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correlation of LRD M/G/∞-based processes Flexible adjustment of the short-term

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Abstract

Video represents a larger and larger portion of the traffic in Internet. This traffic is characterized by a high burstiness and a strong short- and long-range correlation, that is very important from the performance point of view. Particularly, the efficient generation of synthetic sample paths is fundamental because real traces are usually of limited length and lack the necessary diversity required to perform such analysis. In this paper, we focus on the M/G/∞ process due to its theoretical simplicity, its flexibility to exhibit both Short- and Long-Range Dependence and its advantages for simulation studies when compared to other types of processes, and we improve the adjustment of the short-term correlation of LRD M/G/∞-based processes adding autoregressive filters.

*Keywords:* Video traffic modeling; M/G/∞ process; Correlation; Synthetic efficient on-line generation

# Introduction

With the increasing popularity of multimedia applications, video data represents a larger and larger portion of the traffic in Internet. Consequently, adequate models of video traffic, characterized by a high burstiness and a strong positive correlation, are very important for the performance evaluation of network architectures and protocols.

In the last decade several traffic studies have convincingly shown the existence of persistent correlations in several kinds of traffic as VBR video [[14](#_bookmark33),[3](#_bookmark23),[23](#_bookmark44),[4](#_bookmark25),[7](#_bookmark28),[32](#_bookmark53)]. These experimental findings stimulated the opening of a new branch in the stochas- tic modeling of traffic, since the impact of the correlation on the performance metrics may be drastic [[17](#_bookmark38),[15](#_bookmark34),[22](#_bookmark43),[16](#_bookmark35),[8](#_bookmark29)]. So, working with classes of stochastic processes that

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can display diverse forms of correlation by making use of few parameters (parsimo- nious modeling) is essential for traffic modeling purposes. Some of these processes are Fractional Gaussian Noise (FGN), Fractional AutoRegressive Integrated Moving

Average (F-ARIMA) and M/G/∞.

The use of these processes in performance evaluation has opened new problems

and research issues in simulation studies, where the efficient generation of synthetic sample paths is fundamental because real traces are usually of limited length and lack the necessary diversity required to perform such analysis.

different stochastic methods [[10](#_bookmark31),[13](#_bookmark36),[18](#_bookmark39),[20](#_bookmark41),[21](#_bookmark42),[9](#_bookmark30),[25](#_bookmark46)]. We focus on the M/G/∞ pro- Several works have been conducted in modeling VBR video traffic, based on cess [[5](#_bookmark26)], due to its theoretical simplicity, its flexibility to exhibit both Short-Range

Dependence (SRD) and Long-Range Dependence (LRD) in a parsimonious way and its advantages for simulation studies [[13](#_bookmark36),[24](#_bookmark45)], such as the possibility of on-line gen- eration (the main drawback of FGN and F-ARIMA processes is that only off-line methods for trace generation are efficient enough to be of practical use [[19](#_bookmark40)]) and the lower computational cost [[27](#_bookmark48)].

M/G/∞-based processes adding autoregressive filters. In this paper, we improve the adjustment of the short-term correlation of LRD

In order to apply a model to the synthetic generation of traces with a corre-

lation structure similar to that of real sequences, a fundamental problem is the estimation of the parameters of the model. Among the methods proposed in the literature [[31](#_bookmark52),[1](#_bookmark22),[30](#_bookmark51)], those based on the Whittle estimator are especially interesting because they permit both fitting the whole spectral density and obtaining confi- dence intervals of the estimated parameters. Moreover, in [[26](#_bookmark47)] we have presented a method based on the prediction error of the Whittle estimator to choose, among

several models for compressed VBR video traffic based on the M/G/∞ process,

the one that gives rise to a better adjustment of the spectral density, and therefore

of the correlation structure, of the traffic to model. In this paper we extend this method in order to deal with the new models. Moreover, we check if the numerically better adjustment is significant or not.

main concepts related to the M/G/∞ process in Section [2](#_bookmark1) and those related to the The remainder of the paper is organized as follows. We begin reviewing the Whittle estimator in Section [3](#_bookmark3). In Section [4](#_bookmark6) we explain the models for VBR video

traffic that we propose and compare in this work. In Section [5](#_bookmark11) we explain how to use the Whittle estimator to check that the new models are more flexible in order to fit the autocorrelation function measured from empirical VBR traces. Finally, concluding remarks and guidelines for further work are given in Section [6](#_bookmark20).

# M/G/∞ process

The M/G/∞ process [[5](#_bookmark26)], *X*, is a stationary version of the occupancy process of an M/G/ queueing system. Let *λ* be the arrival rate to the system, and denote by

∞

*S* the service time distribution, with finite mean value E [*S*].

Considering a discrete time analysis [[29](#_bookmark50)], if the initial number of users is a

Poisson random variable of mean value *λ*E [*S*], and their service times are mutually independent and have the same distribution as the residual life of *S*, *S*^:

Pr *S*^ = *k* = Pr [*S* ≥ *k*]

E [*S*]

then the stochastic process *X* is strict-sense stationary, ergodic, and enjoys the

following properties:

* it has Poissonian marginal distribution, with mean value:

E [*X*]= *λ*E [*S*] *,*

* the autocorrelation function is:

*r*[*k*]= Pr *S > k* ∀*k.*

^

So, the autocorrelation structure of *X* is completely determined by the distri-

bution of *S*.

If the autocorrelation function is summable, Σ∞ *r*[*k*] *<* ∞, the process exhibits

*k*=0

Σ∞

SRD and conversely, if the autocorrelation function is not summable,

*k*=0 *r*[*k*]=

∞ the process exhibits LRD. In particular, the M/G/∞ process exhibits LRD when

*S* has infinite variance, as it happens in heavy-tailed distributions.

lation function of the stationary M/G/∞ process, with integrable *S*, if and only if In [[13](#_bookmark36)] the authors show that an R+-valued sequence *r*[*k*] can be the autocorre- it is decreasing and integer-convex, with *r*[0] = 1 and lim*k*→∞ *r*[*k*] = 0. In such a

case the probability mass function of *S* is given by:

Pr [*S* = *k*]= *r*[*k* − 1] − 2*r*[*k*]+ *r*[*k* + 1]

1 − *r*[1]

∀*k >* 0*,* (1)

and its mean value is:

# Whittle estimator

E [*S*]= 1 *.*

1 − *r*[1]

Let *f θ* (*λ*) be the spectral density function of a zero-mean Gaussian stochastic pro-

cess, *X* = {*X*

2

; *n* = 1*,* 2*,...* } and let *I*

(*λ*) = 1

ΣN−1 *X*

e−*jλi* be the

*n X*N|

2*π*N

*i*=0

*i*+1

periodogram of a sample of size N of *X*. *θ* = {*θk*; *k* = 1*,... ,* M} is the vector of

^ ^ ,

parameters to be estimated.

The approximate Whittle MLE [[31](#_bookmark52)] is the vector *θ* = *θk*; *k* = 1*,... ,* M that

N|

minimizes, for a given sample *x* of size N of *X*, the statistic:

Δ 1

*QX*N| (*θ*) = 2*π*

*π I* N| (*λ*)

*X*

∫

−*π f θ* (*λ*)

d*λ* +

*π*

log *f θ* (*λ*) d*λ*

∫

−*π*

*.* (2)

Moreover, if *θ*o is the real value of *θ*:

Pr *θ*ˆ *θ*o *< ϵ* 1 *ϵ >* 0*,*

− −−−−→ ∀

N→∞

then √N(*θ*ˆ *θ*o) converges in distribution to *ζ*, as N , where *ζ* is a zero-mean Gaussian vector with matrix of covariances C (*θ*o)= 2 D−1 (*θ*o), being:

— → ∞

*π*

1

∫

D*ίj* (*θ*o)=

*∂ ∂*

log *f ϴ* (*λ*)

log *f ϴ* (*λ*) d*λ*

*.* (3)

2*π* −*π ∂θί*

*∂θj*

*ϴ*=*ϴ*o

So, confidence intervals of the estimated values can be obtained.

A simplification of ([2](#_bookmark4)) can be achieved by choosing a special scale parameter *θ*1, such that:

*y*

and:

∫

*f ϴ* (*λ*)= *θ*1 *f ϴ*∗ (*λ*)= *θ*1 *f* ∗ (*λ*) *,*

*π*

log *f ϴ*∗ (*λ*) d*λ* =

−*π*

*π*

log *f* ∗ (*λ*) d*λ* = 0*,*

∫

*y*

−*π*

where *η* = {*θί*; *i* = 1*,* 2*, ...,* M} and *θ*∗ = (1*, η*).

Thus:

∫

*θ*1 = exp

1 *π*

2*π* −*π*

*σ*2

log *f ϴ* (*λ*) d*λ* = *є ,*

2*π*

of the innovations of the AR(∞) representation of the process [[2](#_bookmark24)]: where *σ*2 is the optimal one-step-ahead prediction error, that is equal to the variance

*є*

∞

Σ

*Xί* = *βjXί*−*j* + *ϵί.*

*j*=1

Equation ([2](#_bookmark4)) therefore simplifies to:

∗ ∗ ∫ *π*

*Q* N| (*θ* )= *Q*

*X*

N| (*η*)=

−*π*

*X*

*I* N| (*λ*)

*X*

d*λ* =

(*λ*)

*f*

*ϴ*∗

*π I* N| (*λ*)

*X*

*f* ∗ (*λ*) d*λ.*

∫

−*π*

*y*

Additionally [[2](#_bookmark24)]:

*є*

*σ*^2 = *Q*

*є*

We use *σ*^2 as a measure of the suitability of a model, since smaller values of *σ*^2

*X*N| (*η*^) *.*

*є*

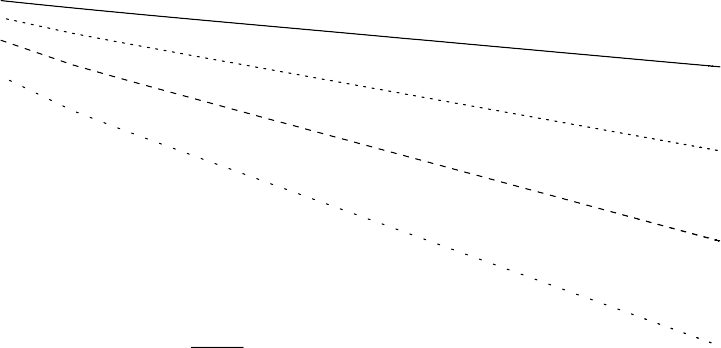
(or *Q*˜ *X*N| (*η*^)) mean better adjustment to the actual correlation of the sample.

# M/G/∞-based LRD models

∗

First, we remember the main features of M/G/∞-based processes able to exhibit LRD that we have proposed in previous works.

1



H = 0*.*9

H = 0*.*7

H = 0*.*8

H = 0*.*6

0.1

0.01

0.001

1e−04

1 4 16 64 256 1024

*k*

Fig. 1. Autocorrelation function of the M/F/∞ process.

* 1. *LRD-1 model: M/F/*∞

This model fixes the autocorrelation function of the resulting M/G/∞ process within the class of exactly second order self-similar processes:

*r*[*k*]= *r*H[*k*] =Δ 1 (*k* + 1)2H − 2*k*2H + (*k* − 1)2H ∀*k,*

2

i.e., the same one that the autocorrelation function of the FGN process.

H is the Hurst parameter [[12](#_bookmark37)]. For 0*.*5 *<* H *<* 1 the process is LRD.

From ([1](#_bookmark2)) the probability mass function of the service time in a M/G/∞ system generating an occupancy process with such correlation function is obtained. We

denote this random variable by F.

In Fig. [1](#_bookmark7) we show the autocorrelation function of the resulting M/F/ process for several values of the parameter H.

∞

* 1. *LRD-2 model: M/S/*∞

service time of the M/G/∞ queueing system. Now, the model considers the heavy-tailed distribution, S, proposed in [[29](#_bookmark50)] as the

The resulting autocorrelation function is:

Δ

m*α*

⎧⎪⎨1 − *α* − 1 *k* ∀*k* ∈ (0*,* m]

*r*[*k*]= *r*{m*,α*}[*k*] = ⎪⎩ 1 m *α*−1

with:

*α* =3 − 2H*,*

⎧(*αr*[1]) 1 *r*

*α*

*k*

∀*k* ≥ m*,*

⎨

*α*−1 ∀

*α* − 1

∈

*α*

[1]

m =

⎩ *α* − *αr*[1] ∀*r*[1] ∈ *α,* 1 *.*

0*,*  1

1

1

H = 0*.*9

H = 0*.*7

H = 0*.*8

H = 0*.*6

0.1

0.01

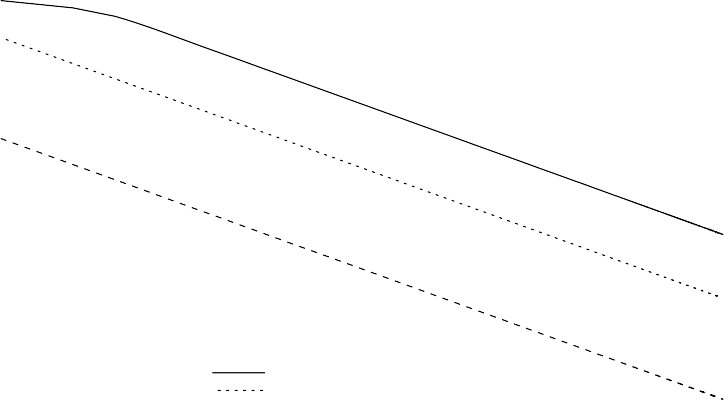
0.001

1

1 4 16 64 256 1024

*k*

0.1



r1 = 0*.*9

r1 = 0*.*5

r1 = 0*.*1

0.01

0.001

1 4 16 64 256 1024

*k*

Fig. 2. Autocorrelation function of the M/S/∞ process for r1 = 0*.*5 (top) and H = 0*.*7 (bottom).

If *α* ∈ (1*,* 2), then H ∈ (0*.*5*,* 1) and Σ∞ *r*{m*,α*}[*k*]= ∞. Hence, in this case, this

*k*=0

correlation structure gives rise to an LRD process.

In Fig. [2](#_bookmark8) we show the autocorrelation function of the resulting M/S/ process for several values of the parameters H and r1 = *r*{m*,α*}[1].

∞

In Fig. [3](#_bookmark9), where we represent the queue of the autocorrelation function of dif- ferent LRD processes with the same value of the Hurst parameter, H = 0*.*7, we can

see that only the M/G/∞ process scales the tail of the queue.

* 1. *On improving the adjustment of the short-term correlation*

In order to improve the adjustment of the short-term correlation of the previous processes, we propose to add an autoregressive filter. Specifically, we focus on the particular case of an AR(1) filter.

0.01

M/S/∞, r1 = 0*.*9

M/S/∞, r = 0*.*5

1

M/S/∞, r = 0*.*1

FGN

1

F-ARIMA

0.001

1e−04

16384 32768 65536

*k*

Fig. 3. Tail of the autocorrelation function of several types of LRD processes.

If *Y* is the M/F/∞ or M/S/∞ original process, the new one is obtained as:

*Xn* = *α*1*Xn*−1 + *Yn,*

with its mean values and covariances related by:

E [*X*]= E [*Y* ] *,*

1 − *α*1

*γX* =

*k*

1

1 − *α*2

1

*k*

1

*α*

1

*γY* +

Σ*ί*=1

*Y*

*k*+*ί*

*γ*

*αί* +

Σ*ί*=1

*Y*

*k*−*ί*

*γ*

*ί .* (4)

It is important to stand out that this will not modify the behavior of the tail

∞

∞

of the autocorrelation function, because its influence decays exponentially, as we observe in equation ([4](#_bookmark10)).

We denote the resulting processes as M/F/∞-AR and M/S/∞-AR (and the models based on them as LRD-1-AR and LRD-2-AR respectively).

If the resulting autocorrelation function is decreasing and convex, we can obtain the service time distribution of the process *X* by means of ([1](#_bookmark2)). In other case, we will have to use an autoregressive filter to generate *X* from *Y* , discarding as many samples as necessary to initialize the generator approximately in steady state.

* 1. *Application of the Whittle estimator to the proposed models*

For the LRD-1 model the spectral density:

+∞

2

Σ

Δ

−2 −1

*f* (*λ*)= *f* H (*λ*) = c*f*

e*jλ* −1

H

|2*πi* + *λ*|

*ί*=−∞

∀*λ* ∈ [−*π, π*]*,*

can be computed efficiently with the Euler’s formula.

For the LRD-2 model, the spectral density is given by [[28](#_bookmark49)]:

Δ m*α*−1 1

*f* (*λ*)= *f* {m*,α*} (*λ*) = Var [*X*]

*f* h (*λ*)+

*α* cos(*λ*) − *α* 2*π*

[m♩

Σ

1

*α*(*m* − *k*)+ *k*

1 m *α*−1

+ 

*π*

*k*=1

cos(*λk*)

m*α* − *α k*

∀*λ* ∈ [−*π, π*]*,*

where *f*h is the spectral density of the FGN process with h = (1−*α*) , scaled by the

2

variance.

For LRD-1-AR and LRD-2-AR models, if *f Y* (*λ*) is the spectral density of the process *Y* , the spectral density of *X* can be obtained as:

*f* (*λ*)= *f Y* (*λ*)

|1 − *α*1 e*jλ* |2

*X*

∀*λ* ∈ [−*π, π*]*.*

# Using the Whittle estimator for the selection of the

best adjustment of the correlation

In this section we explain, by means of two examples, how to use the Whittle estima- tor to check that the new models are more flexible in order to fit the autocorrelation function obtained from empirical VBR traces.

We consider the following empirical traces of the Group of Pictures (GoP) sizes of the MPEG encoded videos: *“Robin Hood”* (T-1) and *“Ice Age I”* (T-2), both of length N = 9000 and available in [[11](#_bookmark32)].

In order to adjust simultaneously the marginal distribution and the autocorrela- tion, as the marginal distribution in all cases is approximately Lognormal, we apply a change of distribution.

In each case, *A* denotes the process we want to generate and *C* the M/G/∞ process from which we start off, that should have a high enough mean value so as

the Poissonian marginal distribution can be considered approximately Gaussian (we

select *σ*2 = *μC* = 106). Moreover, we consider the intermediate process *B* = log(*A*),

*C*

from which we estimate the parameters.

If *A* has Lognormal marginal distribution, then *B* = log(*A*) has Gaussian marginal distribution, with mean, variance and autocorrelation given by [[6](#_bookmark27)]:

*μ*2

*A*

*μB* = log *,*

*μ*2 + *σ*2

*σ*2 = log

*B*

*A A*

2

*σ*

1+ *A ,*

*μ*

2

*A*

LRD-1 4*.*5173 · 10−2 7*.*6194 · 10−2 Model T-1 T-2

LRD-1-AR 4*.*1202 · 10−2 7*.*6157 · 10−2

LRD-2 4*.*1296 · 10−2 7*.*6135 · 10−2

LRD-2-AR 4*.*1287 · 10−2 7*.*6128 · 10−2

c2 Table 1

*σc* with each model.

1 r*B*[*k*]= 2

*σ*

*B*

log 1+

2

*A .* (5)

r*A*[*k*]*σ*

*μ*

2

*A*

The estimations of the parameters of each model, computed via the Whittle estimator, are as follows:

* T-1: *“Robin Hood”*
  + Model LRD-1: H 1.

^

* + Model LRD-1-AR: *α*1 = 0*.*541 and H = 0*.*799.

· Model LRD-2: r^1 = 0*.*892 and H^ = 0*.*869.

^ ^

* + Model LRD-2-AR: *α*1 = 0*.*016, r1 = 0*.*887 and H = 0*.*861.

^ ^ ^

* T-2: *“Ice Age I”*
  + Model LRD-1: H = 0*.*879.

^

* + Model LRD-1-AR: *α*1 = −0*.*042 and H = 0*.*894.

· Model LRD-2: r^1 = 0*.*696 and H^ = 0*.*873.

^ ^

* + Model LRD-2-AR: *α*1 = −0*.*131, r1 = 0*.*749 and H = 0*.*849.

^ ^ ^

The variance (or confidence intervals) of the estimations can be computed from ([3](#_bookmark5)).

*є є*

We use *σ*^2 as a measure of the suitability of a model, since smaller values of *σ*^2

mean numerically better adjustment to the empirical correlation of the sample.

In Table [1](#_bookmark12) we show the estimations of the prediction error.

The results show that increasing the number of parameters leads to smaller prediction errors (better fit of the spectral density) within each family of models.

In Figs. [4](#_bookmark14) and [5](#_bookmark15) we show the adjustment of the autocorrelation function for each empirical trace after the transformation. The case when H 1 in the LRD-1 model is not represented. In this case the strong short term correlation, that we also observe in the high value of *α*1 in the LRD-1-AR model, makes the H parameter tend to be one.

^

Once we have generated a sample of the process *C*, to obtain Lognormal marginal distribution with the mean value and variance of the empirical traces, we apply the inverse transformation:

*σC*

*B*

*B*

*B* = *T* (*C*)= *C* − *μC* *σ*^ + *μ*^ *,*

1

LRD-1-AR

LRD-2-AR

LRD-2

Empirical trace (transf.)

0.9

0.8

0.7

0.6

0.5

0.4

0.3

0.2

0.1

0

0 50 100 150 200

*k*

Fig. 4. Adjustment of the autocorrelation function of the empirical trace “*Robin Hood*” (T-1) with each model, after the transformation.

1

LRD-1-AR

LRD-1

LRD-2-AR

Empirical trace (transf.)

LRD-2

0.9

0.8

0.7

0.6

0.5

0.4

0.3

0.2

0.1

0

0 50 100 150 200

*k*

Fig. 5. Adjustment of the autocorrelation function of the empirical trace “*Ice Age I*” (T-2) with each model, after the transformation.

*A* = exp (*B*)*,*

being *σ*^2 the estimation of the variance of *B* computed with the Whittle estimator,

*B*

that is, considering the autocorrelation structure.

From ([5](#_bookmark13)) we obtain the relationship between the autocorrelation of *A* and *B*:

with:

*A*

r*A*[*k*]=

*μ*^2 exp *σ*^2 r*B*[*k*] − 1

*μ*

*B*

*σ*^2

*A*

*,*

*σ*^2 !

*B*

^*A* = exp *μ*^*B* + 2 *,*

*σ*^2 = exp 2*μ*^*B* + 2*σ*^2 − exp 2*μ*^*B* + *σ*^2 *.*

^

*A*

*B*

*B*

In Figs. [6](#_bookmark16) and [7](#_bookmark17) we represent the adjustment of the autocorrelation function

of each empirical trace, that is, GoP sizes. Again, the case when H represented.

1 is not

An open issue to use this criterion for the selection of the best model is the following:

1

LRD-1-AR

LRD-2-AR

LRD-2

Empirical trace

0.9

0.8

0.7

0.6

0.5

0.4

0.3

0.2

0.1

0

0 50 100 150 200

*k*

Fig. 6. Ajustment of the autocorrelation function of the empirical trace with each model. “*Robin Hood*” (T-1).

1

LRD-1-AR

LRD-1

LRD-2-AR

Empirical trace

LRD-2

0.9

0.8

0.7

0.6

0.5

0.4

0.3

0.2

0.1

0

0 50 100 150 200

*k*

Fig. 7. Ajustment of the autocorrelation function of the empirical trace with each model. “*Ice Age I*” (T-2).

* Increasing the number of parameters supposes a major flexibility in the adjust- ment of the autocorrelation function, and therefore a reduction of the estimation of the prediction error but, is this improvement significant in order to compensate the increase of complexity of the model?
  1. *Hypothesis test over the spectral density*

To solve this question we apply an hypothesis test over the spectral density that we have proposed in a previous work.

Considering two models, 0 and 1, we set out the following null hypothesis:

*H*0 : *f* (*λ*)= *f* 0 (*λ*) *,*

being *f* 0 (*λ*) the spectral density of the process 0, base of the model 0, and the alternative one:

*H*1 : *f* (*λ*)= *f* 1 (*λ*) *,*

being *f* 1 (*λ*) the spectral density of the process 1, base of the model 1.

We define the test statistic as the difference between the estimation of the predic- tion error when the null hypothesis is supposed and the estimation of the prediction

error when the alternative hypothesis is supposed divided by the sample variance:

Δ *σ*^2(*H*0) − *σ*^2(*H*1)

*σ*^2

*E* = *є є .*

We apply the hypothesis test in the classical way, choosing as process 0 the

one with fewer parameters, and we select a critical region with a fixed degree of significance *ϵ*. In this way, if we can not reject the null hypothesis we should consider that the apparently better adjustment of the model with more parameters is not significant, and consequently we should consider the model with fewer parameters. As an example, we apply it to the decision between the models LRD-1 and

LRD-1-AR.

So, we consider as null hypothesis:

and as alternative hypothesis:

0

*H*0 : *f* (*λ*)= *f* Hc (*λ*) *,*

*H*1 : *f* (*λ*)= *f α*c1 *,*Hc (*λ*) *,*

1

being H0 the value obtained for the parameter H using the Whittle estimator and considering the M/F/ process as the base process, and *α*1 and H1 the values obtained for the parameters *α*1 and H using the Whittle estimator and considering

^

∞ ^ ^

the M/F/∞-AR process as base process.

We generate 500 synthetic traces of different sizes:

N ∈ {4096*,* 8192*,* 16384*,* 32768*,* 65536*,* 131072}

of the M/F/∞ process for three different values of the Hurst parameter H ∈

{0*.*6*,* 0*.*75*,* 0*.*9}and we calculate the value of the statistic.

The estimated critical region, *W* , for a degree of significance *ϵ* = 5% is shown

in Table [2](#_bookmark18).

thetic traces of different sizes of the M/F/∞-AR process, and for different com- To verify the power of the test in several concrete cases, we generate 500 syn- binations of H and *α*1. Specifically, N ∈ {4096*,* 8192*,* 16384*,* 32768*,* 65536*,* 131072}, H ∈ {0*.*6*,* 0*.*75*,* 0*.*9} and *α*1 ∈ {−0*.*3*,* 0*.*3}; we check if for the estimated value of the parameter supposing the null hypothesis is rejected, interpolating the border of

H

the critical region. We show the results in Table [3](#_bookmark19).

For each one of the empirical traces we estimate the H parameter assuming the null hypothesis and interpolate the border of the critical region in order to check if the null hypothesis is rejected. The results are shown in Table [4](#_bookmark21).

For T-1 we can observe that the null hypothesis is rejected, so the improvement is significant.

For T-2 the test fails to reject the null hypothesis, so for this trace the nu- merically better adjustment does not compensate the increase of complexity of the model.

N H = 0*.*6 H = 0*.*75 H = 0*.*9

4096 (9*.*82 · 10−4,∞) (9*.*4 · 10−4,∞) (1*.*5 · 10−4,∞)

8192 (4*.*531 · 10−4,∞) (4*.*302 · 10−4,∞) (2*.*881 · 10−4,∞)

16384 (2*.*27 · 10−4,∞) (2*.*12 · 10−4,∞) (1*.*4 · 10−4,∞)

32768 (1*.*251 · 10−4,∞) (1*.*008 · 10−4,∞) (5*.*6 · 10−5,∞)

65536 (5*.*39 · 10−5,∞) (4*.*35 · 10−5,∞) (3*.*8 · 10−5,∞)

131072 (2*.*911 · 10−5,∞) (2*.*231 · 10−5,∞) (1*.*211 · 10−5,∞)

Table 2

Estimated critical region in the test to decide between LRD-1 and LRD-1-AR.

|  |  |  |  |
| --- | --- | --- | --- |
| N | H = 0*.*6 | H = 0*.*75 | H = 0*.*9 |
| 4096 | *α*1 = −0*.*3  500 | *α*1 = −0*.*3  500 | *α*1 = −0*.*3  500 |
| 8192 | 500 | 500 | 500 |
| 16384 | 500 | 500 | 500 |
| 32768 | 500 | 500 | 500 |
| 65536 | 500 | 500 | 500 |
| 131072 | 500 | 500 | 500 |

|  |  |  |  |
| --- | --- | --- | --- |
| N | H = 0*.*6  *α*1 = 0*.*3 | H = 0*.*75  *α*1 = 0*.*3 | H = 0*.*9  *α*1 = 0*.*3 |
| 4096 | 498 | 500 | 500 |
| 8192 | 499 | 500 | 500 |
| 16384 | 499 | 500 | 498 |
| 32768 | 500 | 500 | 500 |
| 65536 | 500 | 500 | 500 |
| 131072 | 500 | 500 | 500 |

Table 3

Power of the test to decide between LRD-1 and LRD-1-AR.

TRACE *E*^ H^0 *W*Hc

0

T-1 2*.*45 · 10−2 ∼ 1 (5*.*21 · 10−4,∞)

T-2 2*.*64 · 10−4 0*.*879 (4*.*48 · 10−4,∞)

Table 4

Application of the test to decide between LRD-1 and LRD-1-AR to the empirical traces.

It is important to highlight that in order to select the best model for each trace the test must be extended to other combinations of models.

# Conclusions and further work

M/G/∞-based processes that we have proposed in previous works adding autore- In this paper we have improved the adjustment of the short-term correlation of LRD gressive filters. Moreover, we have checked if the numerically better adjustment is

significative or not.

The resulting processes enjoy several interesting features as the original ones: highly efficient, on-line generation and the possibility of capturing the whole corre- lation structure in a parsimonious way.

In the same way as we have done with an AR(1) filter, the order can be in- creased, and so the number of parameters of the models, until the improvement is not significant.

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