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ORIGINAL ARTICLE

A solution procedure for preemptive multi-mode project scheduling problem with mode changeability to resumption



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Abstract Extensive research has been devoted to the multi-mode resource constrained project scheduling problem (MRCPSP). However, little attention has been paid to problems where preemp- tion is allowed. This paper involves the preemptive multi-mode resource constrained project scheduling problem (P-MRCPSP) to minimize the project makespan subject to mode changeability after preemption. This problem is a more realistic model and extended case of multi-mode resource constrained project scheduling problem. A binary integer programing formulation is proposed for the problem. The problem formed in this way is an NP-hard one forcing us to use the Simulated Annealing (SA) algorithm to obtain a global optimum solution or at least a satisfying one. The per- formance of the proposed algorithm is evaluated on 480 test problems by statistically comparing in term of the objective function and computational times. The obtained computational results indi- cate that the proposed algorithm is efficient and effective. Also, it is concluded from the results that mode change is very effective to improve the optimal makespan of the project.

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KEYWORDS

Project scheduling; Mode change; Simulated Annealing; Preemption; Resumption

1. Introduction

Resource constrained project scheduling problem (RCPSP) is one of the most important problems in the context of project scheduling which is an NP-hard problem [[7]](#_bookmark22). The decision vari- ables for the RCPSP are the starting times of activities while

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the resources availabilities are considered given. The objective is then to minimize the completion time of the project. In the literature there are several algorithms that solve the RCPSP; recent reviews about exact methods and heuristics can be found in Kolisch and Hartmann [[30]](#_bookmark23), Hartmann and Kolisch [[19]](#_bookmark36), Hartmann and Kolisch [[20]](#_bookmark37), Zhang et al. [[48]](#_bookmark49), Zhang et al. [[49]](#_bookmark50), Jairo et al. [[23]](#_bookmark42), Hartmann and Briskorn [[17]](#_bookmark31), Agar- wal et al. [[3]](#_bookmark20), Fang and Wang [[15]](#_bookmark29), Kone´ [[31]](#_bookmark23), Paraskevopoulos et al. [[38]](#_bookmark32).

In RCPSP it is assumed that activities could only be per- formed in one possible execution mode. In practice, however, it often happens that multiple execution modes can be defined for the project activities. Each activity may be executed in one or more execution modes, each requiring a specific amount of

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resources consumption and resulting in different durations for an activity completion. More exactly, each execution mode defines as a trade-off between time/cost, time/resource, speed/resource etc. The multi-mode problem (MRCPSP) is a generalized version of the RCPSP, where each activity can be performed in one out of a set of modes, with a specific activ- ity duration and resource requirements. The standard multi- mode resource constrained project scheduling problem involves the selection of an execution mode for each activity and the determination of the activity start or finish times such that the precedence and resource constraints are met and the project duration is minimized. As this problem is a generaliza- tion of the RCPSP, the MRCPSP is also NP-hard. Several algorithms that solve the MRCPSP have been proposed in recent years: Hartmann and Drexl [[18]](#_bookmark34), Sprecher and Drexl [[43]](#_bookmark41), Knotts et al. [[28]](#_bookmark23), Nonobe and Ibaraki [[37]](#_bookmark30), Jozefowska et al. [[25]](#_bookmark47), Alcaraz et al. [[4]](#_bookmark22), Bouleimen and Lecocq [[8]](#_bookmark22), Heil- mann [[22]](#_bookmark39), Zhu et al. [[50]](#_bookmark50), Zhang et al. [[48]](#_bookmark49), Zhang et al. [[49]](#_bookmark50), Lova et al. [[32]](#_bookmark23), Jarboui et al. [[24]](#_bookmark44), Ranjbar et al. [[42]](#_bookmark40), Lova et al. [[33]](#_bookmark23), Coelho and Vanhoucke [[10]](#_bookmark22), Ranjbar [[41]](#_bookmark38), Barrios et al. [[6]](#_bookmark22), Afshar-Nadjafi et al. [[2]](#_bookmark21), Nabipoor Afruzi et al. [[36]](#_bookmark24).

The basic RCPSP and MRCPSP assume that each activity, once started, will be executed until its completion. This assumption can be justified only for activities in which their interruption essentially is inapplicable. For example, in order to integrity of foundation, concrete placement cannot be pre- empted. However, for activities in which their interruption is applicable, the optimal makespan can be improved by allowing preemption, because the solution space is extended as a result of the constraint relaxation. Welding can be mentioned as a preemptive activity. Preemptive multi-mode resource con- strained project scheduling problem (P-MRCPSP) refers to a generalization of the multi-mode resource constrained project scheduling problem (MRCPSP) which allows activities to be preempted at any time instance and restarted later on at no additional cost. The literature on solution methods for the pre- emptive resource constrained project scheduling problem is rel- atively scant. For the single-mode case, one can refer to Kaplan [[26]](#_bookmark23), Demeulemeester and Herroelen [[13]](#_bookmark25), Ballestin et al. [[5]](#_bookmark22), Vanhoucke and Debels [[46]](#_bookmark45), Damay et al. [[11]](#_bookmark22). For the multi-mode case, Buddhakulsomsiri and Kim [[9]](#_bookmark22) proved that preemption is very effective to improve the optimal pro- ject makespan in the presence of resource vacations and tem- porary resource unavailability and that the makespan improvement is dependent on the parameters that impact resource utilization. Van Peteghem and Vanhoucke [[47]](#_bookmark48) have proposed a genetic algorithm for the multi-mode resource-

time. In these cases, modeling and solving such a problem as a classical MRCPSP, especially in the preemptive case may lead to poor solutions. To the best of our knowledge, no research has been performed on the P-MRCPSP with permitted mode change.

Therefore, the contribution of this paper is fourfold: first, a binary integer programing formulation is developed for the preemptive multi-mode project scheduling problem of mini- mizing the project makespan subject to resource constraints and precedence relations, where execution mode of each activ- ity can be changed after being preempted. This problem is called P-MRCPSP-MC. This model is not considered in the past literature. Second, an efficient meta-heuristic solution procedure based on SA is developed for the problem due to NP-hardness of the problem. In proposed SA, the activity list representation is used to encode a project schedule and the serial schedule generation scheme (SSGS) embedded with a new dynamic heuristic to translate the schedule representation to a schedule. Third, the effectiveness of proposed SA for the P-MRCPSP-MC will be analyzed. Finally, the effect of mode changeability on project makespan is analyzed.

The remainder of this paper is organized as follows: Sec- tion [2](#_bookmark1) is devoted to the presentation of the problem. In Sec- tion [3](#_bookmark7) the steps of our algorithm to solve the problem is explained. Computational results are represented in Section [4](#_bookmark13). Finally, Section [5](#_bookmark18) contains the conclusions.

1. Problem description

In continuation the project is represented by an activity on the node (AON) network *G*(*N*, *A*) where the set of nodes, *N*, rep- resents activities and the set of arcs, *A*, represents finish-start precedence constraints with a time-lag of zero. The preempt- able activities are numbered from the dummy start activity 1 to the dummy end activity *n* and are topologically ordered, i.e., each successor of an activity has a larger activity number than the activity itself. The set of activities is to be scheduled on a set *Rq* of renewable and *Rm* of nonrenewable resource types. For each activity *i* e *N*, instead of a fixed duration and known resource requirements, a fixed work content *Wi* is given which essentially indicates how much work has to be per- formed. This work content can be performed in a mode *mi*, which is chosen out of a set of *Mi* different execution modes, i.e., with different speeds and resource requirements as long as the required work content is met. The accomplishing of an activity can be temporarily interrupted at discrete time

instants, and restarted at a later time with a same or different mode. The progress of activity *i* during each time unit of its

constrained project scheduling problem and its extension to

the preempted case.

execution in mode *mi* , is *wimi*

(measured by same unit of

*q*

The basic MRCPSP and P-MRCPSP assume that activities assigned modes cannot change during the execution of the pro- ject. This assumption is one of the classical MRCPSP and P- MRCPSP shortcomings. This common assumption can be jus- tified as long as essence and materials of modes are different and mode change is inapplicable. However, if execution modes

of an activity have the same essence and materials, the optimal

*Wi*). Each activity *i* in mode *mi* requires *rimik* renewable

resource units (*k* e *Rq*) during each time unit of its execution. For each renewable resource*k* e *Rq*, the availability *aq* is con- stant throughout the project horizon. Activity *i*, executed in mode *mi*, will also use *rm* nonrenewable resource units (*l* e *Rm*) of the total available nonrenewable resource *am*. Logically, it is assumed that mode *mi* with higher *wimi* requires more renew-

*k*

*imil*

*l*

makespan can be improved by allowing mode change. This is

able *rq*

*i*

*im k*

and nonrenewable resources *rm* .

*i*

*im l*

probable especially in the presence of resource vacations and temporary resource unavailability. However, it is likely that in reality, execution mode of an activity is changed, especially when an activity is preempted and it will be restarted at a later

The objective of the P-MRCPSP-MC is to find a feasible

schedule in order to minimize the makespan of the project. However, changeable execution modes *mi* for activities and preemption plan for activities have to be determined. A sched-

ule is defined as a sequence of start (finish) times for the pro- ject’s activities. A schedule which satisfies the specified prece- dence and resource constraints is called feasible schedule. Also, a feasible schedule which meets as much as possible the objectives set forward by project management is called

optimal. The following notation is used for P-MRCPSP-MC:

appropriately. The resulting schedule may be transferred into a schedule for the original problem by removing the dummy start and end activity, and one time unit left shifting.

Using the above notation, P-MRCPSP-MC can be mathe- matically formulated as follows:

*LFTn*

min *Z* = X *t*.*xnmnt* (3)

*t*=*ESTn* +1

*A*

*N*

*n Rq K*

*Rm*

*L*

*Mi Wi wimi*

*rq*

*imik*

*rm*

*imil*

*aq*

*k*

*l*

*ESTi LFTi*

*sn*

*Z*

*ximit*

*am*

Set of arcs of acyclic digraph representing the project

Set of nodes of acyclic digraph representing the project,

|*N*|= *n*

Number of activities, index by *i*

Set of renewable resource(s), |*Rq*| = *K* Number of renewable resource(s), index by *k* Set of nonrenewable resource(s), |*Rm*|= *L*

Number of nonrenewable resource(s), index by *l* Number of execution modes for activity *i* , index by *mi* Total work content of activity *i*, *i* e *N*

Progress of activity *i* during each time unit of its execution in mode *mi* , *i* e *N*, *mi* = 1,.. .,*Mi*

Renewable resource type *k* requirement of activity *i* in mode

*mi* , *i* e *N*, *k* e *Rq*, *mi* = 1,.. .,*Mi*

Nonrenewable resource type l requirement of activity *i* in mode *mi*, *i* e *N*, *l* e *Rm*, *mi* = 1,.. .,*Mi*

Constant availability of renewable resource type *k*

throughout the project horizon, *k* e *Rq*

Total availability of nonrenewable resource type *l*, *l* e *Rm*

Earliest start time of activity *i* Latest finish time of activity *i* Deadline of the project

Objective function (project makespan)

1, if activity *i* in mode *mi* is in progress in period *t*, 0, otherwise (binary decision variable)

Subject to:

*L*X*FTi*

X*Mi*

*wimi* .*ximi t* ≥ *Wi* 6*i* ∈ *N* (4)

*t*=*ESTi* +1 *mi* =1

X*Mi*

*ximi t* ≤ 1 6*i* ∈ *N*, 6*t* ∈ [*ESTi* + 1, *LFTi*] (5)

*mi* =1

*ximit* + *xim*'(*t*+1) ≤ 1 6*i* ∈ *N*,

*i*

6*t* ∈ [*ESTi* + 1, *LFTi* — 1], 6*mi*–*m*'

*i*

(6)

*t Mi Mj*

XX*wimi* .*ximis* ≥ *Wi*

X*xjmj* (*t*+1) 6(*i*, *j*)∈ *A*,

*s*=1 *mi* =1

*mj* =1

6*t* ∈ [*ESTi* + 1, *LFTj* — 1] (7)

*n Mi*

XX

*rq* .*xim t* ≤ *aq*

6*k* ∈ *Rq*, *t* = 1, ... , *sn* (8)

*imik i k*

*i*=1 *mi* =1

X*n* X*Mi*

*E*X*FTi*

*rm* .*xim t* ≤ *am*

6*l* ∈ *Rm* (9)

*imil i l*

*i*=1 *mi* =1*t*=*ESTi* +1

In our formulation, 0–1 variables *ximi t* are defined, which specify whether an activity *i* in mode *mi* is in progress in period *t* or not. These variables can only be defined over the time

interval of the activity in question *t* e [*ESTi* + 1, *LFTi*]. These limits are determined using the traditional forward and back- ward pass calculations considering duration of activity *i* based on high speed mode as follows:

2 *Wi* 3

*ximi t* = 0, 1 6*i* ∈ *N*, 6*t* ∈ [*ESTi* + 1, *LFTi*],

*mi* = 1, ... , *Mi* (10)

The objective function in Eq. [(3)](#_bookmark2) minimizes the project duration. Remember, however, that this value exceeds the optimal project length because of the unit duration of both the dummy start and dummy end activity. The constraints in inequality [(4)](#_bookmark2) assure that work content of each activity is met. The constraints in Eq. [(5)](#_bookmark2) assure that each activity is

not assigned more than one mode for each time period. The

*di* = 4max *w*

*m*

*i*

*imi*

5 (1)

assumption that mode change is not allowed without preemp-

tion is modeled in Eq. [(6)](#_bookmark2). Eq. [(7)](#_bookmark2) denotes the precedence

The backward pass calculation is started from a fixed pro-

ject deadline *sn*. In this paper, earliest finish time of dummy end activity, *EFTn*, is considered as project deadline. *EFTn* is computed using the traditional forward calculations consider- ing duration of activity *i* based on low speed mode as follows:

*d* = 2 *Wi* 3 (2)

*i* min *w*

6 *im* 7

*mi*

*i*

relations-constraints. Constraints [(8) and (9)](#_bookmark3) take care of the

renewable and nonrenewable resource limitations, respectively. Finally, Eq. [(10)](#_bookmark4) imposes binary values on the decision vari- ables. This formulation requires the definition of at most *n\**

max(*Mi*)*\* sn* binary decision variables. Also, the number of constraints of the formulation amounts to at most *n* + *n sn* [1 + max(*Mi*)(max*(Mi*)-1)/2]+ *sn* (|*A*|+*K*)+*L.*

[Fig. 1](#_bookmark5) shows an example of P-MRCPSP-MC with 7 activi-

ties where 1 and 7 are dummy activities.

It is clear that an activity with work content of 0 is never in

progress and thus does not have a corresponding decision vari- able which is set to 1. This problem, however, can be easily overcome: the dummy start and end activity are assigned a dummy mode with work content of 1. Also, the parameters

, *r*

Each activity has two execution modes. For each mode, 1

renewable resource and 1 nonrenewable resource is indicated. The availability for the renewable (nonrenewable) resource is 5 (132). Problem instance parameters are given in [Table 1](#_bookmark6).

[Fig. 2](#_bookmark8)(a) depicts a schedule with a makespan of 8 days. This

*m*

*r*

*imil*

*q*

*imik*

and *wimi*

for dummy modes are assumed as 1. All

schedule is feasible because it uses exactly 130 nonrenewable

other activities with zero work content can be eliminated, pro- vided that the corresponding precedence relations are adjusted

resource units. Also, precedence relations are met and renew-

able resource availability (5) is not violated. [Fig. 2](#_bookmark8)(b) shows



2

5

1

3

7

4

6

Fig. 1 An example network.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 1 Problem instance information.  Activity *i Wi* Mode *mi wim rq rm*  *i imik imil* | | | | | |
| 1 | 1 | 1 | 1 | 0 | 0 |
| 2 | 14 | 1 | 6 | 5 | 10 |
|  |  | 2 | 4 | 4 | 7 |
| 3 | 8 | 1 | 5 | 2 | 13 |
|  |  | 2 | 3 | 1 | 6 |
| 4 | 11 | 1 | 8 | 4 | 23 |
|  |  | 2 | 5 | 2 | 10 |
| 5 | 14 | 1 | 7 | 3 | 7 |
|  |  | 2 | 5 | 2 | 5 |
| 6 | 6 | 1 | 6 | 2 | 22 |
|  |  | 2 | 3 | 1 | 11 |
| 7 | 1 | 1 | 1 | 0 | 0 |
|  |  |  |  |  |  |

a feasible schedule with a makespan of 7 days, in which pre- emption is allowed. If the problem is relaxed to the P- MRCPSP-MC, a feasible schedule as shown in [Fig. 2](#_bookmark8)(c) can be generated.

1. Proposed SA to solve P-MRCPSP-MC

Simulated Annealing (SA) algorithm has been successfully applied to a noticeable number of project scheduling problems

[[1,8,21,25,35,40]](#_bookmark19). In this section an SA algorithm is proposed to solve P-MRCPSP-MC. In order to increase quality of the proposed SA, an efficient dynamic heuristic algorithm is imple- mented to construct a schedule. Also exact solutions obtained from Lingo 11 are considered to provide comparable computa- tional efforts for SA.

* 1. *Basic Simulated Annealing*

Simulated Annealing (SA) which has been successfully applied to various difficult combinatorial optimization problems is a random search method that is based on Monte Carlo iterative strategy. The origins of SA are in statistical mechanics (Metro- polis algorithm) and it initially was presented as a search algo- rithm for combinatorial optimization by Kirkpatrick et al. [[27]](#_bookmark23). SA is useful for problems with a very large discrete search space, which is too large for an enumeration search method. SA algorithm starts by generating an initial solution and by initializing the so-called temperature parameter *T*. Then, at each iteration a solution *s*' is randomly created in the neighbor- hood of the current solution and if it is better than the current solution, it replaces the current solution. If the new solution is not an improvement upon the current solution, it replaces the current solution with a probability generally computed follow-

ing the Boltzmann distribution exp(— *f*(*s*')—*f*(*s*)) where *T* is the current temperature and *f*(*s*')- *f*(*s*) is the change in objective function value obtained by moving from previous solution to new solution. The temperature *T* is decreased during the search process, thus at the beginning of the search the probability of accepting uphill moves is high and it gradually decreases, con- verging to a simple iterative improvement algorithm. Regard- ing the search process, this means that the algorithm is the result of two combined strategies: random walk and iterative improvement. In the first phase of the search, the bias toward improvements is low and it permits the exploration of the search space; this erratic component is slowly decreased thus

*T*

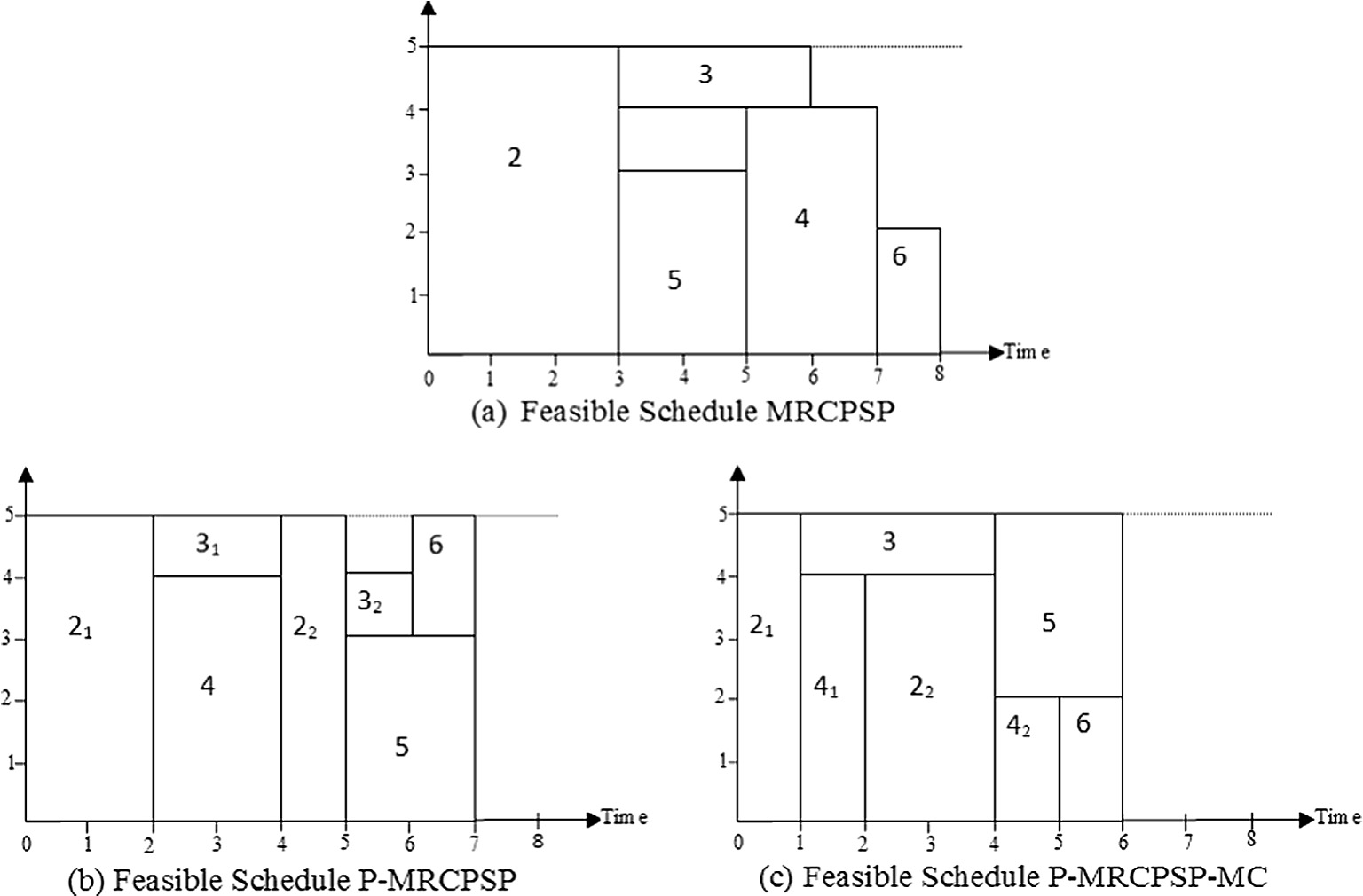


Fig. 2 Schedules for problem instance.

leading the search to converge to a (local) minimum. The prob- ability of accepting uphill moves is controlled by two factors: the difference of the objective functions *f*(*s*')- *f*(*s*) and the tem- perature *T*. On the one hand, at fixed temperature, the higher the difference *f*(*s*')- *f*(*s*), the lower the probability to accept a move from *s* to *s*'. Whereas, the higher *T*, the higher the prob- ability of uphill moves.

The choice of an appropriate cooling schedule is crucial for the performance of the algorithm. One of the most used ones follows a geometric law *Tk*+1 = *aTk* where *a* e (0, 1) which corresponds to an exponential decay of the temperature. The cooling rule may vary during the search, with the aim of tuning the balance between diversification and intensification. For example, at the beginning of the search, *T* might be constant or linearly decreasing, in order to sample the search space; then, *T* might follow a rule such as the geometric one, to con- verge to a local minimum at the end of the search. The cooling schedule and the initial temperature should be adapted to the particular problem instance, since the cost of escaping from local minima depends on the structure of the search landscape. The description of SA indicates that a basic SA does not use the history of the search process. This is one of the reasons why SA is often outperformed by other meta-heuristics. How- ever, due to its simplicity, it is generally very fast and it can be successfully integrated into other search techniques.

* 1. *Preprocessing*

schedule is obtained. In each run, the first un-scheduled activ- ity in the activity list is chosen and the first possible starting time is assigned for that activity such that precedence or resource constraints are preserved.

A feasible solution is represented by a vector which is a precedence-feasible permutation of activities:

*I* = (*jI*, *jI* , ... , *jI* ) (11)

*i* 2 *n*

[Fig. 3](#_bookmark10) shows an example of solution representation related to the mentioned instance in [Fig. 1](#_bookmark5).

Having got a feasible solution represented by the vector described above, the starting times of all activities (sub- activities) are then defined by using the serial SGS. The SGS determines how a feasible schedule is constructed by assigning starting times to the activities. It sequentially adds activities in the activity list to the schedule until a feasible complete sched- ule is obtained such that no precedence or resource constraint is violated. In this paper however, execution modes are deter- mined using a dynamic heuristic embedded into serial SGS. Our proposed heuristic is derived from part period lot sizing heuristic (DeMatteis [[12]](#_bookmark26)); algorithm chooses the number of periods covered by the replenishment order such that the total holding costs are made as close as possible to the setup cost.

To solve P-MRCPSP-MC, an initial feasible activity list is generated. Then, ratio of work to resource (*RWR*) is computed as follows:

P*n W*

*RWR* = *i*=1 *i*

(12)

P*L am* + *T*P*K aq*

In order to reduce the search space, preprocessing is used

*l*=1

*l*

*k*=1 *k*

before the execution of SA. The data reduction procedure has originally been proposed by Sprecher et al. [[44]](#_bookmark43) to increase the speed of their branch and bound algorithm for the MRCPSP. The idea behind this procedure is to omit all non- executable and inefficient modes from the project data without affecting the optimal makespan. An execution mode *mj* is called *non-executable* if its execution would violate the renew- able resource constraints in any schedule. Also, a mode is called *inefficient* if there is another mode of the same activity with the same or higher speed and no more requirements for all resources. Hence, non-executable and inefficient modes may be excluded from the project data without losing optimality.

* 1. *Solution representation*

In the previous researches, various representations for sched- ules in the construction of heuristics for the RCPSP are devel- oped (Kolisch and Hartmann [[30]](#_bookmark23)). The two most important ones are the random-key (RK) representation and the activity-list (AL) representation. Hartmann and Kolisch [[19]](#_bookmark36) deduced from experimental tests that procedures based on AL representations outperform the other procedures. The

*RWR* is a representation of work-resource balancing. In our proposed SA, mode assignment is done by comparing real- ized ratio of work to resources so far with *RWR*. For selected activity *i*, mode *mi* with realized ratio of total completed work content to the spent resources so far as close as to *RWR* is assigned.

After obtaining an initial activity list *I*, the corresponding schedule is computed by the following procedure:

Starting from time period 1, for each time period *t*, set of activities that are executable at a certain time period *t* (i.e., all their predecessors have completed), is identified. This set of activities is denoted by *Ia* which should consist of at least one activity. Activities in *Ia* are arranged according their sequence in *I*. In each iteration, first activity *j* is selected from *Ia* and deleted from it. If activity *j* was in progress at period *t*-1, same execution mode should be assigned to it if possible. Else, procedure is continued by selecting first activity *j* from *Ia* again.

If activity *j* was in progress at period *t*-1, all *impossible* and *dominated* modes of activity *j* are considered inactive. A mode *mj* is called *impossible* if its renewable resource requirement exceeds remaining availability for at least one renewable resource type. Also, A mode *mj* is *dominated* by another mode

*m*' if remaining work of activity *j* is less than or equal to *wjm*'

AL representation is used to encode a project schedule and *j j*

*j*

the serial schedule generation scheme (SSGS) to translate the schedule representation to a schedule. Since the minimum pro- ject makespan criterion is a regular performance measure, i.e., a measure which is non-decreasing in activity completion times, one may use the serial SGS rule to construct the sched- ule. As a result, there is no danger of omitting an optimal schedule by using the serial SGS here. The serial SGS sequen- tially adds activities to the schedule until a feasible complete

whereas *wjmj* > *wjm*' . Then, for remaining modes (if exist) of

Activity list

*j1 j2 j3 j4 j5 j6 j7*

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 4 | 3 | 5 | 6 | 7 |

Fig. 3 Solution representation.

activity *j*, *Pjmj* is computed as follows which is a representation of realized ratio of work to resources, so far.

(13)

chooses an activity randomly and for that activity, a different

For an infeasible generated schedule (*ERR*(*l*) > 0) the local search procedure of Hartmann [[16]](#_bookmark33) is applied to trans- form infeasible solutions into feasible ones. The procedure

P P P*t*

= *i mi s*=1 *i i*

*wim* .*xim s*

*jmj*

*P*

*s*=1 *imil*

P P P P*t*

*i*

*mi*

*l*

*rm x*

+ *t*P*K aq*

mode is chosen. If the *ERR*(*l*) remains the same or decreases,

Finally, the mode *mj*

for which *P*

*jmj*

*imis*

*k*=1 *k*

most nearly equals to

the mode for that activity is changed. This step is repeated

until the mode assignment is feasible (*ERR*(*l*) = 0) or until *J*

*RWR* is assigned to activity *j* and activity *j* set to be in progress

at period *t* . If there are no remaining modes for activity *j*, pro- cedure continues by selecting first activity *j* is from *Ia*. Also, if *Ia* is empty, algorithm restarts by setting *t* = *t* + 1. Above procedure is continued until dummy activity *n* be a scheduled activity. The time complexity of this procedure is the same as the basic serial SGS, *O*(*n*2*K*) (Pinson et al. [[39]](#_bookmark35)). The pseudo- code for decoding a solution to a schedule is shown in [Table 2](#_bookmark12). For example, in [Fig. 1](#_bookmark5) consider the following activity list:

*j*1

1

*j*2

2

*j*3

4

*j*4

3

*j*5

5

*j*6

6

*j*7

7

Decoding this schedule as follows results in the feasible sched- ule shown in [Fig. 2](#_bookmark8)(c):

*t* = 1 activity 2 in mode m1 *t* = 2 activity 4 in mode m1 *t* = 2 activity 3 in mode m2 *t* = 3 activity 2 in mode m2

(activity 2 is preempted at time point *t* = 1)

*t* = 3 activity 3 in mode m2 *t* = 4 activity 2 in mode m2 *t* = 4 activity 3 in mode m2 *t* = 5 activity 4 in mode m1

(activity 4 is preempted at time point *t* = 2)

*t* = 5 activity 5 in mode m1

*t* = 6 activity 5 in mode m1

consecutive unsuccessful trials to improve the mode assign- ment have been made. In this paper, *J* equals to four times the number of activities in the project. This procedure acts only on mode assignment and do not change the activity list. Also, this procedure stops as soon as it reaches to a feasible solution; i.e., resulting solution is close to the inner border of nonrenew- able feasibility. So this procedure may not generate a feasible solution that is very different from the original unfeasible one.

* 1. *Starting solution*

In the proposed SA an initial solution is created by setting all activities on the activity list based on the latest finish time (LFT) which is an efficient priority rule (Kolisch [[29]](#_bookmark23)). Then, the procedure described in Section [4.2](#_bookmark14) is used to determine exe- cution modes and execution time of activities.

* 1. *Neighborhood generation structure*

In order to generate a neighborhood of current solution the following method is used. Let *I* = (*jI*, *jI* , ... , *jI* ) be the current solution. Neighborhood generation mechanism is applied to the activity list of the solution. For activity list of *I*, neighbor- hood generation mechanism operates as follows: A random activity *jI* , is selected from the activity list with position *a*. Last

*a*

*i* 2 *n*

predecessor and first successor’s position of *jI* is identified in the activity list. Subsequently, a random position *x* between the last predecessor and first successor’s position of *jI* is selected, and *jI* is moved to position *x*. Finally, all activities

*a*

*a*

*a*

*a*

*t* = 6 activity 6 in mode m1

between position of *jI*

and position *x* are shifted to the left

The number of requested nonrenewable resource units that exceeds the capacity *am*, *l* ∈ *Rs*, is defined as the excess of resource request *ERR*(*l*) (Van Peteghem and Vanhoucke [[47]](#_bookmark48)). An *ERR*(*l*) = 0 means that the solution is feasible. If

*l*

*ERR*(*l*) is larger than 0, the solution is infeasible with respect to nonrenewable resources. The formula of the *ERR*(*l*) can be adjusted in our problem as follows:

XX

X

or right depending on relative position of *jI* and position *x.*

* 1. *Cooling scheme*

*a*

The cooling scheme is the main factor that needs to be orga- nized when designing the Simulated Annealing algorithm. The temperature is initially set at a large value and then grad-

ually decreased under the cooling schedule function until it

(*rm*

.*xjm t*)— *am*

(14)

X*L*

max 0,

*ERR*(*l*)=

*l*=1

*n sn Mj*

*j*=1 *t*=1 *mj* =1

!! reaches the thermal equilibrium. After each move (neighbor-

*jmjl*

*j*

*l*

hood generation), the temperature is reduced according to

Table 2 Pseudo-code for decoding a solution to a schedule.

1. Let *t* = 1, compute *RWR* from Eq. [(12)](#_bookmark9)
2. Determine the list of uncompleted activities which are admissible to be in progress at period *t* with respect to the precedence constraints, (*Ia*). Arrange activities in *Ia* according their sequence in *I*
3. If *Ia* is empty set *t* = *t* + 1 and go to step 2, else, select first activity *j* from *Ia* , Delete *j* from *Ia* , If *j* = *n* go to step 8
4. If activity *j* was in progress at period *t*-1, assign the same execution mode to it *if* possible and go step 7 *else*, go to step 3
5. Delete all *impossible* and *dominated* modes of activity *j*. If the remaining mode list of activity *j* is empty go to step 3
6. For remaining modes of activity *j* compute *Pjmj* from Eq. [(13)](#_bookmark11), assign the mode *mj* to activity for which *Pjmj* is most nearly equal to *RWR*
7. Set activity *j* in assigned mode to be in progress at period *t*, go to step 2
8. Stop

the cooling schedule suggested by Lundy and Mees [[34]](#_bookmark27) as fol- lows where *a* is chosen close to zero.

*Tk*

the CPU-time limit to the size of the problem, i.e., use of the low CPU-time for small problems and high CPU-time for lar- ger problems. Taguchi [[45]](#_bookmark46) divides factors into controllable

*Tk*+1 = 1 + *aT*

*k*

(15)

and noise factors and offer a set of orthogonal arrays for designing experiments of quality improvement. Although there is no direct control of noise factors, the Taguchi method deter-

* 1. *Stopping criterion*

In theory the SA procedure should be continued until the final temperature *Tf* is zero, but in practice other stopping criteria are used. In this paper, the procedure is continued until a pre- determined CPU time is reached.

1. Performance evaluation
   1. *The test problems*

A set of 480 problems was generated by the project generator ProGen/px developed by Drexl et al. [[14]](#_bookmark28) in order to validate the proposed SA algorithm for the P-MRCPSP-MC. To do this the parameters given in [Table 3](#_bookmark15) are used. The indication [*x*,*y*] means that the value is randomly generated on the inter- val [*x*,*y*]. Renewable resource availability is constant over time. For each combination of the parameter values, 4 instances were generated. The resource factor *RF* reflects the average portion of resource required per activity. The resource strength *RS* reflects the scarceness of the resource. The problem set was extended by generating project deadline *sn* in the same way as described in Section [2](#_bookmark1).

* 1. *Parameters setting*

The values of parameters used in Simulated Annealing (SA) algorithms must be carefully selected since parameter values may have a significant influence on the performance of the algorithm. In this paper, the Taguchi experimental design is used to tune the parameters of SA. CPU-time limit was speci- fied as a stopping criterion which is selected through the com- putational experiments. We obtained good results by indexing

mines the optimal level of controllable factors and minimizes the effect of noise. In the proposed SA, the factors that should be tuned are temperature control parameter *a*, initial tempera- ture and number of milliseconds per activity *CPU*. A set of 27 randomly generated problems with 40 non-dummy activities are used for parameter tuning. Using MINITAB software ver- sion 16, based on a L27 orthogonal array design the optimal levels (in Bold) of the parameters are reported in [Table 4](#_bookmark16).

* 1. *Experimental results*

The procedure has been programed in Borland C++ 5.02 and executed on a personal computer with an Intel Core2Dou,

2.5 GHz processor and 3 GB memory. Since we could not find any algorithm for P-MRCPSP-MC, the proposed SA is com- pared with the optimal solution obtained by Lingo 11. [Table 5](#_bookmark17) presents the computational results of the proposed algorithm where it is compared with the optimal solution obtained by Lingo 11 (or the best obtained solution by SA if Lingo is not able to solve the problem). Proposed SA executed 10 times for each problem to obtain more reliable data. The experimen- tal results demonstrate that control parameter calibration pro- vides high quality solutions. Following notations are used in [Table 5](#_bookmark17):

NPO: Number of problems for which Lingo was able to find optimum solution in 1000 s.

NPM: Number of runs of problems for which SA was able to find optimum solution.

ACNT-L: Average convergence time for Lingo (in seconds).

ACNT-SA: Average convergence time for SA (in seconds). ARD: Average relative deviation percentages.

Relative deviation (*RD*) percentage for each problem is obtained by following formula:

*Z* — *Z*\*

Table 3 The parameter settings for the problem set.

Control parameter Value

Number of activities (non-dummy) (*n*) 20, 30, 40, 60,

90

*RD* = *Z*\* (16)

where *Z* is the value of objective function obtained by SA and

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Number of execution modes 2, 3  Activity work contents [10,20]  Progress of activities (per period) [1,10]  Number of initial and terminal activities 3  Maximal number of predecessors and successors 3  Coeﬃcient of network complexity (*CNC*) 1.5  Resource factor (*RF*) 0.5, 1  Renewable and nonrenewable resource strength 0.25, 0.5 (*RS*)  Number of renewable and nonrenewable resource 1, 2, 3 types  Constant availability of renewable resources *n* | | *Z\** is the optimal solution obtained by Lingo or the best  obtained solution by SA.  From [Table 5](#_bookmark17) it can be observed that when the number of activities is less than or equal to 30, all 192 problems can be solved to optimality by Lingo within the allowed time limit. Also, [Table 5](#_bookmark17) shows that when number of activities is greater  Table 4 Factors levels and the tuned values for *a*, *T0* and  *CPU*. | | | | |
| Total availability of nonrenewable resource  Activity renewable resource (per period) demand | 25*n*  Integer[1,10] | Factor  *CPU* | Number of  3 | Levels Level 1  50 | Level 2  70 | Level 3  100 |
| Activity nonrenewable resource (per period) | Integer[1,10] | *a* | 3 | 0.0045 | 0.0055 | 0.0065 |
| demand |  | *T0* | 3 | 12 | 17 | 25 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 5 Computational results of the SA and Lingo. | | | | | | | | |
| #Activities | #Modes | #Problems | SA |  |  |  | Lingo |  |
|  |  |  | NPM | ARD (%) | ACNT-SA |  | NPO | ACNT-L |
| 20 | 2 | 48 | 480 | 0.00 | 0.014 |  | 48 | 3.96 |
| 20 | 3 | 48 | 480 | 0.00 | 0.023 |  | 48 | 5.62 |
| 30 | 2 | 48 | 480 | 0.00 | 0.079 |  | 48 | 37.19 |
| 30 | 3 | 48 | 392 | 0.34 | 0.053 |  | 48 | 68.27 |
| 40 | 2 | 48 | 480 | 0.00 | 0.069 |  | 16 | 107.30 |
| 40 | 3 | 48 | 381 | 0.39 | 0.074 |  | 10 | 132.61 |
| 60 | 2 | 48 | 354 | 0.42 | 0.189 |  | 9 | 265.78 |
| 60 | 3 | 48 | 335 | 0.56 | 0.274 |  | 2 | 324.43 |
| 90 | 2 | 48 | 351 | 0.50 | 0.190 |  | 0 | – |
| 90 | 3 | 48 | 321 | 0.55 | 0.329 |  | 0 | – |
|  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 6 Computational results of the Lingo with and without mode change. | | | | | |
| #Activities | #Modes | #Problems | P-MRCPSP-MC vs. P-MRCPSP |  |  |
|  |  |  | Average improvement (%) | Better | Equal |
| 20 | 2 | 48 | 4.32 | 46 | 2 |
| 20 | 3 | 48 | 5.76 | 48 | 0 |
| 30 | 2 | 48 | 5.11 | 45 | 3 |
| 30 | 3 | 48 | 6.85 | 47 | 1 |
| 40 | 2 | 16 | 7.64 | 15 | 1 |
| 40 | 3 | 10 | 9.46 | 10 | 0 |
| 60 | 2 | 9 | 8.39 | 9 | 0 |
| 60 | 3 | 2 | 9.82 | 2 | 0 |
|  |  |  |  |  |  |

than 30, while there are many instances that the Lingo is unable to solve, there is a solution by SA. However, Lingo obtained optimum solutions for 229 out of 480 problems in 1000 s and SA algorithm solved all problems with low relative deviation and in a very short time (70 ms per activity). Average CPU-time for Lingo indicates that when the number of execu- tion modes is increased the complexity of the problem is increased. ARD for the algorithm shows that proposed SA gives robust solutions. Also, NPM for the algorithm indicates that too many executions of problems reach the optimum solution.

To observe the statistical comparison of the differences between the results of the Lingo and SA, a paired *t*-test is used for 229 problems to which Lingo obtained optimum solutions and the corresponding 95% confidence interval is calculated as [—9.36, 7.82]. Since the lower confidence level is negative and the upper level is positive, then the null hypothesis cannot be rejected as the population mean of the differences could be zero. This implies that the differences between the quality of solutions obtained by Lingo and SA are not statistically significant.

In order to evaluate the effect of changeability assumption, for 229 problems which Lingo obtained optimum solutions, each problem has been solved without mode changeability assumption. [Table 6](#_bookmark17) presents the computational results. From [Table 6](#_bookmark17) it can be observed that mode changeability obviously leads to an overall average makespan improvement. [Table 6](#_bookmark17) also reveals that mode changeability usually leads to better solutions. Average Improvement (%) column shows that the percent loss due to using the P-MRCPSP model instead of the P-MRCPSP-MC is straightly relevant to number of activ- ities and execution modes.

1. Summary and conclusions

The preemptive multi-mode resource constrained project scheduling problem with permitted mode change (P- MPRCPSP-MC), is investigated in this paper. The objective of P-MPRCPSP-MC is to schedule the activities in order to minimize the project makespan subject to the precedence con- straints and resource constraints. In this problem setting, work content concept is used instead of duration. This problem has not been studied ever before. The problem described with an integer programing model, and then the parameters tuned Sim- ulated Annealing (SA) proposed to solve it. The performance of the proposed algorithm on 480 test problems was compared with the results of the Lingo 11. From the computation results, one could clearly see that the SA algorithm could efficiently solve the project scheduling problem. Also, one could find out that mode changeability obviously leads to an average makespan improvement. For further research, we recommend the adapting mode change concept for other extensions of multi-mode project scheduling problems.

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