Elements of Machine Learning
Exercise Sheet 4
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Problem 3 (T, 4 Points). Dependencies.

Consider the following small example with $p = 2$ predictors and $n = 2$ samples. Suppose that $x_{11} = x_{12}, x_{21} = x_{22}$. Furthermore, suppose that $y_1 + y_2 = 0, x_{11} + x_{21} = 0$ and $x_{12} + x_{22} = 0$, so that the estimated intercept in a least squares model is zero, $\hat{\beta}_0 = 0$.

1. [2pts] What is the linear regression solution $\hat{\beta}$ in this case?

Since $x_{11} = x_{12}$ and $x_{21} = x_{22}$, then the solution $\hat{\beta}_1 = -\hat{\beta}_2$ fulfils the conditions trivially. Furthermore, if we derive the OLS solution by our typical partial derivative exercise, we will always end up estimating the same value for $\hat{\beta}_1$ and $\hat{\beta}_2$, thus any $\hat{\beta}_1 = \hat{\beta}_2$ also fulfils the conditions.

2. [2pts] What problem do you see?

X_1 and X_2 are colinear, since $x_{11} = -x_{21}$ and $x_{12} = -x_{22}$, the two variables are perfectly (negatively) correlated, which makes it hard to separate the effects of the predictor coefficients.