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Hausdorff compactifications of topological function spaces via the theory of continuous lattices

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**Abstract**

It is known from the theory of continuous lattices that if *X* is a locally compact Hausdorff space then the set *LSC(X)* of lowersemicontinuous functions defined on X with values on the extended real lineadmits a unique compact Hausdorff topology making the functional *(f, g)to min(f, g)* continuous, namely the Lawson topology of the continuouslattice *LSC(X)*. It is natural to won- der whether the relative topologyon the subset *C(X)* of continuous functions is the compact- opentopology. Unfortunately, it turns out to be strictly weaker. But a relatedconstruction does produce a Hausdorff compactification of *C(X)*. Weshow that if *X* is a locally compact Hausdorff space and *Y* is aHausdorff topological space which is perfectly embedded into a continuouslattice *L* endowed with the Scott topology, then the Lawson topologyon the continuous lattice *LSC(X,L)* of Scott continuous maps from*X* to *L* induces the compact-open topology on the space*C(X,Y)* of continuous maps from *X* to *Y*. Thus, by takingthe closure of the image of *C(X,Y)* in *LSC(X,L)*, one gets aHausdorff compactification of *C(X,Y)*. Three particular cases are ofinterest. (1) If *Y* is the Euclidean real line one can take *L* as the lattice of compact connected subsets of the two- pointcompactification of *Y* ordered by reverse inclusion. In this case,*C(X,Y)* is already dense in *LSC(X,L)*. (2) If *Y* is alocally compact Hausdorff space, one can take *L* as the compactsubsets of the one-point compactification of *Y*. (3) As a furtherparticular case of (2), if *X* and *Y* are compact Hausdorff, oneconcludes that the Vietoris topology on the closed subsets of the cartesianproduct of *X* and *Y* induces the compact-open topology on*C(X,Y)*, by identifying continuous functions with their closed graphs,using the fact that the Lawson topology coincides with the Vietoristopology.

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