1. Find the limit, if it exists, or show that the limit does not exist.

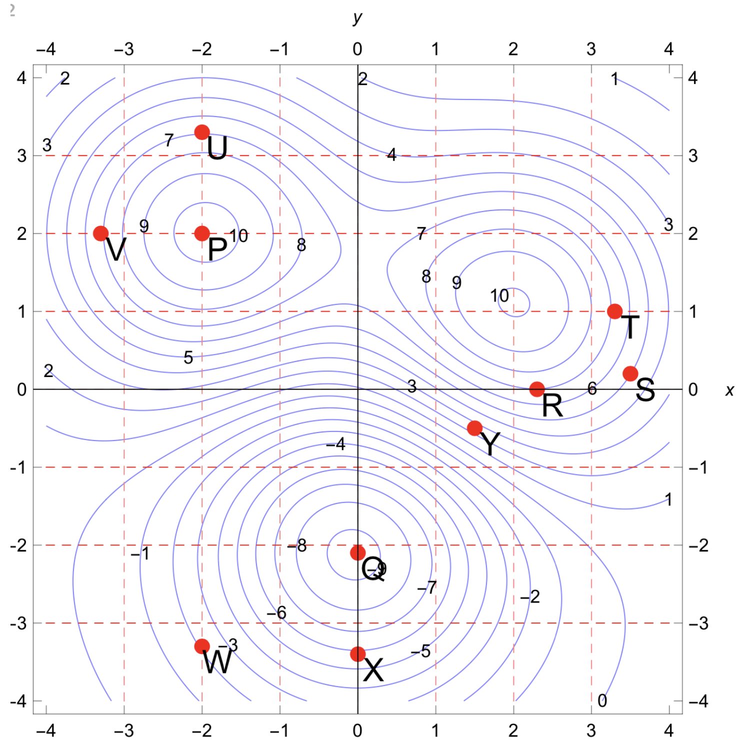
   (a) [3 pts] $\displaystyle\lim_{(x,y)\to(-3,1)} \frac{x^2y - xy^3}{x - y + 2}$

   (b) [3 pts] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x + y}{x^2 + y}$

   (c) [3 pts] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$

   (d) [3 pts] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2 \cos y}{x^2 + y^4}$

2. A contour plot of the function $f(x, y)$ is shown below.



Answer each of the following questions using a subset of the points $P, Q, \ldots, X$. Some of the questions may have more than one answer–list all that apply. No justification is required.

(a) [3 pts] At which point is the length of the gradient vector ∇f maximal? ______

(b) [3 pts] At which point is $f_x > 0$ and $f_y = 0$? ______

(c) [3 pts] At which point is $f_x < 0$ and $f_y > 0$? ______

(d) [3 pts] At which point is the directional derivative $D_{\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle} f = 0$ and $f_x \neq 0$? ______

(e) [3 pts] At which point does f achieve a global minimum on $-4 \leq x \leq 4$ and $-4 \leq y \leq 4$? ______

(f) [3 pts] At wich point is $\nabla f = \vec{0}$ and $f_{xx} < 0$? ______

(g) [3 pts] At which point is ∇f parallel to the vector $\mathbf{j}$? ______

3. Suppose that three quantities x, y, and z, are constrained by the equation $2x^2 + 3y^2 + z^2 = 20$. This equation describes a surface S as a level set.

   (a) [6 pts] Verify that the point $P(2, 1, 3)$ is a point on S and find an equation for the tangent plane to S at P.

   (b) [5 pts] Near $P(2, 1, 3)$ we can think of z as a function of x and y, $z = f(x, y)$. Approximate the value of z corresponding to $x = 2.2$ and $y = 1.4$.

   (c) [5 pts] Find parametric equations for a line ℓ which is orthogonal to the surface S and which passes through the point $P(2, 1, 3)$.

Points earned: ______
out of a possible 16 points

4. [8 pts] Find the critical points of the function $f(x, y) = x^4 + 2y^2 - 4xy$, and classify each as a local maximum, local minimum, or saddle point.