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Dynamics of delayed Cohen-Grossberg neural networks

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**Abstract**

This paper studies the boundedness of Cohen-Grossberg neural networks with discrete delays and distributed delays (CGNN). Applying Lyapunov function and linear matrix inequalities technique (LMI), some novel sufficient conditions on the issue of the uniformly ultimate boundness, the existence of an attractor and the globally exponential stability for CGNN are established, which can be easily checked by the effective LMI toolbox in Matlab in practice.

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*Keywords:* Linear matrix inequalities technique (LMI), Neural Networks, Distributed delay, Boundedness, Attractor, Stability.

#### Introduction

In recent years, much attention has been paid on neural networks since they have been fruitfully applied in signal and image processing [1,2,3]. These applications rely crucially on the analysis of the dynamical behavior [4,5,6,7]. Among them, CGNN [8] can be described as follows

*xi* (*t*)

*i* (*xi* (*t*))[

*i* (*xi* (*t*))

 *aij f j* (*xj* (*t*))]

*j *1

*n*

*Ji* ,

*(1)*

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where *t * 0, *n * 2 ; *n* corresponds to the number of units in a neural network; *xi* (*t*) denotes the potential

(or voltage) of cell *i* at time *t* ; *f j* (*x*(*t*)) denotes a non-linear output function; *i* (*x*(*t*))  0 represents an

amplification function; *i* (*x*(*t*)) represents an appropriately behaved function; the *n * *n* connection matrix

1. ** (*aij* )*nn* denotes the strengths of connectivity between cells, and if the output from neuron *j* excites

(respectively, inhibits) neuron *i* , then *aij * 0 (respectively, *aij * 0) , *Ji* denotes an external input source.

During hardware implementation, time delays do exist due to finite switching speed of the amplifiers and communication time and may lead to an oscillation which is degenerate to the instability of networks furthermore. For model (1), Ye et al. [9] introduced delays by considering the following delay differential

equations

*xi* (*t*)

*i* (*xi* (*t*))[

*i* (*xi* (*t*))

*n*

 *aij f j* (*xj* (*t*

*j *1

*j* ))]

*Ji* , *i*

 1,  , *n*.

*(2)*

Although constant fixed delays in the models of delayed feedback systems serve as a good approximation in simple circuits consisting of a small number of cells, neural networks usually have a spatial extent due to the presence of an amount of parallel pathways with a variety of axon sizes and lengths [10, 12]. Therefore, there will be a distribution of conduction velocities along these pathways.

In this paper, we will consider the following CGNN model with mixed delays (discrete delays and distributed delays):

*xi* (*t*)

*i* (*xi* (*t*))[

*i* (*xi* (*t*))

*n*

 *aij f j* (*xj* (*t*))

*j *1

*n*

 *bij f j* (*xj* (*t*

*j *1

*j* ))

*n t*

*cij*

*f j* (*xj* (*s*))d*s * *Ji* ].

*(3)*

*j *1 *t hj*

Over the past decades, the stability of neural networks has been intensively investigated. In fact, except for stability property, boundedness is also one of the foundational concepts of dynamical neural networks, which plays an important role in investigation for the uniqueness of equilibrium point (periodic solutions), global asymptotic stability, global exponentially stable and its synchronization and so on [14, 15]. To the best of the authors knowledge, few authors have considered on the ultimate boundedness and attractor for CGNN with interval time-varying delays and distributed time-varying delays.

As is well known, compared with linear matrix inequalities (LMI) result, algebraic result is more conservative, and criteria in terms of LMI can be easily checked by using the powerful MATLAB LMI toolbox. This motivates us to investigate the problem of the ultimate boundedness and attractor for CGNN in this paper.

#### Problem formulation

System (3) for convenience can be rewritten as the following vector form



*t*

*t h*

*x*(*t*) (*x*(*t*))[

(*x*(*t*))

 *AF*(*x*(*t*))

 *BF*(*x*(*t*

))  *C F*(*x*(*s*))d*s * *J* ]

*(4)*

(*x*(*t*))*H* (*t*). Where, *x*(*t*)  (*x* (*t*),  , *x* (*t*))*T * *Rn* is the neural state vector; (*x*(*t*)) 



1 *n*

diag( (*x* (*t*)),  ,  (*x* (*t*)))  *Rn n* ; (*x*(*t*)) , *F*(*x*(*t*)) are appropriate dimensions functions;



1 1 *n n*

( ,  ,  )*T* , *h * (*h* ,  , *h* )*T* ; *J * (*J* ,  , *J* )*T* , *H* (*t*) (*x*(*t*))  *AF* (*x*(*t*))  *BF* (*x*(*t *))

1 *n* 1 *n* 1 *n*

*C F*(*x*(*s*))  *J* . The discrete delays and distributed delays are bounded: 0  , \*  max{ };



*t*

*t h*

*i* 1 *i n i*

0  *h* , *h*\*  max{*h* };  max{\*, *h*\*}, here \*, *h*\*,

are scalars. As usual, the initial conditions

*i* 1 *i n i*



associated with system (4) are given in the form vector-valued function.

*x*(*t*) (*t*),

*t * 0, where (*t*) is a differentiable

Throughout this paper, we make the following assumptions.

(**H1** ) We assume that the delay kernels satisfy *f j* (0)  0, *j*

1,  , *n* , and there exist constants *l j* and *Lj* ,

*i *1, 2,  , *n*, such that

*l*



*f j* (*x*)

*f j* ( *y*)

*L* , *x*, *y * *R*, *x * *y*.

*j x * *y j*

(**H2** ) There exist positive constants



*i* , *i* such that

*i * *ai* (*xi* (*t*)) *i* ;



(**H3** ) There exist positive constants *bj* , such that

*x* (*t*) (*x* (*t*))  *b x*2 (*t*).

*j j j j j*

**Remark 2.1** The constants *l j* and *Lj* can be positive, negative or zero. Therefore, the activation functions

*f* (*x*(*t*)) are more general than the forms| *f j* (*u*) |

*K j* | *u* |, *K j*

 0, *j*

 1, 2,  , *n* .

**Definition 2.1** [13] System (4) is uniformly ultimately bounded, if there is

1. ** 0 , for any constant

 0 , there is *t*'

 *t*'(

)  0 , such that

*x*(*t*, *t*0 , )

*B* for all



*t * *t*0

 *t* ',*t*0

 0,  ,

where

*x*(*t*, *t*0 , ) max sup | *xi* (*t * *s*, *t*0 , ) | .

1 *i n s * 0

**Lemma 2.1** [11] For any positive definite constant matrix *W * *Rn n* , scalar *r * 0 , vector function

*u*(*t*) :[*t * *r*, *t*]

 *Rn* ,

*t * 0 , then ( *r u*(*s*)d*s*)*T W* ( *r u*(*s*)d*s*)  *r r uT*

(*s*)*Wu*(*s*)d*s*.

0 0 0

#### Main Results

**Theorem 3.1** For a given constant

*a * 0 , if there is positive-definite matrix *P*

 diag( *p*1, *p*2

 , *pn* ),

*Di * diag(*Di*1, *Di* 2

 , *Din* ) , *i*

 1, 2 , *Q*, *R* such that the following condition holds

11 12 13 *PB*



|  |  |  |  |
| --- | --- | --- | --- |
| 0 | 0 | 0 |  |
| 0 | 0 | 0 |
| 0 | 0 | 0 | 0, |
| 55  56 0  66 0  77 | | |  |

0 *PC* 0

22 0 24

 33 0

44

*(5)*



where *Q*

*Q*11

*Q*12 *Q*22

0, *S*

*S*11

*S*12 *S*22

0, *R*

*R*11

*R*12 *R*22

0, *Di*

 0,*i*

## 1, 2,

  *a* *P * 2*a* *P * *P * *Se a*\* / \*  *h*\**R * *D* ,   *Se a*\* / \*,



11 1 2 11 3 1 12

\*



*PA * *Q * *h R * *D* ,

13 12 12 4 1

*a*\* *a*\* \*

22 11 3 2



*e Q * *D * *Se* /

*Q D h R*

,

\*

*a*\*



*e Q * *D* ,

24 12 4 2

33  22  1  22 ,

*a*\*

44 22 2

*e Q * *D* ,

*R e ah*\* /*h*\*, 

*R e ah*\* /*h*\*, 

*R e ah*\* /*h*\* ,  \*2*S* ,

55 11



56 12

66 22 77

1  diag{1/ 1

 ,1/

*n* }, 2

 diag{*b*1,  , *bn*}, 3

 diag{*l*1*L*1,  , *ln Ln*},

4  diag{(*l*1

 *L*1)/2,  , (*ln*

 *Ln* )/2},

the symbol '\*' within the matrix represents the symmetric term of the matrix, then System (4) is uniformly ultimately bounded.

**Proof.** Choosing the following Lyapunov functional

*V* (*t*)

*V*1 (*t*)

*V*2 (*t*)

*V*3 (*t*)

*V*4 (*t*),

*(6)*

where *V* (*t*) 2*p eat j s* d*s*, *V* (*t*)



*n*



*t*

*t*

*x* (*t* )

*easT* (*s*)*Q*(*s*)d*s*, (*t*)

[ *T T*

]*T* ,

1 *j* 0

2

*j *1

*j* (*s*)

*x* (*t*), *F*

(*x*(*t*))

*V*3 (*t*)



0 *t*

*t*

*eas xT* (*s*)*Sx*(*s*)d*s*d ,

*V*4 (*t*)

0

*h*



*t*

*t*

*easT* (*s*)*R*

(*s*)d*s*d.

Computing the derivative of *V*1 (*t*) along the trajectory of system (4), one can get

*V* (*t*) 2[*ap eat*

*n*

*x j* (*t* )  *s* d*s*

 *p eat x* (*t*) (*x* (*t*))]

1 = *j*

*j *1

0  *j* (*s*)

*j j j j*

[2*xT* (*t*)*PAF*(*x*(*t*))  2*xT* (*t*)*PJ *2*xT* (*t*)*PBF* (*x*(*t*

)) 2*xT* (*t*)*PC F*(*x*(*s*))d*s*]*eat* .

*(7)*

According to Assumption ( *H* 2 ), we obtain the following inequalities



*t*

*t h*

2*ap*

*x j* (*t* )  *s* d*s*

*a p x*2 (*t*).

*(8)*

0  *j* (*s*) *j*



*j*

*j j*

From Assumption ( *H*3 ), inequalities (7) and (8), we obtain

*V*1 (*t*)

*aeat*[*xT* (*t*)

*Px*(*t*)  2*xT* (*t*)

*Px*(*t*)]

[2*xT* (*t*)*PAF*(*x*(*t*))

*(9)*

2*xT* (*t*)*PBF* (*x*(*t*



1 2

)) 2*xT* (*t*)*PC F*(*x*(*s*))d*s*

*xT* (*t*)*Px*(*t*)

*J T PJ* ]*eat* .

Similarly, computing the derivative of $V\_2(t)$ along the trajectory of system (4), one can get



*t*

*t h*

*V*2 (*t*)

*eat*[*xT* (*t*), *FT x*(*t*))]*Q*[*xT* (*t*), *FT x*(*t*))]*T*

*ea*(*t*

)[*xT* (*t*

(*t*)), *FT x*(*t*

))] *Q*[*xT* (*t*

(*t*)), *FT x*(*t*

))]*T*

*(10)*

= *eat*[*xT* (*t*)*Q x*(*t*)  *FT* (*x*(*t*))*QT x*(*t*) *xT* (*t*)*Q F*(*x*(*t*))  *FT* (*x*(*t*))*Q F*(*x*(*t*))]

11 12 12 22

*ea*(*t* \* )[*xT* (*t*

)*Q*11*x*(*t*

) *FT* (*x*(*t*

))*QT x*(*t *)

 *xT* (*t*

12

)*Q*12 *F* (*x*(*t*

)) *FT* (*x*(*t*

))*Q*22*F*(*x*(*t*

))].

Computing the derivative of *V*3 (*t*) along the trajectory of system (4), one can get



*V* (*t*) = 0 [*eat xT* (*t*)*Sx*(*t*)

3

*ea*(*t*

) *xT* (*t*

)*Sx*(*t*

)]d 

 \**eat xT* (*t*)*Sx*(*t*)

*ea*(*t* \* )

*xT* (*s*)*Sx*(*s*)d*s*,

*(11)*

Where \* max{ }. Denote



*t*

*t*



1 *i n i*



Max{1, 2 ,  , *n*} , we obtain

\**eat xT* (*t*)*Sx*(*t*) = \**eat*[

(*x*(*t*))*H* (*t*)]*T S*

(*x*(*t*))*H* (*t*)

\*2*eat HT* (*t*)*SH* (*t*).

*(12)*

Using Lemma 2.1, the following inequality is easily obtained



*t*

*t*



*t*

*t*

*ea*(*t* \* )

*xT* (*s*)*Sx*(*s*)d*s *



*t*

*t*

*ea*(*t* \* )

\*

(

*x*(*s*)d*s*)*T S*(

*x*(*s*)d*s*)

*(13)*

= *ea*(*t* \* ) /

\*[*xT* (*t*)*Sx*(*t*)  2*xT* (*t*

)*Sx*(*t*)

*xT* (*t*

)*Sx*(*t*

)].

Similarly, computing the derivative of *V*4 (*t*) along the trajectory of system (4), one can get

*V*4 (*t*) 

*h*\**eat*[*xT* (*t*)*R x*(*t*)  2*FT* (*x*(*t*))*RT x*(*t*) 

*FT* (*x*(*t*))*R F*(*x*(*t*))]

*(14)*

(



*t*

*t h*

11 12

22

(*s*)d*s*)*T R*(

(*s*)d*s*)*ea*(*t h*\* ) /*h*\*

From Assumption (*H*1 ) , we have



*t*

*t h*

[ *f j* (*xj* (*t*))

*xj* (*t*)

*l j* ][

*f j* (*xj* (*t*))

*xj* (*t*)

*Lj* ]

0, *j*

# 1, 2, , *n*.

*(15)*

Then we obtain

*x*(*t*) *T*

*F* (*x*(*t*))

3 *D*1 4 *D*1

*D*1



*x*(*t*)

*F* (*x*(*t*))

*eat * 0,

*(16)*

and



*x*(*t F* (*x*(*t*



(*t*))

# (*t*)))

*T*

3 2 4 2



*D * *D*

*D*2

*x*(*t F* (*x*(*t*

(*t*))

## (*t*)))

*eat*

 0 . *(17)*

Denote

*MT* (*t*)  (*xT* (*t*), *xT* (*t*

), *FT* (*x*(*t*)), *FT* (*x*(*t*

)), ( (*s*)d*s*)*T* ,(



*t*

*t h*

*x*

*F*(*x*(*s*))d*s*)*T* , *HT* (*x*(*t*)))*T* ,

combing with (9)-(17), we have



*t*

*t h*

*V* (*t*)

*V* (*t*)  *V* (*t*)  *eat M T* (*t*) *M* (*t*)

### *eat J T PJ*.

*(18)*

Therefore, one obtains



1 2 1

*K eat*



1

*x*(*t*) 2

*V* (*x*(0))2

 *a* 1*eat J T PJ* ,

*(19)*

where *K*1

 min{a*np j* /

*j*}, which implies

1 *j n*

1

*x*(*t*)2

 [*e at*

*V* (*x*(0))2

 *a* 1*JT PJ* ]/*K* .

*(20)*

If one choose *B*

 [(1 

*a* 1*JT PJ* )/*K* ]1/2

 0 , then for any constant  0 and   , there

is *t*'

1

 *t*'(

)  0 , such that

*e at*

*V* (*x*(0))2

 1 for all

*t * *t*' . According to Definition 2.1, we have

*x*(*t*, 0, )

*B* for all



*t * *t*' . That is to say, system (4) is uniformly ultimately bounded.

**Theorem 3.2** If all of the conditions of Theorem 3.1 hold, then there exists an attractor *B* for the



solutions of system (4), where *B*



 {*x*(*t*) :

*x*(*t*)

*B*,*t * *t*0}.

**Proof.** If one choose *B*



 [(1

*a* 1*JT PJ* )/*K* ]1/2

 0, Theorem 3.1 shows that for any  , there is *t*'  0 ,

such that

1

*x*(*t*, 0, )

*B* for all *t * *t*' . Let *B* denote by

 {*x*(*t*) :

*x*(*t*)

*B*,*t * *t*0}. Clearly,

*B* is closed, bounded and invariant. Furthermore, lim sup inf



*B*

*x*(*t*; 0, )  *y*

0 . Therefore, *B* is an

attractor for the solutions of system (4).

*t * *y B*

**Theorem 3.3** In addition to all of the conditions of Theorem 3.1 hold, if *J * 0 , then system (4) has a

trivial solution

**Proof.** If *J*

*x*(*t*)  0 and the trivial solution of system (4) is globally exponentially stable.

0 , then system (4) has a trivial solution *x*(*t*)  0 . From Theorem 3.1, one has

2

*x*(*t*; 0, )2  *K e at* for all *(21)*

2

where *K*2 *V* (*x*(0)) /*K* . Therefore, the trivial solution of system (4) is globally exponentially stable.

1

#### Conclusions

In this paper, the dynamics of Cohen-Grossberg neural networks with mixed delays is investigated. Novel multiple Lyapunov-Krasovkii functionals are designed to get new sufficient conditions guaranteeing the uniformly ultimate boundedness, the existence of an attractor and the globally exponential stability. The derived conditions are expressed in terms of LMIs, which are more relax than algebraic formulation and can be easily checked by the effective LMI toolbox in Matlab in practice.

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