1. Let S be the surface parametrized by

$u=3$
$v=\pi/2$
$\langle 0, 3, \pi^2/8 \rangle$

$$\mathbf{r}(u,v) = \left\langle u\cos(v), u\sin(v), \frac{v^2}{2} \right\rangle \quad \text{where } u^2+v^2 \le 9.$$

(a) [5 pts] Find the surface area of S.

$\vec{r}_u = \langle \cos v, \sin v, 0 \rangle$ +2

$\vec{r}_v = \langle -u\sin v, u\cos v, v \rangle$

$$\vec{r}_u \times \vec{r}_v = \mathrm{Det}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & v \end{vmatrix}$$

$$= \begin{bmatrix} v\sin v \\ -v\cos v \\ u\cos^2 v + u\sin^2 v \end{bmatrix} = \begin{bmatrix} v\sin v \\ -v\cos v \\ u \end{bmatrix} \quad +1$$

$$\iint_{u^2+v^2\le 9} \sqrt{v^2+u^2}\, dA \qquad \begin{matrix} v = r\cos\theta \\ u = r\sin\theta \end{matrix} \rightsquigarrow \int_0^{2\pi}\int_0^3 r\cdot r\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{r^3}{3}\Big|_0^3 \, d\theta$$

$$= 18\pi \quad +2$$

(b) [~~4~~ 3 pts] Verify that the point $P = (2, 0, 0)$ $(0, 3, \pi^2/8)$ is a point on the surface S. Find an equation for the tangent plane to S at the point P.

$u=3$
$v=\pi/2$

$$\rightsquigarrow \vec{N}(3, \pi/2) = \begin{bmatrix} \pi/2 \\ 0 \\ 3 \end{bmatrix}$$

Equation of tangent plane:

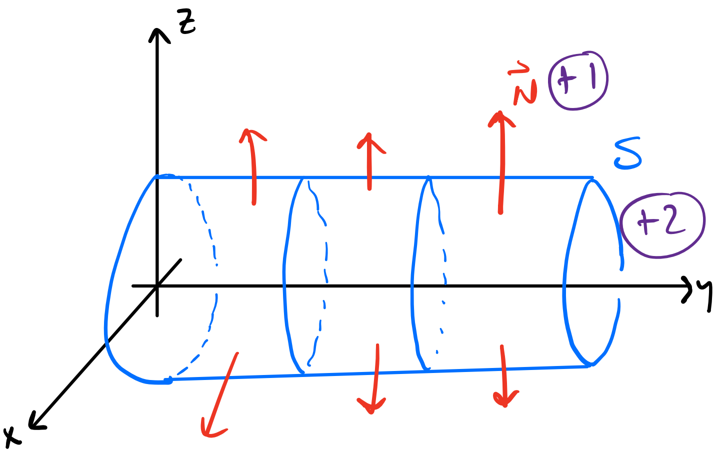
$$\pi/2(x-0) + 0(y-3) + 3(z - \pi^2/8) = 0$$

2. Suppose S is the part of the cylinder $x^2+z^2 = 16$ described in cartesian coordinates by

$$S = \{(x,y,z) \mid 0 \le y \le 4 \text{ and } x^2+z^2 = 16\},$$

and which is oriented via the outward pointing unit normal vector (i.e. the orientation vector is pointing away from the y-axis at every point on S).

(a) [3 pts] Make a sketch of S. Be sure to indicate the orientation of S.



4 ~~(b)~~ [5 pts] Find a parametrization of S. Make sure you clearly indicate the domain of the given parametrization.

Parametrize using cylindrical coordinates:

$$+3 \quad \vec{r}(\theta, y) = \langle 4\sin\theta, y, 4\cos\theta \rangle, \quad \left.\begin{array}{l} 0 \le \theta \le 2\pi \\ 0 \le y \le 4 \end{array}\right) +2$$

$$\rightsquigarrow \vec{r}_\theta = \langle 4\cos\theta, 0, -4\sin\theta \rangle$$

$$\vec{r}_y = \langle 0, 1, 0 \rangle$$

And

$$\vec{r}_\theta \times \vec{r}_y = \text{Det}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4\cos\theta & 0 & -4\sin\theta \\ 0 & 1 & 0 \end{vmatrix} = \begin{bmatrix} 4\sin\theta \\ 0 \\ 4\cos\theta \end{bmatrix}$$

So orientation from parametrization matches that of the surface.

+2

✓ fill

(c) [5 pts] For the vector field $\mathbf{F}(x, y, z) = \langle xe^y, e^{xyz}, ze^y \rangle$, compute the flux $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

Compute:

$$\int_0^{2\pi}\int_0^4 \begin{bmatrix} 4\sin\theta\, e^y \\ e^{16\cos\theta\sin\theta y} \\ 4\cos\theta\, e^y \end{bmatrix} \cdot \begin{bmatrix} 4\sin\theta \\ 0 \\ 4\cos\theta \end{bmatrix} dy\, d\theta$$

+1

$$= \int_0^{2\pi}\int_0^4 \left(16\sin^2\theta\, e^y + 16\cos^2\theta\, e^y\right) dy\, d\theta$$

$$= 16\int_0^{2\pi} d\theta \int_0^4 e^y\, dy = 16(2\pi)(e^4 - 1)$$

$$= 32\pi(e^4 - 1) \,//$$

+2