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Wild Geese Algorithm: A novel algorithm for large scale optimization based on the natural life and death of wild geese

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| A R T I C L E I N F O | A B S T R A C T |
| Keywords:  Large-scale global optimization (LSGO) Wild Geese Algorithm (WGA)  Swarm-based method  Engineering optimization | In numerous real-life applications, nature-inspired population-based search algorithms have been applied to solve numerical optimization problems. This paper focuses on a simple and powerful swarm optimizer, named Wild Geese Algorithm (WGA), for large-scale global optimization whose efficiency and performance are verified using large-scale test functions of IEEE CEC 2008 and CEC 2010 special sessions with high dimensions D ¼ 100, 500, 1000. WGA is inspired by wild geese in nature and models various aspects of their life such as evolution, regular |

cooperative migration, and fatality. The effectiveness of WGA for finding the global optimal solutions of high-dimensional optimization problems is compared with that of other methods reported in the previous literature. Experimental results show that the proposed WGA has an efficient performance in solving a range of large-scale optimization problems, making it highly competitive among other large-scale optimization algorithms despite its simpler structure and easier implementation. The source code of the proposed WGA algorithm is publicly available at [github.com/ebrahimakbary/WGA](http://github.com/ebrahimakbary/WGA).

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| 1. Introduction | problems with different real-world complexities such as nonlinearity, | | | | |
| non-smoothness, | non-convexity, | mixed-integer | nature, | non- |

Many practical optimization problems, which are called Large Scale Global Optimization (LSGO) problems, deal with a lot of decision vari-ables. Some practical LSGO problems are large-scale electronic systems design, scheduling problems, vehicle routing in large-scale traffic net-

differentiability, etc. Some new nature-inspired optimization algo-rithms for solving the practical large-scale optimization problems are listed in Table 1. It should be mentioned that, the boldface rows of this table, show the methods which were used in the comparative study with

works, and inverse problem chemical kinetics. Many real-world optimi- the proposed WGA.

zation problems involve optimization of a large number of control variables with various constraints. However, the classical mathematical programming methods do not generally provide good solutions for different optimization problems with different real-world complexities, due to the huge size of the problems [1]. The global optimization per-formance of the population-based algorithms often becomes weaker in such problems with increasing the dimension and complexity of the problem [2–4]. The practical large-scale optimization problems have been modeled with different benchmark test functions such as those presented in the CEC 2008 [5] and CEC 2010 [6].

Recently, many nature-inspired and population-based meta-heuristic optimization algorithms have been presented to deal with LSGO

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Wild geese have a long-distance, coordinated and organized travel, which can be used as an inspiration for a very appropriate optimization algorithm for high-dimension problems. Based on the general model of wild geese’ lives, a novel algorithm called Wild Geese Algorithm (WGA) is introduced in this paper, which have some main prominent charac-teristics compared to the previous algorithms including:

� It is simple with low computational burden, and its implementation is easily performed.  
� It has proper and satisfactory power for different test functions, from different groups.

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| M. Ghasemi et al. | Table 1 (continued) | Array 11 (2021) 100074 |
| Table 1 |

Summary of some new nature-inspired optimization algorithms for solving the practical large-scale optimization problems.

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| --- | --- | --- | --- | --- | --- |
| Ref. | Year | Abbreviation | Short Description | Dimensions | Real- |
| under study | world |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Ref. | Year | Abbreviation | Short Description | Dimensions | Real- |  |  |  |  |  | problem |
|  |  |  |  | under study | world | [19] | 2013 | GOjDE | A Generalized | 100, 200, | No |
| problem | Opposition based | 500, 1000 |
| [7] | 2008 | MLCC | Multilevel | 100, 500, | No | [20] | 2013 | EOEA | Differential | 1000 | Yes |
| Cooperative | 1000 | Evolution enhanced |
| [8] | 2008 | EPUS-PSO | Coevolution | No | with a self-adapting |
| 100, 500, |
| Efficient Population | parameter tuning |
| strategy |
| Utilization Strategy | 1000 |
| for Particle Swarm | A two-stage based |
| [9] | 2008 | sep-CMA-ES | Optimizer (PSO) | 100–1000 | No | [21] | 2014 | FT-DNPSO | ensemble | 30, 100, | No |
| Covariance Matrix | optimization |
| Adaptation | evolutionary |
| Evolution Strategy | algorithm |
| having diagonal | PSO with dynamic |
| [10] | 2010 | SOUPDE | covariance matrix | 50, 100, | No | [22] | 2014 | CBCC1-DG | neighborhood based | 1000 | No |
| Shuffle or update | on kernel fuzzy |
| parallel differential | 200, 500, | clustering and |
| [11] | 2010 | CCVIL | evolution | 100 | No | variable trust region | 1000 |
| methods |
| Cooperative | 1000 |
| Coevolution with | Two different |
| [12] | 2010 | �DECC-D �DECC-DML | Variable Interaction | 100, 500, | No | [23] | 2015 | CBCC2-DG | versions of | 200, 500, | No |
| Learning | DECC-DG | Contribution Based |
| �Differential  Evolution with | Cooperative Co- |
| 1000 | evolution and |
| Cooperative Co- | Differential |
| evolution using | Evolution with |
| Delta-Grouping | CDE | Cooperative Co- |
| �Differential  Evolution with | evolution, all with |
| differential |
| Multilevel | grouping |
| Cooperative Co- | Continuous |
| [13] | 2010 | GOBL | evolution using | 50, 100, | No | [24] | 2015 | CSO | Differential | 1000 | No |
| Delta-Grouping | Evolution |
| 100, 500, |
| Generalized | A Competitive |
| Opposition-Based | 200, 500, | [25] | 2016 | SOMAQI | Swarm Optimizer | 1000, 2000, | No |
| [14] | 2011 | TSVP | Learning | 100 | No | 5000 |
| Self Organizing |
| Tabu Search with | 100, 400, | 100, 500, |
| [15] | 2011 | SP-UCI | Variable Partitioning | 1000 | No | [26] | 2018 | MWOA | Migrating Algorithm | 1000, 2000, | No |
| Shuffled complex | 10, 50, 100, | with Quadratic | 3000 |
| evolution with | 1000 | Interpolation |
| 100, 300, |
| principal | A Modified Whale |
| [16] | 2012 | LMDEa | components | 1000 | No | [27] | 2019 | EHO | Optimization | 500, 1000 | No |
| analysis–University of California at Irvine | Algorithm |
| 50, 100, |
| Enhanced Elephant |
| Differential | Herding | 200, 500, |
| Evolution with | [28] | 2019 | SFO | Optimization with | 100 | Yes |
| Landscape Modality | Novel Individual |
| [17] | 2012 | DE-CCS | Detection and a | 500,1000 | No | Updating Strategies | 300 |
| Diversity Archive | Sailfish Optimizer |
| Differential | [29] | 2019 | PRO | Poor and rich | 300 | Yes |
| Evolution Algorithm | [30] | 2019 | EBA | optimization | 100, 500, | No |
| with Cooperative | algorithm |
| [2] | 2012 | CCPSO2 | Coevolutionary | 1000 | No | Ensemble Bat |
| Selection Operator | Algorithm | 1000 |
| [31] | 2019 | EO | Yes |
| A new Cooperative | Equilibrium | 10–200 |
| Coevolving Particle | optimizer |
| Swarm | [32] | 2020 | NPO | Nomadic People | 100, 500, | No |
| Optimization with a | [33] | 2020 | ISSA | Optimizer | 2000 | No |
| new position | An improved Social | 100, 500, |
| update rule based | Spider Algorithm | 1000 |

on Cauchy and   
Gaussian

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| [18] | 2012 | LSCBO | distributions | 100, 500, | No | It is worth mentioning that although the proposed WGA may seem |
| Large Scale |
| similar to PSO, especially due to the existence of personal best and global |
| Optimization Based | 1000 |
| best concepts, it has some thorough distinctions of structure and |
| on Co-ordinated |
| Bacterial Dynamics | formulation, the main of which can be listed as follows: |

and Opposite   
Numbers

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1 In WGA, all solutions are sorted based on their objective values so that each member of population moves using information from its adja-cent members in the sorted population.

2 In the proposed method, the formulation for calculating the velocity of each goose is completely different from the PSO and is based on the positions, velocities, and best positions of the goose and its adjacent geese in the sorted population, as well as the global best solution's position. While in PSO, the only parameter that is shared among all solutions is the position of the global best solution.

3 In the proposed WGA, two different solutions are generated per so-lution and are used for creating the next iteration's goose based on a mechanism similar to the crossover operator of differential evolution. 4 Finally, in the proposed algorithm, a population reduction policy is implemented which is accomplished by fatality (elimination) of the weakest goose of the population.

The rest of this paper is organized as follows. Section 2 presents the new proposed algorithm for large-scale optimization problems. Section 3



Fig. 1. An ordered and coordinated migration of wild geese.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| shows | the | experimental | results. | Finally, | Section | 4 | presents | the | where xi;d, pi;d, and vi;d are the dth dimension of the current position, the |
| conclusions. | |
| best position, and the current velocity of the ith wild goose, respectively. |

2. The proposed algorithm: Wild Geese Algorithm

In recent years, some new algorithm inspired from group movement and group search by animals have been proposed for large-scale global continuous optimization [1]. In this paper, based on the different phases of wild geese's lives, including their rhythmic and coordinated group migration, reproduction and evolution and also deaths in the population of geese, a new efficient algorithm, named as Wild Geese Algorithm (WGA), is presented for high-dimensional optimization problems. In Fig. 1, a group ordered migration based on the position of wild geese is shown. In general, the proposed WGA phases are as follows:

1 Ordered and coordinated group migration (or migration and displacement velocity phase)   
2 Walking and searching for food by wild geese.

3 Reproduction and evolution of wild geese.

Note that in this study, rk;d; k ¼ 1; 2; :::; 11 are uniformly distributed random numbers between 0 and 1.

As observed in Eq. (1), the velocity and position changes of each wild goose (for instance i-th wild goose) depend on the velocities of their upfront and rear members, i.e ð vIter iþ1�vIter i�1Þ , and also to the positions of its adjacent members.

According to the model from the migration of wild geese in Fig. 2 and Eq. (1), the wild geese use information from their adjacent individuals in the sorted population, as patterns for their movement and navigation, and tend to reach those members (reduce their distances), i.e. xIter i�1→pIter i ; xIter i → pIter iþ1; xIter iþ1→ pIter iþ2, and xIter iþ2→ � pIter i�1.

Additionally, the global best member is used as another guide for the movements of the whole flock; which is reflected in Eq. (2). This position change is carried out in an ordered form and coordinated with the upfront members in order to model the movement of all members as an ordered series, as shown in Figs. 1 and 2.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 4 Death, migration and ordered evolution of wild geese. | xv i;d¼ pIter i;dþ r7;d � r8;d � | ��gIter d | þ pIter iþ1;d� 2 � pIter | � | þ vIterþ1 | � | (2) |
| First, an initial population of wild geese are created, so that the po- |
| sition vector of the i-th wild goose is equal to xi. The best local position or | where gd is the global best position among all members. | | | | | |

personal best solution pi and migration velocity viare determined. Then, all wild geese populations are sorted from the best to the worst according to their target function.

In this modeling strategy, each wild goose exploits information from its adjacent wild geese in the ordered population, and is directed by those individuals. The phases of WGA are further discussed in the subsequent subsections.

2.1. An ordered and coordinated group migration (or migration and displacement velocity phase)

As it is observed in Fig. 1, migration of wild geese is a group, coor-dinated, ordered and under control migration, which is based on reach-ing the upfront and adjacent individuals in the sorted population. Velocity and displacement equations according to the coordinated ve-locity of the geese are given in Eq. (1) and Eq. (2).

2.2. Walking and searching for food by wild geese

This step is modeled in such a way that the i-th wild goose moves towards its upfront member, i.e. the (iþ1)-th goose (pIter→ pIter iþ1). In another word, the i-th goose tries to reach the (iþ1)-th goose (pIter iþ1� pIter ).

i is The equation for walking and searching for food by the wild goose, xW as follows:

xw i;d¼ pIter i;dþ r9;d � r10;d ��pIter iþ1;d� pIter� (3)

2.3. Reproduction and evolution of wild geese

Another stage of wild geese's life is reproduction and evolution. In this paper, its modeling is performed so that a combination between migra-

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| vIterþ1 i;d  þr3;d �  þr5;d �  ¼�r1;d � vIter  ��pIter  pIter  i;d� xIter  iþ2;d� xIter   i;dþ r2;d �  i�1;d  iþ1;d � �  þ r4;d �  � r6;d � �vIter iþ1;d� vIter  ��pIter iþ1;d� xIter  pIter i�1;d� xIter  i�1;d��  iþ2;d � | � | (1) | tion equation (xV i) and walking and search for food equation (xW i) is used. | | | | | |
| The Cr value for the proposed WGA algorithm is 0.5 in total simulations. | | | | | |
| xIterþ1 i;d | ¼ | ( | xv i;dif | r11;d � Cr  otherwise: | (4) |
| xw i;d |

3

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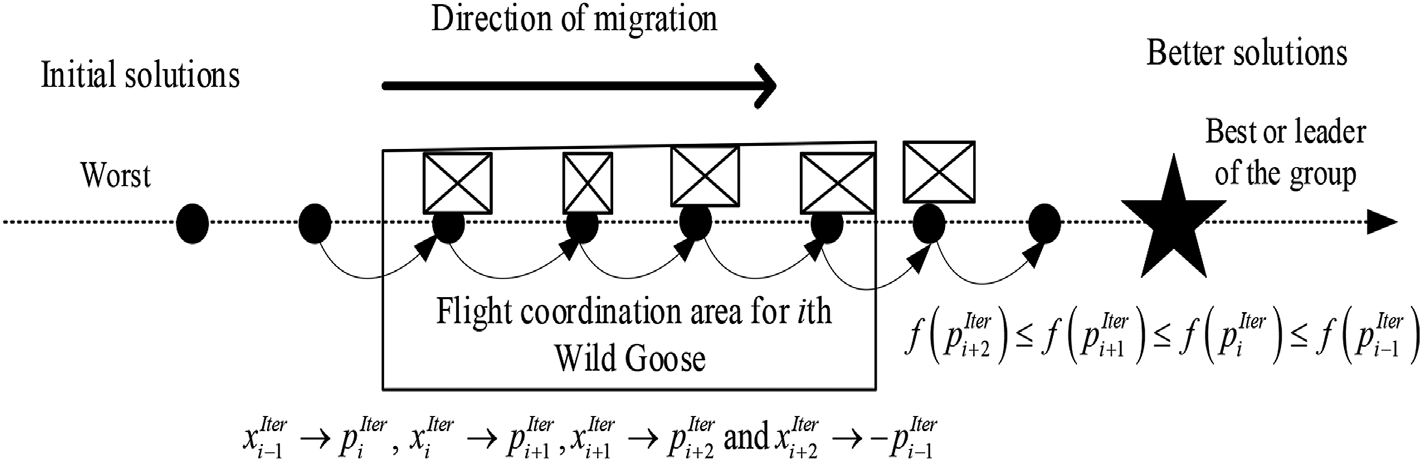


Fig. 2. The model of ordered and coordinated group migration of wild geese.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 2.4. Death, migration and ordered evolution | Algorithm 1 (continued) | | | |
| The previous experiments from the literature show that for different optimization algorithms the population number and the iteration number do not have the same level of influence on solving every types of prob- | 19: XIterþ1 i | | ← Eq. (4); | |
| 20: end for | | | |
| 21: if xIterþ1 i;d | < xmin d | | |
| 22: xIterþ1 i;d | ← xmin d | | ; |

lems. For some functions, the size of algorithm's population is more important and more effective than the number of algorithm's iterations (e.g. F2 and F3 functions), and for some other functions the number of algorithm's iterations is more important and more effective than the size of WGA algorithm's population (e.g. F7 and F8 functions). In this paper, to overcome this problem and establish a compromised solution, the death phase is employed in order to balance algorithm performance for all test functions. In this phase, the algorithm starts with the maximum population number Npinitialand during the algorithm iterations, the weaker members will be removed from the population based on Eq. (5) and the population size will decrease linearly so that it reaches its final value Npfinalin the final iteration.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Np ¼ round 0 B @ | Npinitial  ���Npinitial� Npfinal�  \*� FEs FEsmax | �� | 1 C A | (5) |
| where FEs and FEsmax are the number of function evaluations and its maximum. | | | | |

Algorithm 1   
Demonstrates the optimization process of WGA.

|  |  |
| --- | --- |
| Algorithm 1:  1: to set values of the control parameters of WGA;  2: to generate the initial population (whose number are equal to Npinitial) and VIter¼1 i ½0�;  3: to evaluate the fitness of each population individual and FEs ¼ Npinitial; 4: to find the personal best position of all particles Npinitial(i ¼ 1, 2, …, Npinitial) in swarm Piand the global best position G; | ¼ |

5: while the FEs till FEsmax do   
 6: Wild Goose populations are arranged from the best to the worst according to Fig. 2;   
 7: for i ¼ 1 (best) to Np (worst) do   
 8: Select the sorted members i � 1th; i þ 1th; and i þ 2th;   
 {\*\* An ordered and coordinated group migration based on Eq. (1) and Eq. (2) \*\*}

|  |
| --- |
| 9: for d ¼ 1 to D do   10: VIterþ1 i ← Eq. (1);  11: end for |

12: for d ¼ 1 to D do   
 13: xV i;d← Eq. (2);   
14: end for   
{\*\* Walking and search geese Eq. (3) \*\*}   
15: for d ¼ 1 to D do   
 16: xW i;d← Eq. (3);   
17: end for   
{\*\* Reproduction and evolution Eq. (4) \*\*}

23: end if

|  |  |  |
| --- | --- | --- |
| 24: if xIterþ1 i;d | > xmax d | ; |
| 25: xIterþ1 i;d | ← xmax d |

26: end if   
 27: to evaluate the fitness of XIterþ1 i   
 28: if fðXIterþ1 Þ � fðPIter Þ   
 29: PIterþ1 i ← XIterþ1 i ;   
 30: end if   
 31: if fðPIterþ1 Þ � fðGÞ   
 32: G ← PIterþ1 i   
 ; 33: end if   
 34: end for   
 35: FEs ¼ FEs þ Np;   
 36: Np ← Eq. (5);   
37: end while

3. Results and analysis of experimental evaluation studies

In this section, 20 widely used large scale test functions are exploited to show the efficiency and performance of the proposed algorithm. The formulation and characteristics of all CEC 2010 benchmark test functions are listed in Ref. [6].

The performance and robustness of WGA for solving real and large-scale optimization problems are characterized by two indices: 1) the mean of best values of test function (Mean), and 2) the standard deviation (Std) indices.

|  |
| --- |
| Test functions include 1. Separable functions (F1 � F3), 2. Single-  group m-nonseparable functions (F4 � F8), 3.D 2m-group m-nonseparable  functions (F9 � F13), 4.D m-group m-nonseparable functions (F14 � F18),  and 5. non-separable functions (F19 � F20), where m is the number of variables in each non-separable subcomponent, and D and m are assumed |

as 1000 and 50, respectively. To show the efficiency of WGA, in all simulations of this paper, 25 independent simulations are used in each section for every test function, as in Refs. [6,22]. Furthermore, in all

|  |
| --- |
| simulations, the maximum number of fitness evaluations FEsmax is 3 �106. In all tables, the þ sign means the algorithm outperforms WGA, the– sign means WGA outperforms the algorithm, and the ¼ sign means WGA and the considered algorithm yield the same solution for the given |

problem. It should be mentioned that, in all results tables, the boldface is used to emphasize the algorithm that achieves the best Mean index value for each problem.

3.1. Experimental setup

|  |  |  |
| --- | --- | --- |
| 18: for d ¼ 1 to D do | (continued on next column) | 3.1.1. Influence of death phase on WGA performance |
| At first, to show the performance of the population reduction by death |

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Table 2 functions F3, F6, F7, F11, F12, F16, and F17. Moreover, the convergence

Average fitness values and standard deviations of results for test functions over 25 independent runs.

characteristics of this algorithm for 6 different functions of various types are depicted in Fig. 3, which verify the effectiveness of implementing

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| F | WGA, Np ¼ 30 | WGA, Np ¼ 120 | WGA | death phase in WGA. |
| F1 | 1.68E-21 | 2.33E-24 | 1.05E-26 | 3.1.2. Why Cr ¼ 0.5 in WGA for all test functions?  value for Cr four different constant values other than 0.5, i.e. 0.1, 0.25, In this paper Cr ¼ 0.5 is used for all simulations. To select a suitable |
| F2 | 7.71E-22 | 1.58E-24 | 2.56E-26 |
| 3 | 2 | 1 |
| 7.78Eþ03  7.95Eþ01  3 | 2.18E þ 03  1.14E þ 01  1 | 2.28Eþ03 4.58Eþ01 2 |
| 0.75 and 0.9 are tested, whose results are presented in Table 3. As |
| observed, the constant value 0.5 is the best value for different test |
| F3 | 1.00Eþ01  1.25Eþ01  3 | 1.17E-13 | 1.47E-13 |
| functions of CEC 2010. It should be mentioned that in all simulation |
| 7.40E-15 | 8.94E-15 |
| 1 | 2 | results tables, three values are reported for optimizing each test function |
| F4 | 3.81E þ 11  1.63E þ 11  1 | 9.99Eþ11  1.05Eþ11  3 | 5.15Eþ11 7.89Eþ10 2 | with each algorithm; the first two demonstrate the average and standard |
| deviation of fitness values of the obtained results. The third value shows |
| the rank of that algorithm in terms of the mean index. Furthermore, three |
| F5 | 9.55Eþ07  7.04Eþ06  3 | 5.74Eþ07  3.68Eþ06  2 | 5.47E þ 07 7.93E þ 06 1 |
| parameters are reported for each algorithm in all tables, i.e. Nb, Nw, Mr. |
| Nb and Nw are the number of times the algorithm yields the best and the |
| F6 | 1.98Eþ01  2.50E-02 | 3.56E-09 | 3.55E-09 |
| worst mean index, respectively; and Mr is the average rank of the algo- |
| 1.40E-15 | 5.48E-14 |
| rithm achieved in solving all considered test functions. |
| 3 | 2 | 1 |
| F7 | 8.01E-02 | 4.47Eþ03  1.69Eþ03  3 | 4.60Eþ00 6.28Eþ00 2 | 3.2. Comparing WGA with recent optimization algorithms |
| 2.00E-02 |
| 1 |
| F8 | 8.60E þ 06  3.16E þ 05  1 | 4.30Eþ07  2.74Eþ07  3 | 9.16Eþ06 8.79Eþ06 2 | 3.2.1. CEC 2008 test functions |
| In this section, the results of WGA are compared with those of a series |
| F9 | 2.54Eþ07  1.33Eþ06  2 | 4.55Eþ07  5.50Eþ06  3 | 2.21E þ 07 1.51E þ 06 1 | of the recently proposed optimization algorithms for large-scale prob- |
| lems from CEC 2008 test functions with different high dimensions |
| including D ¼ 100, D ¼ 500 and D ¼ 1000. The formulation and char-acteristics of CEC008 benchmark test functions are listed in Ref. [5] and |
| F10 | 4.67Eþ03  1.60Eþ02  3 | 1.76E þ 03  2.48E þ 01  1 | 2.64Eþ03 2.70Eþ01 2 |
| Table 4: |

|  |  |  |  |
| --- | --- | --- | --- |
| F11 | 8.94Eþ01  7.77Eþ00  3 | 2.34E-13 | 3.06E-13 |
| 1.07E-14 | 5.48E-14 |
| 1 | 2 |
| F12 | 1.62E þ 03  1.30E þ 02  1 | 3.25Eþ04  1.40Eþ03  3 | 4.15Eþ03  2.40Eþ02  2 |
| F13 | 9.11Eþ02  1.93Eþ02  2 | 9.87Eþ02  1.50Eþ02  3 | 6.87E þ 02 2.63E þ 01 1 |
| F14 | 7.51E þ 07  5.36E þ 06  1 | 1.52Eþ08  1.24Eþ07  3 | 7.67Eþ07  4.55Eþ06  2 |
| F15 | 5.28Eþ03  3.79Eþ02  3 | 4.21Eþ03  1.01Eþ02  2 | 3.14E þ 03 5.42E þ 01 1 |
| F16 | 2.69Eþ02  1.37Eþ01  3 | 7.63Eþ00  2.95Eþ00  2 | 3.79E þ 00 6.26E-01 |
| 1 |
| F17 | 1.41E þ 04  6.23E þ 02  1 | 1.47Eþ05  7.77Eþ03  3 | 3.74Eþ04  1.36Eþ02  2 |
| F18 | 2.11Eþ03  1.47Eþ03  2 | 4.15Eþ03  1.56Eþ03  3 | 1.52E þ 03 2.93E þ 02 1 |
| F19 | 8.73E þ 05  1.03E þ 05  1 | 1.35Eþ06  5.17Eþ04  3 | 1.04Eþ06  2.85Eþ04  2 |
| F20 | 1.58Eþ03  7.71Eþ01  3 | 1.15Eþ03  2.42Eþ01  2 | 1.04E þ 03 8.18E þ 01 1 |
| Nb/Nw/Mr | 7/10/2.15 | 4/10/2.3 | 10/0/1.55 |

of Wild Geese, WGA is tested without considering the death phase and is tested with a large population Np ¼ 120 and a small population Np ¼ 30. The suitable results were compared with those of WGA (considering population reduction from Np ¼ 120 (Npinitial¼120) to Np ¼ 30 (Npfinal¼30) using Eq. (5), where the results obtained for each function are listed in Table 2. The results demonstrate that the proposed death phase improves the efficiency of WGA for high-dimensional problems. The positive influence of death phase can be especially observed for test

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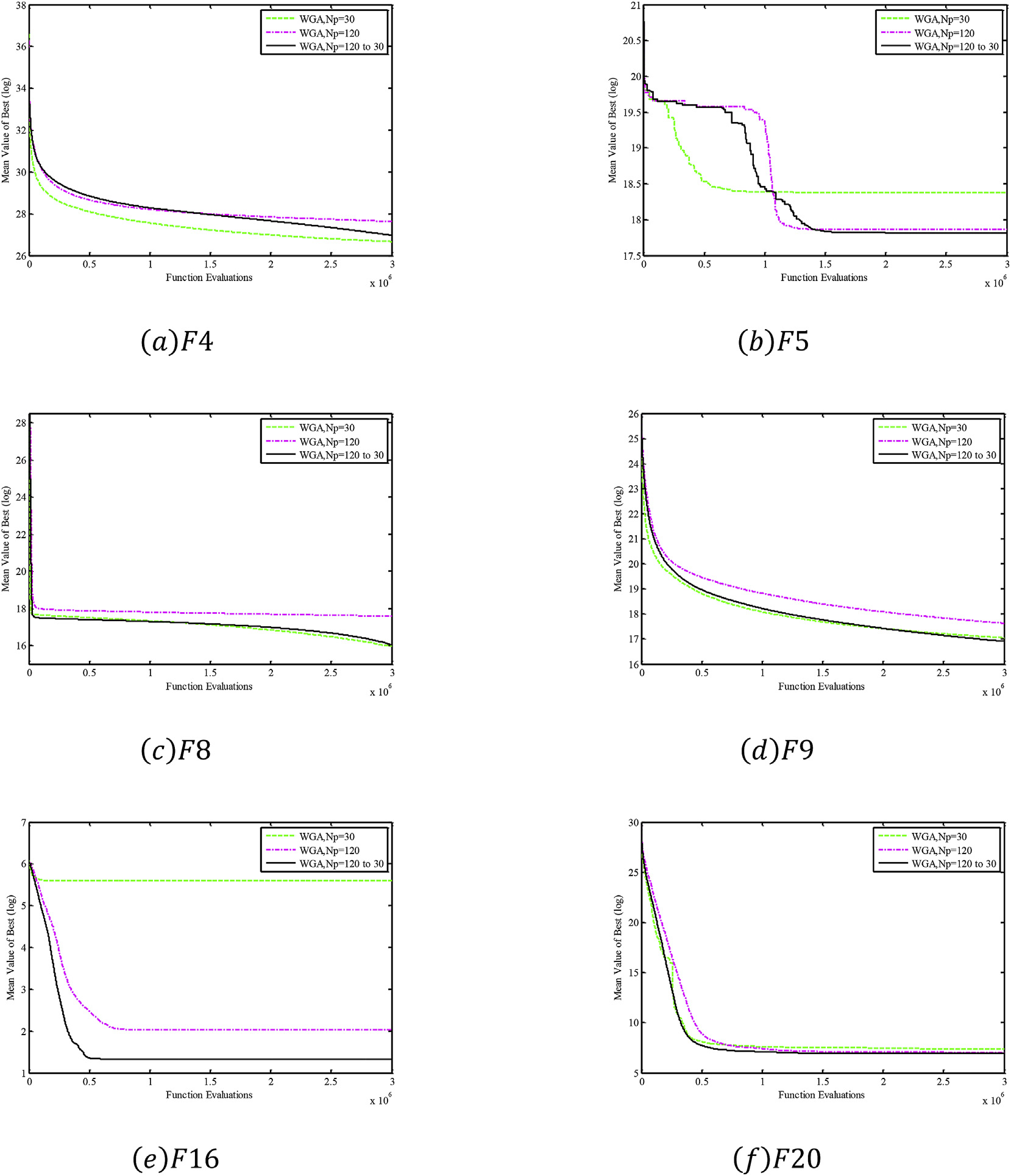


Fig. 3. Average convergence of WGA on nine selected test functions over 25 independent runs.

test functions and dispersion of its results are less than those of the other algorithms. The comparison between WGA and DECC-D algorithm shows that WGA performs better for 18 out of 20 functions. Nonetheless, for functions F2 and F20, it gives a worse result compared to that of DECC-D. For function F2, the average value of WGA is very close to that obtained from DECC-D algorithm. Furthermore, DECC-D algorithm does not pro-vide a good quality solution for different test functions, for example for F2 and F20 it has suitable results, but for F5 � F8, F10 � F12, and F15�F17 its results are not acceptable compared to those of other algorithms. Although DECC-DML algorithm outperforms WGA for five test functions, it has the worst solution for six functions. CBCC1-DG and CBCC2-DG algorithms are more successful than WGA for two and three functions, respectively; however, CBCC1-DG gives the best result for none of the functions and CBCC2-DG yields the best result for only function. DECC-DG algorithm performs better than WGA for 2 out of 20 test functions; however, it gives the worst solution for 4 test functions among all

3.2.3. Test on real-world optimization problems   
 Here, the effectiveness of the proposed algorithm (WGA) was inves-tigated compared to genetic algorithm (GL-25) [34], DE with strategy adaptation (SaDE) [35], DE with control components and composite trial vector generation approaches (CoDE) [36], Standard particle swarm optimization (SPSO2013) [37], and heterogeneous comprehensive learning PSO with improved exploitation and exploration (HCLPSO) [38]

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| on | real-world | usages | including | estimating | the | factor | for |

frequency-modulated (FM) sound waves [39] and large-scale reliabili-ty-redundancy allocation optimization (RRAO) of a gas turbine [40].

1) Estimating the factor for frequency modulated sound waves

The greatly complex multimodal frequency-modulated (FM) sound synthesis optimizing problem plays a key role in various modern music systems for estimating the optimal factors of a FM sound wave synthesis

algorithms. [39]. The estimation of optimal factors of an FM sound wave synthesis is

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|  |  |  |
| --- | --- | --- |
| M. Ghasemi et al.  Table 3  Average fitness values and standard deviations on test functions over 25 inde-pendent runs. | Array 11 (2021) 100074 sound waves for t defined in range of 1–100 are as follows [42]: | |
| yðtÞ ¼ x1 sinðx2tθ þ x3 sinðx4tθ þ x5 sinðx6tθÞÞÞ; | (6) |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| F | Cr ¼ 0.1 | Cr ¼ 0.25 | Cr ¼ 0.75 | Cr ¼ 0.9 | Cr ¼ 0.5 |
| F1 | 3.12Eþ07 7.41Eþ07 3,– 1.17Eþ12 7.40Eþ11 3,– 1.39Eþ10 8.22Eþ09 4,– 3.04Eþ10 2.61Eþ10 4,– 1.50Eþ10 4.20Eþ09 3,– 5/0/0 | 3.77E-06 | 5.24Eþ09 1.03Eþ09 4,– 4.96Eþ13 8.22Eþ13 4,– 1.03Eþ10 7.50Eþ09 3,– 3.19Eþ09 4.54Eþ09 3,– 1.17Eþ11 3.73Eþ09 4,– 5/0/0 | 5.00Eþ10 2.29Eþ10 5,– 2.48Eþ14 5.31Eþ13 5,– 6.71Eþ10 2.56Eþ10 5,– 1.55Eþ11 1.23Eþ11 5,– 6.53Eþ11 3.90Eþ09 5,– 5/0/0 | 1.05E-26 |
| F4 | 1.63E-07 | 2.56E-26 |
| 2,– 7.26Eþ11 9.64Eþ10 2,– 9.04Eþ07 2.76Eþ08 2,– 2.71Eþ09 1.26Eþ09 2,– 1.03Eþ03 5.15Eþ01 1,þ  4/1/0 | 1 |
| 5.15Eþ11 7.89Eþ10 1 |
| F9 | 2.21Eþ07 1.51Eþ06 1 |
| F14 | 7.67Eþ07 4.55Eþ06 1 |
| F20 | 1.04Eþ03 8.18Eþ01 2 |
| �/þ/ ¼  Nb/Nw/Mr | – 4/0/1.2 |
| 0/0/3.4 | 1/0/1.8 | 0/0/3.6 | 0/0/5 |

Table 4   
Summary of CEC 08 Special Session benchmark test functions [5] for large scale global optimization.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Function | Name | Properties | Search | Global |
| space | optimum |
| f1 | Shifted Sphere | Unimodal, separable, | [-100, | 0 |
| f2 | Shifted | scalable | 100] | 0 |
| Unimodal, non- | [-100, |
| f3 | Schwefel's | separable, scalable | 100] | 0 |
| Shifted | Multimodal, non- | [-100, |
| f4 | Rosenbrock's | separable, scalable | 100] | 0 |
| Shifted | Multimodal, | [-5, 5] |
| f5 | Rastrigin's | separable, scalable | [-600, | 0 |
| Shifted | Multimodal, non- |
| f6 | Griewank's | separable, scalable | 600] | 0 |
| Shifted Ackley's | Multimodal, | [-32, 32] |

separable, scalable

an optimization problem with D decision variables. In this work, the case of D ¼ 6 is only considered in accordance with [41,42]. Six components are included in the 6-dimensional parameter vector as x ¼ [x1(a1), x2(ω1), x3(a2), x4(ω2), x5(a3), x6(ω3)] ranging between 6.35 and 6.5 for all variables. The equations provided for the target and approximated

Table 5

|  |  |
| --- | --- |
| y0ðtÞ ¼ 1:0\*sinð0:5tθ � 1:5 \* sinð4:8tθ þ 2:0 \* sinð4:9tθÞÞÞ; | (7) |
| where θ ¼2π 100  The optimization problem objective function is considered as the sum | |

of squared errors between yðtÞ(the approximated wave) and y0ðtÞ (the target wave) with optimal value f(x) ¼ 0 as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| f ðxÞ ¼ | X | ðyðtÞ � y0ðtÞÞ2: | (8) |

2) RRAO constrained problem:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| The | nonlinear | reliability-redundancy | constrained | optimization |

problems are mainly aimed at enhancing the system reliability (maxi-mizing the overall system reliability) through optimizing element re-liabilities vector (r ¼ (r1, r2, …, rm)) and redundancy assignment numbers vector (n¼ (n1, n2, …, nm)) for subsystems of the system. It is possible to formulate this problem as a nonlinear mixed-integer programming model by choosing the system reliability as the objective function to be maxi-mized subjecting to several nonlinear constraints as follows [40]:

|  |  |
| --- | --- |
| Maximize Rs ¼ f ðr; nÞ;  subject to gðr; nÞ � l;  0 � rd � 1; nd 2 Zþ; 0 � d � m: | (9)  (10) |

where Zþis the set of positive integers, Rs represents the reliability of various systems, f(.) and g(.) denote for the objective and constraint functions of RRAO problem for the total parallel-series systems, respec-tively, from which g(.) is usually related to the system cost, weight and volume. n¼ (n1, n2, …, nm) and r ¼ (r1, r2, …, rm) show the redundancy allocation numbers and component reliabilities vectors for system's subsystems including m subsystems, respectively. Moreover, l shows the limit of the system resources.

The overspeed detection was continually offered by the mechanical and electrical systems. By occurring an overspeed, the fuel source must be stopped through control valves (V1 to Vm). Fig. 4 represents a gas turbine's overspeed protection system for RRAO optimizing the mixed-integer non-linear problem. The large-scale test structure involves 40

Results obtained by optimization algorithms for dimension 100 over 25 independent runs.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| F | D ¼ 100 | CSO [24] | sep-CMA-ES [9] | MLCC [7] | EPUS-PSO [8] | ISSA [33] | EO [31] | WGA |
| CCPSO2 [2] |



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| F1 | 7.73E-14 | 9.11E-29 | 9.02E-15 | 6.82E-14 | 7.47E-01 | 0 | 1.31E-20 | 0 |
| F2 | 3.23E-14 | 1.10E-28 | 5.53E-15 | 2.32E-14 | 1.70E-01 | 0 | 5.01E-20 | 0 |
| 6,- | 2,- | 4,- | 5,- | 7,- | 1, ¼  8.31Eþ01  1.91 Eþ01  8,- | 3.- | 1 |
| 6.08Eþ00  7.83Eþ00  2,- | 3.35Eþ01  5.38Eþ00  6,- | 2.31Eþ01  1.39Eþ01  4,- | 2.53Eþ01  8.73Eþ00  5,- | 1.86Eþ01  2.26Eþ0  3,- | 4.29Eþ01 3.69Eþ00 7,- | 2.14E-05 |
| 3.08E-05 |
| 1 |
| F3 | 4.23Eþ02  8.65Eþ02  7,- | 3.90Eþ02  5.53Eþ02  6,- | 4.31E þ 00 1.26E þ 01 1, þ  2.78Eþ02  3.43Eþ01  6,- | 1.50Eþ02  5.72Eþ01  4,- | 4.99Eþ03  5.35Eþ03  8,- | 1.68Eþ02  9.46Eþ01  5,- | 9.21Eþ01 8.97Eþ01 2,þ  6.04Eþ02 8.52Eþ01 8,- | 1.04Eþ02 4.01Eþ01 3 |
| F4 | 3.98E-02 | 5.60Eþ01  7.48Eþ00  4,þ  0 | 4.39E-13 | 4.71Eþ02  5.94Eþ01  7,- | 5.00Eþ00  6.60Eþ00  3,þ  0 | 1.25Eþ02 1.41Eþ01 5 |
| 1.99E-01 | 9.21E-14 |
| 2,þ  3.45E-03 | 1,þ  3.41E-14 |
| F5 | 2.96E-04 | 3.72E-01 | 9.58E-02 | 0 |
| F6 | 4.88E-03 | 0 | 1.48E-03 | 1.16E-14 | 5.60E-02 | 0 | 1.02E-01 | 0 |
| 4,- | 1, ¼  1.20E-014 | 3,- | 2,- | 6,- | 1, ¼  2.09Eþ01  2.99E-02 | 5,- | 1 |
| 1.44E-13 | 2.12Eþ01  4.02E-01 | 1.11E-13 | 2.06Eþ00  4.40E-01 | 2.05Eþ01 1.73E-01 | 1.39E-014 |
| 3.06E-14 | 1.52E-015 | 7.87E-15 | 1.23E-015 |
| �/þ/ ¼  Nb/Nw/Mr | 4,- | 1,þ  3/2/1 | 8,- | 3,- | 6,- | 7,- | 5,- | 2 |
| 5/1/0 | 5/1/0 | 5/1/0 | 6/0/0 | 3/1/2 | 5/1/0 |
| – 3/0/2.333 |
| 0/0/4.167 | 2/0/3.333 | 1/1/4.333 | 1/0/3.333 | 0/4/6.167 | 2/1/4.167 | 0/1/5 |

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Table 6

Results obtained by optimization algorithms for dimension 500 over 25 independent runs.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| F | D ¼ 500 |  |  |  |  |  |  |  |
|  | CCPSO2 [2] | CSO [24] | sep-CMA-ES [9] | MLCC [7] | EPUS-PSO [8] | ISSA [33] | EO [31] | WGA |
| F1 | 3.00E-13 | 6.57E-23 | 2.25E-14 | 4.30E-13 | 8.45Eþ01  6.40Eþ00  8,- | 9.90E-28 | 4.14E-04 | 0.00E þ 00 0.00E þ 00 1 |
| 7.96E-14 | 3.90E-24 | 6.10E-15 | 3.31E-14 | 9.95E-28 | 3.87E-04 |
| 5,- | 3,- | 4,- | 6,- | 2,- | 7,- |
| F2 | 5.79Eþ01 4.21Eþ01 4,- | 2.60E þ 01 2.40E þ 00 1,þ  5.74Eþ02  1.67Eþ02  4,- | 2.12Eþ02  1.74Eþ01  7,- | 6.67Eþ01 5.70Eþ00 5,- | 4.35Eþ01  5.51E-01 | 2.66Eþ02 1.92Eþ01 8,- | 9.34Eþ01 3.01E-01 | 5.73Eþ01 8.72Eþ00 3 |
| 2,þ  5.77Eþ04  8.04Eþ03  5,- | 6,- |
| F3 | 7.24Eþ02 1.54Eþ02 6,- | 2.93E þ 02  3.59E þ 01  1,þ  2.18Eþ03  1.51Eþ02  6,- | 9.25Eþ02 1.73Eþ02 7,- | 8.31Eþ14 3.11Eþ14 8,- | 1.95Eþ03 1.04Eþ03 3,- | 5.22Eþ02 3.60Eþ01 2 |
| F4 | 3.98E-02 | 3.19Eþ02  2.16Eþ01  4,- | 1.79E-11 | 3.49Eþ03  1.12Eþ02  7,- | 2.07Eþ03 5.38Eþ02 5,- | 3.78Eþ03 1.46Eþ02 8,- | 1.25Eþ02 1.41Eþ01 3 |
| 1.99E-01 | 6.31E-11 |
| 2,þ  1.18E-03 | 1,þ  2.13E-13 |
| F5 | 2.22E-16 | 7.88E-04 | 1.64Eþ00  4.69E-02 | 4.48E-02 | 2.42E-01 | 4.12E-16 |
| F6 | 4.61E-03 | 0.00E þ 00 1,þ  4.13E-13 | 2.82E-03 | 2.48E-14 | 1.29E-01 | 6.11E-01 | 5.36E-17 |
| 5,- | 4,- | 3,- | 8,- | 6,- | 7,- | 2 |
| 5.34E-13 | 2.15Eþ01  3.10E-01 | 5.34E-13 | 6.64Eþ00  4.49E-01 | 2.14Eþ01 1.70E-02 | 2.06Eþ01 3.35E-01 | 5.77E-14 |
| 8.61E-14 | 1.10E-14 | 7.01E-14 | 1.58E-15 |
| �/þ/ ¼  Nb/Nw/Mr | 3,- | 2,- | 7,- | 3,- | 4,- | 6,- | 5,- | 1,þ |
| 5/1/0 | 4/2/0 | 5/1/0 | 5/1/0 | 5/1/0 | 6/0/0 | 6/0/0 |
| – 2/0/2 |
| 0/0/4.167 | 2/0/2.5 | 1/2/4.833 | 1/1/4.167 | 0/3/5.667 | 0/2/5.833 | 0/1/6 |

Table 7

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Results obtained by optimization algorithms for dimension D ¼ 1000 over 25 independent runs. | | | | | EPUS-PSO [8] | ISSA [33] | EO [31] | WGA |
| F | D ¼ 1000 | | | |
| CCPSO2 [2] | | CSO [24] | sep-CMA-ES [9] | MLCC [7] |
| F1 | 5.18E-13 | 1.09E-21 | 7.81E-15 | 8.46E-13 | 5.53Eþ02 2.86Eþ01 7,- | 2.09E-18 | 1.35Eþ04 6.94Eþ03 8,- | 1.75E-28 |
| 9.61E-14 | | 4.20E-23 | 1.52E-15 | 5.01E-14 | 3.95E-18 | 1.27E-28 |
| 5,- | | 2,- | 4,- | 6,- | 3,- | 1 |
| F2 | 7.82Eþ01  4.25Eþ01  4,- | 4.15E þ 01 9.74E-01 | 3.65Eþ02  9.02Eþ00  8,- | 1.09Eþ02  4.75Eþ00  5,- | 4.66Eþ01 4.00E-01 | 3.10Eþ02 1.38Eþ01 7,- | 1.64Eþ02 1.16Eþ02 6,- | 7.43Eþ01 4.89Eþ00 3 |
| 1,þ  1.01Eþ03  3.02Eþ01  3,- | 2,þ  8.37Eþ05 1.52Eþ05 6,- |
| F3 | 1.33Eþ03  2.63Eþ02  4,- | 9.10E þ 02  4.54E þ 01  1,þ  5.31Eþ03  2.48Eþ02  5,- | 1.80Eþ03  1.58Eþ02  5,- | 2.17Eþ15 6.89Eþ13 8,- | 2.58Eþ09 2.63Eþ09 7,- | 1.00Eþ03 8.25Eþ01 2 |
| F4 | 1.99E-01 | 6.89Eþ02  3.10Eþ01  3,þ  2.26E-16 | 1.37E-10 | 7.58Eþ03 1.51Eþ02 6,- | 1.49Eþ04 1.93Eþ03 8,- | 7.79Eþ03 1.01Eþ02 7,- | 2.52Eþ03 1.34Eþ02 4 |
| 4.06E-01 | 3.37E-10 |
| 2,þ  1.18E-03 | 1,þ  4.18E-13 |
| F5 | 3.94E-04 | 5.89Eþ00 3.91E-01 | 3.10E-01 | 4.07Eþ01 5.39Eþ01 8,- | 1.22E-15 |
| 3.27E-03 | | 2.18E-17 | 1.97E-03 | 2.78E-14 | 4.51E-01 | 2.91E-16 |
| F6 | 5,- | 1,þ  1.21E-12 | 4,- | 3,- | 7,- | 6,- | 2 |
| 1.02E-12 | 2.15Eþ01  3.19E-01 | 1.06E-12 | 1.89Eþ01 2.49Eþ00 6,- | 2.15Eþ01 7.70E-03 | 2.05Eþ01 1.40E-01 | 1.21E-13 |
| 1.68E-13 | 2.64E-14 | 7.68E-14 | 5.18E-15 |
| 2,- | | 4,- | 5,- | 3,- | 8,- | 7,- | 1 |
| �/þ/ ¼  Nb/Nw/Mr | 5/1/0 | 3/3/0 | 5/1/0 | 5/1/0 | 5/1/0 | 6/0/0 | 6/0/0 | – 2/0/2.167 |
| 0/0/3.667 | 2/0/2.33 | 1/1/4.5 | 1/0/3.833 | 0/0/5.667 | 0/3/6.667 | 0/2/7.167 |

decision variables (m\*2 ¼ 40). The input factors and data for the large-scale test system are provided in Ref. [43] with 20 subsystems.

It is possible to formulate this reliability optimization problem as:

represents the upper volume limit of the products of the subsystem. 2) The system cost limitationg2ðr; nÞ:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Maximize f5ðr; nÞ ¼ Y ½1 � ð1 � rdÞnd�:  0:5 � rd �  1 � nd � 10; 2 Zþ:�1 � 10�6�; 0 � d � m | (11) | g2ðr; nÞ ¼ | X CðrdÞ�nd þ e0:25nd�  ��T ln rd�βd : | � C; | (13) |
| CðrdÞ ¼ αd |
| The system constraints include: |

1) The combined weight, volume, and redundancy allocation con- straintg1ðr; nÞ:

where, C shows the upper cost limit of the system, CðrdÞ is the cost for all element with reliability rd at dth stage, and T is the operating time in

which the components are working.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| g1ðr; nÞ ¼ | X | v2 dn2 d� V | (12) | 3) The system weight limitationg3ðr; nÞ: | | | (14) |
| g3ðr; nÞ ¼ | X | wdnde0:25nd� W |
| where vd shows the volume of dth subsystem for all components and V | | | |

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Table 8

Average fitness values and standard deviations on CEC 2010 functions over 25 independent runs.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| F | MLCC [7] | DECC-D [12] | DECC-DML [12] | CBCC1-DG [22] | CBCC2-DG [22] | DECC-DG [22] | WGA |
| F1 | 1.53E-27 | 1.01E-24 | 1.93E-25 | 1.32Eþ04  6.25Eþ04  7,- | 8.34Eþ03  3.41Eþ04  6,- | 5.47Eþ03  2.02Eþ04  5,- | 1.05E-26 |
| 7.66E-27 | 1.40E-25 | 1.86E-25 | 2.56E-26 |
| 1,þ  5.57E-01 | 4,- | 3,- | 2 |
| F2 | 2.99Eþ02  1.92Eþ01  3,þ  1.81E-13 | 2.17Eþ02  2.98Eþ01  2,þ  1.18E-13 | 4.44Eþ03  1.60Eþ02  6,- | 4.44Eþ03  1.80Eþ02  6,- | 4.39Eþ03  1.97Eþ02  5,- | 2.28Eþ03 4.58Eþ01 4 |
| 2.21E þ 00 1,þ  9.88E-13 |
| F3 | 1.66Eþ01  3.79E-01 | 1.67Eþ01  3.28E-01 | 1.67Eþ01  3.34E-01 | 1.47E-13 |
| 3.70E-12 | 6.68E-15 | 8.22E-15 | 8.94E-15 |
| F4 | 4,- | 3,- | 1,þ  3.58Eþ12  1.54Eþ12  4,- | 5,- | 6,- | 6,- | 2 |
| 9.61Eþ12  3.43Eþ12  7,- | 3.99Eþ12  1.30Eþ12  5,- | 2.31Eþ12  7.43Eþ11  2,- | 2.36Eþ12  7.92Eþ11  3,- | 4.79Eþ12  1.44Eþ12  6,- | 5.15E þ 11 7.89E þ 10 1 |
| F5 | 3.84Eþ08  6.93Eþ07  6,- | 4.16Eþ08  1.01Eþ08  7,- | 2.98Eþ08  9.31Eþ07  5,- | 1.35Eþ08  2.18Eþ07  2,- | 1.36Eþ08  2.46Eþ07  3,- | 1.55Eþ08  2.17Eþ07  4,- | 5.47E þ 07 7.93E þ 06 1 |
| F6 | 1.62Eþ07  4.97Eþ06  6,- | 1.36Eþ07  9.20Eþ06  5,- | 7.93Eþ05  3.97Eþ06  4,- | 1.65Eþ01  3.99E-01 | 1.64Eþ01  3.46E-01 | 1.64Eþ01  2.71E-01 | 3.55E-09 |
| 5.48E-14 |
| 3,- | 2,- | 2,- | 1 |
| F7 | 6.89Eþ05  7.37Eþ05  5,- | 6.58Eþ07  4.06Eþ07  6,- | 1.39Eþ08  7.72Eþ07  7,- | 1.81Eþ04  4.59Eþ04  4,- | 1.35Eþ04  3.92Eþ04  3,- | 1.16Eþ04  7.41Eþ03  2,- | 4.60E þ 00 6.28E þ 00 1 |
| F8 | 4.38Eþ07  3.45Eþ07  7,- | 5.39Eþ07  2.93Eþ07  6,- | 3.46Eþ07  3.56Eþ07  5.- | 3.34Eþ06  2.29Eþ06  2,þ  6.79Eþ07  6.92Eþ06  5,- | 8.70E þ 05  1.71E þ 06  1,þ  7.97Eþ07  1.08Eþ07  6,- | 3.04Eþ07  2.11Eþ07  4,- | 9.16Eþ06 8.79Eþ06 3 |
| F9 | 1.23Eþ08  1.33Eþ07  7,- | 6.19Eþ07  6.43Eþ06  4,- | 5.92Eþ07  4.71Eþ06  2,- | 5.96Eþ07  8.18Eþ06  3,- | 2.21E þ 07 1.51E þ 06 1 |
| F10 | 3.43Eþ03  8.72Eþ02  2,- | 1.16Eþ04  2.68Eþ03  6,- | 1.25Eþ04  2.66Eþ02  7,- | 4.01Eþ03  1.37Eþ02  3,- | 4.04Eþ03  1.21Eþ02  4,- | 4.52Eþ03  1.41Eþ02  5,- | 2.64E þ 03 2.70E þ 01 1 |
| F11 | 1.98Eþ02  6.98E-01 | 4.76Eþ01  9.53Eþ01  5,- | 1.80E-13 | 1.05Eþ01  9.31E-01 | 1.03Eþ01  8.47E-01 | 1.03Eþ01  1.01Eþ00  3,- | 3.06E-13 |
| 9.88E-15 | 5.48E-14 |
| 6,- | 1,þ  3.79Eþ06  1.50Eþ05  7,- | 4,- | 3,- | 2 |
| F12 | 3.49Eþ04  4.92Eþ03  5,- | 1.53Eþ05  1.23Eþ04  6,- | 4.19Eþ03  1.25Eþ03  4,- | 4.00Eþ03  8.63Eþ02  2,þ  4.54Eþ03  1.91Eþ03  5,- | 2.52E þ 03  4.86E þ 02  1,þ  4.54Eþ06  2.13Eþ06  7,- | 4.15Eþ03 2.40Eþ02 3 |
| F13 | 2.08Eþ03  7.27Eþ02  4,- | 9.87Eþ02  2.41Eþ02  2,- | 1.14Eþ03  4.31Eþ02  3,- | 9.10Eþ03  3.75Eþ03  6,- | 6.87E þ 02 2.63E þ 01 1 |
| F14 | 3.16Eþ08  2.77Eþ07  4,- | 1.98Eþ08  1.45Eþ07  3,- | 1.89Eþ08  1.49Eþ07  2,- | 3.64Eþ08  2.61Eþ07  6,- | 3.69Eþ08  2.42Eþ07  7,- | 3.41Eþ08  2.41Eþ07  5,- | 7.67E þ 07 4.55E þ 06 1 |
| F15 | 7.11Eþ03  1.34Eþ03  4,- | 1.53Eþ04  3.92Eþ02  5,- | 1.54Eþ04  3.59Eþ02  6,- | 5.89Eþ03  9.10Eþ01  3,- | 5.88Eþ03  8.81Eþ01  2,- | 5.88Eþ03  1.03Eþ02  2,- | 3.14E þ 03 5.42E þ 01 1 |
| F16 | 3.76Eþ02  4.71Eþ01  7,- | 1.88Eþ02  2.16Eþ02  6,- | 5.08E-02 | 3.08E-12 | 4.44E-12 | 7.39E-13 | 3.79Eþ00 6.26E-01 |
| 2.54E-01 | 3.19E-12 | 4.22E-13 | 5.70E-14 |
| F17 | 4,þ  6.54Eþ06  4.63Eþ05  7,- | 2,þ  4.50Eþ04  3.18Eþ03  3,- | 3,þ  4.73Eþ04  2.77Eþ03  4,- | 1,þ  4.01Eþ04  2.85Eþ03  2,- | 5 |
| 1.59Eþ05  1.43Eþ04  5,- | 9.03Eþ05  5.28Eþ04  6,- | 3.74E þ 04 1.36E þ 02 1 |
| F18 | 7.09Eþ03  4.77Eþ03  4- | 2.12Eþ03  5.18Eþ02  2,- | 2.47Eþ03  1.18Eþ03  3,- | 1.34Eþ09  4.94Eþ08  6,- | 3.47Eþ08  1.39Eþ08  5,- | 1.11Eþ10  2.04Eþ09  7,- | 1.52E þ 03 2.93E þ 02 1 |
| F19 | 1.36Eþ06  7.35Eþ04  2,- | 1.33Eþ07  1.05Eþ06  4,- | 1.59Eþ07  1.72Eþ06  5,- | 1.74Eþ06  8.46Eþ04  3,- | 1.74Eþ06  8.46Eþ04  3,- | 1.74Eþ06  9.54Eþ04  3,- | 1.04E þ 06 2.85E þ 04 1 |
| F20 | 2.05Eþ03  1.80Eþ02  3,- | 9.91E þ 02 2.61E þ 01 1,þ  18/2/0 | 9.91E þ 02  3.51E þ 01  1,þ  15/5/0 | 9.53Eþ04  1.02Eþ05  5,- | 8.42Eþ03  2.36Eþ03  4,- | 4.87Eþ07  2.27Eþ07  6,- | 1.04Eþ03 8.18Eþ01 2 |
| �/þ/ ¼  Nb/Nw/Mr | 18/2/0 | 18/2/0 | 17/3/0 | 18/2/0 | - |
| 2/6/4.5 | 1/1/4.45 | 3/6/3.95 | 0/2/4.05 | 1/3/3.9 | 2/4/3.95 | 12/0/1.75 |

The proposed WGA algorithm and the other 5 algorithms are applied in these two real-world problems. For comparative studies, FEsmax are adjusted to 5.00Eþ04 and a large enough population size is chosen for all algorithms. Table 9 presents the optimization results (mean and standard

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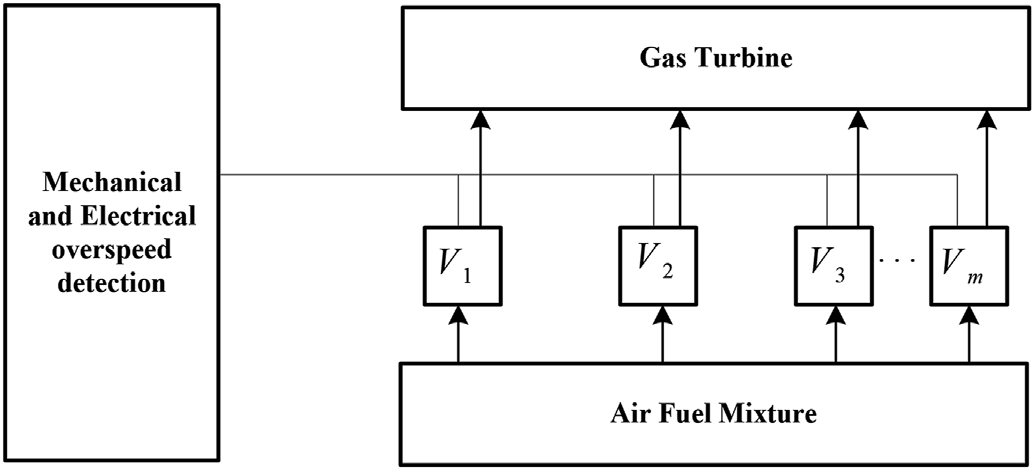


Fig. 4. The diagram block for a gas turbine's overspeed protection system.

Table 9   
Average fitness values and standard deviations on real-world optimization problems.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Algorithms | Problem 1 |  | Problem 2 |  |
|  | Mean | Std | Mean | Std |
| GL-25 | 4.05Eþ000 2.72Eþ000 3.19Eþ000 7.64Eþ000 5.38Eþ000 1.23E-007 | 9.83Eþ000 6.65Eþ000 8.54Eþ000 1.15Eþ001 1.29Eþ001 1.08E-007 | 8.634E-001 | 8.114E-001 |
| SaDE | 8.898E-001 | 2.875E-002 |
| CoDE | 8.882E-001 | 6.155E-001 |
| SPSO2013 | 8.730E-001 | 6.058E-001 |
| HCLPSO | 8.875E-001 | 1.464E-001 |
| WGA | 8.915E-001 | 9.628E-004 |

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The proposed Wild Goose Algorithm (WGA) is a simple and effective algorithm that has been designed and proposed for optimization of high-dimensional problems. This algorithm, which is inspired by wild geese found in nature, includes ordered and coordinated group migration, reproduction and evolution of geese, and also death in the population of geese. To show the performance of the proposed WGA algorithm for optimization of high-dimension problems, it is tested and compared with sep-CMA-ES, CCPSO2, CSO, EPUS-PSO, MLCC, DECCD, DECC-DML, CBCC2-DG, CBCC1-DG and DECC-DG algorithms based on the func-tions of CEC 2008 and CEC 2010. One of the advantages of WGA is that it has only one control parameter, Cr. It is experimentally shown that WGA has better competitive results with respect to other mentioned algo-rithms, and outperforms all other algorithms for most of the test func-tions. Furthermore, WGA is a simple and basic algorithm for large-scale optimization which can be used for various real-world optimization problems. In recent years, numerous studies have been carried out in the area of high-dimension optimization, the most of which focused on cooperative co-evolution technique. In future, WGA may be embedded into the frameworks of different CC methods with various categories in order to improve its performance. Furthermore, WGA can be used for solving other real-world large-scale optimization problems.

Credit author statement

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Mojtaba | Ghasemi: | Conceptualization, | Methodology, | Software, |

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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