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# Qualitative Analysis of a Biological-Chemical Reaction Model of Multi-Molecule Systems

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**Abstract**

This article discuss the limit cycle of the

 .

*p p*1

*x*  1  *x y p*  1, *p*  *Z* , *x*  0, *y*  0,**  0



.

 *y*  ** *y*(*xp y p* 1)

and branching problem of Hopf, whose parameter is Theta,giving the direction and stability Of Hopf branch and its approximate expression based on of small amplitude cycle.

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*Keywords****:***stability ,limit cycle ,Hopfbranch

##### One Introduction

Multi-molecular reaction model among Biochemical reactions is

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[ *A* ] *K*1  *A* , *A* *K*2  0

0 1 1

(Output)

*pA*  *qA* *K*3 ( *p*  *q*) *A*

(Adduct reaction) (1)

1 2 2

*A* *K*4  ( Output) The corresponding mathematical model is:

2

*dx*1  *k x*  *k x*  *pk x p xq*

*dt* 1 0 2 1 3 1 2

*dx*2  *pk x p xq*  *k x*

*dt* This article discusses the situation as follow In the model (1)

3 1 2 4 2

*q*  *p*  1, *p*  1, *p*  *Z* , *k*2  0

1. *k x*  1 *k x*  1 *k x*

Transfore to

**  [ 1 0 ] *p*

### *k x rqk p*

1 0 3

, **  ( 1 0 ) *p* ,

### *rqk p*

3

**  1 0

*k*4

*k k x*  1

**  *t*, *x*  ** *x*, *x*  ** *y* ,to **  4 ( 1 0 ) *p*

1 2

So Model (1) becomes into a systerm as follow

 *dx*  1  *x p y p*1  *P*( *x*, *y*)

 *dt*



*dx*

  ** ( *x p y p*  1)  *Q*( *x*, *y*)

 *dt*

### *k x rqk p*

*x*  0, *y*  0, *p*  1, *p*  *Z*,**  0

1 0 3

In the first quadrant (1)** only get a unique equilibrium,that is point(1,1),and the corresponding characteristic equation of the variational equations is

** 2  *p*(1  ** )**  ** *p*  0

(2)The characteristic roots the second equation is

(2)

**1,2

 1 ( *p*(1  ** ) 

#### 2

*p*2 (1  ** )2  4** *p*

It is obvious that the equilibrium of the system is unstable when

**  1 , Next we focuse on the qualitative form of the (1)** system.

##### Second, nonexistence of the periodic orbit

**  1 ,while the system is stable when

Theorem 1 the (1)** system does not have a limit cycle.when

1

**  2p

Proof: First , if (1)** systerm has limit cycle, then the limit cycle will intersect with the hyperbolic, notes that

The tangent of the system is

*x*  0 ,the solution is y=0, so the limit cycle of the system will not intersect

with two straight lines, so we use Dulac function

*B*( *x*, *y*)  *x* *p y*( *p*1)*e* *p*(** *x* *y* )

[*B*( *x*, *y*)*P*( *x*, *y*)]  *x*( *p*1)*y*( *p*2)*e* *p*(** *x* *y* ) ( *x p y p p* *xy*2  *py*  *p* *xy*)

*x*

[*B*( *x*, *y*)*Q*( *x*, *y*)]  *x*( *p*1)*y*( *p*2)*e* *p*(** *x* *y* ) ( *x p y p p* *xy*2  *p* *xy*  *p* *xy*2 )

*x*

(*BP*)  (*BQ*)  *x*( *p*1)*y*( *p*1)*e* *p*(** *x* *y* ) *p*(** *xy* 1)

*x* *y*

According to Dulac determine theorem, if the (1)** system has a limit cycle it certainly intersect with the hyperbola ** *xy* 1  0.

1

Next,when **  2p , limit cycle of (1)** system and the hyperboa

*L*  *y* 

## 1  0

### *p x*

are tangent and this

1

limit cycle will not intersect with (1)** system .Follow the trajectories of the (1)** system,

*dy*  1

*dL*

*dt*

* 1. *dx*

## 1 1 1 1

  2 *p*  (

** *x*2 

*x*  )

*L*0

### *dt x*2

*dt x*3 1 1 1

1 2

* 1. *p* 2 *p* 2

Here the following discriminant of quadratic algebraic equation,

 1 ** *x*2  1 *x*  1 )=0

1 1 1

1 2

1

when **  2p ,

2 *p* 2 *p* 2 *p*

## 1

2(1

2 *p*

1

** ) 0

2 *p*

  

1

Due to the hyperbola ** *xy* 1  0, when **  2p ,the hyperbola coincides with it in the bottom ,so

1

when **  2p , the limit cycle of (1)** can not intersect the hyperbola ** *xy* 1  0, so there is no limit cycle.

##### Hopf bifurcation

In order to discuss the Hopf bifurcation of the (1)** system,we transform the system into

*d*  *u*   *A* *u*     *f* (*u*, *v*) , ݊Ё *A*    *p*

( *p*  1

#### (1)

*dt*  *v*   *v*  ** *f* (*u*, *v*) 

** *p*

** *p*  **

       

*f* (*u*, *v*) 

*p*( *p*  1)

*u*2 +

*p*( *p*  1)

*v*2 +*p*( *p*  1)*uv* 

*p*2 ( *p*  1)

*uv*2 

*p*( *p*2  1)

*u*2*v*

## 2 2 2 2

 1 *p*( *p*  1() *p*  2)*u*3  1 *p*( *p*2  1)*v*3  *G*(*u*, *v*)

## 6 6

thereinto *G*(*u*, *v*) is a more than three times polynomial about *u*,*v* .we take



**  **(**) 1  **, **  ** (** )=** **

H *H i*

*i*

*i*2

(0  **  *H* ) as a bifurcation parameter, then a pair of conjugate eigenvalues of (2) are

**  **(**) **(**) **(**)*i* , **  **(**) **(**)–**(**)*i*

**(**)= 1 *p*,**(**)= 1

#### 2 2

4(1  **) *p*  *p*2** 2

**(0)=**  0,**( 0)= 1 *p*  0,**(0)=**  *p* ,**( 0)



1 *p*

2

0 2 0

Accordingly.we know that:

\Theorem 3 there is ** *H*  0, when 0  **  *H* ,

(1)

systerm

**

at least has one limit cycle near the

equilibrium point (1,1) (small amplitude periodic solution)

#### (1)

**

Next we discuss the stability of systerm

(**1,** 2)

when

*A*

**  0

,the direction and the stability of Hopf branch,at

**(0) **0*i* 



*pi*

this time, the feature vector

of matrix

(*p* 



which eigenvalue is

*pi*1  ( *p*  1)** 2  0

must fulfill

Then Select

 *p*1  ( *p*  *pi* 2  0

##  1 0 

**  1,**   1 ( *p*  *pi* ) ,And make *B*   *p p* 

1 2 *p*  1

  

 *p*  1 *p*  1 

 

Then transform,we

Get

 *v*   *y* 

   2 

 *u*   *B*  *y*1 

 *dy*1 

 *dt*   0

 *p*  *y* 

 *F*1( *y* , *y* ) 

    

## 1   

1 2 

(3)

 *dy*2 



*p*

2



0  *y*2   *F*

( *y*1, *y*2 ) 

Here we know

 *dt* 

1 *p*(2 *p*  1)



*p p*

2

*p*2 2

*F* ( *y*1, *y*2 ) 

2( *p*  1)

*y*1 

*p*  1 *y*1 *y*2  2( *p*  1) *y*2 

*p*5  *p*4  *p*3  *p*2  *p*

1 ( 2

3

## 1 5 1 4

## 2 3 1 2 1 2

## 3( *p*  1)2



*p*

*p*3

*y*  ( *p*   *p p*  1) 6 2

  *p*   *p*

## 3 2



*p*2 ( *p*  1) *p*

 2 *p*) *y*1 *y*2

##  ( *p*  1)2

1 2

2 1 2

*y y*2 

6( *p*  1)2

*y*3  *G*(*y* , *y* )

*F* 2 ( *y* , *y* )  *F*1( *y* , *y* )



1

*p*

1 2 1 2

Therein, *G*(*u*, *v*) is a more than three times polynomial about *u*,*v*

####  1 2*F*1  2*F*1 

2 *F* 2

2*F* 2 1 

*g*11

4 [ *y*2

*y*2

*i*( *y*2

+ *y*2

)]=  (*p i p* )

#### 4

1 2 1 2

## 1 2*F*1

2 *F*1

2 *F* 2

2 *F* 2

2 *F* 2

2 *F*1

1 3*p*2  *p*

*p*(5*p* 1)

*g*11  4 [ *y*2

 *y*2

 2

*y* *y*



 *i*( *y*2

 *y*2

+2

*y* *y*

)]=  (

## 4

*p*  1  *i*

*p*  1 )

1 2 1 2 1 2 1 2

####  1 2*F*1  2*F*1 

2 *F* 2 

2 *F* 2  2 *F* 2 

2*F*1 1 

*g*20

4 [ *y*2

*y*2

*y* *y*

*i*( *y*2

*y*2

2 *y* *y*

)]=  (3*p i p* )

#### 4

1 2 1 2 1 2 1 2

*C* (0)  *i* (*g g*  2 *g* 2  *g* 2 )  1 *g*



2 *p*

1

3

1 20 11 11 02

2 21

 *p*

16( *p*  1)2



*p*

[2( *p*  1)2  ( 5 *p*4  3 *p*3  4 *p*2  2 *p*  4)

3 3



## 1 3 1 1 5 4 3 3 15 2 11 1

## 16 2 2 (1  *p*)2 3 2 6 6 6

[

*p*  

(

*p*

 *p*



*p*



*p*



*p* 

)]*i*

** *H*   Re *C* 0/ ** ' 0  1 5 *p* 4  3 *p*3  4 *p* 2  2 *p*  4  2*p*  12    *f* *p*

2 1

Among which

8*P*  12 3 3

 8*p*  12

*f* *p*  5 *p* 4  3 *p* 3  4 *p* 2  2 *p*  4  2 *p*  12

As for 3 3

when *p*  *N* there is *f* *p*  0 ,

Therefore, for any *p*  *N* there is ** *H*  0 **  2 Re *C* 0  0

2

2 1

Im*C* 0  1 *p*  3  1  1 *p*5  *p* 4  3 *p*3  5 *p* 2  11 *p*  1 

*p*

## 1  2  

16  2

2 1  *p*  3

## 2 2 6

## 6 

**  Im*C* 0  ** *H * ' 0/ ** 

 *x* 

## 1

2



##  *y*1  

1 2

1  ** cos

0

/ *T*  *O*** 2  

*pt*



   

*pt*

*pt*



### *y*

  *B*

### *y*

  1   **  *p* cos

/ *T* 

*p* sin

/ *T*  *O*** 2 

4

##   1

**

 2  



*p*  1 

from

*H*  0, **

####  0,

see

4

we know that.

1

2

2

Theorem 4 system There is **  ** , when 1  **  **  2 *p* , 1** systerm has a stable limit cycle, and its approximate expression is given by 4 equation.

The above exposition shows that the model 1** of multi-cellular biochemical reactions,

when 0  **  **   1 , the equilibrium 1,1 is globally asymptotically stable in the first quadrant,

1 1

when1  **  **  2 *p* there is a stable oscillations, when **  2 *p* ,the oscillations disappear.

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