[Array 17 (2023) 100265](https://doi.org/10.1016/j.array.2022.100265)

|  |  |  |
| --- | --- | --- |
|  | Contents lists available at [ScienceDirect](https://www.elsevier.com/locate/array) |  |
| Array |
| journal homepage: [www.elsevier.com/locate/array](http://www.elsevier.com/locate/array) |
|  | | |

Nonlinear anisotropic diffusion methods for image denoising problems: Challenges and future research opportunities   
Baraka Maiseli   
*Department of Electronics & Telecommunications Engineering, College of Information & Communication Technologies, University of Dar es Salaam, P.O. Box 33335, Dar es Salaam, Tanzania*

|  |  |  |
| --- | --- | --- |
| A R T I C L E | I N F O | A B S T R A C T  Nonlinear anisotropic diffusion has attracted a great deal of attention for its ability to simultaneously remove noise and preserve semantic image features. This ability favors several image processing and computer vision applications, including noise removal in medical and scientific images that contain critical features |
| Dataset link: [https://www.mathworks.com/ma tlabcentral/fileexchange/116](https://www.mathworks.com/matlabcentral/fileexchange/116260-anisotropic-diffusion-denoising)2[60-anisotropic-d iffusion-](https://www.mathworks.com/matlabcentral/fileexchange/116260-anisotropic-diffusion-denoising)d[e](https://www.mathworks.com/matlabcentral/fileexchange/116260-anisotropic-diffusion-denoising)no[isin](https://www.mathworks.com/matlabcentral/fileexchange/116260-anisotropic-diffusion-denoising)g | |
| *Keywords:*  Anisotropic diffusion  Image processing  Inverse problem  Noise estimation  Restoration method | | (textures, edges, and contours). Despite their promising performance, methods based on nonlinear anisotropic diffusion suffer from practical limitations that have been lightly discussed in the literature. Our work surfaces these limitations as an attempt to create future research opportunities. In addition, we have proposed a diffusion-driven method that generates superior results compared with classical methods, including the popular Perona–Malik formulation. The proposed method embeds a kernel that properly guides the diffusion process across image regions. Experimental results show that our kernel encourages effective noise removal and ensures preservation of significant image features. We have provided potential research problems to further expand the current results. |

**1. Introduction**

Noise, regardless of its source and type, degrades the quality of images, making them less useful in sensitive applications [1–3]. For example, noise may corrupt the details of a medical image and cause doctors to incorrectly interpret results of patients. In computer and machine vision applications, noisy images may negatively impact the accuracy and reliability of the object detection and recognition algo-rithms. Motivated by the quest for quality images in a wide range of applications, there has been intensive efforts to develop different methods for suppressing noise in images [4–13].

Noise may be generated during acquisition, processing, and trans-mission of an image. For instance, if the imaging sensor contains damaged pixels on its surface, random spots (noises) with varied inten-sities will be introduced into the acquired image because such locations cannot register the incident light. Examples of noise in images include salt & pepper [14], Gaussian [15,16], Poisson [17], shot [18], and speckle [2,19], typically modeled as either additive (noise added to the original image) or multiplicative (noise multiplied to the original image).

Several image denoising methods adapted for a range of noise types have been proposed in the literature [20–23]. Of the methods, those based on anisotropic diffusion processes have gained a considerable attention for the ability of such methods to simultaneously remove noise and preserve semantically useful image features (textures, edges,

*E-mail address:* [barakaezra@udsm.ac.tz](mailto:barakaezra@udsm.ac.tz).

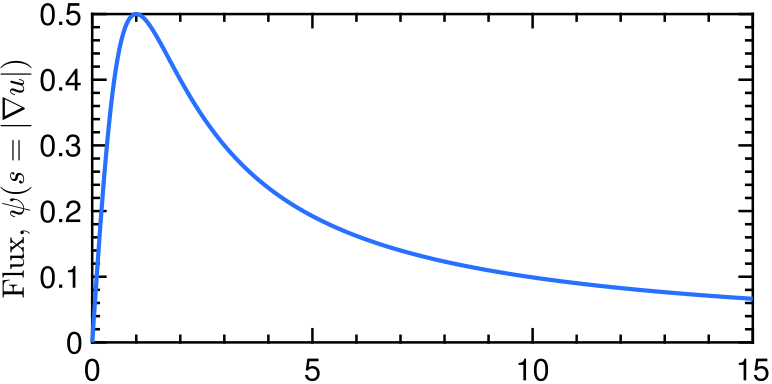
and contours) [24–27]. The seminal work by Perona and Malik [28] has inspired many scholars to develop advanced anisotropic diffusion models for suppressing spurious image features, including noise and undesirable artifacts. These models have demonstrated outstanding results in delicate image denoising applications (e.g., medical imaging) that require quality images with critical features well preserved. Despite the merits of anisotropic diffusion processes in image de-noising, scholars have not adequately unraveled the possible research opportunities to advance the field. This work opens scholarly dis-cussions to address various limitations of the available anisotropic diffusion models. We highlight recommendations for the possible so-lutions of some limitations as guidance for scholars. Furthermore, a superior method based on nonlinear anisotropic diffusion process is proposed. Compared with classical methods, our method demonstrates outstanding results by generating images with higher perceptual and objective qualities. Some recommendations are provided to further expand the our results.

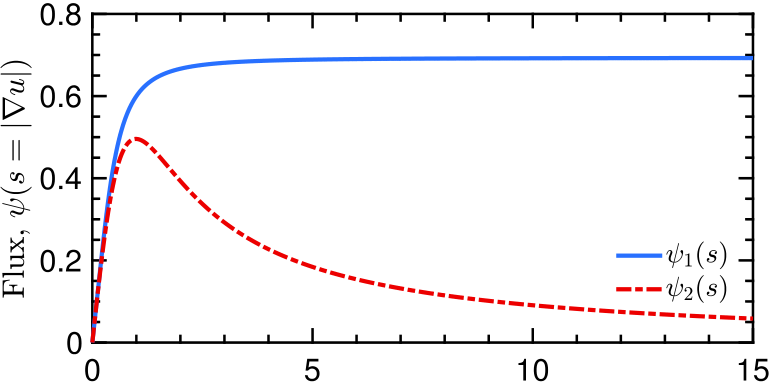
We have organized our paper as follows: the next section exten-sively reviews anisotropic diffusion when applied in image denois-ing; Section 3 gives potential research avenues in anisotropic diffu-sion, providing researchers with immense opportunities to advance this growing field; Section 4 describes our anisotropic diffusion method

<https://doi.org/10.1016/j.array.2022.100265>  
[Received 20 September 2022; Received in re](https://doi.org/10.1016/j.array.2022.100265)vised form 16 November 2022; Accepted 17 November 2022   
Available online 21 November 2022   
[2590-0056](http://creativecommons.org/licenses/by-nc-nd/4.0/)/© 2022 The Author(s). Published by Elsevier Inc. This is an open access article under the CC BY-NC-ND license ([http://creativecommons.org/licenses/by-](http://creativecommons.org/licenses/by-nc-nd/4.0/)

[nc-nd/4.0/](http://creativecommons.org/licenses/by-nc-nd/4.0/)).

*B. Maiseli*  *Array 17 (2023) 100265*





|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | | | |  | − | 1 |  |  |
|  |
| **Fig. 1.** Perona–Malik influence functional (flux) versus image gradient: *𝜓*(*𝑠*) | = | **Fig. 2.** Our influence functionals versus image gradient: *𝜓*1(*𝑠*) = *𝑠* | | | | ( | 1−*𝑎* | (*𝑠 𝐾*) | ) | and |
| |  |  |  | | --- | --- | --- | | *𝑠*  1+ ( *𝐾*  *𝑠* | )2 *,* | *𝐾* = 1. | |  | ( | − | 1 | ) |  |
|  | *𝜓*2(*𝑠*) = *𝑠* | ( | *𝑎*−( *~~𝑠 𝐾~~*)  1+ ( *𝐾*  *𝑠*  )2 | ) . |  | 1+*𝑎* | (*𝑠 𝐾*) |  |  |

and highlights its competitive advantages; Section 5 presents and dis-cusses results; finally, we have concluded the research in Section 6 by summarizing our study and proving perspectives.

**2. Anisotropic diffusion in image processing**

In science and engineering disciplines, diffusion generally means movement of particles (e.g., atoms, ions, and molecules) between re-gions of different concentrations until equilibrium conditions are es-tablished. This physical process may be explained by the *Fick’s law*

*𝑗* = −*𝐷* ⋅ ∇*𝑢,*  (2.1)

which states that the concentration gradient, ∇*𝑢*, generates flux, *𝑗*, with a goal of compensating for this gradient at the rate governed by the diffusion tensor (positive definite symmetric matrix), *𝐷* [29] (Figs. 1 and 2). The process dictates particles from the highly concentrated region to move at the rate determined by *𝐷* towards the region of low concentration. Diffusion obeys the fundamental laws of mass conservation, hence the process does not lead to creation or destruction of mass, *𝑢*, as described by the *continuity equation*

*𝜕𝑢𝜕𝑡*= −div*𝑗,*  (2.2)

with *𝑡* denoting time. Plugging (2.2) into (2.1) gives the diffusion equation

*𝜕𝑢𝜕𝑡*= div(*𝐷* ⋅ ∇*𝑢*) (2.3)

that models several physical processes involving mass transportation (e.g., carburization and construction of semiconductors through dop-ing).

In image processing and computer vision, *𝑢* denotes the intensity of an image. Therefore, applying (2.1) on the image dictates movement of intensity from high-intensity to low-intensity pixels. This process causes smoothing where the intensity of image regions with spurious features (noise) gets reduced.

The directions of *𝑗* and ∇*𝑢* determine the type of diffusion: par-allel, isotropic; and non-parallel, anisotropic. The formulation (2.1) resembles the heat flow equation (or convolution of *𝑢* with a Gaussian kernel) when *𝐷* remains constant over all image regions that undergo isotropic diffusion. Setting *𝐷* constant tends to smudge edges and other critical image features, a consequence not preferred in many sensitive applications. In anisotropic diffusion, *𝐷* becomes a function of *𝑢*, allowing it to change spatially across different image regions. For example, *𝐷* = *𝑓*(|∇*𝑢*|) may be a function of the image gradient to give*𝜕𝑢*  (2.4)

A well-designed *𝑓* leads to image denoising with edge preservation.*𝜕𝑡*= div(*𝑓*(|∇*𝑢*|)∇*𝑢*)*.*

Modeling noise as energy, *𝜌*, in the image, we may regard diffusion as the energy minimization problem with a solution that represents

2

|  |  |
| --- | --- |
| *B. Maiseli* | *Array 17 (2023) 100265*  **Table 1**  Energy functionals for anisotropic diffusion equations: *𝜌*(*𝑠* = |∇*𝑢*|), energy functional; *𝜓*(*𝑠*) = *𝜌*′(*𝑠*), influence function; *𝜙*(*𝑠*), diffusivity.  𝓁2 [30]  𝓁1 [31] *𝑠*   1  2*𝑠*2 *𝑠*  sgn(*𝑠*)   1  1  *𝑠*   Comments  𝓁1 − 𝓁2 [32]  𝓁*𝑝* [33]   *𝐾*2  *𝑠𝜈*   (√1 + ( *𝐾*  *𝑠*  )2− 1 )  sgn(*𝑠*)*𝑠𝜈*−1√1+ (  *𝑠*  *𝐾*  *𝑠*  )2 *𝑠𝜈*−2 √1+   1  ( *𝐾*  *𝑠*  )2   *𝐾* ∈ R+  0 *< 𝜈* ≤ 2  ‘‘Fair’’  Huber [34]  Cauchy [28]  Geman–McClure [35,36]  *𝐾*2 [ ⎧⎪⎨⎪⎩*𝐾*2  2(1+*𝑠*2)  2log  *𝐾*  2*𝑠*2  *𝑠*2  (   *𝐾*− log  *𝑠* −1  ( 1 +   2*𝐾*  (  )  ( 1 +*𝑠*  *𝐾 𝑠* )2)   *𝐾* )]  {  1+*𝑠*  1+  (1+*𝑠*2)2  *𝑠*  (  *𝑠*  *𝐾*sgn(*𝑠*)  *𝑠*  *𝐾*  *𝑠*  *𝐾*  *𝑠*  )2   {   1+*𝑠*  1+  (1+*𝑠*2)2  1  (  1  1  *𝐾*  *𝐾*  *𝑠*  1  *𝐾*  *𝑠*  )2   { if  if   *𝑠* ≤ *𝐾*  *𝑠 > 𝐾*  Welsch [37] *𝐾*2  2 [ 1 − exp (−( *𝐾*  *𝑠* )2)] *𝑠* exp (−( *𝐾*  *𝑠* )2) exp (−( *𝐾*  *𝑠* )2)  Tukey [38]  weighted 𝓁1 − 𝓁2 ⎧⎪⎨⎪⎩*𝐶𝐾*2   *𝐾*2  *𝐾*2  6  6   (  (√(  1 −[  1 +  1 −  (  (  *𝐾*  *𝑠*  *𝐾*  *𝑠*  )2)  )2]3)  − 1 ) ⎧⎪⎨⎪⎩√ *𝑠*  0  1+  [  *𝐶𝑠*  1 −  ( *𝐾*  *𝑠*  )2 ( *𝐾*  *𝑠* )2]2 ⎧⎪⎨⎪⎩√ [  0  1+  1 −  *𝐶*  ( *𝐾*  *𝑠*  (  )2  *𝐾 𝑠* )2]2 {  0 *< 𝐶* ≤ 3*.*5   if  if   *𝑠 < 𝐾*  *𝑠* ≥ *𝐾* |

statistics. In the former direction, scholars strive to design effective energy functionals with better mathematical properties, including con-vexity and uniqueness. Quality of the optimal solution of (2.9) depends on the design of *𝜌*. The later direction requires a solid understanding of the noise statistics in the image. Performance of the anisotropic diffusion model depends upon how well *𝜗* adapts to the noise type (e.g., additive, multiplicative, and mixed).

**3. Potential research avenues in anisotropic diffusion**

*3.1. Design of energy functionals*

|  |
| --- |
| Formulation of an effective anisotropic diffusion equation requires a solid understanding of its corresponding energy functional, *𝜌*. Superior denoising results may be achieved when *𝜌* possesses proper math-ematical properties, such as convexity and uniqueness. specifically, the variational problem governing *𝜌* should guarantee existence and uniqueness of the minimizer, *𝑢* ∈ *𝛺*, in (2.6). Despite these important requirements, no systematic procedures have been established to design *𝜌*. Scholars have proposed different energy functionals (Table 1) with-out clear guidelines on what it takes to derive such functionals. Some interesting questions for further investigation would be (1) does an optimal *𝜌* exist for all denoising applications? (2) what are systematic procedures for designing *𝜌*? (3) what specific mathematical proper-ties should be considered when designing *𝜌* for denoising problems? (4) how does *𝜌* behave at different scale resolutions of the evolving solution, *𝑢*?  We noted a general practice that most authors design diffusivity functionals, *𝜙*(*𝑠*) = *𝜌*′(*𝑠*)∕*𝑠*, directly without considering the properties of *𝜌*. This approach seems straightforward because *𝜙*(*𝑠*) possesses a well-known mathematical property [28]: non-negative monotonically decreasing functional with *𝜙*(0) = 1 and *𝜙*(∞) = 0. Perona and Malik [28] highlight that any well-designed *𝜙*(*𝑠*) with this property may generate satisfactory results. However, it may not always be possible to derive *𝜌* from *𝜙*(*𝑠*) to learn the properties of *𝜌*—an important functional to explain why the anisotropic diffusion equation works. For example, Guo et al. [40] proposed  *𝜙*(*𝑠*) = 1 (3.1)  where *𝐾* denotes the shape-defining tuning constant and *𝛼* ∈ [0*,* 1] 1 + ( *𝐾 𝑠* )*𝛼 ,*  defines an adaptive variable exponent that changes according to the |

3

|  |  |  |
| --- | --- | --- |
| *B. Maiseli* |  | *Array 17 (2023) 100265* |
| *3.2. Design of shape-defining constant* |

The diffusivity Eq. (3.1) contains a shape-defining constant, *𝐾*, the threshold parameter of the gradient magnitude. This tuning constant controls the sensitivity of diffusion to edges, and is usually determined experimentally or as a function of the noise statistics in the image [41–44]. Quality of the denoised image depends, to a greater extent, on the value of *𝐾*. Therefore, effective strategies are needed to optimize *𝐾* for better denoising results. However, the available approaches for determining *𝐾* are manual, hence time-consuming, inconvenient, and error-prone. Scholars may need to investigate the relationship between *𝐾* and the structural information in the images. One interesting finding could be the value of *𝐾* that changes adaptively depending upon the local image features, and this value should work across all types of images.

*3.3. Design of regularization term*

There have been efforts to design regularization (fidelity) terms for different noise types (e.g., additive and multiplicative) [45–50]. Due to the random nature of noise, designing regularization terms remains an open-ended research question and a non-trivial task. Given a noisy image, an interesting question would be to estimate the prob-ability density function (PDF) that generates noise contained in the image. Noise models assume specific PDFs [51–53]: Gaussian, Gamma, Rayleigh, exponential, impulse, uniform, among others. Establishment of the regularization term requires a comprehensive understanding and analysis of these PDFs. In some situations, experiments should be performed to establish the relationship between noise statistics and structural information in the image.

Another aspect worth considering is the fidelity parameter, *𝜆*, of the regularization term, which is usually determined empirically. The manual strategy of tuning *𝜆* introduces challenges in the implemen-tation of anisotropic diffusion models. Some authors have, however, recommended adaptive approaches for determining *𝜆* as the denoising image, *𝑢*, evolves through iterations [40,54]. Guo et al. [40], for example, estimates *𝜆* by evaluating its value from (2.9) under steady conditions (when time tends to infinity). This approach implies that a more accurate value of *𝜆* may be obtained after convergence of the solution. Advanced techniques are needed to accurately determine *𝜆* after every iteration. We may investigate structural features and noise statistics of previous (premature) solutions to estimate *𝜆* iteratively.

*3.4. Applications in inverse problems*

Given observations from the system under investigation, an inverse problem involves determination of parameters characterizing the sys-tem [55–57]. For example, the image denoising problem provides a noisy image and requires determination of the clean (original) image. Inverse problems tend to generate multiple undesirable solutions be-cause of their ill-posed nature: fewer observations than the number of system parameters under investigation. The solution of an inverse problem may be unstable, amplifying errors in the intended solution from the arbitrarily small errors in the measurement data [58]. Anisotropic diffusion has been widely used to address the ill-posed nature of inverse problems. Despite its effectiveness in image denoising, a paucity of studies exist to analyze the impact of anisotropic diffusion in other image processing techniques, such as compression, segmenta-tion, inpainting, and tomographic image reconstruction [29]. Another possible research avenue can be the application of anisotropic diffusion in neural networks for learning a priori information [59].

4

*B. Maiseli*  *Array 17 (2023) 100265*

enhance our understanding on how the diffusion process works. For example, we will understand how and why pixels diffuse across image regions to reduce noise and spurious features. Furthermore, inverse operations may be established to reconstruct the original (noisy) signal after diffusion.

*3.8. Implementation schemes*

For decades, explicit numerical schemes have been widely ap-plied to realize and implement partial differential equations (PDEs) for nonlinear diffusion models. These schemes provide simpler and quicker implementation strategies, but become increasingly inefficient and ineffective in time-sensitive applications that demand larger time steps. Explicit (Euler forward) schemes produce better results under the Courant–Friedrichs–Lewy (CFL) criterion [64] that restricts a time step between 0 and 0.25 to achieve stable and quality results. This restriction makes such schemes ineffective in complex computer vision and image processing tasks.

Given the limitations of explicit schemes, researchers should investi-gate and establish more advanced and efficient numerical methods for PDEs [29]: implicit schemes, splitting and multigrid techniques, and grid adaptation strategies. Weickert [29] argues researchers to develop advanced software packages that implement different nonlinear diffu-sion filters. Supporting the open science [65,66], the packages should be made publicly available across the research community.

*3.9. Noise estimation*

Development of regularization terms require a comprehensive kno-wledge of the noise statistics in an image. Effective methods for noise estimation should be devised as an important milestone to achieve robust nonlinear anisotropic diffusion models [67–70]. However, es-tablishment of such methods seems challenging due to the random nature of noise. This open-ended research problem may be addressed using machine learning methods that can learn complex and random relationships of noise variables [71]. Also, compressive sensing and stochastic techniques [72–74] may be employed to model image noise. The fundamental question is to derive noise PDF from the corrupted image.

*3.10. Image quality evaluation metrics*

Quality of results generated by restoration methods, including the ones discussed in our work, should be gauged using suitable assess-ment metrics. Currently, most authors prefer peak-signal-to-noise ratio (PSNR) [75] and structural similarity (SSIM) [76] metrics for image quality assessment (IQA). However, PSNR and SSIM require complete information of the reference image that may not be available in prac-tical applications [77]. When executing a real denoising problem, the original (clean) image is unavailable. In some few cases, we may have a small portion of information from the original image. Researchers may need to divert their attention to other types of metrics for im-age quality assessment, including reduced-reference and no-reference IQA metrics [78–83]. Subjective IQA approaches, while they remain popularly used, should not be drawn based on the author’s perceptions and experience. Other people should be engaged in the quality assess-ment process, and voting statistics employed as the basis for drawing conclusions.

5

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *B. Maiseli* | **Table 2**  Performance of Perona–Malik method and our method under optimal tuning parameters, {*𝑎, 𝐾*}. | | | | | | | | |  |  |  | *Array 17 (2023) 100265* |
|  | Method | | Pout | SSIM | {*𝑎, 𝐾*} | | Mandril | SSIM | |  | Walkbridge |  |  |
|  | PSNR | PSNR | {*𝑎, 𝐾*} | PSNR | SSIM | {*𝑎, 𝐾*} |
|  | Perona–Malik method [28] Our method | 33.3637 33.5270 | | 0.8738 0.8780 | {−*,* 5} {7*,* 5} | 26.0017 26.0013 | | | 0.8188  0.0.8189 | {−*,* 87} {6*,* 48} | 25.2815  25.2905 | 0.8630 0.8628 | {−*,* 90}  {9*,* 51} |
|  |  |  |  |  |  |  |

**Table 3**   
Peak-signal-to-noise ratio (PSNR) and structural similarity (SSIM) values of different anisotropic diffusion methods for the cat image corrupted by heavy Gaussian noise. (*𝜎* and *𝑡* denotes standard deviation and iteration number, respectively.)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Method | *𝜎* = 40*, 𝑡* = 30 |  | *𝜎* = 50*, 𝑡* = 35 |  | *𝜎* = 60*, 𝑡* = 40 |  |
|  | PSNR | SSIM | PSNR | SSIM | PSNR | SSIM |
| Perona–Malik method [28] Monteil method [84]  Tebini [85]  Gupta method [86]  Rezgui et al. [87] | 20.7438 22.2268 22.9435 23.8332 20.3328 | 0.4500 0.5824 0.6201 0.6618 0.5399 | 17.3968  19.8749  20.2365  20.9656  18.5991 | 0.2915 0.4245 0.4582 0.5078 0.4671 | 16.3042  18.8512  19.0856  21.4205  17.1711 | 0.2502 0.3624 0.3983 0.5433 0.4083 |
| Proposed method (*𝑎* = 13*, 𝐾* = 19) | *𝑡* = 5 |  | *𝑡* = 6 |  |  |  |
|  | 23.8570 | 0.7668 | 22.5464 | 0.7137 | 21.5094 | 0.6683 |

**5. Results and discussions**

The current work demonstrates the capabilities of methods based on nonlinear anisotropic diffusion in restoring degraded images with-out destroying important features. Despite their notable advantages, potential weaknesses of these methods have been discussed to inspire early career researchers in developing more advanced and effective nonlinear anisotropic diffusion methods. We have indicated interesting research avenues in this promising field, the goal being to expand the application domain of such methods in denoising corrupted images. Supported by our experimental results, researchers may raise critical questions leading to improvement of the proposed method or other classical methods in the literature.

Results justify the capability of our approach to generate images with higher perceptual and objective qualities (Fig. 4). Compared with the results generated by the Perona–Malik method, our images contain visually appealing and attractive features. Specifically, the proposed approach suppresses noise and reconstructs visual semantic features. Even for the complex images, such as Baboon (Fig. 4, second row), our approach generates acceptable results by preserving plausible features (e.g., fur, edges, and textures). Intuitively, we can observe from the results (Fig. 4, far-right column) that the current method properly balances between visual appealingness and noise removal in accor-dance with the human visual system. This encouraging observation may be explained from the structure of our diffusion kernel (3.3) that it provides an effective strategy for steering the diffusion process across image regions. However, this benefit comes at the expense of tuning the kernel parameters, *𝑎* and *𝐾*. We noted that the image type dictates the {*𝑎, 𝐾*} search space, which seems non-intuitive as it requires heuristic approaches to determine optimal values. Future studies may consider more efficient algorithms for computing the values of *𝑎* and *𝐾* that produce optimal results.

The proposed method performs well on objective quality metrics (Tables 2 and 35). The method gives competitive values of PSNR and SSIM even for an image with a higher noise density. More importantly, our method converges faster that other methods, thus making it more suitable in real-time computing devices. For a Cat image degraded by Gaussian noise of standard deviation 40, the proposed method converges with only five iterations—sixfold convergence rate faster than the classical methods.

Despite its higher computational speed, the tuning parameters, {*𝑎, 𝐾*}, of our method needs manual tuning to achieve optimal re-sults (Table 2). This time-consuming process may give design and

5 For fair comparison, some results from this Table were adapted from the work by Gupta et al. [86].

6

*B. Maiseli*  *Array 17 (2023) 100265*

**6. Conclusion**

The current work highlights potential advantages of nonlinear anisotropic diffusion methods for noise removal in corrupted images. Given their wide applications in image processing and computer vision, it seems reasonable to further improve the methods and make them more useful in theoretical and practical settings. Therefore, limita-tions of the methods have been extensively discussed as an attempt to create awareness and establish research opportunities within the scholarly community. Furthermore, we have proposed a nonlinear anisotropic diffusion method that generates promising results relative to the classical Perona–Malik formulation.

**Declaration of competing interest**

The authors declare that they have no known competing finan-cial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Data availability**

The link to the implementation source code has been included in the manuscript. [https://www.mathworks.com/matlabcentral/fileexchange](https://www.mathworks.com/matlabcentral/fileexchange/116260-anisotropic-diffusion-denoising)/ [116260-anisotropic-diffusion-denoising](https://www.mathworks.com/matlabcentral/fileexchange/116260-anisotropic-diffusion-denoising)

**Acknowledgment**

This work is not funded by any organization.

**References**

[1] [Shen H, Li X, Zhang L, Tao D, Zeng C. Compressed sensing-based inpainting of aqua moderate resolution imaging spectroradiometer band 6 using adap-tive spectrum-weighted sparse bayesian dictionary learning. IEEE Trans Geosci Remote Sens 2013;52:894–906.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb1)

[2] [Karaoğu O, Bilge HŞ, Uluer İ. Removal of speckle noises from ultrasound images](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb2)  [using five different deep learning networks. Eng Sci Technol 2022;29:101030.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb2) [3] [Pang T, Zheng H, Quan Y, Ji H. Recorrupted-to-recorrupted: Unsupervised dee](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb2)[p](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb3)  [learning for image denoising. In: Proceedings of the IEEE/CVF conference on](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb3)  [computer vision and pattern recognition. 2021, p. 2043–52.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb3)

[4] [Buades A, Coll B, Morel J-M. A review of image denoising algorithms, with a](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb4)  [new one. Multiscale Model Simul 2005;4:490–530.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb4)

[5] [Buades A, Coll B, Morel J-M. Image denoising metho](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb4)[ds. A new nonlocal principle.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb5)  [SIAM Rev 2010;52:113–47.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb5)

[6] [Jain P, Tyagi V. A survey o](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb5)[f edge-preserving image denoising methods. Inf Syst](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb6)  [Front 2016;18:159–70.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb6)

[7] [Tian C, Fei L, Zheng](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb6) [W, Xu Y, Zuo W, Lin C-W. Deep learning on image](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb7)  [denoising: An overview. Neural Netw 2020;131:251–75.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb7)

[8] [Gondara L. Medical image denoising using convolutional](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb7) [denoising autoencoders.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb8)

[In: 2016 IEEE 16th international conference on data mining workshops. IEEE; 2016, p. 241–6.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb8)

[9] [Starck J-L, Cand](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb8)[ès EJ, Donoho DL. The curvelet transform for image denoising.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb9)  [IEEE Trans Image Process 2002;11:670–84.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb9)

[10] [Ilesanmi AE, Ilesanmi TO. Methods for image denoising using convolutional](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb10)  [neural network: A review. Complex Intell Syst 2021;7:2179–98.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb10)

[11] [Shi Q, Tang X, Yang T, Liu R, Zhang L. Hyperspectral image denoising using a 3-D attention denoising network. IEEE Trans Geosci Remote Sens 2021;59:10348–63.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb11)

[12] [Zhang J, Cao L, Wang T, Fu W, Shen W. NHNet: A non-local hierarchical network](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb12)  [for image denoising. IET Image Process 2022a;16:2446–56.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb12)

[13] [Zhang J, Cai Z, Chen F, Zeng D. Hyperspectral image deno](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb12)[ising via adversarial](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb13)  [learning. Remote Sens 2022b;14:1790.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb13)

[14] [Fu B, Zhao X, Song C, Li X, Wang X. A salt and pepper noise image denoising method based on the generative classification. Multimedia Tools Appl 2019;78:12043–53.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb14)

[15] [Russo F. A method](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb14) [for estimation and filtering of gaussian noise in images. IEEE](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb15)  [Trans Instrum Meas 2003;52:1148–54.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb15)

[16] [Luisier F, Blu T, Unser M. Image den](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb15)[oising in mixed Poisson–Gaussian noise.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb16)  [IEEE Trans Image Process 2010;20:696–708.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb16)

[17] [Zhang B, Fadili JM, Starck J-L. Wavelets, r](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb16)[idgelets, and curvelets for Poisson](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb17)  [noise removal. IEEE Trans Image Process 2008;17:1093–108.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb17)

[18] [Jin Q, Grama I, Liu Q. Poisson shot noise removal by an oracular non-local](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb18)  [algorithm. J Math Imaging Vision 2021;63:855–74.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb18)

7

*B. Maiseli*  *Array 17 (2023) 100265*

[57] [Romanov VG. Inverse problems of mathematical physics. In: Inverse problems of](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb57)  [mathematical physics. De Gruyter; 2018.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb57)

[58] [Kabanikhin SI. Definitions and examples](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb57) [of inverse and ill-posed problems. 2008.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb58) [59] [Singh P, Shankar A. A novel optical image denoising technique using con-](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb58)[volutional neural network and anisotropic diffusion for real-time surveillance applications. J Real-Time Image Process 2021;18:1711–28.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb59)

[60] [Landi G, Piccolomini EL, Tomba I. A stopping criterion for iterative regularization](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb60)  [methods. Appl Numer Math 2016;106:53–68.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb60)

[61] [Rao K, Malan P, Perot JB. A stopping criterion](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb60) [for the iterative solution of partial](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb61)  [differential equations. J Comput Phys 2018;352:265–84.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb61)

[62] [Axelsson](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb61)  [O,](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb61)  [Kaporin](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb61)  [I.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb61)  [Error](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb61)  [norm](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb61)  [estimation](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb61)  [and](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb61)  [stopping](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [criteria](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [in](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [preconditioned](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [conjugate](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [gradient](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [iterations.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [Numer](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [Linear](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [Algebra](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [Appl](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)  [2001;8:265–86.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)

[63] [Witkin AP. Scal](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb62)[e-space filtering. In: Readings in computer vision. Elsevier; 1987,](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb63)  [p. 329–32.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb63)

[64] [Courant R](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb63)[, Friedrichs K, Lewy H. On the partial difference equations of](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb64)  [mathematical physics. IBM J Res Dev 1967;11:215–34.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb64)

[65] [Vicente-Saez R, Martinez-Fuentes C. Open science now](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb64)[: A systematic literature](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb65)  [review for an integrated definition. J Bus Res 2018;88:428–36.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb65)

[66] [Foster ED, Deardorff A. Open science framework (OSF). J](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb65) [Med Libr Assoc](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb66)  [2017;105:203.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb66)

[67] [Pimpalkhute](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb66) [VA, Page R, Kothari A, Bhurchandi KM, Kamble VM. Digital image noise estimation using DWT coefficients. IEEE Trans Image Process 2021;30:1962–72.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb67)

[68] [Sarker R, Kaur A](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb67)[, Singh D. Noise estimation using back propagation neural](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb68)  [networks. ECS Trans 2022;107:18761.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb68)

[69] [San-Roman R, Nachmani E, Wolf L.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb68) Noise estimation for generative diffusion models. 2021, arXiv preprint [arXiv:2104.02600](http://arxiv.org/abs/2104.02600).

[70] [Pyatykh S, Hesser J, Zheng](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb70) [L. Image noise](http://arxiv.org/abs/2104.02600) [level estimation by principal](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb70)  [component analysis. IEEE Trans Image Process 2012;22:687–99.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb70)

[71] [Zhang X-P. Thresholding neural network for adaptive noise reduc](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb70)[tion. IEEE Trans](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb71)  [Neural Netw 2001;12:567–84.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb71)

[72] [Leportier T, Park M-C. Filter](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb71) [for speckle noise reduction based on compressive](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb72)  [sensing. Opt Eng 2016;55:121724.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb72)

[73] [Bindilatti AA, Mascarenhas ND. A](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb72) [nonlocal Poisson denoising algorithm based](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb73)  [on stochastic distances. IEEE Signal Process Lett 2013;20:1010–3.](http://refhub.elsevier.com/S2590-0056(22)00098-4/sb73)

8