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# Asymptotic Stability of a Class of Fourth Order Nonlinear Systems

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**Abstract**

In this paper, two asymptotic stability criterions of the trivial solutions to a class of forth order nonlinear systems are improved, and an error which exists in the proof of Theorem 2 of the paper [1] is pointed out.

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##### Introduction

Lyapunov stability is named after Aleksandr Lyapunov, a Russian mathematician who published his book "The General Problem of Stability of Motion" in 1892[1]. Lyapunov was the first to consider the modifications necessary in nonlinear systems to the linear theory of stability based on linearizing near a point of equilibrium. His work, initially published in Russian and then translated to French, received little attention

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for many years. Interest in it started suddenly during the Cold War (1953-1962) period when the so-called "Second Method of Lyapunov" was found to be applicable to the stability of aerospace guidance systems which typically contain strong nonlinearities not treatable by other methods. A large number of publications appeared then in the control and systems literature. More recently the concept of Lyapunov exponent (related to Lyapunov's First Method of discussing stability) has received wide interest in connection with chaos theory. Lyapunov stability methods have also been applied to finding equilibrium solutions in traffic assignment problems following the work by MJ Smith and MB Wisten.

Various types of stability may be discussed for the solutions of differential equations describing dynamical systems. The most important type is that concerning the stability of solutions near to a point of equilibrium. This may be discussed by the theory of Lyapunov. In simple terms, if all solutions of the dynamical system that start out near an equilibrium point  and stay near  forever, then  is Lyapunov stable. More strongly, if  is Lyapunov stable and all solutions that start out near  and converge to , then is asymptotically stable. The notion of exponential stability guarantees a minimal rate of decay, i.e., an estimate of how quickly the solutions converge. The idea of Lyapunov stability can be extended to infinite-dimensional manifolds, where it is known as structural stability, which concerns the behavior of different but "nearby" solutions to differential equations. Input-to-state stability (ISS) applies Lyapunov notions to systems with inputs.

In the paper[3], Liapunov function for a fourth order linear system with constant coefficients is given by means of the Barbashin’s formula, and then stability of the trivial solutions to a class of forth order nonlinear systems is studied by applying the comparison method. But, we think it necessary to point out the following two problems:

First, the two stable theorems in [2] are treaded

*yz*  0 or (*az*  *u*)sgn *y*  0,(*dx*  *cy*)sgn *z*  0 as

stability condition, therefore the given conditions in [2] cannot ensure that the function *V* (*x*, *y*, *z*,*u*) is positive in arbitrarily small neighborhood of the origin.

Next, an error exists in the proof of theorem 2 in [2]. Because we cannot assert that

(*a*(

*y* )2

#### *z*

* *c*) *z*

 (*ah* 2

* *c*)*l*

by *a*  0, *c*  0 and *k*  *nl*

Theorem 2.

in the region

*y*  *k*, *z*  *l* , the paper [2] hasn’t finished the proving of

In this Paper, we improve upon the two asymptotic stability criterions in [2] and obtain the following conclusions.

**Theorem 1**. Consider the fourth order nonlinear equation

*x*(4)  *ax*(3)  *bx*(2)  *cx*(1)  *f* (*x*)  0

or the equivalent system of (1)

(1)

 *x*(1)  *y*, *y*(1)  *z*, *z* (1)  *u*,

 (1)

*u*

  *f* (*x*)  *cy*  *bz*  *au*

(2)

where constants *a*  0,*b*  0, *c*  0 and *f* (0)  0.

If there exists a constant *h*  0 , the function satisfies the following conditions

*f* (*x*) which has continuous derivative of second order

*abc*  *c* 2  *a* 2 *f* (*x*)  **  0, *f* (*x*)  **

1

* 0,

*f* **** (*x*)  2**

#### *ah*

where 0  **  1 in the neighborhood stable.

*x*  *h* , then the trivial solution of the system(2) is asymptotically

**Proof.** By the given conditions we easily obtain

*xf* (*x*)  0 and

Form the function

*x*[*abc*  *c*2  *a*2 *f* (**)] *f* (**)*d*  0

0



, when 0 

*x*  *h* .

*V* (*x*, *y*, *z*, *u*) 

*a* [ *f* (*x*)  *cy* 

### *c*

3

*c z*]2 *a*

[(*ab*  *c*) *y*  *a* 2

*z*  *au*]2

 2*a* *x* [*abc*  *c* 2  *a* 2 *f* (**)]*f* (**)*d*  [*abc*  *c* 2  *a* 2 *f* (*x*)]*y* 2

*c* 0

Obviously, *V* (*x*, *y*, *z*,*u*) is positive in the neighborhood the system (2) is

(*x*, *y*, *z*,*u*)

 *h* , the total derivative of *V* for

*dV*  2*a*[*abc*  *c* 2  *a* 2 *f* (*x*)]*y* 2  *a* 2 *f* **** (*x*) *y* 3

### *dt*

 -2*a*[*abc*  *c* 2 - *a* 2 *f* (*x*)  *a*

## 2

*f* **** (*x*) *y*]*y* 2

 2*a*[**  *f* **** (*x*) ]*y* 2  2*a*(1  ** )*y* 2  0

*ah*

2

and *dV* / *dt*  0 can only be achieved by (*x*, *y*, *z*,*u*)  (0,0,0,0) .Hence the trivial solution of (2) is asymptotically stable by the BarBashin-Krasovskii’s theorem.

**Theorem 2**. Consider the fourth order nonlinear equation

*x*(4)  *ax*(3)  ** (*x*(1) )*x*(2)  *cx* (1)  *dx*  0

or the equivalent system of (3)

(3)

 *x*(1)  *y*, *y*(1)  *z*, *z* (1)  *u*,

 (1)

*u*

 *dx*  *cy*  ** ( *y*)*z*  *au*

(2)

where constants *a*  0, *c*  0 and *d*  0.

If there exists a constant *h*  0 , the function ** (*x*) which has continuous derivative of second order satisfies the following conditions

*ac* ( *y*)  *c* 2  *a* 2 *d*  **  0, ** ( *y*)  2**

#### *ch*

where 0  **  1 in the neighborhood stable.

**Proof .** Form the function

*y*  *h* , then the trivial solution of the system(4) is asymptotically

*V* (*x*, *y*, *z*, *u*)  *a*[*dx*  *cy* 

*c z*]2 *a*

 *a* 2

### *c*

[*dy*  *cz* 

*c u*]2 *a*

 2*d* *y* [*ac* (**)  *c* 2  *a* 2*d* ]*d*  1 [*ac* (**)  *c* 2  *a* 2*d* ]*z* 2

*c* 0 *a*

It is easy to see that *V* (*x*, *y*, *z*,*u*) is positive in the neighborhood of *V* for the system (4) is

(*x*, *y*, *z*,*u*)

 *h* , the total derivative

*dV*  2[*ac* ( *y*)  *c* 2  *a* 2*d* ]*z* 2  *c* **** ( *y*)*z* 3

### *dt*

 -2[*ac* (**)  *c* 2 - *a* 2*d*  *c * **** ( *y*)*z*]z2

## 2

 2[**  ** **** ( *y*) ]*z* 2  2(1  ** )*z* 2  0

*ch*

2

and *dV* / *dt*  0 can only be achieved by (*x*, *y*, *z*,*u*)  (0,0,0,0) .Therefore, the trivial solution of (4) is

asymptotically stable by the BarBashin-Krasovskii’s theorem.

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