

1. Determine if the following statements are true or false.

(a) [2 pts] If  $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ , and  $\mathbf{a}$  is not the zero-vector (i.e.  $\mathbf{a} \neq \mathbf{0}$ ), then  $\mathbf{b} = \mathbf{c}$ .

(b) [2 pts] If the vector  $\mathbf{a}$  satisfies the equation  $\mathbf{a} \cdot \mathbf{b} = 0$  for all vectors  $\mathbf{b}$ , then  $\mathbf{a}$  is the zero-vector.

2. [3 pts] If  $\mathbf{a} = \langle 2, -1, 2 \rangle$  and  $\mathbf{b} = \langle 1, -1, 2 \rangle$ , find a non-zero vector  $\mathbf{c}$  such that  $\mathbf{a} \cdot \mathbf{c} = \mathbf{b} \cdot \mathbf{c} = 0$ .

3. [3 pts] Determine the projection vector  $\text{proj}_{\mathbf{a}}(\mathbf{b})$  of  $\mathbf{b}$  onto  $\mathbf{a}$  where  $\mathbf{a} = \langle 1, 2, 3 \rangle$  and  $\mathbf{b} = \langle 1, 2, 2 \rangle$ .

4. [3 pts] If the scalar projection of  $\mathbf{b}$  onto  $\mathbf{a}$  is  $\|\text{proj}_{\mathbf{a}}(\mathbf{b})\| = 1$ , determine the value of  $\|\text{proj}_{2\mathbf{a}}(3\mathbf{b})\|$ .
5. Consider the points  $P(3, 1, 1)$ ,  $Q(4, 1, 2)$ , and  $R(4, 4, 1)$  in  $\mathbb{R}^3$ .
- (a) [3 pts] Find an equation for the plane containing the points  $P$ ,  $Q$ , and  $R$ .
- (b) [2 pts] Find the area of the triangle with vertices  $P$ ,  $Q$ , and  $R$ .
6. [4 pts] Find an equation of the plane which passes through the points  $(2, 2, 1)$  and  $(-1, 1, -1)$  and is perpendicular to the plane  $2x - 3y + z = 3$ .
7. [4 pts] Find a set of parametric equations for the line of intersection of the planes:

$$\begin{cases} 6x - 3y + z = 5 \\ -x + y + 5z = 5 \end{cases}$$