Elements of Machine Learning
Assignment 1

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Problem 4 (T, 8 Points). **Least Squares Error**

1. [2pts] Let Y be a random variable. Show that

$$\mathbb{E}(Y) = argmin_c \mathbb{E}[(Y - c)^2].$$

Explain why this might be an important result for Linear Regression.

The expected predicted error of $Y - E(Y)$–can be computed as the average of the squared difference of Y and some term c. If we minimize with respect to c, we choose $c = Y$ to give us an expected error of 0.
In the case of Linear Regression, the corrolary is choosing an estimated $\hat{f}(X)$ such that $\hat{f}(X) = Y$, that is, picking the predicted function that is close or equal to, the true underlying function.

2. [6pts] The R^2 statistic is a common measure of model fit corresponding to the fraction of variance in the data that is explained by the model. In general, R^2 is given by the formula:

$$R^2 = \frac{TSS - RSS}{TSS} = 1 - \frac{RSS}{TSS}$$

- Show that for univariate regression, $R^2 = Cor(X, Y)^2$ holds.
- Show that in the univariate case, $R^2 = Cor(Y, \hat{Y})^2$ holds.

COR(X,Y) $= \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2}\sqrt{\Sigma(y_i - \bar{y})^2}}$
$TSS = \Sigma(y_i - \bar{y})^2$
$RSS = \Sigma(y_i - \hat{y}_i)^2$

$R^2 = Cor(X, Y)^2$:

ISLR 3.18 defines $Cor(X, Y)$ as $\frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2}\sqrt{\Sigma(y_i - \bar{y})^2}}$, thus $Cor(X, Y)^2 = \frac{(\Sigma(x_i - \bar{x})(y_i - \bar{y}))^2}{\Sigma(x_i - \bar{x})^2 \Sigma(y_i - \bar{y})^2}$

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$R^2 = Cor(Y, \hat{Y})^2$:

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