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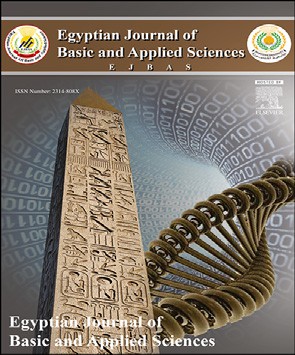
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Full Length Article

Thermosolutal convection in a viscoelastic dusty fluid with hall currents in porous medium



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## a b s t r a c t

An incompressible Oldroydian viscoelastic fluid layer heated and soluted from below in the presence of suspended (dust) particles and uniform vertical magnetic field to include the effect of Hall currents in porous medium is considered. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. For the case of stationary convection, Oldroydian viscoelastic fluid behaves like an ordinary Newtonian fluid. Dust particles and Hall currents are found to have a destabilizing effect on the thermosolutal convection, whereas magnetic field is found to have a stabilizing effect on the thermosolutal convection. Medium permeability has both stabilizing and destabilizing effect on the thermosolutal convection under certain conditions. Graphs have been plotted by giving numerical values to the parameters to depict the stability characteristics. The case of overstability is also considered.

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# Introduction

The growing importance of the use of viscoelastic fluids in technology and industries has led various researchers to attempt diverse flow problems related to several non- Newtonian fluids. Recently an attention has been drawn by calculations of the rheological behavior of dilute suspensions and emulsions to the idealized incompressible viscoelastic liquids whose behavior at small variable shear stresses is characterized by three parameters coefficient of viscosity m, a relaxation time l, and a retardation time l0(<l). A theoretical

model is proposed by Oldroyd [[1]](#_bookmark20) for a class of viscoelastic

fluids. An experimental demonstration by Toms and Straw- bridge [[2]](#_bookmark21) revealed that a dilute solution of methyl methacry- late in n-butyl acetate agrees well with the theoretical model of the Oldroyd fluid. Sharma [[3]](#_bookmark22) studied the problem of the ther- mal instability in a viscoelastic fluid layer in hydromagnetics. The problem of thermal instability of a Maxwellian visco- elastic fluid in the presence of magnetic field is studied by Bhatia and Steiner [[4]](#_bookmark23). The effect of magnetic field on ther- mosolutal instability of an Oldroydian viscoelastic fluid in porous medium is considered by Sharma and Bhardwaj [[5]](#_bookmark24). They found that magnetic field has a stabilizing effect on the system while medium permeability has dual effect. In thermal

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Notations

*a*

*c d D*

l l0

dimensionless wave number

speed of light depth of layer

derivative with respect to *z* (= *d*/*dz*)*e* Charge of

an electron

relaxation time retardation time ( < l)

g(0; 0; —*g*) acceleration due to gravity field

H uniform magnetic field having components

(0; 0; *H*)

(*hx*; *hy*; *hz*) perturbation in magnetic field having components

*kx*; *ky* horizontal wave numbers

*k* wave number

*kT*| thermal diffusivity

*ks* Solute diffusivity

1. Hall current parameter
2. electron number density

*n* growth rate

1. fluid pressure

*p*1 Prandtl number

*p*2 magnetic Prandtl number

1. velocity of fluid

(*u, v, w*) perturbations in fluid velocity

1. Chandrasekhar number
2. Rayleigh number

*t* Time coordinate

*T* temperature

*C* concentration

*x*(*x*; *y*; *z*) space coordinates

*Greek Symbols*

h electrical resistivity

a coefficient of thermal expansion

b uniform temperature gradient

q perturbation in temperature,

g perturbation in concentration,

d*p* perturbation in pressure*p*;

r fluid density

dr perturbation in densityr

m*e* magnetic permeability

V del operator

v curly operator

and thermosolutal convection problems, the Boussinesq approximation is used, which is well justified in the case of incompressible fluids. Usually the magnetic field has a stabi- lizing effect on the instability. A numerical study of the hy- dromagnetic thermal convection in a viscoelastic dusty fluid in a porous medium is discussed by Goel and Agrawal [[6]](#_bookmark25).

The Hall Effect is likely to be important in many geophys- ical situations as well as in flow of laboratory plasma. There is growing importance of non-Newtonian fluids in chemical technology, industry and geophysical fluid dynamics. The Hall currents have relevance and importance in geophysics, MHD generator and industry. Hall effect on thermosolutal insta- bility of Rivlin-Ericksen fluid with varying gravity field in

porous medium is discussed by Sharma and Kishor [[7]](#_bookmark26). Sunil et al. [[8]](#_bookmark27) investigated the Hall effects on thermosolutal insta- bility of Walters' (model B') fluid in porous medium and found that magnetic field has a stabilizing effects, whereas the Hall

currents have a destabilizing effect on the system. Kumar et al. [[9]](#_bookmark28) studied the RayleigheTaylor instability of rotating Oldroydian viscoelastic fluids in porous medium in the pres- ence of a variable magnetic field.

The problem on a couple-stress fluid heated from below in hydromagnetics has been studied by Kumar and Kumar [[10]](#_bookmark29). They found that magnetic field has both stabilizing and destabilizing effects on the thermal convection under certain conditions. Singh and Dixit [[11]](#_bookmark30) considered the stability of stratified Oldroydian fluid through porous medium in hydro- magnetics in presence of suspended particles. Vikrant et al.

[[12]](#_bookmark31) studied the problem of thermal convection in a compressible Walters' (model B’) elastico-viscous dusty fluid with Hall currents and found that Hall currents have desta- bilizing effect on the system. The effect of Hall currents on

thermal instability of compressible dusty viscoelastic fluid in porous medium is discussed by Kumar [[13]](#_bookmark32) and found that Hall currents have destabilizing effect on the thermal con- vection. The instability of the plane interface between two viscoelastic Kuvshiniski superposed fluids in porous in the presence of uniform rotation and variable magnetic field has been considered by Kumar [[14]](#_bookmark33).

Wang and Tan [[15]](#_bookmark34) considered the stability analysis of Soret-driven double-diffusive convection of Maxwell fluid in a porous medium. Bishnoi and Goyal [[16]](#_bookmark35) studied the problem of Soret-Dufour driven thermosolutal instability of Darcy- Maxwell fluid and found that the Dufour number enhances the stability of Darcy-Maxwell fluid for stationary convection as well as overstability. Kumar and Mohan [[17]](#_bookmark36) included the double-diffusive convection in an Oldroydian viscoelastic fluid under the simultaneous effects of magnetic field and suspended particles through porous medium.

In the past studies, instability in an Oldroydian viscoelastic fluid layer in porous medium heated and soluted from below has been investigated including the external constraints such as magnetic field and/or rotation. During the survey it was noticed that effect of Hall currents is completely neglected from the studies of Oldroydian viscoelastic dusty fluid in porous medium. Further, magnetic field and medium perme- ability have dual character. Therefore, an attempt has been made to study the effect of thermosolutal convection in an Oldroydian viscoelastic dusty fluid in presence of Hall cur- rents in porous medium.

# Formulation on the problem

Consider an infinite layer of an incompressible, finitely con- ducting (electrically and thermally both) Oldroydian visco- elastic dusty fluid, confined between two horizontal planes

situated at *z* = 0 and *z* = *d*, acted upon by a uniform vertical

magnetic field H(0, 0, *H*) and gravity field g(0, 0, —*g*). The fluid layer is heated and soluted from below leading to an adverse temperature gradient b = *T*0 —*T*1 ; where *T*0 and *T*1 are the constant temperatures of the lower and upper boundaries with *T*0 > *T*1 andb' = *C*0 —*C*1 ; where *C*0 and *C*1 are the constant

*d*

*d*

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concentrations of the lower and upper boundaries with

*C*0 > *C*1. When the fluid permeated a porous material, the

small and therefore ignored and *mN*0 being mass of particles per unit volume. *d* =  v + *q*.V stands for the convective

actual path of individual particles of fluid cannot be followed analytically. The gross effect is represented by Brinkman equation as the fluid slowly percolates through the pores of

derivative.

*dt* v*t*

the rock. If ε is the porosity and *k*1 is medium permeability then the hydromagnetic equations relevant to the physical model, following Boussinesq approximation, are

# Basic state and perturbation equations

In the undisturbed state, let the fluid be at rest. Constants temperatures and concentrations are maintained in the fluid

1 1 + l v vq

+ 1 (q.V)q

1 v v

=— 1 + l V*p* + g 1 + l

and a constant vertical magnetic field is applied, therefore, the

ε v*t* v*t* ε

r0 v*t*

× 1 + dr + *v* 1 + l v

v*t* steady state solution is

q = (0, 0, 0), H = (0, 0, *H*), *T* = *T*(*z*), *C* = *C*(*z*), r = r(*z*),

0

r0 ε v*t*

V2 — ε q + 1 + l v *K*'*N*0 q — q

*k*1

v*t*

εr0

d



with *T*(*z*)= *T*0 — b*z*, *C*(*z*)= *C*0 — b'*z*, r = r0[1 + ab*z* — a'b'*z*]

and *N*0 = *N*1 = constant

(10)

To use linearized stability theory and normal mode tech-

+ 1 + l v m*e* [(V × H) × H], (1)

v*t* 4pr0

V.q = 0, (2)

nique, here we assume small perturbations on the steady state solution. Let q(*u*, *v*, *w*), h(hx, hy, hz), dr, d*p*, q and g denote, respectively the perturbations in the fluid velocity, magnetic field, density, pressure and temperature and concentration, then the linearized perturbation equations are

ε *d*H = (H.V)q + εhV2H — *c*ε V × [(V × H)× H], (3)

*dt*

4p*Ne*

r0

1 v vq 1 v v dr *v*

Vd*p* + g

1 + l

+

1 + l0

ε

v*t*

v*t*

v*t*

v*t*

ε

v

v*t*

V.H = 0. (4)

1 + l

=—

r0

1 + l

*mN* vqd + 1 q .V q = *K*'*N* q — q , (5)

V2 — ε q + 1 + l v *K*'*N*1 q — q

+ 1 + l v m*e* [(V × *h*)× H],

*k*1

v*t*

εr0

d

0 v*t* ε d d 0 d

v*t* 4pr0

(11)

ε v*N*0 + V. *N* q = 0, (6)

v*t*

0

d

When the fluid flows through a porous medium, the equation of heat conduction is

V.q = 0, (12)

ε v*h* = (H.V)q + εhV2*h* — *c*ε *h*)× H], (13)

V × [(V ×

v*t* 4p*Ne*

r*c* ε + r *c* (1 — ε) v*T* + r*c* (q.V)*T* + *mN c*

*f*

*s s*

v*t*

*f*

ε v + q .V *T*

V.h = 0, (14)

= *kT*V2*T*. (7)

0 *pt*

v*t*

*d*

An analogous solute concentration equation is

(*E* + *hd*ε) v

v*t*

q

= b(*w* + *hds*)+ *kT*V2q, (15)

' v*C*

r*cf* ε + r*scs*(1 — ε)

2

v*t*

v *E* + *h*' ε vg = b' *w* + *h*' *s* + *k* V2g, (16)

v*t*

= *ks*V *C*. (8)

+ r*cf* (q.V)*C* + *mN*0*cpt*

ε

+ q*d*.V

*C*

*d*

v*t*

*d*

*s*

Since density variations are mainly due to variations in temperature and solute concentration, the equation of state for the fluid is given by

*m* v

*K*' v*t* + 1 qd = q, (17)

ε v*N*1 + *N* V.q = 0, (18)

r = r [1 — a(*T* — *T*0)+ a'(*C* — *C* )], (9)

v*t* 0 d

*mN*

*cpt c*'

0 0 Here

1 = *f* is the mass fraction, *hd* = *f* , *h*' = *f pt* ,

r0 *cf d cf*

where q, r, *p*, *T* and *C* denote, respectively the fluid velocity, density, pressure, temperature and concentration and *kT*, *ks*, a, a' m*e*, *N*, *e*, *c* and h stands for the thermal diffusivity, solute diffusivity, thermal coefficient of expansion, an analogous

coefficient of expansion, magnetic permeability, electron number density, charge of an electron, speed of light and electrical resistivity. The suffix zero refers to the values at the

reference level *z* = 0. Here, we assume that the distance be-

tween particles is quite large as compared with their diameter so that inter-particle reactions need not to be considered. The

*E* = ε — (1 — ε) r*scs* and r0, *cf*; r*s*, *cs* stand for density and heat

0 *f*

r *c*

capacity of fluid and solid matrix, respectively and *E*' is an

analogous solute parameter. The change in density dr caused by the perturbations q,g in temperature and solute concen- tration at the lower boundary z = 0 is given by

dr = —r0(aq — a'g). (19)

Analyzing the perturbations into normal modes, we as- sume that the perturbation quantities are of the form

effect of pressure, gravity and magnetic field on the sus- pended particles, assuming large distance apart, is negligibly

[*w*, q, g, *hz*, z, x] =[*W*(*z*), Q(*z*), G(*z*), *K*(*z*), *Z*(*z*), *X*(*z*)]exp

× *ikxx* + *ikyy* + *nt*},

(20)

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where *kx* and *ky* are the wave numbers in *x*' and *y*' directions

*D*2 — *a*2 — *E*' s*q* G =—

*d* 1

*W*, (26)

*x*

*y*

respectively,*k* = (*k*2+*k*2)1/2 is the resultant wave number of

b'*d*2 *H*' + st

1

propagation and *n* is the frequency of any arbitrary distur-

bance which is, in general, a complex constant. z = v*v* — v*u*

where we have put *a* = *kd*, s = *nd*2 ,*F* =  ln,*F*

= l0 n,

*ks*

1 + st1

v*x* v*y*

n *d*2 0 *d*2

andx = v*hy* — v*hx* are the z-components of the vorticity and cur- *p*1 = n ,*q* = n , *pl* = *k*1 , *p*2 = n,t = n,t1 = tn,*Hd* = *hd*+1,

v*x* v*y*

*kT ks*

*d*2 h *K d*2

rent density respectively. Using equation [(20)](#_bookmark3), equations

*H*' = *h*' + 1, *E*1 = *E* + *hd*ε, *E*' = *E*' +*h*' ε and *D*\*=*dD*[(\*) is dropped for

*d d* 1 *d*

[(11)](#_bookmark2)e[(18)](#_bookmark2) in non-dimensional form become

convenience]. On eliminating various physical parameters from equations [(21)](#_bookmark5)e[(26)](#_bookmark5), we obtain the final stability governing equation as

s(1 + s*F*) 1 + *f*

— (1 + s*F*0) *D*2 — *a*2 + (1 + s*F*0) n *D*2 — *a*2 — s*p* 2 + *M* *D*2 — *a*2 *D*2o

ε

*Q*(1 + s*F*) 2

+

*D*

ε

1 + st1

2

— *a*

— s*p*2 *D*

ε *pl*

2 s(1 + s*F*) *f*

×

1 +

1 + st1

ε

2

(1 + s*F*0) 2 2

—

*D*

— *a*

ε

(1 + s*F*0)

*pl*

*D*2 — *a*2 — *E* s*p* *D*2 — *a*2 — *E*'s*q* *D*2 — *a*2 *W* — *Ra*2(1 + s*F*) *Hd* + st1 *D*2 — *a*2 — *E*'s*q* *W*

+

1

1

1 + st1

*H*' + st1

+*Sa*2(1 + s*F*)

*d*

1 + st1

*D*2 — *a*2 — *E*s*p*1 *W*

+

1 +

*Q*(1 + s*F*) s(1 + s*F*) *f*

*D*2

ε

ε

1 + st1

(27)

(1 + s*F* )

0

+

—

ε

*D*2 — *a*2

+ (1 + s*F*0)

*pl*

*D*2 — *a*2

— s*p*2

*Q*(1 + s*F*)*D*2

ε

* *a*2
* *E*s*p*1

(*D*2 — *a*2 — *E*'s*q* *D*2 — *a*2 *D*2*W* = 0.

where *R* = *g*ab*d*4 is the Rayleigh number, *S* = *g*a'b'*d*4 is the analo-

s

(1 + s*F*)

1 + *f*

1 1 + s*F* ) *D*2 — *a*2

1

+ (1 + s*F* )

n*kT*

n*ks*

ε

1 + st1

—

ε

(

0

*pl*

0

gous solute Rayleigh number, *Q* = m*e H*2 *d*2 the Chandrasekhar

0

(*D*2 — *a*2 *W*

(21)

number and *M* =

4pr nh

2

4 *cH* is the non-dimensional number ac-

+(1 + s*F*) *ga d* (aQ — a'G)— (1 + s*F*) *e*  *D*2 — *a*2 *DK* = 0,

2 2 m *Hd*

p*Ne*h

n 4pr0n

s (1 + s*F*) 1 + *f* 1 1 + s*F* ) *D*2 — *a*2 + 1 (1 + s*F* ) *Z*

ε

1 + st1

— (

ε

0

*p*

0

*l*

counting for Hall currents.

Here we consider the case of two free boundaries, and the medium adjoining the fluid is electrically non-conducting. The case of two free boundaries is slightly artificial, except

in stellar atmospheres and in certain geophysical situations

= (1 + s*F*) m*e Hd DX*,

4pr0n

(22)

where it is most appropriate, but it allows for an analytical solution. Since both the boundaries are maintained at con- stant temperature and so the perturbations in the tempera- ture are zero at the boundaries therefore, the appropriate boundary conditions are

*D*2 — *a*2 — s*p*2 *K*

*Hd*  *cHd*

=— *DW* + *DX*, (23)

εh 4p*Ne*h

*W* = 0, *D*2*W* = 0, *DZ* = 0, Q = 0, G = 0, *X* = 0 (28)

and *hx*, *hy*, *hz* and are continuous at *z* = 0, 1

*cH*

*Hd*  2 2

2 2 2

*D* — *a* — s*p X* =— *DZ* — *D* — *a DK*, (24)

εh 4p*Ne*h*d*

2 2 b*d*2 *Hd* + st1

*D*

— *a*

— *E*1s*p*1 Q =—

*kT*

*W* (25)

1 + st1

The proper solution of equation [(27)](#_bookmark6) characterizing the lowest mode is

*W* = *W*0 sin p*z*, (29)

where *W*0 is constant. Using equation [(29)](#_bookmark7), equation [(27)](#_bookmark6) gives

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1 + *x* 1 + *i*s1p2t1 *i*s1 1 + *i*s1p2*F* *f* (1 + *i*s1p2*F*0 (1 + *x*)

*R*1 1 + *i*s1p2*F*

=

*Hd* + *i*s1p2t1

1 +

1 + *i*s1p2t1

+

*x*

ε

ε

(1 + *i*s1p2*F*0

+

*P*

1 + *x* + *i*s1*E*1*p*1

*Q*1 1 + *i*s1p2*F* *i*s1 1 + *i*s1p2*F* *f*

(1 + *i*s1p2*F*0 (1 + *i*s1p2*F*0 (1 + *x*)

+

ε

ε

1 + 1 + *i*s p2t

+

*P* +

ε

1 1

*Q*1 1 + *i*s1p2*F*

1 + *x* + *i*s1*p*2

+

1 + *x* + *i*s1*E*1*p*1 ×

(30)

ε

*i*s1 1 + *i*s1p2*F* *f*

1 + 1 + *i*s p2t

+

(1 + *x*)+

ε

1 + *x* + *i*s1*p*2

+ *M*(1 + *x*)

ε

1

(1 + *i*s1p2*F*0 (1 + *i*s1p2*F*0 n

1

*d* 1 1

1

1 1

1

*P*

2 o

*Q* (1 + *i*s p *F*)

2

+

1

1

1 + *x* + *i*s1*p*2

+ *S*1 1 + *i*s1p2*F*

ε

—1

*H*' + *i*s p2t 1 + *x* + *i*s *E p*

*Hd* + *i*s1p2t1

1 + *x* + *i*s1*E*' *q*

where *R*

= *R* , *S*

= *S* , *i*s

=  s , *x* = *a*2 , *P* = p2*p*

and

which is positive, therefore solute gradient has a stabilizing

1 p4

1 p4

1 p2 p2 *l*

*Q*1 = *Q* . Equation [(30)](#_bookmark8) is the required dispersion relation

p2

including the parameters characterizing the dust particles, solute gradient, Hall currents, magnetic field and medium permeability.

effect on the thermosolutal convection in an Oldroydian viscoelastic fluid.

From equation [(31)](#_bookmark9), we have

(1+*x*) *Q*1 (1+*x*) + 1 ((1+*x*)2 + (1+*x*) + *Q*1 )

*dR*

*Hdx*

ε

ε

*P*

ε

*P*

ε

# Stationary convection

1 =—

*dM*

1+*x* + 1

2

(1 + *x* + *M*)+ *Q*1

, (34)

ε *P* ε

When the instability sets in as stationary convection (s = 0), equation [(30)](#_bookmark8) reduces to

*R* = (1 + *x*) (1 + *x*) + 1 (1 + *x Q*1 (1 + *x*) + 1 (1 + *x Q*1

1

*Hdx*

ε

*P*

)+

ε

ε

*P*

)+

ε

which is negative, therefore Hall currents have a destabilizing effect on the thermosolutal convection in an Oldroydian viscoelastic fluid.

From equation [(31)](#_bookmark9), we have

(1 + *x*) + 1

*dR* (1 + *x*) (1 + *x*) 1 1 + *x* (1 + *x*) 1

ε *P*

*Q*1 —1 *H*'

1 =

*dQ*1

*Q*

*Hdx*

ε + *P* ε

ε + *P*

(1 + *x* + *M*)

(1 + *x* + *M*)+

ε

+ *S*1 *d*,

*Hd*

(31)

+ 1 (1 + *x* + 2*M*)

ε2

*Q*1 *Q*2 1 + *x* 1

*Q*1 —2

Thus, for the case of stationary convection, the relaxation time parameter *F* and the strain retardation time parameter *F*0 vanishes with s and Oldroydian viscoelastic fluid behaves like

+ ε2

(1 + *x*) + 1

ε3

ε + *P*

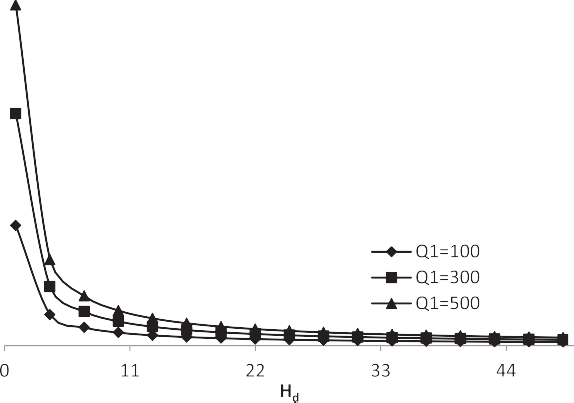
(1 + *x* + *M*)+  ,

ε

(35)

an ordinary Newtonian fluid. The above relation expresses the modified Rayleigh number *R*1 as a function of the parameters *Hd*, *S*1,*M*, *Q*1, *P* and dimensionless wave number *x*. To study the effect of dust particles, solute gradient, Hall currents, mag- netic field and medium permeability, we examine the nature

of *dR*1 , *dR*1 , *dR*1 , *dR*1 and *dR*1 analytically.



which shows that magnetic field has a stabilizing effect on the

thermosolutal convection in an Oldroydian viscoelastic fluid.



*dHd dS*1 *dM dQ*1 *dP*

From equation [(31)](#_bookmark9), we have

*dR*1 (1 + *x*)"( (1 + *x*) 1

*dH* = — *H*2*x*

+

ε *P*

(1 + *x*)+

ε

*Q*1 2

*d d*

(1 + *x*)

1 2) (1 + *x*) 1

*Q*1 —1

+ *M*(1 + *x*)

+ *S*1*H*' ,

*d*

ε + *P*

ε + *P*

(1 + *x* + *M*)+

ε

(32)

which is negative, therefore dust particles have a destabilizing effect on the thermosolutal convection in an Oldroydian viscoelastic fluid.

From equation [(31)](#_bookmark9), we have

Fig. 1 e Variations of critical Rayleigh number*R*1 with*Hd* for

*d*

*dR*1 *dS*1

*H*'

= *d*, (33)

*Hd*

fixed value of ε = 0.5, *M* = 5, *P* = 0.05, *S*1

*Q*1 ¼ 100,200,300.

= 20, *H*'

= 10 and

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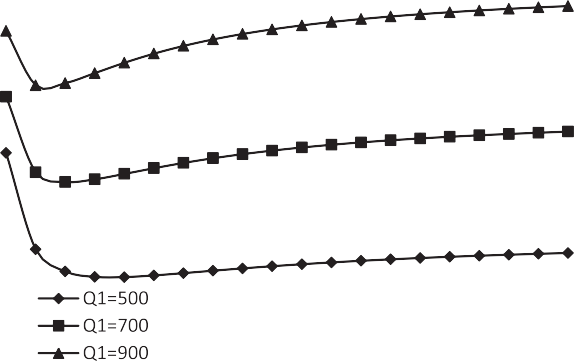
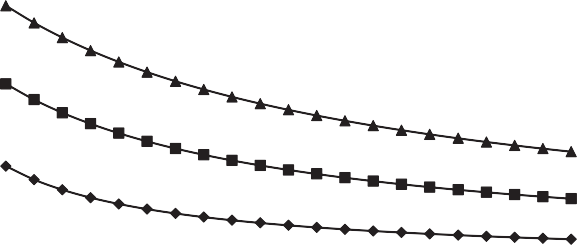






Fig. 2 e Variations of critical Rayleigh number *R*1 with *M* for



fixed value of ε = 0.5, *P* = 0.05, *S*1 = 20, *Hd* = 5, *H*'

*d*

= 10 and

Fig. 4 e Variations of critical Rayleigh number *R*1 with *P* for

*Q*1 ¼ 100,200,300.

fixed value of ε = 0.5, *M* = 10, *S*1 = 20, *Hd* = 10, *H*'

= 20 and

*Q*1 ¼ 500,700,900.

*d*

In the absence of Hall currents, equation [(35)](#_bookmark10) reduces to

*dR*1 = (1 + *x*)

*d*

*dQ*1

*H x*ε , (36)

*dR*1

(1 + *x*)2

predicting that magnetic field has also stabilizing effect on thermosolutal convection in an Oldroydian viscoelastic fluid in the absence of Hall currents.

Further equation [(31)](#_bookmark9) yields

*dP* =— *H xP*2 , (38)

which clearly shows that medium permeability has a desta- bilizing effect on the thermosolutal convection. Thus medium permeability has a dual character, in the absence of Hall

*d*

*MQ*2 —

1

(1+*x*) + 1

(1 + *x* + *M*)+ *Q*1

2

(1 + *x*)

currents it has destabilizing effect while in the presence of

*dR*1 *dP*

=

(1 + *x*) ε2 ε *P*

*HdxP*2

1+*x* + 1

(1 + *x* + *M*)+ *Q*1

ε Hall currents, it has both stabilizing and destabilizing effects

2 on the system.

,

ε *P* ε

(37)

which shows that medium permeability has stabilizing or destabilizing effect on the thermosolutal convection accord- ing as

# Numerical computation

*MQ*2

1 > *or* <

ε2

(1 + *x*) + 1

(1 + *x* + *M*)+

*Q*1 2

ε

(1 + *x*)

For the stationary convection critical thermal Rayleigh num- ber for the onset of instability is determined for critical wave

In the absence of Hall currents, equation [(37)](#_bookmark14) reduces to

ε

*P*

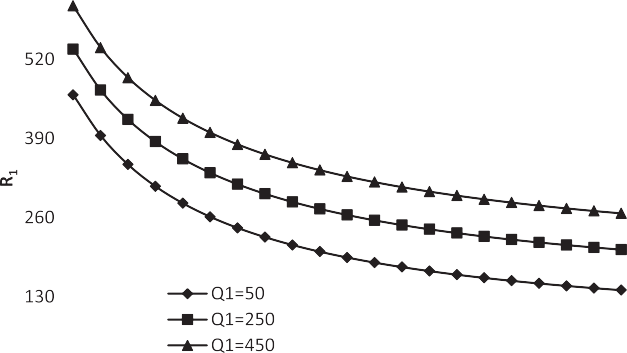
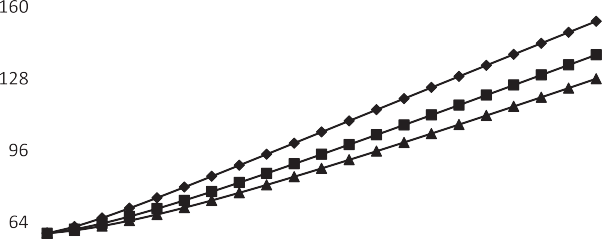


Fig. 3 e Variations of critical Rayleigh number*R*1 with*Q*1 for Fig. 5 e Variations of critical Rayleigh number *R*1 with *P* for

fixed value of ε = 0.5, *S*1 = 20, *P* = 0.05, *Hd* = 5, *H*'

*d*

*M* ¼ 5,10,15.

= 10 and

fixed value of ε = 0.5, *M* = 0, *S*1 = 20, *Hd* = 10, *H*'

*Q*1 ¼ 50,250,450.

*d*

= 20 and

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number obtained by the condition *dR*1 = 0 and analyzed numerically using NewtoneRaphson method.

*dx*

In [Fig. 1](#_bookmark11), critical Rayleigh number *R*1 is plotted against dust particles parameter*Hd* for fixed values of ε = 0.5, *M* = 5, *P* = 0.05, *S*1 = 20, *H*' = 10 and *Q*1 = 100,200,300

*d*

The critical Rayleigh number *R*1 decreases with increase in

dust particles parameter which shows that dust particles have

# Case of overstability

Here, we discuss the possibility as to whether instability may occur as overstability. Equating real and imaginary parts of equation [(26)](#_bookmark4) and eliminating *R*1 between them, we obtain

*A*9*c*9 + *A*8*c*8 + *A*6*c*7 + *A*6*c*6 + *A*5*c*5 + *A*4*c*4 + *A*3*c*3 + *A*2*c*2 + *A*1*c*1 + *A*0 = 0,

destabilizing effect on the system.

In [Fig. 2](#_bookmark12), critical Rayleigh number *R*1 is plotted against Hall

1 1 1 1

1 1 1 1

(39)

currents parameter *M* for fixed values of

*d*

where *c*1 = s2, *b* = 1 + *x* and

ε = 0.5, *P* = 0.05, *S*1 = 20, *Hd*

1

= 5, *H*' = 10 and *Q*1

= 100,200,300.

p18*F*3t5*q*2*p*4*E*'2 *bF*p2t

*E* p2t*F p* *b* 1

The critical Rayleigh number *R*1 decreases with increase in

Hall currents parameter*M* which shows that Hall current has a

*A*9 =

2 1

ε2 ε

+ 1 0 1

ε

ε + *P*

destabilizing effect on the system.

+ *FE*1*p*1 (1 + *f* — *H* ) *b*(*b* — 1), (40)

In [Fig. 3](#_bookmark15), critical Rayleigh number *R*1 is plotted against ε

*d*

2 2*Mb*10

*A*0 = (*b* — 1) (*Hd* — 1)p t

+

+

+

+

ε3

2*Mb*7

+

*P*3

*Q*3*b*5

6*Mb*8

*P*ε

3*MQ*1*b*7

ε3

3*Q*2*b*6

*MQ*2*b*5

3*MQ*1*b*6

+

+

*P*ε

*b*10

+

+

+

ε2

2*b*9

*P*ε

*b*8 *M*2*b*8

*P*2

ε2

2*M*2*b*7

+

*P*ε

*M*2*b*6

1

ε3

1

ε2

1

ε2

+

+

3*Q*1*b*7

+

3*Q*1*b*8 *b*

+ 1 +

*E*1*p*1 — *p*2

*HdQ*1 *b*8

2*b*7 *b*6

+ +

2*Q*1*b*6

+

+

2*Q*1*b*5

+

*Q*2*b*4

+

1

*HdMQ*2p2(*F* — *F*0) *b*

+ 1 *b*5

*P*2 *P*ε

1

ε2 ε *P*

ε ε2 *P*ε *P*2 ε2

*P*ε ε2

ε2 ε *P*

*Hd*(1 + *f* ) 10

+

ε3 *b*

*P*ε

ε

*P*2

2*Hd* *M* 1

9 *Hd* *M*2

2*Q*1 4*M* 1

8 2*MQ*1 *E*1*p*1 + *p*2 6

+

*HdMQ*1 *E*1*p*1 + *p*2

+ *Q*1 *E*1*p*1 — 1 *b*5

+

ε2

+

ε *P*

(1 + *f* )*b*

*HdMQ*2 *E*1*p*1

— *fb* *b*4

+ 6*HdM* + *Hd*

+ 2*HdMQ*1

' 2

+

ε

ε2 +

(1 + *f* )*b*

3*HdM*2

+

ε2 + *P*ε + *P*2

+

+ 4*HdQ*1 *b*7

+ 2*HdM* +

*HdQ*2

*b*

+

1 +

4*HdMQ*1

3*HdM*2

+

ε2 ε2 *P* ε

1

+

*P*2ε *P*3 ε3

*P*ε2

*P*ε2

*P*3 ε3

*P*ε2

*P*2ε

+ 2*HdQ*1 *b*6 +

*P*2ε

*HdM*2 *P*3

*HdQ*2

+ 1 +

*P*ε2

2*HdMQ*1 *b*5

*P*2ε

+ *S*1(*b* — 1)

*b*6 *Hd* — *Hd* p t *P*

2*Mb*5

+ *P*

*M*2*b*4

+ *P*

*b*7

+ *P* +

2*Mb*6

ε

*M*2*b*5

+

ε

+

+

2*Q*1*b*5

+

ε

2*MQ*1*b*4 *b* 1

+

+

+ *HdHd E*1*p*1 — *E*1*q*

+

+

+

+

+

+

ε

ε

*P*

' p t*Q b* '

' *b*5

*b*6 2*Mb*5

ε

ε

2*Mb*4

*P*

*M*2*b*3

*P*

*M*2*b*4

ε

2*Q*1*b*4

ε

2*MQ*1*b*3 *b* 1

ε

ε

*P*

' *HdH*' *Q*2*b*3

2 2 4

+

*Hd* — *Hd*

ε2

1

*P*

+ *E*1*p*1 — *E*1*q d* 1

ε2

(41)

magnetic field parameter *Q*1 for fixed value of

ε = 0.5, *S*1 = 20, *P* = 0.05, *Hd* = 5, *H*' = 10 and *M* = 5,10,15. The

*d*

critical Rayleigh number *R*1 increases with increase in mag- netic field parameter which shows that magnetic field has stabilizing effect on the system.

In [Fig. 4](#_bookmark13), critical Rayleigh number *R*1 is plotted against medium permeability *P* for fixed value of ε = 0.5, *M* = 10, *S*1 = 20, *Hd* = 10, *H*' = 20 and *Q*1 = 500,700,700.

*d*

The critical Rayleigh number *R*1 decreases up to certain values

of *P* and gradually increases there after which shows that medium permeability has both destabilizing and stabilizing

and the coefficients A1eA8 being quite lengthy and not needed in the discussion of stability, have not been written here.

Since s1 is real for overstability, the nine values of *c*1 (=s2)

1

are positive. The product of the roots = —*A*0 is negative and this is to be positive.

*A*9

It is clear from [(40)](#_bookmark17) and [(41)](#_bookmark18) that *A*0 and *A*9 are always positive if

1 + *f* > *Hd*, *Hd* > 1, *Hd* > *H*' , *F* > *F*0, *E*1*p*1 > *b*p2(*F* — *F*0), *E*1*p*1 > 1,

*d*

*E p* > *p* , *E p* > *Pfb* and *E p* > *E*' *q*

effect on the system.

In [Fig. 5](#_bookmark16), critical Rayleigh number *R*1 is plotted against

1 1 2

1 1 ε

1 1 1

(42)

medium permeability *P* for fixed value of

ε = 0.5, *M* = 0, *S*1 = 20, *Hd* = 10, *H*' = 20 and *Q*1 = 50,250,450.

*d*

The critical Rayleigh number *R*1 decreases with increase in

The inequalities [(42)](#_bookmark19) imply that the sufficient conditions for non-existence of overstability are

1 + *f* > *H* , *H* > 1, *H* > *H*' , *F* > *F* , *E p* > *b*p2(*F* — *F* ), *E p* > 1,

medium permeability which shows that medium perme-

ability has destabilizing effect on the system.

*d d d d*

*E p* > *p* , *E p* > *Pfb E p* > *E*' *q*

0 1 1

0 1 1

1 1 2 1 1 ε 1 1 1

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But *F* > *F*0, as l > l0, therefore, the sufficient conditions for r e f e r e n c e s

non-existence of overstability becomes

1 + *f* > *Hd*, *Hd* > 1, *Hd* > *H*' , *E*1*p*1 > *b*p2(*F* — *F*0), *E*1*p*1 > 1, *E*1*p*1 > *p*2,

*d*

*E*1*p*1 >

*Pfb*

, *E*1*p*1 > *E q*

'

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ε 1

i.e

*c* > *c* , *c* > *c*' , *E*  n > (1 + *x*)p2 ln — l0n , *E* n > n, *E* n > 1,

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*f pt pt pt*

n ' n

*kT d*2 *d*2

n *mN*1(1 + *x*)p2*k*1

*kT* h *kT*

'

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*E*1*k*

> *E*1*k* , and *E k* >

r ε*d*2 . *i*.*e*. *cf* > *cpt*, *cpt* > *cpt*

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*T s T*

n

and *E k*

> max

(1 + *x*)p

*T*

0

2 ln l0n n *mN*1(1 + *x*)p2*k*1

*d*2 — *d*2

, , 1,

h

r ε*d*2 , *E*1*k*

0

' n

*s*

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[1973;41:271](http://refhub.elsevier.com/S2314-808X(15)00022-6/sref4)e[83](http://refhub.elsevier.com/S2314-808X(15)00022-6/sref4).

These are, therefore, the sufficient conditions for the non- existence of overstability.

# Conclusions

In the present paper, we have investigated the effect of Hall currents on an electrically conducting Oldroydian viscoelastic dusty fluid heated and soluted from below in porous medium. Dispersion relation governing the effects of dust particles, solute gradient, Hall currents, magnetic field and medium permeability is derived. The main results obtained from the analysis of this paper are as follows:

* 1. For stationary convection, the relaxation time param- eter *F* and the strain retardation time parameter *F*0 vanishes with s and thus an Oldroydian viscoelastic fluid behaves like an ordinary Newtonian fluid.
  2. For the case of stationary convection, suspended (dust) particles and Hall currents are found to have destabi- lizing effects whereas magnetic field has stabilizing ef- fect on the system.
  3. It is also found, for stationary convection, that the me- dium permeability has both stabilizing and destabiliz- ing effects on the system in contrast to its destabilizing effect in the absence of the Hall currents. Solute gradient has a stabilizing effect on the thermosolutal convection.
  4. It is also observed from [Figs. 1](#_bookmark11)e[5](#_bookmark11) that suspended (dust) particles and Hall currents have destabilizing effects whereas the magnetic field has stabilizing effect on the system. The medium permeability, however, has both stabilizing and destabilizing effects in contrast to its destabilizing effect in the absence of Hall Currents.
  5. The conditions

*E*  n > max 1 + *x*)p2 ln — l0 n , n, 1, *mN*1 (1+*x*)p2 *k*1 , *E*' n ,

*kT*

*d*2

*d*2

h

r0 ε*d*2

1*ks*

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*cf* > *cpt* and *cpt* > *c*' are the sufficient conditions for the

*pt*

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non-existence of overstability.