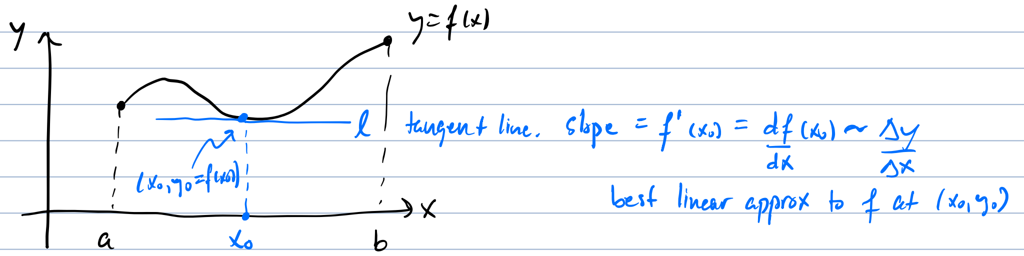
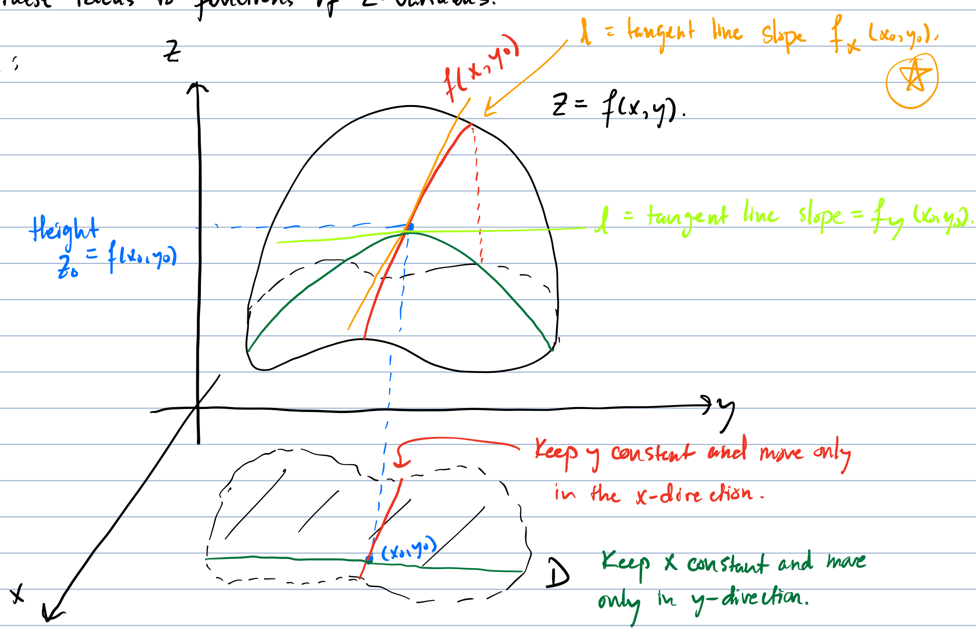
TODAY: PARTIAL DERIVATIVES

In Calc I: if $f : [a,b] \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ real-valued function f on $[a,b]$.



GOAL: Extend these ideas to functions of 2-variables.

Big Pic:



Notice: Can now move in TWO independent directions in domain D.

- By keeping y at a fixed value, say $y = y_0$, then $z = f(x, y_0)$ is really a fn of one variable, x.

only x changes.

$\rightsquigarrow$ Define $f_x(x_0, y_0) := \lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0, y_0)}{h}$ provided this limit exists.

Called "Partial Derivative of f w.r.t. x".

Lots of symbols: f_x, $\frac{\partial f}{\partial x}$

del

- Likewise, keeping x fixed ...... $z = f(x_0, y)$ only a fn of y.

Define: $f_y(x_0, y_0) = \frac{\partial f}{\partial y}(x_0, y_0) = \lim_{h\to 0} \frac{f(x_0, y+h) - f(x_0, y_0)}{h}$.

- Symbollically: lots of fun!!

Ex:

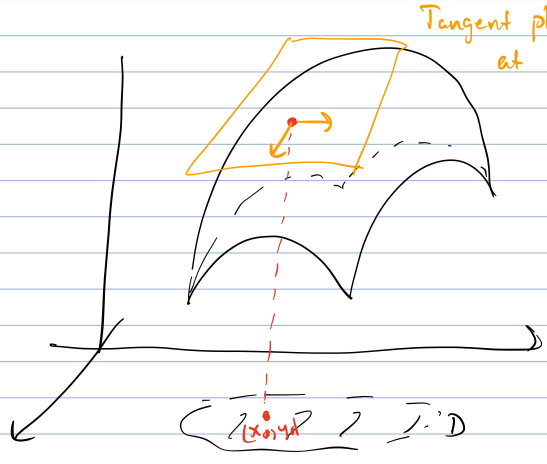
Means: treat y as a constant.

$f(x,y) = x^2 + \sin(xy) \rightsquigarrow f_x = \frac{\partial f}{\partial x} = 2x + y\cos(xy)$

$\rightsquigarrow f_y = \frac{\partial f}{\partial y} = x\cos(xy)$. //

- The analogue of "tangent line" $\rightsquigarrow$ is Now the tangent PLANE

Tangent plane to the surface at $z_0 = f(x_0, y_0)$



- To describe eqn for tangent plane:

→ Any plane can be written as

$$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$

→ Divide by C, letting $a = -A/C$, $b = -B/C$

$$\rightsquigarrow \quad z - z_0 = a(x-x_0) + b(y-y_0)$$

0

Setting $y = y_0$:

- $z - z_0 = a(x - x_0)$ eqn for tangent line

Conclusion:

a must in fact be $a = f_x(x_0, y_0)$.

By similar argument:

b must in fact be $b = f_y(x_0, y_0)$

So: Eqn for tangent plane at a point $(x_0, y_0, z_0 = f(x_0, y_0))$ given by

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

→ Useful for approximations, differentials, etc....