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Incremental Rebinding with Name Polymorphism [1](#_bookmark2)

Davide Ancona[a](#_bookmark0)*,*[2](#_bookmark2) Paola Giannini[b](#_bookmark1)*,*[3](#_bookmark2) Elena Zucca[a](#_bookmark0)*,*[4](#_bookmark2)

a *DIBRIS, Universit`a di Genova, Italy*

b *CS Institute, DISIT, Universit`a del Piemonte Orientale, Alessandria, Italy*

**Abstract**

We propose an extension with *name variables* of a calculus for incremental rebinding of code introduced in previous work. Names, which can be either constants or variables, are used as interface of fragments of code with free variables. Open code can be dynamically rebound by applying a *rebinding*, which is an association from names to terms. Rebinding is *incremental*, since rebindings can contain free variables as well, and can be manipulated by operators such as overriding and renaming. By using name variables, it is possible to write terms which are parametric in their nominal interface and/or in the way it is adapted, greatly enhancing expressivity. The type system is correspondingly extended by *constrained name-polymorphic types*, where simple inequality constraints prevent conflicts among parametric name interfaces.

*Keywords:* open code, incremental rebinding, name polymorphism, metaprogramming

# Introduction

Our previous work [[1](#_bookmark17),[2](#_bookmark18)] smoothly integrates static binding of the simply-typed lambda-calculus with a mechanism for dynamic and incremental rebinding of code. Fragments of open code to be dynamically *rebound* are values. Rebinding is done on a *nominal* basis, that is, free variables in open code are associated with *names* which do not obey *α*-equivalence. Moreover, rebinding is *incremental*, since rebind- ings, which are associations between names and terms, can in turn contain free variables to be rebound. Rebindings are first class values, and can be manipulated by operators such as overriding and renaming.

In this paper, we propose an extension of this previous work which supports, besides name constants, *name variables*, making it possible to write terms which are parametric in their nominal interface and/or the way it is adapted. For instance,

1 This work has been partially funded by “Progetto MIUR PRIN CINA Prot. 2010LHT4KM”.

2 Email: [davide.ancona@unige.it](mailto:davide.ancona@unige.it)

3 Email: [giannini@di.unipmn.it](mailto:giannini@di.unipmn.it)

4 Email: [elena.zucca@unige.it](mailto:elena.zucca@unige.it)

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it is possible to write a term which corresponds to the selection of an arbitrary component of a module. We summarize here below the language features.

* *Unbound terms*, of shape *⟨x*1 *'→ X*1*,..., xm '→ Xm | t⟩* are values representing “open code”. That is, *t* may contain free occurrences of variables *x*1*,..., xm* to be dynamically bound through the global nominal interface *X*1*,..., Xm*. To be used, open code should be combined with a *rebinding X*1 *'→ t*1*,..., Xm '→ tm*.
* Rebinding application is *incremental*, that is, an unbound term can be par- tially rebound, and a rebinding can be open in turn. For instance, the term

*⟨x'→X , y'→Y | x* +*y⟩* can be combined with the rebinding *⟨y'→Y | X '→y, Z '→y⟩*, getting *⟨y '→ Y , yj '→ Y | yj*+*y⟩*. This makes possible code specialization, simi- larly to what partial application achieves for positional binding.

* Rebindings are first-class values as well, and can be manipulated by operators such as overriding and renaming.
* A *name X* can be either a *name constant N* or a name variable *α*, and *name abstraction* Λ*α.t* and *name application t X* can be used analogously to lambda- abstraction and application to define and instantiate name-parametric terms.

The type system in [[2](#_bookmark18)], supporting both *open* (*non-exact* ) and *closed* (*exact* ) types for rebindings, is correspondingly extended to handle name variables. Notably, types are extended with *constrained name-polymorphic types* of shape *∀α*:*c.T* , where *c* is a set of *inequality constraints X /*=*Y* among names. Such constraints are necessary to guarantee that for each possible instantiation of *α* we get well-formed terms and types. For instance, the term Λ*α*:*α /*= *N .⟨ | N* :int *'→* 0*, α*:int *'→* 1*⟩* is a rebinding parametric in the name of one of its two components, which, however, must be different from the constant name *N* of the other component.

In the rest of this paper, we first provide the formal definition of an untyped version of the calculus (Section [2](#_bookmark3)), followed by some examples showing its expressive power (Section [3](#_bookmark5)). We then define a typed version of the calculus (Section [4](#_bookmark10)), for which we state a soundness result. We show typing examples in Section [5](#_bookmark15), and finally in the Conclusion we discuss related and future work.

# Untyped calculus

The syntax and reduction rules of the untyped calculus are given in Figure [1](#_bookmark4), where we leave unspecified constructs of primitive types such as integers, which we will use in the examples. We assume infinite sets of *variables x* , *name constants N* and *name variables α*. We use *X , Y* to range over *names* which are either name constants or name variables.

We use various kinds of sequences which represent finite maps: *unbinding maps u* from variables to names, *rebinding maps r* from names to terms, *renamings σ* from names to names, and *substitutions s* from variables to terms. We assume that order and repetitions are immaterial in such sequences. Moreover, in a term *t* which is well-formed, written *▶ t* , they actually represent maps, e.g., in *X*1 *'→ t*1*,..., Xm '→ tm*, if *Xi* = *Xj* then *ti* = *tj*. Hence, we can use the following notations: *dom* and *rng* for the domain and range, respectively, *u*1 *◦ u*2 for map composition, assuming *rng* (*u*2) *⊆ dom*(*u*1), (*u*1*, u*2) for the union of two maps with disjoint

domains, and *u*1[*u*2] for the map coinciding with *u*2 wherever the latter is defined, with *u*1 elsewhere.

* 1. :: = *... | v | x* term

*| t*1 *t*2 application

*| t X* name application

*| t*1>*t*2 rebinding operator

*|*!*t* run

*| t*1 *t*2 overriding

*| σ*1n*t* u*σ*2 renaming operator

* 1. :: = *x*1 *'→ X*1*,..., xm '→ Xm* unbinding map

1. :: = *X*1 *'→ t*1*,..., Xm '→ tm* rebinding map

*σ* :: = *X*1 *'→ Y*1*,..., Xm '→ Ym* renaming

*X , Y* :: = *N | α* names

* 1. :: = *... | λx.t | ⟨u | t⟩ | ⟨u | r⟩ |* Λ*α.t* value

*E* :: = [ ] *| ... | E t | v E | E X | E* >*t | v* >*E |*!*E | E t* evaluation context

*| v E | σ*1n*E* u*σ*2

1. :: = *x*1 *'→ t*1*,..., xm '→ tm* substitution

*t −→ t j*

(Ctx) *E* [*t* ] *−→ E* [*t j*]

(App)

(*λx.t* ) *v −→ t{x '→ v}*

(Name-App) *▶ t{α '→ N }*

(Λ*α.t* ) *N −→ t{α '→ N }*

(Reb-App) *rng* (*u*2)*∩dom*(*r* )= *$*

*⟨u|r⟩*>*⟨u*1*, u*2*|t⟩ −→ ⟨u, u*2*|t{x '→ r* (*u*1(*x* ))*|x∈dom*(*u*1)*}⟩*

(Run) (Over)

!*⟨ | t⟩ −→ t ⟨u*1 *| r*1*⟩ ⟨u*2 *| r*2*⟩ −→ ⟨u*1*, u*2 *| r*1[*r*2]*⟩*

(Rename)

*σ*1n*⟨u | r⟩*u*σ*2 *−→ ⟨σ*1 *◦ u | r ◦ σ*2*⟩*

**Fig. 1:** Untyped calculus: syntax and reduction rules

Besides lambda-abstractions and values of primitive types, there are three new kinds of values in the calculus: *unbound terms ⟨u | t⟩*, *rebindings ⟨u | r⟩* and *name abstractions* Λ*α.t* .

An unbound term, e.g., *⟨x '→ N | x* +1*⟩*, represents code which is not directly used but, rather, “boxed”, as the brackets suggest. This boxed code is possibly open, and can be dynamically rebound through a nominal interface.

Conversely, a rebinding represents code which can be used to dynamically rebind open code. A rebinding can be unbound as well, that is, its code can be open, as in *⟨x '→ N | N*1 *'→* 0*, N*2 *'→* 1+*x⟩*. According to the sequence notation, an unbound term with an empty unbinding map is simply written *⟨ | t⟩*, and analogously for a rebinding.

Name abstractions can be used to write terms which are parametric w.r.t. the nominal interface, e.g., Λ*α.⟨x '→ α | x* +1*⟩* is the parametric version of the above unbound term. Note that, differently from, e.g., [[11](#_bookmark27)], we take a stratified approach where *names are not terms*, to keep separate the conventional language, which is here lambda-calculus for simplicity, from the meta-level constructs, whose semantics is in principle independent. Hence, we have ad-hoc constructs for name abstraction and name application.

Besides values and variables, terms include compound terms constructed by the following operators: application, name application, rebinding, run, overriding, and renaming. They are illustrated together with reduction rules given in Figure [1](#_bookmark4).

Rule (Ctx) is the usual contextual closure.

Rule (App) is standard. The *application of a substitution to a term*, *t{s}*, is defined in the standard way. Note that a variable occurrence in the domain of an unbinding map behaves like a *λ*-binder. Hence, the variables in *dom*(*u*) are not free in *⟨u | t⟩*, and not subject to substitution.

In a name application *t X* , *t* and *X* are expected to reduce to a name abstraction, and a name constant, respectively. The name abstraction is applied to the name constant, as modeled by rule (Name-App). The *name substitution*, *t{α '→ N }*, that is, substitution of a name variable with a name constant, is defined in the standard way. In particular, the only construct that introduces binders is name abstraction, whereas name substitution has to be propagated also to unbinding maps, rebinding maps, and renamings. Note that, by name substitution, we could obtain ill-formed terms, e.g., *⟨ | α '→* 0*, N '→* 1*⟩{α '→ N }* gives *⟨ | ⟨ | N '→* 0*, N '→* 1*⟩⟩*. In this case, the rule cannot be applied, as formally denoted by the side condition *▶ t{α '→ N }*.

In a term *t*1>*t*2, the arguments of the rebinding operator *t*1 and *t*2 are expected to reduce to a rebinding and to an unbound term, respectively. When the rebinding is applied to the unbound term, rule (Reb-App), all the variables associated with

names provided by the rebinding (side condition *rng* (*u*2) *∩dom*(*r* )= *∅*) are replaced by the corresponding terms, and are therefore removed from the unbinding map of the unbound term. However, the unbinding map of the resulting unbound term is augmented with the unbinding map of the rebinding term. The condition *dom*(*u*) *∩ dom*(*u*2) = *∅*, implicitly required for the well-formedness of *u, u*2, can be always satisfied by applying a suitable *α*-renaming to one of the two terms. We also tacitly assume that the rule is applicable only when *r* (*u*1(*x* )) is defined for all *x ∈ dom*(*u*1), that is, *rng* (*u*1) *⊆ dom*(*r* ). For instance,

*⟨y '→ N*2 *| N*1 *'→ y* +2*, N*3 *'→ y⟩*>*⟨x '→ N*1*, y '→ N*2 *| x* +*y⟩*

reduces to *⟨y '→ N*2*, yj '→ N*2 *|* (*y* +2)+*yj⟩*.

In a term !*t* , the argument of the run operator is expected to reduce to an

unbound term with no names to be rebound, which can be unboxed, rule (Run). For instance, !*⟨ |* 0+1*⟩* reduces to 0+1, which can then be evaluated. Unbound terms can be “unboxed” and executed through the run operator only after their open code has been completed through one or more applications of rebindings so that they do not contain unbound variables; for instance, the unbound term *⟨x '→ N | x* +1*⟩* can be made self-contained with the rebinding *⟨ | N '→* 0*, N j '→* 1*⟩*.

In a term *t*1 *t*2, the arguments of the overriding operator are expected to reduce to two rebindings. Rule (Over) allows one to merge the two rebindings giving preference to the right one in case of conflict. Unbinding maps *u*1 and *u*2 are simply merged together (hence, names are shared). As it happens for rule (Reb-App), the implicit condition *dom*(*u*1) *∩ dom*(*u*2)= *∅* can be always satisfied by applying a suitable *α*-renaming to one of the two terms. For instance,

*⟨x '→ N*1 *| N*2 *'→ x* 1*, N*3 *'→* 1*⟩ ⟨x '→ N*1 *| N*3 *'→* 2*, N*4 *'→ x* 2*⟩* reduces to *⟨x '→ N*1*, x j '→ N*1 *| N*2 *'→ x* 1*, N*3 *'→* 2*, N*4 *'→ x j* 2*⟩*.

In a term *σ*1n*t* u*σ*2, the argument of the rebinding operator is expected to reduce

to a rebinding *⟨u | r⟩*; we use the more coincise notation *σ*1n*t* and *t* u*σ*2 when *σ*2 and *σ*1 are the identity renamings, respectively. The renaming operator is used for adapting the nominal interfaces of the unbinding and rebinding map *u* and *r* , respectively, rule (Rename). With the renaming *σ*1 it is possible to merge names, while with *σ*2 one can duplicate and remove terms; for instance

(*N*1 *'→ N*2*, N*2 *'→ N*2)n*⟨x '→ N*1*, y '→ N*2 *| N*1 *'→* 0*, N*3 *'→* 1*⟩*u(*N*1 *'→ N*1*, N*2 *'→ N*1)

reduces to *⟨x '→ N*2*, y '→ N*2 *| N*1 *'→* 0*, N*2 *'→* 0*⟩*. As for rule (Reb-App), we tac- itly assume that rule (Rename) is applicable only when *rng* (*u*) *⊆ dom*(*σ*1) and *rng* (*σ*) *⊆ dom*(*r*2) respectively hold.

Renamings are essential for adapting unbound terms and rebindings; renamings and name abstractions favor dynamic software adaptation and reuse. For instance the term

*t* = Λ*α*1*.*Λ*α*2*.λxr .*(*xr* u(*N*1 *'→ α*1*, N*2 *'→ α*2))>*⟨x*1 *'→ N*1*, x*2 *'→ N*2 *| x*1 *x*2*⟩*

is expected to take a rebinding *xr* with generic shape *⟨| α*1 *'→ t*1*, α*2 *'→ t*2*,.. .⟩*, adapt it by renaming and then apply it to the unbound term *⟨x*1 *'→ N*1*, x*2 *'→ N*2 *| x*1 *x*2*⟩*; as an example, *t N*3 *N*4 *⟨ | N*3 *'→ λx.x* +1*, N*4 *'→* 1*⟩* reduces (in some steps) to 2.

# Examples of use of name abstraction

In previous work [[2](#_bookmark18)] we have already analyzed the expressive power of the constructs for building unbound terms and rebindings, for overriding and renaming of rebind- ings, and for rebinding application to unbound terms. Such constructs support several programming notions, as dynamic scoping, rebinding, meta-programming and component-based programming. For instance, if we assume to extend the cal- culus with the let rec construct to define recursive functions, then the following declarations define a function pow supporting program specialization via generative programming:

let rec aux\_pow = lambda n.

if n >0 then <x *'→* X, y *'→* Y | Y *'→* x\*y> > aux\_pow (n -1) else <y *'→* Y | y>

let pow = lambda n.

let f = < | Y *'→* 1> > ( aux\_pow n) in lambda x. !( < | X *'→* x> > f)

For instance, pow 3 evaluates to

lambda x. !( < | X*'→*x> > <x1 *'→* X, x2 *'→* X, x3 *'→* X | x3\*x2\*x1 \*1 >).

Therefore, pow 3 2 rewrites to !(< | X*'→*2> > <x1 *'→* X, x2 *'→* X, x3 *'→* X | x3\*x2\*x1\*1>), which rewrites to ! < | 2\*2\*2\*1>, which rewrites to 2\*2\*2\*1, and, finally, to 8.

Here we focus on the expressive power of the newly introduced constructs for name manipulation, and show how they favor generic and meta-programming.

## Module/component selection

Rebinding terms directly support the notion of module/component. We have already shown [[2](#_bookmark18)] how member selection of closed (that is, where all dependencies have been resolved) modules/components can be encoded. For instance, the fol- lowing term encodes an operator which selects the *Y* member of a (closed) module represented by a rebinding:

*ts* =lambda x. !( x > < y *'→* Y | y >)

For instance the term *ts* < | X*'→*0, Y*'→*42> evaluates to 42. However, in this way selection can be encoded only for a single fixed name constant (Y in this specific case).

With the newly introduced construct of name abstraction, a generic definition of the selection operator can be provided by a single term of the calculus.

*t′* =Lambda *α*. lambda x. !( x > < y *'→ α* | y >)

*s*

In this way, the same term *t′* can be used for selecting members associated with

*s*

*s*

arbitrary names. For instance, if *t* =< | F*'→*lambda n.n+1, N*'→* 41>, then (*t′*

*s*

F *t*) (*t′*

N *t*)

evaluates to 42.

In mainstream object-oriented languages such meta-programming facilities are supported either by specific libraries for reflection, or by more flexible constructs, as the JavaScript bracket notation. In all cases, no static checking is performed to ensure that the selected names will be always defined at runtime.

For instance, with the use of the bracket notation in JavaScript [5](#_bookmark6) it is possible to define the following function:

function select ( name , object ){ return object [ name ]}

The notation *e*1[*e*2] allows programmers to access properties of the object denoted by *e*1 whose name is defined by the arbitrary expression *e*2. So, select ("val",{val:42}) returns 42, whereas select ("foo",{val:42}) is undefined.

As we will see in Section [5](#_bookmark15), the term *t′* =Lambda *α*. lambda x. !(x > < y *'→ α* | y >)

*s*

can be typed statically, to ensure that only defined members are selected.

5 All examples presented here are compliant with the ECMAScript 5 syntax, although some of them could be written in a slightly more concise way by using the new features and shorthands introduced with the recently released specification of ECMAScript 6.

## Dynamic adaptation of mixins

Mixin classes [[3](#_bookmark19)] and mixin modules [[4](#_bookmark20)] are notions commonly employed in generic programming to support software reuse.

Among statically typed mainstream object-oriented programming languages, mixins are only supported by C++, with templates, see [[14](#_bookmark30)]. The following class template defines class CheckedMixin which is parametric in its base class, represented by the template parameter B.

template <class B>

class CheckedMixin : public B { public :

static int checked\_op ( int value ) { if( B:: in\_bounds ( value ))

return B:: op( value ); else

throw std :: logic\_error (" Illegal argument ");

}

};

The mixin adds the static method checked\_op, and can be instantiated with classes defining op(int) and in\_bounds(int), as in the following code fragment:

class Sqrt { public :

static int op( int value ) { return sqrt ( value ); } static bool in\_bounds ( int value ){ return value >= 0; }

};

class Checked\_sqrt : public CheckedMixin <Sqrt > { }; int main () {

assert ( Checked\_sqrt :: checked\_op (4)==2) ; assert ( Checked\_sqrt :: op ( -4)!=2) ; assert ( Checked\_sqrt :: checked\_op ( -4)!=2) ; *// throws logic\_error*

}

Thanks to the generic code defined by CheckedMixin, class Sqrt is extended with the static method checked\_op with checks whether the argument is non negative, before applying the static method op which, in turn, applies the library function [6](#_bookmark7) sqrt.

The main limitation of mixins implemented with C++ class templates is their inability to be adapted to classes where their methods do not match the name con- vention imposed by the mixin; in the case of CheckedMixin, the parametric base class must provide the static methods op(int) and in\_bounds(int). Furthermore, typecheck- ing of C++ templates is not compositional, therefore such constraints are checked every time the template is instantiated.

Dynamic languages, as JavaScript [[9](#_bookmark25)], allow dynamic adaptation of mixins.

In this case the mixin is defined by a function [7](#_bookmark8) taking three arguments that are expected to contain strings: op denotes the name of the operation that has to be checked, in\_bounds denotes the name of the operation that performs the check, and new\_op denotes the name of the newly added operation corresponding to the checked version of op.

function CheckedMixin ( op , in\_bounds , new\_op ){ this [ new\_op ] = function ( x){

6 Function sqrt does not perform any check, unless math\_errhandling has the constant MATH\_ERREXCEPT

set.

7 We recall that JavaScript is a prototype-based language where objects are dynamically created through functions, although recently an equivalent class-based notation has been introduced in ECMAScript 6.

if (! this [ in\_bounds ]( x)) throw " Illegal argument "

return this [ op ]( x)

}

}

Thanks to the bracket notation the programmer can pass to the CheckedMixin function the proper strings to adapt the instances of CheckedMixin.

sqrt ={ *// a new object with two properties*

sqrt : Math . sqrt , check\_arg : function ( x){ return x >=0}

}

CheckedMixin . prototype = sqrt

*// all instances of CheckedMixin will have sqrt as prototype* chk\_sqrt = new CheckedMixin (" sqrt "," check\_arg "," checked\_sqrt ") chk\_sqrt . sqrt ( -4) *// evaluates to NaN* chk\_sqrt . checked\_sqrt (4) *// evaluates to 2* chk\_sqrt . checked\_sqrt ( -4) *// throws " Illegal argument "*

The same function CheckedMixin can be used to extend an object which computes the

log function.

log ={ *// a new object with two properties*

log : Math . log10 , check\_arg : function ( x){ return x >=0}

}

CheckedMixin . prototype = log

*// all instances of CheckedMixin will have log as prototype* chk\_log = new CheckedMixin (" log "," check\_arg "," safe\_log ") chk\_log . log ( -10) *// evaluates to NaN*

chk\_log . safe\_log (10) *// evaluates to 1*

chk\_log . safe\_log ( -10) *// throws " Illegal argument "*

Thanks to the support for name manipulation, mixin adaptation and application can be expressed in our calculus; furthermore, as shown in Section [4](#_bookmark10), compositional typechecking ensures the type correctness of mixin adaptation and application. The JavaScript example given above can be recast [8](#_bookmark9) in our calculus as follows:

*tm* =Lambda *αop* . Lambda *αin b* . Lambda *αn op* . lambda r. let *n op* =

!( r > < *op '→ αop* , *in b '→ αin b* | lambda x. if ( not *in b* ( x)) -1 else *op* (x) >) in r *Q* < | *αn op '→ n op* >

As in the previous example, the mixin takes three names *αop*, *αin b*, and *αn op*, corresponding to the name of the operation that has to be checked, the name of the operation that performs the check, and the name of the newly added operation which is the checked version of the operation *αop*. Then it takes a rebinding *r*, which is expected to provide a definition for the operations *αop*, and *αin b*, and that is applied to an unbound term which defines the new operation in terms of the operations *αop*, and *αin b* provided by the rebinding. The result of the application of the rebinding is run to get the value corresponding to the new operations, and, finally, the rebinding (which plays the role of a module) is extended with the new component by means of the overriding operator.

8 Since the calculus does not support exceptions, in case the bounds are not verified the function simply returns the conventional value -1.

# Typed calculus

Figure [2](#_bookmark11) shows the syntax of the typed calculus, which is extended by annotating variables and names with types, and name variables with constraints, as explained in detail below.

*t*

*u r σ*

:: = *... | λx* :*T.t |* Λ*α*:*c.t | x | t*1 *t*2 *| t X | t*1>*t*2 *|*!*t | t*1*Qt*2 *| σ*1n*t* u*σ*2 term

:: = *x*1:*T*1 *'→ X*1*,..., xm*:*Tm '→ Xm*

:: = *X*1:*T*1 *'→ t*1*,..., Xm*:*Tm '→ tm*

:: = *X*1 *'→ Y*1*,..., Xm '→ Ym*

*X , Y* :: = *N | α*

*T* :: = *... |T*1 *→ T*2 *| ∀α*:*c.T | ⟨*Δ *| T⟩ | ⟨*Δ1 *|* Δ2*⟩ν*

*c* :: = *X*1*/*=*Y*1 *... Xm/*=*Ym*

Δ :: = *X*1:*T*1*,..., Xm*:*Tm*

*ν* :: = *◦ |* +

Σ :: = *A*; *c*; Γ

*A* :: = *α*1 *... αn*

Γ :: = *x*1:*T*1*,..., xm*:*Tm*

unbinding map

rebinding map renaming name

type constraints name context

(variance) annotation typing context

name variables variable context

**Fig. 2:** Typed calculus: syntax

*Constraints* are of shape *X /*= *Y* . A set of constraints *c* is consistent under name variables *A*, written *A ▶ c*, if variables occurring in *c* belong to *A*, and, moreover, *X /*=*X /∈ c* for all *X* . We say that *X could be equal to Y under c*, written *c |*= *X* =? *Y* , if *X /*= *Y /∈ c*.

Types include function types, constrained name-polymorphic types, unbound types *⟨*Δ *| T⟩*, and rebinding types *⟨*Δ1 *|* Δ2*⟩ν*. For simplicity we omit basic types for primitive values such as integers or booleans. In the explanations in the follow- ing, we illustrate in more detail the new feature of the type system represented by constrained name-polymorphic types. The reader can refer to our previous work for more explanations and examples on unbound types and open/closed rebinding types.

A type *T* is well-formed under the set of name variables *A* and constraints *c* if the judgment *A*; *c |*= *T* OK is derivable by the rules of Figure [3](#_bookmark12). We write *A ▶ X* to indicate that *X* belongs to *A*, if it is a name variable.

*A*; *c |*= *T* OK *A*; *c |*= *T′* OK *A ∪ {α}*; *c, c′ |*= *T* OK

(WF-arrow-type) (WF-name-arrow-type)

*A*; *c |*= *T → T′* OK *A*; *c |*= *∀α*:*c′.T* OK

*A*; *c |*= *T* OK (1 *≤ k ≤ m*) *A ▶ X* (1 *≤ k ≤ m*) *c |*= *Xi* ? *X ⇒ T* = *T* (1 *≤ i, j ≤ m*)

*k*

*k*

= *j*

*i j*

(WF-name-ctx)

*A*; *c |*= *X*1:*T*1*,..., Xn*:*Tm* OK

*A*; *c |*=Δ OK *A*; *c |*= *T* OK *A*; *c |*= Δ*′* OK *A*; *c |*=Δ OK

(WF-unb-type) (WF-reb-type)

*A*; *c |*= *⟨*Δ *| T⟩* OK *A*; *c |*= *⟨*Δ*′ |* Δ*⟩ν* OK

*c |*= *Xi* ? *Xj ⇒ ti* = *tj* (1 *≤ i, j ≤ m*)

=

(WF-reb-map)

*c |*= *X*1 *'→ t*1*,..., Xm '→ tm* OK

*A ▶ X* (1 *≤ k ≤ m*) *A ▶ Y* (1 *≤ k ≤ m*) *c |*= *Xi* ? *X ⇒ Y* = *Y* (1 *≤ i, j ≤ m*)

*k*

*k*

= *j*

*i j*

(WF-ren)

*A*; *c |*= *X*1 *'→ Y*1*,..., Xm '→ Ym* OK

**Fig. 3:** Well-formed types, rebinding maps, and renamings

Function types correspond to lambda abstractions, where the variable is now

annotated with a type.

Constrained name-polymorphic types correspond to name abstractions, where the name variable is now annotated with constraints. Constraints are necessary to guarantee that for each possible instantiation of *α* we get well-formed terms and types. For instance, the term Λ*α*:*α /*= *N .⟨ | N* :int *'→* 0*, α*:int *'→* 1*⟩* is a rebinding parametric in the name of one of its two components, which, however, must be different from the constant name *N* of the other component.

Unbound types *⟨*Δ *| T⟩* correspond to open code: Δ is a sequence *X*1:*T*1*, .., Xm*:*Tm* called name context. The type specifies that the open code needs the rebinding of the names *Xi* to terms of type *Ti* (1 *≤ i ≤ m*) in order to correctly produce a term of type *T* . An unbound type is well-formed under name variables *A* and constraints *c* only if types occurring in the sequence are well-formed, name variables occurring in the sequence belong to *A*, and names which could be equal under *c* are mapped in the same type, as modeled by rules (WF-unb-type) and (WF-name-ctx) in Figure [3](#_bookmark12).

Rebinding types *⟨*Δ1 *|* Δ2*⟩ν* correspond to rebindings; the name context Δ1 specifies the names which the rebinding depends on, while the name context Δ2 = *X*1:*T*1*,..., Xm*:*Tm* specifies that the rebinding map associates each name *Xi* with a term of type *Ti* (1 *≤ i ≤ m*). If the type is annotated with *ν* = +, then we say that the type is *open* (or *non-exact* ), and the rebinding map is allowed to contain more associations than those specified in the name context. The annotation *ν* = *◦* is used for *closed* (or *exact* ) types, to enforce that the domain of the rebinding map exactly coincides with the domain of Δ2. In the typing rules we will use the binary operator *H* over annotations, defined by *◦H ν* = *ν H◦* = *ν*, and + *H* + = +. A rebinding type is well-formed under name variables *A* and constraints *c* only if types occurring in the sequences Δ1 and Δ2 are well-formed, name variables occurring in the sequences belong to *A*, and names which could be equal under *c* are mapped in the same type, analogously to what is required for an unbound term, as modeled by rules (WF-reb-type) and (WF-name-ctx) in Figure [3](#_bookmark12).

Renamings, as well as values, evaluation contexts, substitutions, and name sub- stitutions are defined as for the untyped language. Figure [3](#_bookmark12) also defines well- formedness of rebinding maps under constraints *c*, and of renamings under name variables *A* and constraints *c*. (Untyped) rebinding maps are well-formed if names which could be equal under *c* are mapped in the same term, as modeled by rule (WF-reb-map). Note that well-formedness of type annotations is separately checked by rule (WF-name-ctx). Well-formedness of renamings requires that name vari- ables belong to *A*, and names which could be equal under *c* are mapped in the same name, as modeled by rule (WF-ren).

The subtyping relation is defined in Figure [4](#_bookmark13).

Subtyping between function types is standard. A constrained polymorphic type can be made more specific by adding more constraints or making more specific the type obtained by instantiation.

Subtyping between unbound types obeys a rule similar to that for function types: the relation is contravariant in the name context, and covariant in the type returned after rebinding. Subtyping between name contexts is defined by the usual rule for record subtyping: both width and depth subtyping are allowed. Width and depth

**Fig. 4:** Typed calculus: subtyping rules

(Sub-arr)

*T′ ≤ T*1 *T*2 *≤ T′*

1 2

*T*1 *→ T*2 *≤ T′ → T′*

(Sub-name-arr)

1 2

*c′⊆c T ≤ T′*

*∀α*:*c.T ≤ ∀α*:*c′.T′*

(Sub-unb)

Δ*′ ≤* Δ *T ≤ T′*

*⟨*Δ *| T⟩ ≤ ⟨*Δ*′ | T′⟩*

(Sub-open-reb)

Δ*′ ≤* Δ1 Δ2 *≤* Δ*′*

*⟨*Δ1 *|* Δ2*⟩ν ≤ ⟨*Δ*′ |* Δ*′ ⟩*+

1 2

Δ*′ ≤* Δ1 *Ti ≤ T′* (1 *≤ i ≤ n*)

(Sub-closed-reb)

1

*i*

*⟨*Δ1 *| X*1:*T*1*,..., Xn*:*Tn⟩ ≤ ⟨*Δ*′ | X*1:*T′ ,..., Xn*:*T′ ⟩*

*◦*

*◦*

1 1 *n*

(Sub-name-ctx)

*∀i* (1 *≤ i ≤ n*) *∃j* (1 *≤ j ≤ m*) *X ′* = *Xj ∧ Tj ≤ T′*

*i i*

*X*1:*T*1*,..., Xm*:*Tm ≤ X ′*:*T′ ,..., X ′* :*T′*

1 1

*n n*

*A*; *c |*= *T* OK *A*; *c |*= *T′* OK *T ≤ T′*

(Sub-constr)

*A*; *c |*= *T ≤ T′*

1 2

subtyping are also allowed between rebinding types, in case the right-hand-side (rhs for short) type in the relation is open, because a closed type can always be considered as an open type, but not the other way around. This is a consequence of the fact that closed types express more restrictive constraints on rebinding maps. For instance, the rebinding *⟨| X* :*TX '→ tx , Y* :*TY '→ ty ⟩* has, for any Δ, type *⟨*Δ *| X* :*TX, Y* :*TY ⟩ν* for both *ν* = + and *ν* = *◦*, whereas it has type *⟨*Δ *| X* :*TX⟩ν* only for *ν* = +; note also that the most precise type for this term is *⟨ | X* :*TX, Y* :*TY ⟩◦*. When the rhs type in the subtyping relation is a closed rebinding type, then the lhs type must be closed as well, and, therefore, it must define the same set of names; in this case only depth subtyping is allowed.

Finally, rule (Sub-constr) models subtyping under name variables and con- straints.

The typing judgment has shape *A*; *c*;Γ *▶ t* : *T* , meaning that the term *t* has type *T* under the name variables *A*, constraints *c*, and context Γ providing types for the free variables. The typing rules are given in Figure [5](#_bookmark14).

The type system supports subsumption, rule (T-Sub). Note that the second premise implies both types to be well-formed.

Rule (T-Abs) for lambda abstractions is standard.

In rule (T-Name-Abs), the term Λ*α*:*cj.t* is well-typed if the introduced con- straints *cj* are consistent under the current name variables augmented by *α*, and *t* is well-typed taking the union of the constraints.

In rule (T-Unb), the term *⟨u | t⟩* is well-typed if the name context extracted from *u* by the auxiliary function *name ctx* , say, *X*1:*T*1*,..., Xm*:*Tm* , is well-formed under the current name variables and constraints, that is, *Xi* belongs to *A* if it is a name variable, and, if *Xi* could be equal to *Xj* under *c*, then they are mapped in the same type. The resulting type *T* is obtained by typing *t* in the context updated by that extracted from *u* by the auxiliary function *ctx* . Both auxiliary functions are defined at the bottom of Figure [5](#_bookmark14).

In rule (T-Reb), the term *⟨u | r⟩* is well-typed if the name contexts extracted from *u* and *r* are well-formed under the current name variables and constraints. Moreover, *r* must be well-formed under the current constraints, that is, names which could be equal are mapped in the same term. Finally, for each name in the domain of *r* , annotated with type, say, *T* , the associated term must have type *T* in the context updated by that extracted from *u* by the auxiliary function *ctx* . Note

**Fig. 5:** Typed calculus: typing rules that an exact type can be always deduced.

*A*; *c*;Γ *▶ t* : *T A*; *c |*= *T ≤ T′ A*; *c |*= *T*1 OK *A*; *c*; Γ[*x* :*T*1] *▶ t* : *T*2

(T-Sub) (T-Abs)

*A*; *c*;Γ *▶ t* : *T′ A*; *c*;Γ *▶ λx* :*T*1*.t* : *T*1 *→ T*2

*A ∪ {α} ▶ c′ A ∪ {α}*; *c, c′*;Γ *▶ t* : *T*

(T-Name-Abs)

*A*; *c*;Γ *▶* Λ*α*:*c′.t* : *∀α*:*c′.T*

*A*; *c |*=Δ OK *A*; *c*; Γ[Γ*′*] *▶ t* : *T*

(T-Unb)

*A*; *c*;Γ *▶ ⟨u | t⟩* : *⟨*Δ *| T⟩*

*name ctx* (*u*)=Δ

*ctx* (*u*)= Γ*′*

*c |*= *X*1 *'→ t*1*,..., Xm '→ tm* OK

*A*; *c |*= *⟨*Δ1 *|* Δ2*⟩◦* OK *A*; *c*; Γ[Γ*′*] *▶ ti* : *Ti* (1 *≤ i ≤ m*)

(T-Reb)

*A*; *c*;Γ *▶ ⟨u | X*1:*T*1 *'→ t*1*,..., Xm*:*Tm '→ tm⟩* : *⟨*Δ1 *|* Δ2*⟩*

*name ctx* (*u*)= Δ1

* *ctx* (*u*)= Γ*′*

Δ2 = *X*1:*T*1*,..., Xm*:*Tm*

Σ *▶ t*1 : *T*1 *→ T*2 Σ *▶ t*2 : *T*1

(T-Var) Γ(*x* )= *T* (T-App)

*A*; *c*;Γ *▶ x* : *T* Σ *▶ t*1 *t*2 : *T*2

*A ▶ c′{α '→ X } A*; *c*;Γ *▶ t* : *∀α*:*c′.T A ▶ X*

(T-Name-App)

*A*; *c*;Γ *▶ t X* : *T{α '→ X }*

*A*; *c |*= Δ1*,* Δ2 OK

*A*; *c*;Γ *▶ t* : *⟨*Δ *|* Δ *,* Δ *⟩ A*; *c*;Γ *▶ t* : *⟨*Δ *|* Δ *⟩*

(T-Over)

1

1

*′* 1

*ν*

1

2

2

*ν*2

(Δ = *$* or *ν* = *◦*) and

1

2

*A*; *c*;Γ *▶ t*1*Qt*2 : *⟨*Δ *|* Δ1*,* Δ2*⟩ν*1 *Hν*2

Σ *▶ t* : *⟨ | T⟩*

(T-Run)

Σ *▶*!*t* : *T*

*dom*(Δ*′* ) *⊆ dom*(Δ2)

1

(T-Reb-App)

*A*; *c |*= Δ1*,* Δ2 OK

*A*; *c*;Γ *▶ t*1 : *⟨*Δ*′,* Δ1 *|* Δ*,* Δ2*⟩ν A*; *c*;Γ *▶ t*2 : *⟨*Δ*,* Δ1 *| T⟩*

*A*; *c*;Γ *▶ t*1>*t*2 : *⟨*Δ*′,* Δ1 *| T⟩*

(Δ = *$* or *ν* = *◦*) and

1

*dom*(Δ1) *∩ dom*(Δ2)= *$*

*A*; *c |*= *σ*1 OK *A*; *c |*= *σ*2 OK *A*; *c |*= *σ*1 *◦* Δ1 OK *A*; *c*;Γ *▶ t* : *⟨*Δ1 *|* Δ2*⟩ν*

(T-Rename)

*A*; *c*;Γ *▶ σ*1n*t* u*σ*2 : *⟨σ*1 *◦* Δ1 *|* Δ2 *◦ σ*2*⟩◦*

*ctx* (*x*1:*T*1 *'→ X*1*,..., xm*:*Tm '→ Xm*)= *x*1:*T*1*,..., xm*:*Tm*

*name ctx* (*x*1:*T*1 *'→ X*1*,..., xm*:*Tm '→ Xm*)= *X*1:*T*1*,..., Xm*:*Tm*

*σ ◦* Δ=

⎧⎪⎨ Δ*′*

⎪

⎩

if *dom*(Δ) *⊆ dom*(*σ*)

where *X* :*T ∈* Δ*′* iff *∃Y Y* :*T ∈* Δ *∧ σ*(*Y* )= *X*

⎧

undefined otherwise

*′*

⎪⎨ Δ

if *rng* (*σ*) *⊆ dom*(Δ)

Δ *◦ σ* =

⎪⎩ undefined otherwise

where *X* :*T ∈* Δ*′* iff *X ∈ dom*(*σ*) *∧ T* = Δ(*σ*(*X* ))

Rules (T-Var) and (T-App) are standard.

In rule (T-Name-App), the term *t X* is well-typed if *X* belongs to *A* if it is a name variable, *t* has a constrained polymorphic type *6α*:*cj.T* , and by replacing *α* by *X* in the constraints *cj* we do not get inequalities of shape *Y /*=*Y* . In this case, the resulting type is obtained by replacing *α* by *X* in *T* . The obvious definitions of replacing a name variable by a name in constraints and types are omitted.

In rule (T-Over), overriding *t*1 *t*2 is well-typed only if *t*1 and *t*2 have rebinding types; the name context of the type of *t*1 is deterministically split in two parts.

The part Δ*j* corresponds to names which are also defined in *t*2, as expressed by

1

the side condition *dom*(Δ*j* ) *⊆ dom*(Δ2), hence are overridden, whereas the part Δ1 corresponds to names which are not defined in *t*2. If Δ1 = *∅*, then *t*1 is fully

1

overridden, hence the name context of the result is that of *t*2; in this particular case the type of *t*2 is allowed to be open, whereas if Δ1 */*= *∅*, then *t*2 is required to have a closed type, otherwise it would not be possible to correctly identify Δ1.

The previously defined operator *H* combines the two annotations *ν*1 and *ν*2 so that the resulting type is closed if and only if both types of *t*1 and *t*2 are closed.

Note that, due to the presence of name variables, besides names which are neces- sarily overridden, there are names which *could* be overridden in some instantiation. For instance, in the term Λ*α*:*α /*= *N*1*.⟨ | N*1:*T*1 *'→ t*1*, N*2:*T*2 *'→ t*2*⟩ ⟨ | α*:int *'→* 1*⟩*, the name *N*1 is never overridden, whereas the name *N*2 could be overridden for *α* = *N*2. The name context which is assigned to the overriding term is that cor- responding to the case of no overriding, that is, *N*1:*T*1*, N*2:*T*2*, α*:int in this case. However, since this name context must be well-formed under the constraints *α /*= *N*1, the type *N*2 must necessarily be int, so that we get a well-formed type even for the instantiation *α* = *N*2.

Rule (T-Run) states that a term of unbound type can be safely run only if its name context is empty, that is, all variables have been already properly bound in the code.

The typing rule (T-Reb-App) for rebinding application *t*1>*t*2 is similar to the typing rule for overriding: to correctly identify the names in *t*1 that are not neces- sarily bound, denoted by Δ1, the rule requires an exact type for *t*2, except when

Δ1 = *∅* (that is, all names are bound) for which an open type is allowed as well. This is due to the fact that the bound names of *t*1 must have the same type of the corresponding names in *t*2, while additional names in *t*2 not specified in the open type of *t*2 might be used for binding names of *t*1 with incompatible types. Note that by applying subsumption, it is always possible to bind a name with a term whose type is a subtype of the expected type.

Finally, in rule (T-Rename) for renaming, the two renamings must be well- formed under current name variables and constraints, that is, the newly introduced names must exist, and names which could be equal are mapped in the same name. The name contexts of the resulting type are propagated from the original ones by the auxiliary operators *σ ◦* Δ and Δ *◦ σ*, both partial, defined at the bottom of Figure [5](#_bookmark14). Note that if two names *X* and *Y* are mapped by *σ*1 in two names which could be equal, then *X* and *Y* must have the same type, as formally expressed by requiring the well-formedness of the name context *σ*1 *◦* Δ1.

Soundness of the type system w.r.t. the operational semantics states that *well- typed terms do not get stuck*. This is derived from the *subject reduction* and *progress* properties that follows.

**Theorem 4.1 (Subject Reduction)** *Let t be such that, for some* Σ *and T we have* Σ *▶ t* : *T. If t −→ t j, then* Σ *▶ t j* : *T.*

**Theorem 4.2 (Progress)** *Let t be such that, for some T we have ∅*; *∅*; *∅ ▶ t* : *T. Then either t is a value or for some t j, we have that t −→ t j.*

# Examples of typing

In this section we consider the typed version of the examples in Section [3](#_bookmark5).

## Module/component selection

We can define the typed version of the term which corresponds to generic selec- tion of closed modules in the following way:

*t′′* =Lambda *α*:*$*. lambda x:*⟨ | α*:*T⟩*+ . !( x > < y:*T '→ α* | y >)

*s*

The parameter x must be a rebinder without dependencies, otherwise the run oper- ator ! could not be safely applied; its type is open, because additional components are allowed to be present; the only component that is required to be defined must have the name denoted by the name variable *α*, otherwise the rebinder application x > < y *'→ α* | y > would not return an unbound term without dependencies, and the application of the run operator would be unsafe. The type *T* associated with *α* is arbitrary, but must be fixed once and for all; a more generic definition could be given if the calculus could support standard parametric polymorphism, besides name polymorphism. We leave for further investigation an extension of the calculus and its type system towards this direction.

No constraints have to be imposed on *α*, since no name conflicts can ever arise in this case.

According to the rule (T-Name-Abs), *∅*; *∅*; *∅ ▶ tjj* : *6α*:*∅.⟨ | α*:*T⟩*+ *→ T* , be-

*s*

cause

1. *{α}*; *∅*; *∅▶ λx*:*⟨ | α*:*T⟩*+*.*!(*x*>*⟨y*:*T '→ α | y⟩*): *⟨ | α*:*T⟩*+ *→ T*
2. *{α}*; *∅*; *x*:*⟨ | α*:*T⟩*+ *▶*!(*x*>*⟨y*:*T '→ α | y⟩*): *T*
3. *{α}*; *∅*; *x*:*⟨ | α*:*T⟩*+ *▶ x*>*⟨y*:*T '→ α | y⟩* : *⟨ | T⟩*
4. *{α}*; *∅*; *x*:*⟨ | α*:*T⟩*+ *▶ x* : *⟨ | α*:*T⟩*+
5. *{α}*; *∅*; *x*:*⟨ | α*:*T⟩*+ *▶ ⟨y*:*T '→ α | y⟩* : *⟨α*:*T | T⟩*

In particular, the judgment ([iii](#_bookmark16)) is derivable by instantiation of rule (T-Reb- App) where Δ*j*, Δ1, and Δ2 are empty, and Δ = *α*:*T* .

## Dynamic adaptation of mixins

The example of dynamic mixin adaptation shown in Section [3](#_bookmark5) can be annotated with types in the following way, where *T*1 = int *→* int, *T*2 = int *→* bool:

*tm* =Lambda *αop* :*$*. Lambda *αin b* :*αin b /*=*αop* . Lambda *αn op* :*αn op /*=*αop , αn op /*=*αin b* . lambda r: *⟨ | αop* :*T*1*, αin b* :*T*2*⟩*+ .

let *n op*:*T*1 =

!( r > <*op*:*T*1 *'→ αop* , *in b*:*T*2 *'→ αin b* | lambda x:int. if( not *in b* ( x)) -1 else *op* (x) >) in r *Q* < | *αn op* :*T*1 *'→ n op* >

The constraints *αin b/*=*αop*, and *αn op/*=*αin b* are necessary to ensure that the term *tm* is well-typed, since the type *T*1 associated with *αop*, and *αn op* is different from the type *T*2 associated with *αin b*. On the other hand, the constraint *αn op/*=*αop* is not strictly required to ensure type safety, since both name variables are associated with the same type *T*1. However, it guarantees that the mixin defined by *tm* is additive, in the sense that the component *αop* required to be provided from *r* will not be overridden by the addition of the component *αn op*; if the constraint *αn op/*=*αop* is removed, then the mixin can be applied in a more permissive way, since the user is free to decide whether to override or not component *αop* with *αn op*.

The typing judgment

*∅*; *∅*; *∅▶ tm* : *6αop*:*∅.6αin b*:*c*1*.6αn op*:*c*2*.*

*⟨ | αop*:*T*1*, αin b*:*T*2*⟩*+ *→⟨ | αop*:*T*1*, αin b*:*T*2*, αn op*:*T*1*⟩*+

can be derived for the term *tm*, with *c*1=*αin b/*=*αop*, and *c*2=*αn op/*=*αop, αn op/*=*αin b*.

In particular, by rule (T-Reb) it is possible to derive

*A*; *c*1*, c*2; *n op*:*T*1 *▶⟨ | αn op*:*T*1 *'→ n op⟩* : *⟨ | αn op*:*T*1*⟩○*

where *A* = *{αop, αin b, αn op}*, and, by rule (T-Over) is possible to derive

*A*; *c*1*, c*2; *n op*:*T*1 *▶ r ⟨ | αn op*:*T*1 *'→ n op⟩* : *⟨ | αop*:*T*1*, αin b*:*T*2*, αn op*:*T*1*⟩*+

by instantiating the rule with Δ and Δ*j* empty, Δ1 = *αop*:*T*1*, αin b*:*T*2, and Δ2 =

1

*αn op*:*T*1. The rule is applicable because the judgment *A*; *c*1*, c*2 *|*= Δ1*,* Δ2 OK is derivable from rule (WF-name-ctx), thanks to the two constraints *αin b/*=*αop*, and *αn op/*=*αin b* in *c*1*, c*2.

# Conclusion

We proposed a calculus which integrates standard static binding with incremental rebinding of code based on a *parametric* nominal interface. That is, names, which can be either constants or variables, are used as interface of fragments of code with free variables, which can be passed around and rebound. By using name variables, it is possible to write terms which are parametric in their nominal interface and/or in the way it is adapted, greatly enhancing expressivity. The type system is based on *constrained name-polymorphic types*, where simple inequalities constraints prevent conflicts among parametric name interfaces. We have shown how to express type- safe dynamic adaptation of code, in particular, we showed how to express mixins. Similar results can be achieved in dynamically typed languages, such as JavaScript or through the use of reflection. However, in these settings we loose the possibility of expressing type constraints that can be statically checked. In C++ with multiple inheritance and templates we can define mixins, but we have to know the names of the methods that will be mixed in.

This work continues a stream of research on foundations of binding mechanisms, started with [[8](#_bookmark24),[7](#_bookmark23)]. The goal was to provide a unifying foundation for dynamic scoping, rebinding of marshalled computations, meta-programming features, and operators present in calculi for modules. Classical (ad-hoc) models for dynamic scoping are [[10](#_bookmark26)] and [[6](#_bookmark22)], whereas the *λ*marsh calculus of [[5](#_bookmark21)] supports rebinding w.r.t. named contexts (not individual variables). The meta-programming features of our calculus are orthogonal to the one of MetaML [[15](#_bookmark31)], since, on one side, we do not have the analogous of the *escape* annotation of MetaML forcing evaluation inside boxed code, but on the other, our rebinding construct avoids the problem of unwanted variable capturing. Module calculi are described, e.g., in [[4](#_bookmark20)].

In future work we plan to add polymorphic types, so that name polymorphism can be more effectively used and also explore the relations between our name ab- straction and the one provided by languages of the family of FreshML [[13](#_bookmark29),[12](#_bookmark28)], where

it is possible to compute with syntactical data structures involving names and name binding in a statically typed setting.

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