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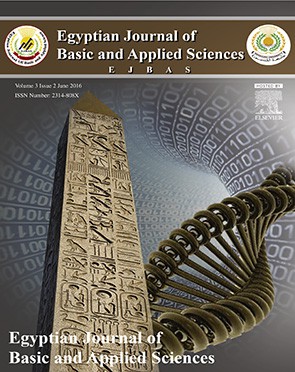
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**Full Length Article**

**Traveling wave solutions of generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony and simplified modified form of Camassa–Holm equation exp(–*φ*(*η*)) – Expansion method**



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In this article, we established abundant traveling wave solutions for nonlinear evolution equa- tions. The exp(–*φ*(*η*))-expansion method is used to construct traveling wave solutions for the generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony equation and Simplified Modi- fied form of Camassa–Holm equation. The traveling wave solutions are expressed in terms of the hyperbolic functions, the trigonometric functions and the rational functions. The pro- posed solutions are found to be important for the explanation of some practical physical problems in mathematical physics and engineering.

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# Introduction

It is well known that seeking exact solutions [[1–53]](#_bookmark20) for non- linear evolution equations (NLEES) plays an important role in mathematical physics. For instance, nonlinear evolution equa- tions (NEEs) are widely used as models to describe complex physical phenomena in various fields of sciences, especially

in fluid mechanics, solid-state physics, plasma physics, plasma waves and biology. One of the basic physical problems for those models is to obtain their travelling wave solutions. In particu- lar, various methods have been utilized to explore different kinds of solutions of physical models described by nonlinear partial differential equations (NPDEs). In the past few decades or so, many effective methods have been presented, which contain the inverse scattering transform method, the Backlund

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transformation [[1]](#_bookmark20), bilinear transformation, the tanh-sech method [[2]](#_bookmark21), the extended tanh method, the pseudo-spectral method [[3,8–10,14,15]](#_bookmark22), trial function and the sine-cosine method [[4,5]](#_bookmark23), Hirota method [[6]](#_bookmark24), tanh-coth method [[2,7,11,13]](#_bookmark21), the ex- ponential function method [[16–24]](#_bookmark25), the (*G´*/*G*)-expansion method [[25–29]](#_bookmark26), the homogeneous balance method [[30,31]](#_bookmark27), F-expansion method [[33–35]](#_bookmark28) and the Jacobi elliptic function expansion method [[36–38]](#_bookmark29) and so on. In a subsequent work, Ma et al. [[39]](#_bookmark30)

where *u*  *u**x*, *t* is an unknown function, *P* is a polynomial in *u**x*, *t* and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In order to solve Eq. [(2)](#_bookmark6) by using the exp(–*φ*(*η*))-expansion method we have to follow the following steps.

**Step 1.** Combining the real variables *x* and *t* by a com- pound variable *η* we assume

developed the complexiton solutions for Toda lattice equa- tion through the Casoratian formulation and hence obtained

*u**x*, *t*  *u* **,

**  *x*  *Vt*

(3)

a set of coupled conditions which guaranteed Casorati deter- minants to be the solution of Toda Lattice which consequently produced complexiton solutions. Moreover, Ma and You [[40]](#_bookmark31) used variation of parameters for solving the involved non-

where *V* is the speed of the traveling wave. Using the travel- ing wave variable [(3)](#_bookmark1), Eq. [(2)](#_bookmark6) is reduced to the following ODE for *u*  *u* **

homogeneous partial differential equations and obtained

solution formulas helpful in constructing the existing solu- tions coupled with a number of other new solutions including

*Q* *u*, *u*, *u*, *u*, *u*, …  0,

(4)

rational solutions, solitons, positions, negatons, breathers, com- plexions and interaction solutions of the KdV equations. It is needed to be highlighted that the basic spirit of the exp- function method which is the conversion of nonlinear partial differential equations into integrable ordinary differential equa-

where Q is a function of *u* ** and its derivatives, prime denotes

derivative with respect to *η*.

**Step 2.** Suppose the solution of (4) can be expressed by a polynomial in exp(–*φ*(*η*)) as follows

tions was explicitly presented and minutely analyzed in 1996 by Ma and Fuchssteiner [[41]](#_bookmark32). In fact, the exp-function method

*u* **  *an* exp** ***n*  *an*1 exp** ***n*1  ⋯,

(5)

is restricted to produce rational solutions in the form of trans- formed variables and such solutions can be obtained easily by making use of other techniques including Wronskian and Casoratian [[41–43]](#_bookmark32). Recently, Ma, Wu and He [[44]](#_bookmark33) presented a much more general idea to yield exact solutions to nonlinear wave equations by searching for the so-called Frobenius trans- formations. Some recently developed methods, such as, the

modified simple equation [[45–49]](#_bookmark34), the enhanced Exp(–*φ*(*(*))- expansion method [[50,51]](#_bookmark35), the Enhanced (G′/G)-Expansion method [[52,53]](#_bookmark36), etc. which provide useful exact solutions to

where *an*, *an*1, ⋯ and V are constants to determined later such that *an* ≠ 0 and ** ** satisfies Eq. [(1)](#_bookmark5).

**Step 3.** By using the homogenous principal, we can evalu- ate the value of positive integer *n* between the highest order linear terms and nonlinear terms of the highest order in Eq.

[(4)](#_bookmark2). Our solutions now depend on the parameters involved in Eq. [(1)](#_bookmark5).

Case 1. *h*2 − 4*μ* > 0 and *μ* ≠ 0,

NLEEs have been discussed.

 1 



  

The objective of this article is to apply the exp(–*φ*(*η*))-

** **  ln    **2  4** tanh

**  *c*1   ** ,

(6)

expansion method to construct the exact solutions for nonlinear evolution equations in mathematical physics via generalized

**2  4**

2** 

 2   

Zakharov–Kuznetsov–Benjamin–Bona–Mahony equation and Simplified Modified form of Camassa–Holm equation. The subject matter of this method is that the traveling wave so- lutions of a nonlinear evolution equation can be expressed by

where *c*1 is a constant of integration.

Case 2. *h*2 − 4*μ* < 0 and *μ* ≠ 0,

a polynomial in exp(–*φ*(*η*)), where *φ*(*η*) satisfies the ordinary dif-





** **  ln  1  **  **2  4** tan

**2  4** **  *c*  

ferential equation (ODE):

**  **  exp** **  ** exp** **  **

Where *η* = *x* − *Vt*.

(1)

2**   2

Case 3. *μ* = 0 and *h* ≠ 0,

** **   ln ** ** 

 

exp** **  *c*1   1

1   

(7)

(8)

 

# Description of exp(–*φ*(*η*))-expansion

**method**

Now we explain the exp(–*φ*(*η*))-expansion method for finding traveling wave solutions of nonlinear evolution equations. Let us consider the general nonlinear partial differential equa-

Case 4. **2  4**  0, **  0, and *μ* ≠ 0,

** **  ln 2 ** **  *c*1   2

 2 

** **  *c*1 





(9)

tion of the form.

*P**u*, *ut*, *ux*, *utt*, *uxx*, *uxxx*,…,

(2)

Case 5. *h* = 0, and *μ* = 0,

** **  ln **  *c*1 

(10)

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**Step 4.** Substitute Eq. [(5)](#_bookmark3) into Eq. [(4)](#_bookmark2) and using Eq. [(1)](#_bookmark5), the

*C*  0, **  1  2

*ab**V*  1

*a* 2*bV*  *V*  1  2*b*

*a* 2*bV*  *V*  1  2*b* ,

left hand side is converted into a polynomial in exp** **. Equating each coefficient of this polynomial to zero, we obtain a set of algebraic equations for *an*, ⋯ *V*, **, ** .

**Step 5.** Eventually, solving the algebraic system of equa-





 a1 



*ab**V*  1

*a*



2 *ab**V*  1

, a0  *a* ,



tions obtained in Step 4 by the use of Maple or Mathematica, we obtain the values of the constants *an*, ⋯, *V*, ** and *μ*. Sub- stituting *an*, ⋯, *V* and the general solution of Eq. [(1)](#_bookmark5) into solution

where *h* and *μ* are arbitrary constants. Now substituting the values into Eq. [(14)](#_bookmark12), we obtain

of Eq. [(5)](#_bookmark3), we obtain some valuable traveling wave solutions of Eq. [(2)](#_bookmark6).

*u*  2**  1 

2*e*****, (16)

# Solution procedure

## *Generalized Zakharov–Kuznetsov–Benjamin–Bona–* Mahony equation

Let us consider the generalized Zakharov–Kuznetsov–Benjamin–

where **  *x*  *y*  *Vt*. Now substituting Eq. [(6)](#_bookmark4) to Eq. [(10)](#_bookmark7) into Eq.

[(16)](#_bookmark8) respectively, we get the following five traveling wave so- lutions of generalized Zakharov–Kuznetsov–Benjamin–Bona– Mahony equation.

Case 1. When *h*2 − 4*μ* > 0 and *μ* ≠ 0, we obtain the hyperbolic function traveling wave solution.

 2 2**

Bona–Mahony equation.

*u*1 ** 

2**  1  



**2  4** tanh

,

**2  4** **  *c*   **

ut  ux  au3   buxt  uyy   0, (11)

  2

1  

x x

where *η* = *x* + *y* − V*t* and where *c*1 is an arbitrary constant.

where *a* and *b* are some nonzero parameters. We utilize the

traveling wave variable

*u**x*, *t*  *u* **, **  *x*  *y*  *Vt*, we can

Case 2. When *h*2 − 4*μ* < 0 and *μ* ≠ 0, we obtain trigonometric so-

convert Eq. [(11)](#_bookmark9) into an ordinary differential equation.

*Vu*  *u*  3au2*u*  *bu*3*V*  *bu*  0, (12)

lution.

u   2  1  2 2 ,

2    

2  4

 2  4 tanh   c1   

where the prime denotes the derivative with respect to *η*. Now integrating Eq. [(12)](#_bookmark10) we have,

*Vu*  *u*  *bVu*  *au*3  *bu*  *C*  0, (13)

Balancing the *u*’’ and *u*2 by using homogenous principal, we have

  2  

where *η* = *x* + *y* − V*t* and where *c*1 is an arbitrary constant.

Case 3. When *μ* = 0 and *h* ≠ 0, we obtain exponential solu- tion.

3*M*  *M*  2,

*u*3 ** 

2  1  2** ,

exp**  *c*1 **  1

*M*  1.

Then the trial solution of Eq. [(12)](#_bookmark10) can be expressed as follows,

where *η* = *x* + *y* − V*t* and where *c*1 is an arbitrary constant.

Case 4. When **2  4**  0, **  0 and *μ* ≠ 0, we obtain rational function solution.

*u* **  **1 exp** **  **0,

(14)

*u*4 ** 

2  1  ,

where *α*1 ≠ 0 and *α*0 is a constant to determined, while *h*, *μ* are arbitrary constants.



2 **  *c*1 **2

2 **  *c*1 **  2

Substituting *u*, *u*, *u*, *u*2 into Eq. [(13)](#_bookmark11) and then equating the coefficients of *exp*** ** to zero, we get

*a*0  *ba*1**  *C*  *bVa*1** *aa*3  *Va*0  0,

0

where *η* = *x* + *y* − V*t* and where *c*1 is an arbitrary constant.

Case 5. When *h* = 0, and *μ* = 0, we obtain rational function so- lution.

*a*1  *bVa*1**2  2*ba*1**  2*bVa*1**  3*aa a*2  *ba*1**2 *Va*  0,



2

1 0 1

1

3*aa a*2  3*ba *  3*bVa *  0,

(15)

*u*5 ** 

2  1  **  *c*  ,

0 1 1 1

*aa*3  2*ba*  2*bVa*  0

1 1 1

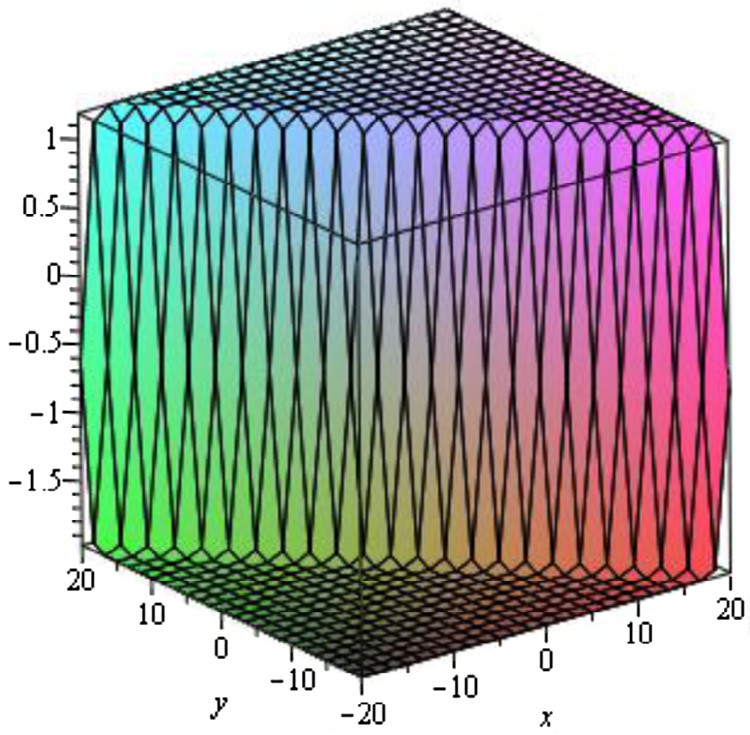
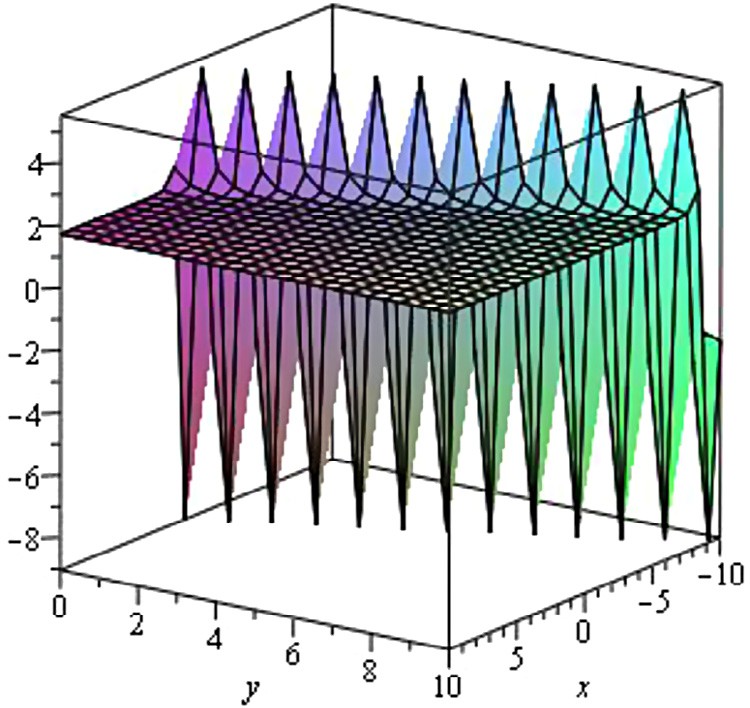
Solving the set of algebraic equations, we obtain the fol- lowing solution.

where *η* = *x* + *y* − V*t* and where *c*1 is an arbitrary constant.

### Graphical representation of the solutions:

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### Fig. 1 – Kink wave solution *u*1 ** when

***a*2**  **1, *a*0**  **2, *y***  **0,** **  **3,** **  **2, *c*1**  **1 .**

### Fig. 3 – Singular Kink wave solution *u*3 ** when

***a*2**  **1, *a*0**  **2, *y***  **0,** **  **1, *c*1**  **1 .**

The graphical illustrations of the solutions are given below in the figures with the aid of Maple ([Figs. 1–5](#_bookmark13)).

## *Simplified Modified form of Camassa–Holm equation*

Let us consider Simplified Modified form of Camassa–Holm equation.

*ut*  2*ux*  *uxxt*  *u*2*ux*  0, (17)

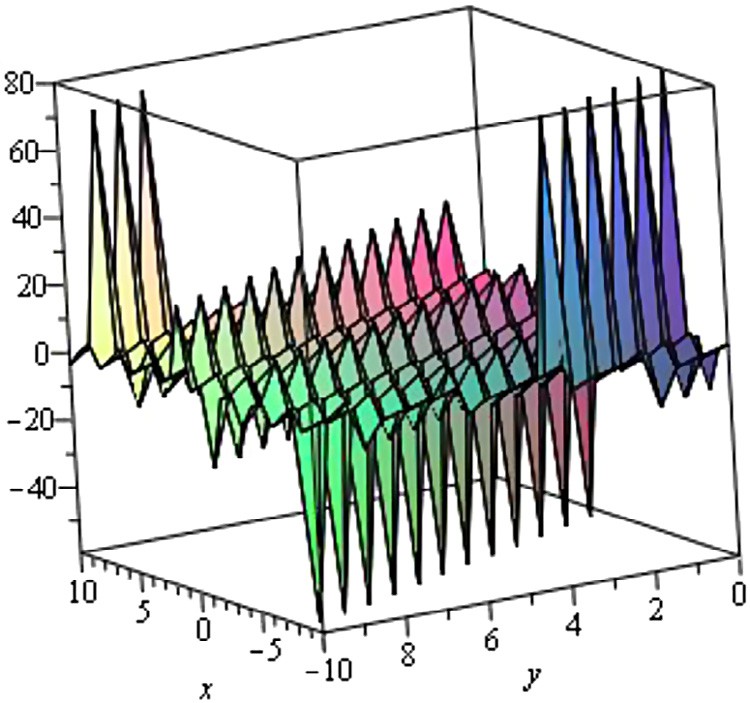
where *β* and *δ* are some nonzero parameters.

We utilize the traveling wave variable *u**x*, *t*  *u* **, *η* = *x* − *Vt*, we can convert Eq. [(17)](#_bookmark14) into an ordinary differential equa-

tion.

*Vu*  2*u*  *Vu*  *u*2*u*  0, (18) where the prime denotes the derivative with respect to *η*.

Now integrating Eq. [(18)](#_bookmark15) we have,



### Fig. 2 – Singular Kink wave solution *u*2 ** when

***a*2**  **10, *a*0**  **8, *y***  **0,** **  **7,** **  **5, *c*1**  **10 .**

*Vu*  2*u*  *u**V*  1 *u*3  *C*  0, (19)

3

Balancing the *u*’ and *u*2 by using homogenous principal, we have

3*M*  *M*  2,

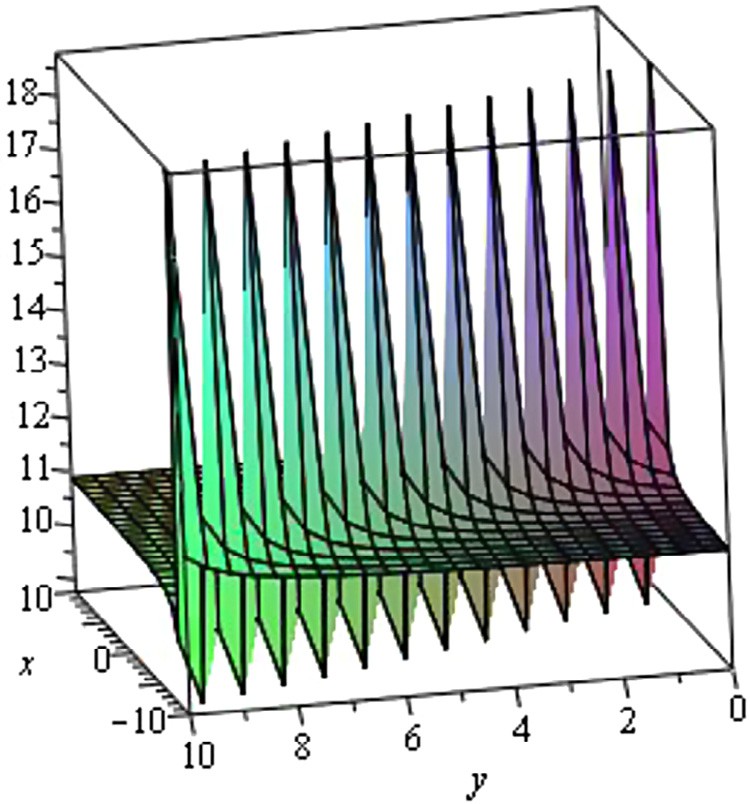
*M*  1.

Then the trial solution of Eq. [(18)](#_bookmark15) can be expressed as follows,

*u* **  **1 exp** **  **0, (20)

where *α*1 ≠ 0, *α*0 is a constant to determined, while *h*, *μ* are ar- bitrary constants.

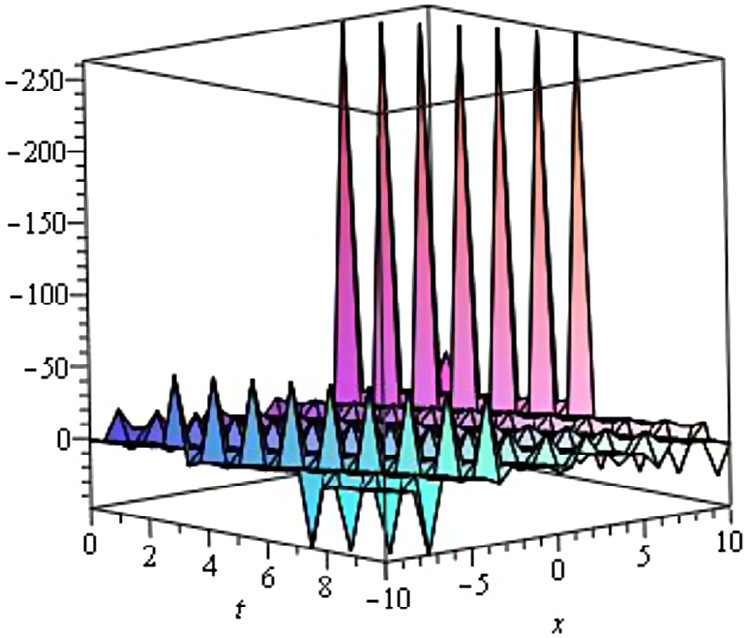
Substituting *u*, *u*, *u*, *u*2 into Eq. [(19)](#_bookmark16) and then equating the coefficients of *exp*** ** to zero, we get



### Fig. 4 – Singular Kink wave solution *u*4 ** when

***a*2**  **3, *a*0**  **2, *y***  **0,** **  **5,** **  **4, *c*1**  **2 .**

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### Fig. 5 – Singular Kink wave solution *u*5 ** when

***a*2**  **0.5, *a*0**  **0.2, *y***  **0,** **  **0.1, *c*1**  **0.1 .**

### Fig. 6 – Kink wave solution *u*1 ** when

***C***  **1, *a*0**  **0.1, *y***  **0,** **  **0.2,** **  **0.5, *c*1**  **0.3 .**

where *η* = *x* − V*t* and where *c*1 is an arbitrary constant.

1 *a*3  2*a *  *C*  *Va *  *Va*

 0,

Case 3. When *μ* = 0 and *h* ≠ 0, we obtain exponential solu- tion.

3 0 0 1 0

2*Va*1**  *a*2*a*  2*a *  *C*  *Va *2  *Va*

 0,

6**

0 1 1 1 1

*a a*2  3*Va *  0,

0 1

1

(21)

*u*3 **  *a*0

 *exp***  *c* **  1,

1 *a*3  2*Va*  0

1

3 1 1

where *η* =

*x* − V*t*

and where *c*1

is an arbitrary constant.

Solving the set of algebraic equations, we obtain the fol- lowing solution.

Case 4. When **2  4**  0, **  0, and *μ* ≠ 0, we obtain rational function solution.

** 

6*V*

 6*V*



1 *a*0

, ** 

1 3*V*  6**  *a*2

,

0

a1  

, C  0,



6 **  *c*1 **2

2 **  *c*1 **  2

 3 *V* 6 *V * *u*4 **  *a*0  ,

where *h* and *μ* are arbitrary constants.

Now substituting the values into Eq. [(20)](#_bookmark17), we obtain,

*u*  



*a*0  6 –*Ve*****

**

(22)

where *η* = *x* − V*t* and where *c*1 is an arbitrary constant.

Case 5. when *h* = 0, and *μ* = 0, we obtain rational function so- lution.

Where *η* = *x* − *Vt*.

Now substituting Eq. [(6)](#_bookmark4) to Eq. [(10)](#_bookmark7) into Eq. [(22)](#_bookmark19) respec- tively, we get the following five traveling wave solutions of the Simplified Modified form of Camassa–Holm equation.

Case 1. When *h*2 − 4*μ* > 0 and *μ* ≠ 0, we obtain the hyperbolic function traveling wave solution.

 2 6**

*u*5 **  *a*0   6 ,

where *η* = *x* − V*t* and where *c*1 is an arbitrary constant.



**  *c*1 

### Graphical representation of the solutions:

The graphical illustrations of the solutions are given below in the figures with the aid of Maple ([Figs. 6–10](#_bookmark18)).

**Conclusions:** The exp** **-expansion method is very im-

*u*1 **  *a*0  

 

**2  4** tanh

  ,

**  *c*1   **

**2  4**

portant in finding the exact solutions of nonlinear evolution equations. In this article, we have successfully formulated the

  2  



where *η* = *x* − V*t* and where *c*1 is an arbitrary constant.

Case 2. When *h*2 − 4*μ* < 0 and *μ* ≠ 0, we obtain trigonometric so- lution.

*u*2 **  *a*0  2 6** ,

  **2  4** tan **2  4** **  *c*   **

exact and traveling wave solutions to the generalized Zakharov– Kuznetsov–Benjamin–Bona–Mahony equation and Simplified Modified form of Camassa–Holm equation. The wave solu- tions are obtained through the hyperbolic, trigonometric, exponential and rational functions. The calculation proce- dure is simple, direct and constructive. This study shows that the method is quite efficient and much effective for finding exact solutions of nonlinear evolution equations (NLEEs). Also, we observe that the method is straightforward and can be

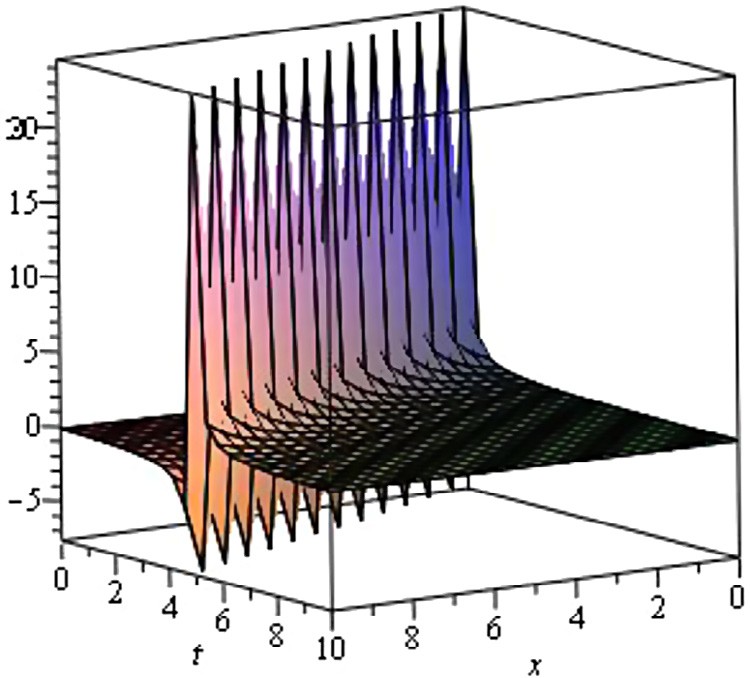
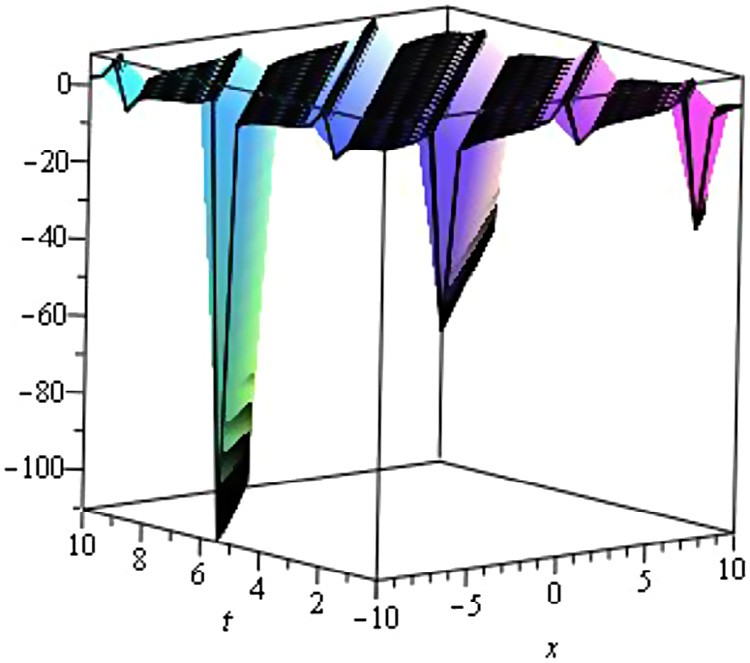
  2

1  

applied to many other nonlinear evolution equations.

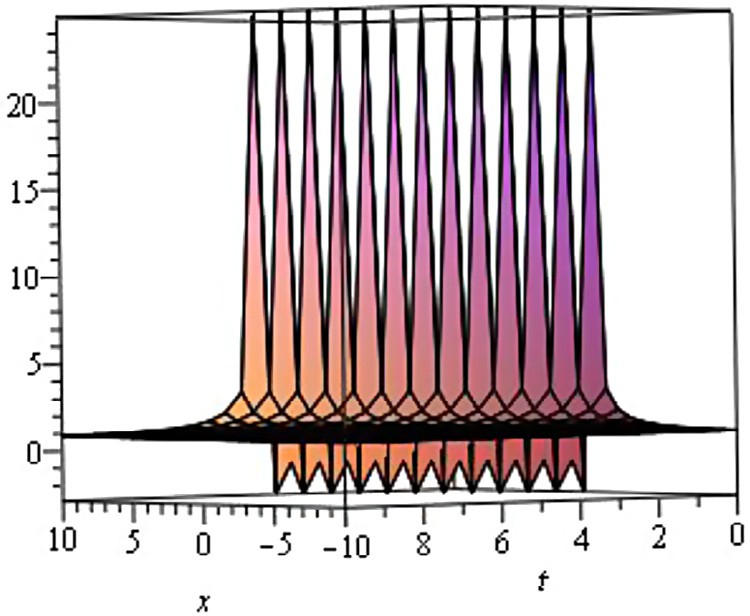
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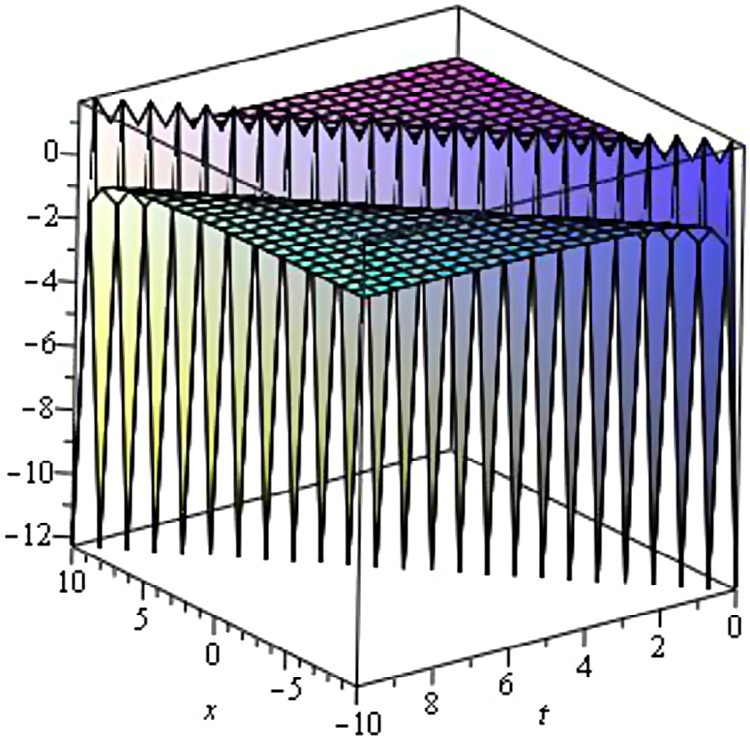
### Fig. 7 – Periodic solution *u*2 ** when

***a*0**  **1, *C***  **1, *y***  **0,** **  **0.1,** **  **0.2, *c*1**  **0.1 .**



### Fig. 8 – Singular Kink wave solution *u*3 ** when

**  **0.1, *C***  **1, *y***  **0,** **  **0.3, *a*0**  **0.1, *c*1**  **0.1 .**



### Fig. 9 – Singular Kink wave solution *u*4 ** when

***a*0**  **0.1, *C***  **1, *y***  **0,** **  **0.1,** **  **1, *c*1**  **0.1 .**

### Fig. 10 – Singular Kink wave solution *u*5 ** when

***C***  **1, *y***  **0,** **  **0.1, *a*0**  **0.1, *c*1**  **0.1 .**

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