1. Consider the integral
$$\mathbf{I} = \iint_R \frac{dxdy}{x+y},$$
where R is the region bounded by $x = 0$, $y = 0$, $x + y = 1$, and $x + y = 4$.

   (a) [6 pts] Define $\mathbf{T}$ to be the transformation
$$\mathbf{T} = \begin{cases} x = u - uv \\ y = uv \end{cases}$$
   Sketch the region S in the uv-plane which maps onto the region R under the transformation $\mathbf{T}$.

   (b) [3 pts] Compute the Jacobian of the transformation $\mathbf{T}$.

   (c) [4 pts] Set up, but do not evaluate an expression for $\mathbf{I}$ as an iterated integral in terms of the variables u and v.

2. [4 pts] Compute the work done by the vector field

$$\mathbf{F}(x,y) = \left\langle \frac{x+1}{(x+1)^2+y^2}\,,\ \frac{y}{(x+1)^2+y^2} \right\rangle$$

in moving a particle along the line segment $1 \leq y \leq 4$ on the y-axis, oriented upward.

3. Consider the vector field

$$\mathbf{F}(x,y) = \left\langle 4x\ln(y)\,,\ \frac{2x^2-1}{y} \right\rangle$$

defined on the domain $D = \{(x,y) \mid y > 0\}$.

(a) [6 pts] Carefully explain whether or not $\mathbf{F}$ is a conservative vector field. If $\mathbf{F}$ is conservative, then find a potential function $f(x,y)$ defined on D.

(b) [3 pts] Determine the value of $\displaystyle\int_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{r}$ where $\mathcal{C}$ is the circle centered at $(0,3)$ with radius $r=1$. Justify your answer.

Points earned: ______
out of a possible 13 points