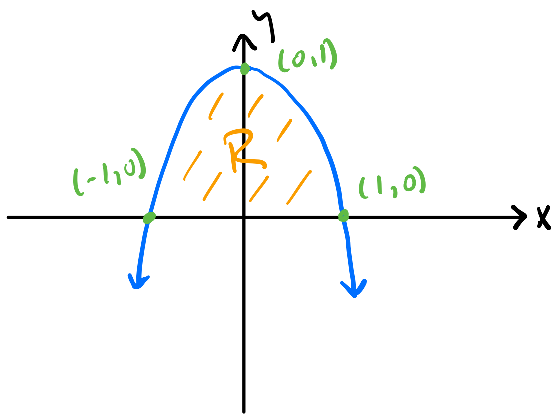
1. [6 pts] Consider the double integral

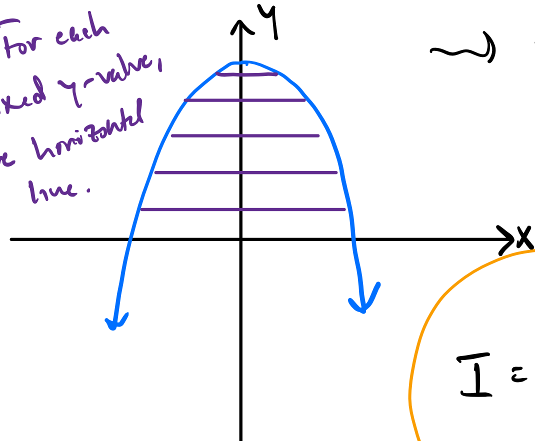
$$\mathbf{I} = \int_{-1}^{1} \int_{0}^{1-x^2} \sqrt{1-y}\, dy\, dx$$

(a) [3 pts] Sketch the region R of integration.



(b) [3 pts] Switch the order of integration and rewrite the integral $\mathbf{I}$ with the y-variable being outermost and x-variable being innermost.

For each fixed y-value, have horizontal line.



$\rightsquigarrow -1 \le x \le 1$

$0 \le y \le 1-x^2 \rightsquigarrow 0 \le y \le 1$

and $y = 1-x^2 \Rightarrow x^2 = 1-y$

$\Rightarrow -\sqrt{1-y} \le x \le \sqrt{1-y}$

$$I = \int_0^1 \int_{-\sqrt{1-y}}^{\sqrt{1-y}} \sqrt{1-y}\, dx\, dy$$

(c) [3 pts] Evaluate the integral $\mathbf{I}$.

Use answer from (b)

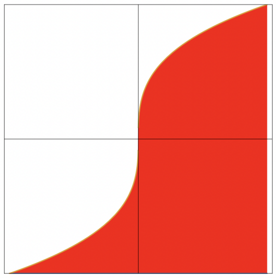
$$I = \int_0^1 x\sqrt{1-y}\,\Big|_{x=-\sqrt{1-y}}^{x=\sqrt{1-y}} dy = \int_0^1 (1-y)+(1-y)\, dy = \int_0^1 2-2y\, dy$$

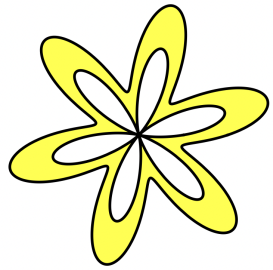
$$= 2y - y^2 \Big|_0^1$$

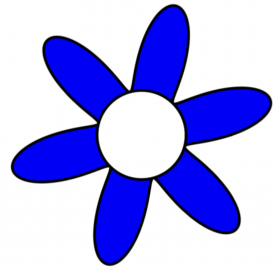
$$= 2(1)-(1)^2$$

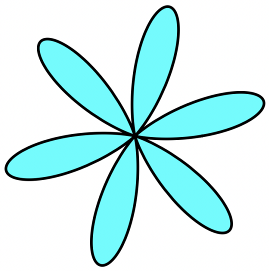
$$= 1$$

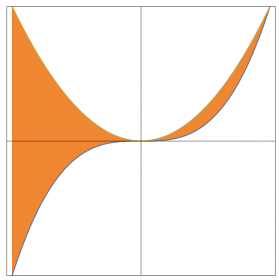
2. [12 pts] Match each picture below with the double integral that computes the area of the region.

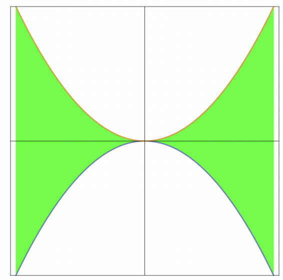
A 

B 

C 

D 

E 

F 

Enter A – F	Integral
D	$\int_0^{2\pi}\int_0^{1+\sin(6\theta)} r dr d\theta$
A	$\int_{-1}^{1}\int_{y^3}^{1} dx dy$
E	$\int_{-1}^{1}\int_{x^3}^{x^2} dy dx$
F	$\int_{-1}^{1}\int_{-x^2}^{x^2} dy dx$
B	$\int_0^{2\pi}\int_{1+\sin(6\theta)}^{2+\sin(6\theta)} r dr d\theta$
C	$\int_0^{2\pi}\int_1^{2+\sin(6\theta)} r dr d\theta$