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*MGS: a Rule-Based Programming Language for* Complex Objects and Collections

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*Abstract*

*We present the rst results in the development of a new declarative program- ming language called MGS. This language is devoted to the simulation of biological processes, especially those whose state space must be computed jointly with the current state of the system. MGS proposes a uni ed view on several computational mechanisms initially inspired by biological or chemical processes (Gamma and the CHAM, Lindenmayer systems, Paun systems and cellular automata). The basic computation step in MGS replaces in a collection A of elements, some subcollection B, by another collection C. The collection C only depends on B and its adjacent elements in A. The pasting of C into A B depends on the shape of the involved collections. This step is called a transformation. The speci cation of the collection to be substituted can be done in many ways. We propose here a pattern language based on the neighborhood relationship induced by the topology of the collection. Several features to control the transformation applications are then presented.*

# *1 Motivations*

*1.1 Dynamical Systems and their State Structures*

*A dynamical system (or DS in short) corresponds to a phenomenon that evolves* in time. The phenomenon is located on a system characterized by \observ- ables". The observables are called the variables of the system, and are linked by some relations. The value of the variables evolves with the time. The set of the values of the variables that describe the system constitutes its state. The state of a system is its observation at a given instant. The state has often a spatial extent (the speed of a uid in every point of a pipe for example). The temporal sequence of state changes is called the trajectory of the system.

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*Intuitively, a DS is a formal way to describe how a point (the state of the* system) moves in the phase space (the space of all possible states of the sys- tem). It gives a rule telling us where the point should go next from its current location (the evolution function). These notions are illustrated in Fig.1.

*We are interested in the simulation of such systems. This requires the* speci cation of the system state and the evolution function. This speci cation

*Trajectories x(t) and y(t)*

20

15

*Evolution Constraints for x and y dx*

*= Ax Bxy*

*dt*

*dy*

*= Cy + Dxy*

*dt*

10

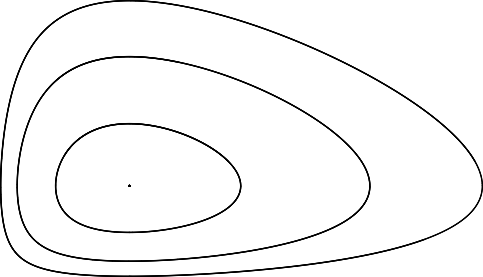
5

50 100 150 200 250 300

*Solving the constraints gives the trajectory of x(t) and y(t) starting*

*from some initial state. The evolution of a variable is periodic.*

*Evolution in the phase space (x; y)*



12

10

8

6

4

2

5 10 15 20

*The three curves correspond to the cyclic evolution of the system starting from three di erent initial conditions. A point in this plot corresponds to a state (x; y). A curve corresponds to the evolution (x; y)(t). The periodicity of the trajectories of x and y gives a closed curve. There is a fourth curve reduced to a xed point. The image by the evolution function of this point is itself. This point is characterized by dx=dt = dy=dt = 0 (no change).*

*Fig. 1. Example of the evolution of a predator-prey system (this DS has a static structure). The system is characterized by two variables: x corresponds to the num- ber of predators and y to the number of preys in some ecological system. The num- ber of preys changes because of the growth of the population and because the preys are eaten by the predators. The number of births is proportional to the number of preys and the decrease is proportional to the number of prey-predator encounters, which is itself proportional to the product xy. The number of predators decreases because the competition between predators and the increase is proportional to the chance of prey-predator encounters. The resulting di erential equations specify the evolution function. They can be integrated to plot the trajectory of x and y (top picture) and the state evolution (bottom picture). The structure of the system is static in the sense that the state of the system is always described as an element of R2.*

*can be very diÆcult to achieve because of the complexity of the description* of the phase space and of the evolution function. However, the more we know about the phase space, the more we know about the DS. For example, if the phase space is nite, every trajectory is nally cyclic.

*Very often the phase space has some structure and this structure can be* used to simplify the description of the state and its evolution and to gain some knowledge about the system. For example, one may specify the evolution function hi for each observable oi and recover the global evolution function h as a product of the \local" hi.

*Standard DS exhibit a static structure, that is, the exact phase space of* the DS can be known statically before the simulation. For instance, in the example of a uid owing through a pipe, since the geometry of the pipe is not subject to change, the structure of the state is not a function of time (and the phase space corresponds to the vector elds on the static volume of the pipe).

*1.2 DS with a dynamical structure*

*The a priori determination of the phase space cannot always be done. This is* a common situation in biology [9,7,8]. Such DS can be found in the modeling of plant growing, in developmental biology, integrative cell models, protein transport and compartment simulation, etc. This accounts for the fact that the structure of the phase space must be computed jointly with the current state of the system. In this case, we say that the DS has a dynamical structure. The description of DS with a dynamical structure are especially hard.

*In this kind of situation, the dynamic of the system is often speci ed as* several local competing transformations occurring in an organized set of sim- pler entities. The organization of this set is subject to possible drastic changes in the course of time and is a plain part of the state of the DS.

*1.3 Unifying Several Biologically Inspired Computational Models*

*One of our additional motivations is the ability to describe generically the* basic features of four models of computation: and the CHAM, P systems, L systems and cellular automata (CA). They have been developed with various goals in mind, e.g. parallel programming for , semantic modeling of nonde- terministic processes for the CHAM, calculability and complexity issues for P systems, formal language theory and biological modeling for L systems, paral- lel distributed model of computation for CA (this list is not exhaustive). We assume that the reader is familiar with the main features of these formalisms but a short description of these computational models is given in section 5 for the readers convenience.

*All these computational models rely on a biological or biochemical metaphor.*

*It is then natural to require their integration in a uniform framework.*

# *2 The Basic Ideas*

*Our goal is to provide a general support for the notions of \organized set" and*

*\local competing transformations" that can be used to describe uniformly the* computation mechanisms of , P and L systems and CA.

*We call collection a set of elements with some \organization" (to be clari-*

*ed later). Several kind of organizations are used in programming languages* and give raise to several data structures: sets, multisets (or bags), sequences (or list), arrays, trees, terms, etc. The collection type underlying the compu- tations in , CHAM and P system is the multiset, L systems rely on sequences and CA on arrays.

*2.1 A Uni ed Description of , P and L system and CA*

*A program, a P or a L system and a CA can be themselves viewed abstractly* as a discrete dynamical system: a running program can be characterized by a state and the evolution of this state is speci ed through evolution rules. From this point of view, the following characteristics have to be stressed.

*Discrete space and time. The structure of the state (the multiset in , the* membranes hierarchy in a P system, the word in a L system and the array in a CA) consists of a discrete collection of values. This discrete collection of values evolves in a sequence of discrete time steps.

*Temporally local transformation. The computation of a new value in the* new state depends only on values for a xed number of preceding steps (and usually just one step).

*Spatially local transformation. The computation of a new collection is* done by a structural combination of the results of more elementary compu- tations involving only a small and static subset of the initial collection.

*\Structural combination", means that the elementary results are combined* into a new collection, irrespectively of their precise value. \Small and static subset" makes explicit that only a xed subset of the initial elements are used to compute a new element value (this is measured for instance by the diameter of the evolution rule of a P systems, the local neighborhood of a CA, the number of variables in the right hand side of a reaction or the context of a rule in a L system).

*Considering these shared characteristics, the main di erence between the* four formalisms appears to be the organization of the collection. The abstract computational mechanism is always the same:

*(i) a subcollection A is selected in a collection C;*

*(ii) a new subcollection B is computed from the collection A;*

*(iii) the collection B is substituted for A in C.*

*see Fig. 2. We call these three basic steps a transformation . In addition* to transformation speci cation, there is a need to account for the various

*constraints in the selection of the subcollection A and the replacement B.* This abstract view makes possible the uni cation in the same framework of various computational devices. The trick is just to change the organization of the underlying collection.

*C T(C)*

*A*

*B*

*T*

*x y = f(x’)*

*Fig. 2. The basic mechanism of the transformation of a collection. Collection C is of some kind. A rule T speci es that a subcollection A of C has to be substituted by a collection B computed from A. The right hand side of the rule is computed from the subcollection matched by the left hand side x and its possibles neighbors x0 in the collection C.*

*Constraining the Subcollections*

*There is a priori no constraint in the case of : one element or many* elements are replaced by zero, one or many elements. In the case of P sys- tems, the evolution of a membrane may a ect only the immediate enclosing membrane (by expelling some tokens or by dissolution): there is a localization of the changes. This is also the case for L systems: the new collection B is inserted at the place of A and not spread out over C. For CA, the changes are not only localized, but also A and B are constrained to have the same shape: usually A is restricted to be just one cell in the array and B is also one cell to maintain the array structure.

*2.2 Collections as Spaces*

*Considering these constraints and their expression, it is very natural to see* a collection as a set of places or positions organized by a topology de ning the neighborhood of each element in the collection and also the possible sub- collections. To stress the importance of the topological organization of the collection's elements, we call them topological collection .

*For instance, one may decide that neighbors of an element in a sequence* are their two adjacent elements (except for the rst and the last element in the sequence which have only one neighbor). The neighborhood can be speci ed by a relation denoted by \;". That is to say, x; y means that x is a neighbor of y. If S is a subset of the elements of the collection C, then we say that S is connected if the quotient of S by the transitive closure of \;" is reduced to only one element. A subsequence C0 of C is a connected subset of the elements of C. This means that the possible subsequences of a sequence ` are the intervals of

*`. Additional conditions can be put to constrain the possible subcollections.*

*For instance, one may want to consider only the sequence pre xes or the* sequence suÆxes for the subcollections. However, a subcollection is always a connected subset of the main collection.

*This topological approach formalizing the notion of collection is part of a* long term research e ort [12] developed for instance in [13] where the focus is on the substructure and in [10] where a general tool for uniform neighborhood de nition is developed. The topology needed to describe the neighborhood in a set or a sequence, or more generally the topology of the usual data struc- tures, are fairly poor. They are sketched in section 5. So, one may ask if the machinery needed is worthwhile. Actually, more complex topologies are needed for some biological modeling applications [11]. And more importantly, the topological framework unify various situations. Our ultimate goal is to develop a generic implementation based on these notions, see [11].

*Now, we come back to our initial goal of specifying the dynamical structure* of a DS. A collection is used to represent the state of a DS. The elements in the collection represent either entities (a subsystem or an atomic part of the DS) or messages (signal, command, information, action, etc.) addressed to an entity. A subcollection represents a subset of interacting entities and messages in the system. The evolution of the system is achieved through transformations, where the left hand side of a rule typically matches an entity and a message addressed to it, and where the right and side speci es the entity's updated state, and possibly other messages addressed to other entities. If one uses a multiset organization for the collection, the entities interact in a rather unstructured way, in the sense that an interaction between two objects is enabled simply by virtue of their both being present in the multiset. More organized topological collections are used for more sophisticated spatial organization.

*2.3 The MGS Project and the Organization of the Rest of this Paper*

*We do not claim that topological collection are a useful theoretical framework* encompassing all the previous formalisms. We advocate that few notions and a single syntax can be consistently used to allow the merging of these formalisms for programming purposes. This leads to the development of an experimental programming language called MGS. MGS is the acronym of \ (encore) un Mod ele G eneral de Simulation (de syst eme dynamique) " (yet another General Model for the Simulation of dynamical systems). MGS is a vehicle used to investigate general notions of collections and transformations and to study their adequacy to the simulation of various biological processes.

*The MGS language is presented informally in section 3 through some exam-* ples. We review rst the notions of collections and then their transformations. Simple examples of MGS programs are given in section 4. All examples are pro- cessed using the current version of the MGS interpreter. Then, in section 5, we sketch how the previous formalisms can be emulated in MGS.

*3 An MGS Quick Tour*

*MGS embeds the idea of topological collections and their transformations into* the framework of a simple dynamically typed functional language. Collec- tions are just new kinds of values and transformations are functions acting on collections and de ned by a speci c syntax using rules. MGS is an applicative programming language: operators acting on values combine values to give new values, they do not act by side-e ect.

*In our context, dynamically typed means that there is no static type check-* ing and that type errors are detected at run-time during evaluation. Although dynamically typed, the set of values has a rich type structure used in the def- inition of pattern-matching, rule and transformations.

*We give here informally the main constructs concerning collections, trans-* formations and their applications. Elements of the MGS syntax are given through examples.

*3.1 Collections*

*In addition to basic values like integers, oats, strings, lambda-expressions,* etc., MGS handles records and several kinds of collections. The elements in a collection can be any kind of values: basic, records or arbitrary nesting of collections. The values of the record's elds are also of any kind, thus achieving complex objects in the sense of [5]. Collections are (sub-)typed. The tree in Fig. 3 gives the type hierarchy of collections.

collection

monoidal

record

pair

array

**...**

*MySet*

set

bag

*AnotherSet*

seq

*Fig. 3. The subtyping hierarchy of collection kinds. MySet and AnotherSet are user-de ned collection types, Cf. below. The types collection and monoidal do not correspond to concrete data structures, but to predicates, Cf. below. Concep- tually, a record is a set of pairs ( eld-name, eld-value) but it is managed through dedicated operators.*

*Monoidal Collections*

*Several kinds of topological collections are supported by MGS. We focus* here on sets, multisets and sequences. These kinds of collection are called monoidal because they can be build as a monoid with operator join \;": a sequence corresponds to a join that has no special property (except associativ- ity), multisets are obtained with commutative joins and sets when the operator

*is both commutative and idempotent. The join operator with its properties* induces the topology of the collection and the neighborhood relationship.

*There is a large amount of generic operations available for all collection* kinds, based on the function algebra developed for instance in [5]. We do not detail these features as they are not relevant for our purpose here. The table 1 gives the main construction operations for structural recursion.

*Table 1*

*Main constructions operations for monoidal collections. The line (\*) gives an overloaded syntax (the type of the arguments is used for desambiguation).*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *empty* | *addition* | *singleton* | *merge* |
| *Set* | *set : ()* | *insert* | *single set(x)* | *union* |
| *Bag* | *bag : ()* | *increment* | *single bag(x)* | *sum* |
| *Seq* | *set : ()* | *::* | *single seq(x)* | *@* |
| *(\*)* |  | *;* |  | *;* |

*User-De ned Subtypes*

*Often there is a need to distinguish several collections of the same kind* (e.g. several multisets nested in one other multiset). Various ways can be used to achieve the distinction. For instance, in the P system formalism, each multiset is labeled by a unique integer to reference them unambiguously. We chose to distinguish between collections of the same kind by types. The type of a collection must be thought as a label that does not change the structure of the collection. Types are organized by a subtyping relationship. The subtyp- ing relation organizes types into a poset. The kind of a collection constitutes the maximal elements of this hierarchy. Collection type declarations look like:

*collection MySet = set; collection AnotherSet = set;*

*collection AnotherMySet = MySet ;*

set

*MySet AnotherSet AnotherMySet*

*These three declarations specify a hierarchy of three types. Type AnotherMy-*

*Set isa subtype of MySet which is a subtype of set. The type set is prede ned* and corresponds to a collection kind (other prede ned types are seq for se- quences and bag for multisets). The type AnotherSet is also a subtype of set but is not comparable with MySet.

*A type introduced by a type declaration can later be used in pattern-* matching (Cf. section 3.3) or as a predicate to test if a value is of a given type. A monoidal collection type can also be used in the building of a collection by

*the enumeration of its elements:*

*1; 1+ 1; 2+ 1; 2 2; MySet : ()*

*is an expression evaluating to the set of four integers: 1, 2, 3 and 4. The* collection kind is a set, and its type is MySet. Actually, expression \Myset : ()" denotes the empty MySet and \;" is the overloaded join operator: x; X creates a new collection with element x merged with the elements of collection X; and expression X, Y creates a new collection with elements of both collections *X* and Y .

*The type of a collection is taken into account for several collection op-* erations. For instance, the join of two collections of type A and B gives a collection with type C corresponding to the common ancestor of A and B (with the previous example, set is the common ancestor of MySet and An- otherSet ). Other example, MySet is the common ancestor of AnotherMySet ) and itself.

*3.2 Records*

*An MGS record is a special kind of collection. An MGS record is a map that* associates a value to a name called eld. The value can be of any type, including records or other collections. Accessing the value of a eld in a record is achieved with the dot notation: expression fa = 1; b = "red"g:b evaluates to the string "red".

*Records can be merged with the overloaded + operator. Expression r1 +r2* computes a new record r having the elds of both r1 and r2. Then r:a has the value of r2:a if the eld a belongs to r2, else the value of r1:a (asymmetric merge [18]).

*For records, type declarations look like* state R = fag;

*state S = fb; ~cg + R;*

*state T = S + fa = 1; d : stringg;*

*(state is the keyword used to introduce the de nition of a record type in MGS).* The rst declaration speci es a record type R which consists of the records with at least a eld named a. Types can be used as predicates:

*R(fa = 2; x = 3g) or equivalently fa = 2; x = 3g : R*

*evaluates to true because the record fa = 2; x = 3g) has a eld a. The second* declaration de nes S which has all the elds of R plus a eld b and no eld

*c. The + operator between record types emulates a kind of inheritance. The* de nition T specializes type S by constraining the eld a to the value 1 and saying that an additional eld d must be present and be a string.

*3.3 Pattern, Rule and Transformations* A transformation T is a set of rules:

*trans T = f ::: rule ; ::: g*

*When there is only one rule in the transformation, the enclosing braces can* be dropped. A rule is a basic transformation taking the following form:

*pattern => expression*

*where pattern in the left hand side (lhs) of the rule matches a subcollection A* of the collection C on which the transformation is applied. The subcollection A is substituted in C by the collection B computed by the expression in the right hand side (rhs) of the rule. There are also several kinds of rules, as detailed below.

*3.3.1 Patterns*

*We present the pattern expressions that have a generic meaning, that is, they* can be interpreted against any collection kind. The grammar of the patterns expression is:

*P at ::= x j f:::g j p; p0 j p + j p j p : P j p=exp j p as x j (p)*

*where p; p0 are patterns, x ranges over the pattern variables, P is a predicate* and exp is an expression evaluating to a boolean value. The explanations below give an informal semantics for these patterns.

*variable: a pattern variable x matches exactly one element. The variable* *x* can then occur elsewhere in the rest of the rule.

*state pattern: f:::g are used to match one element which is a record. The* content of the braces can be used to match records with or without a spe- ci c eld (eventually constrained to a given eld type or eld value). For instance, fa; b : string; c = 3; ~dg is a pattern that matches a record with

*elds a, b of type string and c with value 3, but no eld d.*

*neighbor: p; p0 is a pattern that matches two connected collections p and* p0. For example, x; y matches two connected elements (i.e., x must be a neighbor of y). The connection relationship depends of the collection kind.

*repetition: pattern p+ (resp. p ) matches a non empty subcollection of* elements matched by p (resp. a possibly empty subcollection).

*binding: a binding p as x gives the name x to the collection matched by p.* This name can be used anywhere in the rest of the rule. E.g., the pattern x; x matches two connected elements with the same value (each occurrence of x in a rule denotes the same value).

*guard: p=exp matches the collections matched by p verifying exp. Pattern p : P is a syntactic suggar for ((p as x)=P (x)) where x is a fresh variable.*

*For instance, x : MySet lters an element of type MySet. Another example:* y= y > 3 matches an element y provided that y > 3 holds.

*Here is a contrived example. Pattern*

*(x : int=x < 3)+ as S = (card(S) < 5) & (fold[+](S) > 10)*

*selects a subcollection S of integers less than 3, such that the cardinality of S is* less than 5 and the sum of the elements in S is greater than 10. If this pattern is used against a sequence (resp. a set, a multiset), S denotes a subsequence (resp. a subset, a sub-multiset).

*Some pattern constructs are speci c to a collection kind. For example, the* construct \^; x" is used to select an element which has no left neighbor in a sequence. Such pattern has no meaning when the transformation is applied for instance to a set, and an error is raised. Another example of a speci c construct are the operators left and right. They can be used in the guard of a pattern (or in the rhs of a rule) to refer to the element to the right or to the left of a matched subsequence. These constructions depend on the topology of the collection and we plan to develop a generic and systematic speci cation of these operators using the notion of boundary.

*3.3.2 Rules*

*A transformation is a set of rules. When a transformation is applied to a* collection, the strategy is to apply as many rules as possible in parallel. A rule can be applied if its pattern matches a subcollection. Several features are used to have a ner control over the choice of the rules applied within a transformation.

*Exclusive and inclusive rules*

*Exclusive rules consume their argument: that is, a subcollection matched* by an exclusive rule cannot intersect a subcollection matched by any other rule. Inclusive rules don't have this kind of constraint. They are mainly used to transform independent parts of a complex object. Currently, only a rhs matching a record is allowed in an inclusive rule, but the idea must be extended to nested collections. The concept of inclusive rule may appear very speci c; however, it is a very e ective way to cut down the combinatorial explosion of the behavior speci cations. Inclusive rules are better explained by an example. Suppose we have to manipulate records having at least a eld x and y. Then,

*fx as vg +=> fx = v + 1g and fy as vg +=> fy = 2 vg*

*are two inclusive rules (because the arrow is +=>) matching respectively a* record with at least a eld x and a record with at least a eld y. So they can both apply to the record fx = 2; y = 3g. An inclusive rule of form

*r+ => r0 where r is a record pattern and r0 an expression evaluating to a* record, replaces the matched record R by R + r0. So, the result of applying the two previous rules to fx = 2; y = 3; z = 0g is fx = 3; y = 6; z = 0g. This result is computed as

*fx = 2; y = 3; z = 0g + fx = 2 + 1g + fy = 2 3g or fx = 2; y = 3; z = 0g + fy = 2 3g + fx = 2+ 1g*

*and is independent of the order of application of the two rules. Indeed, the*

*rules work on independent parts of the record, both for accessing or updating* the value of a eld.

*Priority*

*Exclusive rules are applied before any inclusive rules. A priority can be* associated to each rule, to specify a precedence order within each class (the priority of inclusive rules may be used to specify the relative order of their applications).

*Local variables and conditional rules*

*MGS is a functional language with some imperative features. Imperative* local variables can be attached to a transformation and updated by side e ects in the rhs of the rules. These variables can be used in a rule guard allowing the conditional use of a rule. For instance, the transformation

*trans T [a = 0] = f::: ; R = x =f on a < 5 g=> (a := a + 1; 2 x); ::: g*

*speci es a rule R which is applied at most 5 times (within the evaluations trig-* gered by one application of T ). The semicolon in the rhs of the rule denotes the sequencing of two evaluations. As a consequence, the local imperative variable a, initialized to 0 when T is applied, counts the number of rule ap- plications. The initial value of a variable local to a transformation can be overridden when the transformation is applied; for instance the evaluation of T [a = 3](:::) triggers at most 2 uses of rule R.

*3.4 Managing the Applications of a Transformation*

*A transformation T is a function like any other function and a rst-class* value. For instance, a transformation can be passed as an argument to an- other function or returned as a result. It allows to sequence and compose transformations very easily.

*The expression T (c) denotes the application of one transformation step of* the transformation T to the collection c. As said above, a transformation step consists in the parallel application of the rules (modulo the rule application's features). A transformation step can be easily iterated:

*T [n] (c) denotes the application of n transformation steps to c* T [fixpoint] (c) application of T until a xpoint is reached

*T [fixrule] (c) idem but the xpoint is detected when no rule applies*

*In addition to the standard transformation step strategy, two other appli-* cation modes exist. In the stochastic mode, the choice of the exclusive rule to apply is made randomly. The priorities of the exclusive rules are then considered as the relative probability of their e ective application (when they can apply). In asynchronous mode, only one exclusive rule is applied in one transformation step.

# *4 Examples of MGS Programs*

*The following example are freely inspired by examples given for , P systems* and L systems.

*Sorting a Sequence*

*A kind of bubble-sort is immediate:*

*trans Sort = (x; y = y < x) => y; x;*

*(This is not really a bubble-sort because swapping of elements can take at* arbitrary places; hence an out-of-order element does not necessarily bubble to the top in the characteristic way.)

*Eratosthene's Sieve on a Set*

*The idea is to generate a set with integers from 2 to N (with rules Generate* and Succed ) and to replace an x and an y such that x divides y by x (rule Eliminate). The result is the set of the prime integers less than N.

*trans Generate = fx; trueg => x; fx + 1; trueg; trans Succed = fx; trueg => x;*

*trans Eliminate = (x; y = y mod x = 0) => x;*

*With these de nition, the expression*

*Eliminate [fixrule] Succed Generate[N](f2; trueg; set : ()) computes the primes up to N.*

*Eratosthene's Sieve on a Sequence*

*The idea is to re ne the previous algorithm using a sequence. Each element* i in the sequence corresponds to the previously computed ith prime Pi and is represented by a record fprime = Pig. This element can receive a candidate number n, which is represented by a record fprime = Pi; candidate = ng. If the candidate satis es the test, then the element transforms itself to a record

*r = fprime = Pi; ok = ng. If the right neighbor of r is of form fprime = Pi+1g,* then the candidate n skips from r to the right neighbor. When there is no right neighbor to r, then n is prime and a new element is added at the end of the sequence. The rst element of the sequence is distinguished and generates the candidates.

*trans Eratos = f*

*Genere1 = n : integer = ~right n*

*=> n; fprime = ng;*

*Genere2 = n : integer; fprime as x;~candidate;~ok g*

*=> n + 1; fprime = x; candidate = ng;*

*Test1 = fprime as x; candidate as y;~ok g = y mod x = 0*

*=> fprime = xg;*

*Test2 = fprime as x; candidate as y;~ok g = y mod x <> 0*

*=> fprime = x; ok = yg;*

*Next = fprime as x1; ok as yg; fprime as x2;~ok ;~candidateg*

*=> fprime = x1g; fprime = x2; candidate = yg; NextCreate = fprime as x; ok as yg as s= ~right s*

*=> fprime = xg; fprime = yg;*

*g*

*We have given an explicit name to each rule. The expression*

*Erasto[N]((2; seq : ()))*

*executes N steps of the Erastothene's sieve. For instance Erasto[100]((2; seq : ())) computes the sequence: 42; fcandidate = 42; prime = 2g; fok = 41; prime = 3g; fprime = 5g; fprime = 7g; fprime = 11g; fprime = 13g; fok = 37; prime = 17g; fprime = 19g; fprime = 23g; fprime = 29g; fprime = 31g; seq : ().*

# *5 Comparison with Other Approaches*

*We want to show that and the CHAM, P systems, L systems and cellular* automata (CA) can been handled in MGS. Because they t harmoniously, we gain con dence that the underlying concepts of topological collection may reveal unifying and covering a broad class of biological DS with a dynamical structure.

*5.1 Sets and Multisets: The programming language and the CHAM*

*The computational model underlying [2,1] is based on the chemical reaction* metaphor; the data are considered as a multiset M of molecules and the computation is a succession of chemical reactions according to a particular rules. A rule (R; A) indicates which kind of molecules can react together (a subset m of M that satis es predicates R) and the product of the reaction

*(the result of applying function A to m). Several reactions happen at the same* time. No assumption is made on the order on which the reactions occurs. The only constraint is that if the reaction condition R holds for at least one subset of elements, at least one reaction occurs (the computation does not stop until the reaction condition does not hold for any subset of the multiset).

*The CHemical Abstract Machine (CHAM) extends these ideas with a focus* on the expression of semantic of non deterministic processes [3].

*The Topology of Sets*

*A set V is organized such that each element is neighbor of any other* elements in the set (with this de nition, an element of V is connected with any other element).

*A multiset M of elements e 2 E can be represented by a set M^*

*N E.*

*If e 2 M with multiplicity n, then the n elements (1; e); (2; e); :::; (n; e) belong*

*to M^ . The multiset M is represented as the set associated to M^*

*and any*

*element in the multiset is neighbor of any other element.*

*With this representation, the application of one rule on a multiset M is* also the application of an MGS rule. The connection between any two multiset elements accounts the fact that any sub-multiset can be matched and replaced in a rule.

*5.2 Nesting of Multisets: P systems*

*P systems [17,16] are a new distributed parallel computing model based on* the notion of a membrane structure. A membrane structure is a nesting of cells represented, e.g, by a Venn diagram without intersection and with a unique superset: the skin. Objects are placed in the regions de ned by the membranes and evolve following various transformations: an object can evolve into another object, can pass trough a membrane or dissolve its enclosing membrane. As for , the computation is nished when no object can further evolve.

*The P Systems Topology*

*The case of P systems is more interesting, because the topology can be used* to take into account the nesting of multisets and the locality of a computation step. In this approach, the region associated to a membrane would be a

*2 dimensional object (surfaces) and the membranes would be 1 dimensional* (curves).

*A cruder and simpler approach just associates a multiset M to the region* associated with the skin of a P system. The di erence with is that the elements of M can be multiset themselves, associated to the inner membranes. In this approach, P systems are viewed as a theory of nested (opposed to at) multiset rewriting. We can handle also this approach, because MGS values can be arbitrary combinations of other values.

*5.3 Sequences: L systems*

*L systems are a formalism introduced by A. Lindenmayer in 1968 for simulat-* ing the development of multicellular organism. Related to abstract automata and formal language, this formalism has been widely used for the modeling of plants. A L system can be roughly described as a grammar with an axiom and a set of derivation rules. The productions are applied in parallel in a non deterministic manner. 0L systems are context-free grammars. D0L systems are deterministic context-free grammars: given a letter A there is at most one production rule that can be applied. Parametric L systems deal with modules instead of letters: a module is a letter associated with a list of parameters. The production rules are extended with side-conditions on the parameters. For example,

*A(x; y): x 3 ! A(2x; x + y)*

*is a rule that can be applied to the module A(2; 5) to gives the module A(4; 7).* This rule cannot be applied on A(7; 1) because the rst parameter x does not match the condition.

*The Topology of Sequences*

*The topology of a sequence has been sketched in paragraph 2.2. It is the* intuitive view of the sequence has a sequence of contiguous cells.

*The application of only one production a ! b of a D0L system is similar* to the application of a simple MGS rule (x=x == a) => b on a sequence.

*5.4 Cellular Automata*

*Cellular automata (CA) have been invented many times under di erent names:* tessalation automata, cell spaces, iterative arrays, etc. However, a fair fraction of the computer research on two-dimensional cellular automata has its ultimate origins in the work of J. Von Neumann to provide a more realistic model for the behavior of complex systems in biology [19].

*In a simple case, a 2D cellular automaton consists in a grid of cells or* sites, each with a value taken in a nite set V. The values are updated in a sequence of discrete time steps, according to a de nite, xed, rule. Denoting the value of a site at position (i; j) by ai;j, a simple rule gives its new value

*as a0 = '(ai;j; ak ; :::; ak ), where ' is a function from Vp+1 to V and where*

*i;j 1 p*

*the akj are the values of the p neighbors of site (i; j). For example, the Von*

*Neumann neighbors of a cell (i; j) are the four cells (i 1; j); (i+1; j); (i; j 1)*

*and (i; j + 1).*

*Many variations are possible: organization of the cells in a regular lattice* of any dimensions or even in a general graph, variable neighborhood, various

*nite set V. However the main characteristics of CA are largely una ected by* such additional complications.

*The Topology of Arrays*

*The organization of the cells of an array is the natural one (Von Neuman* or Moore neighborhood). A rule of a cellular automata is an MGS rule applying on only one cell. The conditions on the neighbor cells can be expressed using guards and the speci c neighbors accessors.

# *6 Conclusion and Future Work*

*The technical report [11] gives more details on the topological formalization of* collections and transformations and outlines several examples of MGS programs (the tokenization of a sequence of letters, the computation of the convex hull of a set of points in R3, the computation of the maximal segment sum, a Turing di usion-reaction process, etc.).

*Currently, it exists two versions of an MGS interpreter: one written in OCAML* (a dialect of ML and one written in C++. There is some slight di erences be- tween the two versions. For instance, the OCAML version is more complete with respect to the functionnal part of the language. These interpreters are freely available 3 . In this current MGS implementations, only sets, multisets and sequences of elements are supported. Elements are of any types, allowing arbitrary nesting. Implementation of arrays is in progress and group-based data elds (GBF which generalizes functional arrays, Cf. [12,10]) are planed in a short term. We also begin the study of a generic implementation of topo- logical chain complex, a suitable formalization of our topological collection, using G-maps [14] to represent arbitrary join/neighborhood structure.

*At the language level, the study of the topological collections concepts must* continue with a ner study of transformation kinds. Several kinds of restriction can be put on the transformations, leading to various kind of pattern languages and rules. The complexity of matching such patterns has to be investigated. We also want to develop a type system that can handle nested collections, along the lines developed in [4]. At last but not least, we want to known if the topological spaces built by transformations, can be characterized through a non standard type system. The eÆcient compilation of a MGS program is a long-term research e ort.

*The applications opened by this preliminary work are numerous. From the* applications point of view, we are targeted by the simulation of the topological changes at the early development of the embryo. This is an actual example of tissues formation and fusion requiring complex topology beyond what is accessible using simple data-structures. Another motivating application is the case of a spatially distributed biochemical interaction networks, for which some extension of rewriting have been advocated, see [6,15].

*3 see* [*www.lami.univ-evry.fr/mgs.*](http://www.lami.univ-evry.fr/mgs)

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# *References*

*[1] Banatre, J. P., A. Coutant and and D. Le Metayer, Parallel machines for multiset transformation and their programming style, Technical Report RR- 0759, Inria, 1987.*

*[2] Banatre, J. P. and D. Le Metayer, A new computational model and its discipline of programming, Technical Report RR-0566, Inria, 1986.*

*[3] Berry, G., and G erard Boudol, The chemical abstract machine, Theoretical Computer Science, 96:217{248, 1992.*

*[4] Blelloch, G., NESL: A nested data-parallel language (version 2.6), Technical Report CMU-CS-93-129, School of Computer Science, Carnegie Mellon University, April 1993.*

*[5] Buneman, P., S. Naqvi, Val Tannen, and L. Wong, Principles of programming with complex objects and collection types, Theoretical Computer Science, 149(1):3{48, 18 September 1995.*

*[6] Fisher, M., G. Malcolm, and R. Paton, Spatio-logical processes in intracellular signalling, BioSystems, 55:83{92, 2000.*

*[7] Fontana, W., and L. Buss, The Arrival of the Fittest": Toward a theory of biological organization, Bulletin of Mathematical Biology, 1994.*

*[8] Fontana, W., and L. Buss, \Boundaries and Barriers", Casti, J. and Karlqvist,*

*A. edts. Chapter The barrier of Objects: from dynamical systems to bounded organizations, pages 56{116. Addison-Wesley, 1996.*

*[9] Fontana, W., Algorithmic chemistry. In Christopher G. Langton, Charles Taylor, J. Doyne Farmer, and Steen Rasmussen, editors, \Proceedings of the Workshop on Arti cial Life (ALIFE '90)", volume 5 of Santa Fe Institute Studies in the Sciences of Complexity, pages 159{210, Redwood City, CA, USA, February 1992. Addison-Wesley.*

*[10] Giavitto, J.-L., and O. Michel, Declarative de nition of group indexed data structures and approximation of their domains, In \Proceedings of the 3nd International ACM SIGPLAN Conference on Principles and Practice of Declarative Programming (PPDP-01)". ACM Press, September 2001.*

*[11] Giavitto, J.-L., and O. Michel, MGS: a programming language for the transformations of topological collections, Technical Report 61-2001, LaMI { Universit e d'E vry Val d'Essonne, May 2001. 85p.*

*[12] Giavitto, J.-L., O. Michel, and J. Sansonnet, Group-based elds, In \Parallel Symbolic Languages and Systems (International Workshop PSLS'95)", volume 1068, pages 209{215, 1996.*

*[13] Giavitto, J.-L., A framework for the recursive de nition of data structures, In*

*\Proceedings of the 2nd International ACM SIGPLAN Conference on Principles and Practice of Declarative Programming (PPDP-00)", pages 45{55. ACM Press, September 20{23 2000.*

*[14] Lienhardt, P., Topological models for boundary representation : a comparison with n-dimensional generalized maps, Computer-Aided Design, 23(1):59{82, 1991.*

*[15] Manca, V., Logical string rewriting, Theoretical Computer Science, 264:25{51, 2001.*

*[16] Paun, G., From cells to computers: Computing with membranes (P systems). In \Workshop on Grammar Systems", Bad Ischl, austria, July 2000.*

*[17] Paun, G., Computing with membranes, Technical Report TUCS-TR-208, TUCS - Turku Centre for Computer Science, November 11 1998.*

*[18] R emy, R., Syntactic theories and the algebra of record terms, Technical Report 1869, INRIA-Rocquencourt, BP 105, F-78 153 Le Chesnay Cedex, 1992.*

*[19] Von Neumann, J., \Theory of Self-Reproducing Automata", Univ. of Illinois Press, 1966.*