University of Virginia **MATH 2310**
Department of Mathematics **Quiz 1** Spring 2022

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else during the exam:

Name (sign): Solutions Name (print): ______________________

1. [4 pts] Given the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$. Determine the vector projection $\mathsf{proj}_{\mathbf{a}}(\mathbf{b})$ of $\mathbf{b}$ onto $\mathbf{a}$.

$$\text{proj}_{\vec{a}}(\vec{b}) = \frac{\begin{bmatrix}1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}1\\-1\\1\end{bmatrix}}{\begin{bmatrix}1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}} \begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{1-1+1}{3} \begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{1}{3} \begin{bmatrix}1\\1\\1\end{bmatrix}$$

2. True/False. If the statement is true, give an explanation why you think so. If a statement is false, provide a counter-example.

(a) [3 pts] The cross product of two unit vectors is a unit vector.

False. $\vec{u} = \begin{bmatrix}1\\0\\0\end{bmatrix}$ $\rightarrow \vec{u} \times \vec{v} = \vec{i} \times \frac{1}{\sqrt{2}}(\vec{i} + \vec{j})$

$\vec{v} = \frac{1}{\sqrt{2}} \begin{bmatrix}1\\1\\0\end{bmatrix}$ $= \frac{1}{\sqrt{2}} \vec{i} \times \vec{j} = \frac{1}{\sqrt{2}} \vec{k}$

Not length $= 1$!!

(b) [3 pts] If $\mathbf{u}$ is a scalar multiple of $\mathbf{v}$, then $\mathbf{u} \times \mathbf{v} = \mathbf{0}$.

True.

if $\vec{u} = c\vec{v} \Rightarrow$ angle btw $\vec{u}$ and $\vec{v}$ is $\theta = 0$ or $\theta = \pi$

in either case $\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \|\vec{v}\| \underbrace{\sin\theta}_{=0}, \Rightarrow \vec{u} \times \vec{v} = \vec{0}$.

(c) [3 pts] If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are all non-zero vectors in space and $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

False.

Choose $\vec{u} = \vec{i}$, $\vec{v} = \vec{j}$, $\vec{w} = \vec{k}$

Then $\vec{i} \cdot \vec{j} = 0 = \vec{i} \cdot \vec{k}$

but $\vec{j} \neq \vec{k}$.

(d) [3 pts] The vector equation

$$\langle x, y, z \rangle \times \langle 1, 1, 1 \rangle = \langle 0, 1, 0 \rangle$$

has a solution in $\mathbb{R}^3$.

Solve $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = \begin{bmatrix} y-z \\ -(x-z) \\ x-y \end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$ $\rightarrow y = z$, $\rightarrow x = y$

So $x = y = z$ but then no solution to $x - z = -1$ //

No solutions //