LAST TIME:

- Discussed cross product of two vectors in $\mathbb{R}^3$. (Determinants, right-hand-rule, geometric properties....) — §12.4

TODAY:

- Equations of lines and planes in $\mathbb{R}^3$ (§12.5)
- Equations of Quadric surfaces in $\mathbb{R}^3$ (§12.6)

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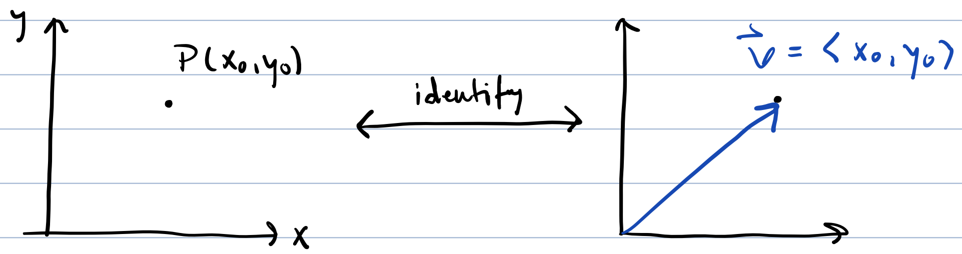
§12.5: Equations of lines and planes in $\mathbb{R}^3$

Lines in 2D: It's easiest if we make a mental shift, and "identify"

[Points in plane $\mathbb{R}^2$] $\longleftrightarrow$ [Arrow pointing from origin to point P]

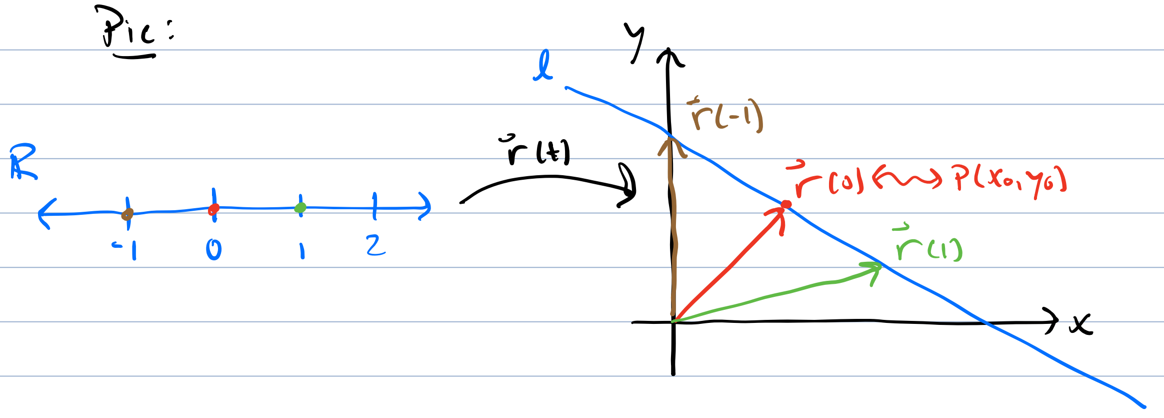


Purpose of mental shift: We can do ARITHMETIC w/ vectors.

- Having made the above identification, can think of a line as being traced out by a **VECTOR-VALUED** function $\vec{r}(t)$:

$\hookrightarrow$ $\vec{r}(t): \begin{bmatrix}\text{Real} \\ \text{Numbers}\end{bmatrix} \longrightarrow \begin{bmatrix}\text{Vectors}\end{bmatrix}$

Pic:



To describe a line ℓ via $\vec{r}(t)$:

1. Pick a point $P(x_0, y_0)$ lying on ℓ
2. Pick a direction parallel to ℓ.

$\longrightarrow$ Then every point on ℓ can be described as:

$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$

Note:

Vectors are often written as "column matrices", so

$$\vec{v} = \langle v_1, v_2 \rangle = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

So: often write formula for vector-valued function as

$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$

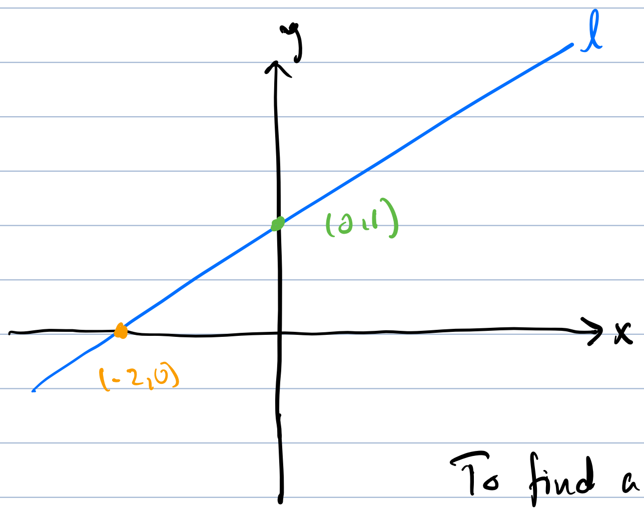
$$= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + t\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

A vector $\vec{v}$ which is parallel to line ℓ.

The point $p(x_0, y_0)$ thought of as a vector

Example:

Express ℓ drawn below as $\vec{r}(t)$:



Choose base point on ℓ to be

$p(x_0, y_0) = (0,1)$

Which we identify w/ the

Vector $\vec{r}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

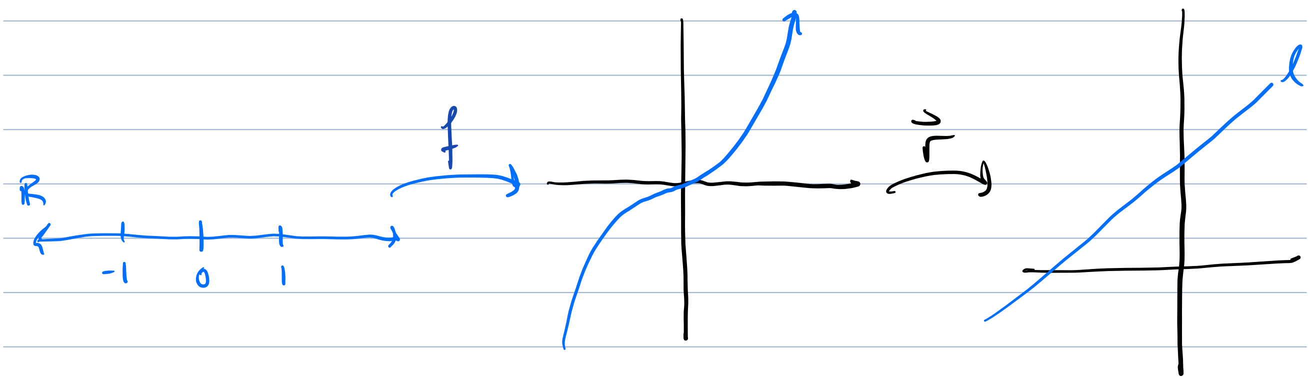
To find a vector $\vec{v}$ which is parallel to ℓ, use slope $= 1/2 \rightsquigarrow \vec{v} = \langle 1, 2 \rangle = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

So: $\vec{r}(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad -\infty < t < \infty$

is a parametrization of the line ℓ.

Warning! There can be <u>many</u> parametrizations of the same line!!

e.g. Precompose the previous parametrization w/ $f(t) = t^3$ so that



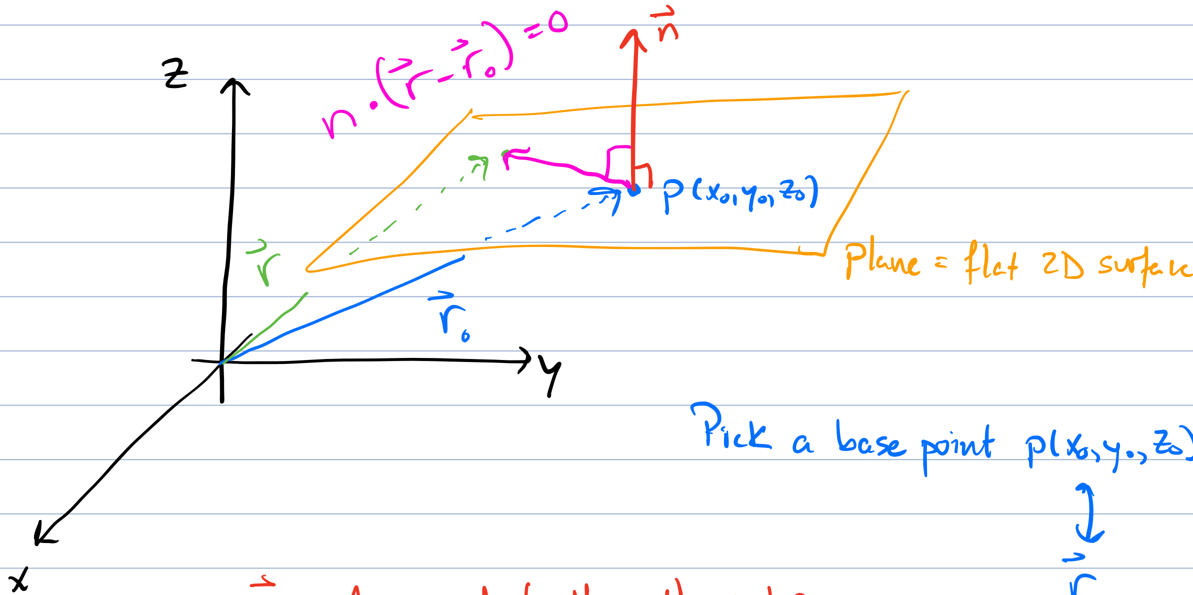
Since the function $t \mapsto t^3$ is <u>one-to-one</u>, it follows that

$$\vec{r} \circ f\,(t) = \vec{r}\,(f(t))$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t^3 \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad -\infty < t < \infty$$

is another parametrization of the line ℓ.

PLANES IN $\mathbb{R}^3$: Consider a 2D plane in $\mathbb{R}^3$:



Pick a base point $p(x_0,y_0,z_0)$

$\updownarrow$

$\vec{r}_0$

- $\vec{n}$ = A normal (orthogonal) vector to the plane

Notice: Any other point $\vec{r}$ is on the plane

only when $(\vec{r}-\vec{r}_0) \perp \vec{n}$

$\leadsto$ i.e. only when $(r-\vec{r})\cdot\vec{n}=0$