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The Approximate Correctness of Systems Based on δ-bisimulation

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Abstract

The correctness of system is an important attribute to quantify the quality. δ-bisimulation based on complete lattices have been proposed to generalize the classical bisimulation. To analyze the implementations of system approximates its specification step by step, the infinite evolution mechanism of δ- bisimulation is established. Firstly, the relations between the implementations and specification under δ-bisimulation are analyzed, δ-limit bisimulation is defined and some examples of δ-limit bisimulations are given. Then, δ-bisimulation limit is proposed to state the specification is the limit of implementations. Some algebraical properties of δ-bisimulation limit are proved. Finally, in order to use the flexible hierarchic development and modular design methods to archive the limit, the continuous of δ-bisimulation limit under various combinators are showed.

Keywords: limit, correctness, bisimulation, complete lattice, fuzzy system

1 Introduction

Correctness is always very important to evaluate the quality of the system. According the theory of process algebra, the correctness can be described as the equivalence relation between implementation and specification, such as bisimulation equivalence,

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trace equivalence, and so on. However, in real world situations, there exist many hardware equipments in a system, it is difficult to guarantee the completely equivalence. For a lots of systems, the implementations are approximately satisfied by specification [1,2,3]. In Ying's opinions, there are at least two ways to loosen the correctness of system: the static approach and the dynamic approach [4,5]. In the static opinion, a certain small error between the implementation of a system and its specification is allowed. In this case, the system may be seen as an approximate implementation of the specification. A number of theories have been presented to describe approximate correctness in this method (for example [6,7,8,9]). The idea of these theories is to generalize the concept of equivalence with the psedometric, aiming at measuring the behavioral dissimilarities between nonequivalent models.

During the design of system, it is necessary to verify the correctness during the development. However, many reasons can lead to the first implementation can not satisfy specification, such as the inconsistence of program language and experience of developer, and so on. Hence the implementation should be modified step by step. An evolution sequence of implementations is obtained. From the course of design, we notice that the specification is the finial aim for the sequence of implementations. It means that the system is more and more correctness during the design. In order to describe this characterization, Ying proposed the dynamic approach to research the correctness [5]. It means that a sequence of implementations approximates the specification closer and closer. This sequence of implementations can be seen as an evolution toward the specification. Ying [5] proposed strong/weak limit bisimulations and strong/weak bisimulation limits to describe the infinite evolution of system correctness. In order to characterize the influence of environment, the first author of this paper presented parameterized limit bisimulation and two-thirds limit bisimulations [10,11]. Recently, ϵ-limit bisimulation and ϵ-bisimulation limit in deterministic probabilistic processes were presented by the first author to characterize the dynamic correctness of a probabilistic system [12].

In order to obtain the approximate of system, the behavioral equivalences are extended to the quantitative cases and limit cases. However, most of these extensions mainly focus on numerical behavioral equivalence semantics of a labeled transition systems(LTSs), where the truth values of metric between label actions come from *real numbers*. From the view of partial order set, the truth value in real numbers are comparable. But, Pan et al [13,14] showed some example to state that it is possible that the truth values in transition system are not numbers and are not linearly ordered. Based on this case, they extend the notion of simulation to the residuated lattice-valued setting[15]. Firstly, they generalize the labeled transition system to quantitative transition system (QTS). Then, δ-simulation is defined on QTS, which relaxes the equality of labels transitions in the classical simulation by allowing to perform transitions induced by different labels, as long as the truth degree between the labels is greater than or equal to the threshold δ taken from a certain lattice. Naturally, we can achieve the δ-bisimulation to offer a convenient co-inductive proof technique for establishing behavioral equivalence.

Although some interesting aspects of δ-simulation proposed by Pan et al. were explored, δ-bisimulation generalize the classical bisimulation and the infinite evolution of system correct remains unexplored when δ-bisimulation is chosen to verify the approximate correctness of the system. In real world situations, if the truth values of a system depending on are not numbers and are not linearly ordered, δ-bisimulation can be chosen to verify the approximate correctness of the system. Therefore, we need to characterize the infinite evolution mechanism of implementations. The goal of this paper is to generalize δ-bisimulation to δ-limit bisimulation and δ-bisimulation limit. Furthermore, their algebraic properties will be discussed.

In Section 2, some concepts and results of CCS and δ-bisimulation are reviewed. In Section 3, δ-limit bisimulation is defined and some examples are shown. δ-bisimulation limit is proposed in Section 4. The uniqueness of δ-bisimuation limit and consistence with δ-bisimulation are proved. The substitutivity laws of δ-bisimulation limit under various combinators are stated in Section 5. Finally, we conclude the paper in Section 6.

2 Preliminaries

2.1 CCS summary

Let $Act = \Gamma \cup \{\tau\}$ be the set of *actions*, where τ is the silent or perfect actions, Γ be the set of labels. Define that $l, l', \cdots$ range over Γ. The detail definition of the syntax of Communication and Concurrency System(CCS) can be found in [16]. Next, we only show the definition of process expressions.

Definition 2.1 [Process expression][16] The set of process expressions $\mathcal{E}$ includes $\aleph, \mathcal{K}$ and the following expressions:

$$E ::= \alpha.E \mid \sum_{i \in I} E_i \mid E_1 \mid E_2 \mid E\backslash X \mid E[f],$$

where $X \subseteq \Gamma$,$f : \Gamma \to \Gamma$ is a relabeling function, $\aleph$ is the set of processes variables, $\mathcal{K}$ is the set of processes constants and I is indexing set.

Generally, we call processes is the set of process expressions without variables, written by the sign $\mathcal{P}$. The transitional semantics of CCS is presented in [17], which is the structural operational semantics.

Definition 2.2 [Labeled transition system(LTS)] [16] The transition relations $\overset{\alpha}{\to}$ $(\alpha \in Act)$ of a labeled transition system $\sigma = (\mathcal{E}, Act, \{\overset{\alpha}{\to} : \alpha \in Act\})$ are showed in the following rules:

$$\textbf{Act}\frac{}{\alpha.E \xrightarrow{\alpha} E} \qquad \textbf{Sum}_\textbf{j}\frac{E_j \xrightarrow{\alpha} E'_j}{\sum_{i\in I} E_i \xrightarrow{\alpha} E'_j}$$

$$(j \in I)$$

$$\textbf{Com}_\textbf{1}\frac{E \xrightarrow{\alpha} E'}{E \mid F \xrightarrow{\alpha} E' \mid F} \quad \textbf{Com}_\textbf{2}\frac{F \xrightarrow{\alpha} F'}{E \mid F \xrightarrow{\alpha} E \mid F'}$$

$$\textbf{Com}_\textbf{3}\frac{E \xrightarrow{l} E'\ F \xrightarrow{\bar{l}} F'}{E \mid F \xrightarrow{\tau} E' \mid F'} \quad \textbf{Res}\frac{E \xrightarrow{\alpha} E'}{E\backslash X \xrightarrow{\alpha} E'\backslash X}$$

$$(\alpha, \bar{\alpha} \notin X)$$

$$\textbf{Rel}\frac{E \xrightarrow{\alpha} E'}{E[f] \xrightarrow{f(\alpha)} E'[f]} \qquad \textbf{Con}\frac{P \xrightarrow{\alpha} P'}{A \xrightarrow{\alpha} P'}(A \overset{def}{=} P)$$

2.2 δ-simulation

Pan et al. presented a new extension of LTS called quantitative transition systems (QTS), where the set of labels is equipped with a residuated lattice-valued version of classical equivalence relation. And, in order to extend the classical bisimulation to residuated lattice, δ-simulation has been defined in [15]. Next, we will review some definitions of lattice and δ-simulation. For the notations of lattice, the detailing contents can be found in the literature [18].

The lattice as a poset will be denoted by $(L, \preceq)$, and the lattice as an algebra $(L, \wedge, \vee)$. Sometimes we write simply L to denote the lattice in both senses. According to the theory of lattice, let $(L, \wedge, \vee)$ be a complete lattice and Y be a nonempty set, then any function $f : Y^2 \to L$ is an L-valued (lattice-valued) relation on Y.

For an L-valued relation f, if $f(x, x) = 1$ for all $x \in Y$, then f is reflexive; if $f(x, y) = f(y, x)$ for all $x, y \in Y$, then f is symmetric; if $f(x, y) \wedge f(y, z) \preceq f(x, z)$ for all $x, y, z \in Y$, then f is transitive. By the definition of equivalence relation, we know that an L-valued relation on Y is called an L-valued equivalence relation if f is reflexive,symmetric and transitive. An L-valued equivalence relation on Y satisfying that $f(x, y) = 1$ implies $x = y$ will be called an L-valued equality relation.

Next, we will recall the definition of quantitative transition system and δ-simulation that was proposed by Pan et al [15].

Definition 2.3 [15] A quantitative transition system (for short, QTS) is $\mathcal{Q} = (\mathcal{A}, \theta)$, where

- $\mathcal{A} = (S, \Sigma, R)$, which is called the support set of QTS, is an LTS, where S is the set of states, Σ is the set of labels, and R is the set of transition relations;
- θ is an L-valued equality relation on Σ.

An LTS can be viewed as a degenerate QTS when θ is crisp. Because QTS generalizes LTS, we can use QTS to get the transitional semantics of process expresses in order to describe the correctness of system more generally when the truth values of labels are not numbers. The transition rules are same as LTS. The differences are that the set of states in QTS is set of process expresses, the set of labels is Act, the transition relations is $\xrightarrow{\alpha}$ $(\alpha \in Act)$ and there is an L-valued equality relation θ on Act. Thus, we can rewrite the δ-simulation on processes $\mathcal{P}$ according to the δ-simulation of Pan. etal [15]

Definition 2.4 [δ-simulation][15] Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS and $\delta \in L$. A relation $S_\delta \subseteq \mathcal{P}\times\mathcal{P}$ is called a δ-simulation if for any $(W, V) \in S_\delta$ and for each $\alpha \in Act$, when $W \xrightarrow{\alpha} W'$, there exist $\beta \in Act$ and $V \xrightarrow{\beta} V'$ such that $\theta(\alpha, \beta) \succeq \delta$ and $(W', V') \in S_\delta$.

In order to offer a convenient co-inductive proof technique for establishing behavioral equivalence, according to the definition of δ-simulation, naturally, we can define δ-bisimulation.

Definition 2.5 [δ-bisimulation] Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS and $\delta \in L$. A relation $S_\delta \subseteq \mathcal{P} \times \mathcal{P}$ is called a δ-bisimulation if for any $(W, V) \in \mathcal{S}_\delta$ and for each $\alpha \in Act$,

- if $W \xrightarrow{\alpha} W'$, there exist $\beta \in Act$ and $V \xrightarrow{\beta} V'$ such that $\theta(\alpha, \beta) \succeq \delta$ and $(W', V') \in S_\delta$;
- if $V \xrightarrow{\beta} V'$, there exist $\alpha \in Act$ and $W \xrightarrow{\alpha} W'$ such that $\theta(\alpha, \beta) \succeq \delta$ and $(W', V') \in S_\delta$.

As usual, we use the symbol $W \sim_\delta V$ to express W and V are δ-bisimilar if there exists a δ-bisimulation S such that $(W, V) \in S$. In other words, δ-bisimilarity $\sim_\delta$ is defined as $\sim_\delta = \bigcup\{\mathcal{S}_\delta : \mathcal{S}_\delta \text{ is } \delta - \text{bisimulation}\}$.

3 δ-limit bisimulation

When δ-bisimulation is chosen as the criteria of verifying the correctness between specification and implementation, the implementation may not satisfy the specification. To ensure that the implementation satisfies the specification, the implementation will have to be modified.

A series of implementations are produced. For a simple system, these implementations can form a sequence $\{W_n : n \in \omega\}$, where ω is natural number set. In real world situations, $W_1, W_2, \cdots$ may not satisfy the specification V, but there exists $n_0 \in \omega$ such that for any $n \geq n_0$, W_n can satisfy the specification V. According to the view of topology, specification V can be treated as the limit of these implementations. Generally, for a complex system, it is consist of several modular that the modifications of implementation might be done by different modular at the same time. In this case, implementations after modification can form a partial order. Therefore, we can use the nets of processes to characterize this case. By the theory of domain, the net is a mapping from a directed set to a nonempty set. The net of processes is also a mapping from a directed set to processes set. For more details on directed set, cofinality, cofinality subset, net and subnet, the reader may refer to the literature [19,20].

We write $\mathcal{P}_N$ for the class of all nets on $\mathcal{P}$. For any $\{P_n : n \in D\} \in \mathcal{P}_N$, where D is directed set, $P_n \in \mathcal{P}$ for any $n \in D$. Now, we give the definition of δ-limit bisimulation as follows.

Definition 3.1 [δ-limit bisimulation] Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS and $\delta \in L$, where $\mathcal{A} = (\mathcal{P}, Act, \{\xrightarrow{\alpha}: \alpha \in Act\})$, θ is an L-valued equality relation over Act. A relation $S_\delta \subseteq \mathcal{P} \times \mathcal{P}_N$. If $(W, \{V_n : n \in D\}) \in S_\delta$ for any $\alpha \in Act$ satisfied the following conditions, then S_δ is called a δ-limit bisimulation:

- if $W \xrightarrow{\alpha} W'$, then there are $\beta_n \in Act$ for every $n \in D$, $\{V'_n : n \in D\} \in \mathcal{P}_N$, and $n_0 \in D$ such that $V_n \xrightarrow{\beta_n} V'_n$, $\theta(\alpha, \beta_n) \succeq \delta$ for any $n \geq n_0$ and $(W', \{V'_n : n \in D\}) \in S_\delta$;
- if C is a cofinal subset of D, $V_m \xrightarrow{\beta_m} V'_m$ for any $m \in C$, then there are $\alpha \in Act$, $W' \in \mathcal{P}$, and a cofinal subset B of C such that $W \xrightarrow{\alpha} W'$, $\theta(\alpha, \beta_k) \succeq \delta$ for any $k \in B$ and $(W', \{V_k : k \in B\}) \in S_\delta$.

Compared to Definition2.5, the relation $S_\delta \subseteq \mathcal{P} \times \mathcal{P}_N$ in Definition 3.1 is more complicated than $S_\delta \subseteq \mathcal{P} \times \mathcal{P}$ in Definition 2.5. If specification of a system is abstracted as W, and the implementations which are obtained during the design are expressed as $\{V_n : n \in D\}$, then S_δ state the relation between specification and implementations. Furthermore, the conditions of δ-limit bisimulation need that W and V_n are related by δ-bisimulation for every $n \geq n_0$. Therefore, δ-limit bisimulation is an extension of δ-bisimulation.

Example 3.2 Let $P = a.0 + b.0$, $Q_n = a_n.0 + b_n.0$, $n \in \omega$. Then the transition diagrams are shown in Fig.1. Since $(\omega, \leq)$ is a directed set, $\{Q_n : n \in \omega\} \in \mathcal{P}_N$, $P \in \mathcal{P}$. Let $L = ([0, 1], \succeq)$, the partial order is great than and equal to on the set of the natural number order, and θ is an L-valued equality relation over the set of actions, where $\delta = 0.9 \in [0, 1]$.

$\theta(a, a) = \theta(b, b) = \theta(a_n, a_n) = \theta(b_n, b_n) = 1$,
$\theta(a, a_0) = \theta(b, b_0) = \frac{1}{4} \times 0.9$,
$\theta(a, a_1) = \theta(b, b_1) = \frac{2}{4} \times 0.9$,
$\theta(a, a_2) = \theta(b, b_2) = \frac{3}{4} \times 0.9$,·
$\theta(a, a_n) = \theta(b, b_n) = (n - (n - 1)) \times 0.9$ when $n \geq 3$, except the value of θ that have been defined, for any other actions $\theta(\alpha, \beta) = 0$, where $\alpha \neq a$ or b, $\beta \neq a_i$ or b_i, $i = 0, 1, \cdots, n$. The relation on $\mathcal{P} \times \mathcal{P}_N$ is $S_\delta = \{(W, \{V_n : n \in \omega\}), (W_1 = 0, \{W_n = 0 : n \in \omega\}), (P_2 = 0, \{V_n = 0 : n \in \omega\}), (0, \{0 : b \in B\})\}$, where B is any cofinal subset of ω.

According to Definition 3.1, we can find that $\mathcal{R}_\delta$ is a δ-limit bisimulation.

In fact, if $P \xrightarrow{a} P_1$, then there exist $a_n \in Act$ for $n \in \omega$, $\{Q'_n = W_n : n \in \omega\} \in \mathcal{P}_N$, and $n_0 = 3$ such that $Q_n \xrightarrow{a_n} Q'_n$, $\theta(a, a_n) \succeq \delta$ for any $n \geq n_0$ and $(0, \{Q'_n : n \in \omega\}) \in S_\delta$. If $P \xrightarrow{b} 0$, then there exist $b_n \in Act$ for $n \in \omega$, $\{Q'_n = V_n : n \in \omega\} \in \mathcal{P}_N$ and $n_0 = 3$ such that $Q_n \xrightarrow{b_n} Q'_n$, $\theta(b, b_n) \succeq \delta$ for any $n \geq n_0$ and $(0, \{Q'_n : n \in \omega\}) \in S_\delta$.

On the other hand, if C is a cofinal subset of ω and $Q_m \stackrel{a_m}{\rightarrow} Q'_m = 0$ for any $m \in C$, then we suppose that $C = \{n_l : l \in \omega\}$. Thus $Q_m = Q_{n_l} \stackrel{a_{n_l}}{\rightarrow} Q'_{n_l}$ for every $m \in C$. So, there exist $a \in Act$, $P' = 0$, and a cofinal subset $B = \{n_l + 4 : l \in \omega\}$ of C, such that $P \stackrel{a}{\rightarrow} P'$, $\theta(a, a_b) \succeq \delta$ for $b \in B$ and $(P', \{Q'_b : b \in B\}) \in S_\delta$. Finally, we prove that S_δ is a δ-limit bisimulation.

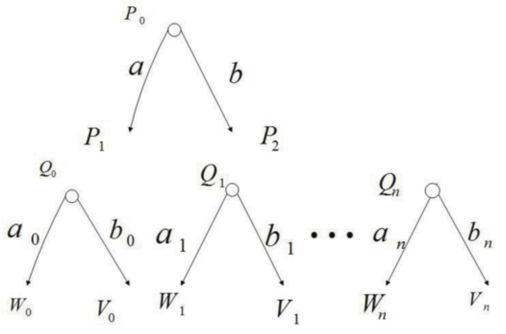

Fig. 1. An example of δ-limit bisimulation

According to the definition of δ-limit bisimulation, the identical relation between processes can be extended to limit case.

Proposition 3.3 *Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS and $\delta \in L$. Then,*

$$Ilim_{S_\delta} = \{(W, \{V_n : n \in D\}) : W \in \mathcal{P}, \{V_n : n \in D\} \in \mathcal{P}_N, \textit{and there exists}$$
$$n_0 \in D \textit{ such that } V_n = W \quad \textit{for each} n \geq n_0\}$$

is a δ-limit bisimulation.

Proposition 3.3 states that if the obtained implementations are same to specification after some modifications, then the relation between implementations and specification is δ-limit bisimulation.

Next, we will provide some attendant results, which are useful to describe the relation between specification and implementation of system. We consider the subnet closure of a relation between processes and nets of processes.

Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS, and $\delta \in L$, $S_\delta \subseteq \mathcal{P} \times \mathcal{P}_N$. Then we define

$$sub(S_\delta) = \{W, \{V_n : n \in D\}) : \text{ there exists } (W, \{W_m : m \in C\}) \in S_\delta \text{ such that}$$
$$\{V_n : n \in D\} \text{ is a subnet of } \{W_m : m \in C\}\}.$$

Proposition 3.4 *Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS, $\delta \in L$ and $S_\delta \subseteq \mathcal{P} \times \mathcal{P}_N$. Then $sub(S_\delta)$ is a δ-limit bisimulation if and only if for $(W, \{V_n : n \in D\}) \in S_\delta$ and $\alpha \in Act$,*

- *if $W \stackrel{\alpha}{\rightarrow} W'$, then there exist $\beta_n \in Act$ for every $n \in D$, $\{V'_n : n \in D\} \in \mathcal{P}_N$, and $n_0 \in D$ such that $V_n \stackrel{\beta_n}{\rightarrow} V'_n$, $\theta(\alpha, \beta_n) \succeq \delta$ for all $n \geq n_0$, and $(W, \{V_n : n \in D\}) \in sub(S_\delta)$;*

- *if C is a cofinal subset of D and $V_m \stackrel{\beta_m}{\to} V'_m$ for all $m \in C$, then there exist $\alpha \in Act$, $W' \in \mathcal{P}$, and a cofinal subset B of C such that $W \stackrel{\alpha}{\to} W'$, $\theta(\alpha, \beta_k) \succeq \delta$ for $k \in B$, and $(W', \{V_k : k \in B\}) \in sub(S_\delta)$.*

Proof. For the "only if": it follows from Definition 3.1.

For the "if" part: we need to prove $sub(S_\delta)$ is δ-limit bisimulation. Let $(W, \{V_n : n \in D\}) \in sub(S_\delta)$. Then by the construction of $sub(S_\delta)$, we can get $(W, \{W_m : m \in C\}) \in S_\delta$ such that $\{V_n : n \in D\}$ is a subnet of $\{W_m : m \in C\}$. Furthermore, according to the definition of subnet, that there is a mapping $f : D \to C$ such that (D, f) is a cofinality of C and $V_n = Q_{f_n}$ for each $n \in D$. We can suppose that the mapping f is increasing by the theory of domain, which means that $n_1 \leq n_2$ implies $f(n_1) \leq f(n_2)$.

If $W \stackrel{\alpha}{\to} W'$, then from the condition of Proposition 3.4, there are $\beta_m \in Act$ for every $m \in C$, $\{V'_m : m \in C\} \in \mathcal{P}_N$ and $m_0 \in C$ such that $V_m \stackrel{\beta_m}{\to} V'_m$, $\theta(\alpha, \beta_m) \succeq \delta$ for all $m \geq m_0$ and $(W', \{V'_m : m \in C\}) \in sub(S_\delta)$. Since (D, f) is a cofinality of C, there exists some $n_0 \in D$ with $f_n \geq m_0$ for each $n \geq n_0$. Let $V'_n = W'_{f_n}$ for every $n \in D$, we have that $V_n = W_{f_n} \stackrel{\beta_{f_n}}{\to} W'_{f_n} = V'_n$ for all $n \geq n_0$, where $\{V'_n : n \in D\}$ is a subnet of $\{W'_m : m \in C\}$. By the construction of $sub(S_\delta)$, $(W', \{V'_n : n \in D\}) \in sub(S_\delta)$. Therefore, if $W \stackrel{\alpha}{\to} W'$, then there exist $\gamma_n = \beta_{f_n} \in Act$ for every $n \in D$, $\{V'_n : n \in D\} \in \mathcal{P}_N$, and $n_0 \in D$ such that $V_n \stackrel{\gamma_n}{\to} V'_n$, $\theta(\alpha, \gamma_n) \succeq \delta$ for all $n \geq n_0$, and $(W', \{V'_n : n \in D\}) \in sub(S_\delta)$.

If B is a cofinal subset of D and $V_b \stackrel{\beta_b}{\to} V'_b$ for all $b \in B$, then $f(B)$ is a cofinal subset of C. Let $W'_{f_b} = V'_b$ for each $b \in B$. Then we can get that $W_{f_b} = V_b \stackrel{\beta_b}{\to} V'_b = W'_{f_b}$ for all $b \in B$. Furthermore, there exist $\alpha \in Act$, $W' \in \mathcal{P}$ and a cofinal subset H of $f(B)$ such that $W \stackrel{\alpha}{\to} W'$ and $(W', \{W'_r : r \in H\}) \in sub(S_\delta)$. Since f is increasing, $f^{-1}(H)$ is a cofinal subset of B. Thus, we have that $\{W'_r : r \in H\} = \{V'_b : b \in f^{-1}(H)\}$ and $(W', \{V'_b : b \in f^{-1}(H)\}) \in sub(S_\delta)$. □

From Proposition 3.4, it is easy to get the following corollary.

Corollary 3.5 *Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS, $\delta \in L$. If S_δ is a δ-limit bisimulation, then $sub(S_\delta)$ is also a δ-limit bisimulation.*

According to Definition 3.1, we can obtain that any union of δ-limit bisimulation is δ-limit bisimulation.

Corollary 3.6 *If $\delta \in L$, $(S_\delta)_i$ is a δ-limit bisimulation for each $i \in I$, then $\bigcup_{i \in I} (S_\delta)_i$ is also a δ-limit bisimulation.*

4 δ-Bisimulation Limit

In this section, we will present the definition of δ-bisimulation limit in order to describe the converge mechanism of system under δ-bisimulation. The δ-bisimulation limit characterizes the specification of system is the limit of implementations under δ-bisimulation.

Definition 4.1 Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS, $\delta \in L$, $W \in \mathcal{P}$ and $\{V_n : n \in D\} \in \mathcal{P}_N$.

- $W \in \mathcal{P}$ is called a δ-bisimulation limit of $\{V_n : n \in D\}$ if there is a δ-limit bisimulation S_δ such that $(W, \{V_n : n \in D\}) \in S_\delta$. We can use symbol $W \overset{\delta}{\sim} \lim\limits_{n \in D} V_n$ to express this limit.

Suppose that

$$\overset{\delta}{\sim} \lim = \bigcup \Big\{(W, \{V_n : n \in D\}) : W \in \mathcal{P}, \{V_n : n \in D\} \in \mathcal{P}_N \text{ and } W \overset{\delta}{\sim} \lim_{n \in D} V_n\Big\},$$

Then Corollary 3.6 tell us $\overset{\delta}{\sim} \lim$ is the greatest δ-limit bisimulation.

The following proposition provides the recursive mechanism of δ-bisimulation limit.

Proposition 4.2 *(Recursive definition) Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS, $\delta \in L$. Then $W \overset{\delta}{\sim} \lim\limits_{n \in D} V_n$ if and only if for any $\alpha \in Act$,*

(i) *if $W \overset{\alpha}{\to} W'$, then there are $\beta_n \in Act$ for $n \in D$, $\{V'_n : n \in D\} \in \mathcal{P}_N$ and $n_0 \in D$ such that $V_n \overset{\alpha}{\to} V'_n$ and $\theta(\alpha, \beta_n) \succeq \delta$ with $n \geq n_0$ and $W' \overset{\delta}{\sim} \lim\limits_{n \in D} V'_n$;*

(ii) *if C is a cofinal subset of D, $V_m \overset{\beta_m}{\to} Q_{m'}$ for any $m \in C$, then there are $\alpha \in Act$, $W' \in \mathcal{P}$ and B is a cofinal subset of C such that $W \overset{\alpha}{\to} W'$ and $\theta(\alpha, \beta_k) \succeq \delta, k \in B$ and $W' \overset{\delta}{\sim} \lim\limits_{k \in B} V'_k$.*

Proof. For the "only if" part: if $W \overset{\delta}{\sim} \lim\limits_{n \in D} V_n$, then according to Definition 3.1, we can easily get the conclusion.

For the "if" part: let

$$S_\delta = \{(W, \{V_n : n \in D\}) : for\ \alpha \in Act \text{ the conditions in Proposition 4.2 hold}\}.$$

In the following, we need to show S_δ is a δ-limit bisimulation. In fact, if $(W, \{V_n : n \in D\}) \in S_\delta$, then the conditions in Proposition 4.2 hold for all $\alpha \in Act$. Therefore, $W' \overset{\delta}{\sim} \lim\limits_{n \in D} V'_n (W' \overset{\delta}{\sim} \lim\limits_{k \in B} V'_k)$ holds. According to the "only if" part, $(W', \{V'_n : n \in D\}) \in S_\delta((W', \{V_k : k \in B\}) \in S_\delta)$. By the definition of δ- limit bisimulation, S_δ is a δ-limit bisimulation. So, $W \overset{\delta}{\sim} \lim\limits_{n \in D} V_n$.

□

According to the topology theories, the limit is unique. For δ-bisimulation limit, we also need to prove it is unique. The following proposition tells us if W and U are the limit of $\{V_n : n \in D\}$, then W and U are δ-bisimulation.

Proposition 4.3 *Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS, $\delta \in L$. $W, U \in \mathcal{P}$ and $\{V_n : n \in D\} \in \mathcal{P}_N$. If $W \overset{\delta}{\sim} \lim\limits_{n \in D} V_n$, $U \overset{\delta}{\sim} \lim\limits_{n \in D} V_n$, then $W \overset{\delta}{\sim} U$.*

Proof. Since $W \overset{\delta}{\sim} \lim\limits_{n\in D} V_n$, $U \overset{\delta}{\sim} \lim\limits_{n\in D} V_n$, we can find S_δ^1 and S_δ^2 are δ-limit bisimulations such that $(W, \{V_n : n \in D\}) \in S_\delta^1$, $(U, \{V_n : n \in D\}) \in S_\delta^2$. Next, we need to construct a δ-limit bisimulation $S_\delta \subseteq \mathcal{P} \times \mathcal{P}$ such that $(W, U) \in S_\delta$. By Proposition 3.4, we know that $sub(S_\delta^1)$ and $sub(S_\delta^2)$ are δ-limit bisimulations. Therefore, let $S_\delta = sub(S_\delta^1) \circ sub(S_\delta^2)$, where the composition of relations is defined as follows:

$$sub(S_\delta^1) \circ sub(S_\delta^2) = \{(W, U) : \text{there exists } \{V_n : n \in D\} \in \mathcal{P} \text{ such that } (W, \{V_n : n \in D\}) \in sub(S_\delta^1) \text{ and } (U, \{V_n : n \in D\}) \in sub(S_\delta^2)\}, \text{for any} S, S' \subseteq \mathcal{P} \times \mathcal{P}_N.$$

Now assume that $(W, \{V_n : n \in D\}) \in sub(S_\delta^1)$, $(V, \{V_n : n \in D\}) \in sub(S_\delta^2)$. Since S_δ^1 and S_δ^2 are δ-limit bisimulations, we know that $sub(S_\delta^1)$ and $sub(S_\delta^2)$ are also δ-limit bisimulations from Corollary 3.5. If $W \xrightarrow{\alpha} W'$, then there are $\{V_n' : n \in D\} \in \mathcal{P}_N$, $\beta_n \in Act$ for $n \in D$ and $n_0 \in D$ such that $V_n \xrightarrow{\beta_n} V_n'$ and $\theta(\alpha, \beta_n) \succeq \delta$ for every $n \geq n_0$, and $(W', \{V_n' : n \in D\}) \in sub(S_\delta^1)$. Since $D[n_0)$ is a cofinal subset of D, and $V_n \xrightarrow{\beta_n} V_n'$ for $n \geq n_0$, $(V, \{V_n : n \in D\}) \in sub(S_\delta^2)$ implies that there exist $V' \in \mathcal{P}$, $\gamma \in Act$ and a cofinal subset B of $D[n_0)$ satisfy $V \xrightarrow{\gamma} V'$, $(V', \{V_b : b \in B\}) \in sub(S_\delta^2)$ and $\theta(\gamma, \beta_b) \succeq \delta$ for $b \in B$. Thus, B being a cofinal subset of $D[n_0)$ leads to B is a subnet of D, and $\{V_b : b \in B\}$ is a subnet of $\{V_n' : n \in D\}$. Furthermore, $(W', \{V_b' : b \in B\}) \in sub(S_b^1)$. So, $(W', V') \in sub(S_b^1) \circ sub(S_\delta^2)$. Since θ is L-valued equality relation, $\theta(\alpha, \beta_n) \succeq \delta$ for $n \geq n_0$, $\theta(\gamma, \beta_n) \succeq \delta$ for $n \in B \subset D[n_0)$. So, $\theta(\alpha, \gamma) \succeq \delta$ for $n \in B$. Thus, if $V \xrightarrow{\alpha} V'$, then there are $W' \in \mathcal{P}$, $\beta \in Act$ such that $(W', V') \in sub(S_\delta^1) \circ sub(S_\delta^2)$ and $W \xrightarrow{\beta} W'$ and $\theta(\alpha, \beta) \succeq \delta$. □

Proposition 4.4 states if two specifications W and V are very similar, and the specification W is the limit of some implementations, then V is also the limit of these implementations.

Proposition 4.4 *Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS, $\delta \in L$, $W, V \in \mathcal{P}$, and $\{V_n : n \in D\} \in \mathcal{P}_N$. If $W \overset{\delta}{\sim} V$, and $W \overset{\delta}{\sim} \lim\limits_{n\in D} V_n$, then $V \overset{\delta}{\sim} \lim\limits_{n\in D} V_n$.*

Proof. Since $W \overset{\delta}{\sim} \lim\limits_{n\in D} V_n$, there exist $S_\delta \subseteq \mathcal{P} \times \mathcal{P}_N$ is a δ-limit bisimulation such that $(W, \{V_n : n \in D\}) \in S_\delta$. We can construct the following set.

$$S_\delta' = \{(V, \{V_n : n \in D\}) : \text{there exists } W \in \mathcal{P} \text{ such that } W \overset{\delta}{\sim} V \text{ and } (W, \{V_n : n \in D\}) \in S_\delta\}.$$

We only need to show S_δ' is a δ-limit bisimulation. In fact, let $(V, \{V_n : n \in D\}) \in S_\delta'$, then we have $W \in \mathcal{P}$ such that $W \overset{\delta}{\sim} V$ and $(W, \{V_n : n \in D\}) \in S_\delta$. If $V \xrightarrow{\alpha} V'$, then $W \overset{\delta}{\sim} V$ leads to there are $W' \in \mathcal{P}$ and $\beta \in Act$ such that $W \xrightarrow{\beta} W'$, and $\theta(\alpha, \beta) \succeq \delta$, and $W' \overset{\delta}{\sim} V'$. Since $(W, \{V_n : n \in D\}) \in S_\delta$, there exist $\gamma_n \in Act$ for $n \in D$, $\{V_n' : n \in D\} \in \mathcal{P}_N$, and $n_0 \in D$ such that $V_n \xrightarrow{\gamma_n} V_n'$, $\theta(\beta, \gamma_n) \succeq \delta$ for every $n \geq n_0$ and $(W', \{V_n' : n \in D\}) \in S_\delta$. Thus we get that $(V', \{V_n' : n \in D\}) \in S_\delta'$. Since L is a complete lattice and θ is an L-valued equality relation, the transitive is satisfied, i.e., $\delta \preceq \theta(\alpha, \beta) \wedge \theta(\beta, \gamma_n) \preceq \theta(\alpha, \gamma_n)$, i.e., $\theta(\alpha, \gamma_n) \succeq \delta$ for every $n \geq n_0$.

On the other hand, if B is a cofinal subset of D and $V_b \xrightarrow{\beta_b} V'_b$ for each $b \in B$, then $(W, \{V_n : n \in D\}) \in S_\delta$ implies that there exist $\beta \in Act$, $W' \in \mathcal{P}$, and a cofinal subset H of B such that $W \xrightarrow{\beta} W'$, $\theta(\alpha_k, \beta) \succeq \delta$ for $k \in H$, and $(W', \{V_k : k \in H\}) \in S_\delta$. Since $W \overset{\delta}{\sim} V$, there is $\gamma \in Act$ and $V' \in \mathcal{P}$ such that $V \xrightarrow{\gamma} V'$, $\theta(\beta, \gamma) \succeq \delta$, and $W' \overset{\delta}{\sim} V'$. Thus, since L is a complete lattice and θ is an L-valued equality relation, the transitive property can leads to $\theta(\alpha_k, \gamma) \succeq \theta(\alpha_k, \beta) \wedge \theta(\beta, \gamma) \succeq \delta$ for $k \in H$. Thus, we obtain that $(V', \{V'_k : k \in H\}) \in S'_\delta$. Therefore, S'_δ is a δ-limit bisimulation and $(V, \{V_n : n \in D\}) \in S'_\delta$, i.e., $V \overset{\delta}{\sim} \lim_{n \in D} V_n$. □

The following Proposition 4.5 states that if the implementations of two teams are very similar from some modification and the implementation of one team is obtained based on the specification W, then the implementations of another team is also based on the same specification.

Proposition 4.5 *Let $\mathcal{Q} = (\mathcal{A}, \theta)$ be a QTS, and $\delta_n \in L$ for $n \in D$, $W \in \mathcal{P}$, $\{W_n : n \in D\} \in \mathcal{P}_N$ and $\{V_n : n \in D\} \in \mathcal{P}_N$. If for some $n \geq n_0$, $V_n \overset{\delta_n}{\sim} W_n$ for each $n \geq n_0$, and $W \overset{\bigwedge_{n \in D[n_0)} \delta_n}{\sim} \lim_{n \in D} W_n$, then $W \overset{\bigwedge_{n \in D[n_0)} \delta_n}{\sim} \lim_{n \in D} V_n$.*

Proof. Since $W \overset{\bigwedge_{n \in D[n_0)} \delta_n}{\sim} \lim_{n \in D} W_n$, there exists $S_{\bigwedge_{n \in D[n_0)} \delta_n} \subseteq \mathcal{P} \times \mathcal{P}_N$ such that $(W, \{W_n : n \in D\}) \in S_{\bigwedge_{n \in D[n_0)} \delta_n}$. We construct the set $S'_{\bigwedge_{n \in D[n_0)} \delta_n} = \{(W, \{V_n : n \in D\}) :$ there exists $\{W_n : n \in D\} \in \mathcal{P}_N$ such that $(W, \{W_n : n \in D\}) \in S_{\bigwedge_{n \in D[n_0)} \delta_n}$ and for some $n_0 \in D, W_n \overset{\delta_n}{\sim} V_n$ for $n \geq n_0\}$.

We only need to show $S'_{\bigwedge_{n \in D[n_0)} \delta_n}$ is a $\bigwedge_{n \in D[n_0)} \delta_n$-limit bisimulation. In fact, suppose $(W, \{V_n : n \in D\}) \in S'_{\bigwedge_{n \in D[n_0)} \delta_n}$. Then we have $(W, \{W_n : n \in D\}) \in S_{\bigwedge_{n \in D[n_0)} \delta_n}$ and there exists $n_0 \in D$ such that $W_n \overset{\delta_n}{\sim} V_n$ for $n \geq n_0$. If $W \xrightarrow{\alpha} W'$, then $(W, \{W_n : n \in D\}) \in S_{\bigwedge_{n \in D[n_0)} \delta_n}$ implies that there exist $\beta_n \in Act$ for $n \in D$, $\{W'_n : n \in D\} \in \mathcal{P}_N$ and $n_1 \in D$ such that $W_n \xrightarrow{\beta_n} W'_n$, $\theta(\alpha, \beta_n) \succeq \bigwedge_{n \in D[n_0)} \delta_n$ for $n \geq n_0$, and $(W', \{W'_n : n \in D\}) \in S_{\bigwedge_{n \in D[n_0)} \delta_n}$. Since D is directed set, there exists $n_2 \in D$ such that $n_2 \geq n_0$ and $n_2 \geq n_1$. Thus, we can get $W_n \xrightarrow{\beta_n} W'_n$ and $W_n \overset{\delta_n}{\sim} V_n$ for each $n \geq n_0$. So, there exist $V'_n \in \mathcal{P}_N$ and $\gamma_n \in Act$ such that $V_n \xrightarrow{\gamma_n} V'_n$, $\theta(\gamma_n, \beta_n) \succeq \delta_n$, and $W'_n \overset{\delta_n}{\sim} V'_n$ for each $n \geq n_2$. Thus, for $\{V_n : n \in D\} \in \mathcal{P}_N$, there exist $n_2 \in D$, $\{V'_n : n \in D\} \in \mathcal{P}_N$, and $\gamma_n \in Act$ such that $V_n \xrightarrow{\gamma_n} V'_n$, $\theta(\alpha, \beta_n) \succeq \bigwedge_{n \in D[n_0)} \delta_n$ for every $n \geq n_2 \geq n_1$, and $\theta(\beta_n, \gamma_n) \succeq \delta_n \succeq \bigwedge_{n \in D[n_0)} \delta_n$ for every $n \geq n_2$. Furthermore, since θ is an L-valued equality relation, we have that

$\theta(\alpha,\gamma_n) \succeq \theta(\alpha,\beta_n) \wedge \theta(\beta_n,\gamma_n) \succeq \bigwedge\limits_{n\in D[n_0)} \delta_n$ for every $n \geq n_2$.

Conversely, if C is a cofinal subset of D and $V_m \overset{\alpha_m}{\to} V'_m$ for each $m \in C$, then since $C[n_0) \subseteq C$ is a cofinal subset, we can obtain $V_m \overset{\alpha_m}{\to} V'_m$ for each $m \in C[n_0)$. Thus, $W_n \overset{\delta_n}{\sim} V_n$ for $n \geq n_0$ leads to $W_m \overset{\delta_m}{\sim} V_m$ for $m \in C[n_0)$. Therefore, there exists $W'_m \in \mathcal{P}_N$, $\beta_m \in Act$ such that $W_m \overset{\beta_m}{\to} W'_m$, $\theta(\alpha_m,\beta_m) \succeq \delta$ for $m \in C[n_{0)}$. Since $(W,\{W_n : n \in D\}) \in S_{\bigwedge\limits_{n\in D[n_0)} \delta_n}$ and $C[n_0) \subseteq D$ is a cofinal subset, there exist $W' \in \mathcal{P}$, $\gamma \in Act$, and a cofinal subset B of $C[n_{0)}$ satisfies $W \overset{\gamma}{\to} W'$, $\theta(\beta_b,\gamma) \succeq \bigwedge\limits_{n\in D[n_0)} \delta_n$ for $b \in B$, and $(W',\{W'_b : b \in B\}) \in S_{\bigwedge\limits_{n\in D[n_0)} \delta_n}$. Thus we can get a cofinal subset B of C such that $(W',\{W'_b : b \in B\}) \in S'_{\bigwedge\limits_{n\in D[n_0)} \delta_n}$. Since θ is an L-valued equality relation, for $b \in B$, $\theta(\beta_b,\gamma) \succeq \bigwedge\limits_{n\in D[n_0)} \delta_n$ and $\theta(\alpha_m,\beta_m) \succeq \delta$ when $m \in C[n_0)$. So, $\theta(\alpha_b,\beta_b) \succeq \delta \succeq \bigwedge\limits_{n\in D[n_0)} \delta_n$ when $b \in B$. By the transitive property of L-valued equality relation, we have that $\theta(\alpha_n,\gamma) \succeq \bigwedge\limits_{n\in D[n_0)} \delta_n$ for $n \in B$. Therefore, $S'_{\bigwedge\limits_{n\in D[n_0)} \delta_n}$ is a $\bigwedge\limits_{n\in D[n_0)} \delta_n$-limit bisimulation. □

5 The substitutivity laws of δ- bisimulation limit

The design of system needs many different modular. In order to archive the flexible hierarchic development and modular design methods, in this section, we will mainly discuss this substitutivity laws of δ-bisimulation limit under various combinators. Firstly, we will introduce the following definitions that are useful for the following theory.

Definition 5.1 [δ-round] Let L be a complete lattice, and θ be an L-valued equality relation on X. $\delta \in L$, $Y \subseteq X$, $x \in Y$, $\theta(x,y) \succeq \delta$ implies $y \in Y$, then Y is said to be δ- round.

Definition 5.2 [non-contractile] Let L be a complete latticea, and θ be an L-valued equality relation on X. f is a mapping from X into itself. If for any $x, y \in X$, $\theta(f(x), f(y)) \succeq \theta(x,y)$, then f is said to be non-contractile.

Theorem 5.3 *Let $\mathcal{Q} = (\mathcal{A},\theta)$ be a QTS, and $\delta_n \in L$, and θ be an L-valued equality relation on Act.*

(i) *If $W \overset{\delta}{\sim} \lim\limits_{n\in D} V_n$, then $\alpha.W \overset{\delta}{\sim} \lim\limits_{n\in D} \alpha.V_n$;*

(ii) *If $W \overset{\delta}{\sim} \lim\limits_{n\in D} V_n$, then $W + R \overset{\delta}{\sim} \lim\limits_{n\in D}(V_n + R)$;*

(iii) *If $W \overset{\delta}{\sim} \lim\limits_{n\in D} V_n$, then $W[f] \overset{\delta}{\sim} \lim\limits_{n\in D} V_n[f]$ when f is non-contractile.*

(iv) *If $W \overset{\delta}{\sim} \lim\limits_{n\in D} V_n$, then $W\backslash L \overset{\delta}{\sim} \lim\limits_{n\in D} V_n\backslash L$ when Act $L \cup \bar{L}$ is δ-round*

Proof. (i) Firstly, we prove the first item in Theorem 5.3. If $\alpha.W \xrightarrow{\alpha} W$, then according to the transitive rule 'Act' in Definition 2.2, we can get that $\alpha.V_n \xrightarrow{\alpha} V_n$, $\theta(\alpha,\alpha) = 1 \succeq \delta \in L$ for any $n \in D$. Furthermore, since $W \overset{\delta}{\sim} \lim_{n\in D} V_n$, the condition (i) in Proposition 4.2 holds.

On the other hand, if $C \subseteq D$ is a cofinal subset, and $\alpha.V_m \xrightarrow{\alpha} V_m$ for every $m \in C$, then we have $W \in \mathcal{P}$, $\beta = \alpha \in Act$ and a cofinal subset $B = C$ of C such that $\alpha.W \xrightarrow{\alpha} W$ and $\theta(\alpha,\alpha) = 1 \succeq \delta$ for every $m \in C$ and $W \overset{\delta}{\sim} \lim_{n\in D} V_n$. So, the condition (ii) in Proposition 4.2 holds. Therefore, Proposition 4.2 can guarantee $\alpha.W \overset{\delta}{\sim} \lim_{n\in D} \alpha.V_n$ holds

(ii) We prove the second item in Theorem 5.3. If $W + R \xrightarrow{\alpha} W'$, then there are two cases $W \xrightarrow{\alpha} W'$ or $R \xrightarrow{\alpha} W'$.

Case 1: if $W \xrightarrow{\alpha} W'$, then $W \overset{\delta}{\sim} \lim_{n\in D} V_n$ leads to there are $\{V_n' : n \in D\} \in \mathcal{P}_N$, $\beta_n \in Act$ for every $n \in D$ and $n_0 \in D$ such that $V_n \xrightarrow{\beta_n} V_n'$ and $\theta(\alpha,\beta_n) \succeq \delta$ for $n \geq n_0$ and $W' \overset{\delta}{\sim} \lim_{n\in D} V_n'$. By the transitive rule $'Sum_j'$ in Definition 2.2, we have that $R + V_n \xrightarrow{\beta_n} V_n'$. So, there exist $\{V_n' : n \in D\} \in \mathcal{P}_N$ and $n_0 \in D$, such that $R + V_n \xrightarrow{\beta_n} V_n'$, $\theta(\alpha,\beta_n) \succeq \delta$ for each $n \geq n_0$ and $W' \overset{\delta}{\sim} \lim_{n\in D} V_n'$. Thus we have that $W + R \overset{\delta}{\sim} \lim_{n\in D}(V_n + R)$.

Case 2: if $R \xrightarrow{\alpha} W'$. By the transitive rule $'Sum_j'$ in Definition 2.2, we have $R + V_n \xrightarrow{\alpha} W'$ for every $n \in D$. Let $V_n' = W'$ for every $n \in D$. Then we can find a process net $\{V_n' : n \in D\} \in \mathcal{P}_N$ and $n_0 = n$ for some $n \in D$, $R + V_n \xrightarrow{\alpha} V_n'$, $\theta(\alpha,\alpha) = 1 \succeq \delta$ and $W' \overset{\delta}{\sim} \lim_{n\in D} V_n'$.

Conversely, if $C \subseteq D$ is a cofinal subset and $R + V_m \xrightarrow{\beta_m} V_m'$ for each $m \in C$. Then by the transition rule "Sum_j" in Definition 2.2, we obtain that $V_m \xrightarrow{\beta} V_m'$ for each $m \in C$ or $R \xrightarrow{\beta_m} V_m'$ for each $m \in C$.

Case 1: if $V_m \xrightarrow{\beta_m} V_m'$ for each $m \in C$. Since $C \subseteq D$ is a cofinal subset and $W \overset{\delta}{\sim} \lim_{n\in D} V_n$, there is a cofinal subnet B of C, $W' \in \mathcal{P}$ and $\alpha \in Act$ such that $W \xrightarrow{\alpha} W'$, $\theta(\alpha,\beta_m) \succeq \delta$ for $m \in B$ and $W' \overset{\delta}{\sim} \lim_{m\in B} V_m'$. The transition rule "$Sum_j$" in Definition 2.2 can tell us $W + R \xrightarrow{\alpha} W'$. So, we have that $W + R \overset{\delta}{\sim} \lim_{n\in D}(R + V_n)$.

Case 2, if $R \xrightarrow{\beta_m} V_m'$ for each $m \in C$, then since $R \overset{\delta}{\sim} \lim_{n\in D} R$ and $C \subseteq D$ is a cofinal subset, we have a cofinal subset $B \subseteq C$, $\alpha \in Act$ and $W' \in \mathcal{P}$ such that $R \xrightarrow{\alpha} W'$, $\theta(\alpha,\beta_m) \succeq \delta$ for $m \in B$ and $W' \overset{\delta}{\sim} \lim_{n\in B} V_m'$. Furthermore, $W + R \xrightarrow{\alpha} W'$. Thus, we can get that $W + R \overset{\delta}{\sim} \lim_{n\in D}(R + V_n)$

(iii) We prove the third item in Theorem 5.3. Since $W \overset{\delta}{\sim} \lim_{n\in D} V_n$, there exist

S_δ is a δ-bisimulation such that $(W, \{V_n : n \in D\}) \in S_\delta$. Next, we construct a δ-bisimulation $S'_\delta \subseteq \mathcal{P} \times \mathcal{P}$. We only need to prove $(W, \{V_n : n \in D\}) \in S'_\delta$. Let $S'_\delta = \{(W[f], \{V_n[f] : n \in D\}) : (W, \{V_n : n \in D\}) \in S_\delta\}$. If $W[f] \xrightarrow{\alpha} W'$, then we have that $W \xrightarrow{\beta} W''$, $\alpha = f(\beta)$ and $W' = W''[f]$. Since $(W, \{V_n : n \in D\}) \in S_\delta$, there exist $\{V''_n : n \in D\} \in \mathcal{P}_N$ and $\gamma_n \in Act$ for every $n \in D$ such that $V_n \xrightarrow{\gamma_n} V''_n$, $\theta(\beta, \gamma_n) \succeq \delta$ for each $n \geq n_0$, and $(W'', \{V''_n : n \in D\}) \in S_\delta$. By the transition rule $'Ref'$ in Definition 2.2, we have that $V_n[f] \xrightarrow{f(\gamma_n)} V''_n[f]$ for each $n \geq n_0$. Thus, we can find a processes net $\{V'_n = V''_n[f] : n \in D\} \in \mathcal{P}_N$ and $n_0 \in D$ such that $V_n[f] \xrightarrow{f(\gamma_n)} V'_n$ for every $n \geq n_0$. Since f is non-contractile, $\theta(\alpha, f(\gamma_n)) = \theta(f(\beta), f(\gamma_n)) \succeq \theta(\beta, \gamma_n) \geq \delta$ for every $n \geq n_0$. At the same time, according to the definition of S'_δ, we know that $(W''[f], \{V''_n[f] : n \in D\}) \in S'_\delta$.

On the other hand, if $C \subseteq D$ is a cofinal subset, $V_m[f] \xrightarrow{\beta_m} V''_m$, then $V_m \xrightarrow{\gamma_m} V''_m$, $\beta_m = f(\gamma_m)$, $V'_m = V''_m[f]$ for each $m \in C$. Since $(W, \{V_m : m \in D\}) \in S_\delta$, there exist $W'' \in \mathcal{P}$,a cofinal subset B of C and $\alpha \in Act$ satisfy $W \xrightarrow{\alpha} W''$, and $\theta(\alpha, \gamma_m) \succeq \delta$ for $m \in B$ and $(W'', \{V''_m : m \in B\}) \in S_\delta$. By the transition rule $'Rel'$ in Definition 2.2, $W[f] \xrightarrow{f(\alpha)} W''[f]$. Thus, let $W' = W''[f]$. Then $W[f] \xrightarrow{f(\alpha)} W'$ and $(W', \{V'_m : m \in B\}) \in S'_\delta$. f is non-contractile can guarantee $\theta(f(\alpha), f(\gamma_m)) \succeq \theta(\alpha, \gamma_m) \succeq \delta$ for $m \in B$.

(iv) We prove the fourth item in Theorem 5.3. $W \overset{\delta}{\sim} \lim_{n \in D} V_n$ leads to there exists a δ-limit bisimulation S_δ such that $(W, \{V_{n:n \in D}\}) \in S_\delta$. Next, we construct a new relation $S'_\delta \subseteq \mathcal{P} \times \mathcal{P}_N$.

Let $S'_\delta = \{(W \backslash L, \{V_n \backslash L : n \in D\}) : (W, \{V_n : n \in D\}) \in S_\delta\}$. We only need to prove that S'_δ is δ-bisimulation.

In fact, if $W \backslash L \xrightarrow{\alpha} W'$, then we have that $W \xrightarrow{\alpha} W''$ and $\alpha, \bar{\alpha} \notin L \cup \bar{L}$, $W' = W'' \backslash L$. Since $(W, \{V_n : n \in D\}) \in S_\delta$, there exist $\{V''_n : n \in D\}$ and $\beta_n \in Act$ and $n_0 \in D$ such that $V_n \xrightarrow{\beta_n} V''_n$ and $\theta(\alpha, \beta_n) \succeq \delta$ for any $n \geq n_0$, and $(W'', \{V''_n : n \in D\}) \in S_\delta$. Since $\alpha, \bar{\alpha} \notin L \cup \bar{L}$, $\alpha, \bar{\alpha} \in Act \backslash L \cup \bar{L}$. And $Act \backslash L \cup \bar{L}$ is δ- round, and $\theta(\alpha, \beta_n) \succeq \delta$ lead to $\beta_n, \bar{\beta_n} \in Act \backslash L \cup \bar{L}$. Thus $\beta_n, \bar{\beta_n} \notin L$. So, by the transitive rule $'Res'$ in Definition 2.2, we have that $V_n \backslash L \xrightarrow{\beta_n} V''_n \backslash L$ for $n \geq n_0$. And, we obtain the processes net $\{V'_n = V''_n \backslash L : n \in D\} \in \mathcal{P}_N$, $\beta_n \in Act$ and $n_0 \in D$ such that $V_n \backslash L \xrightarrow{\beta_n} V'_n$, $\theta(\alpha, \beta_n) \succeq \delta$ for any $n \geq n_0$ and $(V_n \backslash L, \{V''_n \backslash L : n \in D\}) \in S'_\delta$.

On the other hand, if $C \subseteq D$ is cofinal subset, $V_m \backslash L \xrightarrow{\beta_m} V'_m$, then by the transition rule, we can obtain $V_m \xrightarrow{\beta_m} V''_m$, $V'_m = V''_m \backslash L$, $\beta_m, \bar{\beta_m} \notin L \cup \bar{L}$. Thus, $\beta_m, \bar{\beta_m} \in Act \backslash L \cup \bar{L}$. Since $V_m \xrightarrow{\beta_m} V''_m$, we find a cofinal subset $B \subseteq C$, $W'' \in \mathcal{P}$ and $\gamma \in Act$ satisfy $W \xrightarrow{\gamma} W''$, $\theta(\gamma, \beta_m) \succeq \delta$ for $m \in B$ and $(W'', \{V''_m : m \in B\}) \in S_\delta$. Since $Act \backslash L \cup \bar{L}$ is δ- round, and $\beta_m, \bar{\beta_m} \in Act \backslash L \cup \bar{L}$, $\gamma, \bar{\gamma} \in Act \backslash L \cup \bar{L}$.Thus,$\gamma, \bar{\gamma} \notin L \cup \bar{L}$. Therefore, $W \backslash L \xrightarrow{\gamma} W'' \backslash L$. Let $W' = W'' \backslash L$, we can find a cofinal subset B of C, $W' \in \mathcal{P}$ and $\gamma \in Act$, such that $W \backslash L \xrightarrow{\gamma} W'$ and $(W', \{V''_m \backslash L : m \in B\}) \in S'_\delta$. □

6 Conclusion and Future Work

In this paper, we mainly discuss the infinite evolution of implementation under δ-bisimulation in order to describe the dynamic correctness of system. δ-limit bisimulation and δ-bisimulation limit are presented. They reflect the convergence mechanism of implementation under δ-bisimulation, and state that the specification of system is the limit of a series of implementations under δ-bisimulation. From the the point of view of mathematics, the limit can produce a topology. As a future work, we will try to establish the topological theory of δ-bisimulation limit.

References

[1] A. Girard, G. J. Pappas. Approximate bisimulations for constrained linear systems. In Proceedings of the 44th IEEE Conference on Decision and Control and European Control Conference, Seville, 2005, 4700-4705.

[2] A. Di Pierro, C. Hankin, and H. Wiklicky. Quantitative relations and approximate process equivalences. In Proceedings of 14th International Conference on Concurrency Theory, Marseille, Springer-Verlag, LNCS, 2003, 2761:498-512.

[3] T.Chen, T.Han, J. Lu. On metric for probabilistic systems: definitions and algorithms. Computers and Mathematics with Applications, 2009, 57:991-999.

[4] Mingsheng Ying. Topology in Process Calculus: Approximate Correctness and Infinite Evolution of Concurrent systems. Springer-Verlag, New York, 2001.

[5] M. S. Ying. Bisimulation indexes and their applications. Theoretical computer science, 2002, 275(1-2): 1-68.

[6] U. Fahrenberg, A. Legay. The quantitative linear-time branching-time spectrum. Theoretical Computer Science, 2014, 538:54-69.

[7] T. A. Henzinger. Quantitative reactive modeling and verification. Computer Science Research and Development, 2013, 28:331-344.

[8] A. Henzinger,T. R.Majumdar, V.S.Prabhu. Quantifying similarities between timed systems. Formal Modeling and Analysis of Timed Systems, Lecture Notes in Computer Science, 2005, 3829:226-241.

[9] J.Desharnais, F. Laviolette, M.Tracol. Approximate analysis of probabilistic processes: logic, simulation and games. In Proceedings of Fifth International Conference on Quantitative Evaluation of Systems, 2008, 264-273.

[10] Yanfang Ma, Min. Zhang. Topological construction of parameterized bisimulation limit. Electronic Notes in Theoretical Computer Science, 2009, 257: 55-70.

[11] Yanfang Ma, Min Zhang, Yixiang Chen. Formal sescription of software dynamic correctness. Journal of Computer Research and Development, 2013, 50(3):626-635.

[12] Yanfang Ma, Min Zhang. The infinite evolution mechanism of ϵ-bisimilarity. Journal of Computer Science and Technology, 2013, 28(6):1097-1105.

[13] Haiyu Pan, Yongzhi Cao, Min Zhang, Yixiang Chen. Simulation for lattice-valued doubly labeled transition systems. International Journal of Approximate Reasoning, 2014, 55:797-811.

[14] Haiyu Pan, Min Zhang, Hengyang Wu, Yixiang Chen. Quantitative analysis of lattice-valued Kripke structures. Fundamenta Informaticae, 2014, 135: 269-293.

[15] Haiyu Pan, Yongming Li, Yongzhi Cao. Lattice-valued simulations for quantitative transition systems. Internatinal Journal of Approximate Reasoning, 2015, 56:28-42.

[16] R. Milner. Communication and Concurrency. Prentice Hall, New York, 1989.

[17] G. D. Plotkin. A Structural Approach to Operational Semantics. Technical Report DAIMI FN-19, Computer Science Department, Aarhus University, 1981.

[18] R.Belohlavek. Fuzzy Relational Systems: Foundations and Principles. Springer-Verlag, New York, 2002.

[19] R. Engelking. General Topology. Polish Scientific Publisher, Warszawa, 1977.

[20] John L.Kelley, General Topology. Springer-Verlag, 1975.