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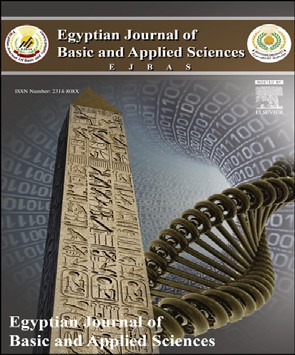
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Modified Laguerre Wavelets Method for delay differential equations of fractional-order

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## a b s t r a c t

In this article, Laguerre Wavelets Method (LWM) is proposed and combined with steps Method to solve linear and nonlinear delay differential equations of fractional-order. Computational work is fully supportive of compatibility of proposed algorithm and hence the same may be extended to other physical problems also. A very high level of accuracy explicitly reflects the reliability of this scheme for such problems.

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# Introduction

Fractional differential equations are applied to model wide range of physical problems including nonlinear oscillation of earth quakes [[1]](#_bookmark13), fluid-dynamic traffic [[2]](#_bookmark14), frequency depen- dent damping behavior of many viscoelastic materials, signal processing [[5]](#_bookmark16) and control theory [[6]](#_bookmark17). Moreover, in several areas of applied mathematics [[1,7](#_bookmark13)e[11]](#_bookmark13) fractional differential equations are often used. These are also used in the study of

epidemics, age-structured population growth [[12]](#_bookmark19), automa- tion, traffic flow and in many engineering problems. The basic motivation of this paper is to develop a Laguerre Wavelets Method (LWM) and combine it with the steps Method [[13]](#_bookmark20) to solve linear and nonlinear delay differential equations [[4]](#_bookmark15) of fractional-order. It is observed that proposed method is fully compatible with the complexity of such problems and is very user-friendly. The error estimates explicitly reveal the very high accuracy level of the suggested technique.

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# Laguerre Wavelets

from dilation and translation of a single function j(*x*) called Wavelets [[2,3,5]](#_bookmark14) constitute a family of functions constructed the mother wavelet. When the dilation parameter *a* and the

translation parameter *b* vary continuously we have the following family of continuous wavelets as [[10]](#_bookmark18)

j (*x*)= |*a* —1 *x* — *b* ; *a*; *b*2*R*; *a*s0.

*a*;*b*

| 2 j

*a*

*y*a(*x*)= *f* (*y*)+ *g*(*x*)*y* *x* — *c* ; 0 < *x* < *b*; 1 < a ≤ 2; (3)

*y*(*x*)= *p*(*x*); —*b* ≤ *x* ≤ 0.

*a*

where *g*(*x*) is a source term function, *f* (*y*) is a given continuous linear or nonlinear function.

According to the proposed method, first use the method of step to convert the delay differential equation [(3)](#_bookmark2) to inhomo- geneous ordinary differential equation by using initial func-

tion, *p*(*x*), Equation [(3)](#_bookmark2) implies

If we restrict the parameters *a* and *b* to discrete values as

*a* = *a*—*k*; *b* = *nb*0*a*—*k*; *a*0 > 1; *b*0 > 0; we have the following family

*a*

*y*a(*x*)= *f* (*y*)+ *g*(*x*) *p* *x* — *c* ; 0 < *x* < *b*; 1 < a ≤ 2; (4)

0 0

of discrete wavelets

which is a fractional differential equation and

The solution of the Equation [(4)](#_bookmark3) can be expanded as a

j*n*;*m*(*x*);

*n*=1

*m*=0 *n*;*m*

*k*

j (*x*)= |*a akx* — *nb* ; *k*; *n*2Z;

*k*;*n*

|2 j

0

0

Laguerre wavelets series as follows:*y*(*x*)= P∞ P∞ *c*

where j*k*;*n* form a wavelet basis for *L*2(*R*). In particular, when

*a*0 = 2 and *b*0 = 1, then j*k*;*n*(*x*) form an orthonormal basis.

The Laguerre wavelets j*n*;*m*(*x*)= j(*k*; *n*; *m*; *x*) involve four

where j*n*;*m*(*x*) is given by the Equation [(1)](#_bookmark4). We approximate

*y*(*x*) by the truncated series

2*k*—1 *M*—1

X X

arguments *n* = 1; 2; /; 2*k*—1; *k* is assumed any positive integer,

*m* is the degree of the Laguerre polynomials and it is the

*yk*;*M*

(*x*) = *cnm*

*n*=1 *m*=0

j*n*;*m*

(*x*). (4a)

normalized time. They are defined on the interval [0; 1) as

Then a total number of 2*k*—1*M* conditions should exist for

8>>< 2*k L*~ 2*kx* — 2*n* + 1 ; *n* — 1 ≤ *x* < *n* ;

j*n*;*m*(*x*)=

2 *m*

2*k*—1

2*k*—1

(1)

determination of 2*k*—1*M* coefficients *c*10;

*c*20; *c*21; …; *c*2*M*—1; …; *c*2*k*—10; *c*2*k*—11; /; *c*2*k*—1 *M*—1.

Since two conditions are furnished by the initial condi-

*c*11

; ….; *c*1*M*—1;

> 0; otherwise

:

where

*L*~ (*x*  1 *L* (*x*); (2)

)=

tions, namely

2*k*—1 *M*—1

X X

*yk*;*M*(0)= *cnm*j*n*;*m*(0)= *p*(0);

*n*=1 *m*=0

*k*—1

X

(5)

*m m*! *m*

d d 2

*yk*;*M*(0)=

*M*X—1

*cnm*j*n*;*m*(0)= *p*'(0).

*m* = 0; 1; 2; /; *M* — 1. In eq. [(2)](#_bookmark5) the coefficients are used for orthonormality. Here *Lm*(*x*) are the Laguerre polynomials of degree *m* with respect to the weight function *w*(*x*)= 1 on the interval[0; ∞], and satisfy the following recursive formula

d*x* d*x n*=1 *m*=0

We see that there should be 2*k*—1*M* — 2 extra conditions to recover the unknown coefficients *cnm*. These conditions can be

obtained by substituting Equation [(4)](#_bookmark3) in Equation [(3)](#_bookmark2);

*L*0(*x*)= 1; *L*1(*x*)= 1 — *x*;

*Lm*+2(*x*)=

((2*m* + 3 — *x*)*Lm*+1(*x*)— (*m* + 1)*Lm*(*x*))

*m* + 2 ; *m* = 0; 1; 2; 3; /.

da 2*k*—1 *M*—3

*n*=1 *m*=0

X X

d*x*a

2*k*—1 *M*—3

*n*=1 *m*=0

*cnm*j*n*;*m* (*x*)= *f*

*cnm*j*n*;*m*(*x*)

X X

! *x*

(6)

+ *g*(*x*) *p a* — *c* .

Modified Laguerre Wavelet Method (MLWM): In the present paper, we consider the Delay Differential Equation of the form

We, now assume Equation [(6)](#_bookmark7) is exact at 2*k*—1*M* — 3 points *xi*

as follows:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Table 1 e Numerical results of Example 1. | | | | | | |
| *t* | Exact solution | Solution by proposed method | Error in | proposed method  *M* = 5 | Error in proposed method  *M* = 10 | Error in proposed method  *M* = 20 |
| 0.0 | 1.00000000000 | 1.00000000200 |  | 2.00000E-09 | 2.70000E-08 | 1.00000E-09 |
| 0.1 | 0.90031699980 | 0.90033016590 |  | 1.31661E-05 | 2.04000E-08 | 9.00000E-10 |
| 0.2 | 0.80241064730 | 0.80244245410 |  | 3.18068E-05 | 1.75000E-08 | 1.10000E-09 |
| 0.3 | 0.70773067800 | 0.70774110310 |  | 1.04251E-05 | 1.78000E-08 | 9.00000E-10 |
| 0.4 | 0.61740564790 | 0.61737305130 |  | 3.25966E-05 | 2.83000E-08 | 1.00000E-09 |
| 0.5 | 0.53228073020 | 0.53222793850 |  | 5.27917E-05 | 4.12000E-08 | 8.00000E-10 |
| 0.6 | 0.45295378910 | 0.45293810650 |  | 1.56826E-05 | 4.87000E-08 | 5.00000E-10 |
| 0.7 | 0.37980938990 | 0.37987859860 |  | 6.92087E-05 | 5.91000E-08 | 5.00000E-10 |
| 0.8 | 0.31305050400 | 0.31316715980 |  | 1.16656E-04 | 7.68000E-08 | 3.00000E-10 |
| 0.9 | 0.25272775330 | 0.25266423690 |  | 6.35164E-05 | 9.57000E-08 | 1.00000E-10 |
| 1.0 | 0.19876611040 | 0.19797297850 |  | 7.93132E-04 | 1.30400E-07 | 7.00000E-10 |

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# Solution procedure

Example 1. Consider the Fractional Delay Differential Equa- tion of the form

|  |  |  |  |
| --- | --- | --- | --- |
| Table 2 e Numerical results of Example 2. | | | |
| *t* | Exact solution | Solution by proposed method | Error in proposed method  *M* = 5 |
| 0.0 | 0.00000000000 | —0.00000000141 | 1.41421E-09 |
| 0.1 | 0.01000000000 | 0.01000004758 | 4.75800E-08 |
| 0.2 | 0.04000000000 | 0.04000009693 | 9.69300E-08 |
| 0.3 | 0.09000000000 | 0.09000014701 | 1.47010E-07 |
| 0.4 | 0.16000000000 | 0.16000019820 | 1.98200E-07 |
| 0.5 | 0.25000000000 | 0.25000025090 | 2.50900E-07 |
| 0.6 | 0.36000000000 | 0.36000030550 | 3.05500E-07 |
| 0.7 | 0.49000000000 | 0.49000036240 | 3.62400E-07 |
| 0.8 | 0.64000000000 | 0.64000042200 | 4.22000E-07 |
| 0.9 | 0.81000000000 | 0.81000048480 | 4.84800E-07 |
| 1.0 | 1.000000000000 | 1.00000055100 | 5.51000E-07 |

a —*t*

*t* *t*

—3*t*

*t*

*t* *t*

*u* (*t*)= — *u*(*t*)— *e* 2 sin 2 *u* 2

— 2*e* 4 cos

4 sin 4 *u* 4 ,

0 ≤ *t* ≤ 1, 0 < a ≤ 1,

da 2*k*—1 *M*—3

X X

X2*k*—1 *M*X—3

! *x*

subject to the initial condition *u*(0)= 1.

The exact solution of the above system is *u*(*t*)= *e*—*t*cos(*t*).

[Table 1](#_bookmark8) shows the comparison of the absolute error be-

*M* = 5, 10, 20 and *K* = 1 by Modified Laguerre Wavelet Method tween exact solution and approximate solution for (MLWM).

d*x*a

*n*=1 *m*=0

*cnm*j

*n*,*m*

(*xi*) = *f*

*n*=1 *m*=0

*cnm*j

*n*,*m*

(*xi*)

+ *g*(*xi*) *p*

*i* — *c* .

*a*

(7)

Example 2. Consider the Fractional Delay Differential Equa- tion of the form

The best choice of the *x* points are the zeros of the shifted *u*a(*t*)= 3 *u*(*t*)+ *u* *t* — *t*2 + 2, 0 ≤ *t* ≤ 1, 1 < a ≤ 2,

*i* 4 2

Laguerre polynomials of degree 2*k*—1*M* — 2 in the interval [0, 1]

that is *xi* = *si* +1 , where *si* = cos (2*i*—1)p , *i* = 1, …, 2*k*—1*M* — 2.

2

2*k*—1 *M*—1

Combine Equations [(5) and (7)](#_bookmark6) to obtain 2*k*—1*M* linear equations from which we can compute values for the un-

known coefficients, *cnm*. Same procedure is repeated for delay differential equations of first and second order also.

subject to the initial conditions *u*(0)= 0, *u*'(0)= 0.

The exact solution of the above system is *u*(*t*)= *t*2.

tween exact solution and approximate solution for *M* = 5, [Table 2](#_bookmark9) shows the comparison of the absolute error be- and *k* = 1 by Modified Laguerre Wavelet Method (MLWM).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Table 3 e Numerical results of Example 3. | | | | | |
| *t* | Exact solution | Solution by proposed method | Error in proposed method  *M* = 5 | Error in proposed method  *M* = 10 | Error in proposed method  *M* = 20 |
| 0.0 | 1.00000000000 | 0.99999999950 | 5.00000E-10 | 6.30000E-09 | 0.00000E+00 |
| 0.1 | 0.90483741800 | 0.90483743970 | 2.17000E-08 | 1.49000E-08 | 1.00000E-10 |
| 0.2 | 0.81873075310 | 0.81872927080 | 1.48230E-06 | 2.35000E-08 | 1.00000E-10 |
| 0.3 | 0.74081822070 | 0.74081168180 | 6.53890E-06 | 3.17000E-08 | 1.00000E-10 |
| 0.4 | 0.67032004600 | 0.67031009600 | 9.95000E-06 | 4.00000E-08 | 0.00000E+00 |
| 0.5 | 0.60653065970 | 0.60653917080 | 8.51110E-06 | 4.82000E-08 | 1.00000E-10 |
| 0.6 | 0.54881163610 | 0.54890279800 | 9.11619E-05 | 4.82000E-08 | 1.00000E-10 |
| 0.7 | 0.49658530380 | 0.49689410400 | 3.08800E-04 | 6.50000E-08 | 0.00000E+00 |
| 0.8 | 0.44932896410 | 0.45009544900 | 7.66485E-04 | 7.39000E-08 | 1.00000E-10 |
| 0.9 | 0.40656965970 | 0.40817842780 | 1.60877E-03 | 8.27000E-08 | 3.00000E-10 |
| 1.0 | 0.36787944120 | 0.37090386940 | 3.02443E-03 | 8.62000E-08 | 2.00000E-10 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Table 4 e Numerical results of Example 4. | | | | | | |
| *t* | Exact solution | Solution by proposed method | Error in proposed method  *M* = 5 | Error in proposed method  *M* = 10 | Error in | proposed method  *M* = 20 |
| 0.0 | 1.00000000000 | 0.99999999950 | 5.00000E-10 | 5.20000E-08 |  | 2.10000E-08 |
| 0.1 | 0.99500416530 | 0.99500423090 | 6.56000E-08 | 6.33000E-08 |  | 2.11000E-08 |
| 0.2 | 0.98006657780 | 0.98006756710 | 9.89300E-07 | 7.78000E-08 |  | 2.09000E-08 |
| 0.3 | 0.95533648910 | 0.95533840950 | 1.92040E-06 | 9.32000E-08 |  | 2.09000E-08 |
| 0.4 | 0.92106099400 | 0.92106174490 | 7.50900E-07 | 1.10700E-07 |  | 2.08000E-08 |
| 0.5 | 0.87758256190 | 0.87757914590 | 3.41600E-06 | 1.27600E-07 |  | 2.06000E-08 |
| 0.6 | 0.82533561490 | 0.82532877020 | 6.84470E-06 | 1.44500E-07 |  | 2.04000E-08 |
| 0.7 | 0.76484218730 | 0.76484536080 | 3.17350E-06 | 1.62800E-07 |  | 2.03000E-08 |
| 0.8 | 0.69670670930 | 0.69676024620 | 5.35369E-05 | 1.79200E-07 |  | 2.00000E-08 |
| 0.9 | 0.62160996830 | 0.62180134050 | 1.91372E-04 | 1.95600E-07 |  | 1.99000E-08 |
| 1.0 | 0.54030230590 | 0.54079314330 | 4.90837E-04 | 2.21800E-07 |  | 1.97000E-08 |

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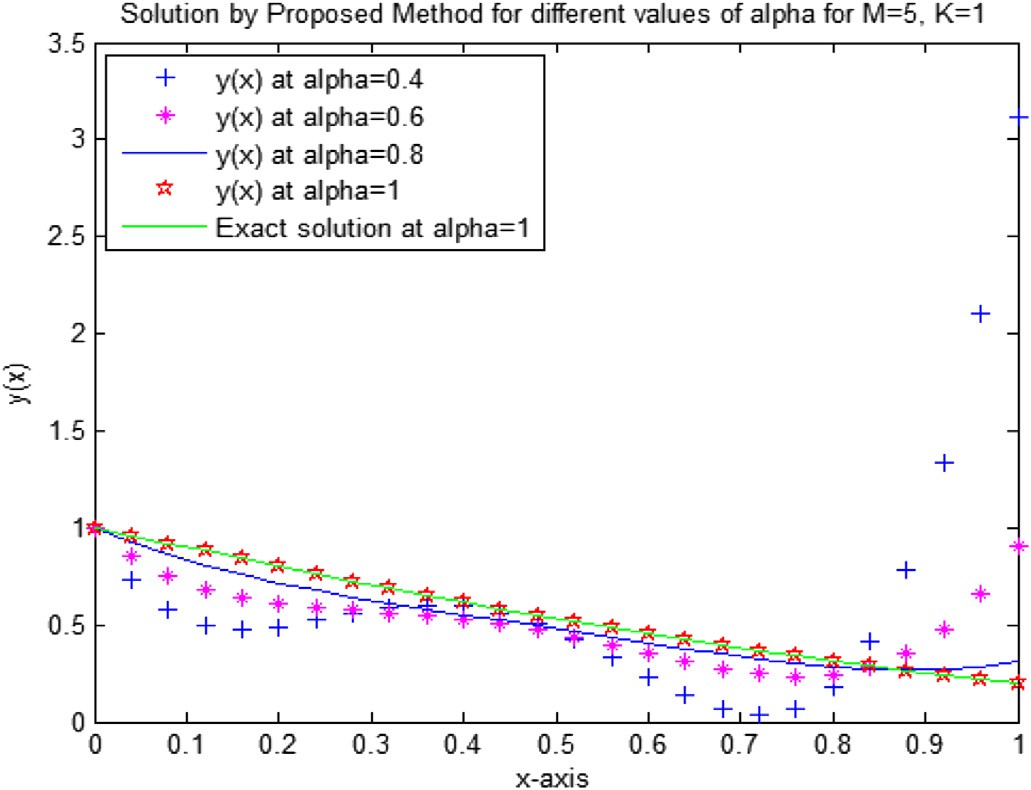


Fig. 1 e Modified Laguerre Wavelets Method solution for fractional DDE given in Example 1 and its comparison with exact solution.

Example 3. Consider the Fractional Delay Differential Equa- tion of the form

*u*a(*t*)= —*u*(*t*)— *u*(*t* — 0.3)+ *e*—*t*+0.3, 0 < *t* ≤ 1, 2 < a ≤ 3,

subject to the initial condition *u*(0)= 1,

*u*'(0)= —1, *u*'' (0)= 1.

The exact solution of the above system is *y*(*x*)= *e*—*t*.

[Table 3](#_bookmark10) shows the comparison of the absolute error be-

*M* = 5, 10, 20 and *k* = 1 by Modified Laguerre Wavelet Method tween exact solution and approximate solution for (MLWM).

Example 4. Consider the Nonlinear Fractional Delay Differ- ential Equation of the form

*u*a(*t*)= 1 — 2*u*2 *t* , 0 ≤ *x* ≤ 1, 1 < a ≤ 2,

2

subject to the initial condition *u*(0)= 1, *u*'(0)= 0.

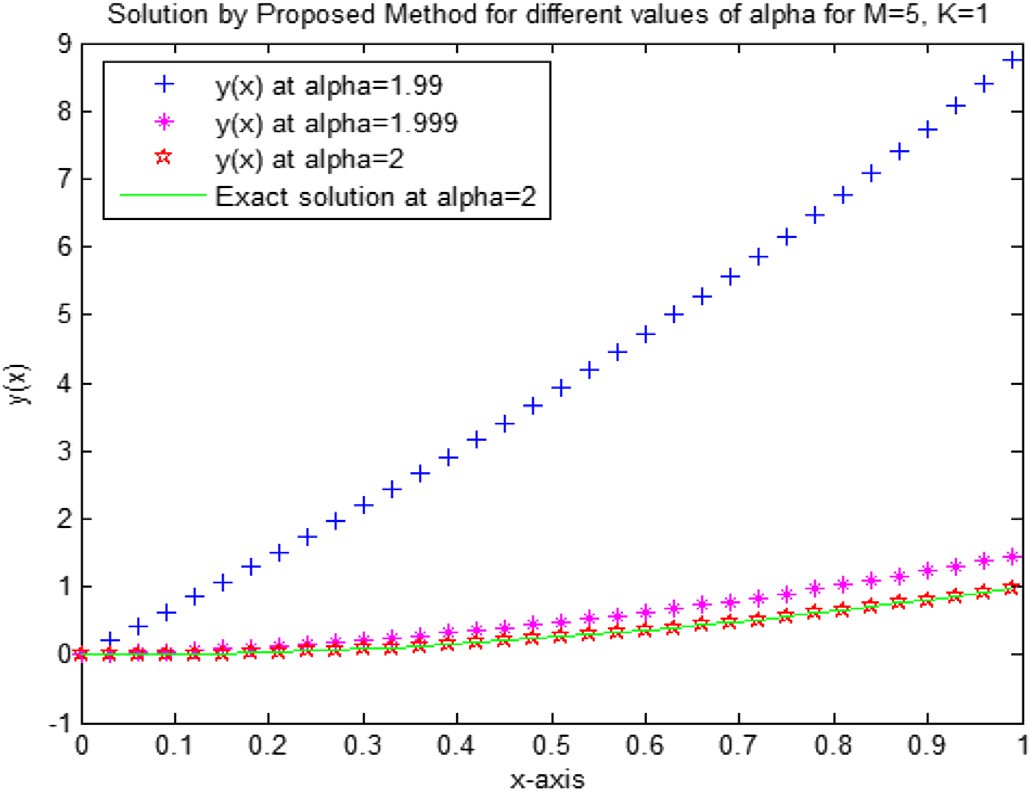
The exact solution of the above system is *u*(*t*)= cos(*t*). exact solution and approximate solution for *M* = 5, 10, 20 and [Table 4](#_bookmark11) shows the comparison of the absolute error between

*k* = 1 by Modified Laguerre Wavelet Method (MLWM).

# Conclusion

Linear and Nonlinear Delay Differential Equations of fractional-order are successfully tackled by Modified Laguerre Wavelets Method (MLWM). The solutions of the fractional delay differential equation converge to the solution of integer delay differential equation, as shown in [Figs. 1](#_bookmark12)e[4](#_bookmark12). According to the Tables, we get more accurate results while increasing

M. Computational work and numerical results explicitly reflect that the proposed method (MLWM) is very user-friendly but extremely accurate.

Fig. 2 e Modified Laguerre Wavelets Method solution for fractional DDE given in Example 2 and its comparison with exact solution.

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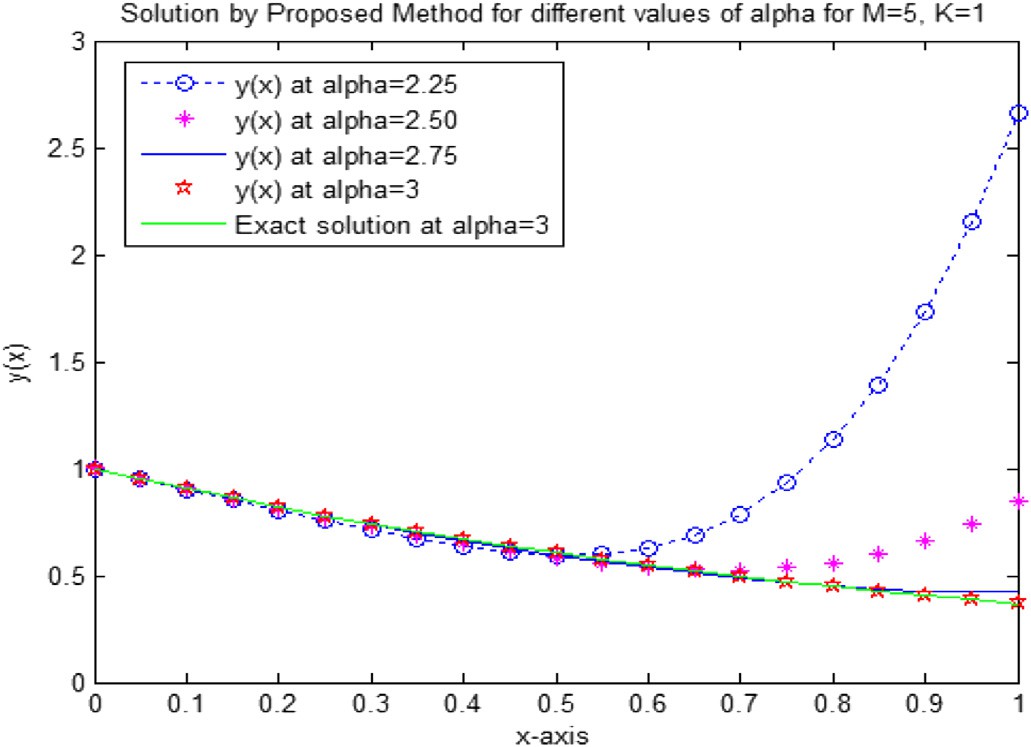


Fig. 3 e Modified Laguerre Wavelets Method solution for fractional DDE given in Example 3 and its comparison with exact solution.

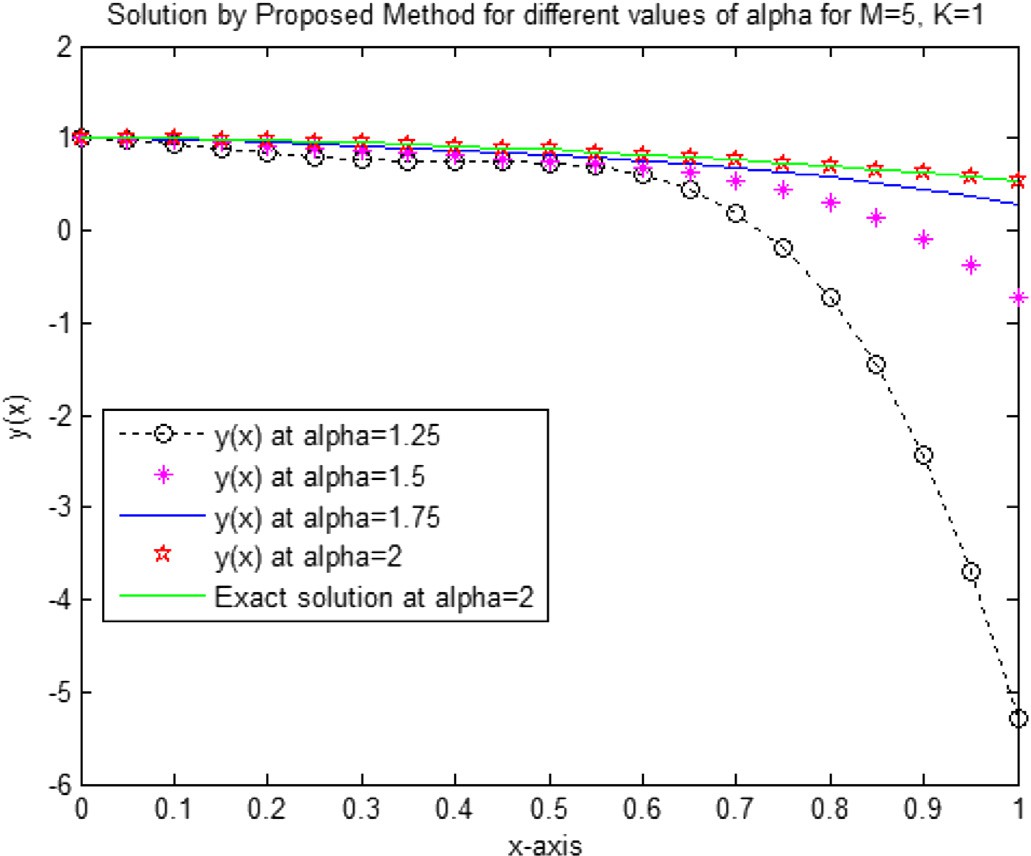


Fig. 4 e Modified Laguerre Wavelets Method solution for fractional DDE given in Example 4 and its comparison with exact solution.

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