

Locating Self-Organization at the Edge of Chaos

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Abstract

The classification of cellular automata rules by Wolfram into four universality classes is based on quantitative statistical properties of the flows produced by the rules. Of particular interest is rule class 4 which is regarded as the class of rules capable of carrying out nontrivial computation. Langton proposed a parametric characterization of CA rules via a single tunable numerical parameter with values in the unit interval and argued that Wolfram's class 4 rules occur with parameter values near critical values corresponding to phase transitions in the chaotic properties of the rules. Evidence against Langton's thesis emerged with computational experiments and statistical analysis by Mitchell, Hraber and Crutchfield. More evidence emerged with techniques subsequently developed by Crutchfield for the detection of rules that exhibit non-chaotic complex flows and that have parameter values well inside the range of values proposed by Langton as characteristic of chaos. Nevertheless, a parametric characterization of rules based straightforwardly on Lyapunov exponents produces a plentiful supply of rules determining flows with complex nonchaotic, i.e. class 4-like behavior. Langrangian interpolation of rules yields numerical CA rules determined by several tunable parameters. A Lyapunov exponent can be associated with each point of the parameter space. The exponent measures the instability of flows produced by the corresponding rule. The resulting scalar field typically exhibits an extraordinary geometric structure that shows a tendency for stable rules to cluster into regions with apparently sharp boundaries. Sampling shows that class 4-like rules occur near these boundaries at locations corresponding to moderate local rates of change in the Lyapunov exponents. A combination of sampling and binary search can estimate the locations of these boundaries which can then be searched along in a variety of ways to find class 4-like rules. This heuristic typically finds suitable rules rapidly. We present an example of a rule found in this manner that yields self-organizing persistent noise-tolerant dynamical structures within CA trajectories that have non-periodic interactions. Although these rules are found by numerical heuristics, their stability implies a noise tolerance that in turn permits the recovery of discrete class 4-like CA-rules. Finally, Boolean-valued CA's can be recovered by using an encoding governed by the Fibonacci numbers.
