1. Multiple choice. Clearly mark your answers. No justification is required but may result in partial credit if provided.

   (a) [3 pts] If $|\mathbf{v} \times \mathbf{w}| = 0$ for two unit vectors $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

   (a) True.
   (b) False.
   (c) Indeterminable.

   (b) [3 pts] If $\mathbf{a} \cdot \mathbf{b} > 0$, and $\mathbf{b} \cdot \mathbf{c} > 0$, then $\mathbf{a} \cdot \mathbf{c} > 0$.

   (a) True.
   (b) False.
   (c) Indeterminable.

   (c) [3 pts] If the acceleration of a curve $\mathbf{r}(t)$ is zero for all time t and the velocity $\mathbf{v}(t)$ is nonzero at time $t = 0$, then the curve parametrized by $\mathbf{r}(t)$ is a line.

   (a) True.
   (b) False.
   (c) Indeterminable.

   (d) [3 pts] Let $f(x, y) = xye^{xy}$. Which of the following is a unit vector pointing in the direction of the maximal rate of increase at the point $(1, 1)$?

   (a) $\langle 2e, 2e\rangle$
   (b) $\langle 1, 1\rangle$
   (c) $\frac{1}{\sqrt{2}}\langle 1, 1\rangle$
   (d) $\frac{1}{\sqrt{2}}\langle -1, -1\rangle$
   (e) None of the above.

   (e) [3 pts] If a function $f(x, y, z)$ has gradient satisfying $|\nabla f(x, y, z)| = 1$ everywhere, then the level surface $f(x, y, z) = 1$ is a sphere.

   (a) True.
   (b) False.
   (c) Indeterminable.

   (f) [3 pts] Find the area of the triangle with vertices $(4, 2, 2)$, $(3, 3, 1)$, and $(5, 5, 1)$.

   (a) 0
   (b) 4
   (c) $\sqrt{3}$
   (d) $\sqrt{6}$

(g) [4 pts] Which of the following best describes the critical points of the function

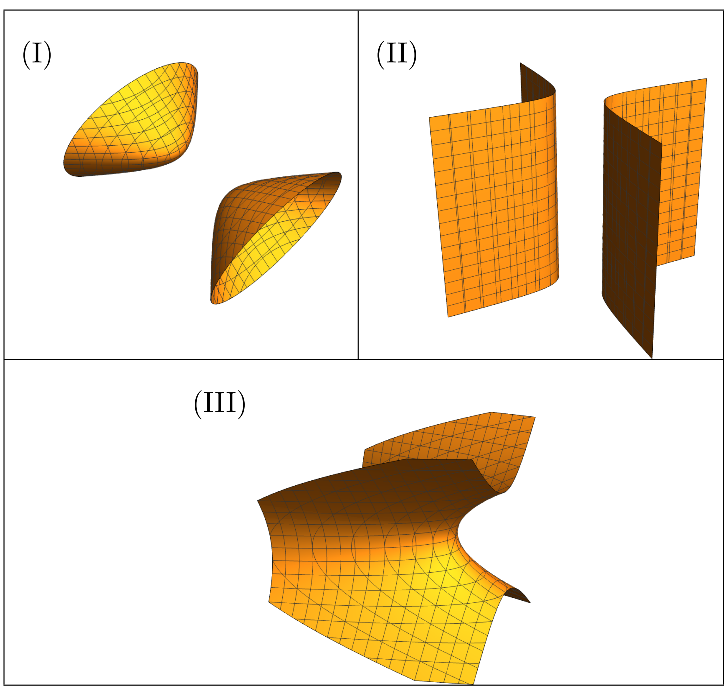
$$f(x,y) = x + \frac{y^3}{3} + \frac{y^2}{2} - \frac{x^2}{2}?$$

(a) One critical point.
(b) 2 critical points. One local minimum and one local maximum.
(c) 2 critical points. One saddle point and one local maximum.
(d) 2 critical points. One saddle point and one local minimum.
(e) 3 critical points. One saddle point, one local maximum, and one local minimum.

(h) [4 pts] Consider the space curve $\mathbf{r}(t) = \langle 2\cos t, e^t, t\rangle$, $-\infty < t < \infty$, which of the following points lies on the tangent line to the curve at the point $(2, 1, 0)$?

(a) $(1, 1, 1)$
(b) $(2, 1, 1)$
(c) $(1, 2, 0)$
(d) $(2, 2, 1)$
(e) $(0, 1, 2)$

2. [4 pts] Match the contour surfaces with their equations. Enter O if there's no match.



$g(x,y,z) =$	O, (I), (II), (III)
$x^2 - y^2 + z^2 = 1$	
$x^2 - y^2 = 1$	
$x^4 + z = 1$	
$x^2 + y - z^2 = 1$	

Points earned: ______
out of a possible 12 points

3. A leaf tumbles down along the curve

$$\mathbf{r}(t) = \langle t^2 \cos(t), t^2 \sin(t), 16 - 2t \rangle$$

in space.

(a) [4 pts] What is the speed of the leaf at time $t = \pi$?

(b) [4 pts] Find the distance the leaf travels along $\mathbf{r}(t)$ from $t = -8$ to $t = 8$.

 Points earned: ______
out of a possible 8 points

4. For each of the following, determine whether the limit exists. If so, compute the limit. If not, explain why.

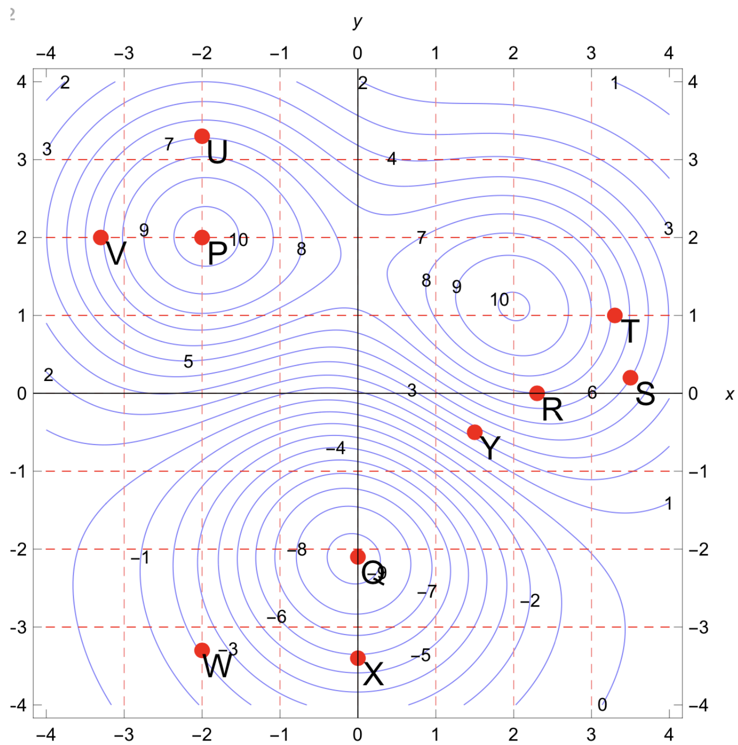
   (a) [5 pts] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y e^y}{x^4 + 3y^2}$

   (b) [5 pts] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2(1-\cos(2x))}{x^4 + y^2}$

   (c) [5 pts] $\displaystyle\lim_{(x,y)\to(0,0)} (x^2 + y^2)\ln(x^2 + y^2)$

Points earned: ______
out of a possible 15 points

5. A contour plot of the function $f(x, y)$ is shown below.



Answer each of the following questions using a subset of the points $P, Q, \ldots, X$. Some of the questions may have more than one answer–list all that apply. No justification is required.

(a) [3 pts] At which point is the length of the gradient vector ∇f maximal? ______

(b) [3 pts] At which point is $f_x > 0$ and $f_y = 0$? ______

(c) [3 pts] At which point is $f_x < 0$ and $f_y > 0$? ______

(d) [3 pts] At which point is the directional derivative $D_{\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle} f = 0$ and $f_x \neq 0$? ______

(e) [3 pts] At which point does f achieve a global minimum on $-4 \leq x \leq 4$ and $-4 \leq y \leq 4$? ______

(f) [3 pts] At wich point is $\nabla f = \vec{0}$ and $f_{xx} < 0$? ______

(g) [3 pts] At which point is ∇f parallel to the vector $\mathbf{j}$? ______

6. Suppose that three quantities x, y, and z, are constrained by the equation $2x^2 + 3y^2 + z^2 = 20$. This equation describes a surface S as a level set.

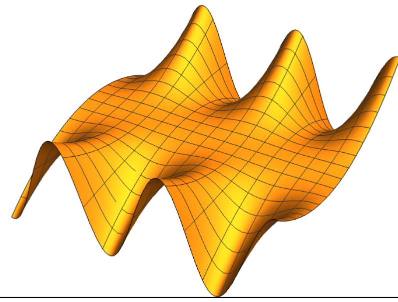
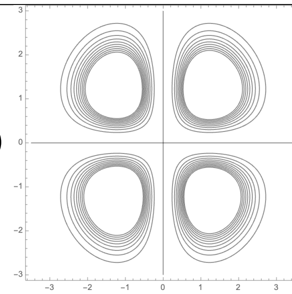
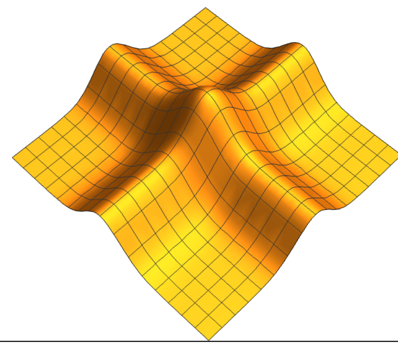
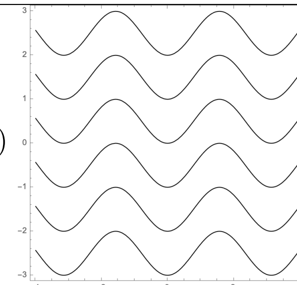
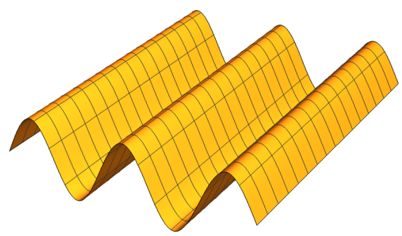
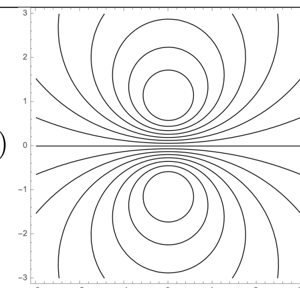
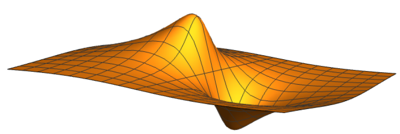
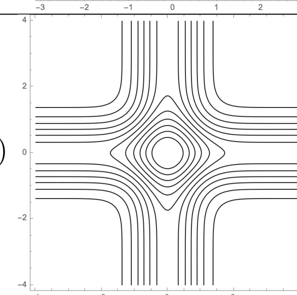
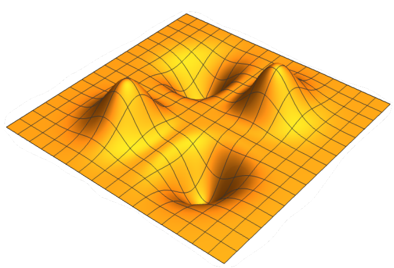
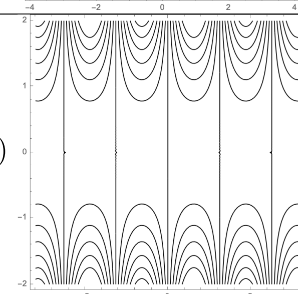
   (a) [6 pts] Verify that the point $P(2, 1, 3)$ is a point on S and find an equation for the tangent plane to S at P.

   (b) [5 pts] Near $P(2, 1, 3)$ we can think of z as a function of x and y, $z = f(x, y)$. Approximate the value of z corresponding to $x = 2.2$ and $y = 1.4$.

   (c) [5 pts] Find parametric equations for a line ℓ which is orthogonal to the surface S and which passes through the point $P(2, 1, 3)$.

Points earned: ______ out of a possible 16 points

7. [10 pts] Consider the following table of functions $f(x, y)$, their graphs, and their level curves. On the following page, fill out the table provided by matching the functions with their graphs and level curves.

(a) $x^3y^3\exp(-x^2-y^2)$	(A) 	(I) 
(b) $\dfrac{-10y}{x^2+y^2+1}$	(B) 	(II) 
(c) $\cos(x)^2+y$	(C) 	(III) 
(d) $y^2\sin(x)$	(D) 	(V) 
(e) $e^{-x^2}+e^{-y^2}$	(E) 	(VI) 

Points earned: ____
out of a possible 10 points

By filling in the table below, correctly match these functions, graphs, and level curves.

Function	**Graph**	**Level Curves**
$x^3y^3\exp\left(-x^2-y^2\right)$		
$\dfrac{-10y}{x^2+y^2+1}$		
$\cos(x)^2+y$		
$y^2\sin(x)$		
$e^{-x^2}+e^{-y^2}$		