1. Find the limit, if it exists, or show that the limit does not exist.

(a) [3 pts] $\lim_{(x,y)\to(-3,1)} \frac{x^2y - xy^3}{x - y + 2}$ → Plug in values

$$= \frac{(-3)^2(1) - (-3)(1)^3}{-3-1+2} = \frac{9+3}{-2} = -6$$

(b) [3 pts] $\lim_{(x,y)\to(0,0)} \frac{x+y}{x^2+y}$ Approach (0,0) along the x-axis ⤳ $\gamma(t) = (t,0)$

$$\lim_{t\to 0} \frac{t+0}{t^2+0} = \lim_{t\to 0} \frac{1}{t} = \text{DNE}.$$

Can also approach (0,0) along y-axis ⤳ limit = 1

And find that these two limits don't agree.

OR something like this

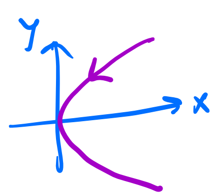
(c) [3 pts] $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$

Use squeeze theorem: Since $\sqrt{x^2+y^2} \geq \sqrt{x^2} = |x|$

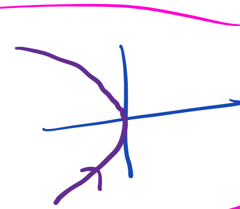
$$\Rightarrow \underbrace{\lim_{(x,y)\to 0} -\frac{xy}{\sqrt{x^2}}}_{=0} \leq \lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} \leq \underbrace{\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2}}}_{=0}$$

(d) [3 pts] $\lim_{(x,y)\to(0,0)} \frac{xy^2\cos y}{x^2+y^4}$

Approach (0,0) along the curve (t^2, t) ⤳



$$\lim_{t\to 0} \frac{t^4\cos(t)}{t^4+t^4} = \frac{1}{2}$$



Approach (0,0) along the curve $(-t^2, t)$ ⤳

$$\lim_{t\to 0} \frac{-t^4\cos(t)}{t^4+t^4} = -\frac{1}{2}$$

Since these limits don't agree ⇒ limit = DNE

(e) [3 pts] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2\sin^2(x)}{x^4+y^4}$

Approach (0,0) along $y=0 \rightarrow \displaystyle\lim_{t\to 0} f(t,0) = \lim_{t\to 0} 0 = 0.$

Don't agree

$\Rightarrow$ lim = DNE

But approach (0,0) along $y=x$

$$\rightarrow \lim_{t\to 0} f(t,t) = \lim_{t\to 0} \frac{t^2\sin^2(t)}{2t^4}$$

$$= \frac{1}{2}\lim_{t\to 0} \frac{\sin(t)}{t}\cdot\frac{\sin(t)}{t} = 1/2$$

(f) [3 pts] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$

Change into polar coords: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

Then limit becomes:

$$\lim_{r\to 0} \frac{r^3\cos^3\theta + r^3\sin^3\theta}{r^2} = \lim_{r\to 0} r\cos^3\theta + r\sin^3\theta = 0 //$$

(g) [3 pts] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2}$

Use polar coords!

$$\lim_{r\to 0} \frac{e^{-r^2}-1}{r^2} \xrightarrow[\text{L'Hopital's}]{} \lim_{r\to 0} \frac{-2re^{-r^2}}{2r} = -1. //$$

2. Consider the following level curves. Answer the questions below.

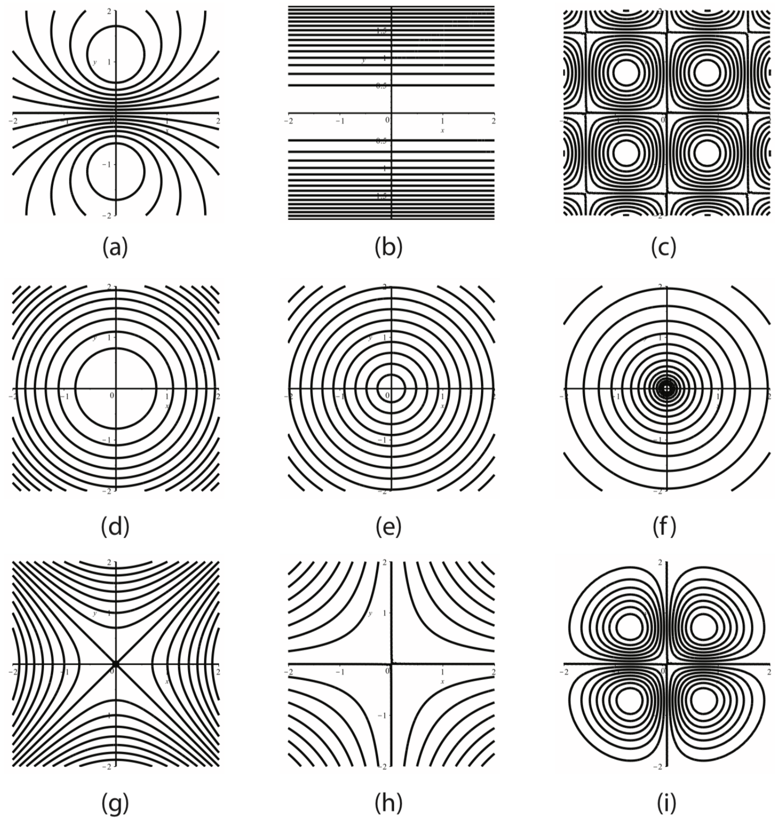


Figure 1:

(a) [2 pts] Which picture represents the level curves of $f(x, y) = y^2$ b

(b) [2 pts] Which picture represents the level curves of $f(x, y) = \sin(2x)\sin(2y)$? c

(c) [2 pts] Which picture represents the level curves of $f(x, y) = xy$? h

(d) [2 pts] Which picture represents the level curves of $f(x, y) = \sqrt{x^2 + y^2}$? e

(e) [2 pts] Which picture represents the level curves of $f(x, y) = (x^2 + y^2)^2$? D

Points earned: ____
out of a possible 10 points