PREVIOUSLY: Discussed lines and Planes in $\mathbb{R}^3$

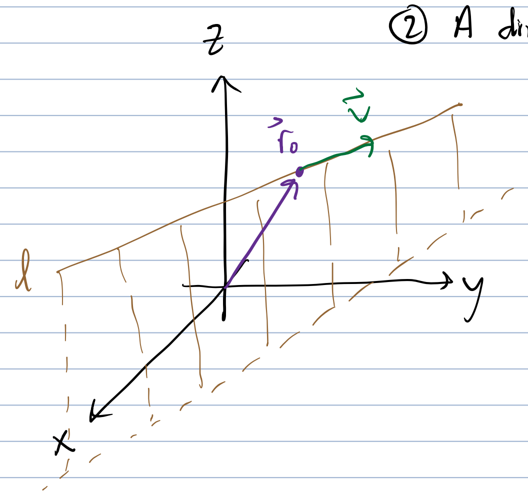
LINES:

$\vec{r}(t) = \vec{r}_0 + t\vec{v}$: THINK: "Translation Vector" + "Direction vector"

i.e.: Same as 2D, to describe a line need two pieces of info

① A point on the line $\vec{r}_0$

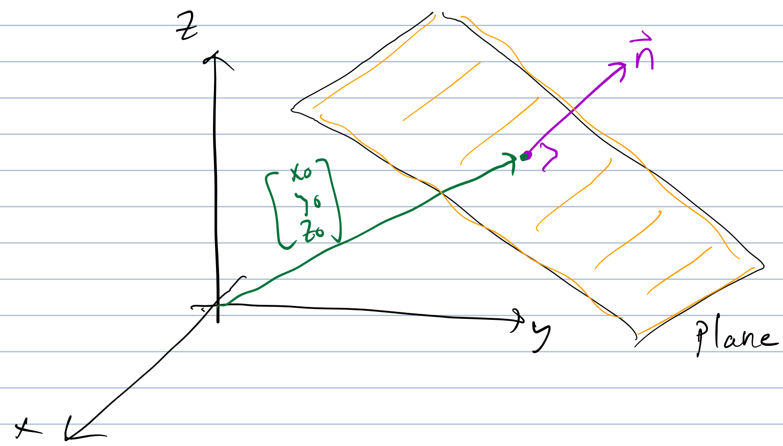
② A direction: $\vec{v}$



PLANES: $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

THINK: "Translation vector" + "Tilt"

$\langle x_0, y_0, z_0 \rangle$ $\vec{n} = \langle a, b, c \rangle$

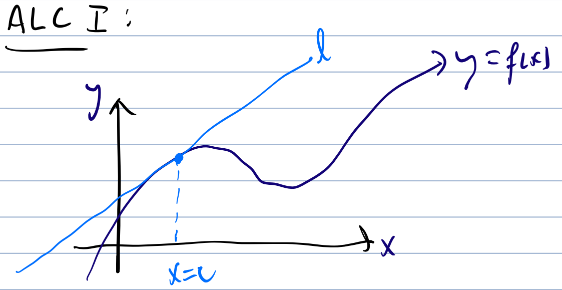


- Lines and planes are "linear objects"

<u>TODAY</u>: Discuss "Quadric Surfaces"

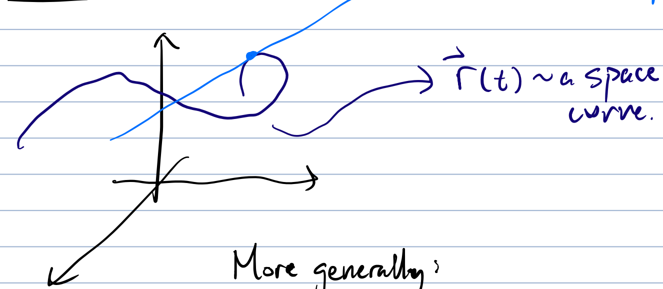
<u>CALC I</u>:

$\ell \sim$ tangent line to $y=f(x)$ at $x=c$ "best linear approx"



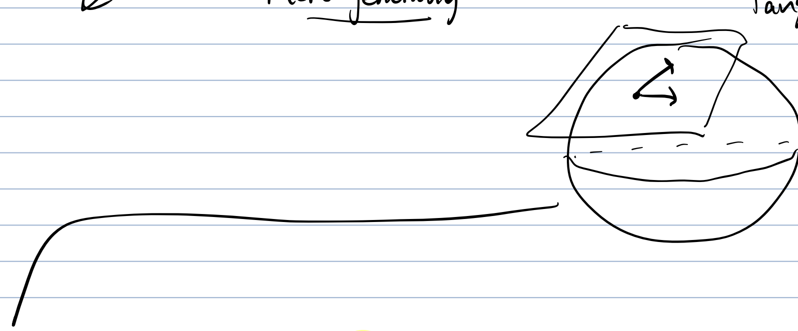
<u>CALC III</u>:

$\ell \sim$ best "linear approx" in $\mathbb{R}^3$.



<u>More generally</u>:

Tangent plane to the surface $\sim$ "best linear approx"



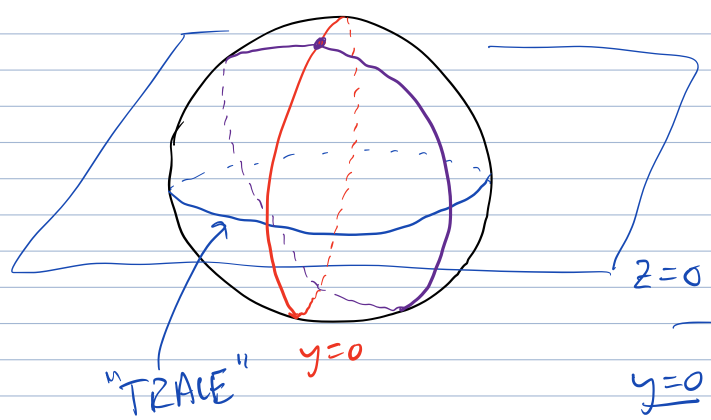
First, should describe some surfaces in $\mathbb{R}^3$.

HOW TO SKETCH:

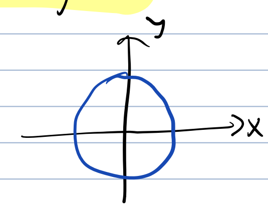
- When sketching surfaces in $\mathbb{R}^3$, it is helpful to determine the curves of intersection of the surface w/ planes parallel to the coordinate planes

  $\rightsquigarrow$ Resulting curves are called TRACES or CROSS-SECTIONS of the surface.

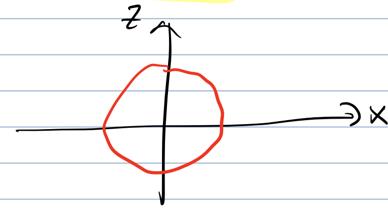
Ex: Sphere $x^2+y^2+z^2=1$



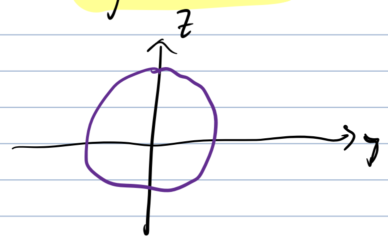
$\underline{z=0}$: $x^2+y^2=1$



$\underline{y=0}$: $x^2+z^2=1$



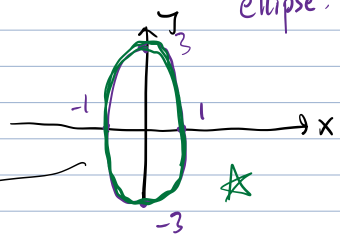
$\underline{x=0}$: $y^2+z^2=1$



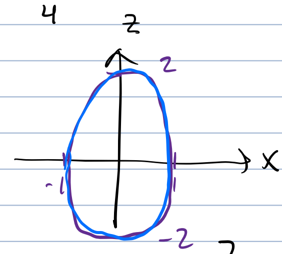
Ex: (Ellipsoid) $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

(In general: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, $a,b,c>0$)

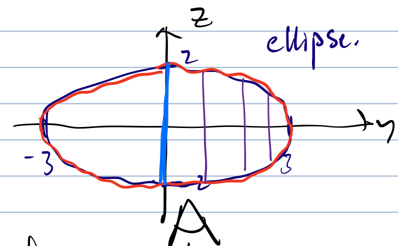
$z=0$: $x^2 + \frac{y^2}{9} = 1$ ellipse.

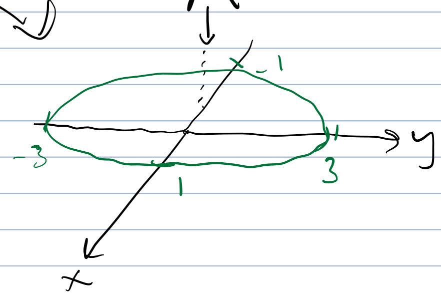


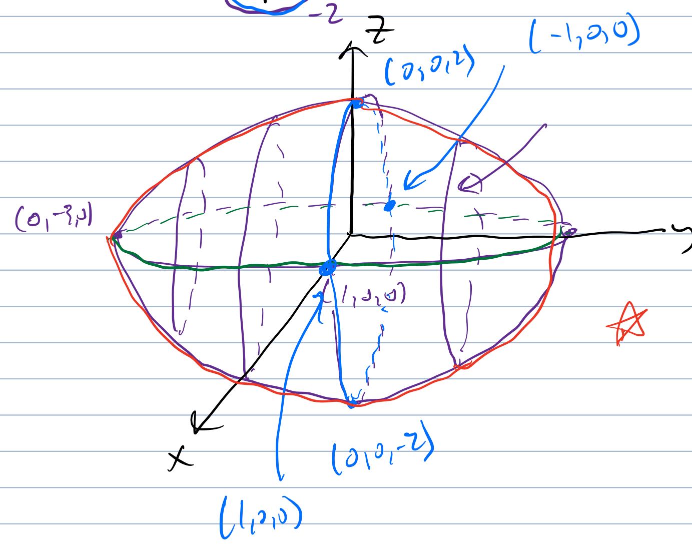
$y=0$: $x^2 + \frac{z^2}{4} = 1$



$x=0$: $\frac{y^2}{9} + \frac{z^2}{4} = 1$ ellipse.







Ex: (Hyperboloid) $\pm \frac{x^2}{a^2} \pm \frac{y^2}{b^2} \pm \frac{z^2}{c^2} = 1$

- if all "+" $\rightsquigarrow$ ellipsoid.
- if all "−" $\rightsquigarrow$ No solns.

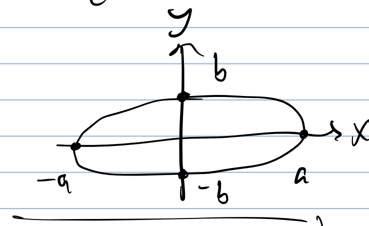
Two cases:

① 2 "+" and 1 "−"

② 1 "+" and 2 "−"

Case ①: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

$z = 0$: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

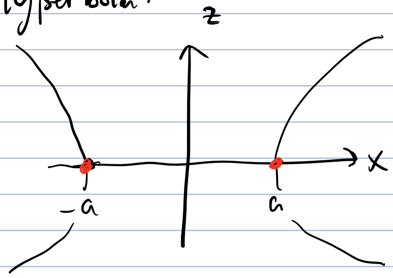


Notice: in fact, for any constant value of z, still get an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \underbrace{1 + \frac{z_0^2}{c^2}}_{\text{"radius"}}$$

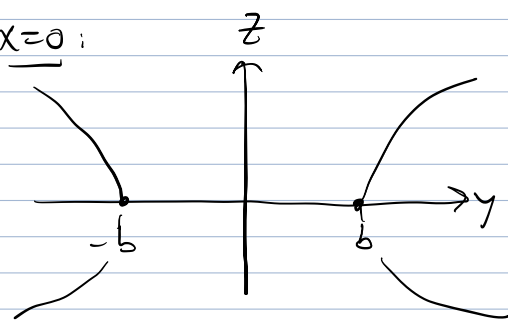
$y = 0$: $\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$

Hyperbola:

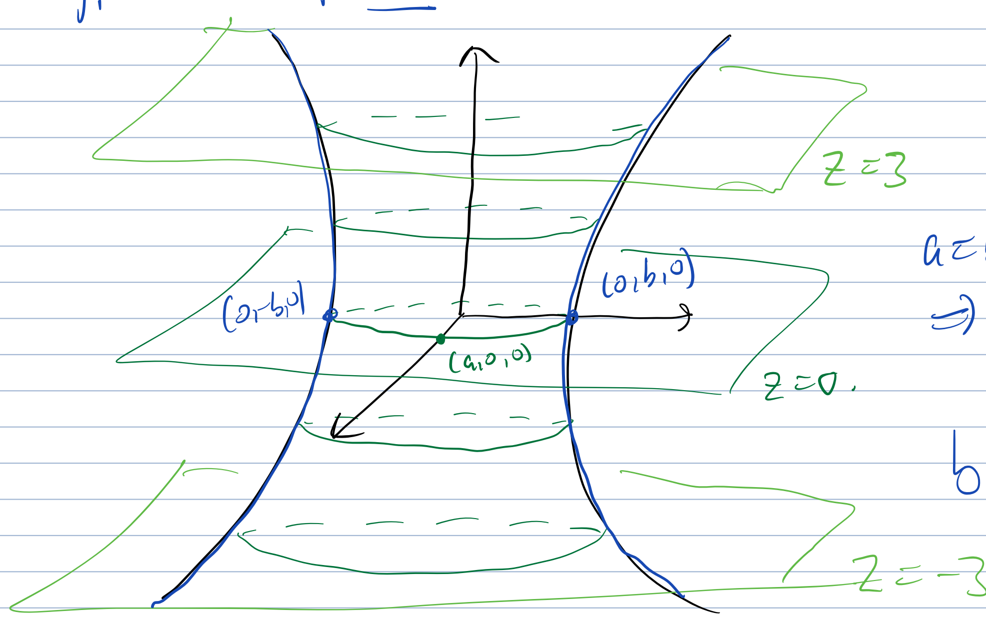


When $z = 0$ $\frac{x^2}{a^2} = 1 \Rightarrow x = \pm a$

$x = 0$:



Hyperboloid of One Sheet



$a = b = 1$

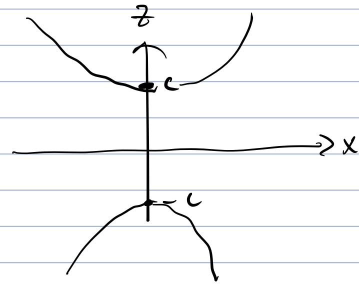
$\Rightarrow$ Surface of revolution.

$b > a$.

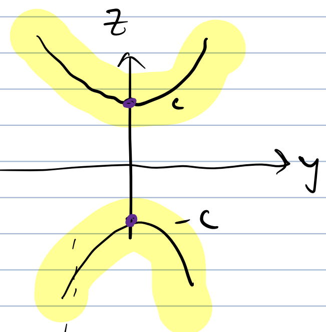
CASE ② : Two "−" signs $\quad -\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$z=0$: $\quad -\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \rightsquigarrow$ No solns!

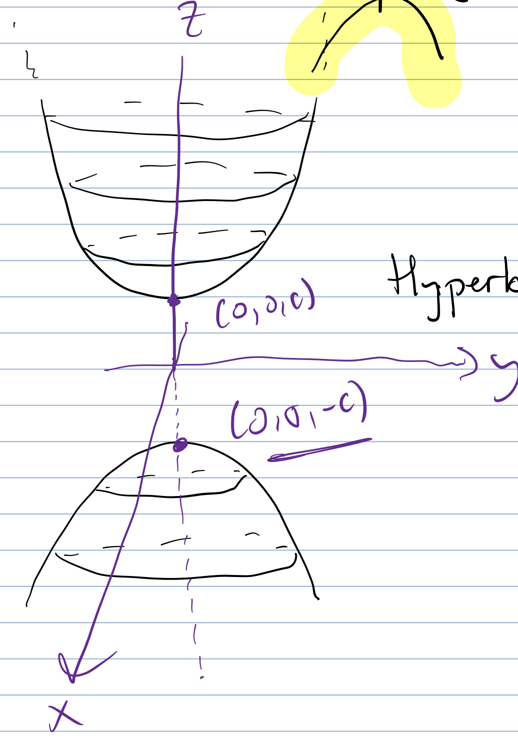
$y=0$: $\quad -\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$



$x=0$: $\quad -\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



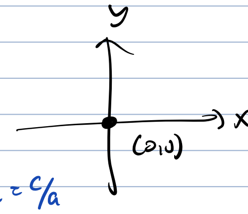
$\frac{z^2}{c^2} = 1 \Rightarrow z = \pm c$



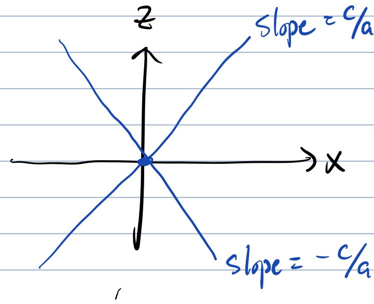
Hyperboloid of Two Sheets.

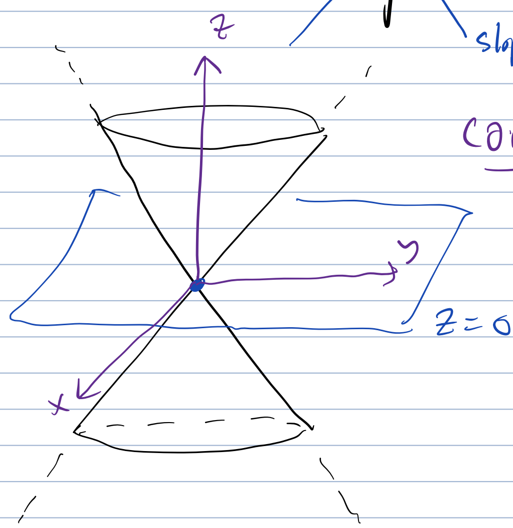
Example: CONE $\dfrac{z^2}{c^2} = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$ (All terms Quadratic $a, b, c > 0$)

$\underline{z=0}$: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 0 \rightsquigarrow x=0, y=0.$

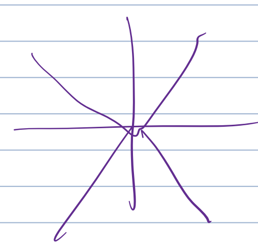


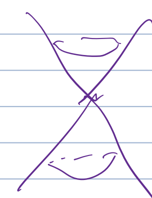
$\underline{y=0}$: $\dfrac{z^2}{c^2} = \dfrac{x^2}{a^2}$





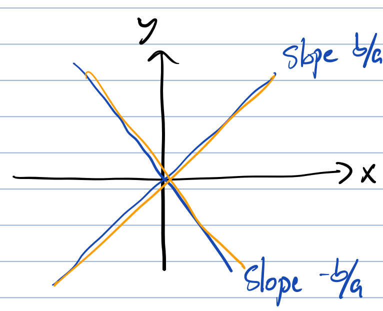
CONE.



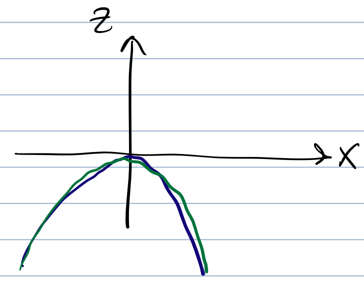


Ex: (Hyperbolic Paraboloid) $\dfrac{z}{c} = -\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$, $a, b, c > 0.$

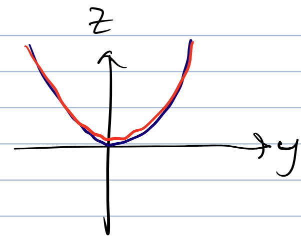
$\underline{z=0}$: $-\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 0$



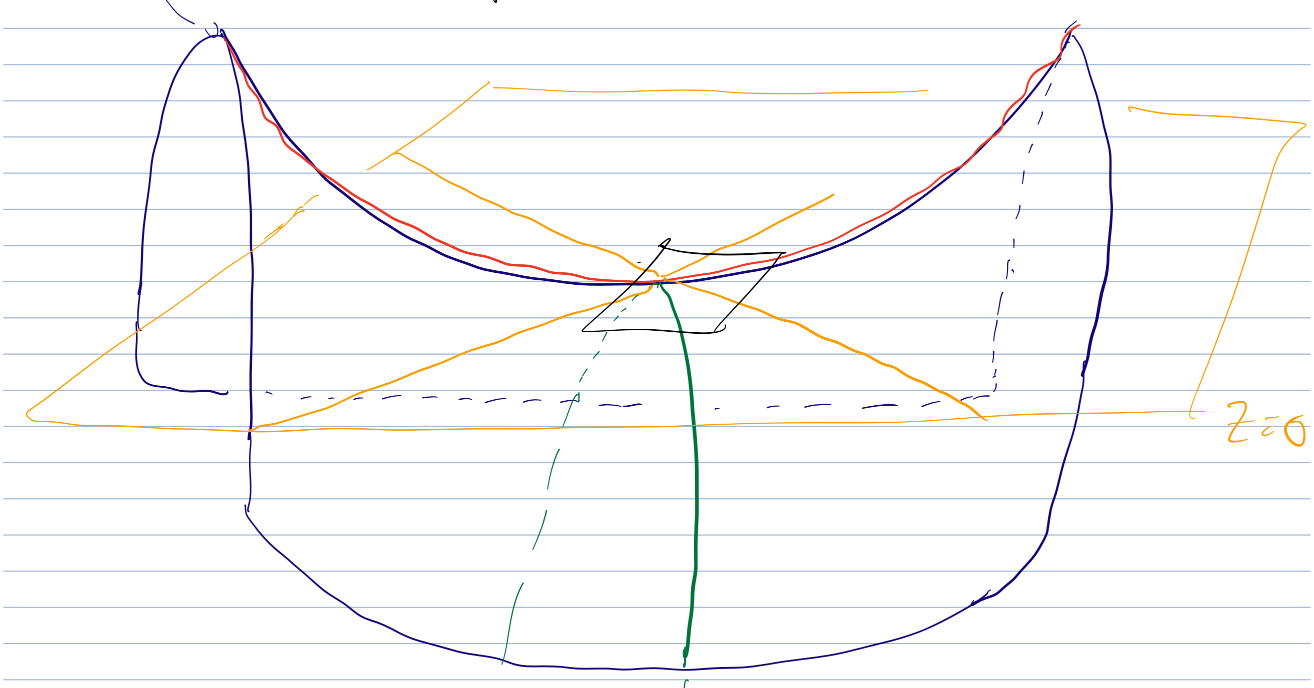
$y=0$: $\dfrac{z}{c} = -\dfrac{x^2}{a^2}$



$x=0$: $\dfrac{z}{c} = \dfrac{y^2}{b^2}$



SADDLE.



A critical point

but not max/min.