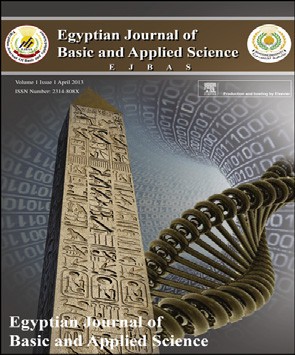
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Full Length Article

Reduced differential transform method for solving (1 + n) e Dimensional Burgers' equation



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## a r t i c l e i n f o

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## a b s t r a c t

differential transform method (RDTM) for solving the (1 + *n*) e dimensional Burgers' This paper discusses a recently developed semi-analytic technique so called the reduced equation. The method considers the use of the appropriate initial or boundary conditions

and finds the solution without any discretization, transformation, or restrictive assump- tions. Four numerical examples are provided in order to validate the efficiency and reli- ability of the method and furthermore to compare its computational effectiveness with other analytical methods available in the literature.

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# Introduction

Let us take the following (1 + *n*) e dimensional Burgers' equation

*u*(*x*1, *x*2, *x*3, ..., *xn*, 0)= *u*0(*x*1, *x*2, *x*3, ..., *xn*), (2)

where a*i*, *i* = 1, 2, 3, ..., *n*, and b are constants.

Eq. [(1)](#_bookmark5) is used in the study of cellular automata, and

interacting particle systems. Eq. [(1)](#_bookmark5) describes the flow pattern

v*u* v2*u*

v2*u*

v2*u*

v2*u*

v*u*

of the particle in a lattice fluid past an impenetrable obstacle

v*t* = a1v*x*2 + a2v*x*2 + a3v*x*2 + ...... + a*n*v*x*2 + b*u* v*x* . (1)

1 2 3 *n* 1

Subject to the initial conditions

[[1,2]](#_bookmark21); it can be also used as a model to describe the water flow in soils.

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The one dimensional Burgers' equation is quite popular in wave theory, which has wide applications such as in gas dy- namics [[3]](#_bookmark22), plasma physics etc. Due to its broad range of ap-

where*U*(*k*1,*k*2,....,*kn*,*km*)= *F*1(*k*1)×*F*2(*k*2)×.... ×*Fn*(*kn*)×*Fm*(*km*) is called the spectrum of *u*(*x*1,*x*2,...,*xn*,*t*).

Let *R*D denotes the reduced differential transform oper-

D

plications, various studies have been made to generalize it to

ator and *R*—1

the inverse reduced differential transform

higher dimension. Richard's equation [(1)](#_bookmark5) is a nonlinear PDE and as we know that obtaining the solutions of nonlinear PDEs are more difficult than those of linear differential equations. In

most of cases, these nonlinear PDEs do not admit analytical solutions, so these equations should be solved by using special methods. In recent years, some researchers used new tech- niques for solving these types of problems [[4](#_bookmark23)e[8]](#_bookmark23). In the past few decades, traditional integral transform methods such as the Fourier and the Laplace transforms have commonly been used to solve engineering problems. These methods transform differential equations into algebraic equations which are easier to deal with. However, these integral transform

methods are more complex and difficult when applying to

operator. The basic definition and operation of the RDTM method is described below. The basic definitions and oper- ations of reduced differential transform are introduced below.

Definition 2.1 If *u*(*x*1, *x*2, ..., *xn*, *t*) is analytic and continuously differentiable with respect to space and time in

the domain of interest, then the spectrum function

*R*D[*u*(*x*1, *x*2, ..., *xn*, *t*)]z*Uk*(*x*1, *x*2, ..., *xn*)

1. " v*k* #

nonlinear problems. Recently, the (1 + *n*) e dimensional Bur- gers' equation has been solved by Srivastava and Awasthi [[9]](#_bookmark26)

= *k*!

v*tk u*(*x*1, *x*2, ..., *xn*, *t*)

*t*=*t*0

, (4)

using Homotopy perturbation method (HPM), Adomian

decomposition method (ADM) and Differential transform method (DTM). However we notice that these methods involve complex computations. In recent years, Keskin et al. intro- duced a reduced form of DTM as reduced DTM (so called RDTM) and they applied it to approximate some PDE [[10]](#_bookmark27) and factional PDEs [[11]](#_bookmark28). More recently, Abazari and Ganji [[12]](#_bookmark29) extended RDTM to study the PDE with proportional delay. Futhermore, Gupta [[13]](#_bookmark30) used RDTM to fractional BenneyeLin equation, and Abazari and Abazari [[14]](#_bookmark31) applied the RDTM on generalized HirotaeSatsuma coupled *K*d*V* equation, while Srivastava et al. [[15](#_bookmark32)e[18]](#_bookmark32) used RDTM to solve the various problems arising in telecommunication systems and biological population model. The reduced differential transform recursive equations pro- duce exactly all the Poisson series coefficients of solutions, whereas the differential transform recursive equations pro- duce exactly all the Taylor series coefficients of solutions.

In this work, RDTM method is employedto solve the (1 + *n*) e

is the reduced transformed function of *u*(*x*1, *x*2, ..., *xn*, *t*). Here the lowercase *u*(*x*1, *x*2, , *xn*, *t*) represents the original func-

tion while the uppercase *Uk*(*x*1, *x*2, , *xn*) stands for the reduced

transformed function.

Poisson series form of the input expression *u*(*x*1, *x*2, , *xn*, *t*) We notice that the relationship introduced in [(4)](#_bookmark6) is the

with respect to the variables *x*1, *x*2, , *xn*, *t*, to order *N*, using the

variable weights *Uk*(*x*1, *x*2, , *xn*).

The differential inverse reduced transform of

*Uk*(*x*1, *x*2, , *xn*) is defined as

*R*—1[*Uk*(*x*1, *x*2, , *xn*)]z*u*(*x*1, *x*2, , *xn*, *t*)

X

D

∞

= *Uk*(*x*1, *x*2, ..., *xn*)(*t* — *t*0)*k*. (5)

*k*=0

Combining Eqs. [(4) and (5)](#_bookmark6), it can be seen that

X 1 " v*k* # *k*

∞

dimensional Burgers' equation. Four numerical examples are

carried out to validate and illustrate the proposed method. As

*u*(*x*1, *x*2, ..., *xn*, *t*)=

*k*=0 *k*!

v*tk u*(*x*1, *x*2, ..., *xn*, *t*)

*t*=*t*0

(*t* — *t*0) . (6)

an important observation, RDTM overcomes the demerit of complex calculation of classical DTM, capable of reducing the size of calculation.

# Reduced differential transform method

Consider a function *u*(*x*1, *x*2, ..., *xn*, *t*) of (*n* + 1) e product of (*n* + 1) single-variable function, i.e. variables and assume that it can be represented as a

*u*(*x*1, *x*2, ..., *xn*, *t*)= *F*1(*x*1)*F*2(*x*2)...*Fn*(*xn*)*Fm*(*t*). On the basis of the

properties of the one-dimensional differential transform, the

function *u*(*x*1, *x*2, ..., *xn*, *t*) can be represented as

From the above definition it can be seen that the concept of

the reduced differential transform is derived from the two- dimensional differential transform method.

Definition 2.2 If *u*(*x*1, *x*2, .....*xn*, *t*)= *R*—1[*Uk*(*x*1, *x*2, .....*xn*)], *v*(*x*1,

D

*x*2, .....*xn*, *t*)= *R*—1[*Vk*(*x*1, *x*2, .....*xn*)], and the convolution 5 denotes the reduced differential

D

transform version of multiplication, then the fundamental operations of the reduced differ- ential transform are expressed as follows:

(i).

*R*D[*u*(*x*1, *x*2, ..., *xn*, *t*)*v*(*x*1, *x*2, ..., *xn*, *t*)]

∞ ∞ ∞

*u*(*x*1,*x*2,...,*xn*,*t*)= X *F*1(*k*1)*xk*1 X *F*2(*k*2)*xk*2 ... X *Fn*(*kn*)*xkn*

= *Uk*(*x*1, *x*2, ..., *xn*)5*Vk*(*x*1, *x*2, ..., *xn*)

*k*1 =0

X

∞

1

*k*2 =0

1. *n*

*kn* =0

*k*

= *Uk*(*x*1, *x*2, ..., *xn*)*Vk*—*r*(*x*1, *x*2, ..., *xn*). (7)

X

× *Fm*(*km*)*tkm*

*km* =0

(ii).

*r*=0

∞ ∞ ∞ ∞

= X X... X X *U*(*k*1,*k*2,...,*kn*,*km*)*xk*1 *xk*2 ...*xkn tkm* ,

*k*1 =0 *k*2 =0

*kn* =0 *km* =0

1

2

*n*

(3)

*R* [a*u*(*x* , *x* , ..., *x* , *t*)±b*v*(*x* , *x* , ..., *x* , *t*)]

= a*Uk*(*x*1, *x*2, ..., *xn*)±b*Vk*(*x*1, *x*2, ..., *xn*). (8)

D

1

2

*n*

1

2

*n*

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(iii).

v*r* v*r*

*U*1 = (*x*1 + *x*2 + *x*3 + ... + *xn*), *U*2 = (*x*1 + *x*2 + *x*3 + ... + *xn*), …, *Un*

= (*x*1 + *x*2 + *x*3 + ... + *xn*).

*R*D v*xr u*(*x*1, *x*2, ..., *xn*, *t*)

*i*

*i*

(iv).

= v*xr Uk*(*x*1, *x*2, ..., *xn*), *i* = 1, 2, ..., *n*; *r*

= 1, 2, 3, ...

(9)

Then, using the differential inverse transformation we get

∞

*u*(*x*1, *x*2, ..., *xn*, *t*)= *Uktk*

X

*k*=0

= (*x*1 + *x*2 + *x*3 + ... + *xn*)+ (*x*1 + *x*2 + *x*3 + ...

v*r*

*R*D v*tr u*(*x*1, *x*2, ..., *xn*, *t*)

+ *xn*)*t* + (*x*1 + *x*2 + *x*3 + ... + *xn*)*t*2 + ...

(*k* + *r*)!

= (*k* + 1)(*k* + 2)...(*k* + *r*)*Uk*+*r*(*x*1, *x*2, ..., *xn*)

= *k*! *U*

= 1, 2, 3, ...

*k*+*r*(*x*1, *x*2, ..., *xn*), *r*

The exact solution, in closed form, is given by

*u*(*x* ,*x* ,*x* ,...,*x* ,*t* (*x*1 + *x*2 + *x*3 +...+ *xn*) provided0 ≤ *t*<1, (18)

(17)

(10)

1 2 3

*n* )=

1— *t* ,

(v).

v*r*1 +*r*2 +...+*rn* +*rm*

*R*D v*xr*1 v*xr*2 v*xr*3 ...v*xrn* v*trm u*(*x*1, *x*2, ..., *xn*, *t*)

which is the same solution as obtained by HPM, ADM and DTM [[9]](#_bookmark26).

1 2 3 *n*

(*k* + *rm*)!

= *k*!

v*r*1 +*r*2 +...+*rn*

v*xr*1 v*xr*2 v*xr*3 ...v*xrn Uk*+*rm* (*x*1, *x*2, ..., *xn*). (11)

Example 3.2 Let us take the (1 + 3) e dimensional Burgers'

equation

(vi).

1 2 3 *n*

v*u* =

v2*u*

+

v2*u*

v2*u*

+

v*u*

+ *u *, (19)

*R* *xa*1 *xa*2 *xa*3 ...*xan tam* = *xa*1 *xa*2 *xa*3 ...*xan* d(*k* — *a* )

D

1

2

3

*n*

1

2

3

*n*

*m*

*m*

v*t* v*x*2 v*y*2 v*z*2 v*x*

= *xa*1 *xa*2 *xa*3 ...*xan* , *km* = *am* . (12) 0, otherwise

1

2

3

*n*

# Numerical examples

with the initial condition

*u*(*x*, *y*, *z*, 0)= *u*0(*x*, *y*, *z*)= *x* + *y* + *z*. (20)

Applying the RDTM to Eq. [(19)](#_bookmark7), we obtain the following

recurrence relation

In this section, we describe the method explained in the pre-

v2 v2

v2 X*k*  v

vious sections by the following four examples to validate the efficiency of the proposed method.

(*k* + 1)*Uk*+1 = v*x*2 *Uk* + v*y*2 *Uk* + v*z*2 *Uk* +

*r*=0

*Ur Uk*—*r*. (21)

v*x*

Example 3.1 Consider the (1 + *n*) e dimensional Burgers' equation

From the initial condition [(20)](#_bookmark8), we can write

*U*0 = *x* + *y* + *z*. (22)

Using Eq. [(22)](#_bookmark10) into Eq. [(21)](#_bookmark9), we get the following *Uk* values

successively

v*u*  v2*u* v2*u* v2*u* v2*u* v*u*

*U*1 = (*x* + *y* + *z*), *U*2 = *x* + *y* + *z*, …, *Un* = *x* + *y* + *z*.

1 2 3 *n* 1

=

v*t*

v*x*2 + v*x*2 + v*x*2 + ... + v*x*2

+ *u* , (13)

v*x*

Now, using the differential inverse transformation, we get

∞

X

subject to the initial condition

*u*(*x*1, *x*2, *x*3, ..., *xn*, 0)= *u*0(*x*1, *x*2, *x*3, ..., *xn*)= *x*1 + *x*2 + *x*3 + ... + *xn*.

(14)

*u*(*x*, *y*, *z*, *t*)= *Uktk* = (*x* + *y* + *z*)+ (*x* + *y* + *z*)*t*

*k*=0

+ (*x* + *y* + *z*)*t*2 + ... + (*x* + *y* + *z*)*tn* + ...

In closed form, the exact solution is given by

(23)

According to the RDTM, we construct the following recur- rence equation for the Eq. [(13)](#_bookmark11)

*u*(*x*, *y*, *z*, *t*

(*x* + *y* + *z*)

)= 1 — *t* ,

provided 0 ≤ *t* < 1, (24)

v2 v2 v2 v2

(*k* + 1)*Uk*+1 = *Uk* + *Uk* + *Uk* + ... + *Uk*

v*x*2 v*x*2 v*x*2 v*x*2

1 2 3 *n*

X*k*  v

+

*Ur*

*Uk*—*r*. (15)

*r*=0

v*x*1

which is same as obtained by HPM, ADM and DTM [[9]](#_bookmark26).

Example 3.3 Consider the (1 + 2) e dimensional Burgers' equation

From the initial condition [(14)](#_bookmark12), we can write

*U*0 = *x*1 + *x*2 + ... + *xn*. (16)

Substituting the initial condition [(16)](#_bookmark14) in Eq. [(15)](#_bookmark13), we obtain

the following *Uk* values successively:

v*t* = v*x*2 + v*y*2 + *u* v*x* , (25)

under the initial condition

v*u*

v2*u*

v2*u*

v*u*

*u*(*x*, *y*, 0)= *u*0(*x*, *y*)= *x* + *y*. (26)

According to the RDTM, we construct the following itera-

tion formula to Eq. [(25)](#_bookmark15)

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2 2 *k*

(*k* + 1)*U* = v *U* + v *U* + X *U*  v *U* . (27)

equation. The solutions obtained by the method are an infinite

+ v*x*2

*k* 1 *k k r k r*

—

v*y*2

*r*=0 v*x*

powerseries for appropriate initial condition, which can, inturn, be expressed in a closed form, the exact solution. The results

From the initial condition [(26)](#_bookmark16), we can write

*U*0 = *x* + *y*. (28)

Using Eq. [(28)](#_bookmark18) in Eq. [(27)](#_bookmark17), we obtain the following *Uk* values

successively

*U*1 = *x* + *y*, *U*2 = *x* + *y*, …, *Un* = *x* + *y*.

Then, using the differential inverse transformation, we get

∞

*u*(*x*, *y*, *t*)= *Uktk*

X

*k*=0

= (*x* + *y*)+ (*x* + *y*)*t* + (*x* + *y*)*t*2 + ... + (*x* + *y*)*tn* + ... (29)

The exact solution, in closed form, is given by

*u*(*x*, *y*, *t*) = (*x* + *y*) provided 0 ≤ *t* < 1, (30) 1 — *t*

,

which is the same result as obtained by HPM, ADM and DTM [[9]](#_bookmark26).

Example 3.4 Consider the (1 + 1) e dimensional Burgers' equation

v*u* v*u* v2*u*

v*t* + *u* v*x* = v*x*2 , (31)

Subject to the initial condition

*u*(*x*, 0)= *u*0(*x*)= 2*x*. (32)

Using the RDTM to Eq. [(31)](#_bookmark19), we get the following iteration

formula

and quite accurate mathematical tools for solving the (1 + *n*) e reveal that the proposed method is very effective, convenient dimensional Burgers' equation. It can be observed that the so-

lution approach of RDTM is much simpler than differential transform method (DTM) and it needs less computational effort than DTM. In other word, RDTM is an alternative approach to overcome thedemeritof complex calculation of DTM, capable of reducing the size of calculation. As a special advantage of RDTM rather than DTM, the reduced differential transform recursive equations produce exactly all the Poisson series coefficients of solutions, whereas the differential transform recursive equa- tions produce exactly all the Taylor series coefficients of solu- tions. We notice that the RDTM technique is highly accurate, rapidly converge and is very easily implementable mathemat- ical tool for the multidimensional physical problems emerging in various domains of engineering and allied sciences.

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X

(*k* + 1)*U* =— *U*  v *U* + v *U* . (33)

*k* 1 *r k r k*

—

+

v*x* v*x*2

*r*=0

From the initial condition [(32)](#_bookmark20), we can write

*U*0 = 2*x*. (34)

Using Eq. [(34)](#_bookmark25) in Eq. [(33)](#_bookmark24), we get the following *Uk* values

successively

*U*1 = (—4*x*), *U*2 = (8*x*), *U*3 = (—16*x*), …

Then, using the differential inverse transformation, we get

*u*(*x*, *y*, *t*)= *Uktk* = (2*x*)+ (—4*x*)*t* + (8*x*)*t*2 + (—16*x*)*t*3 + ... (35)

∞

X

*k*=0

The exact solution, in closed form, is given by

*u*(*x*, *t*  2*x* (36)

1 + 2*t*

)= ,

which is the same solution as obtained by HPM, ADM and DTM

[[9]](#_bookmark26).

# Conclusions

is introduced for solving the (1 + *n*) e dimensional Burgers' In this work, the reduced form of differential transform method

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