equal importance. Flow inside syringes and nasogastric tube is also a kind of squeezing flows. One can find more than enough literature on these flows in

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Refs. [2,3] and references therein. Electrically conducting flows are also very important as a slight change in magnetic field may cause to flow to disperse or often to go smoothly for some time. It was therefore essential to discuss the flow under the influence of magnetic field to see how it affects the flow behavior. Refs. [4–6] studied the effects of magnetic field on squeezing flow for different geometries and pointed out some important aspects of these flows.

Flows through nonparallel walls gain importance in early 19th century after the pioneering works of Jeffery [7] and Hamel [8]. Since then, there are many studies available that discussed the different practical and industrial applications of these flows and reported that flow characteristics vary by changing the angle between two channels [9–11]. Flows through rivers and channels, different biological flows such as flow through arteries and veins are some practical applications of these types of flows. Due to nonlinearity of the problems involved in fluid mechanics, exact solutions are unlikely. Many numerical and analytical techniques are available to solve these problems [12–15].

Wavelet methods are also one of the relatively new techniques for obtaining approximate solutions of differential equations. Commonly used wavelet schemes are Haar wavelets, Legendre wavelets and Chebyshev wavelets. Islam et al. used Haar wavelet collocation method for obtaining the numerical solutions of boundary layer flow problem [17] and Hariharan applied Haar wavelet method for solving Sine-Gordon and Klein–Gordon equations [18]. Rawashdeh implemented Legendre wavelet method to obtain solution of fractional integro-differential equations [16]. Ali et al. used Chebyshev wavelets to obtain solutions for linear and nonlinear boundary value problems [19]. Iqbal et al. obtained solutions for fractional delay differential equations using Chebyshev wavelets [20]. Since the models for which we are approximating solutions are of nonlinear in nature, so for better results we are also using quasilinearization technique.

The quasilinearization technique was first introduced by Bellman and Kalaba [21] as a generalization of the Newton–Raphson method [22] to tackle the single or systems of nonlinear ordinary or partial differential equations.

The proposed method formed by merging Chebyshev wavelets method with quasilinearization technique is fully compatible for solving such nonlinear physical models. To the best of our knowledge, this is the first article on Chebyshev wavelet methods in fluid mechanics. Two non-linear problems are taken into account. A well-known numerical method Runge–Kutta order 4 method is used to solve the same problems. Comparison is made among the solutions to verify the accuracy of the proposed solutions.

2. Mathematical formulation

2.1. MHD squeezing flow between two infinite plates

The equations of motion for the flow are given by [23],

$$\nabla \cdot V = 0, \tag{2.1}$$

$$\rho\left[\frac{\partial V}{\partial t} + (V\cdot\nabla)V\right] = \nabla\cdot T - f_B, \tag{2.2}$$

where V is velocity vector, ρ density constant and T is the Cauchy Stress tensor given by,

$$T = -\rho I + A_1,$$

where $A_1 = (\nabla V) + (\nabla V)^T$.

While f_B is a source term arising due to applied magnetic field, i.e., the so called magnetic or Lorentz force. This force is known to be a function of the imposed magnetic field $\mathbf{B}$, the induced electric field $\mathbf{E}$ and the fluid velocity vector $\mathbf{V}$, that is

$$f_B = \sigma(\mathbf{E} + \mathbf{V}*\mathbf{V})*\mathbf{B}.$$

Detailed derivation of the considered model is discussed in Ref. [23]. Using compatibility equation

$$\rho\left[\frac{1}{r}\frac{\partial E^2\Psi}{\partial t} - \frac{\partial\left(\Psi, \frac{E^2\Psi}{r^2}\right)}{\partial(r,z)}\right] = \frac{\mu}{r}E^4\Psi - \sigma\frac{B_0^2}{r}\frac{\partial\Psi}{\partial z}, \tag{2.3}$$

where

$$E^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} = 0,$$

from Eq. (2.3), after simplification we get

$$-\rho\left[\frac{\partial\left(\Psi, \frac{E^2\Psi}{r^2}\right)}{\partial(r,z)}\right] = \frac{\mu}{r}E^4\Psi - \sigma\frac{B_0^2}{r}\frac{\partial\Psi}{\partial z}, \tag{2.4}$$

with associated auxiliary conditions

$$z = H, \ \text{then } u = 0, \ w = -V, \tag{2.5a}$$

$$z = 0, \ \text{then } w = 0, \ \frac{\partial u}{\partial z} = 0. \tag{2.5b}$$

We can now define stream function as

$$\Psi(r,z) = r^2 f(z). \tag{2.6}$$

Replacing value from Eq. (2.6) into Eq. (2.4) reduces Eq. (2.4) into a nonlinear ordinary differential equation

$$f^{iv}(z) + \frac{2\rho}{\mu}f(z)f'''(z) - \frac{\sigma B_0^2}{\mu r}f''(z) = 0. \tag{2.7}$$

Subject to the boundary conditions

$$f(0) = 0, \ f''(0) = 0,$$

$$f(H) = \frac{V}{2}, \ f'(H) = 0. \tag{2.8}$$

Nonlinear differential equation in Eq. (2.7) along with boundary conditions in Eq. (2.8) can be made dimensionless by using the following non-dimensional parameters

$$F = \frac{f}{V/2}, \ x = \frac{z}{H}, \ Re = \frac{\rho H}{\mu/V}, \ M = \sqrt{\frac{\sigma H B_0^2}{\mu}}, \tag{2.9}$$

$$F^{iv}(x) + Re\, F(x)F'''(x) - M^2F''(x) = 0, \tag{2.10}$$

with boundary conditions converted into the form

$F(0) = 0,\ F''(0) = 0,\ F(1) = 1,\ F'(1) = 0.$

2.2. *Jeffery–Hamel flows*

For the flow of an incompressible viscous fluid due to either a source or a sink that is present at the intersection of two rigid, nonparallel plane walls; angle between walls is 2α as shown in Fig. A. Flow is assumed to be symmetric and purely radial. These assumptions mean that the velocity field is of the form $V = [u_r, 0, 0]$, where μ_r is a function of both r and θ.

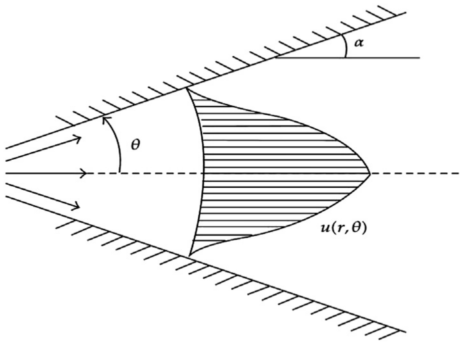

Fig. A – Schematic diagram of the problem.

In polar coordinates equations of motion in the absence of body forces given in Ref. [24] are

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) = 0, \tag{2.11}$$

$$u_r\frac{\partial u_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \upsilon\left[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{u_r}{r^2}\right], \tag{2.12}$$

$$-\frac{1}{pr}\frac{\partial p}{\partial r} + \frac{2\upsilon}{r^2}\frac{\partial u_r}{\partial \theta} = 0, \tag{2.13}$$

where ρ, is constant density; μ, is dynamic viscosity and p is pressure.

Boundary condition at the center line of the channel is

$$\psi_{n,m}(x) = \begin{cases} 2^{\frac{k}{2}}\sqrt{\dfrac{4}{(b-a)\pi}}T_m(2^k x - \widehat{n}), & a + (b-a)\dfrac{\widehat{n}}{2^k} \le x \le a + (b-a)\dfrac{\widehat{n}+1}{2^k}, \\ 0, & \text{otherwise} \end{cases} \tag{3.1}$$

$$\frac{\partial u_r}{\partial \theta} = 0, \tag{2.14}$$

and on the walls are $u_r = 0$.

From continuity equation (2.11), we have

$$f(\theta) = ru_r. \tag{2.15}$$

For making equations dimensionless parameters are defined as

$$F(x) = \frac{f(\theta)}{f_{\max}},\quad x = \frac{\theta}{\alpha}. \tag{2.16}$$

From Eqs. (2.12) and (2.13) after eliminating pressure terms and using Eqs. (2.15) and (2.16), we get a nonlinear ordinary differential equation for normalized velocity profile $F(x)$

$$F'''(x) + 2\alpha ReF(x)F'(x) + 4\alpha^2 F'(x) = 0. \tag{2.17}$$

Accordingly the boundary conditions (2.14) are

$$F(0) = 1,\ F'(0) = 0,\ F(1) = 0, \tag{2.18}$$

where Re is Reynolds number given by

$$Re = \frac{f_{\max}}{\upsilon} = \frac{U_{\max} r\alpha}{\upsilon}\left(\begin{array}{l} \text{Divergent Channel}: \ \alpha > 0,\ U_{\max} > 0 \\ \text{Convergent Channel}: \ \alpha < 0,\ U_{\max} < 0 \end{array}\right), \tag{2.19}$$

$U_{\max}$ here is center line velocity.

3. Shifted Chebyshev wavelets

In the present work, we use the shifted Chebyshev polynomials on $[a,b]$, so the shifted Chebyshev nodes are

$$x_k = \frac{b-a}{2}\cos\left(\frac{(2k+1)\pi}{2M}\right) + \frac{a+b}{2},\quad k = 0, 1, 2, \ldots, M-1,$$

where a and b are real numbers with $a < b$. The shifted Chebyshev polynomials $T_m(x)$ of order m are defined on the interval $[a,b]$ and are given by the following recurrence formulae,

$$T_0(x) = 1,\ T_1(x) = \frac{2x-(b+a)}{b-a},\ T_{m+1}(x) = 2\left(\frac{2x-(b+a)}{b-a}\right)T_m(x) - T_{m-1}(x),\quad m = 1, 2, 3, \ldots.$$

The orthogonality conditions is

$$\int_a^b \frac{1}{\sqrt{1-\left(\frac{2x-(b+a)}{b-a}\right)}}T_m(x)T_n(x)dx = \begin{cases} 0, & m \neq n; \\ \left(\dfrac{b-a}{2}\right)\pi, & m = n. \end{cases}$$

Shifted Chebyshev wavelets defined on the interval $[a,b]$ as2

k = 1,2,3,..., is the level of resolution, n = 1,2,3,...,2^{k-1} is the translation parameter, m = 1,2,3,...,$M-1$ is the order of the Chebyshev polynomials, $M > 0$. The solution obtained by Chebyshev wavelets is of the form

$$y(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\psi_{n,m}(x),$$

where $\psi_{n,m}(x)$ is given by equation (3.1). We approximate $y(x)$ by the truncated series

$$y(x) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x). \tag{3.2}$$

Then a total number of conditions $2k^{-1}M$ should exist for determination of $2k^{-1}M$ coefficients $c_{10}, c_{11}, \ldots, c_{1M-1}, c_{20}, c_{21}, \ldots, c_{2M-1}, \ldots, c_{2^k-10}, c_{2^k-11}, \ldots, c_{2^{k-1}M-1}$.

Some conditions are furnished by the initial or boundary conditions, while for remaining conditions we replace $y_{K,M}$ in our differential equation to recover the unknown coefficients $c_{n,m}$.

4. Quasilinearization

The quasilinearization [25,26] approach is a generalized Newton–Raphson technique for differential equations. The quasilinearization technique converges quadratically to the exact solution if there is convergence at all and it has monotone convergence.

Consider a nonlinear second order differential

$$y''(x) = f(y(x), x) \tag{4.1}$$

with the boundary conditions

$$y(a) = \alpha \text{ and } y(b) = \beta, \quad a \le x \le b, \tag{4.2}$$

where f may be a function of x or $y(x)$. Let $y_0(x)$, be an initial approximation of the function $y(x)$. The Taylor's series expansion of f about $y_0(x)$ is

$$f(y(x), x) = f(y_0(x), x) + (y(x) - y_0(x))f_{y_0}(y_0(x), x) + O\left((y(x) - y_0(x))^2\right). \tag{4.3}$$

Ignoring second and higher order terms and replacing in Eq. (4.1), we get

$$y''(x) = f(y_0(x), x) + (y(x) - y_0(x))f_{y_0}(y_0(x), x) \tag{4.4}$$

solving Eq. (4.4) and calling answer $y_1(x)$. Using $y_1(x)$ and again expanding Eq. (4.1) about $y_1(x)$, we have

$$y''(x) = f(y_1(x), x) + (y(x) - y_1(x))f_{y_1}(y_1(x), x) \tag{4.5}$$

after simplification we get $y_2(x)$, second approximation to $y(x)$. Continuing this process we obtain the desired accuracy if the problem converges. Generally we can write the recurrence relation in the following form

$$y''_{n+1}(x) = f(y_n(x), x) + (y(x) - y_n(x))f_{y_n}(y_n(x), x) \tag{4.6}$$

in which $y_n(x)$ is known and after solving we get $y_{n+1}(x)$. The boundary condition in (4.2) is also converted into the form $y_{n+1}(a) = \alpha$ and $y_{n+1}(b) = \beta$. Same procedure can be applied on other higher order nonlinear problems also.

5. Convergence analysis

Since we are using both quasilinearization technique and Chebyshev wavelets method so we discuss the convergence for both of these.

5.1. *Convergence of quasilinearization technique*

The convergence of quasilinearization technique is derived and discussed in Ref. [25] which shows that the convergence of quasilinearization technique is second order if there is convergence at all.

5.2. *Convergence of Chebyshev wavelets method*

The convergence of Chebyshev wavelets method is derived in Ref. [20] which shows that the series solution by Chebyshev wavelets method converges to $y(x)$ for differential equation of any order.

6. Solution procedure

6.1. *For MHD squeezing flow between two infinite plates*

Differential equation of MHD squeezing flow (2.10) after applying quasilineariztion technique becomes

$$\frac{d^4}{dx^4}F_{n+1}(x) + ReF_n(x)\frac{d^3}{dx^3}F_{n+1}(x) + ReF_{n+1}(x)\frac{d^3}{dx^3}F_n(x) - M^2\frac{d^2}{dx^2}F_{n+1}(x) = ReF_n(x)\frac{d^3}{dx^3}F_n(x), \tag{6.1}$$

with boundary conditions converted into the form

$$F_{n+1}(0) = 0, \quad \frac{d^2}{dx^2}F_{n+1}(0) = 0, \quad F_{n+1}(1) = 1, \quad \frac{d}{dx}F_{n+1}(1) = 0.$$

For applying Chebyshev wavelets technique now substitute,

$$F_{n+1}(x) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x)$$

Eq. (6.1) becomes (see Tables 1 and 2, Figs. 1 and 2),

$$\frac{d^4}{dx^4}\left(\sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x)\right) + ReF_n(x)\frac{d^3}{dx^3}\left(\sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x)\right) + Re\left(\sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x)\right)\frac{d^3}{dx^3}F_n(x) - M^2\frac{d^2}{dx^2}\left(\sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x)\right) = ReF_n(x)\frac{d^3}{dx^3}F_n(x). \tag{6.2}$$

6.2. *For Jeffery–Hamel flow*

Eq. (2.17) representing Jeffery–Hamel flow after applying quasilinearization becomes,

$$\frac{d^3}{dx^3}F_{n+1}(x) + 2\alpha ReF_{n+1}(x)\frac{d}{dx}F_n(x) + 2\alpha ReF_n(x)\frac{d}{dx}F_{n+1}(x) + 4\alpha^2\frac{d}{dx}F_{n+1}(x) = 2\alpha ReF_n(x)\frac{d}{dx}F_n(x). \tag{6.3}$$

Accordingly the boundary conditions are converted into

Table 1 – Comparison of solutions for MHD squeezing flow (6.2) for $M = 1$ and $Re = 1$, for different polynomial values of proposed method with RK-4.

x	$y_{m=5}$	$y_{m=10}$	$y_{m=15}$	$y_{RK\text{-}4}$	Error at $y_{m=5}$	Error at $y_{m=10}$	Error at $y_{m=15}$
0.0	0.000000	0.000000	0.000000	0.000000	8.8356E−10	2.0000E−13	2.0000E−17
0.1	0.179073	0.149529	0.150294	0.150294	2.8779E−02	7.6409E−04	9.7110E−07
0.2	0.350522	0.296018	0.297481	0.297481	5.3041E−02	1.4620E−03	1.0152E−06
0.3	0.508061	0.436441	0.438468	0.438468	6.9594E−02	2.0258E−03	1.5166E−06
0.4	0.646863	0.567797	0.570190	0.570190	7.6674E−02	2.3914E−03	2.0176E−06
0.5	0.763564	0.687121	0.689626	0.689626	7.3940E−02	2.5020E−03	2.2772E−06
0.6	0.856258	0.791478	0.793798	0.793798	6.2462E−02	2.3179E−03	2.4621E−06
0.7	0.924503	0.877942	0.879781	0.879781	4.4724E−02	1.8369E−03	2.7129E−06
0.8	0.969314	0.943567	0.944697	0.944697	2.4618E−02	1.1287E−03	1.6322E−06
0.9	0.993167	0.985322	0.985707	0.985707	7.4602E−03	3.8448E−04	5.7870E−07
1.0	1.000000	1.000000	1.000000	1.000000	1.0250E−06	2.3752E−10	6.7291E−13

Table 2 – Comparison of solutions for MHD squeezing flow (6.2) when $M = 5$ and $Re = 5$, for different polynomial values of proposed method with RK-4.

x	$y_{m=5}$	$y_{m=10}$	$y_{m=20}$	$y_{RK\text{-}4}$	Error at $y_{m=5}$	Error at $y_{m=10}$	Error at $y_{m=20}$
0.0	0.000000	0.000000	0.000000	0.000000	1.5195E−12	3.0000E−14	2.0000E−17
0.1	0.139490	0.124266	0.131396	0.131398	8.0912E−03	7.1321E−04	2.5254E−06
0.2	0.277545	0.247984	0.261827	0.261830	1.5712E−02	2.3848E−03	5.5279E−06
0.3	0.412278	0.370508	0.390154	0.390153	2.2116E−02	5.9653E−03	7.6114E−06
0.4	0.541307	0.490960	0.514882	0.514878	2.6415E−02	2.3931E−02	9.0962E−05
0.5	0.661754	0.608014	0.633961	0.633959	2.7782E−02	2.5957E−02	1.0881E−05
0.6	0.770250	0.719551	0.744547	0.744549	2.5691E−02	5.5007E−03	1.1488E−05
0.7	0.862929	0.822111	0.842734	0.842736	2.0183E−02	3.0634E−03	1.1001E−05
0.8	0.935432	0.910033	0.923211	0.923213	1.2212E−02	2.3186E−03	8.5481E−06
0.9	0.982904	0.974168	0.978825	0.978827	4.0748E−03	4.6613E−04	4.7314E−06
1.0	0.999997	0.999999	1.000000	1.000000	3.0000E−06	1.0000E−06	3.0000E−09

$$F_{n+1}(0) = 1,\ \ \frac{d}{dx}F_{n+1}(0) = 0,\ \ F_{n+1}(1) = 0.$$

For applying Chebyshev wavelets technique now substitute,

$$F_{n+1}(x) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x).$$

Eq. (6.3) becomes (see Tables 3 and 4, Figs. 3 and 4)

$$\frac{d^3}{dx^3}\left(\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x)\right) + 2\alpha Re\left(\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x)\right)\frac{d}{dx}F_n(x) + 2\alpha ReF_n(x)\frac{d}{dx}\left(\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x)\right) + 4\alpha^2\frac{d}{dx}\left(\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x)\right) = 2\alpha ReF_n(x)\frac{d}{dx}F_n(x). \quad (6.4)$$

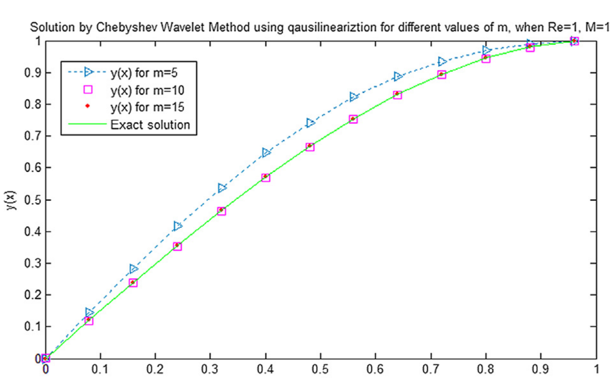

Fig. 1 – Comparison of MHD squeezing flow solutions when $M = 1$ and $Re = 1$ for different values of m for proposed method with RK-4.

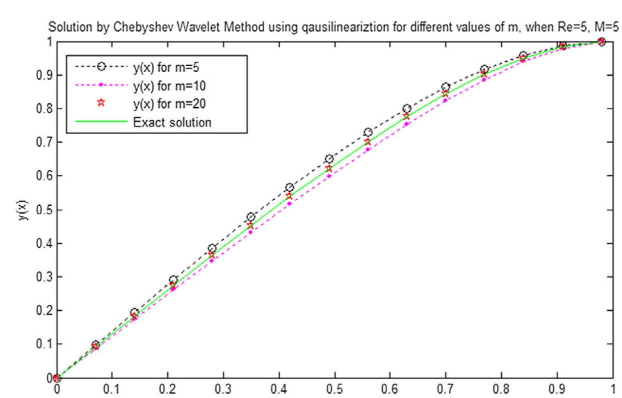

Fig. 2 – Comparison of MHD squeezing flow solutions when $M = 1$ and $Re = 1$ for different values of m for proposed method with RK-4.

Table 3 – Comparison of solutions for Jeffery–Hamel flow (6.4) when $\alpha = 3$ and $Re = 10$ (diverging channel), for different polynomial values of proposed method with RK-4.

x	$y_{m=5}$	$y_{m=10}$	$y_{m=15}$	$y_{RK\text{-}4}$	Error at $y_{m=5}$	Error at $y_{m=10}$	Error at $y_{m=15}$
0.0	1.00000000	1.00000000	1.00000000	1.000000	0.0000E+00	8.0000E−15	0.0000E+00
0.1	0.99157877	0.98926632	0.98927742	0.989277	2.3017E−03	1.0674E−05	4.2882E−07
0.2	0.96567510	0.95717777	0.95722187	0.957222	8.4531E−03	4.4222E−05	1.2284E−07
0.3	0.92129433	0.90406280	0.90416150	0.904162	1.7132E−02	9.9194E−05	4.9197E−07
0.4	0.85739549	0.83044063	0.83061596	0.830616	2.6779E−02	1.7536E−04	3.1634E−08
0.5	0.77289138	0.73698202	0.73725696	0.737257	3.5634E−02	2.7497E−04	3.5067E−08
0.6	0.66664851	0.62445984	0.62485587	0.624856	4.1792E−02	3.9615E−04	1.2573E−07
0.7	0.53748710	0.49369572	0.49422070	0.494221	4.3266E−02	5.2527E−04	2.9941E−07
0.8	0.38418113	0.34551120	0.34612473	0.346125	3.8056E−02	6.1379E−04	2.6389E−07
0.9	0.20545829	0.18069489	0.18122899	0.181229	2.4229E−02	5.3410E−04	2.0706E−09
1.0	0.00000000	0.00000000	0.00000000	0.000000	2.0797E−09	1.0512E−11	2.5103E−15

Table 4 – Comparison of solutions for Jeffery–Hamel Flow (6.4) when $\alpha = -15$ and $Re = 50$ (converging channel), for different polynomial values of proposed method with RK-4.

x	$y_{m=5}$	$y_{m=10}$	$y_{m=20}$	$y_{RK\text{-}4}$	Error at $y_{m=5}$	Error at $y_{m=10}$	Error at $y_{m=20}$
0.0	1.000000	1.000000	1.000000	1.00000	0.0000E+00	1.0000E−15	0.0000E+00
0.1	0.995146	0.994182	0.994204	0.994204	9.4273E−04	2.1705E−05	2.1293E−07
0.2	0.979819	0.976258	0.976335	0.976335	3.4846E−03	7.6169E−05	1.1514E−10
0.3	0.952012	0.944758	0.944924	0.944925	7.0877E−03	1.6684E−04	2.4876E−08
0.4	0.908579	0.897127	0.897461	0.898461	1.1118E−02	3.3309E−04	4.8795E−07
0.5	0.845234	0.829676	0.830308	0.830309	1.4925E−02	6.3271E−04	2.1740E−07
0.6	0.756551	0.737579	0.738653	0.738653	1.7898E−02	1.0734E−03	5.2853E−07
0.7	0.635962	0.615016	0.616549	0.616549	1.9413E−02	1.5325E−03	9.1173E−07
0.8	0.475762	0.455492	0.457191	0.457191	1.8571E−02	1.6981E−03	8.0688E−07
0.9	0.267103	0.252419	0.253604	0.253604	1.3500E−02	1.1832E−03	1.2011E−06
1.0	0.000000	0.000000	0.000000	0.000000	1.0000E−07	2.0000E−10	1.0000E−16

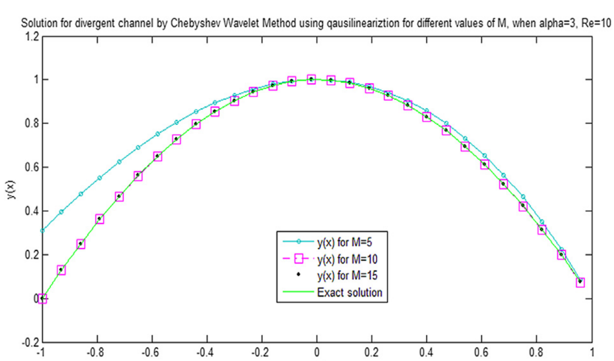

Fig. 3 – Comparison of Jeffery–Hamel flow when $\alpha = 3$ and $Re = 10$ for different values of M for proposed method with RK-4.

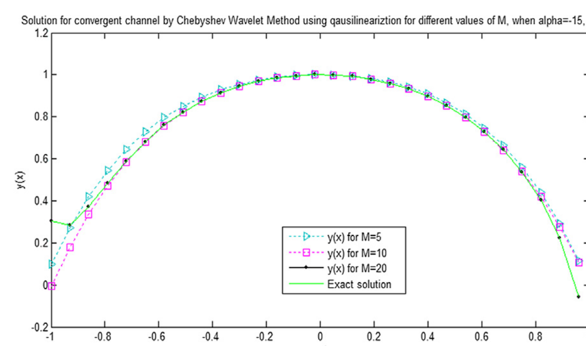

Fig. 4 – Comparison of Jeffery–Hamel flow when $\alpha = -15$ and $Re = 50$ for different values of M for proposed method with RK-4.

7. Conclusion

This article investigates the MHD flow between two parallel plates and Jeffery–Hamel flow. Shifted Chebyshev wavelets-quasilinearization technique is applied to solve the equations of flow. In order to check the accuracy of the solution obtained by shifted Chebyshev wavelets-quasilinearization technique at different values of polynomials figures and tables are drawn which shows that the accuracy of method is increased when we increase the order of the polynomial. Numerical solution is also sought out for the sake of comparison. It is clear from Figures and Tables that this method can be applied successfully to different problems of physical nature.

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