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Bound Analysis for Whiley Programs

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**Abstract**

The Whiley compiler can generate naive C code, but the code is inefficient because it uses infinite integers and dynamic array sizes. Our project goal is to build up a compiler that can translate Whiley programs into efficient OpenCL code with fixed-size integer types and fixed-size arrays, for parallel execution on GPUs. This paper presents an abstract interpretation-based bound inference approach along with symbolic analysis for Whiley programs. The source Whiley program is first analyzed by using our symbolic analyzer to find the matching pattern and make any necessary program transformation. Then the bound analyzer is used to analyze the transformed program to make use of primitive integer types rather than third-party infinite integer type (e.g. using GMP arbitrary precision library). The bound analysis results provide conservative estimates of the ranges of integer variables and array sizes so that efficient code can be generated and integer overflows avoided. The bound analyzer combines the bound consistency technique along with a widening operator to give fast time of solving program constraints and of converging to the fixed point. Several example programs are used to illustrate the bound analyzer algorithm and the program transformation.

*Keywords:* Static Analysis, Range Analysis, Abstract Interpretation, Bound Consistency, Widening Operator, Symbolic Analysis, Pattern matching, Program Transformation.

# Introduction

Static program analysis techniques validates the consistency between software spec- ifications and program behaviors using mathematical methodologies. For example, the bound consistency technique is widely used to solve the finite constraint do- main problem (a.k.a constraint satisfaction problem)[[17](#_bookmark34)]. However, the problems of object-oriented program languages, such as side-effect problems or non-deterministic results, makes it a grand challenge[[15](#_bookmark32)] to create such a compiler, with automated

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mathematical and logical reasoning, that can verify the specifications and detect the errors at compile-time.

Whiley[[20](#_bookmark35)] is a new and verification-friendly programming language with the aim of resolving verification issues that arise from object-oriented programming languages. Whiley verifying compiler can detect bugs at compile-time and convert the program into bug-less Java or C code. However, translating high-level Whiley programs into efficient implementations has some challenges, for instance, the use of unbounded integers causes substantial slowdown on the performance of Whiley implementations.

This paper aims to describe the design of bound analyzer along with symbolic analyzer to assist the code generator to produce behavior-predicable C code that makes use of efficient integer data types in the implementation. The main objectives are summarized as below:

* Recognize patterns of a Whiley program to make any necessary program trans- formation.
* Analyze the transformed program to produce the bound constraints.
* Infer the bounds using propagation rules and speed up bound analysis using the widening operator.
* Determine the efficient integer data types for the code generator.

This paper is organized as follows. Section 2 reviews some related works about static analysis, bound analysis and symbolic loop bounds. Section 3 describes the bound inference procedure, fixed-point approximation using widen operator, and pattern matching along with program transformation. Section 4 illustrates the algorithm of bound analysis with example programs and shows the performance of generated C code with/without program transformation. And the final section concludes the future work.

# Related Work

* 1. *Static Bound Analysis*

Many automatic static program analyzers have been developed to improve the program correctness and produce the high-quality software, such as ESC/Java Checker[[10](#_bookmark27)] and Microsoft Spec# Static Verifier[[2](#_bookmark19)].

The static analysis using abstract interpretation, which approximates the ab- stract semantics of a computer program without executing all the calculation, al- lows the compiler to detect errors and find applicable optimization. For example, Microsoft Research Clousot[[9](#_bookmark26)] can statically check the absence of run-time er- rors and infer facts to discharge assertions.

However, computing the fixed-point in abstract domain is iterative and some- times time-consuming. The abstract interpretation-based widening operator[[6](#_bookmark23)] can rely on bound results at earlier iterations, and then widen the open-ended bounds to *±* inf, so as to accelerate the converging time to the fixed point. But the widening

operator may result in over-approximated analysis results. Thus, Su and Wanger[[22](#_bookmark39)] proposed the first polynomial algorithm to solve integer constraint problems with- out widening or narrowing operator, and showed that their method can achieve precise bounds in polynomial time whilst the termination is guaranteed.

The static analysis in LLVM (Low Level Virtual Machine) also gains popularity for its write-once-run-anywhere property. Campos et al.[[4](#_bookmark21)] used Su and Wagner’s approach to implement an industrial-quality range analysis in LLVM compiler. Su’s approach did not explicitly describe how to solve loop constraints, so the Campos system adapted Gawlitza approach[[12](#_bookmark29)] to observe the decrease or increase in cycles and then saturate the cycles using the widening operator. But compared to source- code level analyzer, the LLVM bound analyzer would more easily introduce overflow problems for the lost signed information at LLVM level. Thus, Navas, Schachte, et al.[[18](#_bookmark36)] designed a signedness-agnostic bound analyzer to handle both signed and unsigned LLVM variables.

Pearce[[19](#_bookmark37)] presented a forward propagation algorithm in Whiley to restrict the ranges of integer variables by exploiting type and loop invariant. Our approach does not require such explicit type declarations to infer the bounds, but uses the widening operator to compute the fix-point and ensure the loop termination.

* 1. *Loop Bound Analysis*

Loop bound analysis is a compiler optimization technique to obtain the number of loop iterations and prove the loop termination. It also can unroll the loop to reduce the compiler heap consumption. The commonly used techniques include *pattern-matching* and *counter increment*.

CodeStatistics, developed by Fulara et al[[11](#_bookmark28)], was applied with the pattern- matching technique to prove the loop termination by finding all for loop patterns in Java programs, and generating and inserting termination conditions as annotation into existing code. Their experimental results show that their method can efficiently prove 80% of for loops and detect error-prone loops in large-scaled applications, including Google App Engine, Apache Hadoop, TomCat and Oracle Berkeley DB. Shkaravska et al.[[21](#_bookmark38)] presented an counter-incremented approach to obtain the linear and non-linear loop-bound function (LBF), that binds the numeric loop con- dition to the number of loop iterations. Shkaravska’s approach can handle very complicated loops to infer polynomial LBFs but also ensure the correctness of de- rived LBFs using an external verifying tool. Due to inefficiency on simple loops, it

is usually considered as a complementary approach to other existing ones.

However, the above approaches both fail to handle multi-path loops of different effects or non-trivial patterns. Gulwani et al.[[14](#_bookmark30)] used *control-flow reﬁnement* tech- nique to transform a multi-path loop into one or more explicit interleaving loops to simplify the analysis, and then uses *progress invariant* technique to compute precise symbolic loop bounds. Their experimental results show that Gulwani approach can find 90% of loop bounds in a large Microsoft product.

# Methodology

* 1. *Overview*

This paper presents the bound analyzer along with symbolic analyzer to perform the analysis on Whiley programs and provide complementary analysis results to improve the quality of generated code with some optimization.

The source Whiley program is first compiled into WyIL (Whiley Intermediate Language) representation, produced by Whiley compiler, and then processed by our analyzers. Rather than from source level, the advantages of WyIL code allow the analyzers to:

* Separate from Whiley compiler and include other kinds of analysis, e.g. unique type analysis.
* Simplify the analysis for fewer control-flow constructs in WyIL.
* Optimize the program with additional program transformation and assertions without re-compilation.

Finally, the code generator translates the optimized WyIL code into bug-less and efficient C or OpenCL code that can be executed across heterogeneous platforms.

* 1. *Bound Analyzer*

The bound analyzer is implemented as a Java plug-in on top of the Whiley project. It infers the bounds of integer variables in two phases. First, the analyzer evaluates each WyIL code semantics to extracts the constraints on the abstract domain. Then the analyzer computes bounds with the bound consistency technique and converge the termination time by using the abstract interpretation-based widening operator.

* + 1. *Bound Consistency Check*

Bound consistency technique[[17](#_bookmark34)] restricts the variables to a finite set of values and satisfies the arithmetic constraints. This technique allows the bound analyzer to propagate lower or upper bounds among variables in the form of constraints and ensure that lower bounds never exceed upper bounds.

The bound analyzer takes the function code block as input, goes through the bound constraint and inference procedure (see Algorithm [1](#_bookmark5)), and produce the ag- gregated inferred bounds as output. It starts at the *main* function and in-lines all the function calls whenever necessary. The steps include:

**CFG Construction.** The analyzer processes each WyIL code semantics to build up a control flow graph (CFG) for each function and add it to a map for later use. Based on WyIL code type, the analyzer creates different types of CFG blocks (see CFG Definitions[[1](#_bookmark18)]), then extracts the bound constraints and place them into the CFG block.

**Bound Inference.** The analyzer initializes the bounds of each variable with nat- ural number domain [*−∞,* +*∞*] and infer the bounds from one block to another,

**Algorithm 1** Context-Sensitive Bound Inference Algorithm

**Input:** *graph*: CFG graph of function; *in bounds*: bounds of input parameters.

**Output:** *bound*: bounds of function return variables.

1: **procedure** InferBounds(*graph*, *in bounds*)

2: Initialize(*bounds*, *in bounds*)

3: iteration:=0

4: **while** any bounds has changed **do**

5: before bounds := bounds

6: **for** each block *b* **do** *// Block order does not matter*

7: *// Take the union of parents’ bounds*

S

8: b.bounds := *{* p.bounds *|* p *∈* b.parents*}*

9: **for** each constraint *c* **do** *// Ordered by byte-code order*

10: **if** *c* is FunctionCall(func name, in, out) **then**

11: bounds[out] := InferBounds(CFG(func name), bounds[in])

12: **else**

13: c.inferBound(b.bounds)

14: **end if**

15: **end for**

16: *// Check if lower bound ≤ upper bound.*

17: b.consistency := checkconsistency()

18: **end for**

19: iterations++

20: **if** iterations % 3 == 0 **then**

21: *//Widen the bounds every three iterations*

22: bounds := before bounds *∇* bounds

23: **end if**

24: **end while**

25: **return** bounds[return vars]

26: **end procedure**

and produces the bounds consistent with all the constraints[[16](#_bookmark33)]. The detailed procedure is shown in Algorithm [1](#_bookmark5).

* + 1. *Widening Operator*

Abstract interpretation-based widening operator[[6](#_bookmark23)] is an over-approximation tech- nique to speed up the time to the fixed point without executing all loop iterations. In this project, the widening operator can be operated in naive or gradual mode. The former follows Cousot’s original design to jump straight to *±∞* whilst the latter widens the bounds against a list of thresholds.

**Definition 3.1** *The naive widening operator ∇ can be used to extrapolate the un- stable bounds of an interval to ± inﬁnity.*

*⊥∇x* = *x x∇⊥* = *x*

[*l ,u* ] *∇* [*l , u*

*′ ′*

n n

*where:*

n+1

n+1]= *l ,u*

*l′* = *−∞ if l*n+1 *< l*n *l*n *Otherwise*

*u′* = *∞ if u*n+1 *> u*n *u*n *Otherwise*

The naive widening operator observes the increases of upper bounds at each iteration and decides whether to blow out the bounds to ultimately stationary *∞*. In the same manner, the operator converges decreasing lower bounds to *−∞*. Within finite steps, the widening operator can stabilize the bounds and accelerate the bound inference time.

|  |  |  |
| --- | --- | --- |
|  | Table 1 Threshold Values |  |
| **Threshold** | **Description** | **Value** |
| + inf |  | inf |
| *I*64max | *max*(long long Integers) | 9,223,372,036,854,775,807 |
| *I*32max | *max*(**int** Integers) | 2,147,483,647 |
| *I*16max | *max*(**short** Integers) | 32,767 |
| *I*16min | *min*(**short** Integers) | -32,768 |
| *I*32min | *min*(**int** Integers) | -2,147,483,648 |
| *I*64min  *−∞* | *min*(**long long** Integers) | -9,223,372,036,854,775,808  *−∞* |

*Widening with thresholds*, introduced by Blanchet et al.[[3](#_bookmark20)], can improve the precision of interval analysis and proves the boundedness of variables. This project redefines the thresholds with integer ranges defined in Microsoft Visual Studio C compiler (see Table [1](#_bookmark6)).

**Definition 3.2** *The gradual widening operator ∇*¯

*goes through thresholds and ﬁnds*

*an interval that stabilizes the bounds and reaches the ﬁxed point.*

*⊥ ∇*¯ *x* = *x x∇*¯ *⊥* = *x*

[*l*n*, u*n] *∇*¯ [*l*n+1*, u*n+1]= *l*th*, u*th

*where:*

*l*th = max(*th*min *∈ TH*min *| th*min *≤ l*n+1) *if l*n+1 *≤ l*n *l*n *Otherwise*

*u*th = min(*th*max *∈ TH*max *| th*max *> u*n+1) *if u*n+1 *> u*n *u*n *Otherwise*

The gradual widening operator broadens an increasing upper bound to the min- imum of possible maximal thresholds. Otherwise, the upper bound *u* stays un- changed. And the operator widens the decreasing lower bound *l* to the maximum of minimal thresholds. The widening operator is repeatedly applied on the bounds until all the bounds have no changes. The inferred bounds allows the code generator to determine the smallest fixed-sized integer data type. For example, *short* integers can be used to store the variables in the range of *I*16min and *I*16max.

* 1. *Symbolic Analyzer*

Static bound analysis provides variable bounds for the code generator to choose efficient integer data types without overflows. However, inferring a dynamic array size using a while-loop requires symbolic analysis[[21](#_bookmark38)]. By using pattern-matching techniques, the symbolic analyzer can infer symbolic loop bounds in Whiley program and then apply applicable transformation rules to convert one pattern into another.

* + 1. *Pattern Matching*

**Algorithm 2** Pattern Matching Algorithm

**Input:** WyIL code block

**Output:** Pattern

1: **for** each pattern *p* in available patterns **do**

2: Split code block into several parts to construct the pattern *p*.

3: **if** *p* is successfully constructed **then**

4: **return** *p* // *Return the matching pattern*

5: **end if**

6: **end for**

7: **return** NULL pattern. // *Return no matching patterns.*

Given a code block, the pattern matching analyzer iterates all the patterns and returns the matching one (see Algorithm [2](#_bookmark7)). The symbolic analyzer has been built in with several loop patterns, including *for all* pattern, *while loop* increment or decrement pattern and *build list* pattern.

Table 2

List of Predicates of Loop Pattern

**Predicates Return Value**

*for loop*() *True* if the code block contains a *for all* loop. *while loop*() *True* if the code block contains a *while* loop. *loop var*(*V* ) *True* if loop variable *V* is NOT null.

*init*(*V, Init*) *True* if initial value of *V* (*Init*) is NOT null.

*cond*(*V, OP, B*) *True* if while loop condition (*V* , *OP* and *B*) is NOT null.

*range*(*V, Init, B*) *True* if the range (*V* , *OP* and *B*) is NOT null.

*decr*(*V,* 1) *True* if *V* is only updated with a decrement of one (*V − −*). *incr*(*V,* 1) *True* if *V* is only updated with an increment of one (*V* + +). *list var*(*L*) *True* if the code block contains a *list* variable (*L*).

*list init*(*L*) *True* if *L* is initialized with an empty list.

*list add*(*L, Item*) *True* if *L* is appended with a new item (*Item*).

Each loop pattern consists of a loop variable *V* , initial value *Init*, loop condition (loop bound *B* and comparing operator *OP* ), and the number of loop iterations *loop iters*(*V* ). The pattern predicates are defined in Table [2](#_bookmark8).

**Theorem 3.3 (For All Pattern)** *Let a code block contain a for all loop which iterates over a range and satisfy the below predicates:*

*∃v* [ *for loop*() *∧ range*(*v, Init, B*) ] *⇒ loop iters*(*v*) := *B − Init*

**Theorem 3.4 (While Loop Decrement Pattern)** *Let a code block contain a* *while loop with a decrementing counter and satisfy the below predicates:*

*∃v* [ *while loop*() *∧ loop var*(*v*) *∧ init*(*v, Init*) *∧ cond*(*v, OP, B*) *∧ decr*(*v,* 1) ]

*⇒ loop iters*(*V* ) := *Init − B if OP is >*

*Init − B* +1 *if OP is ≥*

**Theorem 3.5 (While Loop Increment Pattern)** *Let a code block contain a* *while loop with an incrementing counter and satisfy the below predicates:*

*∃v* [ *while loop*() *∧ loop var*(*v*) *∧ init*(*v, Init*) *∧ cond*(*v, OP, B*) *∧ incr*(*v,* 1) ]

*⇒ loop iters*(*v*) := *B − Init if OP is <*

*B − Init* +1 *if OP is ≤*

**Theorem 3.6 (Build List Pattern)** *Let a code block builds up a list L and*

*list size*(*L*) *using a while loop and satisfy the below predicates:*

*∃v∃l* [(*while loop increment pattern*(*v*) *∨ while loop decrement pattern*(*v*) *∧*

*list var*(*l*) *∧ list init*(*l*) *∧ list add*(*l, Item*)]

*⇒ list size*(*l*) := *loop iters*(*v*)

A typical *while-loop* is classified as incremented or decremented loop pattern (see Theorem [3.4](#_bookmark9) and Theorem [3.5](#_bookmark10)). *Build List* pattern is also an extension of while- loop pattern with an additional list and list predicates. As the list is initialized with an empty array and inserted with only one item at each iteration, build list pattern suggests that the list size can be estimated by inferring the number of loop iterations (see Theorem [3.6](#_bookmark11)).

* + 1. *Program Transformation*

Program transformation technique converts one program into another valid one that makes efficient use of limited system resources, such as memory spaces. Currently, the program transformer operates on while-loop pattern and build list pattern.

**Theorem 3.7 (From While-Loop Pattern to For All Pattern)** *A typical while-loop increment or decrement pattern can be transformed into a for all pattern.*

*S*1; *v* := *init*; *S*2; ***while***(*cond*)*{S*3; *update S*4; *} S*5;

*⇒ S*1; *S*2; ***for***(*v* := *init*; *cond*; *update*)*{S*3; *S*4; *} S*5;

*where: Si*=1*...*5 *represents a list of statements. init is the initial assignment of v,* *cond is the loop condition, and update is the increment or decrement of v.*

The transformer changes the while-loop structure to the for all pattern and divides the program into several parts, including pre-loop, loop header, loop body and loop exit. The pre-loop part is split into *S*1 and *S*2 by *init*; the loop body part is grouped into *S*3 and *S*4 by *update*; *S*5 is the loop exit. *S*1 *... S*5 can be preserved and put into the transform program in order without changes (see Theorem [3.7](#_bookmark12)).

**Theorem 3.8 (From Build List Pattern to Build List First Pattern)** *A typical build list pattern can be transformed into a static build list ﬁrst pattern.*

*S*1; *v* := *|ls|*; *r* := []; *S*2;

***while***(*v >* 0) ***where***(*v ≤ |ls|*)*{*

*S*3; *v* + +; *r*+= [*ls*[*v*]]; *S*4;

*} S*5;

*⇒ S*1; *v* := *|ls|*; *rc* := *|ls|*; *r* := *ls*; *rs* := 0; *S*2;

***while***(*v >* 0) ***where***((*v ≤ |ls|*)&&(*rs ≥* 0))*{*

*S*3; *v* + +; *r*[*rs*]= *ls*[*v*]; *rs* + +; *S*4;

*} assert rs* == *rc*; *S*5;

*where: v is the loop variable. ls is the input list. |ls| is the length of ls. r is the output list. rc is the capacity of r. rs is the size of r. Si*=1*...*5 *is the list of statements.*

A build list pattern can be transformed to the *build list ﬁrst* pattern for better performance. Rather than starting with an empty array, the new build list pattern

first copies the input list and then gradually fills each item in the list using the loop. Due to the use of fixed-sized list, the transformed program avoids re-sizing list capacity and gains speedups from in-place update.

# Evaluation

The bound analyzer algorithm is illustrated with three test cases, including nested if-else, while-loop and loop-bounded list programs. Each contains a ’main’ method along with one or two functions.

An external industrial analyzer, **Frama-C** value analyzer[[7](#_bookmark24)], is used to verify our bound analyzer and remove the chances of false alerts from our analyzer because it has been developed and used for more than 10 years and is relatively bug-free. To make use of Frama-C analyzer, the Whiley program is first translated into the provisional C code, using the *long long* type for every integer without type checking, and then perform the range analysis.

* 1. *Test Case: Nested If-Else Program*

1: **function** f(int x) *→* int:

2: **if** x *<* 10:

3: **return** 1

4: **else**:

5: **if** x *>* 10:

6: **return** 2

7: **return** 0

8: **method** main(System.Console sys) *→* **void**: 9: sys.out.println(f(10))

Table 3 Bound Results

|  |  |  |  |
| --- | --- | --- | --- |
| Domain | Naive | Gradual | Frama-C |
| D(f(10)) | 0 | 0 | 0 |
| D(f(11)) | 2 | 2 | 2 |
| D(f(1212)) | 2 | 2 | 2 |
| D(f(-1212)) | 1 | 1 | 1 |

This test case shows that the bound analyzer produces context-sensitive bounds. For example, when the bound analyzer encounters a function call, it passes the bounds of x ([10 *...* 10]) to ’f’ function and then performs the bound inference pro- cedure (see Algorithm [1](#_bookmark5)) to obtain the bounds of return value of ’f’ function and propagate the return bounds back to ’main’ function. The results in Table [3](#_bookmark14) show the bound analyzer can provide exactly the same results as Frama-C.

* 1. *Test Case: While Loop Program*

*// Sum up all the integers from 0 upto the given limit.*

1: **function** f(int limit) *→* int: 2: **requires** limit *<*= 1000000: 3: int i=0

4: int sum=0

5: **while** i*<*limit

6: sum = sum + i

7: i=i+1

8: **return** sum

9: **method** main(System.Console sys)

*→* **void**:

10: sys.out.println(f(50000))

Entry limit:=50,000 i:=0

sum:=0

Loop Header

Apply Widening Operator on ’i’ and ’sum’



Loop Body sum := sum + i i := i + i

Loop Condition i < limit

Loop Exit i >= limit

Exit

return sum

Fig. 1. Control Flow Graph

Table 4

Bound Results of While Loop Program

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Domain | Naive Widening  Operator | Gradual Widening  Operator | Frama-C | Frama-C(slevel=  50,000) |
| *D*(*limit*)entry | [50,000*.. .*50,000] | [50,000*.. .*50,000] | [50,000*.. .*50,000] | [50,000*.. .*50,000] |
| *D*(*i*)loop header | [0*...* 50,000] | [0*...* 50,000] | [0*...* 50,000] | [0*.. .*50,000] |
| *D*(*i*)exit | [50,000*.. .*50,000] | [50,000*.. .*50,000] | [0*.. .*50,000] | [0*.. .*50,000] |
| *D*(*sum*)exit | [0*...* inf] | [0*...* inf] | [*−* inf *...* inf] | 1,249,975,000 |

This test case shows the naive and gradual operator can widen the bounds and force the bound analyzer to terminate the while-loop in *f* function. Table [4](#_bookmark15) shows that on this simple example the naive and gradual widening operators give the same results, but on some more complex loop updates, such as *x* = *x/*2 + 100[[3](#_bookmark20)], the gradual widening operator gives more accurate results than the naive widening operator.

Compared to our bound analyzer, Frama-C provides a more precise range over ’i’ variable and with ’slevel’ option set to 50,000 [4](#_bookmark16) produces concrete results.

4 The *slevel* option allows FramaC to unroll the loop[[5](#_bookmark22)], and thus produce accurate results. But it has the side effect of long computing time and does not help on many programs because it is performing concrete execution rather than abstract interpretation.

* 1. *Test Case: Loop-bound List Program Original* Program

1: **function** reverse([int] ls) *→* [int]:

2: int i = *|*ls*|*

3: [int] r = []

4: **while** i *>* 0 **where** i *≤ |*ls*|*: 5: i = i - 1

6: r = r ++ [ls[i]] 7: **return** r

8: **method** main(System.Console sys)

*→* void:

9: [int] xs = []

10: int limit = 300000000

11: //Create input list using a for loop 12: **for** i **in** 0.. limit:

13: xs = xs ++ [i]

14: [int] rs = reverse(xs) 15: sys.out.println(rs)

*Transformed* Program

1: **function** reverse([int] ls) *→* [int]: 2: int i = *|*ls*|*

3: int r capacity = *|*ls*|*

4: [int] r = ls

5: int r size = 0

6: **while** i *>* 0 **where** i *≤ |*ls*|*

**&&** r size *>*= 0: 7: i = i - 1

8: r[r size] = ls[i]

9: r size = r size +1

10: **assert** r size == r capacity 11: **return** r

12:...

13://main() remains the same.

60

*.*

*.*

85

10

*.*62

10

*.*91

21

*.*05

35

65

57

55

50

45

40

35

Speedup

30

25

20

15

10

106 107 108 108*.*3108*.*48 109

List Size(*log*10)

Fig. 2. Performance of Program Transformation

The original ’reverse’ function is a typical build list pattern. Inside the loop, each single insertion adds one item to the list and thus constraints the list size to the loop bound (see Theorem [3.6](#_bookmark11)). The symbolic analyzer first performs the pat- tern matching. Once the program is matched with build list pattern, it transforms the original program and generates a new program (see Theorem [3.8](#_bookmark13)). The trans- formed program initializes the list with the input list, instead of an empty list, and

imposes bounds on the list size with additional *r capacity* and *r size* variables and an assertion so that out-of-bound errors can be avoided.

The code generator translates the program into C code using *long long* integers rather than arbitrary precision GNU MP[[13](#_bookmark31)] integers because the primitive integers would give more speedups to the generated implementation. Future versions of the code generator will generate short or int types where the inferred bounds of a variable fit within that range, and will reject Whiley programs that contain variables whose bounds are larger than long long.

The benchmark experiment repeatedly runs the generated C code of original and transformed ’reverse’ function for 10 times against 5 kinds of list sizes: 1 million, 10 million, 100 million, 200 million and 300 million. The average execution time ignores the first run and averages the remaining ones. The speedup is the average execution time of the original reverse function, divided by the average execution time of the transformed reverse function.

Benchmarks are run on the i5 2.4GHz CPU machine with 8G memory and the speedup performance is plotted in Figure [2](#_bookmark17). The experiment results show that the program transformation improves the efficiency of the program upto 57 times faster but the speedup reduces to 10 times as the list size increases to 200 million.

# Conclusions

The bound analyzer and symbolic analyzer analyze a Whiley program at byte-code level and provides complementary analysis results for the code generator to produce efficient and safe C code.

In *Nested If-else* test cases, the bound results show that the bound analyzer propagates the bounds for each function call and produces context-sensitive bounds that are as precise as the bounds from the industrial Frama-C. In the *while-loop* test case, the bounds analysis result show that the bound analyzer using the grad- ual widening strategy achieves more precise bound results than Frama-C (without unroll-loop option). In the *Dynamic-sized List* test case, the performance chart shows that the program transformation speeds up the generated C code, but allows the bound analyzer to replace the dynamic list by a fixed-size array with in-place updates. This will be a good basis for generating parallel code in the future.

The symbolic and bound analysis enable the code generator to make use of fixed- sized integers and fixed-sized lists in translation so the generated code avoids integer overflows and out-of-range list errors and reduces memory usage. This makes our project similar to the *w* constraint operator[[8](#_bookmark25)]. But our gradual widening operator increases the precision of bound analysis, and the support for program transforma- tions improves the efficiency of generated code.

The future work includes solving the extra copying of data types and the use of inefficient data structures. And the code generator needs further improvement and code optimization to generate the OpenCL code for parallel execution on GPUs.

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