1. [3 pts] Determine the arc length along the curve

$$\mathbf{r}(t) = \langle \cos(t) + t\sin(t), \sin(t) - t\cos(t), t^2 \rangle$$

from $t = 0$ to $t = \pi/2$.

2. [3 pts] Interestingly, the notion of arc length can be defined in any dimension. A curve in four-dimensional space $\mathbb{R}^4$ is parametrized as

$$\mathbf{r}(t) = \langle x(t), y(t), z(t), w(t) \rangle, \quad a \leq t \leq b.$$

Find the arc length of $\mathbf{r}(t) = \langle t, \ln(t), 1/t, \ln(t) \rangle$ and $1 \leq t \leq 4$.

3. Suppose that the trajectory of a particle in $\mathbb{R}^3$ is described by the vector-valued function $\mathbf{r}(t)$, and let $\mathbf{v}(t) = \mathbf{r}'(t)$ and $\mathbf{a}(t) = \mathbf{r}''(t)$ be its velocity and acceleration vectors, respectively. For each of the following statements, either give a proof or exhibit a counter-example.

   (a) [2 pts] Let $\mathcal{C}$ be the space-curve in $\mathbb{R}^3$ which is parametrized by $\mathbf{r}(t)$, $-\infty < t < \infty$. If the velocity vector $\mathbf{v}(t)$ is constant, then the curve $\mathcal{C}$ lies entirely in a single plane.

(b) [2 pts] Define the *speed* of the particle at time t to be the length of its velocity vector $s(t) = |\mathbf{v}(t)|$ at time t. If the speed is a constant function, then the curve lies entirely in a plane.

(c) [2 pts] If the acceleration vector $\mathbf{a}(t)$ is constant, then the curve $\mathcal{C}$ lies entirely in a single plane.

(d) [2 pts] If the velocity vector $\mathbf{v}(t)$ is *orthogonal* to the acceleration vector $\mathbf{a}(t)$ for all time t, then the curve $\mathcal{C}$ lies entirely in a single plane.

(e) [2 pts] Prove that $\mathbf{r}(t) \times \mathbf{a}(t) = \mathbf{0}$ implies $\mathbf{r}(t) \times \mathbf{v}(t) = \mathbf{c}$, where $\mathbf{c}$ is a constant vector.

(f) [2 pts] If $\mathbf{r}(t) \times \mathbf{v}(t) = \mathbf{c}$ for all time t, prove that the motion takes place in a plane (i.e. that the space curve parametrized by $\mathbf{r}(t)$ lies entirely in a plane). Consider both $\mathbf{c} = \mathbf{0}$ and $\mathbf{c} \neq \mathbf{0}$.

 Points earned: ______
out of a possible 10 points

4. [4 pts] Prove that the space curve

$$\mathbf{r}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 2t - \cos t + 3 \\ \sin^2 t + 4t \\ \frac{1}{2}(-\cos^2 t + 2\cos t + 1) \end{bmatrix}$$

lies entirely in a single plane. Find an equation for the plane.

5. [4 pts] Find a vector-valued function, $\mathbf{r}(t)$, that represents the curve of intersection between the cylinder $\{(x, y, z) \mid x^2 + y^2 = 9\}$ and the surface $\{(x, y, z) \mid z = xy\}$.

Points earned: ______
out of a possible 4 points