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Rational trigonometric cubic spline to conserve convexity of 2D data

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Abstract Researchers in different fields of study are always in dire need of spline interpolating function that conserve intrinsic trend of the data. In this paper, a rational trigonometric cubic spline with four free parameters has been used to retain convexity of 2D data. For this purpose, constraints on two of free parameters β_i and γ_i in the description of the rational trigonometric function are derived while the remaining two α_i and δ_i are set free. Numerical examples demonstrate that resulting curves using the technique of the underlying paper are C^1.

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1. Introduction

Representation of data in the form of congenial curves and surfaces is of great importance in many areas of scientific research such as computer graphics and data visualization. This visual display of data provides prompt cognition and insight into data. Shape preservation and smoothness are the most desirable features required by a researcher in the field of data visualization. Spline interpolating function demonstrates the incredible result in this regard.

Properties that quantify shape of data are positivity, monotonocity, and convexity. Plenty of spline functions exist which can produce a smooth curve but inept to preserve the inherent shape of the given data. The motivation of this paper is to preserve the intrinsic attribute of data that is convexity.

Convexity pervades everyday life. Whether it is manufacturing of lenses, modeling of cars, analysis of indifference curves, nonlinear programming and approximation of functions, convexity stays the part and parcel of all. Loss of convexity is irreconcilable in all such practical problems.

Over the years, many milestones have been achieved in the field of shape preservation when data under consideration exhibit convex trend. Various schemes have been developed to reach the epitome of abstraction. Brodlie and Butt [1] preserved the convex shape of data by establishing a piecewise cubic function. Their scheme suffered the detriment of insertion of the additional knots in an interval where convexity is lost.

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Convexity preserving properties of rational Bezier and non-uniform rational B-spline were studied geometrically by Carnicer [2]. Explicit representation of rational cubic function with one free parameter was developed by Delbourgo and Gregory [3] to retain convexity. Passcow and Roulier [4] formed a spline interpolant by constructing an auxiliary set of points and using convexity preserving properties of Bernstein polynomials. Schumaker [5] preserved the convexity by piecewise quadratic polynomial which was economical but again an extra knot had to be inserted. A rational cubic function with two shape parameters was introduced by Sarfraz and Hussain [6] to retain convexity. Sarfraz et al. [7] developed rational cubic functions with four parameters. In [7], the authors derived the shape preserving constraints on two parameters and two parameters are left free for the user to refine the shape of the curves.

The algorithm presented in this paper is a noteworthy addition to already existing schemes and is a leap forward in many ways. The use of trigonometric function strengthens a designer to frame conics accurately. It does not require insertion of extra knots to conserve the intrinsic trend of the data. Four free parameters have been used in the specification of the rational trigonometric cubic spline. Constraints on two of free parameters are derived to envision convex data. The remaining two free parameters give enough freedom to the user for the refinement of convex shape of data according to the requirement.

The remainder of the paper is organized as follows. Section 2 is a review of the rational trigonometric cubic function [8] to be used for convexity. Section 3 deals with the development of convexity conserving constraints. In Section 4, numerical examples have been demonstrated. Section 5 summarizes the contributions and concludes the paper.

2. Rational trigonometric cubic function

Let $\{(x_i,f_i),\ i = 0,1,2,\ldots,n\}$ be the given set of data points defined over the interval $[a,b]$ where $a = x_0 < x_1 < x_2 < \cdots < x_n = b$. A piecewise rational trigonometric cubic function is defined over each subinterval $I_i = [x_i, x_{i+1}]$ as

$$S_i(x) = \frac{p_i(\theta)}{q_i(\theta)}, \tag{1}$$

$$\begin{aligned} p_i(\theta) &= \alpha_i f_i(1-\sin\theta)^3 + \left\{\beta_i f_i + \frac{2h_i\alpha_i d_i}{\pi}\right\}\sin\theta(1-\sin\theta)^2 \\ &\quad + \left\{\gamma_i f_{i+1} - \frac{2h_i\delta_i d_{i+1}}{\pi}\right\}\cos\theta(1-\cos\theta)^2 \\ &\quad + \delta_i f_{i+1}(1-\cos\theta)^3, \end{aligned}$$

$$\begin{aligned} q_i(\theta) &= \alpha_i(1-\sin\theta)^3 + \beta_i\sin\theta(1-\sin\theta)^2 + \gamma_i\cos\theta(1-\cos\theta)^2 \\ &\quad + \delta_i(1-\cos\theta)^3, \end{aligned}$$

where

$$\theta = \frac{\pi}{2}\left(\frac{x-x_i}{h_i}\right), \quad h_i = x_{i+1} - x_i, \quad i = 0,1,2,\ldots,n-1.$$

The rational trigonometric cubic function (1) satisfies the following properties:

$$S(x_i) = f_i, \quad S(x_{i+1}) = f_{i+1}, \quad S^{(1)}(x_i) = d_i, \quad S^{(1)}(x_{i+1}) = d_{i+1}.$$

d_i and d_{i+1} are derivatives at the end points of the interval $I_i = [x_i, x_{i+1}]$, α_i, β_i, γ_i and δ_i are the free parameters.

3. Convex rational trigonometric cubic spline

In this section, problem of preserving convexity of 2D is addresses. For this purpose, constraints on free parameters in the description of rational trigonometric cubic function (1) are developed. Let the convex data set defined over the interval $[a,b]$ be $\{(x_i,f_i),\ i = 0,1,2,\ldots,n\}$. The necessary condition for the convexity of data is

$$d_1 < \Delta_1 < \ldots \Delta_{i-1} < d_i < \Delta_i \ldots < \Delta_{n-1} < d_n.$$

The rational trigonometric cubic function (1) is convex if and only if $S_i^2(x) > 0$, where

$$\begin{aligned} S_i^2(x) = \frac{\pi}{2h_i(q_i(\theta))^2}\{ & A_0\sin\theta(1-\sin\theta)^8 + A_1\cos^2\theta(1-\sin\theta)^7 \\ & + A_2\sin^2\theta(1-\sin\theta)^7 + A_3\sin\theta\cos^2\theta(1-\sin\theta)^6 \\ & + A_4\sin\theta\cos^2\theta(1-\sin\theta)^5(1-\cos\theta) \\ & + A_5\sin^2\theta\cos\theta(1-\sin\theta)^6 + A_6\sin\theta\cos^2\theta(1-\sin\theta)^6 \\ & + A_7\sin^2\theta(1-\sin\theta)^6(1-\cos\theta) \\ & + A_8\cos\theta(1-\sin\theta)^6(1-\cos\theta)^2 \\ & + A_9\sin^3\theta\cos\theta(1-\sin\theta)^5 \\ & + A_{10}\sin^2\theta\cos^2\theta(1-\sin\theta)^4(1-\cos\theta) \\ & + A_{11}\sin^3\theta(1-\sin\theta)^5(1-\cos\theta) \\ & + A_{12}\sin^2\theta\cos\theta(1-\sin\theta)^4(1-\cos\theta)^2 \\ & + A_{13}\sin\theta\cos\theta(1-\sin\theta)^5(1-\cos\theta)^2 \\ & + A_{14}\cos^3\theta(1-\sin\theta)^4(1-\cos\theta)^2 \\ & + A_{15}\cos^3\theta\sin\theta(1-\sin\theta)^3(1-\cos\theta)^2 \\ & + A_{16}\cos\theta\sin^2\theta(1-\sin\theta)^3(1-\cos\theta)^3 \\ & + A_{17}\cos^2\theta(1-\sin\theta)^3(1-\cos\theta)^4 \\ & + A_{18}\cos^2\theta\sin\theta(1-\sin\theta)^3(1-\cos\theta)^3 \\ & + A_{19}\sin^2\theta(1-\sin\theta)^3(1-\cos\theta)^4 \\ & + A_{20}\cos^2\theta(1-\sin\theta)^4(1-\cos\theta)^3 \\ & + A_{21}\sin\theta(1-\sin\theta)^5(1-\cos\theta)^3 \\ & + A_{22}\sin^2\theta(1-\sin\theta)^4(1-\cos\theta)^3 \\ & + A_{23}\cos\theta(1-\sin\theta)^3(1-\cos\theta)^5 \\ & + A_{24}\sin^3\theta(1-\sin\theta)^6 + A_{25}\sin^2\theta\cos^2\theta(1-\sin\theta)^5 \\ & + A_{26}\sin^4\theta\cos\theta(1-\sin\theta)^4 \\ & + A_{27}\sin^3\theta(1-\cos\theta)^4(1-\sin\theta)^2 \\ & + A_{28}\sin\theta\cos\theta(1-\cos\theta)^5(1-\sin\theta)^2 \\ & + A_{29}\sin^3\theta\cos\theta(1-\cos\theta)^3(1-\sin\theta)^2 \\ & + A_{30}\sin^3\theta\cos\theta(1-\cos\theta)^2(1-\sin\theta)^3 \\ & + A_{31}\sin^4\theta(1-\cos\theta)(1-\sin\theta)^4 \\ & + A_{32}\sin^3\theta\cos^2\theta(1-\cos\theta)(1-\sin\theta)^3 \\ & + A_{33}\sin\theta\cos^2\theta(1-\cos\theta)^4(1-\sin\theta)^2 \\ & + A_{34}\sin^2\theta\cos^3\theta(1-\cos\theta)^2(1-\sin\theta)^2 \\ & + A_{35}\sin^3\theta(1-\cos\theta)^3(1-\sin\theta)^3 \\ & + A_{36}\sin^2\theta\cos^2\theta(1-\cos\theta)^3(1-\sin\theta)^2 \\ & + A_{37}\sin^2\theta\cos\theta(1-\cos\theta)^6 + A_{38}\cos^2\theta(1-\cos\theta)^7 \\ & + A_{39}\sin^2\theta\cos^2\theta(1-\cos\theta)^4(1-\sin\theta) \\ & + A_{40}\cos^3\theta(1-\cos\theta)^3(1-\sin\theta)^3 \\ & + A_{41}\sin^2\theta\cos^2\theta(1-\cos\theta)^2(1-\sin\theta)^3 \\ & + A_{42}\sin\theta\cos^3\theta(1-\cos\theta)^3(1-\sin\theta)^2 \\ & + A_{43}\sin^3\theta\cos^2\theta(1-\cos\theta)^2(1-\sin\theta)^2 \\ & + A_{44}\cos^3\theta(1-\cos\theta)^6 + A_{45}\cos^4\theta(1-\cos\theta)^4(1-\sin\theta) \\ & + A_{46}\cos^3\theta(1-\cos\theta)^5(1-\sin\theta) \\ & + A_{47}\cos^4\theta\sin\theta(1-\cos\theta)^4 + A_{48}\sin^2\theta\cos\theta(1-\cos\theta)^5(1-\sin\theta) \\ & + A_{49}\sin\theta\cos^3\theta(1-\cos\theta)^5 + A_{50}\sin^2\theta(1-\cos\theta)^3(1-\sin\theta)^4 \\ & + A_{51}\cos\theta(1-\cos\theta)^8 + A_{52}\sin^2\theta(1-\cos\theta)^7 \\ & + A_{53}\sin\theta(1-\cos\theta)^3(1-\sin\theta)^5 + A_{54}\sin\theta(1-\cos\theta)^6(1-\sin\theta)^2 \\ & + A_{55}\sin\theta\cos^2\theta(1-\cos\theta)^3(1-\sin\theta)^3 + A_{56}\cos^2\theta(1-\cos\theta)^6(1-\sin\theta) \\ & + A_{57}\sin^2\theta(1-\cos\theta)^6(1-\sin\theta) + A_{58}\sin\theta\cos^2\theta(1-\cos\theta)^6 \\ & + A_{59}\sin^2\theta\cos^2\theta(1-\sin\theta)^5 + A_{60}\sin^2\theta\cos^2\theta(1-\cos\theta)^3(1-\sin\theta)^2 \\ & + A_{61}\sin^3\theta\cos^2\theta(1-\cos\theta)(1-\sin\theta)^3 + A_{62}\sin^2\theta\cos^3\theta(1-\cos\theta)^3(1-\sin\theta) \\ & + A_{63}\sin^2\theta\cos^2\theta(1-\cos\theta)^4(1-\sin\theta)\} \end{aligned} \tag{2}$$

with

$$\begin{aligned}
A_0 &= -\alpha_i d, \quad A_1 = \alpha_i d + \alpha_i a - 2\beta_i d, \quad A_2 = -\alpha_i a - \beta_i d, \quad A_3 \\
&= 2\alpha_i a - 2\beta_i a - \beta_i d, \quad A_4 = 3\alpha_i k + 2\alpha_i l + 3\alpha_i b - \beta_i k - 4\gamma_i d, \quad A_5 = \alpha_i k \\
&= A_6, \quad A_7 = 2\alpha_i g - \alpha_i k, \quad A_8 = g\alpha_i, \quad A_9 = \alpha_i l + \beta_i k, A_{10} \\
&= 4\alpha_i l + 2\alpha_i e + \beta_i k + 2\beta_i b - 4\gamma_i a, \quad A_{11} = 2\alpha_i h - \alpha_i l + 2\beta_i g - \beta_i k, \quad A_{12} \\
&= \alpha_i(4h - 2e + 3f + 6a) + \beta_i(3c - 2b + g) - 6\delta_i a + \gamma_i a, A_{13} \\
&= \alpha_i(2h + 3g - 2b + 3c) - \beta_i g - 6\delta_i d + \gamma_i d, \quad A_{14} \\
&= \alpha_i(4b + e) - 2\beta_i b + \gamma_i(a - 5d), \quad A_{15} = 5\alpha_i e + \beta_i(2b - e) + 4\gamma_i a, \quad A_{16} \\
&= 4\alpha_i i + \gamma_i(k - 2g) - 5\delta_i k, \quad A_{17} = \alpha_i i + \gamma_i g + \delta_i k, \quad A_{18} \\
&= 5\alpha_i f + \beta_i(2c - f), \quad A_{19} = \alpha_i(5j - i) - \delta_i(k + 4g) + 2\gamma_i g, \quad A_{20} \\
&= \alpha_i(4c + f) - 2\beta_i c + \delta_i(a - 5d), \quad A_{21} = -\alpha_i c, \quad A_{22} = -\alpha_i f - \beta_i c, \quad A_{23} \\
&= \alpha_i j + \delta_i g, \quad A_{24} = -\beta_i a, \quad A_{25} = -4\beta_i a, \quad A_{26} = \beta_i l, \quad A_{27} \\
&= \beta_i(-i + 5j) - \delta_i(4h + l) + 2\gamma_i h, \quad A_{28} \\
&= 6\alpha_i j - \beta_i j + \delta_i(2h - 3g - 2b - 3c) + \gamma_i c, \quad A_{29} \\
&= 4\beta_i i + \gamma_i(l - 2h) - 5\delta_i l, \quad A_{30} = \beta_i(2h - 2e + 3f), \quad A_{31} = \beta_i(2h - l), \quad A_{32} \\
&= 2\beta_i(e - l), \quad A_{33} = 6\alpha_i i - \beta_i i + \gamma_i(2h - 3g - c) + \delta_i(2l - 4b - 3k), \quad A_{34} \\
&= 3\beta_i e, \quad A_{35} = -\beta_i f, \quad A_{36} = -\beta_i f, \quad A_{37} = \gamma_i(3i + j) - 2\delta_i i, \quad A_{38} \\
&= \gamma_i j + \delta_i i, \quad A_{39} = 4\beta_i i + \gamma_i(j - 2h) + 2\delta_i(e - l), \quad A_{40} = \gamma_i k, \quad A_{41} \\
&= -3\gamma_i k, \quad A_{42} = \gamma_i(6b + 2l - 3k), \quad A_{43} = -3\gamma_i l, \quad A_{44} = \gamma_i i, \quad A_{45} \\
&= \gamma_i(e - 2b), \quad A_{46} = \gamma_i(f - 2c) + \delta_i(e - 2b), \quad A_{47} = -\gamma_i e, \quad A_{48} \\
&= 4\beta_i j - \gamma_i f - \delta_i(2e + 3f + 2h), \quad A_{49} = -\gamma_i f - \delta_i e, \quad A_{50} = -\delta_i a, \quad A_{51} \\
&= -\delta_i j, \quad A_{52} = 2\gamma_i j - \delta_i(i + j), \quad A_{53} = -\delta_i d, \quad A_{54} = -\delta_i c, \quad A_{55} \\
&= -4\delta_i a, \quad A_{56} = \delta_i(f - 2c), \quad A_{57} = -\delta_i f = A_{58}, \quad A_{59} = 4\beta_i a, \quad A_{60} \\
&= 4\beta_i f, \quad A_{61} = 4\beta_i l, \quad A_{62} = 4\gamma_i e, \quad A_{63} = 3\delta_i e,
\end{aligned}$$

and

$$\begin{aligned}
a &= -\beta_i F_0 + \alpha_i F_1, \quad b = \gamma_i(F_1 - 3F_0) + F_2(3\alpha_i - \beta_i), \quad c \\
&= \delta_i(F_1 - 3F_0) + F_3(3\alpha_i - \beta_i), \quad d = -\beta_i F_0 + \alpha_i F_1, \quad e \\
&= 2\beta_i F_2 - 2\gamma_i F_1, \quad f = 2\beta_i F_3 - 2\delta_i F_1, \quad g \\
&= \alpha_i(3F_3 - F_2) - F_0(3\delta_i - \gamma_i), \quad h \\
&= \beta_i(3F_3 - F_2) - F_1(3\delta_i - \gamma_i), \quad i = \gamma_i F_3 - \delta_i F_2, \quad j \\
&= \gamma_i F_3 - \delta_i F_2, \quad k = 2\alpha_i F_2 - 2\gamma_i F_0, \quad l = 2\beta_i F_2 - 2\gamma_i F_1,
\end{aligned}$$

where

$$\begin{aligned}
F_0 &= \alpha_i f_i, \\
F_1 &= \beta_i f_i + \frac{2\alpha_i h_i d_i}{\pi}, \\
F_2 &= \gamma_i f_{i+1} - \frac{2h_i \delta_i d_{i+1}}{\pi}, \\
F_3 &= \delta_i f_{i+1}.
\end{aligned}$$

Now, $S_i^2(x) > 0$ if A_i, $i = 0, 1, 2, \ldots, 63$ are all positive as the denominator in (2) is strictly a positive quantity. A_i, $i = 0, 1, 2, \ldots, 63$ are all positive if the free parameters satisfy the following conditions

$$\gamma_i > \frac{2h_i d_{i+1}\delta_i}{\pi f_{i+1}}, \quad \gamma_i > \frac{2d_{i+1}\delta_i}{\pi \Delta_i}, \quad \gamma_i > \frac{2d_{i+1}\delta_i}{\pi(\Delta_i - 3\delta_i \Delta_i)},$$

$$\gamma_i > \frac{6d_{i+1}\delta_i}{3\delta_i \Delta_i},$$

$$\beta_i > \frac{2\alpha_i d_i}{\pi \Delta_i}, \quad \beta_i > \frac{\alpha_i(2d_i + 3\pi\Delta_i)}{\Delta_i \pi}, \quad \beta_i > \frac{2\alpha_i u_i d_i}{\pi u_i \Delta_i - 2\delta_i d_{i+1}},$$

$$\beta_i > \frac{6\alpha_i u_i(\delta_i d_{i+1} + u_i d_i + 5\pi\delta_i \Delta_i)}{\pi u_i \Delta_i - 2\delta_i d_{i+1}},$$

where $u_i = \max\{0, \gamma_i\}$, α_i and δ_i are positive real numbers.

Table 1 A convex data set.

x	1	1.2	5.3	7.36	10.5	12
y	1	0.36	0.035	0.018	0.008	0.0069

Table 2 A convex data set.

x	−1.6	−1.71	−5.3	−7.36	−7.8
y	0.81	0.29	0.30	0.016	0.018

Table 3 A convex data set.

x	0.1	6	10	28
y	4	0.1	15	25

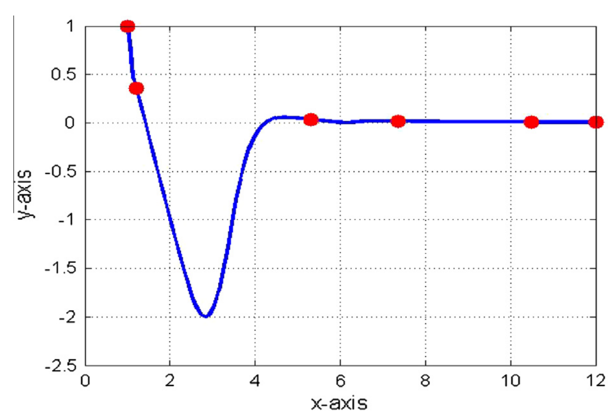

Figure 1 C^1 rational trigonometric cubic function with $\alpha_i = 0.5$, $\beta_i = 1.0$, $\gamma_i = 0.5$, $\delta_i = 1.0$.

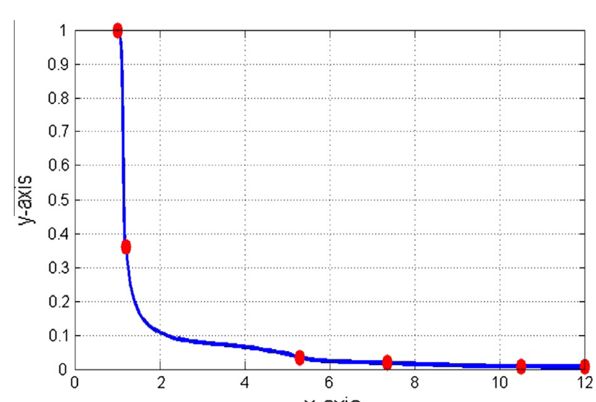

Figure 2 C^1 convex rational trigonometric cubic function with $\alpha_i = 2.1$, $\delta_i = 0.3$.

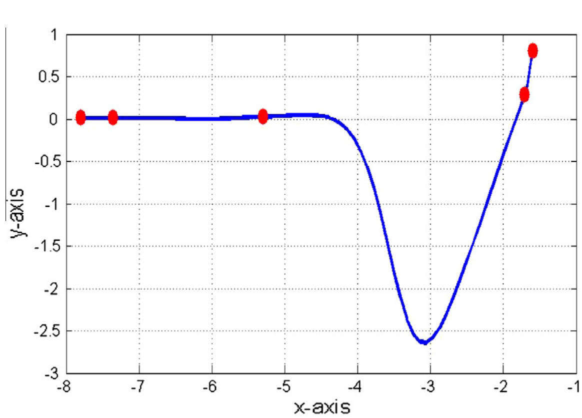

Figure 3 C^1 rational trigonometric cubic function with $\alpha_i = 0.5$, $\beta_i = 1.0$, $\gamma_i = 0.5$, $\delta_i = 1.0$.

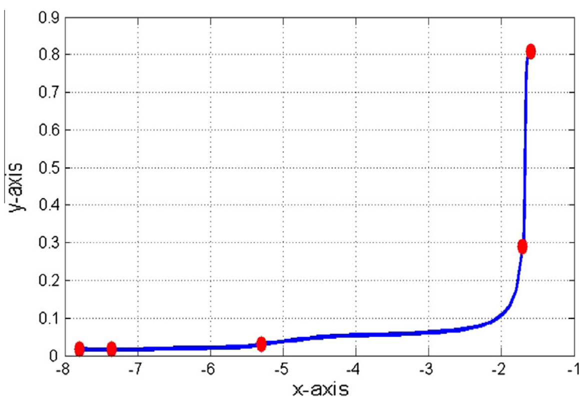

Figure 4 C^1 convex rational trigonometric cubic function with $\alpha_i = 3.5$, $\delta_i = 0.3$.

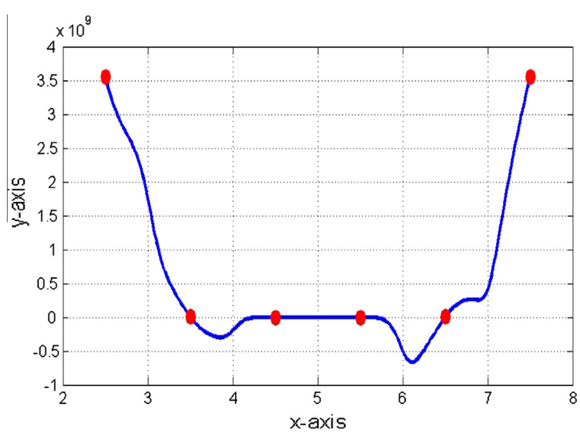

Figure 5 C^1 rational trigonometric cubic function with $\alpha_i = 0.6$, $\beta_i = 1.5$, $\gamma_i = 1.0$, $\delta_i = 1.5$.

The above discussion can be put together as:

Theorem 1. The C^1 piecewise trigonometric rational cubic function (1) preserves the convexity if in each subinterval $I_i = [x_i, x_{i+1}]$, the parameters β_i and γ_i satisfy the following sufficient conditions

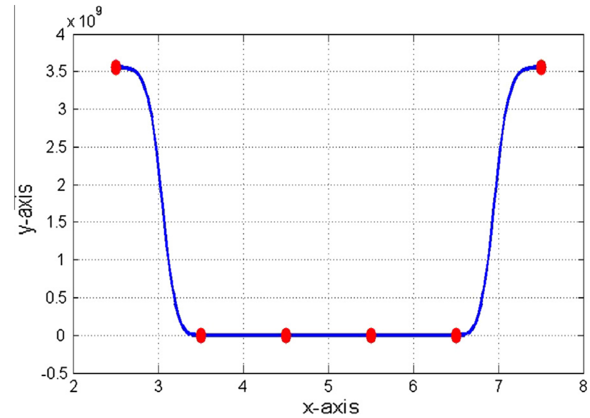

Figure 6 C^1 convex rational trigonometric cubic function with $\alpha_i = 1.5$, $\delta_i = 1.0$.

Table 4 Numerical results corresponding to Fig. 2.

i	1	2	3	4	5	6
d_i	−3.3452	−1.6396	−0.0438	−0.0057	−0.002	0.001
β_i	78.7019	30.691	8.2637	3.1249	3.816	–
γ_i	10.09	1.2543	1.5234	1.3748	0.2	–

Table 5 Numerical results corresponding to Fig. 4.

	1	2	3	4	5
d_i	4.8657	2.3998	0.0396	0.0011	−0.0065
β_i	284.9378	81.0980	17.7179	63.8082	–
γ_i	21.5843	1.2445	0.5162	2.9486	–

Table 6 Numerical results corresponding to Fig. 6.

	1	2	3	4	5	6
d_i	−5.3290e9	−1.7764e9	0	0	1.7764e9	5.3290e9
β_i	6.885e19	0	0	0	6.885e19	–
γ_i	0.3965e20	0	0	0	1.1896e20	–

$$\gamma_i > \frac{2h_i d_{i+1}\delta_i}{\pi f_{i+1}}, \gamma_i > \frac{2d_{i+1}\delta_i}{\pi\Delta_i}, \gamma_i > \frac{2d_{i+1}\delta_i}{\pi(\Delta_i - 3\delta_i\Delta_i)}, \gamma_i > \frac{6d_{i+1}\delta_i}{3\delta_i\Delta_i},$$

and

$$\beta_i > \frac{2\alpha_i d_i}{\pi\Delta_i}, \beta_i > \frac{\alpha_i(2d_i + 3\pi\Delta_i)}{\Delta_i\pi}, \beta_i > \frac{2\alpha_i u_i d_i}{\pi u_i\Delta_i - 2\delta_i d_{i+1}},$$

$$\beta_i > \frac{6\alpha_i u_i(\delta_i d_{i+1} + u_i d_i + 5\pi\delta_i\Delta_i)}{\pi u_i\Delta_i - 2\delta_i d_{i+1}},$$

where $u_i = \max\{0, \gamma_i\}, \alpha_i$ and δ_i are positive real numbers.

The above constraints can be rearranged as

$$\gamma_i = {}_i + \max\left\{0, \frac{2h_i d_{i+1}\delta_i}{\pi f_{i+1}}, \frac{2d_{i+1}\delta_i}{\pi\Delta_i}, \frac{2d_{i+1}\delta_i}{\pi(\Delta_i - 3\delta_i\Delta_i)}, \frac{6d_{i+1}\delta_i}{3\delta_i\Delta_i},\right\}, {}_i > 0,$$

$$\beta_i = w_i + \max\left\{0, \frac{2\alpha_i d_i}{\pi\Delta_i}, \frac{\alpha_i(2d_i + 3\pi\Delta_i)}{\Delta_i\pi}, \frac{2\alpha_i u_i d_i}{\pi u_i\Delta_i - 2\delta_i d_{i+1}}, \frac{6\alpha_i u_i(\delta_i d_{i+1} + u_i d_i + 5\pi\delta_i\Delta_i)}{\pi u_i\Delta_i - 2\delta_i d_{i+1}}\right\}, w_i > 0,$$

where $u_i = \max\{0, \gamma_i\}, \alpha_i$ *and* δ_i *are positive real numbers.*

Algorithm 1.

Step 1. Take a convex data set $\{(x_i, f_i): i = 0, 1, 2, \ldots, n\}$.
Step 2. Estimate the derivatives $d_i's$ at the knots $x_i's$ (if the data are given without derivatives at the knots $x_i's$) with the Arithmetic Mean method.
Step 3. Compute the values of parameters $\beta_i's$ and $\gamma_i's$ using Theorem 1.
Step 4. Substitute the values $d_i's$ at knots $x_i's$, $\beta_i's$ and $\gamma_i's$, α_i and δ_i (positive real numbers) in rational trigonometric cubic function (1) to interpolate and visualize the convex pattern of data.

4. Numerical examples and analysis

This section demonstrates the scheme for convex data developed in Section 3.

The C^1 rational trigonometric cubic function (1) is first used to visualize the convex data sets taken in Tables 1–3, respectively. Arbitrary values are assigned to free parameters α_i, β_i, γ_i and δ_i and resulting curves are shown in Figs. 1, 3 and 5 respectively. It is evident from Figs. 1, 3 and 5 that rational trigonometric cubic function does not preserve the shape of data for arbitrary values of free parameters. The convex curves for the same data sets are produced in Figs. 2, 4, and 6 , respectively, by the convexity preserving rational trigonometric scheme developed Section 3. The corresponding outputs of the derivative and shape parameters, for the convexity conserving curves in Figs. 2, 4, and 6 are given in Tables 4–6, respectively.

5. Conclusion

The problem of retaining convex trend of 2D data is dealt in this paper. A C^1 rational trigonometric cubic spline with four free parameters has been utilized for this purpose. Constraints on two of free parameters β_i and γ_i are derived while the remaining two α_i and δ_i are set free. Though literature is inundated with shape preserving schemes but use of rational trigonometric cubic functions makes the underlying algorithm distinguished and unparalleled. Due to the orthogonality of sine and cosine function, much smoother results are obtained as compared to algebraic spline. Derivative of the trigonometric spline is much lower than that of algebraic spline. Insertion of extra knots to conserve the shape of data is not required. Its applicable for both equally and unequally spaced data. No additional information about derivatives is needed as they are estimated by arithmetic mean method. The process of constructing convex surface is under process.

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