

[Egyptian Informatics Journal (2014) 15, 97–104](http://dx.doi.org/10.1016/j.eij.2014.03.002)

Cairo University

Egyptian Informatics Journal

[www.elsevier.com/locate/eij](http://www.elsevier.com/locate/eij) [www.sciencedirect.com](http://www.sciencedirect.com/science/journal/11108665)

ORIGINAL ARTICLE

Intuitionistic fuzzy entropy and distance measure based TOPSIS method for multi-criteria decision making



Deepa Joshi, Sanjay Kumar [\*](#_bookmark0)

*G. B. Pant University of Agriculture & Technology, Pantnagar 263145, Uttarakhand, India*

Received 11 July 2013; revised 10 March 2014; accepted 23 March 2014

Available online 16 April 2014

Abstract In this paper, an intuitionistic fuzzy TOPSIS method for multi-criteria decision making (MCDM) problem to rank the alternatives is proposed. The proposed method is based on distance measure and intuitionistic fuzzy entropy. The proposed method also uses conversion theorem to convert fuzzy set to intuitionistic fuzzy set given by Jurio et al. (2010). A real case study is taken as an example to find the ranking of four organizations: Bajaj steel, H.D.F.C. bank, Tata steel and Infotech enterprises using real data. In order to compare the different rankingS, they are applied in a portfolio selection problem. Different portfolios are constructed and are analyzed for their risk and return. It is observed that if the portfolios are constructed using the ranking obtained with proposed method, the return is increased with slight increment in risk.

© 2014 Production and hosting by Elsevier B.V. on behalf of Faculty of Computers and Information,

KEYWORDS

MCDM;

Intuitionistic fuzzy entropy; Distance measure;

TOPSIS;

Portfolio selection

Cairo University.

1. Introduction

In multi-criteria (or attribute) decision making (MCDM/ MADM) problem, a decision maker selects or ranks alterna- tives after qualitative or quantitative assessment of a finite set of interdependent or independent criteria. Desirable alter- native can be chosen by providing preference information in terms of exact numerical value or interval. However, prefer- ence information in real life situation can be assessed in a

\* Corresponding author. Tel.: +91 9412404246.

E-mail addresses: [skruhela@hotmail.com](mailto:skruhela@hotmail.com), [skruhela1971@gmail.com](mailto:skruhela1971@gmail.com) (S. Kumar).

Peer review under responsibility of Faculty of Computers and Information, Cairo University.

**Production and hosting by Elsevier**

qualitative way with vague or imprecise knowledge. In such cases, ambiguity caused by vague or imprecise preference information is a big challenge for decision makers. This fact was a great motivation for researchers to extend MCDM tech- niques in fuzzy environment. Technique for order preference by similarity to an ideal solution (TOPSIS), one of the most known classical MCDM methods was developed by Hwang and Yoon [[1]](#_bookmark28) is based upon the concept that the chosen alternative should be the closest from the positive ideal solution and the farthest from the negative ideal solution.

Chen [[2]](#_bookmark29) developed a methodology for solving MCDM problems using TOPSIS in fuzzy environment. Jahanshahloo et al. [[3]](#_bookmark30) extended the concept of TOPSIS to develop a methodology for solving MCDM problems with fuzzy data. Wang and Chang [[4]](#_bookmark31) developed a fuzzy TOPSIS model to choose optimal initial training aircraft for Taiwan Airforce Academy. Various researchers [[5–7]](#_bookmark32) proposed fuzzy multi- criteria decision analysis methods and their applications to portfolio selection problems.

1110-8665 © 2014 Production and hosting by Elsevier B.V. on behalf of Faculty of Computers and Information, Cairo University. <http://dx.doi.org/10.1016/j.eij.2014.03.002>

In 1986, Atansasov [[8]](#_bookmark20) introduced intuitionistic fuzzy set (IFS) as the generalization of fuzzy set proposed by Zadeh [[9]](#_bookmark20). Because of prominent characteristic of IFS to assign each element a membership degree and non-membership degree, this proved to be an ideal tool to handle the non-determinacy in the system. Later on various operators on IFSs were proposed by De et al. [[10]](#_bookmark20), Grzegorzewski and Mrowka [[11]](#_bookmark21). The preferences in TOPSIS are essentially human judgments based on human perceptions, so intuitionistic fuzzy approach allow for a more accurate description of MCDM problems. Many researchers introduced IFS with TOPSIS to give a hybrid method for MCDM problems. Some methods for MCDM problems based on IFS using entropy weights and linear programming were also given by [[12–15]](#_bookmark22).

Some intuitionistic fuzzy aggregation operators were pro- posed by Xu [[16]](#_bookmark25) to aggregate intuitionistic fuzzy information in MCDM problems. Boran et al. [[17]](#_bookmark26) used weighted average operator to aggregate opinions of decision makers and pro- posed an intuitionistic fuzzy TOPSIS method for supplier selection problem. Recently many researchers [[18–21]](#_bookmark27) extended TOPSIS for MCDM and MADM for interval-valued intui- tionistic fuzzy and hesitant fuzzy information.

In the present study, TOPSIS method proposed by [[1]](#_bookmark28) is ex- tended for intuitionistic fuzzy environment by using construc- tion theorem given by [[22]](#_bookmark33) and distance measure given by [[23]](#_bookmark34). In order to see the relative importance of criteria, weights to each criterion have been given by using intuitionistic fuzzy en- tropy measure proposed by [[24]](#_bookmark35). The proposed intuitionistic fuzzy TOPSIS method is applied to rank the four organiza- tions based on some important criteria. Further, the obtained ranking is implemented in portfolio selection problem to ana- lyze the risk and return in different portfolios.

The rest of the paper is organized as follows: Section [2](#_bookmark1) briefly introduces the basic concepts of fuzzy sets, intuitionistic fuzzy set, fuzzy and intuitionistic fuzzy entropy, and construc- tion theorem given by [[22]](#_bookmark33). The different steps in the proposed intuitionistic fuzzy TOPSIS method are presented in section [3](#_bookmark3). In section [4](#_bookmark8), a real case study taken as an example to illustrate the proposed method. In section [5](#_bookmark11), the proposed method is

compared with conventional TOPSIS and fuzzy TOPSIS by

taking a portfolio selection problem along with some basic

*A* = {(*x*; l*A*(*x*)) : ∀*x* ∈ *X*} (1)

where l*A* is the grade of membership of element *x* in the set *A*.

The greater the amount of l*A* the greater is the truth of the statement that ‘the element *x* belongs to the set *A*.

Definition 2. An IFS *I* on a universe *X* is defined as an object of the following form:

*I* = {< *x*; l*I*(*x*); m*I*(*x*) >: ∀*x* ∈ *X*} (2)

where the functions l*I*(*x*):*X* → [0, 1] and m*I*(*x*):*X* → [0, 1] repre-

ship of an element *x* ∈ *I* c *X* respectively. sent the degree of membership and the degree of non-member-

Definition 3. For every *x* ∈ *X*, p*I*(*x*)=1 — l*I*(*x*) — m*I* (*x*) is called degree of uncertainty or non-determinism hesitation of

IFS set *A* in *X*, with the condition 0 6 l*I*(*x*)+ m*I* (*x*) 6 1.

*2.2. Entropy, fuzzy entropy and intuitionistic fuzzy entropy*

Traditional entropy is derived from the concept of probability and measures the discrimination of criteria when applied in MCDM. De Luca and Termini [[25]](#_bookmark36) first approximated non- probabilistic entropy and introduced some requirements to capture intuitive comprehension of the degree of fuzziness. Many researchers [[26–28]](#_bookmark36) proposed many other ways to view the degree of fuzziness. Scmidt and Kacprzyk [[29,30]](#_bookmark36) also introduced few axioms for distance between the IFSs and non-probabilistic entropy measure for them. Following are the definitions of Shanon entropy, fuzzy entropy and intuition- istic fuzzy entropy.

P

Definition 4. Let D*n* = *P* = (*p* ; . ..... ; *p* ) : *p* P 0; *n pi* = 1} be a set of *n*-complete probability distributions. For any probability distribution *P* = (*p*1, .. . , *pn*) ∈ D*n*, Shanon’s entropy [[31]](#_bookmark37) is defined as follows:

1

*n*

*i*

*i*=1

X*n*

*H*(*P*)=—

*i*=1

*p*(*xi*) log *p*(*xi*) (3)

definitions related to portfolio analysis. Finally in section [6](#_bookmark23), findings and conclusions are presented.

1. Preliminaries

Definition 5. Let *A* be a fuzzy set defined in the universe of discourse *X*. Fuzzy entropy for *A* as follows [[24]](#_bookmark35):

1 X*n*

In this section, basic definitions of fuzzy set, intuitionistic fuzzy set, fuzzy entropy and intuitionistic fuzzy entropy are presented. Conversion theorem for IFS given by [[22]](#_bookmark33) is also

*H*(*A*)=—

*n i*=1

[l*A*(*xi*) log l*A*(*xi*)+ (1 — l*A*(*xi*) log(1 — l*A*(*xi*))]

(4)

discussed in this section.

* 1. *Fuzzy set, intuitionistic fuzzy set (IFS)*

Definition 6. Let *I* be a fuzzy set defined in the universe of dis-

course *X*. Intuitionistic fuzzy entropy is given as follows [[30]](#_bookmark37):

*n*

X

In 1965, Zadeh [[9]](#_bookmark20) first introduced the concept of fuzzy set for modeling the vagueness type of uncertainty. In 1986, Atanasov

[[8]](#_bookmark20) gave the concept of IFS to model the uncertainty because of degree of hesitation in system. Following are some basic defi-

*E*(*I*)= 1 min(l*I*(*xi*); m*I*(*xi*)) + p*I*(*xi*)

*n i*=1 max(l*I*(*xi*); m*I*(*xi*)) + p*I*(*xi*)

*2.3. Construction of IFS from fuzzy set*

(5)

nitions of fuzzy and IFSs found in the literature:

Definition 1. A fuzzy set *A* in the universe of discourse *X* is characterized by membership function l*A*:*X* → [0, 1]. A fuzzy set A is represented by the following order pair:

In this subsection, method of construction of IFS from fuzzy sets given by [[22]](#_bookmark33) is presented. Importance of the method lays on the fact that IFSs are built in such a way indeterminacy index for each element is fixed beforehand. The method is as follows:

Let *AF* ∈ *FSs*(*U*), where *FSs*(*U*) denotes the set of all fuzzy sets in the universal set *U* and let p, d:*U* → [0, 1] be two map-

ping. Then

hesitation of the alternative *Aj* with respect to criterion *Ci*. This step includes the following sub steps.

*I* = {< *ui*; *f*(l*AF*

(*ui*); p(*ui*); d(*ui*)); ∀*ui* ∈ *U* (6)

1. Define the universe of discourse for each alternative against each criterion by taking their actual numerical

is an Atanassov [[8]](#_bookmark20) IFS, where the mapping

*f*: [0, 1]2 · [0, 1] → *L*\* given by

*f*(*x*, *y*, d)= (*f*l(*x*, *y*, d), *f*m(*x*, *y*, d)) where

*f*l(*x*; *y*; d)= *x*(1 — d*y*); *f*m(*x*; *y*; d)= 1 — *x*(1 — d*y*)— d*y* (7)

and

*L*\* = {(*x*; *y*) : (*x*; *y*)∈ [0; 1]× [0; 1] and *x* + *y* 6 1} (8)

values.

1. Construct the appropriate fuzzy sets for each criterion.
2. Apply the method given by [[22]](#_bookmark33) to construct IFSs from fuzzy sets obtained in sub step (ii).

Intuitionistic fuzzy decision matrix can be presented as follows:

(l*A*1(*x*1); m*A*1(*x*1); p*A*1(*x*1)) (l*A*1(*x*2); m*A*1(*x*2); p*A*1(*x*2)) ... (l*A*1(*xn*); m*A*1(*xn*); p*A*1(*xn*))

2 3

(l*A*2(*x*1); m*A*2(*x*1); p*A*2(*x*1)) (l*A*2(*x*2); m*A*2(*x*2); p*A*2(*x*2)) ... (l*A*2(*xn*); m*A*2(*xn*); p*A*2(*xn*))

6 7

*D* = . .

.

.

.

.

664 . .

.

.

. 775

.

(l*Am*(*x*1); m*Am*(*x*1); p*Am*(*x*1)) (l*Am*(*x*2); m*Am*(*x*2); p*Am*(*x*2)) ... (l*Am*(*xn*); m*Am*(*xn*); p*Am*(*xn*))

satisfies the following conditions:

(i) If *y*1 6 *y*2 then p(*f*(*x*, *y*1, d)) 6 p(*f*(*x*, *y*2, d)) for all *x*,

d ∈ [0, 1].

(ii) *f*l(*x*, *y*, d) 6 *x* 6 1 — *f*m(*x*, *y*, d) for all *x* ∈ [0, 1].

Step 2: Determination the weights of criteria.

In this step following intuitionistic fuzzy entropy measure given by [[24]](#_bookmark35) is used to obtain the weight vector *w* = (*w*1, *w*2,

P

... , *w* ), where *w* P 0 and *n w* = 1.

*n*

1

*i*

*i*=1

*i*

1. *f*(*x*, 0, d)= (*x*,1 — *x*).
2. *f*(0, *y*, d) = (0, 1 — d*y*).
3. *f*(*x*, *y*, 0) = (*x*,1 — *x*).
4. p(*f*(*x*, *y*, d)) = d*y*.
5. An algorithm for intuitionistic fuzzy TOPSIS

In this section, step wise algorithm for proposed intuitionistic fuzzy TOPSIS method is presented. Let *A* = {*A*1, *A*2, ... , *Am*} be a set of *m* alternatives and decision maker will choose the best one from *A* according to a criterion set *C* = {*C*1, *C*2, *C*3,

... , *Cn*} which include *n* criteria. Various steps in the proposed intuitionistic fuzzy TOPSIS are as follows:

Step 1: Construction of intuitionistic fuzzy decision matrix.

In this step, intuitionistic fuzzy decision matrix *D* = [*dij*]*n*·*m* of intuitionistic fuzzy value *dij* = (l*ij*, m*ij*) is constructed. Here

*wj* = (*n* — *T*) × (1 — *aj*); *j* = 1; 2; ... ; *n* (9)

*m n*

*aj* = *hij*; *T* = *aj* (10)

X

X

*i*=1 *j*=1

Normalized entropy is given by following expression:

*h* = *Eij* (11)

*ij* max(*E* )

*ij*

where *E* is intuitionistic fuzzy entropy.

Step 3: Construction of weighted intuitionistic fuzzy decision matrix.

After the construction of intuitionistic fuzzy decision ma- trix and determining the weights of each criterion, the weighted intuitionistic fuzzy decision matrix is constructed according to the following expression given by Atanassov [[32]](#_bookmark37):

k*I* = (1 — (1 — l )k; (m )k); 0 < k < 1 (12)

l*ij* and m*ij* are degrees of membership and non-membership of *I* *I*

fuzzy index p*ij* =1 — l*ij* — m*ij* shows the decision maker’s the alternative *Aj* satisfying the criterion *Ci*. The intuitionistic

The weighted intuitionistic fuzzy decision matrix can be defined as follows:

(l*A*1*W*(*x*1); m*A*1*w*(*x*1); p*A*1*w*(*x*1)) .. . (l*A*1*w*(*xn*); m*A*1*w*(*xn*); p*A*1*w*(*xn*))

2 3

(l*A*2*w*(*x*1); m*A*2*w*(*x*1); p*A*2*w*(*x*1)) .. . (l*A*2*w*(*xn*); m*A*2*w*(*xn*); p*A*2*w*(*xn*))

6

6 . 7

*D*\* =

64

.

.

.

(l*Amw*(*x*1); m*Amw*(*x*1); p*Amw*(*x*1)) .. . (l*Amw*(*xn*); m*Amw*(*xn*); p*Amw*(*xn*))

775

Step 4. Intuitionistic fuzzy positive-ideal solution (IFPIS) and intuitionistic fuzzy negative ideal solution (IFNIS).

In TOPSIS method, the evaluation criteria can be catego- rized into two categories, benefit and cost. Let *G* be a collec- tion of benefit criteria and *B* be a collection of cost criteria. According to intuitionistic fuzzy theory and the principle of classical TOPSIS method, IFPIS and IFNIS can be defined as follows:

*A*+ =  *Cj*, ⟨(max l*ij* (*Cj*)/*j* ∈ *G*), (min l*ij* (*Cj*)/*j* ∈ *B*)⟩,

*i*

*i*

Enterprises (*A*4) using real data. Further, the obtained ranking is also implemented in portfolio selection problem.

*4.1. Implementation of proposed intuitionistic fuzzy TOPSIS*

Four alternatives (*A*1, *A*2, *A*3, *A*4) are assessed for their perfor- mance on the basis of following five inter-independent criteria (*c*1, *c*2, *c*3, *c*4, *c*5).

(i) *c*1: Earnings per share(EPS).

(ii) *c*2: Face value.

(min m (*C* )/*j* ∈ *G*), (max m (*C* )/*j* ∈ *B*)⟩ *i* ∈ *m* (13)

*i*

*ij*

*j*

*i*

*ij*

*j*

1. *c*4: Dividend.
2. *c*5: P/E (Price-to-earnings) ratio.

(iii) *c*3: P/C (Put–Call) Ratio.

*A*— = *Cj*, ⟨(min l*ij* (*Cj*)/*j* ∈ *G*), (max l*ij* (*Cj*)/*j* ∈ *B*)⟩,

*i*

*i*

First two criteria belong to benefit criteria, i.e., high value

indicates good growth prospects and last three belong to

(max m*ij*(*Cj*)/*j* ∈ *G*), (max m*ij*(*Cj*)/*j* ∈ *B*)⟩

*i*

*i*

*i* ∈ *m*

(14)

non-beneficial criteria, i.e., low value indicates good growth

Step 5: Calculation of distance measures from IFPIS and IFNIS.

prospects. Therefore, *c*1, *c*2 ∈ *G* and *c*3, *c*4, *c*5 ∈ *B*. Actual

To measure distance of each alternative *Ai* from IFPIS and IFNIS, the intuitionistic separation measure given by [[23]](#_bookmark34) is used and expressed by following expression:

*n*

*S*+ = max (|l + (*xi*)— l (*xi*)|, |m*A*+ (*xi*)— m*B*(*xi*)|) (15)

X

*i A B*

*i*=1

X

*n*

*S*— = max (|l*A*— (*xi*)— l*B*(*xi*)|, |m*A*— (*xi*)— m*B*(*xi*)|) (16)

*i*

*i*=1

Step 6. Calculation of relative closeness coefficient (CC)

Finally, relative closeness coefficient of each alternative with respect to intuitionistic fuzzy ideal solutions is computed by using following expression and rank the preference order of all alternatives.

*S*—

*Cj* = *S*+ + *S*— , *j* = 1, 2, 3, ... , *m* (17)

The larger value of relative closeness coefficient *i*ndicates that an alternative is closer to IFPIS and farther from IFNIS simultaneously. Therefore, the ranking order of all the alterna- tives can be determined according to the descending order of relative coefficient values. The most preferred alternative is the one with the highest value.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 4. A real case study  Assessment and evaluations of the organizations based on | Table 2 Fuzzy decision matrix.  *c*1 *c*2 *c*3 *c*4 *c*5 | | | | | |
| some financial criteria is very important in financial system. | *a*1 | 0.2846 | 0.75 | 0.234 | 0.23 | 0.24 |
| In this section, proposed intuitionistic fuzzy TOPSIS method | *a*2 | 0.318 | 0.248 | 0.766 | 0.097 | 0.76 |
| is applied in the ranking of four organizations, Bajaj Steel | *a*3 | 0.759 | 0.75 | 0.315 | 0.32 | 0.278 |
| (*A*1), H.D.F.C. Bank (*A*2), Tata Steel (*A*3) and Infotech | *a*4 | 0.241 | 0.437 | 0.41 | 0.155 | 0.394 |
|  |  |  |  |  |  |  |

numerical values of these five criteria of the four alternatives

are retrieved from [http://www.moneycontrol.com](http://www.moneycontrol.com/) from date 21.9.2012 to 27.9.2012 and average of this information is placed in following table:

Step 1: Crisp numerical values given in [table 1](#_bookmark6) are fuzzified by defining the following fuzzy sets for each criteria and fuzzy decision matrix ([table 2](#_bookmark9)) is obtained.

*A*1 = 0.2846/20.5 + 0.75/10 + 0.234/2.21 + 0.23/2 + 0.24/4.8

*A*2 = 0.318/23.31 + 0.248/2 + 0.766/24.7 + 0.097/0.67 + 0.76/27 *A*3 = 0.759/60.06 + 0.75/10 + 0.315/5.65 + 0.32/2.92 + 0.278/6.5 *A*4 = 0.241/16.86 + 0.437/5 + 0.41/9.7 + 0.155/1.25 + 0.394/11.4

Following intuitionistic fuzzy decision matrix ([Table 3](#_bookmark10)) is constructed the fuzzy decision matrix given by [table 2](#_bookmark9) and construction theorem given by Eqs. [6 and 7](#_bookmark2).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| Table 1 Average of actual numerical value of criteria. | | | | | |
| Alternatives | *c*1 | *c*2 | *c*3 | *c*4 | *c*5 |
| *a*1 | 20.50 | 10.00 | 2.21 | 2.00 | 4.80 |
| *a*2 | 23.31 | 2.00 | 24.7 | 0.67 | 27.00 |
| *a*3 | 60.06 | 10.00 | 5.65 | 2.92 | 6.50 |
| *a*4 | 16.86 | 5.00 | 9.70 | 1.25 | 11.40 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 3 Intuitionistic fuzzy decision matrix. | | | | | |
| Alternatives | *c*1 | *c*2 | *c*3 | *c*4 | *c*5 |
| *a*1 | (0.23, 0.587) | (0.61, 0.2) | (0.192, 0.63) | (0.22, 0.75) | (0.196, 0.62) |
| *a*2 | (0.26, 0.554) | (0.2, 0.61) | (0.63, 0.192) | (0.094, 0.875) | (0.62, 0.196) |
| *a*3 | (0.62, 0.197) | (0.61, 0.2) | (0.259, 0.56) | (0.31, 0.66) | (0.227, 0.59) |
| *a*4 | (0.197, 0.62) | (0.36, 0.454) | (0.337, 0.484) | (0.15, 0.82) | (0.322, 0.50) |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 4 Weighted intuitionistic fuzzy decision matrix. | | | | | |
| Alternatives | *c*1 | *c*2 | *c*3 | *c*4 | *c*5 |
| *a*1 | (0.058, 0.885) | (0.172, 0.725) | (0.042, 0.87) | (0.31, 0.965) | (0.047, 0.90) |
| *a*2 | (0.067, 0.873) | (0.044, 0.91) | (0.18, 0.72) | (0.0123, 0.98) | (0.192, 0.70) |
| *a*3 | (0.20, 0.69) | (0.172, 0.725) | (0.058, 0.89) | (0.045, 0.95) | (0.055, 0.89) |
| *a*4 | (0.049, 0.896) | (0.085, 0.854) | (0.079, 0.865) | (0.020, 0.975) | (0.082, 0.86) |
|  |  |  |  |  |  |

Step 2: The following weights of each criterion are calcu- lated using entropy measure for IFS:

*w*1 = 0.230, *w*2 = 0.200, *w*3 = 0.200, *w*4 = 0.125,

*w*5 = 0.220

Step 3: Weighted intuitionistic fuzzy decision matrix

([Table 4](#_bookmark12)) is obtained by multiplying intuitionistic fuzzy values with weight of the each criterion.

Step 4: Intuitionistic fuzzy positive-ideal solution and intui- tionistic fuzzy negative-ideal solution are calculated by using Eqs. [(13) and (14)](#_bookmark4) and are as follows:

A+ = {(0.20, 0.69)(0.172, 0.725)(0.042, 0.89)(0.0123, 0.98)(0.047, 0.90)}

A— = {(0.049, 0.896)(0.044, 0.91)(0.18, 0.72)(0.045, 0.95)(0.192, 0.90)}

Step 5: Intuitionistic separation measure of each alternative

from the positive-ideal solution and negative ideal solution are calculated using Eqs. [(16) and (17)](#_bookmark5) and are given by [Table 5](#_bookmark13). Step 6: Relative closeness coefficients are calculated using

Eq. [(17)](#_bookmark7) and are given in [Table 6](#_bookmark14).

According to descending order of relative closeness coeffi- cients values four alternatives are ranked as *a*3 > *a*1 > *a*4 > *a*2. The ranking of these four alternatives is also obtained by TOPSIS and fuzzy TOPSIS method proposed by [[1,3]](#_bookmark28) and shown in following table ([Table 7](#_bookmark15)):

It is found that there is no conflict in the preference order- ing of all the alternatives by fuzzy TOPSIS method and proposed intuitionistic fuzzy TOPSIS method. There is only one conflict is found in deciding the preference ordering of *a*1 and *a*4.

1. An application of intuitionistic fuzzy TOPSIS in portfolio selection

To compare the performance of proposed intuitionistic fuzzy TOPSIS with TOPSIS [[1]](#_bookmark28) and fuzzy TOPSIS method [[3]](#_bookmark30), a portfolio selection problem is taken. Basic definitions related to portfolio and portfolio analysis are also given in this section.

* 1. *Basic definitions related to portfolio analysis*

Some basic definitions related to portfolio analysis given by Markowitz [[33,34]](#_bookmark37) are presented in this subsection.

Definition 7. An asset is a valuable economic entity from which the future economic benefits are expected to flow to the owner of the asset.

Definition 8. Return on an asset is an indicator of gain/loss in the investment of an asset in financial market. If current price of an asset is *A*(0) and after *T* time period, the asset is sold off at amount *A*(*T*), then return *r*, on asset for *T* time period is given by following:

*A*(*T*)— *A*(0)

|  |  |  |
| --- | --- | --- |
| Table 5 Intuitionistic separation measures.  Alternatives *S*+ *S*—  *i* *i* | | |
| *a*1 | 0.2377 | 0.5060 |
| *a*2 | 0.7380 | 0.2557 |
| *a*3 | 0.0587 | 0.698 |
| *a*4 | 0.4170 | 0.336 |
|  |  |  |

*r* = *A*(0) (18)

Definition 9. Risk is defined as the degree of uncertainty of return on an asset. It signifies the possibility of loss in the investment. The risk can be either zero (risk free asset) or posi- tive (risky asset). There are two kinds of risk associated with risky asset (i) systematic risk which is concerned with money, capital market, credit and fiscal policy and economic policy which govern the market economy; (ii) unsystematic risk is concerned with outcome of unfavorable litigation, sudden dis- covery of deficiencies in a product of a company and natural catastrophe.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Table 6 Relative closeness coefficients. | | | | |
| Alternatives | *a*1 | *a*2 | *a*3 | *a*4 |
| *Ci* | 0.680 | 0.257 | 0.922 | 0.446 |
|  |  |  |  |  |

Definition 10. A portfolio is a collection of two or more assets,

Table 7 Ranking order of alternatives for different methods.

Method

TOPSIS proposed by [[11]](#_bookmark21) Fuzzy TOPSIS proposed by [[12]](#_bookmark22)

Ranking

Best alternative

*a*3 > *a*4 > *a*1 > *a*2 *a*3

*a*3 > *a*1 > *a*4 > *a*2 *a*3

Proposed intuitionistic fuzzy *a*3 > *a*1 > *a*4 > *a*2 *a*3

TOPSIS

(*x*1, *x*2, *x*3, ... , *xn*), where *xi* ∈ *R* is the number of units of say *a*1, *a*2, *a*3, ... , *an*, represented by an ordered *n*-tuple the asset *ai*(*i* = 1, 2, 3, ... , *n*) owned by the investor.

Definition 11. Let *Vi*(0) and *Vi*(*t*) be the values of the *i*th asset at time *T* = 0 and *T* = *t*, respectively. Value of portfolio (*x*1, *x*2, *x*3, ... , *xn*) at *T* = 0 and *T* = *t*, is denoted by *V*(0) and *V*(*t*) and defined as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Table 8 Assets prices (in Rs.) at different equal intervals. | | | | |
| Date of observation | Bajaj Steel (*a*1) | H.D.F.C. Bank (*a*2) | Tata Steel (*a*3) | Infotech Enterprise (*a*4) |
| 22/09/2012 | 92.50 | 607 | 396 | 196.50 |
| 23/09/2012 | 92.50 | 607 | 396 | 196.50 |
| 24/09/2012 | 93.60 | 626 | 410 | 195 |
| 25/09/2012 | 103.30 | 634 | 406.05 | 194 |
| 26/09/2012 | 101.05 | 639.25 | 398.55 | 193 |
| 27/09/2012 | 101 | 634.60 | 396.90 | 193 |
| 28/09/2012 | 100 | 631 | 400.05 | 191 |
|  |  |  |  |  |

*V*(0)=

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X*n* X*n* | Table 9 Weight matrix of the assets by proposed method.  Date of observation *a*1 *a*2 *a*3 *a*4 | | | | |
| Definition 12. The weight *wi* of the asset *ai* is the percentage of | 22/09/2012 | 0.0969 | 0.212 | 0.554 | 0.137 |
| the value of the asset in the portfolio (*x*1, *x*2, *x*3, ... , *xn*) at | 23/09/2012 | 0.0969 | 0.212 | 0.554 | 0.137 |
| *t* = 0, i.e., | 24/09/2012 | 0.0956 | 0.213 | 0.558 | 0.133 |
|  | 25/09/2012 | 0.105 | 0.214 | 0.549 | 0.131 |

*i*=1

*xiVi*(0) and *V*(*t*)=

*i*=1

*xiVi*(*t*) (19)

*w* = *xiVi*(0)

(*i* = 1, 2, 3, .. . , *n*) (20)

*i n*

*x V* (0)

X

*i*=1

*i*

*i*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 26/09/2012 | 0.103 | 0.219 | 0.545 | 0.132 |
| 27/09/2012 | 0.104 | 0.218 | 0.545 | 0.133 |
| 28/09/2012 | 0.103 | 0.217 | 0.549 | 0.131 |

It can be observed that

*w*1 + *w*2 + *w*3 + ... *wn* = 1 (21)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Table 10 Weight matrix of the assets by TOPSIS method.  Date of observation *a*1 *a*2 *a*3 *a*4 | | | | |
| 22/09/2012 | 0.0969 | 0.212 | 0.554 | 0.137 |
| 23/09/2012 | 0.0969 | 0.212 | 0.554 | 0.137 |
| 24/09/2012 | 0.0665 | 0.206 | 0.5397 | 0.1925 |
| 25/09/2012 | 0.0678 | 0.208 | 0.5331 | 0.191 |
| 26/09/2012 | 0.06704 | 0.212 | 0.545 | 0.192 |
| 27/09/2012 | 0.06726 | 0.2113 | 0.5286 | 0.1927 |
| 28/09/2012 | 0.103 | 0.217 | 0.549 | 0.131 |
|  |  |  |  |  |

* 1. *Portfolio analysis*

We construct different portfolios with the assets of companies (*a*1, *a*2, *a*3, *a*4). Further the constructed portfolios are analyzed for their risk and return.

For analyze the portfolio the assets prices of four compa- nies (*a*1, *a*3, *a*4, *a*2) are taken form [www.moneycontrol.com](http://www.moneycontrol.com/) at different days and given in following table ([Table 8](#_bookmark16)) :

Let us assume that total number of units of assets cannot exceed by 100 in the portfolio (*x*1, *x*2, *x*3, *x*4). According to our ranking *a*3 > *a*1 > *a*4 > *a*2, let us take *x*1 = 30, *x*2 = 10, *x*3 = 40, *x*4 = 20, where *xi* represents the number of units of assets of a company *ai*.

By using the Eq. [(20)](#_bookmark17), we find weight of each company cor- responding to each observation and are placed in [Table 9](#_bookmark18) as follows:

In order to follow ranking proposed by TOPSIS method [[1]](#_bookmark28), i.e.,

*a*3 > *a*4 > *a*1 > *a*2, we take *x*1 = 20, *x*2 = 10, *x*3 = 40, *x*4 = 30 and using Eq. [(20)](#_bookmark17), the weight of the asset of each company on different observations are calculated and are placed in [Table 10](#_bookmark19).

We also determine ranking by fuzzy TOPSIS method, which is same as given by our proposed method. So the weight matrix of the assets by fuzzy TOPSIS method will be same as given in [Table 10](#_bookmark19). Return series is formed with the help of weight matrix of companies. Return series is the matrix of or- der *n* · *m*, where *n* is number of observations and *m* is number of assets with equally spaced incremental return observations. The first row is the oldest observation, and the last row is the most recent. The return series can be formed by multiplying the assets prices with corresponding weights. Matrices *A*, *B* and *C* represent the return series for proposed, TOPSIS and fuzzy TOPSIS method respectively:

8.95

6

26.92

7

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 28.963  8.963  6 | 128.7  128.7  133.3  135.68  139.99 | 219.4  219.4  228.78  222.92  217.21 | 26.92 3  25.93  7 | 2 5.70 124.40  5.70 128.40  6 | 211.46 39.10 3  221.315 37.546  7 | 28.963  8.963  6 | 128.7  128.7  133.3  135.68  139.99 | 219.4  219.4  228.78  222.92  217.21 | 26.92 3  26.92  25.93 7  25.414  7 |
| 10.5 | 138.34 | 216.3 | 25.7 | 6.7900 134.09 | 209.815 37.190 | 10.5 | 138.34 | 216.3 | 25.7 |
|  | 136.937 | 219.63 |  |  |  |  | 136.937 | 219.63 |  |

5.767 128.956

6

211.46 39.10

7

6 7

8.95

6

*A* = 10.84

6 10.4

6

4 10.3

25.414

25.47

7

25.02 75

*B* = 7.0047 131.870 216.460 37.050

6.7700 135.500 210.700 37.056

64 6.6600 132.50 214.02 36.480 75

*C* = 10.84

10.4

6

64 10.3

25.47

25.02 75

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 11 Portfolio risk and return. | | | | | |
| Portfolio | By proposed method & fuzzy  Portfolio risk | TOPSIS method [[3]](#_bookmark30)  Portfolio return |  | By TOPSIS method [[1]](#_bookmark28)  Portfolio risk | Portfolio return |
| Portfolio 1 | 0.1384 | 35.894 |  | 0.1336 | 34.4186 |
| Portfolio 2 | 0.3614 | 64.8725 |  | 0.2157 | 64.2829 |
| Portfolio 3 | 0.8725 | 95.9662 |  | 0.7087 | 94.1472 |
| Portfolio 4 | 1.4197 | 217.1047 |  | 1.3688 | 214.5882 |
| Portfolio 5 | 1.9731 | 158.2431 |  | 2.0403 | 153.87587 |
| Portfolio 6 | 2.5315 | 189.3816 |  | 2.7148 | 183.7400 |
| Portfolio 7 | 3.8924 | 220.5200 |  | 3.7753 | 213.6043 |
| Coeﬃcient of correlation (*r*) = 0.843016 Coeﬃcient of correlation (*r*) = 0.835655 | | | | | |

Using above return series, we construct four portfolios. Risk and return associated with each portfolio are placed in [Table 11](#_bookmark24).

1. Conclusion

In this paper, an intuitionistic fuzzy TOPSIS method for MCDM problems has been proposed. The proposed method uses the construction theorem given by [[22]](#_bookmark33) and the weight of each criterion is calculated using intuitionistic fuzzy entropy measure proposed by [[24]](#_bookmark35). A real case study is taken to rank the four organizations based on five criteria using the proposed method. Further, in order to compare the ranking obtained by proposed method with the methods given by [[1,3]](#_bookmark28), a portfolio selection problem is considered. By retrieving the stock prices of four organizations taken in the study from www.moneycon- trol.com and considering the different rankings, different port- folios are constructed and analyzed for their risk and returns. [Table 11](#_bookmark24) shows the return of different portfolios with associ- ated risk and coefficient of correlation between them. As there is no conflict in the ranking the alternatives with the fuzzy TOPSIS method given by [[3]](#_bookmark30) and proposed intuitionistic fuzzy TOPSIS method, return of each portfolio and involved risk is same. But, when we compare the proposed method with the TOPSIS method given by [[1]](#_bookmark28), we find the return of each port- folio is higher with slightly high risk involved in each portfolio. Coefficient of correlation (*r* = 0.843016) also confirms the bet- ter relationship between portfolio risk and return in different portfolios constructed using the ranking obtained by proposed method.

References

1. [Hwang CL, Yoon KS. Multiple attribute decision making:](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0015) [methods and applications. Berlin: Springer-Verlag; 1981](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0015).
2. [Chen CT. Extension of the TOPSIS for group decision making](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0020) [under fuzzy Environment. Fuzzy Sets Syst 2000;114(1):1–9](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0020).
3. [Jahanshahloo GR, Hosseinzadeh LF, Izadikhah M. Extension of](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0025) [the TOPSIS method for decision-making problems with fuzzy](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0025) [data. Appl Math Comput 2006;181(2):1544–51](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0025).
4. [Wang TC, Chang TH. Application of TOPSIS in evaluating initial](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0030) [training aircraft under a fuzzy environment. Exp Syst Appl](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0030) [2007;33(4):870–80](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0030).
5. [Fatma T, Beyza A. Fuzzy portfolio selection using fuzzy analytic](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0035) [hierarchy process. Inform Sci 2009;179(1–2):53–69](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0035).
6. [Kuo MS, Tzeng GW, Huang WC. Group decision-making based](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0040) [on concepts of ideal and anti-ideal points in fuzzy environment.](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0040) [Math Comp Model 2007;45(3–4):324–39](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0040).
7. [Xidonas P, Askounis D, Psarras J. Common stock portfolio](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0045) [selection: a multiple criteria decision making methodology and an](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0045) [application to the Athens Stock Exchange. Oper Res Int J](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0045) [2009;9:55–79](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0045).
8. [Atanassov K. Intuitionistic fuzzy sets. Fuzzy Sets Syst](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0050) [1986;20(1):87–96](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0050).
9. [Zadeh LA. Fuzzy sets. Inform Control 1965;8(3):338–56](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0055).
10. [De SK, Biswas R, Roy AR. Some operations on intuitionistic](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0060) [fuzzy sets. Fuzzy Sets Syst 2000;114(3):477–84](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0060).
11. [Grzegorzewski P, Mrowka E. Some notes on (Atanassov’s)](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0065) [intuitionistic fuzzy sets. Fuzzy Sets Syst 2005;156(3):492–5](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0065).
12. Hung C-C, Chen L-H. A fuzzy TOPSIS decision making model with entropy weight under intuitionistic fuzzy environment. In: Proceedings of the international multi-conference of engineers and computer scientists (IMECS-2009), vol. 1; 2009. p. 13–6.
13. [Lin L, Yuan XH, Xia ZQ. Multicriteria fuzzy decision-making](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0070) [methods based on intuitionistic fuzzy sets. J Comp Syst Sci](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0070) [2007;73(1):84–8](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0070).
14. [Liu HW, Wang GJ. Multi-criteria decision-making methods](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0075) [based on intuitionistic fuzzy sets. Euro J Operat Res 2007;179(1):](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0075) [220–33](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0075).
15. [Yue Z. Extension of TOPSIS to determine weight of decision](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0080) [maker for group decision making problems with uncertain](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0080) [information. Exp Syst Appl 2012;39(7):6343–50](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0080).
16. [Xu Z S. Intuitionistic fuzzy aggregation operators. IEEE Trans](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0085) [Fuzzy Syst 2007;15:1179–87](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0085).
17. [Boran FE, Genc S, Kurt M, Akay D. A multi-criteria intuition-](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0090) [istic fuzzy group decision making for supplier selection with](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0090) [TOPSIS method. Exp Syst Appl 2009;36(8):11363–8](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0090).
18. [Chai J, Liu JNK, Xu Z. A rule-based group decision model for](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0095) [warehouse evaluation under interval-valued intuitionistic fuzzy](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0095) [environments. Exp Syst Appl 2013;40(6):1959–70](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0095).
19. [Park JH, Park Y, Kwun YC, Tan Xuegong. Extension of the](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0100) [TOPSIS method for decision making problems under interval-](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0100) [valued intuitionistic fuzzy environment. Appl Math Model](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0100) [2011;35(5):2544–56](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0100).
20. [Xu Z, Zhang X. Hesitant fuzzy multi-attribute decision making](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0105) [based on TOPSIS with incomplete weight information. Knowl-](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0105) [Based Syst 2013;52:53–64](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0105).
21. [Zhang H, Yu L. MADM method based on cross-entropy and](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0110) [extended TOPSIS with interval-valued intuitionistic fuzzy sets.](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0110) [Knowl-Based Syst 2012;30:115–20](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0110).
22. Jurio A, Paternain D, Bustince H, Guerra C, Beliakov G. A construction method of Atanassov’s intuitionistic fuzzy sets for image processing. In: Proceedings of the fifth IEEE conference on intelligent systems; 2010. p. 337–42.
23. [Grzegorzewski P. Distances between intuitionistic fuzzy sets and/](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0115) [or interval-valued fuzzy sets based on the Hausdoff metric. Fuzzy](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0115) [Sets Syst 2004;149(2):319–28](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0115).
24. [Chen TY, Li CH. Determining objective weights with intuition-](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0120) [istic fuzzy entropy measures : A comparative analysis. Inform Sci](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0120) [2010;180(21):4207–22](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0120).
25. [De LA, Tremini S. A definition of a non probabilistic entropy in](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0125) [the setting of fuzzy set theory. Inform Control 1972;20(4):301–12](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0125).
26. [Kaufmann A. Introduction to the theory of fuzzy subsets.](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0130) [Fundamental theoretical elements, vol. 1. Academic Press; 1975](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0130).
27. [Kosko B. Fuzzy entropy and conditioning. Inform Sci 1986;40(2):](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0135) [165–74](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0135).
28. [Yager RR. Measures of entropy and fuzziness related to aggre-](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0140) [gation operators. Inform Sci 1995;82(3):147–66](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0140).
29. [Szmidt E, Kacprzyk J. Distances between intuitionistic fuzzy sets.](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0145) [Fuzzy Sets Syst 2000;114(3):;505–518](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0145).
30. [Szmidt E, Kacprzyk J. Entropy for intuitionistic fuzzy sets. Fuzzy](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0150) [Sets Syst 2001;118(3):467–77](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0150).
31. [Shannon CE. A mathematical theory of communication. Bell Syst](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0155) [Tech J 1948;27:379–423](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0155).
32. [Atanassov K. New operations defined over the intuitionistic fuzzy](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0160) [sets. Fuzzy Sets Syst 1999;61(2):137–42](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0160).
33. [Markowitz HM. Portfolio selection. J Finan 1952;7(1):77–91](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0165).
34. [Markowitz HM. Foundations of portfolio theory. J Finan](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0170) [1991;469(2):469–71](http://refhub.elsevier.com/S1110-8665(14)00014-0/h0170).