Polynomial Ideals, Grö;bner Bases and Constraints

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Abstract

Grö;bner Basis Theory provides algorithms to decide and solve many problems in multivariate polynomial ideal theory in mathematics. Constraint Satisfaction is a paradigm which is widely used for the formulation and solutions of many problems both inside and outside academia. This paper discusses relationships between and applications of Ideal and Grö;bner Basis Theory to constraint satisfaction problems.

It is shown that the notion of a finite discrete constraint network in constraint satisfaction coincides with the notion of a variety in geometry. Varieties correspond to certain classes of polynomial ideals in Ideal Theory. This opens the door for the application of Grö;bner Basis algorithms to decide and solve problems in constraint satisfaction.

Grö;bner Basis Theory can be used to decide and solve the consistency problem in Polynomial Ideal Theory. Because of the relationship between polynomial ideals and finite discrete constraint networks this can be used to solve the satisfiability problem in constraint satisfaction.

Grö;bner Basis Theory provides algorithms to compute generators of elimination ideals. This has an application to constraint satisfaction problems because this can be used to create constraints of lower arity than the original ones. Dedicated algorithms can be applied to these new constraints to prune the search space.

As a special case it is shown that algorithms to compute elimination ideals can be used to compute what are called constraint networks in globally solved form. From an intuitive point of view these networks are easy to solve in the sense that all their solutions can be found without "making mistakes" in search.

The paper is intended as an introduction for members of a mixed audience not previously aware of Polynomial Ideal Theory, Grö;bner Basis Theory, Constraint Satisfaction Theory or their relationship. Motivating examples are given and references to the literature are presented.