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Uniform Topological Spaces Based on BF-ideals in Negative Non-involutive Residuated Lattices

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Abstract

In this paper, the uniform topological spaces are established based on the congruences induced by BF-ideals and some of their properties are discussed in negative non-involutive residuated lattices. The following conclusions are proved: (i) every uniform topological space is first-countable, zero-dimensional, disconnected, locally compact and completely regular. (ii) a uniform topological space is a T_1 space iff it is a T_2 space. (iii) the lattice and adjoint operations in a negative non-involutive residuated lattice are continuous with respect to the uniform topology, which make the negative non-involutive residuated lattice to be topological negative non-involutive residuated lattice. Meanwhile, some necessary and sufficient conditions for the uniform topological spaces to be compact and discrete are obtained. The results of this paper have a positive role to reveal internal features of negative non-involutive residuated lattices at a topological level.

Keywords: Non-classical logic, negative non-involutive residuated lattice, BF-ideal; uniform topological space

1 Introduction

It is well-known that non-classical mathematical logic [1] has become a formal and useful tool for computer science to deal with fuzzy information and uncertain information. Various logical algebras have been proposed as the semantical systems of non-classical mathematical logic systems. Among these logical algebras, residuated lattices introduced by Ward and Dilworth [2] are very basic and important algebraic structures. Some other logical algebras such as MTL-algebra, BL-algebra, MV-algebras [3] and NM-algebra, which is also called R_0-algebra [4] are all able to be considered as particular classes of residuated lattices. Filter and ideal are two

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important concepts for studying of logical algebraic structures. The related research work has attracted much attention from scholars [5,6]. It is noteworthy that in logical algebras with negative involutive (regular) properties, considering that filters and ideals are mutually dual, most people focus their attention on the problem of filters. However, when the negation operation in logical algebra loses its involution, the dual relationship between ideal and filter is also broken. Therefore, it will be a meaningful work to explore ideal and its application in the framework of negative non-involutive logic algebras. In view of this, the concepts of ideal and fuzzy ideal have been introduced in BL-algebra and negative non-involutive residuated lattices in [7,8,9], and some results with theoretical significance and application prospect have been obtained.

The concept of fuzzy sets, a remarkable idea in mathematics, was proposed by Zadeh [10] in 1965. At present, fuzzy sets have been extremely used to deal with the many problems in applied mathematics, control engineering, information sciences, expert systems and theory of automata etc. However, in traditional fuzzy sets, the membership degrees of elements are all restricted to the interval $[0,1]$, which leads to a great difficulty in expressing the difference of the irrelevant elements from the contrary elements in fuzzy sets. If a set representation could express this kind of difference, it would be more informative than the traditional fuzzy set representation. Based on these observations, Lee [11] introduced the notion of bipolar-valued fuzzy sets, abbreviated as BF-sets, which is an extension of the traditional fuzzy sets. In the past few decades, more and more researchers have devoted themselves to applying BF-sets theory to various algebraic structures [12,13,14].

Recently, we introduced the concept of BF-ideals in negative non-involutive residuated lattices and discussed some related properties in [15] and [16]. As a continuation of the above work, in this paper, we consider a collection of BF-ideals and use congruence relation induced by BF-ideal to define a uniform topological space and investigate some of its properties. Some interesting results are obtained.

2 Preliminaries

In this section, we review some concepts and properties regarding negative non-involutive residuated lattices [2,3,8,9] and BF-sets [11,15,16]. For notions about topology, please refer to [17] and [18].

Definition 2.1 A residuated lattice is an algebra $(L, \vee, \wedge, \otimes, \to, 0, 1)$ of type $(2,2,2,2,0,0)$ such that:

(R1) $(L, \vee, \wedge, 0, 1)$ is a bounded lattice with the greatest element 1 and the least element 0,

(R2) $(L, \otimes, 1)$ is a commutative monoid,

(R3) $(\otimes, \to)$ is an adjoint pair on L, i.e., for all $x, y, z \in L$, $x \otimes y \leqslant z$ if and only if $x \leqslant y \to z$.

A negative non-involutive residuated lattice is a residuated lattice $(L, \vee, \wedge, \otimes, \to, 0, 1)$ which satisfies that: there exists $x \in L$ such that $x \neq x''$, where $x' = x \to 0$ for all $x \in L$.

In the sequel, a residuated lattice $(L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ will be denoted by L in short.

Lemma 2.2 *Let L be a negative non-involutive residuated lattice, then for all $x, y, z \in L$,*

(P1) $x \leqslant y$ if and only if $x \rightarrow y = 1$,

(P2) $x \rightarrow x = 1$ and $x \rightarrow 1 = 1$ and $1 \rightarrow x = x$,

(P3) $x \leqslant y$ implies $x \otimes z \leqslant y \otimes z$ and $z \rightarrow x \leqslant z \rightarrow y$ and $y \rightarrow z \leqslant x \rightarrow z$,

(P4) $y \rightarrow z \leqslant (x \rightarrow y) \rightarrow (x \rightarrow z) \leqslant x \rightarrow (y \rightarrow z)$ and $x \rightarrow y \leqslant (y \rightarrow z) \rightarrow (x \rightarrow z)$,

(P5) $x \rightarrow (y \wedge z) = (x \rightarrow y) \wedge (x \rightarrow z)$ and $(y \vee z) \rightarrow x = (y \rightarrow x) \wedge (z \rightarrow x)$,

(P6) $(x \rightarrow y) \otimes (y \rightarrow z) \leqslant x \rightarrow z$ and $x \rightarrow (y \rightarrow z) = (x \otimes y) \rightarrow z = y \rightarrow (x \rightarrow z)$,

(P7) $x \leqslant y \Longrightarrow y' \leqslant x' \Longrightarrow x'' \leqslant y''$ and $x \leqslant x''$ and $x''' = x'$ and $x \otimes x' = 0$,

(P8) $x \rightarrow y' = y \rightarrow x' = (x \otimes y)'$ and $(x \rightarrow y')'' = x \rightarrow y'$ and $(x \rightarrow y'')'' = x \rightarrow y''$,

(P9) $(x \vee y)' = x' \wedge y'$ and $(x \wedge y)' \geqslant x' \vee y'$ and $x \rightarrow y \leqslant y' \rightarrow x'$.

Let X be a non-empty set. Denote $J_{[0,1]} = \{\mu^P | \mu^P : X \rightarrow [0,1]\}$ and $J_{[-1,0]} = \{\mu^N | \mu^N : X \rightarrow [-1,0]\}$. For every $\mu_A^P \in J_{[0,1]}$ and $\mu_A^N \in J_{[-1,0]}$, we call $A = \{(x, \mu_A^P(x), \mu_A^N(x)) | x \in X\}$ a bipolar-valued fuzzy set on X, and abbreviate A is a BF-set on X, where $\mu_A^P(x)$ is called a positive membership degree which denotes the satisfaction degree of an element x to some specific property about the BF-set A, and $\mu_A^N(x)$ is called a negative membership degree which denotes the satisfaction degree of x to some implicit counter-property about the BF-set A. For the sake of simplicity, we shall use the symbol $A = (\mu_A^P, \mu_A^N)$ for the BF-set $A = \{(x, \mu_A^P(x), \mu_A^N(x)) | x \in X\}$. The set of all BF-sets on X is denoted by $\mathbf{BFS}(X)$. Let $\left\{A_\lambda = (\mu_{A_\lambda}^P, \mu_{A_\lambda}^N) | \lambda \in \Lambda\right\} \subseteq \mathbf{BFS}(X)$, we define the BF-intersection $\bigsqcap_{\lambda\in\Lambda} A_\lambda$ and BF-union $\bigsqcup_{\lambda\in\Lambda} A_\lambda$ of $\left\{A_\lambda = (\mu_{A_\lambda}^P, \mu_{A_\lambda}^N) | \lambda \in \Lambda\right\}$ are as follows:

- $\left(\bigsqcap_{\lambda\in\Lambda} A_\lambda\right)(x) = \left(\mu^P_{\bigsqcap_{\lambda\in\Lambda} A_\lambda}(x), \mu^N_{\bigsqcap_{\lambda\in\Lambda} A_\lambda}(x)\right) = \left(\bigwedge_{\lambda\in\Lambda} \mu_{A_\lambda}^P(x), \bigvee_{\lambda\in\Lambda} \mu_{A_\lambda}^N(x)\right)$, $\forall x \in X$;
- $\left(\bigsqcup_{\lambda\in\Lambda} A_\lambda\right)(x) = \left(\mu^P_{\bigsqcup_{\lambda\in\Lambda} A_\lambda}(x), \mu^N_{\bigsqcup_{\lambda\in\Lambda} A_\lambda}(x)\right) = \left(\bigvee_{\lambda\in\Lambda} \mu_{A_\lambda}^P(x), \bigwedge_{\lambda\in\Lambda} \mu_{A_\lambda}^N(x)\right)$, $\forall x \in X$.

In particular, if $A = (\mu_A^P, \mu_A^N), B = (\mu_B^P, \mu_B^N) \in \mathbf{BFS}(X)$, we define $A \sqcap B$ and $A \sqcup B$ are as follows:

- $(A \sqcap B)(x) = (\min\{\mu_A^P(x), \mu_B^P(x)\}, \max\{\mu_A^N(x), \mu_B^N(x)\}) = (\mu_A^P(x) \wedge \mu_B^P(x), \mu_A^N(x) \vee \mu_B^N(x))$, $\forall x \in X$;
- $(A \sqcup B)(x) = (\max\{\mu_A^P(x), \mu_B^P(x)\}, \min\{\mu_A^N(x), \mu_B^N(x)\}) = (\mu_A^P(x) \vee \mu_B^P(x), \mu_A^N(x) \wedge \mu_B^N(x))$, $\forall x \in X$,

and define a partial order relation $\sqsubseteq$ on $\mathbf{BFS}(X)$ such that

- $A \sqsubseteq B$ if and only if $\mu_A^P(x) \leqslant \mu_B^P(x)$ and $\mu_A^N(x) \geqslant \mu_B^N(x)$ for every $x \in X$.

Definition 2.3 [15] Let L be a negative non-involutive residuated lattice. A BF-set $A = (\mu_A^P, \mu_A^N) \in \mathbf{BFS}(L)$ is called a BF-ideal of L if it satisfies the following conditions for all $x, y \in L$,

(BFI1) $\mu_A^P(0) \geqslant \mu_A^P(x)$ and $\mu_A^N(0) \leqslant \mu_A^N(x)$,

(BFI2) $\mu_A^P(y) \geqslant \mu_A^P(x) \wedge \mu_A^P((x' \to y')')$ and $\mu_A^N(y) \leqslant \mu_A^N(x) \vee \mu_A^N((x' \to y')')$.

The set of all BF-ideals of L is denoted by $\mathbf{BFI}(L)$.

Lemma 2.4 *[15] Let L be a negative non-involutive residuated lattice and $A = (\mu_A^P, \mu_A^N) \in \boldsymbol{BFI}(L)$. Then for all $x, y, z \in L$,*

(BFI3) $x \leqslant y$ implies $\mu_A^P(y) \leqslant \mu_A^P(x)$ and $\mu_A^N(y) \geqslant \mu_A^N(x)$,

(BFI4) $\mu_A^P((x \to y)') \geqslant \mu_A^P((x \to z)') \wedge \mu_A^P((z \to y)')$ and $\mu_A^N((x \to y)') \leqslant \mu_A^N((x \to z)') \vee \mu_A^N((z \to y)')$.

3 Uniformity and Uniform Topological Space Based on BF-ideals

In this section, we induce a congruence relation by a BF-ideal firstly, then give the definition of uniformity and uniform topological space base on BF-ideals in negative non-involutive residuated lattices.

Definition 3.1 Let L be a negative non-involutive residuated lattice. A binary relation $R \subseteq L \times L$ is said to be a congruence relation on L if it satisfies the following conditions:

(CR1) R is an equivalence relation on L, i. e., R satisfies reflexivity, symmetry and transitivity,

(CR2) for all $x, y, z \in L$, $(x, y) \in R$ implies $(x * z, y * z) \in R$ and $(z * x, z * y) \in R$, where $* \in \{\wedge, \vee, \otimes, \to\}$.

Theorem 3.2 *Let L be a negative non-involutive residuated lattice and $A = (\mu_A^P, \mu_A^N) \in \boldsymbol{BFI}(L)$. A binary relation $\equiv_A \subseteq L \times L$ is defined as follows:*

$$\text{for all } x, y \in L, x \equiv_A y \Longleftrightarrow \begin{cases} \mu_A^P((x \to y)') = \mu_A^P((y \to x)') = \mu_A^P(0), \\ \mu_A^N((x \to y)') = \mu_A^N((y \to x)') = \mu_A^N(0). \end{cases}$$

Then $\equiv_A$ is a congruence relation on L. We named it the congruence relation induced by BF-ideal A on L.

Proof. Firstly, it is obvious that A satisfies reflexivity and symmetry. For transitivity, assume that $x \equiv_A y$ and $y \equiv_A z$, we have $\mu_A^P((x \to y)') = \mu_A^P((y \to x)') = \mu_A^P(0)$ and $\mu_A^N((x \to y)') = \mu_A^N((y \to x)') = \mu_A^N(0)$, $\mu_A^P((y \to z)') = \mu_A^P((z \to y)') = \mu_A^P(0)$ and $\mu_A^N((y \to z)') = \mu_A^N((z \to y)') = \mu_A^N(0)$. For every $x, y, z \in L$, Since $y \to z \leqslant (x \to y) \to (x \to z) \leqslant (x \to z)' \to (x \to y)' \leqslant (x \to y)'' \to (x \to z)''$ by (P4) and (P9), we have that $(y \to z)' \geqslant ((x \to y)'' \to (x \to z)'')'$ by (P7). according to $A \in \mathbf{BFI}(L)$ and Lemma 2.4, we have that $\mu_A^P((y \to z)') \leqslant \mu_A^P(((x \to y)'' \to (x \to z)'')')$ and $\mu_A^N((y \to z)') \geqslant \mu_A^N(((x \to y)'' \to (x \to z)'')')$, thus by Definition 2.3 we can obtain that $\mu_A^P((x \to z)') \geqslant \mu_A^P((x \to y)') \wedge \mu_A^P(((x \to y)'' \to (x \to z)'')') \geqslant \mu_A^P((x \to y)') \wedge \mu_A^P((y \to z)') = \mu_A^P(0)$ and $\mu_A^N((x \to z)') \leqslant \mu_A^N((x \to$

$y)') \vee \mu_A^N(((x \to y)'' \to (x \to z)'')') \leqslant \mu_A^N((x \to y)') \vee \mu_A^N((y \to z)') = \mu_A^N(0)$, and thus $\mu_A^P((x \to z)') = \mu_A^P(0)$ and $\mu_A^N((x \to z)') = \mu_A^N(0)$. Similarly, we can also get that $\mu_A^P((z \to x)') = \mu_A^P(0)$ and $\mu_A^N((z \to x)') = \mu_A^N(0)$. Therefore, $x \equiv_A z$, i. e., A satisfies transitivity.

Secondly, assume that $x \equiv_A y$, we have $\mu_A^P((x \to y)') = \mu_A^P((y \to x)') = \mu_A^P(0)$ and $\mu_A^N((x \to y)') = \mu_A^N((y \to x)') = \mu_A^N(0)$. Since $(x \wedge z) \to (y \wedge z) = ((x \wedge z) \to y) \wedge ((x \wedge z) \to z) = (x \wedge z) \to y \geqslant x \to y$ by (P4) and (P5), we have that $(x \to y)' \geqslant ((x \wedge z) \to (y \wedge z))'$ by (P7). according to $A \in \mathbf{BFI}(L)$ and Lemma 2.4, we have that $\mu_A^P(0) = \mu_A^P((x \to y)') \leqslant \mu_A^P(((x \wedge z) \to (y \wedge z))')$ and $\mu_A^N(0) = \mu_A^N((x \to y)') \geqslant \mu_A^N(((x \wedge z) \to (y \wedge z))')$, and so $\mu_A^P(((x \wedge z) \to (y \wedge z))') = \mu_A^P(0)$ and $\mu_A^N(((x \wedge z) \to (y \wedge z))') = \mu_A^N(0)$. Similarly, we can also get that $\mu_A^P(((y \wedge z) \to (x \wedge z))') = \mu_A^P(0)$ and $\mu_A^N(((y \wedge z) \to (x \wedge z))') = \mu_A^N(0)$. Therefore $(x \wedge z) \equiv_A (y \wedge z)$. Using similar methods, we can prove that $(x \vee z) \equiv_A (y \vee z)$, $(x \otimes z) \equiv_A (y \otimes z)$, $(x \to z) \equiv_A (y \to z)$ and $(z \to x) \equiv_A (z \to y)$. The proof is completed. □

Theorem 3.3 *Let L be a negative non-involutive residuated lattice, $A = (\mu_A^P, \mu_A^N), B = (\mu_B^P, \mu_B^N) \in$ **BFI**(L), $\mu_A^P(0) = \mu_B^P(0)$ and $\mu_A^N(0) = \mu_B^N(0)$. Then $\equiv_A \cap \equiv_B = \equiv_{A \sqcap B}$.*

Proof. Assume that $A = (\mu_A^P, \mu_A^N), B = (\mu_B^P, \mu_B^N) \in \mathbf{BFI}(L)$, $\mu_A^P(0) = \mu_B^P(0)$ and $\mu_A^N(0) = \mu_B^N(0)$. For every $x, y \in L$, it follows that

$$
\begin{aligned}
(x, y) \in \equiv_A \cap \equiv_B &\Longleftrightarrow x \equiv_A y \text{ and } x \equiv_B y \\
&\Longleftrightarrow \begin{cases} \mu_A^P((x \to y)') = \mu_A^P((y \to x)') = \mu_A^P(0) = \mu_B^P(0) = \mu_B^P((x \to y)') = \mu_B^P((y \to x)') \\ \mu_A^N((x \to y)') = \mu_A^N((y \to x)') = \mu_A^N(0) = \mu_B^N(0) = \mu_B^N((x \to y)') = \mu_B^N((y \to x)') \end{cases} \\
&\Longleftrightarrow \begin{cases} \mu_A^P((x \to y)') \wedge \mu_B^P((x \to y)') = \mu_A^P(0) \wedge \mu_B^P(0) = \mu_A^P((y \to x)') \wedge \mu_B^P((y \to x)') \\ \mu_A^N((x \to y)') \vee \mu_B^N((x \to y)') = \mu_A^N(0) \vee \mu_B^N(0) = \mu_A^N((y \to x)') \vee \mu_B^N((y \to x)') \end{cases} \\
&\Longleftrightarrow \begin{cases} \mu_{A \sqcap B}^P((x \to y)') = \mu_{A \sqcap B}^P(0) = \mu_{A \sqcap B}^P((y \to x)') \\ \mu_{A \sqcap B}^N((x \to y)') = \mu_{A \sqcap B}^N(0) = \mu_{A \sqcap B}^N((y \to x)') \end{cases} \\
&\Longleftrightarrow (x, y) \in \equiv_{A \sqcap B},
\end{aligned}
$$

from Theorem 3.2, therefore $\equiv_A \cap \equiv_B = \equiv_{A \sqcap B}$. □

Let X be a non-empty set, $U, V \subseteq X \times X$, the composition, inverse and diagonal are defined as follows

(1) $U \circ V = \{(x, y) \in X \times X | \exists z \in X, (x, z) \in V, (z, y) \in U\}$,
(2) $U^{-1} = \{(x, y) \in X \times X | (y, x) \in U\}$,
(3) $\Delta = \{(x, x) \in X \times X | x \in X\}$.

Definition 3.4 [17,18] Let X be a non-empty set. A uniformity on X is a non-empty collection Ω of subset of $X \times X$ which satisfies the following conditions:

(U1) for all $U \in \Omega, \Delta \subseteq U$,
(U2) for all $U \in \Omega, U^{-1} \in \Omega$,
(U3) for all $U \in \Omega$, there exists a $V \in \Omega$ such that $V \circ V \subseteq U$,
(U4) for all $U, V \in \Omega, U \cap V \in \Omega$,
(U5) $U \in \Omega$ and $U \subseteq V \subseteq X \times X$ imply $V \in \Omega$.

The pair (X, Ω) is called a uniform space.

Let Ω be a uniformity on X, $x \in X$, $U \in \Omega$ and $U[x] := \{y \in X | (x, y) \in U\}$, the Ω can naturally induce a topology τ on X such that $\tau = \{O \subseteq X | \forall x \in O, \exists U \in \Omega,$ s.t. $U[x] \subseteq O\}$, we named it uniform topology induced by Ω on X, and (X, τ) uniform topology space induced by Ω on X.

Now we define uniformity and induce uniform topology base on BF-ideals in a negative non-involutive residuated lattice L. For this reason, in the next sequel, we suppose that $\mathcal{I}$ is a non-empty family of BF-ideals of L which is closed under BF-intersection and for all $A = (\mu_A^P, \mu_A^N), B = (\mu_B^P, \mu_B^N) \in \mathcal{I}$, $\mu_A^P(0) = \mu_B^P(0)$ and $\mu_A^N(0) = \mu_B^N(0)$.

Proposition 3.5 *Let L be a negative non-involutive residuated lattice and $U_A = \{(x, y) \in L \times L | x \equiv_A y\}$ for every $A = (\mu_A^P, \mu_A^N) \in \mathcal{I}$. Then the subsets family $\omega^* = \{U_A | A \in \mathcal{I}\}$ of $L \times L$ satisfies conditions (U1)-(U4).*

Proof. For every $U_A \in \omega^*$, ω^* satisfies (U1)-(U3) follow from reflexivity, symmetric and transitivity of $\equiv_A$. For (U4), let $U_A, U_B \in \omega^*$, we have that $U_A \cap U_B = U_{A \sqcap B}$ by Theorem 3.3. Since $A, B \in \mathcal{I}$ and $\mathcal{I}$ is closed under BF-intersection, we have that $A \sqcap B \in \mathcal{I}$. Thus $U_A \cap U_B \in \omega^*$. Therefore ω^* also satisfies (U4). □

Theorem 3.6 *Let L be a negative non-involutive residuated lattice. Then $\omega = \{U \subseteq L \times L | \exists U_A \in \omega^*, s.t.\ U_A \subseteq U\}$ is a uniformity on L, and so (L, ω) is a uniform space.*

Proof. By Proposition 3.5 and the definition of ω, we know that ω satisfies the conditions (U1)-(U4). We will show that ω satisfies the condition (U5). Indeed, let $U \in \omega$ and $U \subseteq V \subseteq L \times L$, then there exists $U_A \in \omega^*$ such that $U_A \subseteq U \subseteq V$, thus $V \in \omega$. The proof is completed. □

Combining Definition 3.4 and Theorem 3.6, we give the following definition:

Definition 3.7 Let L be a negative non-involutive residuated lattice. Then $\tau_{\mathcal{I}} = \{O \subseteq X | \forall x \in O, \exists U \in \omega,$ s.t. $U[x] \subseteq O\}$ is a topology on L, which is called a uniform topology on L induced by $\mathcal{I}$, and $(L, \tau_{\mathcal{I}})$ is called uniform topological space induced by $\mathcal{I}$. In particular, if $\mathcal{I} = \{A = (\mu_A^P, \mu_A^N)\}$, then $\omega = \{U \subseteq L \times L | U_A \subseteq U\}$, and the uniform topology on L induced by $\mathcal{I}$ is denoted by τ_A.

Note 1 *Let L be a negative non-involutive residuated lattice. According to the definition of uniform topology $\tau_{\mathcal{I}}$ in Definition 3.7, for every $x \in L$ and $A = (\mu_A^P, \mu_A^N) \in \mathcal{I}$, It is obvious that $U_A[x]$ is an open neighborhood of x.*

Example 3.8 Let lattice $L = \{0, a, b, c, 1\}$, the Hasse diagram of L be defined as Figure 1, and the operators $\rightarrow$ and $\otimes$ of L be defined as following two Tables, respectively,

$\to$	0	a	b	c	1
0	1	1	1	1	1
a	b	1	b	1	1
b	a	a	1	1	1
c	0	a	b	1	1
1	0	a	b	c	1

$\otimes$	0	a	b	c	1
0	0	0	0	0	0
a	0	a	0	a	a
b	0	0	b	b	b
c	0	a	b	c	c
1	0	a	b	c	1

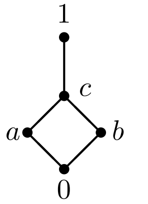

Figure 1: Hasse diagram of L

and $x' = x \to 0$ for every $x \in L$, then $(L, \leqslant, \wedge, \vee, \otimes, \to, 0, 1)$ is a negative non-involutive residuated lattice. Let $A = (\mu_A^P, \mu_A^N) \in \mathbf{BFS}(L)$ be defined as follows

x	0	a	b	c	1
$\mu_A^P(x)$	0.8	0.8	0.3	0.3	0.3
$\mu_A^N(x)$	-0.7	-0.7	-0.4	-0.4	-0.4

By routine calculation, we know that $A = (\mu_A^P, \mu_A^N) \in \mathbf{BFI}(L)$. Taking $\mathcal{I} = \{A = (\mu_A^P, \mu_A^N)\}$, then

$$\begin{aligned}
\omega^* &= \{U_A\} = \{\{(x, y) \in L \times L | x \equiv_A y\}\} \\
&= \{\{(0,0), (a,a), (b,b), (c,c), (1,1), (0,a), (a,0), (1,b), (b,1), (1,c), (c,1)\}\}, \\
\omega &= \{U \subseteq L \times L | U_A \subseteq U\} \text{ and} \\
U_A[0] &= U_A[a] = \{0, a\}, U_A[b] = \{b, 1\}, U_A[c] = \{c, 1\}, U_A[1] = \{b, c, 1\}.
\end{aligned}$$

Thus $\tau_{\mathcal{I}} = \tau_A = \{\emptyset, \{1\}, \{0, a\}, \{b, 1\}, \{c, 1\}, \{0, a, 1\}, \{b, c, 1\}, \{0, a, b, 1\}, \{0, a, c, 1\}, L\}$.

Theorem 3.9 *Let L be a negative non-involutive residuated lattice. Then for every $x \in L$ and $A = (\mu_A^P, \mu_A^N) \in \mathcal{I}$, $U_A[x]$ is a clopen subset of $(L, \tau_{\mathcal{I}})$.*

Proof. For arbitrary $x \in L$ and $A = (\mu_A^P, \mu_A^N) \in \mathcal{I}$, firstly, it follows that $U_A[x]$ is an open subset of $(L, \tau_{\mathcal{I}})$ from Note 1. Secondly, we will show that $U_A[x]$ also is a closed subset of $(L, \tau_{\mathcal{I}})$, it suffices to show that $(U_A[x])^c \in \tau_{\mathcal{I}}$. Let $y \in (U_A[x])^c$, we claim that $U_A[y] \subseteq (U_A[x])^c$. In fact, suppose that $z \in U_A[y]$, then $y \equiv_A z$, thus by Theorem 3.2 we have that $\mu_A^P((y \to z)') = \mu_A^P((z \to y)') = \mu_A^P(0)$ and $\mu_A^N((y \to z)') = \mu_A^N((z \to y)') = \mu_A^N(0)$. If $z \notin (U_A[x])^c$, then $z \in U_A[x]$, i.e., $x \equiv_A z$, thus we have that $\mu_A^P((x \to z)') = \mu_A^P((z \to x)') = \mu_A^P(0)$ and $\mu_A^N((x \to z)') = \mu_A^N((z \to x)') = \mu_A^N(0)$. It follows from $A = (\mu_A^P, \mu_A^N) \in \mathbf{BFI}(L)$ and Lemma 2.4 that

$$\begin{cases} \mu_A^P(0) = \mu_A^P((x \to z)') \wedge \mu_A^P((z \to y)') \leqslant \mu_A^P((x \to y)'), \\ \mu_A^N(0) = \mu_A^N((x \to z)') \vee \mu_A^N((z \to y)') \geqslant \mu_A^N((x \to y)'), \end{cases}$$

and

$$\begin{cases} \mu_A^P(0) = \mu_A^P((y \to z)') \wedge \mu_A^P((z \to x)') \leqslant \mu_A^P((y \to x)'), \\ \mu_A^N(0) = \mu_A^N((y \to z)') \vee \mu_A^N((z \to x)') \geqslant \mu_A^N((y \to x)'), \end{cases}$$

thus $\mu_A^P((x \to y)') = \mu_A^P((y \to x)') = \mu_A^P(0)$ and $\mu_A^N((x \to y)') = \mu_A^N((y \to x)') = \mu_A^N(0)$, and thus $x \equiv_A y$, this shows that $y \in U_A[x]$, it is a contradiction with $y \in (U_A[x])^c$. Therefore $z \in (U_A[x])^c$ and $U_A[y] \subseteq (U_A[x])^c$. Hence $(U_A[x])^c \in \tau_{\mathcal{I}}$ and the proof is completed. □

Proposition 3.10 *Let L be a negative non-involutive residuated lattice and $A = (\mu_A^P, \mu_A^N), B = (\mu_B^P, \mu_B^N) \in \boldsymbol{BFI}(L)$. If $A \sqsubseteq B$, $\mu_A^P(0) = \mu_B^P(0)$ and $\mu_A^N(0) = \mu_B^N(0)$, then $U_A \subseteq U_B$.*

Proof. Let $(x, y) \in U_A$, then $\mu_A^P((x \to y)') = \mu_A^P((y \to x)') = \mu_A^P(0)$ and $\mu_A^N((x \to y)') = \mu_A^N((y \to x)') = \mu_A^N(0)$. Since $A \sqsubseteq B$, $\mu_A^P(0) = \mu_B^P(0)$ and $\mu_A^N(0) = \mu_B^N(0)$, we have that $\mu_B^P(0) = \mu_A^P(0) = \mu_A^P((x \to y)') \leqslant \mu_B^P((x \to y)')$, $\mu_B^P(0) = \mu_A^P(0) = \mu_A^P((y \to x)') \leqslant \mu_B^P((y \to x)')$ and $\mu_B^N(0) = \mu_A^N(0) = \mu_A^N((x \to y)') \geqslant \mu_B^N((x \to y)')$, $\mu_B^N(0) = \mu_A^N(0) = \mu_A^N((y \to x)') \geqslant \mu_B^N((y \to x)')$. Thus $\mu_B^P((x \to y)') = \mu_B^P((y \to x)') = \mu_B^P(0)$ and $\mu_B^N((x \to y)') = \mu_B^N((y \to x)') = \mu_B^N(0)$. Hence $(x, y) \in U_B$ and the proof is completed. □

Theorem 3.11 *Let L be a negative non-involutive residuated lattice and $A = (\mu_A^P, \mu_A^N), B = (\mu_B^P, \mu_B^N) \in \boldsymbol{BFI}(L)$. If $A \sqsubseteq B$, $\mu_A^P(0) = \mu_B^P(0)$ and $\mu_A^N(0) = \mu_B^N(0)$, then $\tau_B \subseteq \tau_A$.*

Proof. Define $\mathcal{I}_1 = \{B\}$, $\omega_1^* = \{U_B\}$, $\omega_1 = \{U \subseteq L \times L | U_B \subseteq U\}$ and $\mathcal{I}_2 = \{A\}$, $\omega_2^* = \{U_A\}$, $\omega_2 = \{U \subseteq L \times L | U_A \subseteq U\}$. Let $O \in \tau_B$, then for all $x \in O$, there exists $U \in \omega_1$ such that $U[x] \subseteq O$, thus $U_B[x] \subseteq U[x] \subseteq O$. Since $A \sqsubseteq B$, $\mu_A^P(0) = \mu_B^P(0)$ and $\mu_A^N(0) = \mu_B^N(0)$, by Proposition 3.10 we have that $U_A \subseteq U_B$. Thus $U_A[x] \subseteq U_B[x] \subseteq U[x] \subseteq O$, therefore $O \in \tau_A$ and $\tau_B \subseteq \tau_A$. □

Theorem 3.12 *Let L be a negative non-involutive residuated lattice. If $B = \bigsqcap_{A \in \mathcal{I}} A$, then $\tau_{\mathcal{I}} = \tau_B$.*

Proof. Let $\mathcal{I}_1 = \{B\}$, $\omega_1^* = \{U_B\}$, $\omega_1 = \{U \subseteq L \times L | U_B \subseteq U\}$ and ω^*, ω be as defined in Proposition 3.5 and Theorem 3.6 respectively. Assume that $O \in \tau_B$, then for all $x \in O$, there exists $U \in \omega_1$ such that $U[x] \subseteq O$, thus $U_B[x] \subseteq U[x] \subseteq O$. Since $B = \bigsqcap_{A \in \mathcal{I}} A$ and $\mathcal{I}$ is closed under BF-intersection, we know $B \in \mathcal{I}$. Thus $U_B \in \omega$, and thus $O \in \tau_{\mathcal{I}}$, this shows that $\tau_B \subseteq \tau_{\mathcal{I}}$. Conversely, let $O \in \tau_{\mathcal{I}}$, then for all $x \in O$, there exists $U \in \omega$ such that $U[x] \subseteq O$. Since $U \in \omega$, there exists $A \in \mathcal{I}$ such that $U_A[x] \subseteq U[x]$. Notice that $B = \bigsqcap_{A \in \mathcal{I}} A \sqsubseteq A$, $\mu_A^P(0) = \mu_B^P(0)$ and $\mu_A^N(0) = \mu_B^N(0)$, we can obtain that $U_B[x] \subseteq U_A[x] \subseteq U[x] \subseteq O$, thus $O \in \tau_B$, and thus $\tau_{\mathcal{I}} \subseteq \tau_B$. Hence $\tau_{\mathcal{I}} = \tau_B$. □

Note 2 *Let L be a negative non-involutive residuated lattice and $B = \bigsqcap_{A \in \mathcal{I}} A$, then we can easily have the following statements:*

(i) By Theorem 3.12, we know that $\tau_{\mathcal{I}} = \tau_B$. For every $x \in L$ and $U \in \omega$, we can obtain that $U_B[x] \subseteq U[x]$. Hence $\tau_{\mathcal{I}} = \{O \subseteq L | \forall x \in O, U_B[x] \subseteq O\}$. This fact shows that $O \subseteq L$ is an open subset of $(L, \tau_{\mathcal{I}})$ if and only if for all $x \in O$, $U_B[x] \subseteq O$ if and only if $O = \bigcup_{x \in O} U_B[x]$.

(ii) For every $x \in L$, by (i), we know that $U_B[x]$ is the smallest open neighborhood of x.

(iii) Let $\mathcal{B}_B = \{U_B[x] | x \in L\}$, By (i) and (ii), it is easy to check that $\mathcal{B}_B$ is a base of uniform topology τ_B.

(iv) For every $x \in L$, $\{U_B[x]\}$ is a countable neighborhood base of x.

Theorem 3.13 *Let L be a negative non-involutive residuated lattice. then $(L, \tau_{\mathcal{I}})$ is discrete space if and only if for all $x \in L$, there exists $A \in \mathcal{I}$ such that $U_A[x] = \{x\}$.*

Proof. Let $(L, \tau_{\mathcal{I}})$ is discrete space. Assume that for any $A = (\mu_A^P, \mu_A^N) \in \mathcal{I}$, there exists $x \in L$ such that $U_A[x] \neq \{x\}$. Taking $B = \prod\limits_{A \in \mathcal{I}} A$, then $B \in \mathcal{I}$ and there exists $x_0 \in L$ such that $U_B[x_0] \neq \{x_0\}$, it follows that there exists $y_0 \in U_B[x_0]$ such that $x_0 \neq y_0$. By Note 2 (ii), $U_B[x_0]$ is the smallest open neighborhood of x_0, hence $\{x_0\} \notin \tau_B = \tau_{\mathcal{I}}$, which is a contradiction with $(L, \tau_{\mathcal{I}})$ is discrete space. Therefore for all $x \in L$, there exists $A \in \mathcal{I}$ such that $U_A[x] = \{x\}$. Conversely, assume for all $x \in L$, there exists $A \in \mathcal{I}$ such that $U_A[x] = \{x\}$. Then $\{x\} \in = \tau_{\mathcal{I}}$. Hence $(L, \tau_{\mathcal{I}})$ is discrete space. $\square$

Definition 3.14 Let L be a negative non-involutive residuated lattice, $A(\mu_A^P, \mu_A^N) \in \mathcal{I}$ and $X \subseteq L$. We define $U_A[X] := \bigcup\limits_{x \in X} U_A[x]$.

Theorem 3.15 *Let L be a negative non-involutive residuated lattice, $A(\mu_A^P, \mu_A^N) \in \mathcal{I}$ and $X \subseteq L$. Then the closure of X is equivalent to $\bigcap\limits_{U_A \in \omega^*} U_A[X]$ and it is denoted by $c(X)$ in the uniform topological space $(L, \tau_{\mathcal{I}})$.*

Proof. Let $x \in c(X)$, then for any $A \in \mathcal{I}$, $U_A[x]$ is an open neighborhood of x and $U_A[x] \cap X \neq \emptyset$. Thus there exists $y \in X$ such that $y \in U_A[x]$, i.e., $(x, y) \in U_A$, it follows that $x \in U_A[y] \subseteq \bigcup\limits_{y \in X} U_A[y] = U_A[X]$. Therefore $x \in \bigcap\limits_{U_A \in \omega^*} U_A[X]$. Conversely, let $x \in \bigcap\limits_{U_A \in \omega^*} U_A[X]$, then for any $A \in \mathcal{I}$, $x \in U_A[X]$, thus there exists $y \in X$ such that $x \in U_A[y]$ and $U_A[x] \cap X \neq \emptyset$. Hence $x \in c(X)$. The proof is completed. $\square$

4 Topological Properties of Uniform Topological Space $(L, \tau_{\mathcal{I}})$

In this section, we study some topological properties of Uniform Topological Space $(L, \tau_{\mathcal{I}})$, such as compactness, connectedness, separation and countability and so on.

Definition 4.1 [17,18] Let (X, τ) is a topological space. A subset Y of X is said to be compact if every open covering of Y contains a finite sub-collection also covers Y. A topological space (X, τ) said to be a compact space if and only if X itself is compact. A topological space (X, τ) said to be locally compact at a point $x \in X$, if x has a compact neighborhood in X, and (X, τ) said to be locally compact if it is locally compact at every point.

Theorem 4.2 *Let L be a negative non-involutive residuated lattice. If $B = \bigsqcap_{A\in\mathcal{I}} A$, Then for any $x \in L$, $U_B[x]$ is a compact subset of uniform topological Space $(L, \tau_{\mathcal{I}})$.*

Proof. For arbitrary $x \in L$, let $\{O_\lambda\}_{\lambda\in\Lambda} \subseteq \tau_{\mathcal{I}}$ such that $U_B[x] = \bigcup_{\lambda\in\Lambda} O_\lambda$. Since $x \in U_B[x]$, then there exists $\lambda \in \Lambda$ such that $x \in O_\lambda \in \tau_{\mathcal{I}}$. It follows from Note 2 (i) that $U_A[x] \subseteq O_\lambda$. Hence $U_B[x]$ is a compact subset of $(L, \tau_{\mathcal{I}})$. □

Lemma 4.3 *[17,18] A topological space (X, τ) is connected if and only if the only subsets of X that are both open and closed in X are empty set and X itself.*

Lemma 4.4 *[17,18] If (X, Ω) is a uniform space, then the corresponding uniform topological Space (X, τ) is a completely regular space.*

Theorem 4.5 *Let L be a negative non-involutive residuated lattice. Then the uniform topological Space $(L, \tau_{\mathcal{I}})$ is a first-countable, zero-dimensional, disconnected, locally compact and completely regular space.*

Proof. Taking $B = \bigsqcap_{A\in\mathcal{I}} A$, by Theorem 3.12, it suffices to show that (L, τ_B) is a first-countable, zero-dimensional, disconnected, locally compact and completely regular space. In fact,

(1) For any $x \in L$, by Note 2 (iv), $\{U_B[x]\}$ is a countable neighborhood base of x. Thus (L, τ_B) is a first-countable space.

(2) By Theorem 3.9 and Note 2 (iii), we know that $\mathcal{B}_B = \{U_B[x] | x \in L\}$ is a clopen base of τ_B. Thus (L, τ_B) is a zero-dimensional space.

(3) By Theorem 3.9 and Lemma 4.3, we have that (L, τ_B) is a disconnected space.

(4) For any $x \in L$, Note 2 (ii) and Theorem 4.2, we know that $U_B[x]$ is a compact neighborhood of x. Thus (L, τ_B) is a locally compact space.

(5) It follows that (L, τ_B) is a completely regular space from Lemma 4.4. □

Definition 4.6 [17,18] A uniform space (X, Ω) is said to be totally bounded if for every $U \in \Omega$, there exists $\{x_i\}_{i=1}^n \subseteq X$ such that $X = \bigcup_{i=1}^n U[x_i]$.

Theorem 4.7 *Let L be a negative non-involutive residuated lattice and $A = (\mu_A^P, \mu_A^N) \in \boldsymbol{BFI}(L)$. Then the following conditions are equivalent:*

(1) Uniform topological Space (L, τ_A) is compact,

(2) Uniform Space (L, ω) is totally bounded,

(3) There exists $P = \{x_i\}_{i=1}^n \subseteq L$, for all $x \in L$, there exists $x_i \in P$ such that $x \equiv_A x_i$.

Proof. (1)$\Longrightarrow$(2): This is clear by Theorem 32 of Chapter 6 in [10].

(2)$\Longrightarrow$(3): Let $U_A \in \omega$, since (L, ω) is totally bounded, by Definition 4.6, there exists $P = \{x_i\}_{i=1}^n \subseteq L$ such that $L = \bigcup_{i=1}^n U_A[x_i]$. Thus for all $x \in L$, there exists $x_i \in P$ such that $x \in U_A[x_i]$, therefore $x \equiv_A x_i$.

(3)$\Longrightarrow$(1): For any $x \in L$, by hypothesis there exists $x_i \in P$ such that $x \equiv_A x_i$.

Therefore $x \in U_A[x_i]$, hence $L = \bigcup_{i=1}^{n} U_A[x_i]$. Now let there exists $\exists\{O_\lambda\}_{\lambda\in\Lambda} \subseteq \tau_A$ such that $L = \bigcup_{\lambda\in\Lambda} O_\lambda$, then for any $x_i \in P = \{x_i\}_{i=1}^n \subseteq L$, there exists $\lambda_i \in \Lambda$ such that $x_i \in O_{\lambda_i}$. Since $O_{\lambda_i} \in \tau_A$, $U_A[x_i] \subseteq O_{\lambda_i}$, we have that $L = \bigcup_{i=1}^{n} U_A[x_i] \subseteq \bigcup_{i=1}^{n} O_{\lambda_i}$. Therefore $L = \bigcup_{i=1}^{n} O_{\lambda_i}$, which means that (L, τ_A) is compact. □

About the separation of Uniform topological Space $(L, \tau_{\mathcal{I}})$, we have the following conclusion:

Theorem 4.8 *Let L be a negative non-involutive residuated lattice. Then the following statements are equivalent:*
(1) Uniform topological Space $(L, \tau_{\mathcal{I}})$ is a T_1-space,
(2) Uniform topological Space $(L, \tau_{\mathcal{I}})$ is a T_2-space.

Proof. (1)$\Longrightarrow$(2): Let $(L, \tau_{\mathcal{I}})$ is a T_1-space and $x, y \in L$ such that $x \neq y$. Then there exists $O_1, O_2 \in \tau_{\mathcal{I}}$ such that $x \in O_1, y \notin O_1$ and $y \in O_2, x \notin O_2$. Hence there exists $A = (\mu_A^P, \mu_A^N), B = (\mu_B^P, \mu_B^N) \in \mathbf{BFI}(L)$ such that $U_A[x] \subseteq O_1 U_B[y] \subseteq O_2$. Let $C = A \sqcap B$, then $C \in \mathcal{I}$. We will show that $U_C[x] \cap U_C[y] = \emptyset$. In fact, let $z \in U_C[x] \cap U_C[y]$, then $z \equiv_C x$ and $z \equiv_C y$, by Definition 3.1 we have that

$$\begin{cases} \mu_C^P((z \to x)') = \mu_C^P((x \to z)') = \mu_C^P(0) = \mu_C^P((z \to y)') = \mu_C^P((y \to z)'), \\ \mu_C^N((z \to x)') = \mu_C^N((x \to z)') = \mu_C^N(0) = \mu_C^N((z \to y)') = \mu_C^N((y \to z)'). \end{cases}$$

It follows from $C \in \mathbf{BFI}(L)$ and Lemma 2.4 that $\mu_C^P(0) = \mu_C^P((x \to z)') \wedge \mu_C^P((z \to y)') \leqslant \mu_C^P((x \to y)')$, $\mu_C^N(0) = \mu_C^N((x \to z)') \vee \mu_C^N((z \to y)') \geqslant \mu_C^N((x \to y)')$ and $\mu_C^P(0) = \mu_C^P((y \to z)') \wedge \mu_C^P((z \to x)') \leqslant \mu_C^P((y \to x)')$, $\mu_C^N(0) = \mu_C^N((y \to z)') \vee \mu_C^N((z \to x)') \geqslant \mu_C^N((y \to x)')$. Hence $\mu_C^P((x \to y)') = \mu_C^P((y \to x)') = \mu_C^P(0)$ and $\mu_C^N((x \to y)') = \mu_C^N((y \to x)') = \mu_C^N(0)$. Since $C = A \sqcap B \sqsubseteq A$ and $\mu_C^P(0) = \mu_A^P(0)$, $\mu_C^N(0) = \mu_A^N(0)$, by Proposition 3.10 we can obtain that $y \in U_C[x] \subseteq U_A[x] \subseteq O_1$, which is a contradiction. Therefore $U_C[x] \cap U_C[y] = \emptyset$. This show that $(L, \tau_{\mathcal{I}})$ is a T_2-space.

(2)$\Longrightarrow$(1): It is obvious. □

Finally, let L a negative non-involutive residuated lattice. we discuss the continuity of lattice operators $\wedge, \vee$ and adjoint operations $\otimes, \to$ in L with respect to uniform topology $\tau_{\mathcal{I}}$.

Definition 4.9 [17,18] Let L be a negative non-involutive residuated lattice and τ a topology on L. Then (L, τ) is called a topological negative non-involutive residuated lattice if the lattice operators $\wedge, \vee$ and adjoint operation $\otimes, \to$ in L are continuous with respect to τ.

Note 3 *Let L be a negative non-involutive residuated lattice, τ a topology on L and $X, Y \subseteq L$. Define*

$$X \star Y := \{x \star y \in L | x \in X \text{ and } y \in Y\}, \star \in \{\wedge, \vee, \otimes, \to\}.$$

Then operations $\star \in \{\wedge, \vee, \otimes, \rightarrow\}$ are continuous with respect to τ equivalent to: for every $x, y \in L$ and $O \in \tau$, $x \star y \in O$ implies there exists $O_1, O_2 \in \tau$ such that $x \in O_1, y \in O_2$ and $O_1 \star O_2 \subseteq O$.

Theorem 4.10 *Let L be a negative non-involutive residuated lattice. Then $(L, \tau_{\mathcal{I}})$ is a topological negative non-involutive residuated lattice.*

Proof. By Definition 4.9, it suffices to show that $\forall \star \in \{\wedge, \vee, \otimes, \rightarrow\}$ are continuous with respect to the uniform topology $\tau_{\mathcal{I}}$. In fact, for every $x, y \in L$ and $O \in \tau_{\mathcal{I}}$, let $x \star y \in O$, then there exists $U \in \omega$ such that $U[a \star b] \subseteq O$ and there exists $A = (\mu_A^P, \mu_A^N) \in \mathcal{I}$ such that $U_A \subseteq U$. We claim that $U_A[x] \star U_A[y] \subseteq U_A[x \star y]$. Indeed, let $z \star q \in U_A[x] \star U_A[y]$, then $z \in U_A[x]$ and $q \in U_A[y]$, thus $z \equiv_A x$ and $q \equiv_A y$, and so $(x \star y) \equiv_A (z \star q)$. It follows that $(x \star y, z \star q) \in U_A \subseteq U$, hence $z \star q \in U_A[x \star y]$, this means that the claim holds. Taking $O_1 = U_A[x]$ and $O_2 = U_A[y]$, then we can obtain that $O_1, O_2 \in \tau_{\mathcal{I}}, x \in O_1, y \in O_2$ and $O_1 \star O_2 = U_A[x] \star U_A[y] \subseteq U_A[x \star y] \subseteq O$. The proof is completed. $\square$

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