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ORIGINAL ARTICLE

An extension of fuzzy decision maps for multi-criteria decision-making

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Abstract This paper presents a new extension to Fuzzy Decision Maps (FDMs) by allowing use of fuzzy linguistic values to represent relative importance among criteria in the preference matrix as well as representing relative influence among criteria for computing the steady-state matrix in the stage of Fuzzy Cognitive Map (FCM). The proposed model is called the Linguistic Fuzzy Decision Networks (LFDNs). The proposed LFDN provides considerable flexibility to decision makers when solving real world Multi-Criteria Decision-Making (MCDM) problems. The performance of the proposed LFDN model is compared with the original FDM using a previously published case study. The result of comparison ensures the ability to draw the same decisions with a more realistic decision environment.

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KEYWORDS

Multi-Criteria Decision- Making (MCDM); Analytic network pro- cess(ANP);

Analytical hierarchical pro- cess(AHP);

Fuzzy cognitive maps (FCM);

Fuzzy Decision Maps (FDM)

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1. Introduction

Multi-Criteria Decision-Making (MCDM) problems are com- mon in everyday life. In personal context, a house or a car one buys may be characterized in terms of criteria such as: price, size, style, safety, and comfort. In business context, MCDM

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problems are more complicated and usually of large scale. Sometimes, MCDM problems are referred to as Multi-Criteria Decision Analysis (MCDA), Multi-Criteria Decision Aid (MCDA), or Multi-Attribute Decision-Making (MADM) [[1,2]](#_bookmark30).

MCDM refers to making preference decisions (such as eval- uation, prioritization, selection, and so on) over the available alternatives that are characterized by multiple, usually conflict- ing criteria [[1,3]](#_bookmark30).

According to many researchers, there are two distinctive types of MCDM problems, namely, Multi-Objective Deci- sion-Making (MODM) and Multi-Attribute Decision-Making (MADM) [[1–3]](#_bookmark30). When the feasible set of alternatives of a deci- sion consists of a finite number of elements that are explicitly known at the beginning of the solution process, these problems are called Multi-Attribute Decision-Making (MADM) [[1,2]](#_bookmark30).

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When the number of alternatives of a decision is uncountably

(*m*—*l*)

8> 0 *x* < *l*

infinite, the alternatives are not specified directly but are de-

>:

Triangle(*x* : *l*, *m*, *u*)=

(1)

(*u*—*x*)

> (*u*—*m*)

fined in terms of decision variables as is usually done in single

optimization problems like linear programming [[1,2]](#_bookmark30). This type

>< (*x*—*l*)

*l* ≤ *x* ≤ *m*

of problem is called a Multi-Objective Decision-Making (MODM).

*m* ≤ *x* ≤ *u*

Many different methods have been developed to solve MCDM. The Analytic Hierarchy Process (AHP) was first introduced by Thomas Saaty in the 1971, and it is one of the most famous methods of MCDM because it is simple and easy to use [[4,5]](#_bookmark31). AHP has been applied to solve many decision problems in political, economic, social, and management sci- ences [[5]](#_bookmark32). The Analytic Network Process (ANP) was developed by Saaty in 1996 to overcome the problem of dependence and feedback among criteria in AHP method [[6]](#_bookmark33). ANP is consid- ered as the general form of the AHP, which has been used for MCDM. The ANP has been applied in many practical decision-making problems, such as project selection, product planning, green supply chain management, and optimal sched- uling problems [[7]](#_bookmark34).

Fuzzy Decision Maps (FDMs) method was proposed in 2006 to deal with the structure of MCDM with dependency and feedback to overcome the complexity drawback of ANP [[7]](#_bookmark34). FDM method is a good technique for MCDM in real sit- uations. In FDM, human judgments for performing the pair- wise comparison and the causal relationship between criteria are usually given by exact numerical values. However, in many practical cases, numerical values are not suitable in the real world problems. Human judgments with preferences are often unclear and hard to be estimated by exact numerical values. A more sensible method is to use linguistic assessments instead of numerical values, in which all assessments of criteria in the problem are evaluated by means of linguistic values.

Fuzzy set theory was first proposed by Lotfi Zadeh in the 1965 [[8]](#_bookmark35). It is a methodology to tackle uncertainty and to han- dle imprecise information in real situations. With fuzzy sets theory, information can be expressed by linguistic or fuzzy val- ues rather than by numbers [[9]](#_bookmark36). In this paper, we propose a methodology that extends the FDM method by applying lin- guistic variables and a fuzzy aggregation method. We call this method Linguistic Fuzzy Decision Networks (LFDNs).

This paper is organized as follows: Section 2 presents fuzzy numbers and linguistic variables. Fuzzy Cognitive Map (FCM) is discussed in Section 3. Section 4 presents the FDM method. The proposed LFDN is introduced in Section 5. A case study is given in Section 6. Discussing the results is given in Section 7, and Section 8 concludes the paper.

1. Fuzzy numbers and linguistic variables

A Fuzzy Number (FN) is a fuzzy set which has some proper- ties such as being: convex and normal, its membership function is piecewise continuous and it is defined over ffi [[9]](#_bookmark36). FN can take various forms such as Triangular Fuzzy Number (TFN), Trapezoidal FN, and Gaussian FN. TFN is the most popular form due to its simple membership function which is represented by three parameters as (*l*, *m*, *u*). The parameters *l*, *m*, and *u* denote the smallest possible value, the most prom- ising value, and the largest possible value of the fuzzy number,

respectively. Let *A*~ = (*l*, *m*, *u*) be a triangular fuzzy number,

then its membership function l (*x*) is defined as follows:

e

*A*

0 *x* > *u*

*A*~1 = (*l*1, *m*1, *u*1), *A*~2 = (*l*2, *m*2, *u*2), and k is positive real num- The fuzzy arithmetic operations of two TFNs bers are defined as follows [[9]](#_bookmark36).

*A*—1 = 1/*A* = ( 1/*u*1, 1/*m*1, 1/*l*1 ) Where *l*1, *m*1, *u*1 > 0

1

e e

(2)

*A*1 + *A*2 = ( *l*1 + *l*2, *m*1 + *m*2, *u*1 + *u*2 ) (3)

e e

*A*1 — *A*2 = ( *l*1 — *u*2, *m*1 — *m*2, *u*1 — *l*2 ) (4)

e e

*A*1/*A*2 = ( *l*1/*u*2, *m*1/*m*2, *u*1/*l*2 ) (5)

e e

*A*1 × *A*2 = ( *l*1 · *l*2, *m*1 · *m*2, *u*1 · *u*2 ) (6)

e e

k × *A*1 = ( k · *l*1, k · *m*1, k · *u*1 ), k > 0 (7)

e

In some situations, it is required to reduce a given fuzzy num-

ber into a single crisp representative value. This is called defuzzification operation. There are several available defuzzifi- cation methods for this purpose such as: Mean Of Maximum (MOM) and centroid method (Center Of Gravity (COG) or Center Of Area (COA)) [[9]](#_bookmark36).

Lotfi Zadeh introduced the concept of linguistic variable in 1975, which allows computation with words instead of num- bers [[10,11]](#_bookmark37). Computing with words can be achieved when lin- guistic variables take linguistic values (i.e., values in the form of words or sentences from natural language). The concept of linguistic variables is very useful to describe situations that are too complex or not well defined in conventional quantita- tive expression. Furthermore, Linguistic variables allow the translation of natural language into logical or numerical state- ments, i.e., computing with words, which provides the tools for further theoretical application, such as fuzzy control and approximate reasoning which is also referred to as human rea- soning [[9,11]](#_bookmark36).

1. Fuzzy Cognitive Map (FCM)

Fuzzy Cognitive Map (FCM) was originally introduced by Kosko in 1986 [[12]](#_bookmark38) as an extension of cognitive map model. The most significant enhancement lies in the way of reflecting

causal relationships. Instead of using only the sign (+ or —),

**W12**

**C1**

**C2**

**W15**

**W25**

**W52**

**C5**

**W41**

**W33**

**W13**

**C3**

**W43**

**C4**

Figure 1 Fuzzy cognitive map [[12]](#_bookmark38).

each edge is associated with a number (weight) that determines the degree of considered causal relation between each two con- cepts. [Fig. 1](#_bookmark5) illustrates a graphical representation of a FCM. It could be considered as a signed weighted graph that consists of nodes and weighted arcs (or edges) with feedback. Each node of the graph stands for a concept that describes one of the behaviors of the system. The value of node (concept) changes over time, usually over the interval [0, 1]. Concepts are con- nected by signed and weighted arcs (Edges) representing the causal relationships between the concepts. Edges or arcs weight usually take values over the fuzzy causal interval

[—1, 1] or the interval [0, 1]. According to [[13]](#_bookmark39), expert describes

the causal relationship between concept by using linguistic

vals, i.e., [—1, 1], rather than the usual binary one, i.e., [0, 1]. variables or fuzzy terms that are defined over a bipolar inter- The linguistic value for each interconnection between two con-

cepts is reduced to a single crisp value through a defuzzifica- tion process. Thus, the weight for any interconnection is

simply crisp number that ranges over [—1, 1]. Consequently,

three possible states arise for each edge weight, first when

(*wij* = 0), which indicates no relationship between *Ci* and *Cj*, second, when (*wij* > 0) indicates direct causality between con- cepts *Ci* and *Cj*, that is, the increase (decrease) in the value of *Ci* leads to the increase (decrease) in the value of *Cj*, third, when (*wij* < 0) indicates inverse (negative) causality between concepts *Ci* and *Cj*. That is, the increase (decrease) in the value of *Ci* leads to the decrease (increase) in the value of *Cj*.

The influence of a specific concept to other concepts can be calculated using the following updating equation [[7,14,15]](#_bookmark34):

*C*(*t*+1) = *f*(*C*(*t*) · *E*), *C*(0) = *In*×*n* (8)

FCMs have advantages such as simplicity, flexibility, and effi- cacy for analyzing and modeling the real world as a collection of concept and causal relationships. This simplicity support experts or decision makers to better understand the underlying formal model and its execution [[17]](#_bookmark40). The main disadvantage of FCMs is the possibility to reach or to settle down in an unde- sired steady state [[18]](#_bookmark41). FCM have been applied in many appli- cations such as, geographic information systems (GISs), fault detection, decision-making, business management, text catego- rization, industrial analysis, and system control [[7,16,19]](#_bookmark34).

1. Fuzzy Decision Maps (FDMs)

FDM was proposed in 2006 for solving the MCDM problem with dependence and feedback. It incorporates the eigenvalue method, FCM, and the weighting equation to overcome the shortcoming of AHP and ANP method [[7]](#_bookmark34). The step of FDM method to derive the priorities of criteria can be summa- rized as follows [[7]](#_bookmark34).

*Step 1*: Compare the importance among criteria to derive the local weight vector using the eigenvalue approach.

*Step 2:* Depict the fuzzy cognitive map to indicate the influ- ence among criteria by the expert.

*Step 3*: Calculate the updating Eq. [(8)](#_bookmark6) to obtain the steady- state matrix.

*Step 4*: Derive the global weight vector. We should first nor- malize the local weight vector (*V*) and the steady-state matrix (*M*) as follows:

where *In*·*n* denotes the identity matrix, *E* = [*Wij*] is *n* · *n* 1 1

(12)

weight matrix, which gathers the values of causal edge weight between concepts *Ci* and *Cj*, *C*(*t*+1) and *C*(*t*) are the state matri- ces at iterations (*t* + 1) and (*t*), respectively, *C*(0) is the initial matrix, and *f* is a threshold transformation function that is used to normalize concept values to a certain binary or bipolar ranges [[13]](#_bookmark39). There are several formulas used as threshold trans- formation functions such as [[7,14,16]](#_bookmark34).

1 *if x* ≥ 1

*Vn* = *k V*, *Mn* = *c M*

where *k* is the largest element of *V* and *c* is the largest row sum

of *M*. Then, the global weight vector (*W*) can be computed as follows:

*W* = *Vn* + *MnVn* (13)

where *Vn* is the normalization of the local weight vector, and

*Mn* is the normalization of the steady-state matrix. Finally,

Hard limit function *f*(*x*)=

0 *if x* < 1

, *x* ∈ [0, 1] (9)

normalize the global weight (*W*).

The advantages of FDM [[7,14]](#_bookmark34) are overcoming the draw- backs of the AHP and ANP, employing the different threshold

Hyperbolic-tangent function *f*(*x*)= tan *h*(*x*)

= (1 — *e*—*x*)/(1 + *e*—*x*), *x* ∈ [—1, 1] (10)

Logistic function *f*(*x*)= 1/(1 + *e*—*x*), *x* ∈ [0, 1] (11)

When FCM starts execution, i.e., using the updating of Eq. [(8)](#_bookmark6),

the resulting transformed state matrix is then repeatedly mul- tiplied by FCM weight matrix and is transformed by a thresh- old function until the system (FCM) settles down to steady state which is one of the following cases[[7,14,15]](#_bookmark34).

* *A hidden pattern or fixed point attractor*: This case is reached when the FCM state matrix remains fixed for successive

iterations, for example, *A*1 → *A*2 → *A*3 → *A*3, where the matrix *A*3 is known as the fixed point attractor.

* *A limit cycle*: A sequence of FCM state matrix keeps repeat-

ing forming a cycle, for example, *A*1 → *A*2 → *A*3 → *A*4

→ *A*5 → *A*3 → *A*4 → *A*5 where the three matrix *A*3 → *A*4

→ *A*5 forming a cycle is known as the limit cycle.

functions to indicate the various kinds of relationship among criteria, dealing with the direct and indirect influences and the ability to solve both the compound and the interaction ef- fects. However, in the FDM, decision makers or experts must quantify a precise value to judge the ratios of weights between criteria, i.e., the process ignores the problem of uncertainty. Since, in practice, problems are usually complicated and uncer- tain, it is even hard for experts to quantify the precise weight values.

1. The proposed Linguistic Fuzzy Decision Networks (LFDNs)

The proposed LFDN uses linguistic values instead of crisp val- ues in computing of both of the pairwise comparison and the FCM stage in the original FDM model. Therefore, LFDN handles the practical uncertainty usually exists in real world MCDM problems in a more efficient and humanistic way rather than that of the FDM model.

|  |  |  |  |
| --- | --- | --- | --- |
| Table 1 Fuzzy preference scale. | | | |
| Intensity of fuzzy scale | Definition of linguistic values | Triangular fuzzy scale | Triangular fuzzy reciprocal scale |
| ~1 | Equal importance (EI) | (1, 1, 1) | (1, 1, 1) |
| ~2 | Equally moderate importance | (1, 2, 3) | (1/3, 1/2, 1) |
| ~3 | Moderate importance (MI) | (1, 3, 5) | (1/5, 1/3, 1) |
| ~4 | Moderately strong importance | (2, 4, 6) | (1/6, 1/4, 1/2) |
| ~5 | Strong importance (SI) | (3, 5, 7) | (1/7, 1/5, 1/3) |
| ~6 | Strongly very strong importance | (4, 6, 8) | (1/8, 1/6,1/4) |
| ~7 | Very strong importance (VSI) | (5, 7, 9) | (1/9, 1/7, 1/5) |
| ~8 | Very strongly extreme importance | (6, 8, 9) | (1/9, 1/8, 1/6) |
| ~9 | Extreme importance (EXI) | (7, 9, 9) | (1/9, 1/9, 1/7) |
|  |  |  |  |

Let us first introduce the concept of local division. If After preparing the fuzzy pairwise comparison matrix, the

*A* = (*l*1, *m*1, *u*1), *B* = (*l*2, *m*2, *u*2) are two triangular positive fuzzy numbers, then we define the local division operation as

e e

follows:

*A*/*B* = (*l*1/*l*2, *m*1/*m*2, *u*1/*u*2) (14)

e e

We introduce this concept to be used only for the case of nor-

malizing a given vector (or matrix) of TFN values. Otherwise,

approximate fuzzy eignvalue method is applied, in order to de- rive the fuzzy local weight vector (*V*) by:

* Sum up each row of fuzzy pairwise comparison matrix *P*~ to get the vector of fuzzy numbers that represents the priorities

e

among criteria without dependence or feedback

the division formula given in Eq. [(5)](#_bookmark4) is the standard one.

TFN. Thus, if *A* = (*l*, *m*, *u*) is a TFN, then it is defuzzified by We use a centroid method as a defuzzification for ranking a

e

computing its center of area as follows:

*V*e*i* =

*n*

*j*=1

X

*aij* =

*n*

*j*=1

X

*lij*,

*T*

*n*

*j*=1

X

*mij*,

*n*

*j*=1

X

*uij*

!, *i* = 1, .. . , *n* (17)

*D*(*A*e)= (*l* + *m* + *u*)/3 (15)

*V*e = [*V*e1, *V*e2, .. . , *V*e*n* ]

(18)

Now, the steps of LFDN method can be described as follows:

* 1. *Step 1: Derive the fuzzy local weight vector*

Elicit the fuzzy judgment matrix of pairwise comparison among the criteria of the decision system. This should be done by domain experts. TFNs are used in pairwise comparison to express the expert preference in the form of linguistics values.

* + - Defuzzifiy the obtained fuzzy vector using Eq. [(15)](#_bookmark9) to get

the highest rank fuzzy value ~*r* of the fuzzy local weight vec-

e

tor *V*

*D*(*r*~)= max(*D*(*V*)) (19)

e

* + - Normalize the fuzzy local weight vector using the local divi- sion formula in Eq. [(14)](#_bookmark10)

Let the preference matrix *P*e = [*aij*] where *P*e represents *n* · *n* e e

1

matrix and *aij* is the importance of criterion *Ci* w.r.t. criterion *Cj* according to the fuzzy preference scale shown in [Table 1](#_bookmark8). The corresponding membership functions can be depicted as shown in [Fig. 2](#_bookmark14).

e

The reciprocals, such as 1/3, 1/5, 1/7, and 1/9, indicate the opposite respectively of the values 3, 5, 7, and 9.

The fuzzy pairwise comparison matrix *P*~ with TFNs ap-

pears as follows:

2 (1, 1, 1) (*l*11, *m*11, *u*11) · · · (*l*1*n*, *m*1*n*, *u*1*n*) 3

6  7

( 1 , 1 ,  1 ) (1, 1, 1) · · · (*l*2*n*, *m*2*n*, *u*2*n*)

*Vn* = *r*~ *V* (20)

where *Vn* is the normalization of the fuzzy local weight vector.

* 1. *Step 2: Derive of the fuzzy influence weight matrix*

In this step, we introduce the modification of the updating Eq. [(8)](#_bookmark6) by using linguistic values in the form of TFNs, so the influence among criteria can be calculated using Eq.

1. as follows:

*P*e = 66

4

*u*11

*m*11

.

.

*l*11

. . . .

. 775

(16)

*C*e(*t*+1) = *F*e(*C*e(*t*) · *E*e), *C*~(0) = *I*e*n*×*n* (21)

( 1 , 1 ,  1 ) ( 1 , 1 ,  1 ) · · · (1, 1, 1)

e

.

.

where *I*e*n*×*n* denotes the identity matrix with TFNs, *E*e = (*W*f*ij*) is

*u*1*n*

**

*EI MI SI*

*VSI EXI*

1 3

5

7

9

1

*m*1*n*

*l*1*n*

*u*2*n*

*m*2*n*

*l*2*n*

Scale

e e e

causal edge weight between concepts *Ci* and *Cj*, *C*(*t*+1) and *C*(*t*) are the state matrices with TFNs at iterations (*t* + 1) *n* · *n* weight matrix with TFNs, which gathers the values of and (*t*), respectively, *C*(0) is the initial matrix with TFNs and

It is important to introduce the modified threshold trans- formation function using linguistic values in the form of TFNs. For example, the hyperbolic-tangent function in Eq.

*F*e is a threshold transformation function with TFNs.

e

[(10)](#_bookmark7) is modified to have the form given in Eq. [(22)](#_bookmark15):

*F*e(*xl*, *xm*, *xu*)= (1 — *e*—*xl* /1 + *e*—*xl* , 1 — *e*—*xm* /1 + *e*—*xm* ,

Figure 2 Membership functions of linguistic values of the fuzzy

preference scale.

1 — *e*—*xu* /1 + *e*—*xu* ) (22)

* Normalize the fuzzy steady-state matrix using Eq. [(14)](#_bookmark10)

Table 2 Linguistic values for causal relationships.

Linguistic values TFN Extremely low (EL) (0, 0.1, 0.2)

Very low (VL) (0.1, 0.2, 0.3)

Low (L) (0.2, 0.3, 0.4)

Medium low (ML) (0.3, 0.4, 0.5)

Medium (M) (0.4, 0.5, 0.6)

Medium high (MH) (0.5, 0.6, 0.7)

High (H) (0.6, 0.7, 0.8)

Very high (VH) (0.7, 0.8, 0.9)

Extremely high (EH) (0.8, 0.9, 1)

1

*C*e*n* = *k*~ *C*e

e

(26)

**

EL VL **L** ML **M** MH **H** VH EH

0

where *Cn* is the normalization of the fuzzy steady-state matrix.

* 1. *Step 3: Derive the fuzzy global weight vector*

The fuzzy global weight *W* can be driven by the following fuz- zy weighting equation

f

*W*f = *V*e*n* + *C*e*n V*e*n* (27)

where *V*e*n* is the normalization of the fuzzy local weight vector

e

and *Cn* is the normalization of the fuzzy steady-state matrix,

and finally, normalize the fuzzy global weight vector as follows:

* + - Defuzzifiy the fuzzy global weight vector *W*e by using Eq.

0.5

e

0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1

Influence

[(15)](#_bookmark9) to get the maximum fuzzy value ~k of the fuzzy global weight vector *W*

~

Figure 3 Membership functions of linguistic values for causal relationships.

The FCM can now be constructed with linguistic values to indicate the influence among criteria. The relationships be-

*D*(k)= max(*D*(*W*f)) (28)

* Normalize the fuzzy steady-state matrix using Eq. [(14)](#_bookmark10)

1

tween concepts are described using the degree of influence. Ex- perts describe this degree of influence using linguistic values as

*W*f*n* = ~k *W*f

(29)

TFNs, see [Table 2](#_bookmark16), and the corresponding membership func- tions are shown in [Fig. 3](#_bookmark17).

Thus, the FCM model can be applied using the updating Eq. [(21)](#_bookmark13) until it reaches a fuzzy steady-state matrix (*C*), which represents the causal relationship among criteria. Then, nor- malize the fuzzy steady-state matrix as follows:

e

* Sum up each row of the fuzzy steady-state matrix *C* to get the fuzzy number vector *S*e

e

X

where *Wn* is the normalization of the fuzzy global weight vec-

tor. Now, *Wn* can be defuzzified using Eq. [(15)](#_bookmark9) to get the cor- responding rank for each criterion.

f

f

1. Case study

In this section, the proposed LFDN model is tested using the same example given in [[7]](#_bookmark34) for testing the FDM model. In this example, a decision maker tries to select the best alternative

*S*e*i* =

X

X

X

*n*

*j*=1

*wij* =

*n*

*j*=1

*lij*,

*n*

*j*=1

*mij*,

*n*

*j*=1

*uij*

!, *i* = 1, ... , *n* (23)

according to five criteria. The original pairwise comparison matrix is given as shown in [Table 3](#_bookmark20). The Original FCM of the five criteria is shown in [Fig. 4](#_bookmark21), while the original matrix

of influence among criteria is given as shown in [Table 4](#_bookmark22).

*S*e = h*S*e , *S*e , ... , *S*e i*T* (24)

1

2

*n*

Now, we start to solve the above case study using our pro-

posed LFDN model as follows:

* Defuzzifiy the fuzzy vector *S* using Eq. [(15)](#_bookmark9) to get the high- est rank fuzzy value *k* of fuzzy number vector *S*e.

e

~

*D*(*k*)= max(*D*(*S*e)) (25)

~

* *The first step*: Relax the hard formulation values of the ori-

ginal pairwise comparison matrix, using the fuzzy scale, given in [Table 1](#_bookmark8), as TFNs linguistic values. The obtained comparison matrix with linguistic values is given in [Table 5](#_bookmark23). Empty cells correspond to locations of inverse linguistic val- ues which will be substituted automatically as reciprocal

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 3 Original pairwise comparison matrix (P) [[7]](#_bookmark34). | | | | | |
|  | Criterion1 | Criterion2 | Criterion3 | Criterion4 | Criterion5 |
| Criterion1 | 1 | 3 | 1 | 5 | 1/3 |
| Criterion2 | 1/3 | 1 | 1/3 | 3 | 3 |
| Criterion3 | 1 | 3 | 1 | 5 | 1/3 |
| Criterion4 | 1/5 | 1/3 | 1/5 | 1 | 1/2 |
| Criterion5 | 3 | 1/3 | 3 | 2 | 1 |
|  |  |  |  |  |  |

0.25

Criterion 1

0.35

0.5

0.3

Criterion 2

Criterion 5

0.45

0.15

0.35

Criterion 3

Criterion 4

0.45

*V*3 = (1 + 1+ 1 + 3+ 1/5, 1 + 3+ 1 + 5+ 1/3, 1+ 5 + 1+ 7 +1)

=(6.2,10.33,15)

e

*V*4 = (1/7 + 1/5 +1/7+ 1 + 1/3, 1/5 + 1/3+ 1/5+ 1+ 1/2, 1/3 + 1+ 1/3 + 1+ 1)

e

=(1.816,2.23,3.66)

*V*5 = (1 + 1/5+ 1 + 1+ 1, 3 + 1/3 + 3+ 2+ 1, 5+ 1 + 5+ 3 +1)

e

=(4.2,9.33,15)

So the fuzzy local weight vector (*V*e) is:

*V* =((6.2,10.33,15),(3.4,7.66,13),(6.2,10.33,15),(1.816,2.23,3.66),(4.2,9.33,15))

*T*

e

Then, using Eq. [(20)](#_bookmark12) to normalize the fuzzy local weight vector

(*V*e), we have:

*T*

Figure 4 Original fuzzy cognitive map for case study [[7]](#_bookmark34).

*V*e*n* =((1,1,1),(0.5483,0.7415,0.8666),(1,1,1),(0.2929,0.2158,0.244),(0.6774,0.9031,1))

fuzzy numbers. After substituting the corresponding TFNs and reciprocal TFNs, the final comparison matrix appears as shown in [Table 6](#_bookmark24).

By using [Table 1](#_bookmark8), we can rewrite pairwise comparison ma- trix *P*~ with TFN as.

By using Eq. [(17)](#_bookmark11), we can obtain the fuzzy local weights vector of each criterion:

– *The second step*: The original hard formulation values of the influence interaction among various criteria, as given in [Fig. 4](#_bookmark21) and [Table 4](#_bookmark22), are relaxed with the linguistic values of causal relationships and their corresponding TFNs forms as given in [Table 2](#_bookmark16). The FCM for the considered case study is shown in [Fig. 5](#_bookmark25). The obtained FCM matrix with influence linguistic values (*E*), and its form after substitution of the corresponding TFNs values are given in [Tables 7 and 8](#_bookmark26).

*V*e*i* =

e

e

5

*j*=1

X

*aij* =

5

*j*=1

X

*lij*,

5

*j*=1

X

*mij*,

5

*j*=1

X

*uij*

!, *i* = 1,...,*n*

Then, we calculate the updating Eq. [(21)](#_bookmark13). The hyperbolic- tangent transformation function with TFN as given in Eq.

*V*1 = (1 + 1+ 1 + 3+ 1/5, 1+ 3 + 1+ 5+ 1/3, 1+ 5 + 1+ 7 + 1)

=(6.2,10.33,15)

*V*2 = (1/5+ 1 +1/5+ 1 +1, 1/3 +1 + 1/3 + 3+ 3, 1 + 1+ 1 + 5+ 5)

e

=(3.4,7.66,13)

1. is used. After eleven iteration cycles of FCM model, i.e., *C*11, a fuzzy steady-state matrix, of the type hidden pattern or fixed point attractor, is reached. [Table 9](#_bookmark27) illustrates the ob-

tained fuzzy steady-state matrix (*C*e) with TFNs.

e

Table 6 Pairwise comparison matrix with TFN (*P*e).

Table 5 Pairwise comparison matrix with linguistic values (*P*e).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 4 Original FCM matrix with exact numerical values influence (E) [[7]](#_bookmark34). | | | | | |
|  | Criterion1 | Criterion2 | Criterion3 | Criterion4 | Criterion5 |
| Criterion1 | 0 | 0.35 | 0.35 | 0 | 0 |
| Criterion2 | 0 | 0 | 0 | 0 | 0.3 |
| Criterion3 | 0 | 0 | 0.15 | 0 | 0 |
| Criterion4 | 0.5 | 0 | 0.45 | 0 | 0 |
| Criterion5 | 0 | 0.45 | 0 | 0 | 0.25 |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Criterion1 | Criterion2 | Criterion3 | Criterion4 | Criterion5 |
| Criterion1 Criterion2 Criterion3 Criterion4 Criterion5 | Equal  Equal Moderate | Moderate Equal Moderate | Equal  Equal Moderate | Strong Moderate Strong Equal  Equally moderate | Moderate  Equal |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Criterion1 | Criterion2 | Criterion3 | Criterion4 | Criterion5 |
| Criterion1 | (1, 1, 1) | (1, 3, 5) | (1, 1, 1) | (3, 5, 7) | (1/5, 1/3, 1) |
| Criterion2 | (1/5, 1/3, 1) | (1, 1, 1) | (1/5, 1/3, 1) | (1, 3, 5) | (1, 3, 5) |
| Criterion3 | (1, 1, 1) | (1, 3, 5) | (1, 1, 1) | (3, 5, 7) | (1/5, 1/3, 1) |
| Criterion4 | (1/7, 1/5, 1/3) | (1/5, 1/3, 1) | (1/7, 1/5, 1/3) | (1, 1, 1) | (1/3, 1/2, 1) |
| Criterion5 | (1, 3, 5) | (1/5, 1/3, 1) | (1, 3, 5) | (1, 2, 3) | (1, 1, 1) |
|  |  |  |  |  |  |

Criterion 1

Low

Low

Low

Criterion 2

Criterion 5

Medium Low

Low

Medium Low

Extremely Low

Criterion 3

Criterion 4

Medium

Figure 5 A fuzzy cognitive map with linguistic values for case study.

Eq. [(26)](#_bookmark18) is used to normalize the fuzzy steady-state matrix

e

(*C*). [Table 10](#_bookmark28) shows the obtained normalization of fuzzy stea- dy-state matrix (*Cn*) with TFNs.

e

* The final step: We use Eq. [(27)](#_bookmark18) to derive the fuzzy global weights vector as follows:

*W* =((1.3629, 1.4799, 1.5697), (0.8583, 1.1828, 1.3905),

(1, 1.0811, 1.1307), (1.1077, 1.0296, 1.0688), (1.3023, 1.7392, 1.9438))*T*

f

Then, we use Eq. [(29)](#_bookmark19) to normalize the fuzzy global weight vec- tor (*W*) as follows:

f

*Wn* =((0.2420, 0.2272, 0.2209), (0.1524, 0.1816, 0.1957),

f

(0.1775, 0.1660, 0.1591), (0.1967, 0.1580, 0.1504), (0.2312, 0.2670, 0.2736))*T*

Then, Eq. [(15)](#_bookmark9) is used to defuzzifiy the normalization of fuzzy global weights vector to get the ranking among criteria as follows:

*D*(*Wn*)= (0.2300, 0.1766, 0.1675, 0.1683, 0.2573)

*T*

[f](#_bookmark29)

[Table 11](#_bookmark29) illustrates the ranking of fuzzy global weights

vector.

From [Table 11](#_bookmark29), it is clear that the highest priority is crite- rion 5, while the lowest one is criterion 3.

Table 10 The normalization of fuzzy steady-state matrix with TFN (*C*e*n* ).

Table 8 FCM matrix with TFN (*E*e).

Table 9 Fuzzy steady-state matrix with TFN (*C*e).

Table 7 FCM matrix with linguistic values influence (*E*).

Criterion1

Criterion2 Criterion3 Criterion4 Criterion5

Criterion1

None None None Medium None

Criterion2

e

Low

None None None

Medium low

Criterion3

Low None

Extremely low Medium low None

Criterion4

None None None None None

Criterion5

None Low None None Low

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Criterion1 | Criterion2 | Criterion3 | Criterion4 | Criterion5 |
| Criterion1 | (0, 0, 0) | (0.2, 0.3, 0.4) | (0.2, 0.3, 0.4) | (0, 0, 0) | (0, 0, 0) |
| Criterion2 | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0.2, 0.3, 0.4) |
| Criterion3 | (0, 0, 0) | (0, 0, 0) | (0, 0.1, 0.2) | (0, 0, 0) | (0, 0, 0) |
| Criterion4 | (0.4, 0.5, 0.6) | (0, 0, 0) | (0.3, 0.4, 0.5) | (0, 0, 0) | (0, 0, 0) |
| Criterion5 | (0, 0, 0) | (0.3, 0.4, 0.5) | (0, 0, 0) | (0, 0, 0) | (0.2, 0.3, 0.4) |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Criterion1 | Criterion2 | Criterion3 | Criterion4 | Criterion5 |
| Criterion1 | (0, 0, 0) | (0.1025, 0.1570, 0.2157) | (0.0996, 0.1565, 0.2182) | (0, 0, 0) | (0.0192, 0.0418, 0.0768) |
| Criterion2 | (0, 0, 0) | (0.0284, 0.0543, 0.0908) | (0, 0, 0) | (0, 0, 0) | (0.1899, 0.2718, 0.3645) |
| Criterion3 | (0, 0, 0) | (0, 0, 0) | (0, 0.0525, 0.1106) | (0, 0, 0) | (0, 0, 0) |
| Criterion4 | (0.1973, 0.2449, 0.2913) | (0.0203, 0.0387, 0.0638) | (0.1681, 0.2439, 0.3284) | (0, 0, 0) | (0.0038, 0.0103, 0.0228) |
| Criterion5 | (0, 0, 0) | (0.1891, 0.2689, 0.3561) | (0, 0, 0) | (0, 0, 0) | (0.2762, 0.3786, 0.4899) |
|  |  |  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Criterion1 | Criterion2 | Criterion3 | Criterion4 | Criterion5 |
| Criterion1 | (0, 0, 0) | (0.2202, 0.2425, 0.2549) | (0.2141, 0.2417, 0.2579) | (0, 0, 0) | (0.0412, 0.0646, 0.0908) |
| Criterion2 | (0, 0, 0) | (0.0611, 0.0838, 0.1073) | (0, 0, 0) | (0, 0, 0) | (0.4080, 0.4197, 0.4307) |
| Criterion3 | (0, 0, 0) | (0, 0, 0) | (0, 0.0811, 0.1307) | (0, 0, 0) | (0, 0, 0) |
| Criterion4 | (0.4241, 0.3782, 0.3442) | (0.0436, 0.0598, 0.0754) | (0.3612, 0.3766, 0.3881) | (0, 0, 0) | (0.0081, 0.0159, 0.0269) |
| Criterion5 | (0, 0, 0) | (0.4064, 0.4153, 0.4209) | (0, 0, 0) | (0, 0, 0) | (0.5935, 0.5846, 0.5790) |
|  |  |  |  |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| Table 11 The ranking of the normalized fuzzy global weights vector. | | | |
| Global weights | LFDN method |  |  |
|  | Fuzzy value | Crisp value (COA) | Ranking |
| Criterion1 | (0.2420, 0.2272, 0.2209) | 0.2300 | 2 |
| Criterion2 | (0.1524, 0.1816, 0.1957) | 0.1766 | 3 |
| Criterion3 | (0.1775, 0.1660, 0.1591) | 0.1675 | 5 |
| Criterion4 | (0.1967, 0.1580, 0.1504) | 0.1683 | 4 |
| Criterion5 | (0.2312, 0.2670, 0.2736) | 0.2573 | 1 |
|  |  |  |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 12 The comparison of proposed method and FDM method. | | | | | |
| Global weights | FDM method[[7]](#_bookmark34)  Crisp value | Ranking |  | LFDN method  Crisp value (COA) | Ranking |
| Criterion1 | 0.2238 | 2 |  | 0.2300 | 2 |
| Criterion2 | 0.1842 | 3 |  | 0.1766 | 3 |
| Criterion3 | 0.1516 | 5 |  | 0.1675 | 5 |
| Criterion4 | 0.1654 | 4 |  | 0.1683 | 4 |
| Criterion5 | 0.2751 | 1 |  | 0.2573 | 1 |
|  |  |  |  |  |  |

1. Discussions

In this paper, we introduce the proposed Linguistic Fuzzy Decision Networks (LFDNs) that combines approximate fuzzy eigenvalue method, FCM with linguistic values and fuzzy weighting equation to deal with MCDM problems. The fuzzy local weights are first derived by using the approximate fuzzy eigenvalue method. Then, the FCM with linguistic values is verified to describe the influence among criteria. Next, the fuzzy steady-state matrix is calculated by using the modification of the updating equation. Finally, the fuzzy global weights vector is derived by using the fuz- zy weighting equation.

[Table 12](#_bookmark29) shows the comparison of the results obtained by both of the original FDM method [[7]](#_bookmark34), and the proposed LFDN method when solving the considered case study. It is clear that we have the same result but with more ability to han- dle uncertainty expressed as linguistic values which provides great flexibility for decision makers when dealing with real MCDM problems. Thus, to summarize, the proposed LFDN is a direct extension of the FDM model with the ability to han- dle uncertainties in the form of linguistic values.

1. Conclusion

Fuzzy Decision Maps (FDMs) approaches have been devel- oped to overcome the difficulties of Analytical Network process (ANP) for solving MCDM problems with depen- dence and feedback. The construction of inner influences among criteria in ANP represents a hard task for decision makers. FDM adopt the methodology of fuzzy cognitive maps (FCM) to simplify getting the final preferential matrix of criteria with inner influences. However, FDM request the decision maker to use crisp weight values to indicate the relative influences among criteria which is not only difficult but also impractical. In this paper, the proposed Linguistic Fuzzy Decision Networks approach (LFDNs) is a direct

extension of the original FDM to allow handling uncertain- ties in the form of linguistic values. The proposed model is tested using a case study that is also used for testing FDM in [[7]](#_bookmark34). The results give the same ranking of criteria which ensures the ability of LFDN to solve MCDM problems with dependence and feedback in a more flexible and humanistic manner.

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