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Dynamic Epistemic Logic with Communication Actions

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**Abstract**

This work proposes a Dynamic Epistemic Logic with Communication Actions that can be performed con- currently. Unlike Concurrent Epistemic Action Logic introduced by Ditmarsch, Hoek and Kooi [[15](#_bookmark47)], where the concurrency mechanism is the so called *true concurrency*, here we use an approach based on process calculus, like CCS and CSP, and Action Models Logic. Our approach makes possible the proof of soundness, completeness and decidability, different from the others approaches. We present an axiomatisation and show that the proof of soundness, completeness and decidability can be done using a reduction method.

*Keywords:* Epistemic Logic, Dynamic Logic, Action Models, Dynamic Epistemic Logic, Concurrent Actions, Communication Action.

# Introduction

Multi-Agent Epistemic Logic has been investigated in Computer Science [[6](#_bookmark36)] to repre- sent and reason about agents (or groups of agents’) knowledge and beliefs. Dynamic Logic aims to reason about actions (programs) and their effects [[8](#_bookmark37)]. Dynamic Epis- temic Logic [[16](#_bookmark48)] is conceived to reason about actions that change agents (or groups of agents’) epistemic state, i.e., actions which change agent’s knowledge and beliefs. The first Dynamic Epistemic Logic was proposed independently by [[11](#_bookmark43)] and [[7]](#_bookmark38)

it is called Public Announcement Logic(PAL) . There are many other approaches but the one that is used in this work is the Action Model Logic proposed by [[1,](#_bookmark33) [2](#_bookmark34)].

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Concurrent Dynamic Epistemic Logic was introduced in [[15](#_bookmark47)] and it was intended to extend Epistemic Action Logic proposed by Van Ditmarsch in [[14](#_bookmark46)] with con- current epistemic actions. In this extension they use a mechanism to deal with concurrency called ”true concurrency” which is inspired on the Concurrent Propo- sitional Dynamic Logic proposed by Peleg in [[10](#_bookmark41)]. An interesting work, entitled Logics of Communication and Knowledge, presented in [[12](#_bookmark44)], proposes a framework for modelling message passing situations that combines properties of dynamic epis- temic semantics and history-based approaches, which consists of Kripke models with records of sent messages in their valuations. Another work that inspired us to represent communication actions as private epistemic action is [[7](#_bookmark38)].

Example: Consider that there are two students waiting for a message from a teacher to send back the homework and that one student does not know if the other received or responded the message. To represent this we need to model the following actions: teacher sending the message (send action), each student receiving (receive action) and responding (response action) the message independently. We also need to guarantee that: the receive action can not be performed before the send action, the response action can not be performed before the receive action and the students actions can be performed concurrently. Can we model this using Action Models Logic? Since this is a very small example one can argue that this can done by using pre conditions and non deterministic choice to model all the possible paths. Now imagine the same situation with 100 students. It would be not so easy to model.

This work proposes a way to deal with concurrency and communication with Dynamic Epistemic Logic. We use an approach based on action models and process calculus, like CCS and CSP, which allow us to prove soundness, completeness and decidability. Different from [[15](#_bookmark47)], that implements concurrency on top of Epistemic Action Logic, we extends Action Models to deal with concurrency and communica- tion. The proofs of soundness, completeness and decidability can be done using a reduction method.

In order to facilitate the proof of soundness, completeness, and decidability we restricted our concurrency approach. We do not deal with ”true concurrency” like in [[15](#_bookmark47)]. Instead, we adopt the interleaving (non-deterministic choices of all possible paths) approach used in process algebras like CCS and CSP. Since we are based on Action Models we can use the pre-conditions to restrict actions that must be executed after another action. We do not deal with Common Knowledge, because this would make the proofs a little more tricky.

In sections [2,](#_bookmark1) [3](#_bookmark5) and [4](#_bookmark15) we give a brief introduction to Multi-agent Epistemic Logic, Action Model Logic and Concurrent Dynamic Epistemic Logic. Next we present the Dynamic Epistemic Logic that we propose in this paper. The last section is the conclusion.

# Multi-Agent Epistemic Logic

This section presents the Multi-Agent Epistemic Logic **S5a**. All the definitions and theorems of this section are based on [[16](#_bookmark48)].

* 1. *Language and Semantics*

**Definition 2.1** The Epistemic language consists of a countable set Φ of proposition symbols, a finite set *A* of agents, a modality *Ka* for each agent *a* and the boolean connectives *¬* and *∧*. The formulas are defined as follows:

*ϕ* ::= *p |T| ¬ϕ | ϕ*1 *∧ ϕ*2 *| Kaϕ*

where *p ∈* Φ, *a ∈ A*.

**Definition 2.2** A multi-agent epistemic *frame* is a tuple *F* = (*S, Ra*) where:

* *S* is a non-empty set of states;
* *Ra* is a binary relation over *S*, for each agent *a ∈ A*;

**Definition 2.3** A multi-agent epistemic *model* is a pair *M* = (*F,* **V**), where *F* is a frame and **V** is a valuation function **V** : Φ *→* 2*S*. We call a rooted multi-agent epistemic model (*M, s*) an epistemic state.

**Definition 2.4** Given a multi-agent epistemic model *M* = *⟨*(*S, Ra*)*,* **V***⟩*. The no- tion of satisfaction *M,s |*= *ϕ* is defined as follows:

1. *M,s |*= *p* iff *s ∈* **V**(*p*)
2. *M,s |*= *¬φ* iff *M,s |*= *φ*
3. *M,s |*= *φ ∧ ψ* iff *M,s |*= *φ* and *M,s |*= *ψ*
4. *M,s |*= *Kaφ* iff for all *sj ∈ S* : *sRasj ⇒ M, sj |*= *φ*
   1. *Axiomatisation*
5. *All instantiations of propositional tautologies*,
6. *Ka*(*ϕ → ψ*) *→* (*Kaϕ → Kaψ*),
7. *Kaϕ → ϕ*,
8. *Kaϕ* *→ KaKaϕ* (+ *introspection*),
9. *¬Kaϕ → Ka¬Kaϕ* (*− introspection*),

## Inference Rules

M.P. *ϕ, ϕ → ψ/ψ* U.G. *ϕ/Kaϕ*

**Theorem 2.5 S5a** *is sound and complete w.r.t its semantics.*

**Example 2.6** This example is from [[16](#_bookmark48)].

Suppose we have a card game with three cards: **0**, **1** and **2**, and three players **a**, **b** and **c**. Each player receives a card and do not know the other players cards.

We use proposition symbols 0*x,* 1*x,* 2*x* for *x ∈ {***a***,* **b***,* **c***}* meaning “player *x* has card **0**, **1** or **2**”. We name each state by the cards that each player has in that state, for instance 012 is the state where player **a** has card **0**, player **b** has card **1** and

player **c** has card **2** [3](#_bookmark9) . The following epsitemic model represents the epistemic state of each agent [4](#_bookmark10) .

*Hexa*1= *⟨*(*S, R*)*,* **V***⟩*:

* *S* = *{*012*,* 021*,* 102*,* 120*,* 201*,* 210*}*
* *R* = *{*(012*,* 012)*,* (012*,* 021)*,* (021*,* 021)*,... }*
* **V**(0*a*)= *{*012*,* 021*}*, **V**(1*a*)= *{*102*,* 120*}*, ...

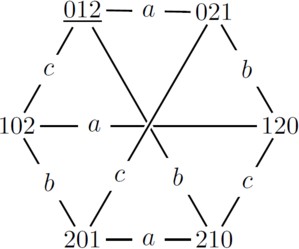


Fig. 1. Epistemic Model *Hexa*1

# Action Models

All the definitions and theorems of this section are based on [[16](#_bookmark48)].

* 1. *Language and Semantics*

**Definition 3.1** *An action model* M *is a structure ⟨*S*, ∼a,* pre*⟩, where:*

* S *is a* finite *domain of action points or events;*
* *∼a is an equivalence relation on* S*, for each agent a ∈ A;*
* pre : S *'→L is a precondition function that assigns a precondition to each* s *∈* S*. Rooted action models is an action model with a distinguished state* (M*,* s)*.*

*Note that* S *is different from S,* M *is different from M and* s *is different from s.*

**Definition 3.2** *The Action Model language consists of a countable set* Φ *of propo- sition symbols, a ﬁnite set A of agents, the boolean connectives ¬ and ∧, a modality* *Ka for each agent a ∈A and a modality* [*α*] *. The formulas are deﬁned as follows:*

*ϕ* ::= *p |T| ¬ϕ | ϕ*1 *∧ ϕ*2 *| Kaϕ |* [*α*]*ϕ, α* ::= (M*,* s) *| α*1; *α*2 *| α*1 *∪ α*2

*where p ∈* Φ*, a ∈ A,* (M*,* s) *a rooted action model and ⟨α⟩↔ ¬*[*¬α*]

**Definition 3.3** *Given an epistemic state* (*M, s*) *with M* = *⟨*(*S, Ra*)*,* **V***⟩ and a rooted action model* (M*,* s) *with* M = *⟨*S*, ∼a,* pre*⟩. The result of executing* (M*,* s) *in* (*M, s*) *is* (*M⊗* M*,* (*s,* s)) *where M⊗* M = *⟨*(*Sj, Raj* )*,* **V***j⟩ such that:*

3 A state name underlined means current state

4 We omit the reflexive loops in the picture.

1. *Sj* = *{*(*s,* s) *such that s ∈ S,* s *∈* S*, and M,s |*= pre(s)*}*
2. (*s,* s)*Raj* (*t,* t) *iff* (*s Ra t and* s *∼a* t)
3. (*s,* s) *∈* **V***j*(*p*) *iff s ∈* **V**(*p*)

## Definition 3.4 Composition of rooted action models

*Given rooted action models* (M*,* s) *with* M = *⟨*S*, ∼,* pre*⟩ and* (M*j,* s*j*) *with* M*j* =

*⟨*S*j, ∼j,* pre*j⟩, their composition is the action model* (M; M*j,* (s*,* s*j*)) *with* M; M*j* =

*⟨*S*jj, ∼jj,* pre*jj⟩:*

* + S*jj* = *{*(s*,* s*j*) *such that* s *∈* S*,* s*j ∈* S*j }*
  + (s*,* s*j*) *∼ajj* (t*,* t*j*) *iff* (s *∼a* t *and* s*j ∼ja* t*j*)
  + pre*jj*(s*,* s*j*)= *⟨*(M*,* s)*⟩*pre*j*(s*j*)

**Definition 3.5** *Given a rooted epistemic state* (*M, s*) *with M* = *⟨*(*S, Ra*)*,* **V***⟩ and a rooted action model* (M*,* s) *with* M = *⟨*S*, ∼,* pre*⟩. The notion of satisfaction M,s |*= *ϕ extends from* [*2.4*](#_bookmark2) *and is deﬁned as follows*

***1,2,3, 4*** *as in deﬁnition* [*2.4*](#_bookmark2)

1. *M,s |*= [(M*,* s)]*φ iff M,s |*= pre(s) *⇒M ⊗* M*,* (*s,* s) *|*= *φ*
2. J*α ∪ β*) *iff* J*α*) *∪* J*β*)
3. J(M*,* s); (M*j,* s*j*)) *iff* (M; M*j,* (s*,* s*j*)) *Composition of action models Where* J*.*) *is the interpretation on a action model.*
   1. *Axiomatisation*

## Epistemic Logic Axioms

*Axioms (i), (ii), (iii), (iv) and (v) of section* [*2.2*](#_bookmark3),

## Action Model Logic Axioms

1. [(M*,* s)]*p ↔* (pre(s) *→ p*),
2. [(M*,* s)]*¬φ ↔* (pre(s) *→ ¬*[(M*,* s)]*φ*)
3. [(M*,* s)](*φ ∧ ψ*) *↔* ([(M*,* s)]*φ ∧* [(M*,* s)]*ψ*)

V

1. [(M*,* s)]*Kaφ ↔* (pre(s) *→* s*~a*t *Ka*[(M*,* t)]*φ*)
2. [(M*,* s)][(M*j,* s*j*)]*φ ↔* [(M*,* s); (M*j,* s*j*)]*φ*
3. [(M*,* s) *∪* (M*j,* s*j*)]*φ ↔* [(M*,* s)]*φ ∧* [(M*j,* s*j*)]*φ*

## Inference Rules

M.P. *ϕ, ϕ → ψ/ψ* U.G. *ϕ/Kaϕ ϕ/*[*α*]*ϕ*

Every formula in the language of action model logic without common knowledge is equivalent to a formula in the language of epistemic logic [[16](#_bookmark48)].

## Example 3.6 Continuation of example [2.6](#_bookmark4)

Suppose now agent **a** wants to perform the action of showing her card to agent

**b**. In fact, we have three actions, agent **a** showing either card **0**, **1** or **2** to agent

**b**. Agents **a** and **b** can distinguish between these three action but agent **c** cannot. This situation can be represented by the action model below.

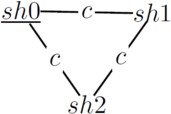


Fig. 2. Action Model for *show*

* S = *{*sh0*,* sh1*,* sh2*}*
* *∼a* = *{*(s*,* s) *|* s *∈* S*}*
* *∼b* = *{*(s*,* s) *|* s *∈* S*}*
* *∼c* = S *×* S
* pre(sh0)= 0*a*
* pre(sh1)= 1*a*
* pre(sh2)= 2*a*

If agent **a** performs the action of showing her card to agent **b** on the epistemic model of example [2.6](#_bookmark4), we obtain:

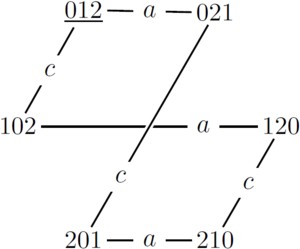


Fig. 3. *Hexa*1 After the Execution of *show*

This new epistemic model, shown in figure [3](#_bookmark16), is obtained by the product of epistemic model of figure [1](#_bookmark6) with the action model of figure [2](#_bookmark14). It is important to notice that the number of states after the product is 18 (6 *×* 3), but most of them are thrown out because they do not satisfy the precondition.

# Epistemic Actions and Concurrent Dynamic Epis- temic Logic

This section provides a brief introduction to the works presented in [[15](#_bookmark47)] and [[16](#_bookmark48)].

Epistemic Actions is an extension of Multi-Agent Epistemic Logic to deal with new information (updates), like Action Models, but it uses a different approach to deal with new information. Concurrent Dynamic Epistemic Logic proposes a way to deal with concurrency in Epistemic Actions.

* 1. *Language and Semantics*

**Definition 4.1** *The Epistemic Actions language consists of a countable set* Φ *of proposition symbols, a ﬁnite set A of agents, the boolean connectives ¬ and ∧, a modality Ka for each agent a ∈A and a modality* [*α*] *. The formulas and the actions are deﬁned as follows:*

*ϕ* ::= *p |T| ¬ϕ | ϕ*1 *∧ ϕ*2 *| Kaϕ |* [*α*]*ϕ,*

*α* ::=?*α | L5β |* (*α*!*α*) *|* (*α¡α*) *|* (*α*; *β*) *|* (*α*1 *∪ α*2)

*where p ∈* Φ*, a ∈ A, B⊆ A, L stands for learning and L5β means ’group B learn that β,* ?*α is a test,* (*α*!*α*) *is called left local choice,* (*α¡α*) *is called right local choice,* (*α*; *β*) *is sequential composition (ﬁrst α then β),* (*α*1 *∪ α*2) *is non-deterministic choice.*

**Definition 4.2** *Given the epistemic model M* = *⟨S, ∼a,V ⟩ and the state s ∈ S. The notion of satisfaction M,s |*= *ϕ extends from* [*2.4*](#_bookmark2) *and is deﬁned as follows*

***1,2,3, 4*** *as in deﬁnition* [*2.4*](#_bookmark2)

1. *M,s |*= [*α*]*φ iff for all* (*Mj, sj*): (*M, s*)[*α*](*Mj, sj*) *implies* (*Mj, sj*) *|*= *φ*
2. (*M, s*)[?*φ*](*Mj, sj*) *iff Mj* = *⟨*[*φ*]*M , ∅,V ∩* [*φ*]*M ⟩ and sj* = *s*
3. (*M, s*)[*LGφ*](*Mj, sj*) *iff Mj* = *⟨Sj, ∼j,V j⟩ and* (*M, s*)[*φ*]*sj*
4. J*α*; *αj*) *=* J*α*) *◦* J*αj*)
5. J*α ∪ αj*) *=* J*α*) *∪* J*αj*)
6. J*α*!*αj*) *=* J*α*)

The Concurrent Dynamic Epistemic Logic language adds the concurrent execu- tion operator to the actions of Epistemic Actions language. The actions are defined as follows:

*α* ::=?*α | L5β |* (*α*!*α*) *|* (*α*¡*α*) *|* (*α*; *β*) *|* (*α*1 *∪ α*2) *|* (*α*1 *∩ α*2) where (*α*1 *∩ α*2) represents a concurrent execution.

**Example 4.3** In order to illustrate the use of the language of Epistemic Actions, we consider the card game presented in section [3.](#_bookmark5)

The Epistemic Model is the same shown in figure [1.](#_bookmark6)

The action of “agent **a** showing her card to agent **b**” can be model as: (*L*(*b*)?0*a ∪ L*(*b*)?1*a ∪ L*(*b*)?2*a*); (*L*(*a,b,c*)?(*Kb*0*a ∨ Kb*1*a ∨ Kb*2*a*))

This means that agent **a** tells agent **b** her card and after that all agents know

that agent **b** knows the card that agent **a** holds. After performing this action, the resulting epistemic model is the same as in figure [3.](#_bookmark16)

# Dynamic Epistemic Logic with Communication Ac- tions

* 1. *Process Calculus*

In this section, we propose a very small process (program) calculus for the programs of Dynamic Epistemic Logic with Communication Actions (DELWCA). It is inspired by [[17](#_bookmark49)].

Let *A* = *{*1*, ..., n}*, denoted by *i, j...*, be a finite set of agents, AMS=*{a*1*, a*2*, a*3 *.. .}* be a finite set of action models and *N* =

*{c*1*, c*2*, c*3*,..., c*1*, c*2*, c*3*,.. .}* be a finite set of communication actions. As a convention, communication actions with one overline represent output and with no overlines represent an input. Communication actions can be combined to form a private action model, by joining an output communication action with its respective input ( [*c*1*, c*1] = *a*1 ). The action model resultant of the join of two communication actions is known as silent action, denoted by *τs* (*.*), that can be interpreted as the result of a communication between agents *i* and *j* [5](#_bookmark19) .

*i,j*

**Definition 5.1** *The language can be deﬁned as follows.*

*η* ::= *α | α.η | η*1; *η*2 *| η*1 + *η*2*, where α ∈ AMS ∪N*

*π* ::= *η | β.π | π*1; *π*2 *| π*1 + *π*2 *| η*1  *η*2 *··· * *ηn*

*where n* = *|A| and ηi denotes the program performed by agent i.*

*We use π and η to denote processes (programs) and α and β to denote action models and communication actions.*

*The* prefix *operator . denotes that the process will ﬁrst perform the action α and then behave as π. The* summation *(or* nondeterministic choice*) operator* + *denotes that the process will make a nondeterministic choice to behave as either π*1 *or π*2*. The* parallel composition *operator * *denotes that the processes η*1*, ..., ηn, performed by agents* 1*, ..., n respectively, may proceed independently or may commu- nicate through a common channel.*

We write *π →α πj* to express that the process *π* can perform the action *α* and

after that behave as *πj*. We write *π →α √* to express that the process *π* successfully

finishes after performing the action *α*. A process finishes when there is no possible

*β √*

action left for it to perform. For example, *β →* . When a process finishes inside

a parallel composition, sequential composition or non-deterministic choice we write

*π* instead of *π|√*, *π*; *√* and *π* + *√*. We also write *√* instead of *√|√*.

Like [[9](#_bookmark42)] we need to restrict the agents to perform some actions. In our case we don’t want to perform communication actions, but we can perform *τ* action which results from the combination of communication actions (*a, a*).

*i,j*

5 As silent actions *τs* (*.*) are interpreted as private action models, the index *s* denotes the root of the action

model *τs* (*.*).

*i,j*

The semantics of our process calculus can be given by the transition rules pre-

sented in table [1](#_bookmark20), where *π* and *η* are process specifications, while *πj* and *ηj* are process specifications or *√*. The *τs* (*.*) action represents an internal communication

*i,j*

action from agent *i* to agent *j*.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *α α √*  *→* | | | | | | | | | |
| *α.π α π*  *→* | | | | | | | | | |
| *π*1 *α ′*  *→ π*1  *α ′*  *π*1; *π*2 *→ π*1; *π*2 | | | | | | | | | |
| *π*1 *α ′*  *→ π*1  *π*1 + *π*2 *α π′*  *→* 1 | | | | | | | | | |
| *β ′*  *π*2 *→ π*2  *β ′*  *π*1 + *π*2 *→ π*2 | | | | | | | | | |
| (*η*1 | *...* | *ηi* | *...* | *α*  *ηi → η′*  *i*  *ηn*) *α* (*η*1  *→* | *...* | *η′*  *i* | *...* | *ηn*) | *, for all i, j ∈A* |
| *c c*  *ηi → η′, ηj → η′*  *i j , for all i, j ∈A*  *τi,j* (*.*) *′ ′*  (*η*1 *... ηi ... ηj ... ηn*) *→* (*η*1 *... ηi ... ηj ... ηn*) | | | | | | | | | |

Table 1 Transition Relation

**Example 5.2** Continuation of the card game example.

Now suppose that the game is online and player **a** sends a message *p* to players **b** and **c**. So after the message *p* players **b** and **c** know all the cards. This problem can be modelled as follows:

* *π*1 = *cab*(*p*); *cac*(*p*)+ *cac*(*p*); *cab*(*p*)
* *π*2 = *cab*(*.*)*.β*
* *π*3 = *cac*(*.*)*.γ*
* *π*1  2  3 = (*π*1  *π*2  *π*3)

Given the programs *π*1*, π*2 and *π*3, initially we have two possible actions: com- munication between **a** and **b** or communication between **a** and **c**. Suppose that the communication between **a** and **b** occurs first, then we will have two possible actions: communication between **a** and **c** or action *β* and so on ...

We can represent this using parallel composition:

*π*1  *π*2  *π*3

\_*c* z*\_*

*τab*

*τac*

*◦*

*◦*

*β*

*j*

*τac*

*τab*

*j*

*γ*

*◦*

*τac*

*◦*

*γ*

*γ*

*j*

*◦*

*γ*

v*z*

*◦*

*β*

*j*

*γ*

v*z*

*◦*

*β*

*◦*

*β*

*◦*

v*z*

*◦*

*β*

*τab*

v*z*

*◦ ◦*

*γ β*

*◦ ◦ ◦ ◦ ◦ ◦*

Fig. 4. Possible Runs of Process *π*1  *π*2  *π*3

So :

* *⟨π*1  2  3*⟩.*(*K*2*p ∧ K*3*p*) is true
* *⟨π*1  2  3*⟩.*(*K*2*p ∨ K*3*p*) is true
* *⟨π*1  2  3*⟩.¬*(*K*2*p ∨ K*3*p*) is false
  1. *Bisimulation*

The concept of bisimulation is a key notion in any process algebra. It is an equiv- alence relation between processes which have mutually similar behavior. The intu- ition is that two bisimilar processes cannot be distinguished by an external observer. Using the notion of bisimulation allows us to transform any process in an equivalent one that is a summation of all their possible actions, that is what the Expansion Law (theorem [5.5](#_bookmark23)) states.tab:semccs

There are two possible semantics for the *τ* action in CCS: it can be regarded as being observable, in the same way as the communication actions, or it can be regarded as being invisible. We adopt the first one, since it is more generic and fits better in our formalism. Whenever the *τ* action is observable the bisimulation relation is called *strong*.

**Definition 5.3 ( [**[**9**](#_bookmark42)**])** *Let* Π *be the set of all processes. A set Z ⊆* Π *×* Π *is a*

strong bisimulation *if* (*π*1*, π*2) *∈ Z implies the following for all α ∈ AMS :*

*α*

*α*

* *If π*1 *→ πj , then there is πj ∈* Π *such that π*2 *→ πj and* (*πj , πj* ) *∈ Z;*

1

2

2

1

2

*α*

*α*

* *If π*2 *→ πj , then there is πj ∈* Π *such that π*1 *→ πj and* (*πj , πj* ) *∈ Z;*

2

1

1

1

2

* *π*1 *→ √ if and only if π*2 *→α √.*

*α*

**Definition 5.4 ( [**[**9**](#_bookmark42)**])** *Two process π and πj are* strongly bisimilar *(or simply* bisimilar*), denoted by π πj, if there is a strong bisimulation Z such that* (*π, πj*) *∈ Z.*

Now, we introduce the Expansion Law, which is very important in the definition of the semantic and in the axiomatisation of our logic. We present a particular case of the Expansion Law, which is suited to our needs. The most general case of the Expansion Law is presented in [[9](#_bookmark42)].

**Theorem 5.5 ( [**[**9**](#_bookmark42)**])** *[Expansion Law (EL)] Let π* = (*η*1  *... * *ηn*)*. Then*

*π ~* Σ *α.*(*η*1  *... * *η′ * *... * *ηn*)+ Σ

*i*

*τi,j* (*.*)*.*(*η*1  *... * *η′*

*i*

*... η′*

*j*

 *... * *ηn*)

*α ' c ' c '*

*ηi→ηi*

(*ηi→ηi* )&(*ηj→ηj* )

*were α is a action model and τi,j is a private action model resulted by the com- bination of two communication actions.*

*We denote the right side of this bisimilarity by Exp*(*π*)*. We also denote by* **0**

*i*

*the processes whose expansion is empty, i.e., there is no* (*ηi →c*

*ηj*) *,* (*ηj →c*

*ηjj* ) *and*

(*ηk →α ηj* ) *for any i, j, k ∈ {*1*, ..., n}.*

*k*

**Proof.** This follows from table [1](#_bookmark20) and definitions [5.3](#_bookmark21) and [5.4](#_bookmark22). A detailed proof for the most general case of this theorem can be found in [[9](#_bookmark42)]. *2*

The Expansion Law is a very useful property of CCS processes. Its intuition is that processes can be rewritten as a summation of all their possible actions.

*def j jj def * *j jj*

Suppose we have a processes *A*

= *c.A*

+ *α.A*

and *B*

= *c.B*

+ *β.B*

, then the

process (*A * *B*) is equivalent, using the Expansion Law, to

(*A * *B*) *α.*(*Ajj * *B*)+ *β.*(*A * *Bjj*)+ *τAB.*(*Aj * *Bj*)

* 1. *Language*

In this section we present the DELWCA language.

**Definition 5.6** *The DELWCA language consists of a set* Φ *of countably many proposition symbols, a set* Π *of programs as deﬁned in* [*5.1*](#_bookmark18)*, a ﬁnite set A of agents,* *the boolean connectives ¬ and ∧, a modality ⟨π⟩ for every program π ∈* Π *(as deﬁned in section* [*5.1*](#_bookmark17)*) and a modality Ka for each agent a. The formulas are deﬁned as follows:*

*ϕ* ::= *p |T| ¬ϕ | ϕ*1 *∧ ϕ*2 *| ⟨π⟩ϕ | Kiϕ*

*where p ∈* Φ*, π ∈* Π*, i ∈ A and ⟨π⟩ϕ means that exists a execution of π that leads to a state where ϕ is true.*

* 1. *Semantics*

For communication actions (actions in *N* ) we need to relax the fact that relations in action models are equivalence relations, we just need them to be relations. For this case all the definitions of action models (def. [3.1](#_bookmark7)), execution (product) of action models (def. [3.3](#_bookmark8)), composition of action models (def. [3.4](#_bookmark11)) can be easily adapted.

**Definition 5.7** *Let A be the set of all agents and i, j ∈ A. The action model*

*τs* (*ϕ*)= (M*,* s)*, with* M = *⟨*S*, ∼,* pre*⟩, is deﬁned as follows:*

*i,j*

* S = *{*s*,* t*}*
* *∼i* = *{*(s*,* s)*,* (t*,* t)*}*
* *∼j* = *{*(s*,* s)*,* (t*,* t)*}*
* *∼k* = *{*(s*,* t)*,* (t*,* t)*}, for all k ∈ A\{i, j}*
* pre(s)= *ϕ*
* pre(t)= *T*

*i,j*

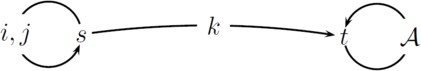


Fig. 5. Action Model for *τs*

In order to obtain the definition of satisfaction for DELWCA we must add the following condition to definition [3.5](#_bookmark12):

*c*1

J(*η*1 *... ηn*)) = *{* J*τi,j*(*.*)); J(*η*1 *... ηij ... ηjj ... ηn*)), for all (*ηi → ηj*) &

*i*

(*ηj →c*1 *ηj* ) *}* *{* J*α*); J(*η*1  *... * *ηj * *... * *ηn*)), for all (*ηi →α*

*j*

*i*

*i*

*ηj*) *}*

*5.4.1 Axiomatisation*

(i) *All instantiations of propositional tautologies*,

## Epistemic Logic Axioms

*Axioms (i), (ii), (iii), (iv) and (v) of section* [*2.2*](#_bookmark3),

## Action Model Axioms

*Axioms (vi), (vii), (viii) and (ix) of section* [*3.2*](#_bookmark13),

## PDL Axioms

1. [*π*](*φ → ψ*) *→* ([*π*]*φ →* [*π*]*ψ*) (K axiom)
2. [*π*1][*π*2]*φ ↔* [*π*1; *π*2]*φ* (Composition)
3. [*π*1 + *π*2]*φ ↔* [*π*1]*φ ∧* [*π*2]*φ* (Non-deterministic Choice)
4. [*α.π*]*φ ↔* [*α*][*π*]*φ* (Prefix) [6](#_bookmark25)
5. [*α.π*]*φ ↔ pre*(*α*) *→* [*π*]*φ*

## Concurrent Action Axiom

1. [*η*1  *... * *ηn*]*φ ↔* [*Exp*(*η*1  *... * *ηn*)]*φ*

## Inference Rules

M.P. *ϕ, ϕ → ψ/ψ* U.G. *ϕ/*[*π*]*ϕ ϕ/Kaϕ*

**Proposition 5.8** *▶* [*α*; *π*2]*φ ↔* [*α*][*π*2]*φ ↔* [*α.π*2]*φ ↔ pre*(*α*) *→* [*π*2]*φ*

**Example 5.9** A supervisor Ane (1) and her two students Bob(2) and Cathy(3) are working in their computer located at their own house. The supervisor wants to book a meeting ”tomorrow at 16:00”. She sends a message asynchronously to Bob and Cathy. We are supposing that the supervisor uses channels *c*12 and *c*13 to communicate with Bob and Cathy respectively. We represent Anne, Bob and Cathy by processes *π*1, *π*2 and *π*3 respectively, and their parallel composition by *π*1  2  3.

* *π*1 = *c*12(*p*); *c*13(*p*)+ *c*13(*p*); *c*12(*p*)
* *π*2 = *c*12(*.*)

6 It is important to notice that Prefix is a special case of Composition

* + *π*3 = *c*13(*.*)
  + *π*1  2  3 = (*π*1  *π*2  *π*3)

We have two possible runs process *π*1  2  3 as shown in the tree in figure [6.](#_bookmark26)

*π*2



*π*1

*τ*12

,*7*



*π*3

*τ*13

*◦*

*τ*13

z

*◦*

*τ*12

*◦ ◦*

Fig. 6. Possible Runs of Process *π*1  *π*2  *π*3

Let propositional symbol *p* represent ”tomorrow at 16:00”. The epistemic model

*Mj* at the begging is as shown in figure [7.](#_bookmark27)

*u ◦* 2*,* 3 *◦ v*

*p ¬p*

Fig. 7. Initial Epistemic Model *n*0

The action models for *τ*12 and *τ*13 are presented in figures [8](#_bookmark28) and [9.](#_bookmark29)

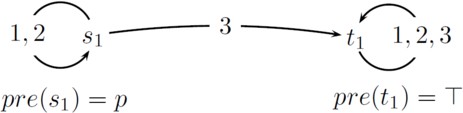


Fig. 8. Action Model for *τ*12

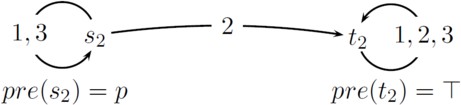


Fig. 9. Action Model for *τ*13

Suppose *τ*12 is performed before *τ*13. After the execution of *τ*12 we obtain the epistemic model picture in figure [10.](#_bookmark30)

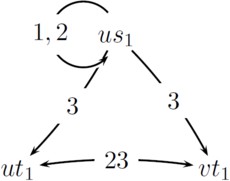


Fig. 10. Epistemic Model *n*1 = *n*0 *⊗ τ*12

It is important to notice that at state *us*1 Ane and Bob knows *p M*1*, us*1 *▶ K*1*p ∧ K*2*p* but Cath doesn’t *M*1*, us*1 *▶ ¬K*3*p*. After the second communication *τ*13 we have the epistemic model of figure [11.](#_bookmark31)

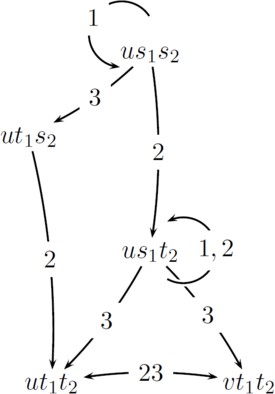


Fig. 11. Epistemic Model *n*2 = *n*1 *⊗ τ*13 = *n*0 *⊗ τ*12 *⊗ τ*13

We can notice, from figure [11](#_bookmark31) that at state *us*1*s*2 Ane, Bob and Cath knows *p M*2*, us*1*s*2 *▶ K*1*p ∧ K*2*p ∧ K*3*p* as expected. If we execute run *τ*13; *τ* 12 we obtain the model *M*3 as shown in figure [12.](#_bookmark32)

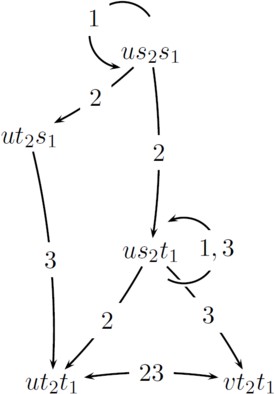


Fig. 12. Epistemic Model *n*3 = *n*0 *⊗ τ*13 *⊗ τ*12

We can show, from figure [12](#_bookmark32), that Ane, Bob and Cath know *p M*3*, us*1*s*2 *▶*

*K*1*p ∧ K*2*p ∧ K*3*p* as expected.

* 1. *Soundness, Completeness and Decidability*
     1. *Soundness*

We need to prove that all axioms are valid. Axioms *i* to *xiii* are standard from Dynamic Epistemic Logic literature and can be found in [[16](#_bookmark48)]. We prove validity only for axiom [6.](#_bookmark24)

**Lemma 5.10** [*η*1  *... * *ηn*]*φ ↔* [*Exp*(*η*1  *... * *ηn*)]*φ is valid.*

**Proof.** This proof follows straightforward from the rules of table [1](#_bookmark20) and theorem

[5.5](#_bookmark23). *2*

A detailed proof of soundness can be found in [[3](#_bookmark35)].

* + 1. *Completeness*

The proof of completeness is similar to the proof for Public Announcement and Action Models Logics introduced in [[13](#_bookmark45)] Dynamic Epistemic Logic. We prove com- pleteness showing that every formula in DELWCA is equivalent to formula in Epis- temic Logic. In order to achieve that we only have to provide a translation function that translate every DELWCA formula to a formula without communication actions and concurrency.

A detailed proof of completeness can be found in [[3](#_bookmark35)].

* + 1. *Decidability*

Decidability follows directly from the decidability of **S5a**.

# Conclusions

In this work we present a Dynamic Epistemic Logic with Communication Actions that can be performed concurrently. In order to achieve that we propose a PDL like language for actions and develop a small process calculus. We show that it’s easy to model problems of communication and concurrency with the proposed dynamic epistemic logic. The main feature of it is the Expansion rule which allows for representing the parallel composition operator. This approach is similar to the one introduced in [[4,](#_bookmark39) [5](#_bookmark40)].

We represent communication actions as private Action Models where the rela- tions are not equivalence relations. We present an axiomatisation and prove com- pleteness using reduction technique.

As future work we would like to investigate the extension with common knowl- edge and/or iteration operators, study other types of communications where agents are not reliable or not trustful, extend this to Dynamic Epistemic Logic With Post- Conditions and change DEMO, or create a new Model Checker, to deal with con- currency and communication.

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