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*Towards Lambda Calculus* Order-Incompleteness

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*Abstract*

*After Scott, mathematical models of the type-free lambda calculus are constructed by order theoretic methods and classi ed into semantics according to the nature of their representable functions. Selinger [48] asked if there is a lambda theory that is not induced by any non-trivially partially ordered model (order-incompleteness problem). In terms of Alexandro topology (the strongest topology whose special-*

*ization order is the order of the considered model) the problem of order-incompleteness can be also characterized as follows: a lambda theory T is order-incomplete if, and only if, every partially ordered model of T is partitioned by the Alexandro topol-*

*ogy in an in nite number of connected components (= minimal upper and lower sets), each one containing exactly one element of the model. Towards an answer to the order-incompleteness problem, we give a topological proof of the following re- sult: there exists a lambda theory whose partially ordered models are partitioned by the Alexandro topology in an in nite number of connected components, each one containing at most one -term denotation. This result implies the incompleteness of every semantics of lambda calculus given in terms of partially ordered models whose Alexandro topology has a nite number of connected components (e.g. the Alexandro topology of the models of the continuous, stable and strongly stable semantics is connected).*

# *1 Introduction*

*Many familiar models of the type-free lambda calculus are constructed by* order theoretic methods. Computational motivations and intuitions justi ed Scott's view of models (see [42] [43]) as partially ordered sets (sets of obser- vations or informations) and of functions as monotonic functions over these sets. After Scott, a large number of mathematical models for the lambda cal- culus, arising from syntax-free constructions, have been introduced in various

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*categories of domains (see [1] [46]) and classi ed into semantics according to* the nature of their representable functions (see [2] [3] [4] [9] [15] [19] [24]). Scott's continuous semantics [43] is given in the category whose objects are complete partial orders and morphisms are continuous functions. The stable semantics introduced by Berry in [10] and the strongly stable semantics intro- duced by Bucciarelli and Ehrhard in [11] are strengthening of the continuous semantics. The stable semantics is given in the category of DI-domains with stable functions as morphisms, while the strongly stable one in the category of DI-domains with coherence, and strongly stable functions as morphisms.

*Lambda theories are consistent extensions of the lambda calculus that in-* clude -conversion. They arise by syntactical considerations, a lambda theory may correspond to a possible operational (observational) semantics of lambda calculus (see e.g. [2] [3] [23]), as well as by semantic ones, a lambda theory may be the theory of a model of lambda calculus (see e.g. [3] [9]). The problem of the completeness/incompleteness of a semantics can be stated as follows: are the set of the lambda theories determined by a semantics equal or strictly included within the set of consistent lambda theories?

*The rst incompleteness result was obtained by Honsell and Ronchi della* Rocca [24] for the continuous semantics via a hard syntactical proof. Gouy

*[20] proved the incompleteness of the stable semantics with a much harder* syntactical proof. Other more semantic proofs of incompleteness for the con- tinuous and stable semantics can be found in [7]. Bastonero [6] provides an incompleteness result for the hypercoherence semantics.

*Recently, the author has introduced in [37] a new technique to prove the* incompleteness of a wide range of lambda calculus semantics (including the strongly stable one, whose incompleteness had been conjectured). Roughly, the technique used in [37] for proving that a class C of models is incomplete is the following. We remark that the partially ordered models of the lambda calculus are topological combinatory algebras w.r.t. the Alexandro topology (the strongest topology whose specialization order is the order of the consid- ered model). Then we nd a (topological) property P veri ed by all models in C and nd a lambda theory whose models do not verify P . The technique was applied to the models of lambda calculus based on domains (continuous, stable, strongly stable models in particular). These models satisfy a strong property of connectedness, while we found a lambda theory whose models satisfy an orthogonal property of separation.

*The problem of the incompleteness of the semantics of lambda calcu-* lus is also related to the open problem of the order-incompleteness of the lambda theories. Selinger [48] asked if there is a lambda theory that is not induced by any non-trivially partially ordered model. He gave a syntacti- cal characterization, in terms of so-called generalized Mal'cev operators, of the order-incomplete lambda theories. Roughly, the problem of the order- incompleteness can be stated as follows: does it exist a sequence M1;::: ; Mn

*of closed -terms such that the lambda theory Tn, axiomatized by* x = M1xyy; Mixxy = Mi+1xyy; Mnxxy = y (1 i < n);

*is consistent? Plotkin and Simpson (see [47]) have shown that T1 is inconsis-* tent, while Plotkin and Selinger (see [47]) obtained the same result for T2. It is

*an open problem whether Tn (n 3) can be consistent. Order-incompleteness*

*is also related to Plotkin's conjecture (see [36] [47] [48]) about the existence of* absolutely unorderable combinatory algebras, where a combinatory algebra is absolutely unorderable if it cannot be embedded in any orderable combinatory algebra.

*The problem of order-incompleteness can be also characterized in terms of* Alexandro topology. A lambda theory T is order-incomplete if, and only if, the Alexandro topology of any partially ordered model of T is the discrete topology if, and only if, the Alexandro topology of any partially ordered model of T partitions the model in an in nite number of connected com- ponents (= minimal upper and lower sets), each one containing exactly one element of the model. Towards an answer to the order-incompleteness prob- lem, in this paper we give a topological proof of the following result: there exists a lambda theory whose partially ordered models are partitioned by the Alexandro topology in an in nite number of connected components, each one containing at most one -term denotation. This result implies the incomplete- ness, that had been conjectured in [37], of every semantics of lambda calculus given in terms of partially ordered models whose Alexandro topology has a nite number of connected components (e.g. the Alexandro topology of continuous, stable and strongly stable semantics is connected).

# *2 Preliminaries*

*To keep this article self-contained, we summarize some de nitions and results* that we will need in the subsequent part of the paper. With regard to the lambda calculus we follow the notation and terminology of Barendregt (see [3]).

*For the general theory of lambda calculus the reader may consult Baren-* dregt [3] and Krivine [28]. For the general theory of universal algebras the reader may consult Burris and Sankappanavar [12] Gratzer [21] and McKen- zie, McNulty and Taylor [29]. The main references for topological algebras are Taylor [50] [51], Gumm [22], Bentz [8] and Coleman [13] [14].

*2.1 Lambda theories*

*denotes the set of -terms, while o denotes the set of closed -terms, where* a -term is closed if it does not admit free occurrences of variables.

*Lambda theories are consistent extensions of the lambda calculus that are* closed under derivation. Remember that an equation is a formula of the form

*M = N with M; N 2 . The equation is closed if M and N are closed -terms.* If T is a set of equations, then the theory + T is obtained by adding to the axioms and rules of the lambda calculus the equations in T as new axioms. If T is a set of closed equations, T + is the set of closed equations provable in

*+ T . T is a lambda theory if T + = T (see [3, Def. 4.1.1]). As a matter* of notation, T ` M = N stands for + T ` M = N; this is also written as

*M =T N. [M]o = fN 2 o : T ` N = Mg denotes the equivalence class of*

*T*

*the closed -term M.*

*2.2 Combinatory algebras and -models*

*An algebra C = (C; ; k; s), where is a binary operation and k; s are con-* stants, is called a combinatory algebra (Curry [16], Schon nkel [41]) if it satis-

*es the following identities (as usual the symbol is omitted, and association* is to the left): kxy = x; sxyz = xz(yz). In the equational language of combi- natory algebras the derived combinator 1 is de ned as 1 s(ki). A function f : C ! C is called representable if there exists an element c 2 C such that cz = f (z) for all z 2 C. If this last condition is satis ed, we say that c represents map f in C.

*Let C be a combinatory algebra and let c be a new symbol for each c 2 C.*

*Extend the language of lambda calculus by adjoining c as a new constant*

*symbol for each c 2 C. Let o(C) be the set of closed -terms with constants* from C. The interpretation of terms in o(C) with elements of C can be de ned by induction as follows (for all M; N 2 o(C) and c 2 C):

*jc jC = c; j(MN)jC = jMjC jNjC; j x:MjC = 1m;*

*where m 2 C is any element representing the following map f : C ! C:* f (c) = jM[x := c ]jC; for all c 2 C.

*The drawback of the previous de nition is that, if C is an arbitrary combi-*

*natory algebra, it may happen that map f is not representable. The axioms* of a subclass of combinatory algebras, called -models or models of lambda calculus (Meyer [30], Scott [45], [3, Def. 5.2.7]), were expressly chosen to make coherent the previous de nition of interpretation. For every -model C, the set T h(C) = fM = N : M; N 2 o; C j= M = Ng constitutes a lambda theory. C is a model of the lambda theory T if T = T h(C).

*We would like to point out here that there exists an algebraic approach to* the model theory of lambda calculus, alternative to combinatory logic, that allows to keep the lambda notation and all the functional intuitions (see [31] [32] [33] [38] [39] [40]).

*2.3 Topology*

*If (A; ) is a topological space (we will occasionally avoid explicit mention of*

*) then the closure of a subset U of A will be denoted by U (if U = fbg is a* singleton set, then we write b for fbg). Recall that a 2 U if U \ V 6= ; for every open neighborhood V of a.

*For any space (A; ) a preorder can be de ned by*

*a b i a 2 b i 8U 2 (a 2 U ) b 2 U ):*

*We have*

*is T0 i is a partial order.*

*For any T0-space A the partial order is called the specialization order of .* Notice that any continuous map between T0-spaces is necessarily monotone and that the order is discrete (i.e. satis es a b i a = b) i A is a T1-space.

*A space A is T2 (or Hausdor ) if for all a; b 2 A there exist open sets U* and V with a 2 U , b 2 V and U \ V = ;.

*The previous axioms of separation can be relativized to pairs of elements.* For example, a and b are T2-separable, if there exist open sets U and V with a 2 U , b 2 V and U \ V = ;. T1-, T0-separability are similarly de ned.

*The connected component of an element a of a space A is the greatest con-* nected subset of A including a. The connected components de ne a partition of the space A.

*Each partition P of any set X into disjoint subsets, together with ;, is* a basis for a topology on X, known as a partition topology. A subset of *X* is then open if and only if it is the union of sets belonging to P and thus its complement is also open; thus a set is open i it is closed. The trivial partitions yield the discrete or indiscrete topologies. In any other cases *X* with a partition topology is not T0.

*Let (A; ) be a partially ordered set (poset). B A is an upper (lower)* set if b 2 B and b a (a b) imply a 2 B. We utilize the notations B" (B#, Bl respectively) for the least upper (lower, upper and lower) set containing a subset B of A. We write a" (a#, al respectively) for fag" (fag#, fagl).

*Given a poset (A; ) we can nd many T0-topologies on A for which is the specialization ordering of (see Johnstone [25, Section II.1.8]). The* Alexandro topology and the weak topology de ned below are the maximal one and the minimal one with this property.

*The Alexandro topology a is constituted by the collection of all upper* sets in A, i.e.,

*U is an Alexandro open (A-open, for short) i U = U ".*

*Then a" is the least open set containing a. A subset U is an Alexandro closed set (A-closed set, for short) i U = U #. A function is continuous w.r.t.*

*the Alexandro topology if, and only if, it is monotone. Every Alexandro space is T0.*

*The weak topology w is constituted by the smallest topology for which* all sets of the form a# are closed, i.e. the topology based by sets of the form A (a1#[ ::: [ ak#):

*Let (A; ) be a poset, be a topology on A. Then is T0 with specializa-* tion order if, and only if, w a.

# *3 The topological theorem*

*Separation axioms in topology stipulate the degree to which distinct points* may be separated by open sets or by closed neighborhoods of open sets. In the main theorem of this Section we prove that every partially ordered com- binatory algebra, under very weak hypotheses, admits elements which can be separated in a very strong way.

*Let (A; ) be a poset with Alexandro topology a. The intersection of* every family of A-open sets is A-open; thus the union of every family of A- closed sets is A-closed. This is a consequence of fact that, for every subset

*V of a poset (A; ), there exist a least upper set V " and a least lower set*

*V #, all of them including V . It follows that the family of A-closed sets of* the Alexandro topology associated with a poset (A; ) is the Alexandro topology a associated with the poset (A; ).

*We now consider the clopen sets, i.e., the sets which are contemporaneously* A-open and A-closed:

*X is A-clopen i X = Xl = X" = X#.*

*Notice that a connected component is a closed set in every topological space.* For the Alexandro topology we have that a subset of a poset is a connected component w.r.t. a i it is such w.r.t. a. So a connected component is a minimal A-clopen set, and the minimal A-clopen sets constitute the partition of the Alexandro space in connected components. In terms of partial ordering they can be described as follows. Let be the symmetric closure of , i.e.,

*a b i either a b or b a.*

*The equivalence relation on A generated by is de ned as follows: a b , (9c0;::: ; cn) a = c0 c1 ::: cn 1 cn = b:*

*Then the equivalence classes of are the partition of A in connected com-*

*ponents w.r.t. the Alexandro topology, i.e.,*

*-equivalence class = minimal A-clopen set = connected component.*

*It is an easy matter to verify that the A-clopen sets of an Alexandro space* constitute a topology, denoted by. It is the partition topology (see Sec- tion 2.3) generated by the partition of the space in connected components.

*Since a map is monotone i the inverse image of an upper set is an upper* set i the inverse image of a lower set is a lower set, then every monotone map is continuous w.r.t. the partition topology.

*A partially ordered combinatory algebra, a po-combinatory algebra for* short, is a pair (C; ) where C is a combinatory algebra and is a partial order on C which makes the application operator of C monotone.

*An Alexandro combinatory algebra is a pair (A; a) where A is a combi-* natory algebra and a is an Alexandro topology on the underlying set A with the property that the application operator of A is continuous (= monotone) with respect to a.

*The category of po-combinatory algebras with monotone maps as mor-* phisms and the category of Alexandro combinatory algebras with continuous maps as morphisms are equivalent.

*In the following we always assume de ned on a po-combinatory algebra* the Alexandro topology.

*In the following theorem we prove that every po-combinatory algebra under* very weak hypotheses admits elements which can be separated by A-clopen sets.

*Theorem 3.1 Let (A; ) be a po-combinatory algebra for which there exist a* combinatory term s(x; y) and a constant 0 such that

*s(x; x) = 0:*

*For all a; b 2 A, de ne a sequence of elements of A as follows:*

*c1 = s(a; b); cn+1 = s(cn; 0):*

*If cn 6= 0 for all n, then there exist an A-clopen set V such that a 2 V and* b 2= V .

*Proof. The proof is divided in claims.*

*Claim 3.2 If c1 and 0 are T2-separated w.r.t. the partition topology then* a and b are also T2-separated w.r.t. the partition topology.

*Let U and S be two A-clopen sets such that U is a neighbourhood of c1, S* is a neighbourhood of 0, and U \ S = ;. Because s(a; b) = c1 2 U and s is continuous w.r.t., then there exist two A-clopen sets V and W such that

*V is a neighbourhood of a, W is a a neighbourhood of b, and s(V; W ) U . If* d 2 V \ W then 0 = s(d; d) 2 U contradicting the choice of U .

*Claim 3.3 For every element z 2 A de ne by induction the following sets:*

*z0 = fzg; z2i+2 = (z2i+1)#; z2i+1 = (z2i)":*

*Then set [i 0zi is equal to the least A-clopen set zl (= connected component)* including z.

*It is suÆcient to check that [i 0zi is an upper and lower set contained within* zl.

*Claim 3.4 For every k 1 we have that*

*s(ck"; 0") ck+1":*

*The relation follows from the monotonicity of s and from the equality ck+1 =* s(ck; 0).

*Claim 3.5 For every k 1 we have that*

*ck 6 0:*

*Assume, by the way of contradiction, that ck 0. Then by monotonicity we* have that

*and*

*0 = s(ck; ck) s(ck; 0) = ck+1*

*ck+1 = s(ck; 0) s(0; 0) = 0:*

*This contradicts the hypothesis that is a partial order.*

*Claim 3.6 For every k 1 we have that ck and 0 are incompatible, i.e.,*

*ck"\ 0" = ;:*

*If there exists an element b such that b ck and b 0 then by monotonicity* we have that

*ck+1 = s(ck; 0) s(b; b) = 0 that contradicts Claim 3.5.*

*Claim 3.7 For every k 1 and every i 1 we have that*

*ci \ 0i = ;; s(ci ; 0i) ci :*

*k k k+1*

*(see the de nition of `( )i' in Claim 3.3).*

*For i = 1 the conclusion follows from Claim 3.6 and Claim 3.4. Assume the*

*conclusion true for i and prove it for i + 1. Let s(ci ; 0i) ci ci+1 . If i is*

*k k+1 k+1*

*k+1*

*odd ci*

*k+1*

*is A-open and ci+1*

*is A-closed. Since s is continuous the pre-image*

*of the A-closed set ci+1 under the map s is A-closed. From s(ci ; 0i) ci+1*

*k+1 k k+1*

*the pre-image of ci+1 , that is closed, contains ci 0i, so s(ci+1; 0i+1) ci+1 .*

*k+1 k k k+1*

*If i is even we make the same reasoning by using the Alexandro topology a*

*associated with the partial ordering on A.*

*We now prove that ci+1 \ 0i+1 = ;. Assume i odd so that ci+1 and 0i+1*

*k k*

*are A-closed sets. Assume, by the way of contradiction, that there is f* *2*

*ci+1 \ 0i+1. It follows that 0 = s(f; f ) 2 ci+1 , because we have already shown*

*k k+1*

*that s(ci+1; 0i+1) ci+1 . But by de nition of closure of a set this is possible*

*k k+1*

*only if for every A-open neighbourhood Z of 0, we have that Z \ ci 6= ;.*

*k+1*

*But this contradicts the induction hypothesis ci \ 0i = ; because 0i is an*

*k+1*

*A-open neighbourhood of 0. A similar reasoning works for an even i by using* the Alexandro topology a associated with the partial ordering on A.

*Claim 3.8 For every k 1 we have that ck and 0 are T2-separated w.r.t. the* partition topology.

*The least clopen sets including ck and 0 are respectively [i 0ci and [i 00i.*

*k*

*Then the conclusion follows from Claim 3.7.*

*Since c1 and 0 are T2-separated w.r.t. the partition topology from* Claim 3.8, then the conclusion of the theorem follows from Claim 3.2. *2*

# *4 Incompleteness*

*In this Section we prove the main theorem of the paper.*

*Consider the (consistent and) semisensible lambda theory axiomatized*

*by*

*where ( x:xx)( x:xx).*

*xx = ;*

*Lemma* *4.1*

*` tu = , ` t = u:*

*Proof. Let ! be the following reduction rule:*

*(1) MN !*

*for every M and N such that ` M = N. The re exive closure of ! satis es the diamond property, and the relations and commute. Then* the reduction rule ! = ! [ ! is Church-Rosser by the Hindley-Rosen Lemma (see Barendregt [3, Prop. 3.3.5]).

*Then we prove that is the lambda theory generated by conversion = from ! , i.e.,*

*(2) ` M = N i M = N:*

*Since MN ! i ` M = N, then it is obvious that M = N implies*

*` M = N. For the opposite direction, it is suÆcient to consider that*

*xx ! for the unique axiom xx = of .*

*If ` tu = then tu = , so that there is a reduction tu .* This is possible only if tu is a -redex i.e. if ` t = u. *2*

*Lemma 4.2 Let t and u be two -terms. De ne the sequence*

*c1 tu; cn+1 (cn) :* If 6` t = u then 6` cn = for all n.

*Proof. The proof is by induction on n. By Lemma 4.1 we have that `*

*tu = i ` t = u, so that our hypothesis 6` t = u implies 6` c1 = .* The remaining part follows from the induction hypothesis and from Lemma 4.1 applied to cn+1 (cn) . *2*

*Theorem 4.3 Every partially ordered model of is partitioned in an in nite* number of connected components (= A-clopen sets), each one containing at most one -term denotation.

*Proof. Let C be a partially ordered model of* *The interpretation of a closed*

*-term t is the element jtjC of C (see Section 2.2). For the sake of simplicity, we* write directly t for jtjC when there is no danger of confusion. De ne 0 and s(x; y) xy. Since ` xx = , then we have that C j= x: xx = x: . This last identity implies cc = ( x: xx)c = ( x: )c = for all c 2 C.

*Let t; u be two -terms such that 6` t = u. Since C is a model of , by* Lemma 4.2 we must have that C j= cn = for all n 1. Then we can apply Lemma 4.1 to get that t and u are separated by two A-clopen sets. Since we have an in nite number of -equivalence classes, then we must have an in nite number of connected components each of them containing at most one term denotation. *2*

*A class C of models of lambda calculus represents a lambda theory T if there* is a model in C whose theory is exactly T . A class of models is incomplete if it does not represent all the lambda theories.

*The models of lambda calculus are classi ed into semantics according to* the nature of their representable functions. A semantics is usually constituted by a class of suitable partially ordered models. This last condition is justi ed by Scott's view of models as sets of sets of observations (or informations) and of computable functions as monotone functions over such sets (see [45]).

*Scott's continuous semantics [43] is the class of the partially ordered models* whose specialization order is a complete partial ordering and the representable functions are all the continuous ones w.r.t. the Scott topology. The graph model semantics (see [44] [18] [34] [35] [9, Section 5.5]) is a subclass of the K- semantics isolated by Krivine (see [28] [9, Section 5.6.2]) within the continuous semantics. The lter model semantics was de ned by Coppo, Dezani, Honsell and Longo in [15] (see also [4]) within the continuous semantics.

*The stable semantics introduced by Berry [10] is the class of the par-* tially ordered models whose specialization order is a DI-domain and the rep- resentable functions are all the stable ones.

*The strongly stable semantics introduced by Bucciarelly and Ehrhard in*

*[11] is the class of the partially ordered models whose specialization order is a* DI-domain with coherence and the representable functions are all the strongly stable ones. The hypercoherence semantics introduced by Ehrhard [17] is a subclass of the strongly stable semantics.

*Stability and strong stability constitute restrictions of continuity to capture*

*the notion of sequentiality.*

*The rst incompleteness result was given by Honsell and Ronchi della* Rocca [24] for the continuous semantics. They proved that the contextual lambda theory induced by the set of essentially closed terms does not ad- mit a continuous model. Following a similar method, Gouy [20] proved the incompleteness of the stable semantics. Other more semantic proofs of in- completeness for the continuous and stable semantics can be found in [7]. Bastonero [6] provides an incompleteness result for the hypercoherence se- mantics. Salibra has recently shown in [37] that the strongly stable semantics is also incomplete.

*Theorem 4.4 (The Incompleteness Theorem) Any semantics of the lambda* calculus given in terms of partially ordered models with a nite number of connected components (= minimal upper and lower sets = A-clopen sets) is incomplete. If constants are admitted then, for every cardinal number Æ, any semantics of the lambda calculus given in terms of partially ordered models with at most Æ connected components is incomplete.

*Proof. From Thm. 4.3. If constants are admitted, it is suÆcient to de ne the* lambda theory in a language with an arbitrary number of constants. *2*

*It follows from Thm. 4.4 that the lambda theory cannot have a model* in the graph model semantics, K-semantics, lter model semantics, stable se- mantics, hypercoherence semantics, strongly stable semantics, and moreover, in the partially ordered models either with a bottom element, or with a top element, or with a structure of complete partial ordering, meet semilattice, join semilattice and lattice.

# *References*

*[1] Abramsky S., Domain theory in logical form, Annals of Pure and Applied Logic*

*51 (1991), 1{77.*

*[2] Abramsky S. and C.H.L. Ong, Full abstraction in the lazy* *calculus,*

*Information and Computation 105 (1993), 159{267.*

*[3] Barendregt H.P., \The lambda calculus: Its syntax and semantics", Revised edition, Studies in Logic and the Foundations of Mathematics 103, North- Holland Publishing Co., Amsterdam, 1984.*

*[4] Barendregt H.P., M. Coppo and M. Dezani-Ciancaglini, A lter model and the completeness of type assignmen, Journal of Symbolic Logic 48 (1983), 931{940.*

*[5] Bastonero O., \Mod eles fortement stables du -calcul et r esultats d'incompl etude", Th ese, Universit e de Paris 7, 1996.*

*[6] Bastonero O., Equational incompleteness and incomparability results for - calculus semantics, manuscript.*

*[7] Bastonero O. and X. Gouy, Strong stability and the incompleteness of stable models of -calculus, Annals of Pure and Applied Logic 100 (1999), 247{277.*

*[8] Bentz W., Topological implications in varieties, Algebra Universalis 42 (1999), 9{16.*

*[9] Berline C., From computation to foundations via functions and application: The*

*-calculus and its webbed models, Theoretical Computer Science 249 (2000), 81{161.*

*[10] Berry G., Stable models of typed lambda-calculi, Proc. 5th Int. Coll. on Automata, Languages and Programming, LNCS vol.62, Springer-Verlag, 1978.*

*[11] Bucciarelli A. and T. Ehrhard, Sequentiality and strong stability, Sixth Annual IEEE Symposium on Logic in Computer Science (1991), 138{145.*

*[12] Burris S. and H.P. Sankappanavar, \A course in universal algebra", Springer- Verlag, Berlin, 1981.*

*[13] Coleman J.P., Separation in topological algebras, Algebra Universalis 35 (1996), 72{84.*

*[14] Coleman J.P., Topological equivalents to n-permutability, Algebra Universalis*

*38 (1997), 200{209.*

*[15] Coppo M., M. Dezani-Ciancaglini, F. Honsell and G. Longo, Extended type structures and lter -models, Logic Colloquium'82, Elsevier Science Publishers (1984), 241{262.*

*[16] Curry H.B. and R. Feys, \Combinatory Logic", Vol. I, North-Holland Publishing Co., Amsterdam, 1958.*

*[17] Ehrhard T., Hypercoherences: a strongly stable model of linear logic, Mathematical Structures in Computer Science 2 (1993), 365{385.*

*[18] Engeler E., Algebras and combinators, Algebra Universalis 13 (1981), 389{392.*

*[19] Girard J.Y., The system F of variable types, fteen years later, Theoretical Computer Science 45 (1986), 159{192.*

*[20] Gouy X., \Etude des th eories equationnelles et des propri et es alg ebriques des mod eles stables du -calcul", Th ese, Universit e de Paris 7, 1995.*

*[21] Gratzer G., \Universal Algebra", Second edition, Springer-Verlag, New York, 1979.*

*[22] Gumm H.P., Topological implications in n-permutable varieties, Algebra Universalis 19 (1984), 319{321.*

*[23] Honsell F. and M. Lenisa, Final semantics for untyped -calculus, LNCS 902, Springer-Verlag, Berlin (1995), 249{265.*

*[24] Honsell F. and S. Ronchi della Rocca, An approximation theorem for topological*

*-models and the topological incompleteness of -calculus, Journal Computer and System Science 45 (1992), 49{75.*

*[25] Johnstone P.T., \Stone Spaces", Cambridge University Press, 1982.*

*[26] Kerth R., Isomorphism and equational equivalence of continuous lambda models, Studia Logica 61 (1998), 403{415.*

*[27] Kerth R., On the construction of stable models of -calculus, Theoretical Computer Science (to appear).*

*[28] Krivine J.L., \Lambda-Calcul, types et mod eles", Masson, Paris, 1990.*

*[29] McKenzie R.N., G.F. McNulty and W.F. Taylor, \Algebras, Lattices, Varieties, Volume I", Wadsworth Brooks, Monterey, California, 1987.*

*[30] A.R. Meyer, What is a model of the lambda calculus?, Information and Control 52, (1982), 87{122.*

*[31] Pigozzi D. and A. Salibra, Lambda abstraction algebras: representation theorems, Theoretical Computer Science 140 (1995), 5{52.*

*[32] Pigozzi D. and A. Salibra, The abstract variable-binding calculus, Studia Logica*

*55 (1995), 129{179.*

*[33] Pigozzi D. and A. Salibra, Lambda abstraction algebras: coordinatizing models of lambda calculus, Fundamenta Informaticae 33 (1998), 149{200.*

*[34] Plotkin G.D., A set theoretic de nition of application, Memorandum MIP-R-95, University of Edinburgh, 1972.*

*[35] Plotkin G.D., Set-theoretical and other elementary models of the -calculus, Theoretical Computer Science 121 (1993), 351{409.*

*[36] Plotkin G.D., On a question of H. Friedman, Information and Computation*

*126 (1996), 74{77.*

*[37] Salibra A., A continuum of theories of lambda calculus without semantics, 16th Annual IEEE Symposium on Logic in Computer Science (2001), Boston, USA.*

*[38] Salibra A., Nonmodularity results for lambda calculus, Fundamenta Informaticae (to appear).*

*[39] Salibra A., On the algebraic models of lambda calculus, Theoretical Computer Science 249 (2000), 197{240.*

*[40] Salibra A. and R. Goldblatt, A nite equational axiomatization of the functional algebras for the lambda calculus, Information and Computation 148 (1999), 71{ 130.*

*[41] Schon nkel M.,Uber die bausteine der Mathematischen Logik, Mathematischen Annalen (english translation in J. van Heijenoort ed.'s book\From Frege to Godel, a source book in Mathematical Logic, 1879-1931", Harvard University Press, 1967), 92 (1924), 305{316.*

*[42] Scott D.S., Some ordered sets in computer science, In: Ordered sets (I. Rival ed.), Proc. of the NATO Advanced Study Institute (Ban , Canada), Reidel (1981), 677{718.*

*[43] Scott D.S., Continuous lattices, In: Toposes, Algebraic geometry and Logic (F.W. Lawvere ed.), LNM 274, Springer-Verlag (1972), 97{136.*

*[44] Scott D.S., Data types as lattices, SIAM J. Computing 5 (1976), 522{587.*

*[45] Scott D.S., Lambda calculus: some models, some philosophy, The Kleene Symposium (J. Barwise, H.J. Keisler, and K. Kunen eds.), Studies in Logic 101, North-Holland, 1980.*

*[46] Scott D.S. and C. Gunter, Semantic domains, Handbook of Theoretical Computer Science, North-Holland, Amsterdam, 1990.*

*[47] Selinger P., \Functionality, polymorphism, and concurrency: a mathematical investigation of programming paradigms", PhD thesis, University of Pennsylvania, 1997.*

*[48] Selinger P., Order-incompleteness and nite lambda models, Eleventh Annual IEEE Symposium on Logic in Computer Science (1996).*

*[49] Steen L.A. and J.A. Seebach, Jr., \Counterexamples in topology", Springer- Verlag, 1978.*

*[50] Taylor W., Varieties of topological algebras, Austral. Math. Soc. 23 (1977) 207{ 241.*

*[51] Taylor W., Varieties obeying homotopy laws, Canad. Journal Math. 29 (1977), 498{527.*