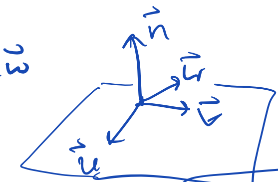
1. Multiple choice. Clearly mark your answer.

(a) [2 pts] If $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{w}$, then $\mathbf{v} \cdot (\mathbf{u} \times \mathbf{w}) = 0$.

(i) True.
(ii) False.

Geometrically, $\vec{u} \times \vec{v} = \vec{v} \times \vec{w}$ means that $\vec{v}$ is contained in the plane spanned by $\vec{u}$ and $\vec{w}$, i.e. that $\vec{v} \cdot (\vec{u} \times \vec{w}) = 0$.

i.e.
Take $\vec{n} = \vec{u} \times \vec{v} = \vec{v} \times \vec{w}$



$(0,0,-4)$ a point on P but NOT a point on Q!!

(b) [2 pts] The two planes $\underbrace{2x + 2y - z = 4}$ and $\underbrace{-4x - 4y + 2z = 3}$ intersect in a line.

(i) True.
(ii) False.

$$\vec{n}_P = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \qquad \vec{n}_Q = \begin{bmatrix} -4 \\ -4 \\ 2 \end{bmatrix} = -2 \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

These planes don't intersect at all!! Totally parallel.

(c) [2 pts] The line $\mathbf{r}(t) = \langle t + 1, 2t - 1, -3t + 16 \rangle$ is perpendicular to which of the following planes?

1. $3z = x + 2(y - 1)$
2. $-x - 2y + 3z = 11$
3. $2x + 4y - 6z = 31$
4. All of them.
5. None of them.

Means $\vec{u} \parallel \vec{v}$

(d) [2 pts] If $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ and $\mathbf{u}$ is not the zero vector, then which of the following is necessarily true?

1. $\cancel{\mathbf{u} \cdot \mathbf{v} = 0}$
2. Either $\mathsf{proj}_{\mathbf{u}}\mathbf{v} = \mathbf{v}$ or $\mathsf{proj}_{\mathbf{u}}\mathbf{v} = -\mathbf{v}$
3. $\cancel{|\mathbf{u}| = |\mathbf{v}|}$
4. All of the above.
5. None of the above.

$$\text{proj}_{\vec{u}}(\vec{v}) = \frac{\vec{v} \cdot \vec{u}}{|\vec{u}|^2} \vec{u}$$

$$= \frac{\vec{v} \cdot \vec{v}}{|\vec{v}|^2} \vec{v} \quad \text{or} \quad \frac{-\vec{v} \cdot \vec{v}}{|\vec{v}|^2} \vec{v}$$

2. [4 pts] Find an equation of the plane which passes through the point $(1, 0, 0)$ and which is orthogonal to both planes given below:

$$\begin{matrix} P \\ Q \end{matrix} \begin{cases} x + y + z = 1 \\ x - y - z = 2 \end{cases}$$

$$\vec{n}_P = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{n}_P \times \vec{n}_Q = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = \begin{bmatrix} -1+1 \\ -(-1-1) \\ -1-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$$

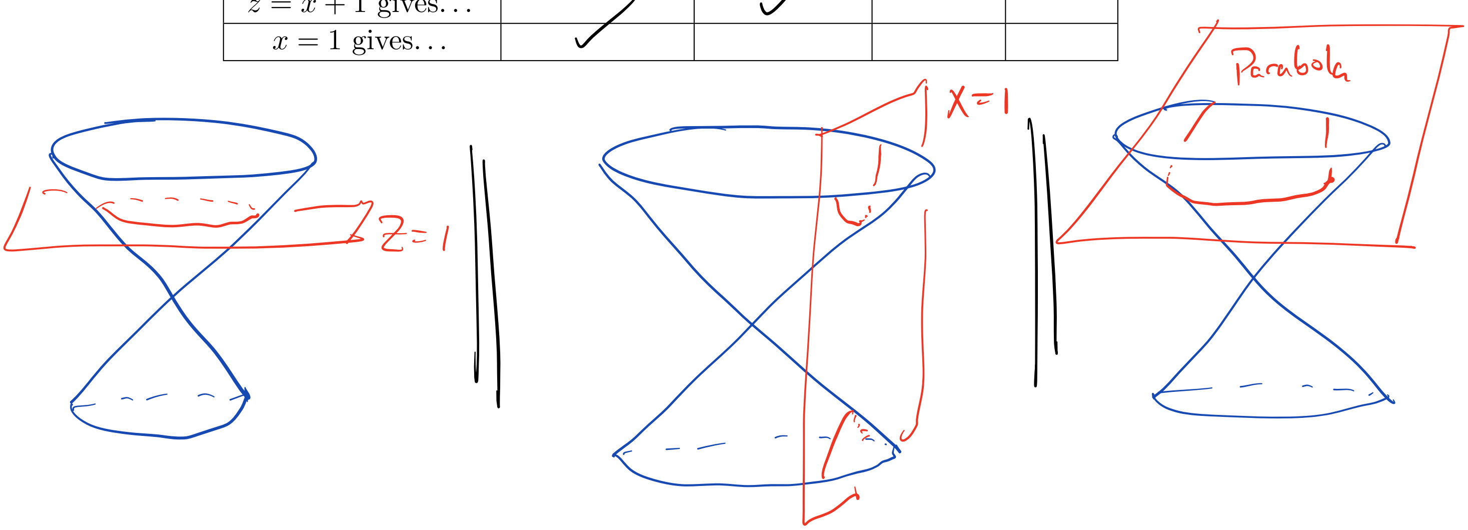
$$\vec{n}_Q = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \qquad = 2\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$\rightsquigarrow 0(x-1) + 1(y-0) - 1(z-0) = 0$$

$$\Rightarrow y - z = 0. //$$

3. [4 pts] The intersection of a plane with the cone $S = \{(x, y, z) : x^2 + y^2 - z^2 = 0\}$ is called a **conic section**. What curve do we get? In each row check only one box.

Intersect S with...	hyperbola(s)	parabola(s)	circle(s)	line(s)
$z = 1$ gives...			✓	
$z = x$ gives...				✓
$z = x + 1$ gives...		✓		
$x = 1$ gives...	✓			



Points earned: ______
out of a possible 8 points