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*Modeling Petri Nets by Local Action Systems* *1*

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# *1 Introduction*

*Among the formal models for describing concurrent systems, Petri Nets are* certainly among the most popular ones. They have an attractive visual pre- sentation, and a rich theory has been developed for them [8,9]. Over the years a wide variety of variants have been described such as Place/Transition Nets [2], Elementary Nets [10], Colored Nets [6], Inhibitor Nets [7], etc. On the other hand, the theory of graph rewriting systems is another important model of computation where the con gurations of a system are graphs, and hence suited for a visual representation. Several types of graph rewriting systems have been proposed, included Algebraic Graph Grammars [3] and Local Ac- tion Systems [5]. The theory of the latter is based on the notion of a process, a description of a system run where the nodes are the nodes of con gurations and where the causal relation is represented by a partial order on these nodes. The aim of the paper is to explore the encoding of Petri Nets by Local Ac- tion Systems rewriting discrete graphs, and in particular the e ect of choosing di erent ways to encode the markings of a Petri Net. It is shown that one gets systems with the same sequential behavior as the net, but with di er- ent concurrent behaviors. It turns out that two particular choices (token and place representation) correspond to well-known classes of Petri Nets. Subse- quently, an application is considered in which the concurrent behavior of a net

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*changes over time, and it is shown that this situation can be handled using* the encoding presented.

*Thus, while it is not surprising that Petri Nets can be modeled by Local* Action Systems, there is evidence that the latter o er suÆcient exibility to serve as a unifying framework for some of the numerous variants of Petri Nets. Moreover, Local Action Systems in their own right o er a powerful language for describing concurrent systems.

# *2 Local Action Systems (LAS)*

*First a few basic ideas of LAS graph rewriting are recalled. Local Action* Systems are a type of graph rewriting systems based on node rewriting and embedding, introduced in [5]. Only the very restricted case where discrete graphs are rewritten is considered in this paper.

*In a LAS, it is assumed that the set of node labels forms a commutative* monoid. A production consists of

*a labeled, discrete graph l,*

*a labeled, discrete graph r,*

*for each pair (x; y), where x is a node of l and y is a node of r, an action* ax;y on node labels.

*In LAS locality of the actions means that both the input and the output of* an action correspond to a node. An action a ects a graph only locally.

*To apply to a graph g, one has (1) to nd a subgraph l0 of g that matches* l, (2) replace the nodes of l0 by new nodes, one for each node of r, and (3) compute the label of the new nodes. If the new node y0 corresponds to node y of r, then its label is the sum of the label of y in r and all the labels a (lab (x0)), where x is a node of l, x0 is the node of g that corresponds to x, and labg(x0) is its label.

*xy g*

*It is assumed that the set of node labels is equipped with a partial order*

*. A graph l matches a graph l0 if there is a bijective function f from the* set of nodes of l into the set of nodes of l0 such that for each node x of l, f (labl(x)) labl0 (x). Hence the notion of matching depends on the choice of

*.*

*A Local Action System consists of a graph and a set of productions.*

# *3 Encoding of Petri Nets*

*Marked Place/Transition Nets with arc-weights, shortly Marked Nets, are the*

*rst type of Petri Nets considered; it is well-known that various variants of* Petri Nets can be modeled by Marked Nets. Fig. 1 depicts a Marked Net. In this abstract we restrict attention to the case where all weights are 1.

*A state of a Marked Net is a marking, while in a graph rewriting system*

*s2 send1 c1 receive1 r2*

*produce queue s3 s1*

*send2*

 *r3*

*c2 receive2*

*dequeue consume r1*

*Fig. 1. A Marked Place/Transition Net.*

*a state is a graph. If one wants to model Marked Nets by a graph rewriting* system, a graph representation is needed for each marking. The following three types of graph representations are considered.

*Each node of a graph represents one token of a marking.*

*Each node of a graph represents a place.*

*Each node of a graph represents a group of places.*

*Fig. 2 shows three di erent graph representations for the marking which as-* signs 1 token in place s2,2 tokens in place s3, and 1 token in place r2. Formally,

|  |  |  |  |
| --- | --- | --- | --- |
| *(a)* | *(b)* | *(c)* |  |
| *s2; 1 s3; 1 s3; 1 r2; 1* | *s1; 0 s2; 1 s3; 2 c1; 0 c2; 0 r3; 0 r1; 0 r2; 1* | *s1; 0*  *s2; 1*  *s3; 2* | *r1; 0*  *c1; 0 r2; 1*  *c2; 0 r3; 0* |

*Fig. 2. Di erent graph representions for a marking.*

*the node labels are elements of v = [S \* Z], i.e. the monoid of partial func-* tions from the set of places S into Z, where the domain of f + g for 2 partial functions f and g is the union of the domains of f and g. In Fig. 2 (c) the set of places is divided into the groups fs1; s2; s3g, fc1; c2g, and fr1; r2; r3g. E.g., the label of the left most node in Fig. 2 (c) denotes the partial function with domain the places fs1; s2; s3g, and mapping s1 to 0, s2 to 1, and s3 to 2.

*In this paper it is shown that di erent graph representations give rise to* Local Action Systems such that their sequential behaviors are the same but their concurrent behaviors di er. In this way one gets di erent concurrent views of the same Marked Net by Local Action Systems.

*All systems considered in this paper, Petri Nets as well as Local Action* Systems, have a process semantics, and each process determines a set of se- quentializations. Since in all cases the systems are either Petri Nets or LAS encodings of a Petri Net, these sequentializations can be viewed as occur- rence sequences in the sense of [2]. Thus one may use the following notion of equivalence to compare systems, and in particular their concurrent behavior.

*De nition* *3.1*

*Let P and P 0 be processes. P and P 0 are equivalent if they determine the* same set of occurrence sequences.

*Let N and N 0 be systems. N and N 0 are process-equivalent if, for each* process P of N , there exists an equivalent process P 0 of N 0, and conversely, for each process P 0 of N 0, there exists an equivalent process P of N .

*The way these di erent representations of a marking lead to di erent Local* Action Systems is now considered in more detail.

*3.1 Token Representation*

*In the rst representation each node of a graph exactly represents one token* of a marking. This representation is perhaps the most straightforward and has already been studied, e.g. in [1]. A transition is modeled by a production such that the left-hand side is a graph representation of the tokens that are consumed by the transition, and the right-hand side is a graph representation of the tokens that are produced by the transition. Fig. 3 depicts the pro- ductions of three transitions of the Marked Net in Fig. 1. The dashed lines

*(a)*

*s2; 1*

*(b)*

*s1; 1*

*(c)*

*r2; 1 r3; 1*

*s1; 1*

*s2; 1 s3; 1*

*r1; 1*

*Fig. 3. Productions modeling the transitions (a) produce, (b) queue, and (c) dequeue.*

*depicts the causality relation. All local actions are 0, i.e. the function on v* mapping every node label into the partial function in v with empty domain, and hence they are not depicted. It turns out that the Local Action System obtained in this way has the same sequential and concurrent behavior as the Marked Net it encodes. (see [4]).

*Theorem 3.2 Let N be a Marked Net and let N 0 be its (token-)encoding. Then N and N 0 are process-equivalent.*

*3.2 Place Representation*

*In the second representation each place of a Marked Net corresponds to one* node of a graph. Each transition changes the amount of tokens in several places and hence a production modeling a transition t has to rewrite each node that corresponds to either an input or an output place of t. In a rst approach, equality is used for the partial order that determines the matching. Since the number of tokens in each place may vary, a transition is modeled by an in nite set of productions. Fig. 4 depicts a few productions modeling the transition queue of the Marked Net in Fig. 1. E.g, the production in Fig. 4(c)

*(a)*

*s1; 1 s2; 0 s3; 0*

*(b)*

*s1; 2 s2; 0 s3; 0*

*(c)*

*s1; 1 s2; 0 s3; 3*

*(d)*

*s1; 2 s2; 1 s3; 5*

*s1; 0*

*s2; 1*

*s3; 1*

*s1; 1*

*s2; 1*

*s3; 1*

*s1; 0*

*s2; 1*

*s3; 4*

*s1; 1*

*s2; 2*

*s3; 6*

*Fig. 4. An in nite number of productions modeling the transition queue.*

*is applicable to a graph modeling 1 token in s1, no tokens in s2, and 3 tokens* in s3. As a result the token of s1 is taken away, and 1 token is added to both s2 and s3 yielding 1 token in s2 and 4 tokens in s3. Obviously, the Local Action System obtained in this way has an in nite set of productions. By introducing nonzero local actions and using another partial order to de ne the matching, one obtains a nite version of it. The in nite set of productions modeling the transition queue can be replaced by one production which is depicted in Fig. 5 (a). A dashed line of the causality relation is replaced by a solid line if the corresponding local action is not 0. In the gure there are 3 solid lines labeled by the local action 1, denoting the identity. As a consequence this production can be applied to any graph which models at least one token in s1 (one chooses for the validity the relation on the set of partial functions from the places into Z). As a result of the combination of the local actions with the right-hand side, one token of s1 is taken away, and one token is added to both s2 and s3. One has the following theorem.

*Theorem 3.3 Let N be a Marked Net. Let N 0 be its in nite place-encoding, and let N 00 be its nite place-encoding. Then N 0 and N 00 are process-equivalent.*

*With this encoding two productions can be applied concurrently only if* they do not rewrite the same node, i.e. only if the places of the transitions they model do not overlap. Hence the places of the Marked Net can be seen as resources that can be accessed only by one transition at a time. This situation also occurs with Elementary Nets and hence contact-free Elementary Nets can be modeled in this way.

*Theorem 3.4 Let N be a contact-free Elementary Net system and let N 0 be* its ( nite or in nite) place-encoding. Then N and N 0 are process-equivalent.

*3.3 Representation of groups of places*

*To illustrate further the exibility of the LAS mechanism a situation is con-* sidered where the concurrent behavior of the Marked Net of Fig. 1 is further restricted: it is assumed that the places are grouped together and that each group of places can be accessed by only 1 transition at a time. This situ- ation arises when one has a certain number of memory blocks managed by processors and one assigns each memory object (place) to a memory block (processor). In this way a transition does not lock a place, but a whole block. In a situation where s1, s2 belong to the same group, but s3 does not, the transition queue can be encoded by the production of Fig. 5 (b).

*One may go one step further and model possible migrations of a place* from one group to another. Then one may use for each place p a migration production (see Fig. 5 (c)). In the gure the local actions p~ and p denote the restrictions of partial functions to fpg and the complement of p, respectively. By applying such a production one changes the grouping of places (and hence the concurrent behavior).

*(a)*

*(b)*

*(c)*

*s1; 1 s2; 0*

*s3; 0*

*s1; 1*

*s2; 0 s3; 0 ; ;*

*1*

*1*

*1*

*1*

*1 1*

*p~*

*p*

*s1; 1 s2; 1*

*s3; 1*

*s1; 1 s3; 1 ; ;*

*s2; 1*

*Fig. 5. Di erent grouping of places in (a) and (b), and migration of a place in (c). Conclusion and Future Work*

*It is shown that Local Action Systems o er a exible framework for modeling*

*di erent kinds of Petri Nets and for modeling Petri Nets with di erent views* of concurrency. Apart from considering other and possibly more complex variants of Petri Nets (e.g., Inhibitor Nets) it may be interesting to develop a theory of processes on a higher level, abstracting away from the concrete details of systems like Petri Nets or Local Action Systems. In this way one may get a framework in which the sequential and concurrent behavior of a large class of systems, including Petri Nets and various types of graph rewriting systems, can be investigated.

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