TODAY: § 14.1 : Multivariable Functions

Functions of Two Variables:

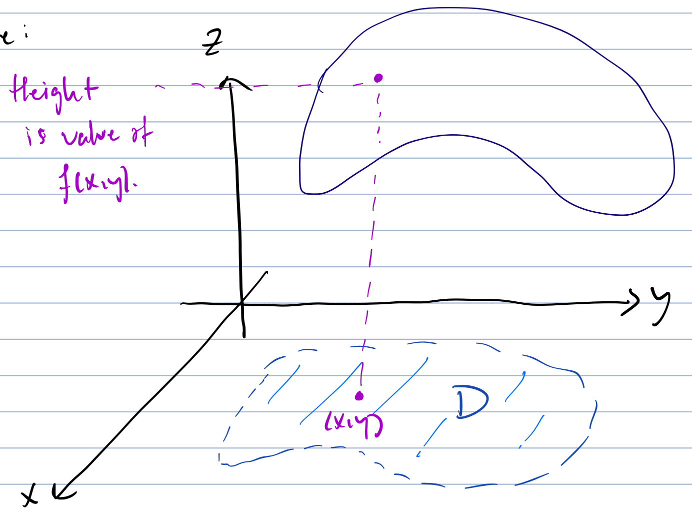
- Let D be a region in the plane $\mathbb{R}^2$.

A function f on D $\quad f: D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$

D is in the plane $\mathbb{R}^2$

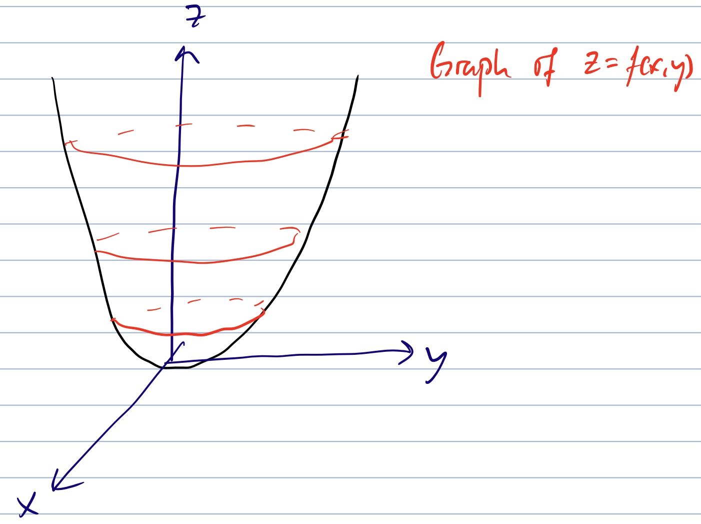
at pt is real #'s.

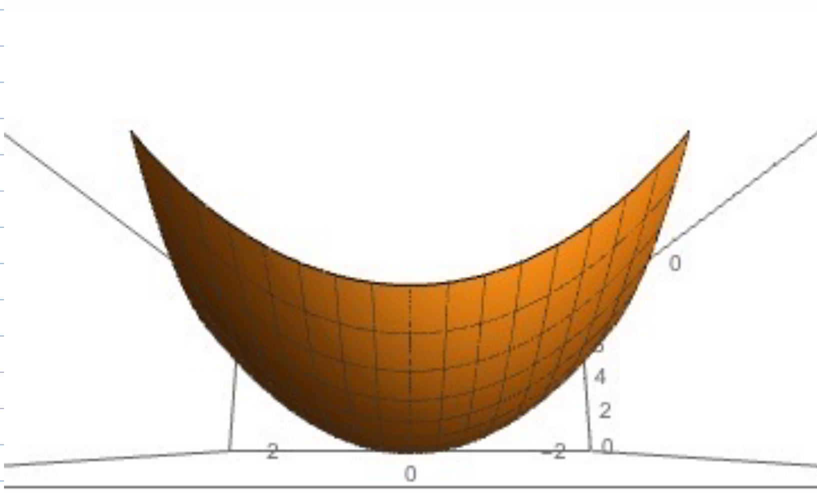
Visualize:



Graph of $z = f(x,y)$.

Ex: Consider $f(x,y) = x^2+y^2$. $\rightsquigarrow$ See the domain is all of $\mathbb{R}^2$.





Ex: Sketch the graph of $z = f(x,y) = \dfrac{1}{x^2+y^2}$

To figure this out: often helpful to study "contour lines" or "level curves"

Another name for xy-traces

Let's study the level curves of $f(x,y) = \dfrac{1}{x^2+y^2}$

Want to solve: $\dfrac{1}{x^2+y^2} = k$

if $k<0$:

$\dfrac{1}{x^2+y^2} = k \rightsquigarrow$ No solutions!!

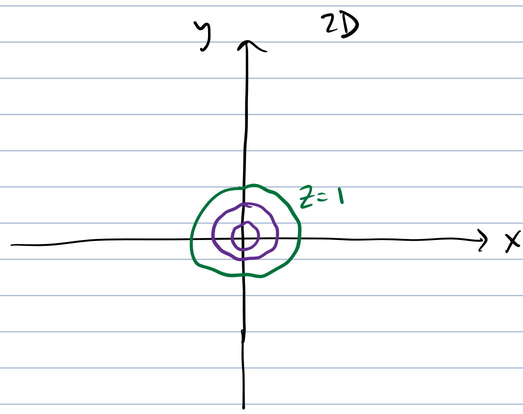
Geometrically: Graph of $z = f(x,y)$ is never below xy-plane.

if $k=0$ $\quad \dfrac{1}{x^2+y^2} = 0 \rightsquigarrow$ No solns!!

"Pick" $k=1$.

if $k>0$: $\quad \dfrac{1}{x^2+y^2} = 1 \implies x^2+y^2=1 \rightsquigarrow$ "Level curve at $z=1$"

2D

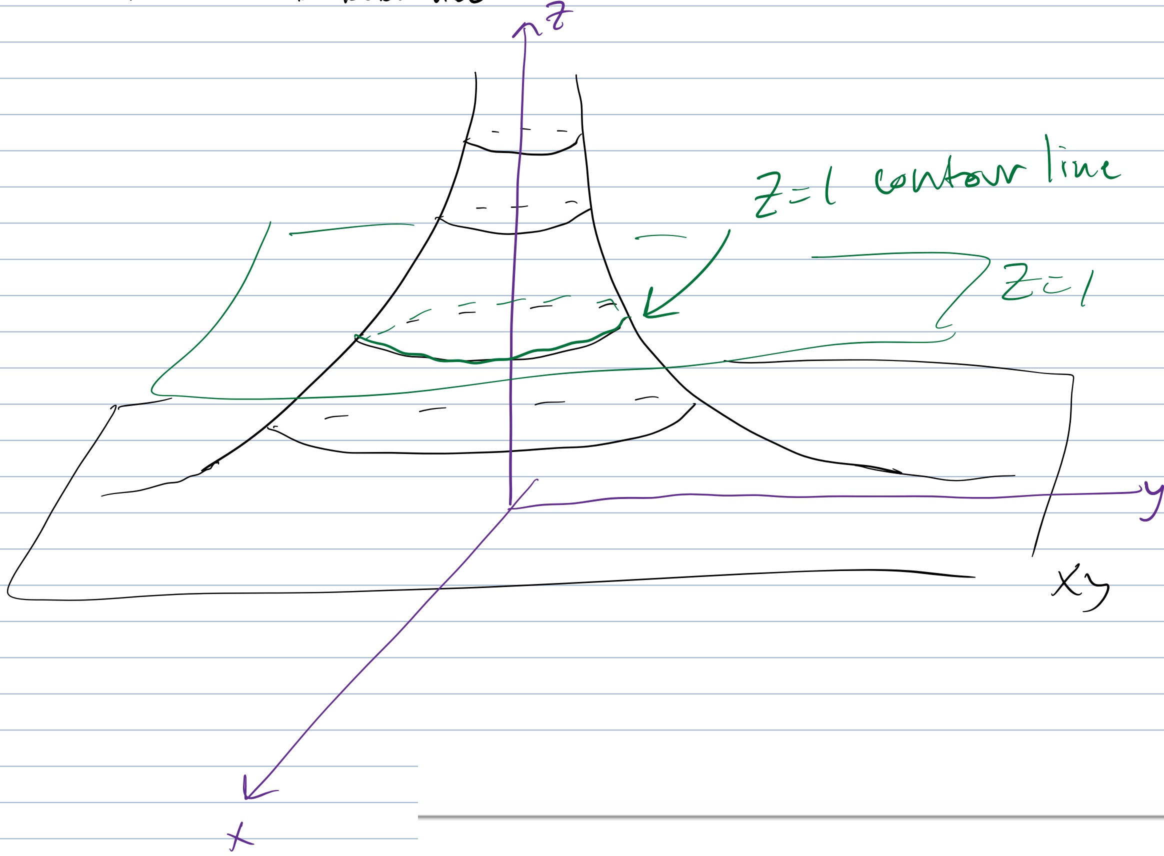


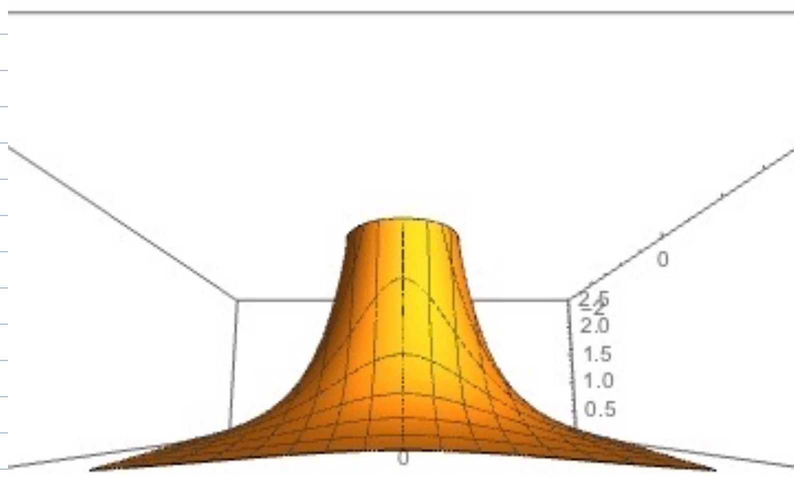
$\rightsquigarrow$ Solving $\dfrac{1}{x^2+y^2} = k \implies x^2+y^2 = \dfrac{1}{k}$

So as $k \to \infty$

The radius of circles $\to 0$.

So Picture in 3D looks like





Ex: Sketch the graph of $f(x,y) = \dfrac{x}{x^2+y^2}$

Consider the level curves:

$\dfrac{x}{x^2+y^2} = k$ Solve this so we can recognize what's happening

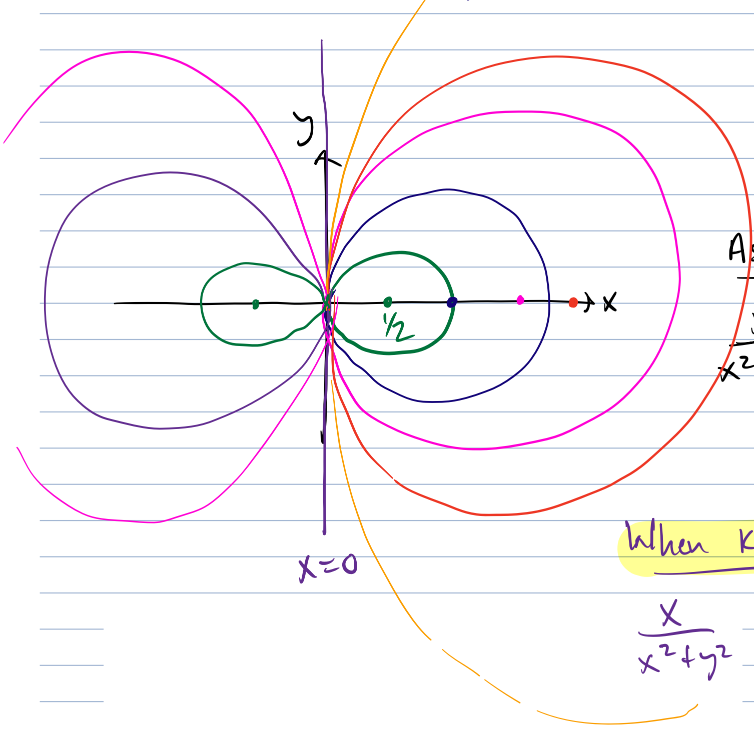
(Means: study intersection of this surface w/ plane $z=1$)

$\rightsquigarrow$ if $k=1$: $\dfrac{x}{x^2+y^2} = 1 \Rightarrow x = x^2+y^2 \Rightarrow x^2 - x + y^2 = 0$

$\Rightarrow (x-\tfrac{1}{2})^2 - \tfrac{1}{4} + y^2 = 0$

$\Rightarrow (x-\tfrac{1}{2})^2 + y^2 = \tfrac{1}{4}$

Center $(\tfrac{1}{2}, 0)$ $r = \tfrac{1}{2}$.



As $k \to 0$; $(k>0)$

$\dfrac{x}{x^2+y^2} = k \Rightarrow x = x^2k + y^2k$

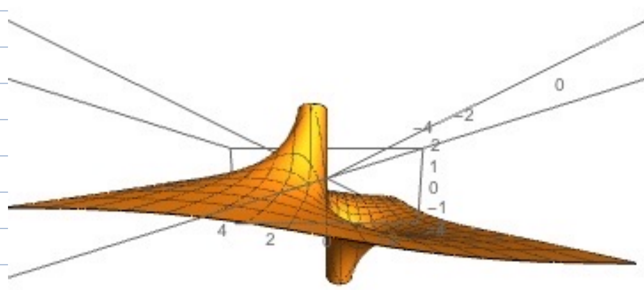
$\Rightarrow kx^2 - x + ky^2 = 0$

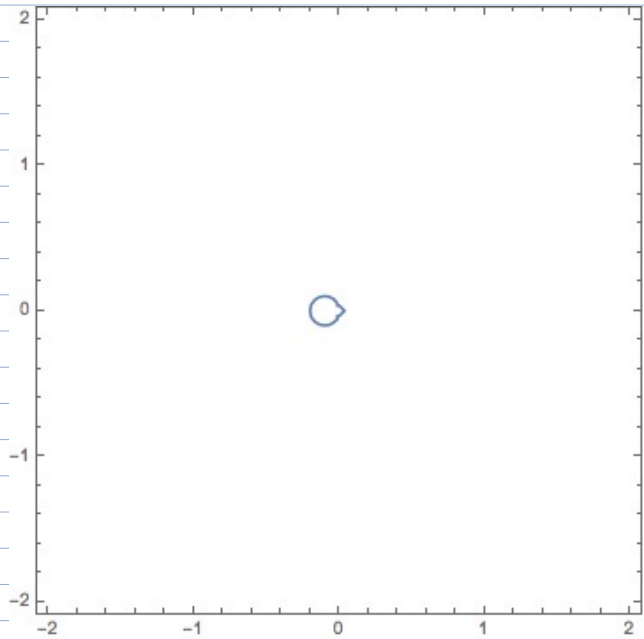
$\Rightarrow k(x^2 - x/k) + ky^2 = 0$

When $k=0$:

$\dfrac{x}{x^2+y^2} = 0 \Rightarrow x = 0.$

As $k \to 0$, looks like family of circles w/ center moving off to the right and radius getting bigger.





Movie starts w/

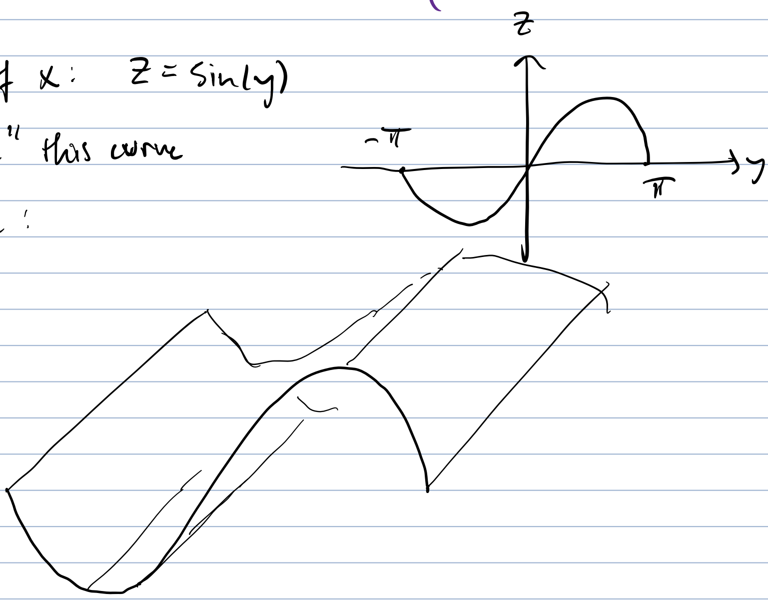
$K<0 \rightsquigarrow K=0$

$\rightsquigarrow K>0.$

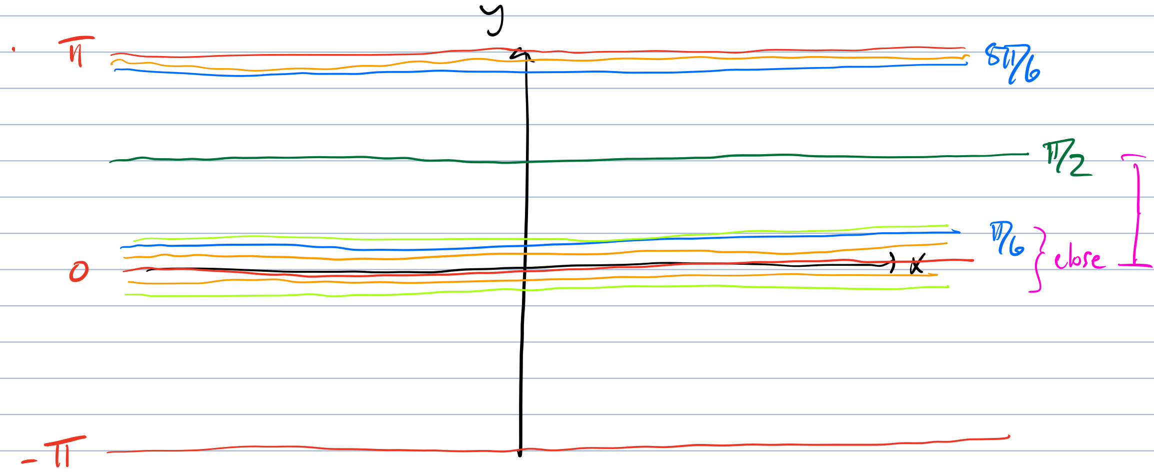
Ex: Sketch the graph of $f(x,y) = \sin(y)$. (Notice: independent of x!!)

Since independent of x: $z = \sin(y)$

Can simply "extrude" this curve in the x-direction:



Just Draw level curves : (Contour lines) $f(x,y) = \sin(y)$



$z = 0$

$z = 1/2 \rightsquigarrow \frac{1}{2} = \sin(y) \Rightarrow y = \pi/6, 5\pi/6$

$z = 1 \longrightarrow 1 = \sin(y) \Rightarrow y = \pi/2$

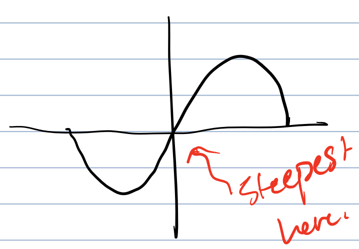
$\Delta z = 1/2$

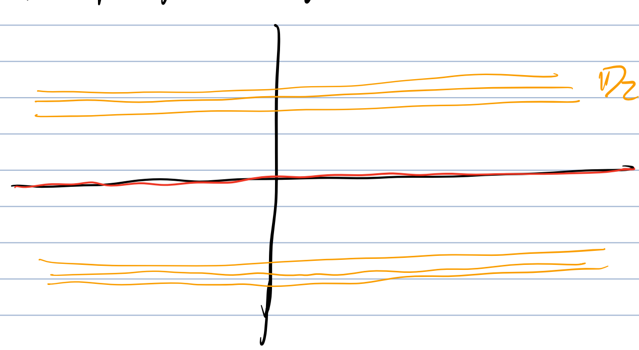
Q: Provided that change in height (Δz) is constant, What does it mean about the surface $z = f(x,y)$ when the contour lines are close together?

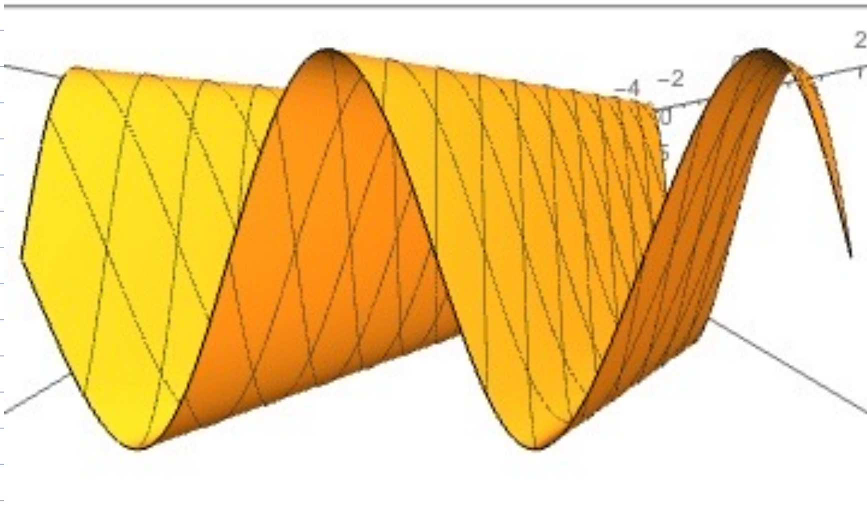
A: When contour lines are close $\rightsquigarrow$ Surface is steeper.

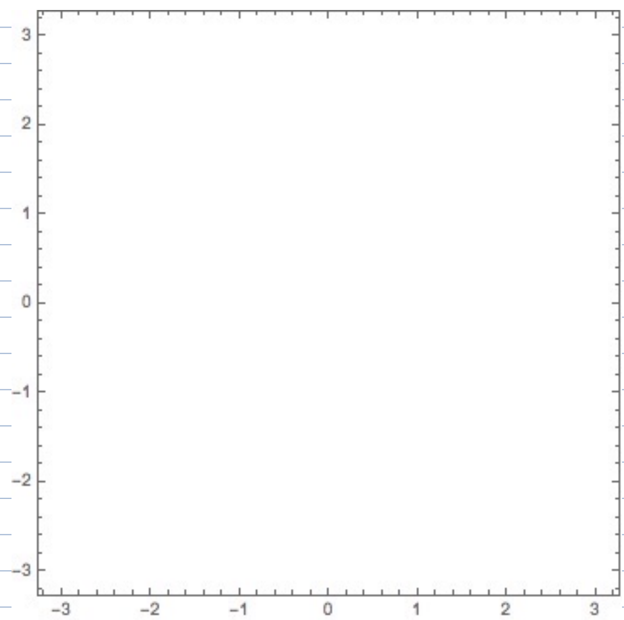
e.g. Can use this to distinguish sine / cosine

Graph $f(x,y) = \cos(y)$









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