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ORIGINAL ARTICLE

Generalized production planning problem under interval uncertainty

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Abstract Data in many real life engineering and economical problems suffer from inexactness. Herein we assume that we are given some intervals in which the data can simultaneously and indepen- dently perturb. We consider the generalized production planning problem with interval data. The interval data are in both of the objective function and constraints. The existing results concerning the qualitative and quantitative analysis of basic notions in parametric production planning problem. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind.

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KEYWORDS

Production planning; Stability;

Linear programming; Interval numbers; Parametric study

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1. Introduction

A production planning problem exists because there are limited production resources that cannot be stored from period to per- iod. Choices must be made as to which resources to include and how to model their capacity and behavior, and their costs. Also, there may be uncertainty associated with the production function and the constraints. The production planning problem starts with a specification of customer demand that is to be met by the production plan. One might only include the most critical or limiting resource in the planning problem, e.g. a bottleneck. Alternatively, when there is not a dominant resource, then one must model the resources that could limit production.

The general references on production planning are Thomas and McClain [[1]](#_bookmark8), Shapiro [[2]](#_bookmark9), Silver et al. [[3]](#_bookmark10), Mula et al. [[4]](#_bookmark11)

and Jamalnia and Soukhakian [[5]](#_bookmark11). Hax and Meal [[6]](#_bookmark11) introduced the notion of hierarchical production planning and provide a specific framework for this problem, where there is an optimization model with each level of the hierarchy. Each optimization model imposes a constraint on the model at the next level of the hierarchy.

The literature in production planning under uncertainty is vast. Different approaches have been proposed to cope with different forms of uncertainty (see, for example [[7,8]](#_bookmark11)). Galbra- ith [[9]](#_bookmark11) defines uncertainty as the difference between the amount of information required to perform a task and the amount of information already possessed. In the real world there are many forms of uncertainty that affect production processes.

Any planning problem starts with a specification of a cus- tomer demand but In most contexts, future demand is partially known. So one relies on a forecast for the future demand but the forecast is inaccurate. This leads to that demand that can- not be met in a period is lost, thus reducing revenue. In our work, we develop a new planning problem to minimize the lost demands and thus maximize revenues. First, we construct the production planning problem with interval numbers as uncer- tainty in both of the objective function and constraints. After that we will treat the uncertain of objective function and con- straints. In Section 4, parametric study for the treatment prob- lem is introduced. Finally, a numerical example is provided to clarify the proposed approach.

Parametric programming investigates the changes in the optimum linear programming solution due to predetermined continuous variations in the model’s parameters. Parametric

where *T* is the number of time periods; *I* is the number of items (raw materials or finished products); *K* is the number of re- sources; *aik* is an amount of resource *k* required per unit of production of item *i*; *bkt* is the amount of resource *k* available in time period *t*; *dit* is the demand for item *i* in time period *t*; *rit* is the unit revenue for item *i* in time period *t*; *cpit* is the unit variable cost of production for item *i* in time period *t*; *cuit* is the unit cost of unmet demand for item *i* in time period *t*; and *cqit* is the unit inventory holding cost for item *i* in time period *t*.

*The decision variables*

*pit*: an amount of production of item *i* during time period *t*; *qit*: an amount of inventory of item *i* at end of time period *t*; *uit*: an amount of unmet demand of item *i* during time per- iod *t*.

For the above model, a linear relationship between the cost and time, and revenue and time is assumed. The objective func- tion [(1)](#_bookmark5) maximizes revenues net of the production, inventory and lost sales costs. Eq. (2) is a set of resource constraints. Pro- duction in each period is limited by the availability of a set of shared resources. Production of one unit of item *i* requires *qit* units of resource *k*, for *k* = 1, 2,.. . , *K*. Typical resources are various types of labor, process and material handling equip- ment. Eq. (3) is a set of inventory balance constraints.

The optimization model of production planning problem that maximize revenues net of the production inventory and lost sales cost with interval numbers is formulated as follows:

study of the mathematical programming problems is impor-

*I*

max

X

*T*

[*rit*([*dL*; *dR*]— *uit*)— *cp p*

X

* *cq q*
* *cuituit*] (5)

tant and enhances the scope of application of the obtained

solutions of those problems. There are three main different ap- proaches to handle the parametric optimization problem, namely the sensitive analysis approach that concerns the minor

*i*=1 *t*=1

subject to

X*I*

*aikp*

6 [*bL* ; *bR* ] *k* = 1; .. . ; *K*; *t* = 1; ... ; *T* (6)

*it it*

*it it*

*it it*

changes in the parameters values and its effects on the ob- tained solutions, the parametric solution approach and finally

*it kt kt*

*i*=1

*q* + *p* — *q* + *uit* = [*dL*; *dR*] *i* = 1; .. . ; *I*; *t* = 1; ... ; *T*

the stability sets approach that deals with the stability of the

optimal solutions in different cases. In our work we exhibit

*i*;*t*—1 *it it*

*it it*

(7)

and apply the last approach [[10]](#_bookmark11). The stability notion plays

an important role in the mathematical programming field. It is important for solver or for the decision maker to preserve

*pit*; *qit*; *uit* P 0 *i* = 1; ... ; *I*; *t* = 1; ... ; *T* (8)

where [*bL* ; *bR* ] is an interval number represents the amount of

*kt kt*

effort and time. Stability in mathematical programming has

resource *k* available in time period *t*; and [*dL*; *dR*] is an interval

*it it*

many types. one of these types depends on making perturba-

tion to the decision space or to the objective space or to both by a parameter. This type is called stability in parametric pro- gramming problems [[11–13]](#_bookmark11). Other types are called internal and external stability.

1. Problem formulation

First, let us exhibit the following production planning problem that in Ref. [[14]](#_bookmark12):

number represent the demand for item *i* in time period *t*.

The superscripts *L* and *R* denote lower and upper bounds of an interval number, respectively.

1. The optimization approach

Based on the proposed approach of Jiang [[15]](#_bookmark12) for treating interval number, we will treat the uncertainty of Eqs. [(5)–(8)](#_bookmark4) as follows.

max

*T*

*t*=1

X

*I*

*i*=1

X

[*rit*(*dit* — *uit*)— *cpitpit* — *cqitqit* — *cuituit*] (1)

* 1. *Treatment of the uncertain objective function*

Let *f*(*pit*; *qit*; *uit*; *dit*)

subject to *I T*

X*I* = X X *rit*ÿ[*dL*; *dR*]— *uit* — *cp p*

*i*=1

*aikpit* 6 *bkt* ∀*k*; *t* (2)

*i*=1

*t*=1

*it*

*it*

*it it*

*it it*

* *cq q*
* *cuituit* (9)

*qi*;*t*—1

+ *pit*

* *qit*

+ *uit*

= *dit*

∀*i*; *t* (3)

in interval mathematics, the uncertain objective function (5)

can be transformed into two objective optimization problem

*pit*; *qit*; *uit* P 0 ∀*i*; *t* (4)

as follows [[16]](#_bookmark12):

*m*(*f*(*p* ; *q* ; *u* ; *d* )) = 1 (*fR*(*p* ; *q* ; *u* ; *d* )+ *fL*(*p* ; *q* ; *u* ; *d* ));

*it it it it* 2 *it it it it it it it it*

(10)

*w*(*f*(*p* ; *q* ; *u* ; *d* )) = 1 (*fR*(*p* ; *q* ; *u* ; *d* )— *fL*(*p* ; *q* ; *u* ; *d* )).

*it it it it* 2 *it it it it it it it it*

(11)

where *m* is called the midpoint value, *w* is called the radius of

tive function is relatively easy provided that the preferences of the objective functions are available. Through the above treat- ments of Eqs. [(5)–(8)](#_bookmark4), it is transformed into the following deterministic form:

max [*c*1*m*(*f*(*pit*; *qit*; *uit*; *dit*)) + *c*2*w*(*f*(*pit*; *qit*; *uit*; *dit*))] (19)

subject to

interval number and the two functions *fL* and *fR* are given as

follows:

*fL*(*p* ; *q* ; *uit*; *dit*) = min *f*(*p* ; *q* ; *uit*; *dit*) (12)

*kt kt*

*I*

*bL* — *aikp*

X

*kt it*

*i*=1

6 *d*

P *kkt*ÿ*bL* — *bR*

*i* = 1; .. . *I*; *t* = 1; ... ; *T*

(20)

*it*

*it it*

*d*∈*D*

*it it*

*qi*;*t*—1

+ *pit*

* *qit*

+ *uit*

*R i* = 1; ... *I*; *t* = 1; .. . ; *T* (21)

and

*qi*;*t*—1

+ *pit*

* *qit*

+ *uit* P *dL*

*i* = 1; ... *I*; *t* = 1; ... ; *T* (22)

*fR*(*p* ; *q* ; *uit*; *dit*) = max *f*(*p* ; *q* ; *uit*; *dit*) (13)

*it*

*c*1; *c*2 P 0; *c*1 + *c*2 = 1 (23)

*it it*

*d*∈*D*

*it it*

*pit*; *qit*; *uit* P 0 *i* = 1; .. . ; *I*; *t* = 1; ... ; *T* (24)

where *d* ∈ *D* = *d dL* < *d* < *dR*}.

* 1. *Treatment of uncertain constraints*

The possibility degree of interval number represents certain de- gree that one interval number is larger or smaller than another [[17]](#_bookmark12). The set of inequality constraints (6) can be written as

*kt kt*

1. The parametric study for Eqs. [(19)–(24)](#_bookmark6)

In this paper we assume that *kkt* are parameters rather than constants. Let *G*(*k*) denotes the decision space of Eqs. [(19)–](#_bookmark6) [(24)](#_bookmark6), *G*(*k*) is defined by:

*G*(*k*)= (*p* ; *q* ; *uit*) ∈ *R*3*IT*; ∀*i*; *t*| satisfies the set of constraints

*it*

*it*

— *aikpit*

*I*

X

*i*=1

Let

X*I*

P — *bL* ; *bR* ∀*k*; *t* (14)

(20)–(24)} (25)

In what follows we give the definition of some basic notions for

Eqs. [(19)–(24)](#_bookmark6). Such notions are the set of feasible parameters, the solvability set and the stability set of the first kind [[13,21]](#_bookmark12).

*x* =—

*i*=1

*aikpit* (15)

*The set of feasible parameters*

The set of feasible parameters of Eqs. [(19)–(24)](#_bookmark6) which is

as in interval linear programming [[18]](#_bookmark12), we can make an

inequality constraint satisfied with a possibility degree level, and formulate the deterministic inequality by the following

possibility degree *Px*P—[*bL* ;*bR* ] :

*kt kt*

1. *x* < —*bL*

=

*kt*

*kt*

*kt*

*kt*

*kt*

*kt*

8>

> ÿ ÿ

denoted by *U*, is defined by:

*U* = *kkt* ∈ *RKT*; ∀*k*; *t*|*G*(*k*) is not empty set (26)

}

*The solvability set*

The solvability set of Eqs. [(19)–(24)](#_bookmark6) which is denoted by*V* is defined by:

< *x* + *bL* / —*bR* + *bL* —*bL* 6 *x* < —*bR*

*R*

*V* = {*k*

∈ *U*; ∀*k*; *t*| Eqs. (19)–(24) has optimal solution}

*kt kt*

>

*P*

*x*P—[*bL* ;*bR* ]

1. *x* P —*b*

>:

(16)

*kt*

*The stability set of the first kind*

(27)

where *Px*P—[*bL* ;*bR* ] P *kkt* is the possibility degree of the *kt*th constraint and 0 6 *kkt* 6 1 is a predetermined possibility degree

*kt kt*

level.

An equality constraint (7) can be transformed into the fol- lowing form:

Suppose that *k*\* ∈ *V* with corresponding optimal solution

(*p*\*, *q*\*, *u*\*) for Eqs. [(19)–(24)](#_bookmark6). The stability set of the first kind

of Eqs. [(19)–(24)](#_bookmark6) that is denoted by *S*(*p*\*, *q*\*, *u*\*) is defined by:

*S*(*p*\*; *q*\*; *u*\*) = {*k*\* ∈ *V*|(*p*\*; *q*\*; *u*\*) is an optimal solution of Eqs. (19)–(24)} (28)

*dL* 6 *q* + *p* — *q* + *uit* 6 *dR*

(17)

*it i*;*t*—1 *it it it*

* 1. *Determination of the stability set of the first kind*

that can be written as:

*qi*;*t*—1

*kt kt*

+ *pit*

* *qit*

+ *uit* P *dL*

and *qi*;*t*—1

+ *pit*

* *qit*

+ *uit* 6 *dR*

Going back to Eqs. [(19)–(24)](#_bookmark6), the Lagrange function is

where *i* = 1; .. . *I*; *t* = 1; ... ; *T* (18)

*it*

*it*

*3.3. The deterministic form of Eqs.* [*(5)–(8)*](#_bookmark4)

*I*

*L*

*LF* = *Z* — *hkt* *b* — X *aikp*

*kt it*

*i*=1

* *kkt*ÿ*bL* — *bR* !

— *ait*ÿ—*qi*;*t*—1

*it*

*it*

— *pit*

+ *qit*

— *uit* + *dR*

The linear combination method [[19,20]](#_bookmark12) is adopted with the

*it*

multiobjective optimization. In multiobjective optimization,

*it*

*i*;*t*—1

— *b* ÿ*q* + *p* — *q* + *uit* — *dL*

applying the linear combination method to integrate the objec-

*it*

— */itpit* — *witqit* — *gituit* (29)

where

1

*Z* = 2 *c*1

*I*

*i*=1

X

*T*

*t*=1

Table 1 The comparison results between the two approaches.

Results in the paper

Objective function value = 41.8 Variables

Results obtained from the model

of Stephen C. Graves

Objective function value = 25

Variables

*u*\* = 2, *u*\* = 3,

21

31

*u*\* = 2, *u*\* = 2,

21

31

*u*\* = 3.3, *p*\* = 1.8,

32

11

*p*\* = 1, *p*\* = 2,

11

12

*p*\* = 1.2, *p*\* = 1,

12

22

*p*\* = 1, *p*\* = 1

22

31

*p*\* = 0.7, *q*\* = 0.8

32

11

and other variables equal zero

and other variables equal zero

X

*rit*ÿ*dR* — *uit* — *cp p*

— *cq q*

— *cuituit* + *rit*ÿ*dL* — *uit* — *cp p* !

× *it*

X X

*I T*

*it it*

*it it it*

—*cqitqit* — *cuituit*

*it it*

1

+ *c*2

2 *i*=1

*rit*ÿ*dR*

*it*

×

*t*=1

* *uit*

*L*

*it it it it it*

— *cp p* — *cq q* — *cuituit* — *rit*ÿ*d*

+*cqitqit* + *cuituit*

* *uit*

+ *cpitpit* !

(30)

The Kuhn–Tucker necessary optimality conditions corre-

sponding to this problem will take the following form:

∂*LF*/∂*p**it* = 0, ∂*LF*/∂*uit* = 0, ∂*LF*/∂*qit* = 0

*i* = 1, ... , *I*, *t* = 1, .. . , *T* (31)

*it*

3, *u*\* = 3.3, *p*\* = 1.8, *p*\* = 1.2, *p*\* = 1, *p*\* = 0.7, *q*\* = 0.8

and all other variables are equal to zero. Objective function

32

11

12

22

32

11

value is equal to 41.8.

The set of feasible parameters, solvability set and stability set of the first kind are calculated where Set of feasible param-

*ait*ÿ—*qi*,*t*—1

* *pit*

+ *qit*

* *uit* + *dL* = 0 *i* = 1, .. . , *I*, *t* = 1, ... , *T*

eters is *U* = {*kkt*

∈ *Rkt*|0 < *kkt*

6 1,*k* = 1, 2,3 and *t* = 1, 2}

*b* ÿ*q*

+ *p* — *q*

(32)

+ *uit* — *dR* = 0 *i* = 1, ... *I*, *t* = 1, .. . , *T*

and the solvability set is *V* = {*kkt* ∈ *U*|*kkt* = 1, *k* = 1, 2,3

and *t* = 1,2} and the stability set of the first kind is

*it i*,*t*—1 *it it it*

*hkt* *bL* — X *aikp* — *kkt*ÿ*bL* — *bR* ! = 0

*I*

(33)

*S*(*p*\*, *q*\*, *u*\*) = {*k*\* ∈ *V*|*k*11 = 0.1, 0 < *k*12 6 1, *k*21 = 0.2, 0 < *k*22 6

1, 0 < *k*31 6 1, *k*32 = 0.6.}.

[Table 1](#_bookmark7) shows the comparison between the results of our

*kt it*

*i*=1

*kt kt*

approach which is based on uncertainty case and one’s of Ste-

phen C. Graves approach which is based on the deterministic

*k* = 1, ... , *K*, *t* = 1, .. . , *T* (34)

*/itpit* = 0 *i* = 1, ... *I*, *t* = 1, .. . , *T* (35)

*witqit* = 0 *i* = 1, ... , *I*, *t* = 1, .. . , *T* (36)

*gituit* = 0 *i* = 1, .. . , *I*, *t* = 1, ... , *T* (37)

*it*

case.

It is clear that the result of the paper is better than the result obtained by Stephen C. Graves especially for the objective function value.

*qi*,*t*—1

*qi*,*t*—1

+ *pit*

+ *pit*

* *qit*
* *qit*

+ *uit* — *dL* P 0 *i* = 1, ... , *I*, *t* = 1, ... , *T*

+ *uit* — *dR* P 0 *i* = 1, .. . , *I*, *t* = 1, ... , *T*

*it*

(38)

(39)

6. Conclusions

In this paper, we have discussed the concepts of stability of

generalized production planning problem under interval data environment. We have defined and characterized some basic

*ait*, *bit*, *hkt*, */*it, *w*it, *g*it P 0 (40)

where *ait*, *bit*, *hkt*, */*it, *w*it, *g*it 6*i*,*k*,*t* are the Lagrange multipliers.

All the relations of the above system are evaluated at the opti- mal solution of Eqs. [(19)–(24)](#_bookmark6).

5. Numerical example

Consider the instance of production planning problem to max- imize net revenues given by

notions for the problem under consideration. These notions are the set of feasible parameters the solvability set and stabil- ity set of the first kind. However, as a point for future research, a comparison study is needed between the interval and fuzzy programming to tackle the production planning problem, where each of fuzzy programming and interval programming are two forms of uncertainty. This point for future research is to determine which of interval and fuzzy programming is more suitable for problem of concern.

*I* = 3, *K* = 3, *T* = 2, (*bL*, *bR*)=

>< 9>=

*it it*

8

(1, 6)(2, 4)

(2, 6)(1, 6)

(4, 13)(1, 12)

(3, 10)(8, 17) ,

8>< 9>=

>: (4, 12)(5, 15) >;

165

31

8>< 344 9>=

, *cp*

=

8>< 12 9>=

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in operations research and management and production and

*it it*

(*dL*, *dR*)=

, *a*

=

*ik*

>: (3, 6)(4, 9) >;

>: 256 >;

>: 21 >;

inventory. Amsterdam: Elsevier Science Publisher B.V.; 1993. p.

333–66.

*cuit* =

8>< 23 9>=

12

>: 21 >;

, *rit* =

8>< 43 9>=

24

>: 44 >;

, *kkt* =

0.90.8

0.60.7

8>< >=9

,

*it*

>: 0.50.9 >;

, *cqit* =

8>< 12 9>=

31

.

>: 12 >;

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By using the optimization approach which is described in

Section 3, the deterministic form for this example is obtained.

31

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Then we get the following optimal solution *u*\*

21

= 2, *u*\* =

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