

1. Find the volume of the solid lying within the sphere $\rho = 4 \cos \phi$ and below the cone $\phi = \pi/4$.

2. Use spherical coordinates to express the following sum of integrals as a single integral:

$$\int_{-2\sqrt{2}}^0 \int_0^{\sqrt{8-x^2}} \int_{-\sqrt{8-x^2-y^2}}^0 x dz dy dx + \int_0^2 \int_x^{\sqrt{8-x^2}} \int_{-\sqrt{8-x^2-y^2}}^0 x dz dy dx$$

3. Express the integral

$$\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \int_0^x (x^2 + y^2) dz dx dy$$

in cylindrical coordinates (do not evaluate the integral).

4. Let \mathbf{I} denote the integral $\int_0^1 \int_0^z \int_x^z z e^{-y^2} dy dx dz$.
- (a) Rewrite the integral in the order $dydzdx$.

(b) Rewrite the integral in the order $dzdydx$.

(c) Rewrite the integral in the order $dx dy dz$.

5. [6 pts] Consider the triple integral

$$\int_0^2 \int_0^{2-y} \int_0^{\sqrt{8-2y^2}} f(x, y, z) dz dx dy$$

Express the integral so that the order of integration is $dx dy dz$.

6. [8 pts] If you take the circle $(y - \frac{1}{2})^2 + z^2 = \frac{1}{4}$ in the yz -plane and rotate it about the z -axis, the resulting surface is called a *torus*. Its equation in spherical coordinates is $\rho = \sin \phi$.

The surface whose equation is $\rho = \cos \phi$ is a sphere.

- (a) Convert the equation of the sphere $\rho = \cos \phi$ to Cartesian coordinates (i.e. x, y, z coordinates) and identify its radius and center.

- (b) Integrate the function $\sigma(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ over the solid region E that lies inside the sphere $\rho = \cos \phi$ and outside the torus $\rho = \sin \phi$.