1. Multiple choice.

   (a) [3 pts] If $\operatorname{div} \mathbf{F}(x, y, z) = 0$ for all (x, y, z) then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for any closed curve C.

   (i) True.
   (ii) False.
   (iii) Indeterminable.

   (b) [3 pts] Which of the following apply to the vector field $\mathbf{F} = (x^2 - y^2)\mathbf{i} - 2xy\mathbf{j} + z^2\mathbf{k}$?

   (i) Its divergence vanishes.
   (ii) It is conservative and its curl vanishes.
   (iii) It is conservative, but its curl does not vanish.
   (iv) Its curl vanishes, but it is not conservative.

   (c) [3 pts] There is a vector field $\mathbf{F}$ on $\mathbb{R}^3$ such that $\operatorname{curl} \mathbf{F} = \langle x \sin(y), \cos(y), 6z - xy \rangle$.

   (i) True.
   (ii) False.
   (iii) Indeterminable.

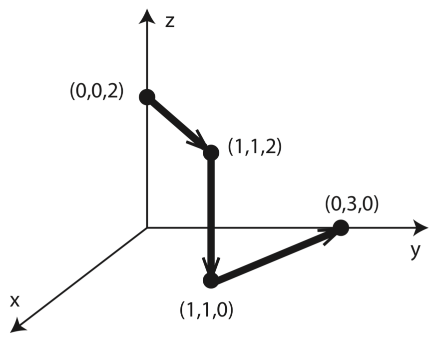
   (d) [3 pts] If $\mathbf{F}(x, y, z) = \langle \sin(z), \cos(z), 0 \rangle$, then $\operatorname{curl}(\operatorname{curl}(\operatorname{curl} \mathbf{F})) = \mathbf{F}$.

   (a) True.
   (b) False.
   (c) Indeterminable.

2. [5 pts] Let

$$\mathbf{F}(x, y, z) = \langle 3x^2yz - 3y, x^3z - 3x, x^3y + 2z \rangle.$$

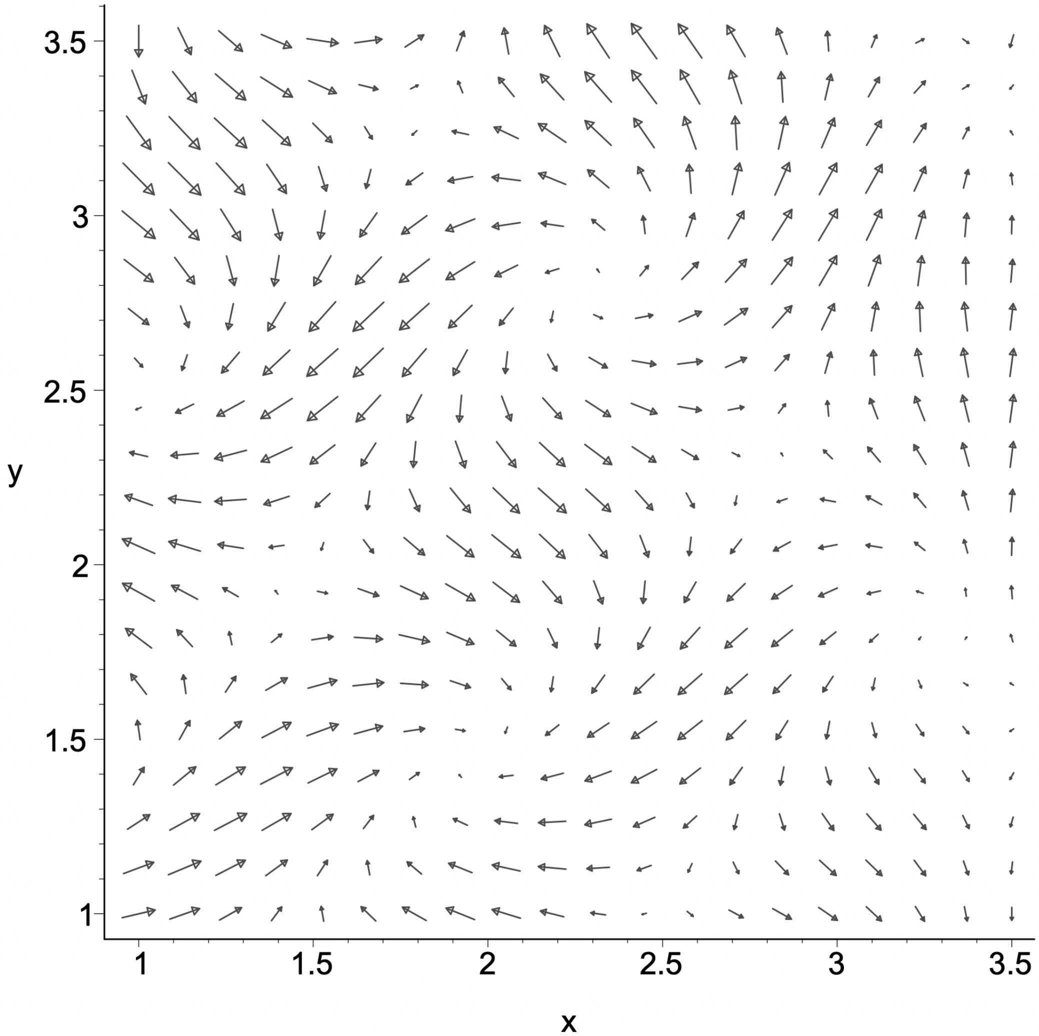
Evaluate the work done by the vector field $\mathbf{F}$ in moving a particle along the following curve from the point $(0, 0, 2)$ to the point $(0, 3, 0)$.



3. [5 pts] Consider the vector field $\mathbf{F}(x, y) = \langle -y^3 + \sin(\cos x), x^3 + y^{2^{y^2+2022}} \rangle$. Calculate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where C is the unit circle centered at the origin oriented counter-clockwise.

Points earned: ______
out of a possible 10 points

4. The plot of a vector field $\mathbf{F}(x, y)$ is drawn below.



(a) [2 pts] Mark a point A on the plot at which $\mathsf{curl}\,\mathbf{F} > 0$.

(b) [2 pts] Mark a point B on the plot at which $\mathsf{div}\,\mathbf{F} < 0$.

(c) [2 pts] Sketch a closed curve C such that the work done by $\mathbf{F}$ along C is positive.

(d) [3 pts] Mark two points on the plot S and T and two curves C_1 and C_2 from S to T such that the work done by $\mathbf{F}$ in moving an object along those paths is positive for C_1 and negative for C_2.

(e) [2 pts] Can the vector field $\mathbf{F}$ be a gradient field? In the space below explain why or why not.