1. Multiple choice. Clearly mark your answers. No justification is required but may result in partial credit if provided.

   (a) [3 pts] If $\mathbf{F}$ and $\mathbf{G}$ are two vector fields which have the same divergence, then $\mathbf{F} - \mathbf{G}$ is a constant vector field.

   (a) True.
   (b) False.
   (c) Indeterminable.

   (b) [3 pts] Every vector field $\mathbf{F}(x, y, z)$ which satisfies the equation $\mathsf{curl}\,\mathbf{F}(x, y, z) = \vec{0}$ on all of $\mathbb{R}^3$ can be written as $\mathbf{F} = \nabla f$ for some scalar function f.

   (a) True.
   (b) False.
   (c) Indeterminable.

   (c) [3 pts] If $\mathsf{div}\,\mathbf{F}(x, y, z) = 0$ for all (x, y, z) then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for any closed curve C.

   (a) True.
   (b) False.
   (c) Indeterminable.

   (d) [3 pts] There is a non-constant function $f(x, y, z)$ such that $\nabla f = \mathsf{curl}(\nabla f)$ everywhere.

   (a) True.
   (b) False.
   (c) Indeterminable.

   (e) [3 pts] The vector field $\mathbf{F}(x, y, z) = \langle x^5, x^6, x^7 \rangle$ is the curl of another vector field defined on all of $\mathbb{R}^3$.

   (a) True.
   (b) False.
   (c) Indeterminable.

   (f) [3 pts] If $\mathbf{F}(x, y, z) = \langle \sin(z), \cos(z), 0 \rangle$, then $\mathsf{curl}(\mathsf{curl}(\mathsf{curl}\,\mathbf{F})) = \mathbf{F}$.

   (a) True.
   (b) False.
   (c) Indeterminable.

2. Consider the double integral
$$\mathbf{I} = \int_1^2 \int_0^{\sqrt{2-y}} \frac{\sin(\pi x)}{1-x^2} dx dy$$

   (a) [4 pts] Sketch the region of integration for $\mathbf{I}$.

   (b) [4 pts] Express the integral $\mathbf{I}$ as an iterated integral with the reversed order of integration.

   (c) [4 pts] Determine the value of $\mathbf{I}$.

Points earned: ______
out of a possible 12 points

3. [8 pts] Evaluate the triple integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{x^2+y^2+z^2} dzdydx$$

4. Let E be the solid which lies inside both the sphere $x^2+y^2+z^2=4$ and the cylinder $(x-1)^2+y^2=1$.

   (a) [5 pts] Sketch a picture of the solid E.

   (b) [5 pts] Express the volume of E as a triple integral. Do not evaluate your expression.

5. Consider the integral

$$\mathbf{I} = \iint_R \frac{dxdy}{x+y},$$

where R is the region bounded by $x = 0$, $y = 0$, $x + y = 1$, and $x + y = 4$.

(a) [6 pts] Define $\mathbf{T}$ to be the transformation

$$\mathbf{T} = \begin{cases} x = u - uv \\ y = uv \end{cases}$$

Sketch the region S in the uv-plane which maps onto the region R under the transformation $\mathbf{T}$.

(b) [4 pts] Compute the Jacobian of the transformation $\mathbf{T}$.

(c) [4 pts] Set up, but do not evaluate an expression for $\mathbf{I}$ as an iterated integral in terms of the variables u and v.

6. Let $\mathbf{F}(x, y) = (xy^2 + 2y)\mathbf{i} + (x^2y + 2x + 2)\mathbf{j}$ be a vector field.

   (a) [5 pts] Carefully explain why $\mathbf{F}$ is a conservative vector field.

   (b) [5 pts] Find a potential function f such that $\nabla f = \mathbf{F}$.

   (c) [4 pts] Evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the path parametrized by $\mathbf{r}(t) = \langle e^t, 1+t\rangle$ for $0 \leq t \leq 1$.

   (d) [4 pts] Evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is a closed curve $\mathbf{r}(t) = \langle 2\sin(t), 2\cos(t)\rangle$ for $0 \leq t \leq 2\pi$.

7. [10 pts] Evaluate the line integral

$$\int_C (e^x + y^2)dx + (e^y + x^2)dy$$

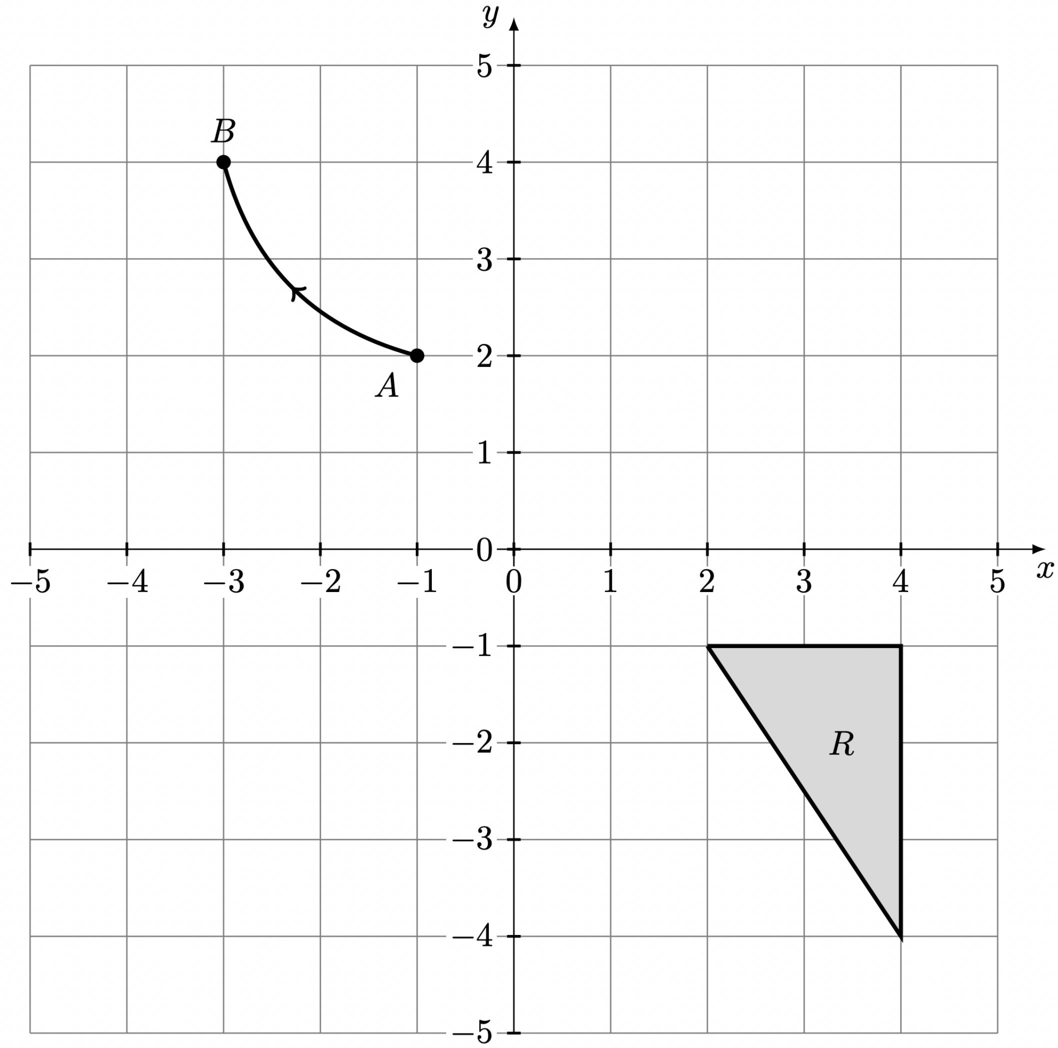
where C is the positively oriented boundary of the region in the first quadrant bounded by $y = x^2$ and $y = 4$.

Points earned: ______
out of a possible 10 points

8. (a) [8 pts] On the coordinate axes below, sketch a smooth vector field $\mathbf{F}(x, y)$ which satisfies the following properties (Note: answers may vary):

- At the point $(3, 3)$, the divergence of $\mathbf{F}$ is positive.
- Let C be the path from the point A to the point B drawn below. Then the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is positive.
- If $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y)\rangle$, and if R is the triangle region drawn in the fourth quadrant below, then the value of the integral $\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$ is non-zero.
- At the point $(-3, -3)$, the curl of $\mathbf{F}$ is non-zero.
- Along the y-axis, the vector field vanishes, i.e. $\mathbf{F}(0, y) = \langle 0, 0\rangle$ for all y.

Hint: To sketch a vector field, you need only draw several representative vectors in the plane. However, the vector field should be *smooth* in that the vectors vary smoothly in the domain.



(b) [2 pts] Comment on whether the vector field $\mathbf{F}$ is conservative or not.

Points earned: ______
out of a possible 10 points