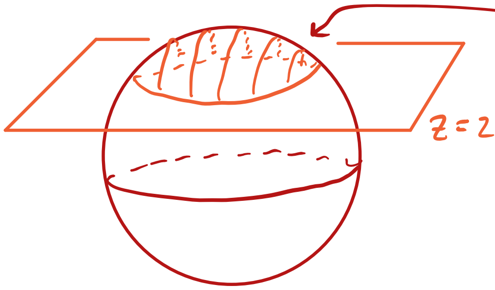
1. [10 pts] Integrate the function

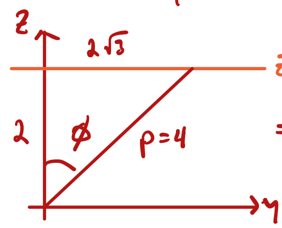
$$f(x,y,z) = z(x^2+y^2+z^2)^{-3/2}$$

over the part of the ball $x^2+y^2+z^2 \le 16$ defined by $z \ge 2$.



Want to integrate over this part of the ball

$\rightsquigarrow$ Use spherical coordinates



$\Rightarrow \cos\phi = 2/4 = 1/2$

$\Rightarrow \phi = \pi/3$

Compute:

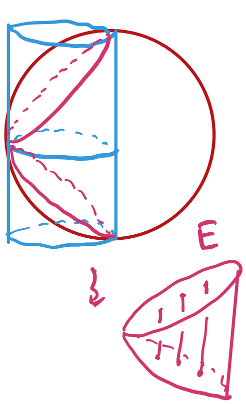
$$\int_0^{2\pi}\int_0^{\pi/3}\int_0^{4} \overbrace{\rho\cos\phi}^{z} \cdot \overbrace{(\rho^2)}^{x^2+y^2+z^2}{}^{-3/2} \cdot \overbrace{\rho^2\sin\phi}^{\text{Magnification factor}}\, d\rho\, d\phi\, d\theta$$

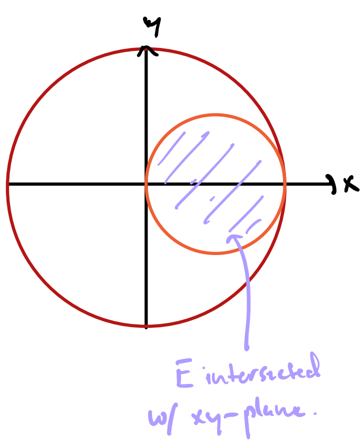
$$= \int_0^{2\pi}\int_0^{\pi/3}\int_0^{4} \frac{\rho^3}{\rho^3}\cos\phi\sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/3} 4\cdot\cos\phi\sin\phi\, d\phi\, d\theta$$

$$= 8\pi\int_0^{\pi/3}\cos\phi\sin\phi\, d\phi = 8\pi\, \frac{\sin^2\phi}{2}\Big|_0^{\pi/3} = 4\pi\cdot\left(\frac{\sqrt{3}}{2}\right)^2 = 3\pi.$$

2. [8 pts] Let E be the solid which lies inside both the sphere $x^2+y^2+z^2=4$ and the cylinder $(x-1)^2+y^2=1$. Express the volume of E as a triple integral. Do **not** evaluate your expression.



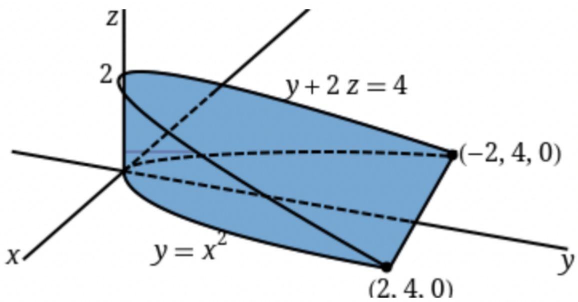


$$\text{Vol}\,E = \int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta}\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$$

3. Let E be the solid region bounded by the three surfaces in $\mathbb{R}^3$:

$$y = x^2, \quad z = 0, \quad y + 2z = 4$$

The solid E is pictured below:



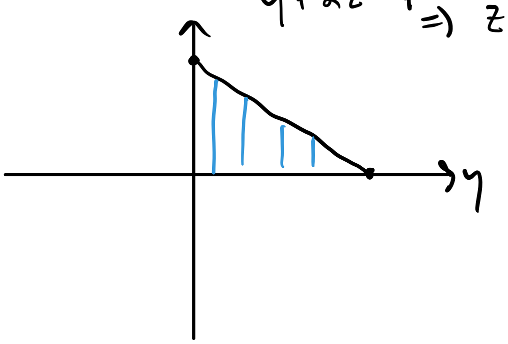
By filling in the empty boxes with the appropriate limit of integration, express the volume of E as an iterated integral in two different ways.

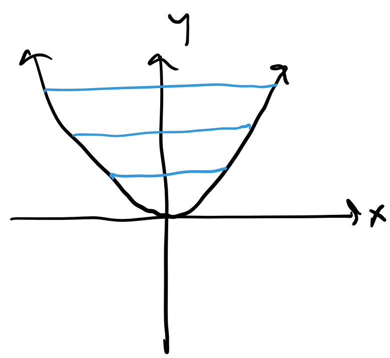
(a) [5 pts] $\displaystyle\int_0^{\boxed{4}} \int_0^{\boxed{2-y/2}} \int_{-\sqrt{y}}^{\boxed{\sqrt{y}}} dx\,dz\,dy$

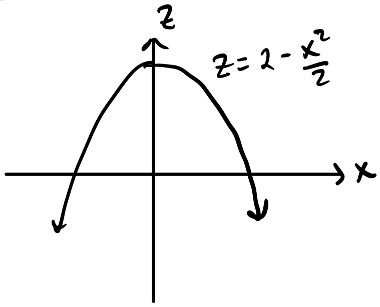
(b) [5 pts] $\displaystyle\int_{\boxed{-2}}^{\boxed{2}} \int_{\boxed{0}}^{\boxed{2-x^2/2}} \int_{\boxed{x^2}}^{\boxed{4-2z}} dy\,dz\,dx$

z

$y+2z=4 \Rightarrow z = \frac{4-y}{2}$







$y=x^2 \Rightarrow z = \frac{4-x^2}{2} = 2 - x^2/2$

Points earned: ______
out of a possible 10 points