

1. [4 pts] A torus is parametrized by the vector-valued function

$$\mathbf{r}(u, v) = \langle (3 + \cos u) \cos v, (3 + \cos u) \sin v, \sin u \rangle \quad 0 \leq u \leq 2\pi \quad 0 \leq v \leq 2\pi$$

Use this parametrization to compute the surface area of the torus.

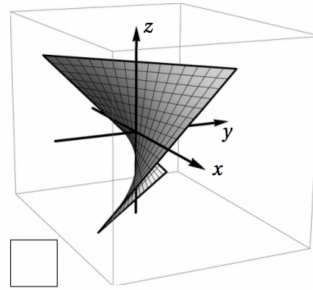
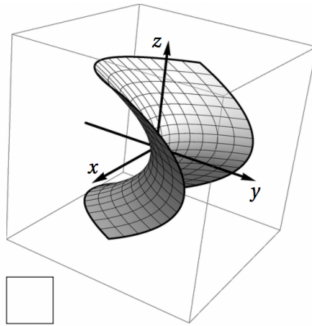
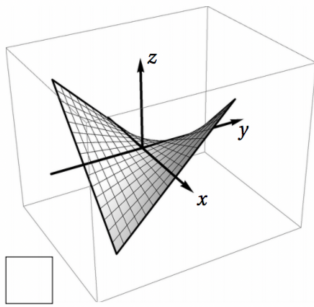
2. [4 pts] Find the tangent plane to the parametrized surface

$$\mathbf{r}(u, v) = \langle u^2 - 1, uv, v^3 \rangle$$

at the point $(3, 4, 8)$. Please give an answer of the form $ax + by + cz = d$.

3. Let S be the surface parametrized by $\mathbf{r}(u, v) = \langle u, uv, v \rangle$ for $-1 \leq u \leq 1$ and $-1 \leq v \leq 1$.

(a) [3 pts] Mark the correct picture of S below.



(b) [3 pts] Completely setup, but do not evaluate, the surface integral $\iint_S x^2 dS$.

4. [4 pts] Evaluate the surface integral $\iint_S z dS$, where S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy -plane.

5. Let S be the surface parametrized by $\mathbf{r}(u, v) = \langle \sin u \cos u, \sin^2 u, v \rangle$ where the domain of the parameters is $D = \{(u, v) \mid 0 \leq u \leq \pi/2, 0 \leq v \leq \sin^2 u\}$.

(a) [4 pts] Find an equation of the tangent plane at the point $(x, y, z) = (\sqrt{3}/4, 1/4, 1/2)$.

(b) [4 pts] Calculate $\iint_S (x + 1) dS$.

6. [4 pts] Let S be the cone given by the equation $z = \sqrt{x^2 + y^2}$ with $0 \leq z \leq 2$, oriented downward. Compute the flux of the vector field $\mathbf{G}(x, y, z) = \langle xz, yz, xy \rangle$ across the surface S .

7. [4 pts] Let S be the helicoid parametrized by

$$\mathbf{r}(u, v) = \langle u \sin v, 2v, u \cos v \rangle \quad 0 \leq u \leq 1, 0 \leq v \leq \pi$$

oriented in the direction of the positive y -axis. Let \mathbf{F} be the vector field defined by

$$\mathbf{F}(x, y, z) = xy\mathbf{i} + (y^2 + 1)\mathbf{j} + yz\mathbf{k}$$

Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

8. [4 pts] Let S be the portion of the cylinder given in cylindrical coordinates by

$$0 \leq z \leq 3, \quad r = 1, \quad 0 \leq \theta \leq \pi/2.$$

Orient S by normal vectors pointing away from the z axis. Compute the flux (surface integral) of $\mathbf{F}(x, y, z) = \langle 2x, y, -3z \rangle$ across S .