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ORIGINAL ARTICLE

Fuzzy multi-criteria decision making model for

different scenarios of electrical power generation in Egypt

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Abstract In the analytic network process (ANP) a hierarchy or network is created to represent a decision and establishes a matrix containing the pair wise comparison judgments for the elements linked under a parent element. A priority vector of relative weights for these elements is derived. Then all the priority vectors are appropriately weighted and summed to obtain the overall priorities for the alternatives of a decision. In this paper we will develop an efficient fuzzy ANP model which helps decision makers to choose among the alternatives for the Egyptian scenarios of electrical power generation.

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KEYWORDS

Fuzzy;

Analytic network process; Gaussian function; Decision-making

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1. Introduction

Fuzzy rule-based systems have been widely used in a variety of engineering areas such as data mining, pattern recognition, system identification, and process control [[1]](#_bookmark16). Fuzzy logic is a key tool to express knowledge of domain experts so that valu- able experience of human beings can be incorporated into con- trollers design and applied to handle real-life situations that

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the classical control approach finds difficult or impossible to tackle [[2]](#_bookmark16).

The analytic network process (ANP) is used for tackling multi-attribute decision-making problems in real situations when there is interrelation among decision criteria or alterna- tives. In the traditional formulation of the ANP, human’s judgments are represented as exact numbers. However, in many practical cases the human preference model is uncertain and decision makers might be unable to assign exact numerical values to the comparison judgments. Since some of the evalu- ation criteria are subjective and qualitative in nature, it is very difficult for the decision-maker to express the preferences using exact numerical values and to provide exact pair-wise compar- ison judgments. It is more desirable for him to use interval or fuzzy evaluations [[3]](#_bookmark17). To improve the ANP method, this paper discusses a fuzzy ANP approach using Gaussian fuzzy num- bers to represent decision makers’ comparison judgments and extent analysis method to decide the final priority of dif- ferent decision criteria. The proposed model uses the linguistic

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variables and Gaussian fuzzy numbers as a pair-wise compar- ison scale for deriving the priorities of different selection attri- butes and sub-attributes. In the last step, the priority weights for main attributes, sub-attributes and alternatives are com- bined to determine the priority weights of the alternatives. The alternative with the highest priority weight is selected as the best alternative.

Erginel and Senturk developed a fuzzy ANP model to rank for three Global Systems for Mobile Communications (GSMs) operators [[4]](#_bookmark18). Yuksel and Dagdeviren used fuzzy ANP to dem- onstrate a process for quantitative Strengths, Weaknesses, Opportunities and Threats (SWOTs) analysis that can be per- formed even when there is dependence among strategic factors for a textile firm [[5]](#_bookmark22).

This paper aims to propose a fuzzy ANP decision-making support system that helps decision-makers of any authority in selecting the best alternatives among several offers. Such a kind of systems often requires highly experienced decision makers to consider vague and uncertain information.

Fuzzy set theory offers a possible means of managing these kinds of data or information. On the other hand, ANP offers a means for dealing with different preferences made to different decision alternatives. The remainder of the paper is organized as follows: In Section 2, an overview on fuzzy sets, linguistic variables, Analytic Hierarchy Process (AHP), and fuzzy AHP applications in literature are given. In Section 3, Analytic network process (ANP) is illustrated. FANP based approach is discussed in Section 4. Gaussian fuzzy ANP (GFANP) pro- posed model to select the best alternative is developed and the steps of each stage of the procedure are explained in detail in Section 5. In Section 6, results which are produced from the model are discussed, and the paper ends with concluding re- marks in Section 7.

Egypt had installed generating capacity of 20 gigawatts (GW) as of 2010, with plans to add 25 GW of additional gen- erating capacity by 2020. Around 90% of Egypt’s electric gen- erating capacity is thermal (natural gas), with the remaining 10% hydroelectric, mostly from the Aswan High Dam. All oil-fired plants have been converted to run on natural gas as their primary fuel. Egypt is also planning to build a part-solar power plant at Kureimat, which will have a total planned capacity of 150 MW. A Netherlands-funded project is building 60 MW of wind power units in the Suez Canal area. Egypt also has a 22-MW nuclear research reactor at Inshas in the Nile Delta, built by INVAP S.A. of Argentina, which began oper- ation in 1997 [[6]](#_bookmark23).

1. Literature survey
   1. *Fuzzy sets*

When fuzzy set theory was presented, researchers considered decision making as one of the most attractive application fields of that theory [[7]](#_bookmark24). Fuzzy decision theories attempt to deal with the vagueness and no specificity inherent in human formula- tion of preferences, constraints, and goals [[8]](#_bookmark25).

A fuzzy set *A* in *X* is formally defined as follows [[9]](#_bookmark26):

*A* = {(*x*; *lA*(*x*))|*xeX*} (1)

where *X* is the universe of discourse and *lA*(*x*) is the member-

ship degree of the element *x* in *A*.

* 1. *Linguistic variables*

The conventional techniques for system analysis are intrinsi- cally unsuitable for dealing with humanistic systems, whose behavior is strongly influenced by human judgment, percep- tion, and emotions [[10]](#_bookmark27). This is a manifestation of what might be called the principle of incompatibility: ‘‘As the complexity of a system increases, our ability to make precise and yet sig- nificant statements about its behavior diminishes until a threshold is reached beyond which precision and significance become almost mutually exclusive characteristics’’. Because of this belief Zadeh proposed the concept of linguistic variables as an alternative approach to modeling human thinking an ap- proach that, in an approximate manner, serves to summarize information and express it in terms of fuzzy sets instead of crisp numbers [[11]](#_bookmark33).

* 1. *The Analytic Hierarchy Process (AHP)*

The AHP approach was developed in the early 1970s in re- sponse to military contingency planning, scarce resources allo- cation, and the need for political participation in disarmament agreements [[12,13]](#_bookmark34). All these problems rely heavily on mea- surement and tradeoff of intangibles in a multi-criteria process. The AHP is a structured method to elicit preference opinion from decision makers. Its methodological procedure can easily be incorporated into multiple objective programming formula- tions with interactive solution process [[12–14]](#_bookmark34). The AHP ap- proach involves decomposing a complex and unstructured problem into a set of components organized in a multilevel hierarchic form [[14]](#_bookmark36). A salient feature of the AHP is to quan- tify decision makers’ subjective judgments by assigning corre- sponding numerical values based on the relative importance of factors under consideration. A conclusion can be reached by letting the judgments determine the overall priorities of variables [[15]](#_bookmark39). The Analytic Hierarchy Process (AHP) finds out the ‘‘best’’ alternative out of several ones by considering a number of conflicting criteria. In the AHP one creates a hier- archy or network to represent a decision and establishes a ma- trix containing the pair wise comparison judgments for the elements linked under a parent element. The hierarchy is formed in a way that it enables the use of elements in a level to compare the elements in the level immediately below. A hierarchy should be rich enough to represent the problem, but simple enough to reflect sensitivity. Paired comparisons are essential. One first makes the paired comparisons, based on the preference table, and then derives the priorities from them. Paired comparisons are the engine for generating rela- tive measurement. One then derives a priority vector of relative weights for these elements. There is one such matrix for every parent element. All the priority vectors are appropriately weighted and summed to obtain the overall priorities for the alternatives of a decision [[16]](#_bookmark40).

* + 1. *The standard AHP*

Satty demonstrated mathematically that the eigenvector solu- tion was the best approach to get a ranking of priorities from a pair wise matrix in the standard AHP [[14]](#_bookmark36). [Table 1](#_bookmark4) represents the standard preference table used by Saaty [[17]](#_bookmark19). [Table 2](#_bookmark4) rep- resents a modified preference table that is used currently in sev- eral cases [[18]](#_bookmark19).

ysis of each object, a fuzzy synthetic degree value can be obtained as follows:

Table 1 The standard preference table of AHP.

Linguistic variable Crisp number

Equally preferred (EP) 1

Equally to Moderately preferred (WMP) 2

Moderately preferred (MP) 3

Moderately to strongly preferred (MSP) 4

Strongly preferred (SP) 5

Strongly to very strongly preferred (SVP) 6

Very strongly preferred (VP) 7

Very strongly to extremely preferred (VEP) 8

Extremely preferred (XP) 9

Let *X* = {*x*1, *x*2,..., *xn*} represents the elements of the alter- natives as an object set, and let *U* = {*u*1, *u*2,..., *um*} represents the elements of the criteria as a goal set. Therefore, *m* extent analysis values for each object can be obtained with the follow- ing signs:

*M*1 ; *M*2 ; ... ; *Mm*

*i* = 1; 2; ... ; *n* (2)

*gi gi gi*

where all the *Mj*

*g*

*i*

(*j* = 1; 2; ... ; *m*) are triangular fuzzy num-

bers (TFNs).

The steps of Chang’s extent analysis were given by Kahr- aman et al. [[27]](#_bookmark28) and Dag˘deviren et al. [[28]](#_bookmark29).

* 1. *Why ANP?*

Table 2 The modified preference table of AHP.

Linguistic variable Crisp number

Equally preferred (EP) 1

Equally to Weakly preferred (EWP) 2

Weakly preferred (WP) 3

Weakly to Moderately preferred (WMP) 4

Moderately preferred (MP) 5

Moderately to strongly preferred (MSP) 6

Strongly preferred (SP) 7

Strongly to very strongly preferred (SVP) 8

Very strongly preferred (VP) 9

Very strongly to extremely preferred (VEP) 10

Extremely preferred (XP) 11

Although the AHP technique removes the deficiencies inherent in the measurement and evaluation steps of the problem anal- ysis, it does not measure the possible dependencies among fac- tors. The AHP method assumes that the factors presented in the hierarchical structure are independent; however, this is not always a reasonable presumption. The possible depen- dency among factors can only be determined as a result of internal and external environmental analyses.

1. Analytic Network Process (ANP) approach

*2.3.2. Extent fuzzy AHP (FAHP)*

The most common method among different FAHP methods is the extent analysis method proposed by Chang [[19]](#_bookmark19). The earli- est work in fuzzy AHP compared fuzzy ratios described by tri- angular membership functions [[20]](#_bookmark19). Cheng and Mon proposed a new algorithm for evaluating weapon systems by Analytical Hierarchy Process (AHP) based on fuzzy scales, which is a multiple criteria decision making approach in a fuzzy environ- ment [[21]](#_bookmark19). Cheng proposed a new algorithm for evaluating na- val tactical missile systems by the fuzzy Analytical Hierarchy Process based on grade value of membership function [[22]](#_bookmark19). This algorithm was applied to a missile system evaluation and selection problem. Kuo et al. developed a decision support system using fuzzy sets theory integrated with analytic hierar- chy process for locating a new convenience store [[23]](#_bookmark19). Altinoz examined supplier selection in general and specifically in the textile sector [[24]](#_bookmark19). Kahraman et al. used fuzzy Analytic Hierar- chy Process (AHP) to select the best supplier firm providing the most satisfaction for the criteria determined in the white good sector [[25]](#_bookmark20). Chan and Kumar discussed a fuzzy AHP ap- proach using triangular fuzzy numbers to represent decision makers’ comparison judgments and fuzzy synthetic extent analysis method to decide the final priority of different deci- sion criteria [[3]](#_bookmark17). The main objective is the selection of best glo- bal supplier for a manufacturing firm. Haq and Kannan proposed a structured model for evaluating vendor selection using the Analytical Hierarchy Process (AHP) and fuzzy AHP [[26]](#_bookmark21). The extent analysis method is used to consider the extent of an object to be satisfied for the goal, that is, satisfied extent. In the method, the ‘‘extent’’ is quantified by using a fuz- zy number. On the basis of the fuzzy values for the extent anal-

ANP is a multi-attribute, decision-making approach based on the reasoning, knowledge, and experience of the experts in the field. ANP can act as a valuable aid for decision making involving both tangible as well as intangible attributes that are associated with the model under study. ANP relies on the process of eliciting managerial inputs, thus allowing for a structured communication among decision makers. Thus, it can act as a qualitative tool for strategic decision-making problems [[29]](#_bookmark30). AHP method does not measure the possible dependencies among factors. It assumes that the factors pre- sented in the hierarchical structure are independent; however, this is not always a reasonable assumption. The possible dependency among factors can only be determined as a result of internal and external environmental analyses. The ANP is a generalization of the AHP. While the AHP represents a frame- work with a uni-directional hierarchical AHP relationship, the ANP allows for complex interrelationships among decision levels and attributes [[5]](#_bookmark22). For instance, not only does the impor- tance of the criteria determine the importance of the alterna- tives, as in a hierarchy, but the importance of the alternatives may also have an impact on the importance of the criteria. Therefore, a hierarchical representation with a lin- ear top-to-bottom structure is not suitable for a complex sys- tem. A system with feedback can be represented by a network.

1. Fuzzy ANP

The inability of ANP to deal with the impression in the pair wise comparison process has been improved in fuzzy ANP. In- stead of a crisp value, fuzzy ANP applies a range of values to incorporate the decision maker’s uncertainly. It enhances the potential of the ANP for dealing with imprecise and uncertain human comparison judgments. Ramik developed a decision

system using ANP and fuzzy inputs [[30]](#_bookmark31). In this paper ex- tended arithmetic operations with fuzzy numbers are proposed as well as ordering fuzzy relations to compare fuzzy outcomes. Kaur and Mahanti developed a fuzzy ANP-based approach for selecting ERP vendors [[31]](#_bookmark32). In this paper ANP equipped with fuzzy logic helps in overcoming the impreciseness and vagueness in the performance. Wu et al. developed a fuzzy ANP-based approach to evaluate medical organizational per- formance [[32]](#_bookmark35). This paper proposes an evaluation model using fuzzy analytic network process (FANP). The proposed model can provide Taiwan’s hospital accreditation policy a reference material, making it highly applicable for academic and govern- ment purposes. Rafiei and Rabbani developed an ordered par- titioning in hybrid MTS/MTO contexts using fuzzy ANP [[33]](#_bookmark37). In this paper, a model based on analytic network process is developed to tackle the addressed decision. Since the regarded decision deals with the uncertainty and ambiguity of data as well as experts’ and managers’ linguistic judgments, the pro- posed model is equipped with fuzzy sets theory. Rouyendegh

and Erol developed the DEA – fuzzy ANP department ranking model applied in Iran Amirkabir University [[34]](#_bookmark38). This research is a two-stage model designed to fully rank the organizational departments where each department has multiple inputs and outputs.

1. A Gaussian fuzzy ANP proposed model (GFANP)
   1. *Problem formulation*

It is required to develop a decision-making system, which helps decision-makers in the Egyptian government to select the best alternatives for the different scenarios of electrical power gen- eration in Egypt. The highest priority would be the best (see [Fig. 1](#_bookmark5)). There are three alternative scenarios:

Alt#1: the current one,

Alt#2: 20% nuclear, 75% petrol, 5% other, and

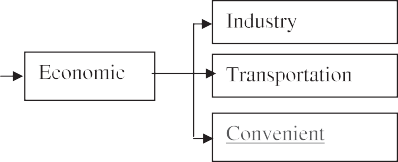
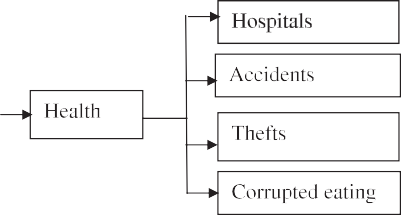


Figure 1 The ANP model.

Alt#3: 25% nuclear, 65% petrol, 5% solar, 5% other.

* 1. *The problem of triangular fuzzy numbers*

The following two cases illustrate the problem with triangular fuzzy numbers.

*Case 1:* It is required to rank the fuzzy numbers shown in [Fig. 2](#_bookmark6) with the FANP methodology. Therefore we have the degree of possibility of (*M*1 = (*l*1, *m*1, *u*1)) P (*M*2 = (*l*2, *m*2, *u*2)) is defined as:

*V*(*M*1 P *M*2)= *l*(*d*1) (3)

And the degree of possibility of (*M*1 = (*l*1, *m*1, *u*1)) P

(*M*3 = (*l*3, *m*3, *u*3)) is defined as

*V*(*M*1 P *M*3)= 0 as *u*1 < *l*3 (4)

Assume that

*d*'(*Ai*)= min *V*(*Si* > *Sk*) for *k* = 1; 2; .. . ; *n*; *k* – *i*. Then(5)

*d*'(*A*1)= min(*l*(*d*1); 0)= 0 (6)

The weight vector is given by

*W*' = (*d*'(*A* ); *d*'(*A* ); *d*'(*A* ))*T* (7)

The degree of possibility of (*M*1 = (*l*1, *m*1, *u*1)) P

(*M*2 = (*l*2, *m*2, *u*2)) is defined as

*V*(*M*1 P *M*2)= *l*(*d*1) (11)

And

The degree of possibility of (*M*1 = (*l*1, *m*1, *u*1)) P (*M*3 = (*l*3, *m*3, *u*3)) is defined as

*V*(*M*1 P *M*3)= 0 as *u*1 < *l*3 (12)

Then *d*'(*A*1)= min(*l*(*d*1); 0)= 0 (13)

The degree of possibility of (*M*2 = (*l*2, *m*2, *u*2)) P (*M*1 =

(*l*1, *m*1, *u*1)) is defined as

*V*(*M*2 P *M*1)= 1 (14)

And

The degree of possibility of (*M*2 = (*l*2, *m*2, *u*2)) P

(*M*3 = (*l*3, *m*3, *u*3)) is defined as

*V*(*M*2 P *M*3)= 0 as *u*2 < *l*3 (15)

Then *d*'(*A*2)= min(1; 0)= 0 (16)

The weight vector is given by

*W*' = (*d*'(*A*1); *d*'(*A*2); *d*'(*A*3))*T* (17)

Then *W*' = (0; 0; 1)*T* (18)

1 2 3

Then *W*' = (0; *l*(*d*2); 1) (8)

Via normalization, the normalized weight vector is

*W* = (0; *w*2; *w*3) where *w*2 and *w*3 are nonzero values. (9)

Then by using triangular fuzzy ANP, the first item is com-

pletely eliminated and its weight over others will be zero.

If there are i items and *u*1 < *l*2,.. . , *li* then the same case will be found and

Via normalization, the normalized weight vector is

*W* = (0; 0; 1)*T* (19)

If there are i items and *li* > *u*1, *u*2,..., *ui*—1 then the same case will be found and

*W* = (0; 0; .. . ; 1)*T*. (20)

Then by using triangular fuzzy ANP, only one item that has a

weight equals to 1, while all other items are wrongly weighted

*W* = (0; *w* ; *w* ; ... ; *w* )*T* where *w* ; *w* ; ... ; *w*

(10)

as 0.

2 3 *n*

2 3 *n*

Thus, from the above discussion, it is clear that triangular

are non zero numbers.

It is possible to have more than one item having weights equal to zero. In such a case more than one fuzzy number will be ranked equally. From the perspective of FANP, this means that some alternatives will be wrongly considered equivalent.

*Case 2:* In case 1, each fuzzy number intersects at least with one fuzzy number. In this case, we assume that some fuzzy numbers do not intersect at all, as shown in [Fig. 3](#_bookmark7).

fuzzy numbers, and even trapezoidal ones, have serious short- age when used as preference values.

* 1. *The proposed model*

The proposed model to overcome the problem of triangular fuzzy numbers depends on replacing them by Gaussian fuzzy numbers. It is clear that defining Gaussian fuzzy numbers over the preference scale results in real intersection between any



M



Figure 2 The fuzzy numbers need to be ranked (case 1). Figure 3 The fuzzy numbers need to be ranked (case 2).

|  |  |  |  |
| --- | --- | --- | --- |
| Table 3 The preference table: *l*=crisp number, *r*=0.5. | | | |
| Linguistic variable | Crisp no. | Triang(*x*, *a*, *b*, *c*) | Gaussian(*x*, *l*, *r*) |
| Equally preferred (EP) | 1 | Triang(*x*, 1, 1, 1) | Gaussian(*x*, 1, 0.5) |
| Equally to Weakly preferred (EWP) | 2 | Triang(*x*, 1.5, 2, 2.5) | Gaussian(*x*, 2, 0.5) |
| Weakly preferred (WP) | 3 | Triang(*x*, 2.5, 3, 3.5) | Gaussian(*x*, 3, 0.5) |
| Weakly to Moderately preferred (WMP) | 4 | Triang(*x*, 3.5, 4, 4.5) | Gaussian(*x*, 4, 0.5) |
| Moderately preferred (MP) | 5 | Triang(*x*, 4.5, 5, 5.5) | Gaussian(*x*, 5, 0.5) |
| Moderately to strongly preferred (MSP) | 6 | Triang(*x*, 5.5, 6, 6.5) | Gaussian(*x*, 6, 0.5) |
| Strongly preferred (SP) | 7 | Triang(*x*, 6.5, 7, 7.5) | Gaussian(*x*, 7, 0.5) |
| Strongly to very strongly preferred (SVP) | 8 | Triang(*x*, 7.5, 8, 8.5) | Gaussian(*x*, 8, 0.5) |
| Very strongly preferred (VP) | 9 | Triang(*x*, 8.5, 9, 9.5) | Gaussian(*x*, 9, 0.5) |
| Very strongly to extremely preferred (VEP) | 10 | Triang(*x*, 9.5, 10, 10.5) | Gaussian(*x*, 10, 0.5) |
| Extremely preferred (XP) | 11 | Triang(*x*, 10.5, 11, 11.5) | Gaussian(*x*, 11, 0.5) |
|  |  |  |  |

number and all the other numbers. This eliminates the problem of getting some alternatives have the same rank and conse- quently be treated equivalently. Thus, after illustrating that idea, we propose a modified preference table, shown in [Table 3](#_bookmark8), in which we introduce Gaussian fuzzy numbers to the eleven- point scale table. It should be noted that the centers (*l*’s) of the Gaussian preference values must be the same as the crisp pref- erence scale values. However, the widths (*r*’s) can be freely as- sumed according to the existing amount of uncertainty.

Gaussian functions have the advantage of being fully deter- mined using only two parameters, i.e. center (*l*) and width (*r*) and its value never equals to zero (within the range of the pref- erence scale values). Thus, the intersection must be existed be- tween every fuzzy number and all the others. In this case, shortages of the triangular fuzzy number are overcome.

The definition of Gaussian function is as follows:

" #

[(25)](#_bookmark9). Once, we get *Si*’s as triangle fuzzy numbers, they can be returned back to Gaussian to perform the ranking step.

For example suppose that

*r* = 0.5 and *a* = 0.1 then

*x*1 = *l* — 0.76 and

*x*2 = *l* + 0.76

The steps of the modified fuzzy ANP (FANP) method are

illustrated as follows:

Let *Gij* be the elements of the preference matrix after per- forming the triangular approximation, then:

Step 1:

*Gij*

P*j*

S*i* = P P *G* (25)

*ij*

*i*

*j*

*j*

*i*

*i*

*i*

(26)

= P P ÿ*lj*, *mj*, *uj*

Gaussian(*x* : *l*, *r*)= exp —(*x* — *l*)

2

*r*2

(21)

P ÿ*lj*, *mj*, *uj*

At any level *a*, as in [Fig. 4](#_bookmark10), it is shown that:

—(*x* — *l*)2

" #

*a* = exp 2

*r*

(22)

*i j i i i*

where

*lj* ﬃ *mj* — *rj*p—ﬃﬃﬃﬃ*L*ﬃﬃﬃ*n*ﬃﬃﬃ(ﬃﬃ*a*ﬃﬃ)**ﬃ**

*i*

*i*

*i*

—*Ln*( ) and (23)

*i*

*i*

*i*

(27)

*l r* pﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃ*a*ﬃﬃﬃﬃ

*x*1 =

—

×

*uj* ﬃ *mj* + *rj* p—ﬃﬃﬃﬃ*L*ﬃﬃﬃ*n*ﬃﬃﬃ(ﬃﬃ*a*ﬃﬃ)**ﬃ**

(28)

*x*2 = *l* + *r* × p—ﬃﬃﬃﬃ*L*ﬃﬃﬃ*n*ﬃﬃﬃ(ﬃﬃ*a*ﬃﬃ)**ﬃ**

(24)

To get good triangular approximation, we choose a low level for *a*.

It is clear that as long as *a* level is small enough, then it is pos-

sible to get a good fuzzy triangular approximation of *G*(*x*, *l*, *r*) by *T*(*x*, *x*1, *l*, *x*2). Such an approximation is useful for perform-

For example let *a* = 0.001

P *lj*, P *mj*, P *uj*

ing the fuzzy arithmetic operations to get S*i* as shown in Eq.

*j i*

*j*

*i*

*j i*

*Si* = P P *j* P P *j* P P

*i*

*jli*,

*i*

*jmi* ,

*i*

*jui*

*j* (29)

P *lj*

=

*j i*

,

*j*

*i*

,

*j i*

P *mj*

P *uj* !

P P *uj* P P *mj* P P *lj*



(30)

*i*

*j i*

*i*

*j*

*i*

*i*

*j i*

X*lj* = X*mj* — X*rj*(p—ﬃﬃﬃﬃ*L*ﬃﬃﬃ*n*ﬃﬃﬃ(ﬃﬃ*a*ﬃﬃ)**ﬃ**) (31)

*j*

*i*

*j*

*i*

*j*

*i*

X*uj* = X*mj* + X*rj* (p—ﬃﬃﬃﬃ*L*ﬃﬃﬃ*n*ﬃﬃﬃ(ﬃﬃ*a*ﬃﬃ)**ﬃ**) (32)

*j*

*i*

*j*

*i*

*j*

*i*

XX*lj* = XX*mj* — XX*rj* p—ﬃﬃﬃﬃ*L*ﬃﬃﬃ*n*ﬃﬃﬃ(ﬃﬃ*a*ﬃﬃ)**ﬃ** (33)

*i*

*j*

*i*

*i*

*j*

*i*

*i*

*j*

*i*

XX*uj* = XX*mj* + XX*rj* (p—ﬃﬃﬃﬃ*L*ﬃﬃﬃ*n*ﬃﬃﬃ(ﬃﬃ*a*ﬃﬃ)**ﬃ**) (34)

*i*

*j*

*i*

*i*

*j*

*i*

*i*

*j*

*i*

m *Si* = *xL*, *ms* , *xR*

Figure 4 Gaussian function A and its approximated triangle B.

*si*

*i*

*si*

where

*si*

*m*

=

*mj*

P P *mj*

*j*

*i*

P

The degree of possibility of *S*2 = *lS* (*x*) P *S*1 = *lS* (*x*) is de- fined as

*V*(*S*2 P *S*1)= hgt(*S*1 ∩ *S*2)= *lS* (*x*int) (44)

2

2 1

*i j i*

P *l*

(35)

*j*

*L j i*

*x* =

8 1 if *ms*

P *ms* , 9

*si* P P *j*

(36)

>< "

#

2 1 >=

*V*(*S*2 P *S*1)=

2

if *ms*

< *ms*

(45)

*xR* = *j i*

*i*

P

*jui*

*uj*

2

1

(37)

*s*2 *s*1

*si* P P *lj*

*i*

*j i*

where Xint is the ordinate of the inner intersection point be-

Now, *Si* can be returned back to a Gaussian fuzzy number (but asymmetric in this case) as follows:

>: exp —

(*ms*2 —*ms*1 )

(*rR* +*rL* )

>;

*m* — *xL*

tween *lS*2 (*x*) and *lS*1 (*x*). To compare *S*1 and *S*2, the values of both *V*(*S*2 P *S*1) and *V*(*S*1 P *S*2) are needed.

*Step 3:*

*rL*  *si si*

—*Ln*(*a*)

*x* — *ms*

*si* = pﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃ**ﬃ**

*R*

*rR*  *si i*

pﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃﬃ**ﬃ**

(38)

(39)

The degree of possibility for a Gaussian fuzzy number *Si* to be greater than k Gaussian fuzzy numbers *Si*(*i* = 1, 2,..., *k*) can be defined by

*si* = —*Ln*(*a*)

where *rL* is the width of the left branch of the Gaussian fuzzy

*V*(*S* > *S*1, *S*2, .. . , *Sk*)= *V*[(*S* > *S*1) and (*S*

*si R* > *S*2) and ··· and (*S* > *Sk*)] = min *V*(*S* > *Si*), *i*

number and *rsi* is the width of the right branch of the Gaussian

fuzzy number.

Now, *Si* becomes an asymmetric Gaussian number as follows:

8 9

> *x*—*m* 2 >

*r*

*l* (*x*)=

>< exp —

*si*

*L*

*si*

if *x* 6 *msi* >=

(40)

Then the weight vector is given by:

= 1, 2, 3, .. . , *k*. (46)

Assume that

*d*'(*Ai*)= min *V*(*Si* > *Sj*) for *j* = 1, 2, .. . , *n*; *j* – *i*. (47)

> *x*—*ms* 2 >

*si*

>: >;

exp — *R i* if *x* > *ms*

*rsi*

*i*

1

2

*n*

*Step 2:*

Let *l*1(*x*) and *l*2(*x*) be two Gaussian fuzzy numbers having the following forms:

8> 2 9>

>< exp — *r*

*x*—*ms*1

*L*

*s*1

if *x* 6 *m*

>=

*s*1

*W*' = (*d*'(*A* ), *d*'(*A* ), ... , *d*'(*A* ))*T*, (48)

where *Ai* (*i* = 1, 2,..., *n*) are n elements.

*Step 4:*

Via normalization, the normalized weight vector is

*W* = (*d*(*A* ), *d*(*A* ), ... , *d*(*A* ))*T*, (49)

1 2 *n*

*i*

*d*'(*Ai*)

*ls*1 (*x*)=

1

>:

>

*x*—*ms* 2

, (41)

>

where *d*(*Ai*)= X*d*'(*A* )

(50)

and

exp — *R*

*s*1

*r*

if *x* > *ms*1 >;

*i*

This gives the required priority weights of one alternative

8> 2 9>

>< exp — *r*

*x*—*ms*2

*L*

*s*2

if *x* 6 *m*

>=

(42)

> *x*—*ms* 2 >

*r*

*l*

(*x*) =

*s*2

>: exp —

2

*R*

*s*2

if *x* > *ms*2 >;

*s*2

among others.

1. Experimental results and discussion

It is needed to rank the alternatives *A*1, *A*2, *A*3, *A*4, *A*5 over Cri-

The intersection point between two Gaussian functions is shown in [Fig. 5](#_bookmark12).

8> " (*m* —*m* ) 2# 9>

teria *C*1, *C*2, *C*3, *C*4. According to [Fig. 1](#_bookmark5), the linguistic prefer- ence matrices of the different criteria nodes are given in [Tables 4–9](#_bookmark11). On the other hand, the inner dependences matrices

exp —

>

*s*1

*s*2

with respect to different criteria nodes are given in [Table 10](#_bookmark15).

<

( *s*1

*s*2 )

=

[35]. (43)

The inner dependence among factors is shown in [Fig. 6](#_bookmark13).

>

*s*2 *s*1

*rL* +*rR*

if *m* > *m*

m = > "

2# >

exp — (*ms*2 —*ms*1 )

>

Operation

0.30

if *m* < *m*

2 3 2 3

: (*rs*1 +*rs*2 ) ;

7 6 7

64 75

*R*

*L*

*s*1

*s*2

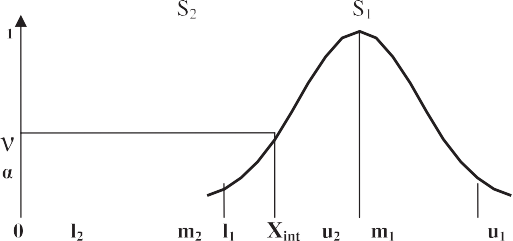
75

>

w

Economic 0.27

= 6 =



1

4

Health

Sources

0.23

0.20

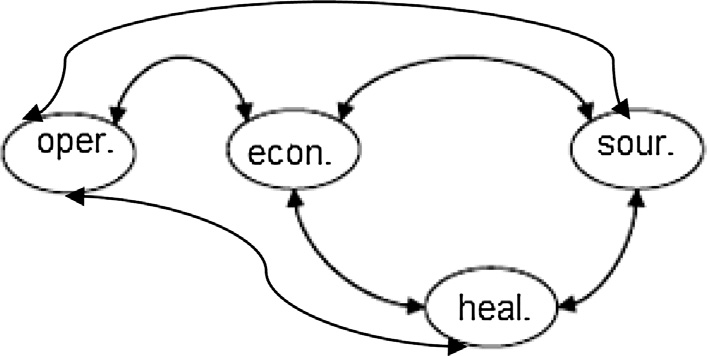
(51)

Figure 5 Intersection point between two Gaussian functions.

|  |  |  |
| --- | --- | --- |
| Table 4 The evaluation matrix with respect to  the Goal. | | |
|  | Risk | Cost |
| Risk | EP | WMP |
| Cost |  | EP |
|  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Table 5 The evaluation matrix with respect to the Risk. | | | | |
|  | Operation | Economic | Health | Sources |
| Operation | EP | WMP | EWP | WP |
| Economic |  | EP | EWP | EWP |
| Health |  |  | EP | EP |
| Sources |  |  |  | EP |
|  |  |  |  |  |

Figure 6 Inner dependence among factors.



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|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Table 6 The evaluation matrix with respect to the Operation Risk. | | | | | | |
|  | Coal | Oil | Nuclear | Gas | Solar | Wind |
| Coal | EP |  |  |  |  |  |
| Oil | WMP | EP | EP |  |  |  |
| Nuclear | EWP |  | EP |  |  |  |
| Gas | WP | EWP | EWP | EP |  |  |
| Solar | MP | WP | EWP | EWP | EP |  |
| Wind | WMP | WP | EWP | EWP | EWP | EP |
|  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Table 11 The evaluation matrix with respect to the Cost and the Coal. | | | | | | | |
|  | Cost |  |  |  | Coal |  |  |
|  | Alt. #1 | Alt. #2 | Alt. #3 |  | Alt. #1 | Alt. #2 | Alt. #3 |
| Alt. #1 | EP | EWP | WP |  | EP |  |  |
| Alt. #2 |  | EP |  |  | EWP | EP |  |
| Alt. #3 |  | EWP | EP |  | WP | EWP | EP |
|  |  |  |  |  |  |  |  |

* 1. *Inner relationship*

Table 7 The evaluation matrix with respect to the Economic Risk.

Industry Transportation Convenient

Industry EP EP EP

Transportation EP EP

Convenient EP

0.25 1.00 0.48 0.48

*W*2 = 0.25 0.48 1.00 0.19

664 775

2 1.00 0.33 0.33 0.33 3

0.50 0.19 0.19 1.00

Table 8 The evaluation matrix with respect to the Health

Risk.

Hospitals Accidents Thefts Corr. Eating

Thefts

Corr. eating

EP

EP

EP

21.00 0.33 0.33 0.333

20.303

20.273

(52)

0.25 1.00 0.48 0.48 0.27 0.28

*V* = *W* \* *w* = 6 \* =

7 6 7 6 7

4 75

64 75

64 75

factors

2

1

0.25 0.48 1.00 0.19

0.50 0.19 0.19 1.00

0.23

0.20

0.23

0.22

|  |  |  |
| --- | --- | --- |
| Hospitals EP | WP EWP | EWP |
| Accidents | EP WP | EWP |

(53)

Sample of the pair-wise comparisons between alternatives Alt.#1, Alt.#2, and Alt.#3 over criteria are given in [Table 11](#_bookmark14).

* 1. *Alternatives*

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Table 9 The evaluation matrix with respect to the sources risk. | | | | | | |
|  | Coal | Oil | Nuclear | Gas | Solar | Wind |
| Coal | EP |  |  |  |  |  |
| Oil | EWP | EP |  |  |  |  |
| Nuclear | WP | EWP | EP |  |  |  |
| Gas | WP | EWP | EWP | EP |  |  |
| Solar | MP | WMP | WP | EWP | EP | EP |
| Wind | MP | WMP | WP | EWP |  | EP |
|  |  |  |  |  |  |  |

Then, the overall normalized priority weight vector of the alternatives is obtained as follows:

*W* = [0.25, 0.33, 0.42]*T*, (54)

which means that Alt.#3 is the best alternative. Therefore 25%

of the generated electricity is come from nuclear power sta- tions, 65% from petrol thermal stations, 5% from solar sta- tions and 5% from other recourses.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 10 The inner dependences matrix.  Operation (Op.) Economic (Ec.) Health (Hel.) Sources (Src.) | | | | | | | | | | | | | | | | | | |
|  | Ec. | Hel. | Src. |  |  | Op. | Hel. | Src. |  |  | Op. | Ec. | Src. |  |  | Op. | Ec. | Hel. |
| Ec. | EP | EP |  |  | Op. | EP |  | EWP |  | Op. | EP |  | EWP |  | Op. | EP |  | EWP |
| Hel. |  | EP |  |  | Hel. | EWP | EP | EWP |  | Ec. | EWP | EP | EWP |  | Ec. | EWP | EP | EWP |
| Src. | EWP | EWP | EP |  | Src. |  |  | EP |  | Src. |  |  | EP |  | Hel. |  |  | EP |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

1. Conclusions

In the proposed model it is possible to benefit from the advan- tages of both interval and fixed value judgments. Shortages caused by each of them can be avoided. GFANP model pro- vides expert judgments the flexibility of using interval values in their preference matrices instead of crisp values. Gaussian fuzzy numbers are used instead of triangular numbers. By using them the case of zero weights will never exist. We recom- mend decision-makers in the Egyptian government to build more nuclear power stations to cover 25% of the generated electricity in Egypt. We also recommend them to construct so- lar power stations to cover 5% of the generated electricity.

References

1. [Chen M, Linkens D. A systematic neuro-fuzzy modeling frame-](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0005) [work with application to material property prediction. IEEE](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0005) [Trans Syst Manuf Cybern Part B 2001;31(5):781–90](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0005).
2. [Wu S, Er M, Gao Y. A fast approach for automatic generation of](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0010) [fuzzy rules by generalized dynamic fuzzy neural networks. IEEE](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0010) [Trans Fuzzy Syst 2001;9(4):578–94](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0010).
3. [Chan F, Kumar N. Global supplier development considering risk](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0015) [factors using fuzzy extended AHP-based approach. Omega](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0015) [2007;35:417–31](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0015).
4. Erginel N, Senturk S. Ranking of the GSM operators with fuzzy ANP. In: Proceedings of the world congress on engineering, vol.II, 2011.
5. [Yuksel I, Dagdeviren M. Using the analytic network process](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0020) [(ANP) in a SWOT analysis – a case study for a textile firm. Inform](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0020) [Sci 2007;177:3364–82](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0020).
6. Electrical power in Egypt. <[http://www.mbendi.com/indy/powr/](http://www.mbendi.com/indy/powr/af/eg/p0005.htm) [af/eg/p0005.htm](http://www.mbendi.com/indy/powr/af/eg/p0005.htm)> [last visit 10.12].
7. [Iwamoto S, Tsurusaki K, Fujita T. Conditional decision-making](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0025) [in fuzzy environment. J Oper Res Soc Jpn 1999;42(2):198–218](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0025).
8. [Turksen I. Operations research and management science applica-](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0030) [tions of fuzzy theory. Chem Comput Simul, Butlerov Commun](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0030) [2001:45–8 [in Russia]](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0030).
9. [Zadeh L. Fuzzy sets. Informat Control 1965;8:338–53](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0035).
10. [Zadeh L. Outline of a new approach to the analysis of complex](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0040) [systems and decision processes. IEEE Trans Syst Man Cybernet](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0040) [1975;3(1):43–80](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0040).
11. [Zadeh L. Quantitative fuzzy semantics. Inform Sci 1971;3:159–76](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0045).
12. [Saaty T. A scaling method for priorities in hierarchical structures.](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0050) [J Math Psychol 1977;15:234–81](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0050).
13. [Saaty T. Priority setting in complex problems. IEEE Trans Eng](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0055) [Manag 1983;EM-30(3):140–55](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0055).
14. [Satty T. Decision making for leaders: the analytical hierarchy](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0060) [process for decisions in a complex world. Belmont (MA):](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0060) [Wadsworth; 1982](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0060).
15. [Saaty T. How to make a decision: the analytic decision process.](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0065) [Interfaces 1994;24(6):19–43](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0065).
16. The analytic hierarchy process (AHP) for decision making.

<[www.fakr.noaa.gov/sustainablefisheries/sslmc/july-06/AHP-](http://www.fakr.noaa.gov/sustainablefisheries/sslmc/july-06/AHPMainLecture.pdf) [MainLecture.pdf](http://www.fakr.noaa.gov/sustainablefisheries/sslmc/july-06/AHPMainLecture.pdf)> [last visit 10.12].

1. McBride J. Analytical hierarchy process (AHP). <http:// [www.freequality.org/sites/www\_freequality\_org/documents/](http://www.freequality.org/sites/www_freequality_org/documents/) Training/Classes%20Spring%202003/PowerPoint%0Slides/ AHP.ppt> [last visit 10.12].
2. [Lee H, Lee S, Lee T, Chen J. A new algorithm for applying fuzzy](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0070) [set theory to evaluate the rate of aggregative risk in software](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0070) [development. Inform Sci 2003;153:177–97](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0070).
3. [Chang D. Extent analysis and synthetic decision. Optimization](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0075) [techniques and applications, vol. 1. Singapore: World Scientific;](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0075) [1992, p. 352](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0075).
4. [Laarhoven V, Pedrycz W. A fuzzy extension of Saaty’s priority](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0080) [theory. Fuzzy Sets Syst 1983;11:229–41](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0080).
5. [Cheng C, Mon D. Evaluating weapon system by analytical](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0085) [hierarchy process based on fuzzy scales. Fuzzy Sets Syst](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0085) [1994;63:1–10](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0085).
6. [Cheng C. Evaluating naval tactical missile systems by fuzzy AHP](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0090) [based on the grade value of membership function. Eur J Oper Res](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0090) [1996;96:343–50](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0090).
7. [Kuo R, Chi S, Kao S. A decision support system for locating](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0095) [convenience store through fuzzy AHP. Comput Ind Eng](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0095) [1999;37:323–6](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0095).
8. Altinoz C. Supplier selection in textiles: a fuzzy approach. Ph.D. thesis. North Carolina State University, Raleigh, USA, 2001.
9. [Kahraman C, Cebeci U, Ulukan Z. Multi-criteria supplier](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0100) [selection using fuzzy AHP. Logist Inform Manag](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0100) [2003;16(6):382–94](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0100).
10. [Haq A, Kannan G. Fuzzy analytical hierarchy process for](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0105) [evaluating and selecting a vendor in a supply chain model. Int J](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0105) [Adv Manuf Technol 2006;29:826–35](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0105).
11. [Kahraman C, Cebeci U, Ruan D. Multi-attribute comparison of](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0110) [catering service companies using fuzzy AHP: the case of Turkey.](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0110) [Int J Prod Econ 2004;87:171–84](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0110).
12. [Dag˘deviren M, Yu¨ksel](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0115) I\_[, Kurt M. A fuzzy analytic network](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0115) [process (ANP) model to identify faulty behavior risk (FBR) in](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0115)

[work system. Safe Sci 2008;46:771–83](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0115).

1. [Ravia V, Shankara R, Tiwarib M. Analyzing alternatives](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0120) [in reverse logistics for end-of-life computers: ANP and](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0120) [balanced scorecard approach. Comput Ind Eng](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0120) [2005;48:327–56](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0120).
2. [Ramik J. A decision system using ANP and fuzzy inputs. Int J](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0125) [Innov Comput Inform Control 2007;3(4):825–37](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0125).
3. [kaur P, Mahanti N. A fuzzy ANP-based approach for selecting](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0130) [ERP vendors. Int J Soft Comput 2008;3(1):24–32](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0130).
4. [Wu C, Chang C, Lin H. A fuzzy ANP-based approach to evaluate](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0135) [medical organizational performance. Inform Manage Sci](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0135) [2008;19(1):53–74](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0135).
5. [Rafiei H, Rabbani M. Order partitioning in hybrid MTS/MTO](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0140) [contexts using fuzzy ANP. World Acad Sci Eng Technol 2009;58](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0140).
6. [Rouyendegh B, Erol S. The DEA – fuzzy ANP department](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0145) [ranking model applied in Iran Amirkabir University. Acta](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0145) [Polytech Hungarica 2010;7(4)](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0145).
7. [Hefny H. Comments on ‘‘Distinguishability quantification of](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0150) [fuzzy sets’’. Inform Sci 2007;177:4832–9](http://refhub.elsevier.com/S1110-8665(13)00020-0/h0150).