Trajectory and Power Control for UAVs

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Outline

1. Introduction
2. Problem Formulation
3. Main Strategies
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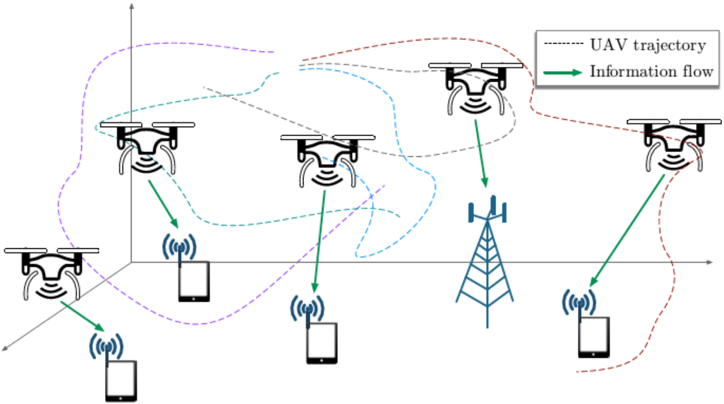
Introduction



Introduction

- Practical constraints
  - Transmission interference
  - Limited speed and altitude
  - Collision

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  - Limited speed and altitude
  - Collision
- TPC Problem
  - Joint trajectory optimization
  - Power control

Problem Formulation

- BS position
  - Fixed base station (BS) position: $\boldsymbol{s} \in \mathbb{R}^{3\times K}$
  - k-th BS position: $\boldsymbol{s}[:, k] \in \mathbb{R}^3, \forall\, k \in \mathcal{K}$, where $\mathcal{K} \triangleq \{1, \ldots, K\}$

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- UAV location
  - UAV location: $\boldsymbol{q} \in \mathbb{R}^{3\times K\times(N+2)}$
  - k-th UAV at time slot n: $\boldsymbol{q}[:, k, n+1] \in \mathbb{R}^3$ for $n \in \mathcal{N}_1^N$, where $\mathcal{N}_i^j = \{i, \ldots, j\}$
  - Initial location: $\boldsymbol{q}[:, k, 1]$
  - Final location: $\boldsymbol{q}[:, k, N+2]$

Problem Formulation

- Altitude constraint
  - Minimum and maximum safe altitude for all UAV: $H_{\min}$ and $H_{\max}$
  - Altitude constraint:

$$H_{\min} \leq \boldsymbol{q}[3, k, n+1] \leq H_{\max} \tag{1}$$

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  - Altitude constraint:

$$H_{\min} \leq \boldsymbol{q}[3, k, n+1] \leq H_{\max} \tag{1}$$

- Speed constraint
  - Level-flight speed, vertical ascending and descending speed: V_L, V_A and V_D
  - Position constraints:

$$\|\boldsymbol{q}[1:2, k, n+1] - \boldsymbol{q}[1:2, k, n]\| \leq V_L T_s \tag{2a}$$

$$-V_D T_s \leq \boldsymbol{q}[3, k, n+1] - \boldsymbol{q}[3, k, n] \leq V_A T_s \tag{2b}$$

Problem Formulation

- Collision avoidance
  - Minimum safety distance between any two UAVs: $d_{\min}$
  - Collision avoidance constraints:

$$\|\boldsymbol{q}[:,k,n+1] - \boldsymbol{q}[:,j,n+1]\| \geq d_{\min} \tag{3}$$

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- Power constraint
  - Transimission power of UAV: $\boldsymbol{p} \in \mathbb{R}^{k\times(N+2)}$
  - k-th UAV at time slot n: $\boldsymbol{p}[k, n+1] \in \mathbb{R}$
  - Maximum transimission power: $P_{\max}$
  - Power constraint:

$$0 \leq \boldsymbol{p}[k, n+1] \leq P_{\max} \tag{4}$$

Problem Formulation

- Objective function
  - Channel capacity (bits/second): $R \in \mathbb{R}^{K\times(N+2)}$
  - k-th UAV at time slot n: $R_{[k,n+1]} \in \mathbb{R}$, for $k \in \mathcal{K}$ and $n \in \mathcal{N}_1^N$
  - TPC optimization problem:

$$\begin{aligned} \max_{\substack{\boldsymbol{p}_{[:,2:N+1]},\\ \boldsymbol{q}_{[:,:,2:N+1]}}} \quad & \sum_{n=2}^{N+1}\sum_{k=1}^{K} R_{[k,n]}(\boldsymbol{p},\boldsymbol{q}) \\ s.t. \quad & (1),(4)\ \forall\, k \in \mathcal{K}, n \in \mathcal{N}_1^N \\ & (2a),(2b),\ \forall\, k \in \mathcal{K}, n \in \mathcal{N}_1^{N+1} \\ & (3),\ \forall\, k,j \in \mathcal{K}, k<j, n \in \mathcal{N}_1^N \end{aligned} \tag{5}$$

Problem Formulation

- $R_{[k,n]}$: Channel capacity
  - Bandwidth of channel (Hz): B
  - Signal-to-noise ratio: SNR
  - Channel capacity:

$$R_{[k,n]} = B \log_2 \left(1 + \mathrm{SNR}_{[k,n]}\right) \tag{5.1}$$

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- SNR: Signal-to-noise ratio
  - Channel gain: $G \in \mathbb{R}^{K \times K \times (N+2)}$
  - Power spectral density of AWGN: N_0
  - SNR:

$$\mathrm{SNR}_{[k,n]} = \frac{G_{[k,k,n]} \boldsymbol{p}_{[k,n]}}{BN_0 + \sum_{j=1, j \neq k}^{K} G_{[j,k,n]} \boldsymbol{p}_{[j,n]}} \tag{5.2}$$

Problem Formulation

- $G_{[k,k,n]}$: Power gain
  - Channel gain beween UAV and BS: $G \in \mathbb{R}^{K \times K \times (N+2)}$
  - Channel gain between any UAV and BS at one meter: G_0
  - Channel gain between j-th UAV and k-th BS at time slot n:

$$G_{[j,k,n+1]} = \frac{G_0}{\|\boldsymbol{q}_{[:,j,n+1]} - \boldsymbol{s}_{[:,k]}\|^2} \tag{5.3}$$

Problem Formulation

- Notice
  - UAV should return to its initial position:

$$\boldsymbol{q}_{[:,k,1]} = \boldsymbol{q}_{[:,k,N+2]}, \ \forall\, k \in \mathcal{K} \tag{6.1}$$

  - Time slot short enough to avoid collision:

$$T_s \leq \frac{d_{\min}}{\sqrt{4V_L^2 + (V_A + V_D)^2}} \tag{6.2}$$

Main Strategies

- Two main difficulties
  - Large problem dimension
  - Non-convex optimization

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- Two main strategies
  - Heuristic dimension-reduced method
  - Successive convex approximation (SCA)

Main Strategies

Fly-hover-fly Strategy

The fly-hover-fly strategy is described as

- UAVs fly to the hovering location.
- UAVs hover over that location.
- UAVs return to the original position from the hovering location.

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- UAVs fly to the hovering location.
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Assumption 1

The whole time horizon T is much longer than the time UAVs need to fly to the hovering location.

Main Strategies

Lemma 1

Assume all UAVs return to the initial locations, and the ascending and descending speed are equal, i.e., $V_A = V_D$*. Denote one of the optimal solution of TPC problem as* $\boldsymbol{q}^*$ *and* $\boldsymbol{p}^*$*, then for all* $k \in \mathcal{K}, n \in \mathcal{N}_2^{N+2}$*,*

$$\boldsymbol{q}^*_{[:,k,n]} = \boldsymbol{q}^*_{[:,k,N+3-n]}, \ \boldsymbol{p}^*_{[k,n]} = \boldsymbol{p}^*_{[k,N+3-n]} \quad (6)$$

Main Strategies

Lemma 2

Using the same notations in Lemma 1*, for some* $M_0 \in \{2, 3, \ldots, N+1\}$ *and* $M \in \{2, 3, \ldots, M_0\}$*, we have*

$$R_{s[n]} \leq R_{s[M]} = R_{s[M+1]} = \cdots = R_{s[M_0]}, \ \forall\, n \in \mathcal{N}_2^M \tag{7}$$

where M_0 *is the time when the UAVs need to return from the hovering location, and* M *is the time when UAVs arrives the hovering location. In particular, if* $V_A = V_D$*,* $M_0 = N + 3 - M$*.*

Main Strategies

- Reformulation 1

$$
\begin{aligned}
\max_{\substack{\boldsymbol{p}_{[:,2:M]},\\ \boldsymbol{q}_{[:,:,2:M]},\\ M\in\mathcal{N}_2^{(N+1)/2}}} \quad & \sum_{n=2}^{M}\sum_{k=1}^{K} R_{[k,n]}(\boldsymbol{p},\boldsymbol{q}) \\
& \quad + \left(\tfrac{N+1}{2} - M\right)\sum_{k=1}^{K} R_{[k,M]}(\boldsymbol{p},\boldsymbol{q}) \qquad (8)\\
s.t. \quad & (1),(2a),(2b),(4),\ \forall\, k\in\mathcal{K}, n\in\mathcal{N}_2^M \\
& (3),\ \forall\, k,j\in\mathcal{K}, k<j, n\in\mathcal{N}_2^M
\end{aligned}
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& \quad + \left(\frac{N+1}{2} - M\right)\sum_{k=1}^{K} R_{[k,M]}(\boldsymbol{p},\boldsymbol{q}) \\
s.t. \quad & (1), (2a), (2b), (4),\ \forall\, k\in\mathcal{K}, n\in\mathcal{N}_2^M \\
& (3),\ \forall\, k,j\in\mathcal{K}, k<j, n\in\mathcal{N}_2^M
\end{aligned}
\tag{8}
$$

- Reformulation 2

$$
\begin{aligned}
\max_{\substack{\boldsymbol{p}_{[:,2:M]},\\ \boldsymbol{q}_{[:,:,2:M]}\\ M\in\mathcal{N}_2^{(N+1)/2}}} \quad & \sum_{n=2}^{M}\sum_{k=1}^{K} R_{[k,n]}(\boldsymbol{p},\boldsymbol{q}) \\
& \quad +\left(\frac{N+1}{2} - M\right) R_s^* \\
s.t. \quad & (1), (2a), (2b), (4),\ \forall\, k\in\mathcal{K}, n\in\mathcal{N}_2^M \\
& (3),\ \forall\, k,j\in\mathcal{K}, k<j, n\in\mathcal{N}_2^M \\
& \boldsymbol{q}_{[:,k,M]} = \boldsymbol{q}_{\boldsymbol{h}}{}^*_{[:,k]},\ \ \boldsymbol{p}_{[k,M]} = \boldsymbol{p}_{\boldsymbol{h}}{}^*_{[k]},\ \ \forall\, k\in\mathcal{K}
\end{aligned}
\tag{9}
$$

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- Successive convex approximation. Find a locally tight convex surrogate function to replace the objective function or constraints.

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- Trick. First-order Taylor's expansion.

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$$-\log(1+x) \geq -\log(1+\bar{x}) - \frac{x-\bar{x}}{1+\bar{x}}$$

$$x^2 \geq 2x\bar{x} - \bar{x}^2$$

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$$x^2 \geq 2x\bar{x} - \bar{x}^2$$

- Objective function.

$$\begin{aligned}\overline{R}[k,n](\boldsymbol{a},\boldsymbol{q}) \triangleq \quad & \log\left(1+\textstyle\sum_{j=1}^{K} \frac{G_0(\boldsymbol{a}[k,j])^2}{BN_0\boldsymbol{d}[j,k,n]}\right) \\ & -\log\left(1+\textstyle\sum_{j=1,j\neq k}^{K} \frac{G_0(\boldsymbol{a}[k,j])^2}{BN_0\boldsymbol{d}[j,k,n]}\right)\end{aligned} \tag{11}$$

Main Strategies

- Surrogate function.

$$\widetilde{R}_{[k,n]}(\boldsymbol{a},\boldsymbol{q};\boldsymbol{a}^r,\boldsymbol{q}^r) \triangleq \log\left(1+\frac{G_0}{BN_0}\sum_{j=1}^{K}\left[\frac{2\boldsymbol{a}^r_{[j,n]}}{\boldsymbol{d}^r_{[j,k,n]}}\boldsymbol{a}_{[j,n]} - \frac{(\boldsymbol{a}^r_{[k,j]})^2}{(\boldsymbol{d}^r_{[j,k,n]})^2}\left\|\boldsymbol{q}_{[:,j,n]}-\boldsymbol{s}_{[:,k]}\right\|^2\right]\right) - \log(1+\boldsymbol{I}^r_{[k,n]}) + \frac{\boldsymbol{I}^r_{[k,n]}}{1+\boldsymbol{I}^r_{[k,n]}} - \frac{G_0}{BN_0(1+\boldsymbol{I}^r_{[k,n]})}\sum_{j=1,j\neq k}^{K}\frac{(\boldsymbol{a}_{[j,n]})^2}{\boldsymbol{d}^r_{[j,k,n]}+2(\boldsymbol{q}^r_{[:,j,n]}-\boldsymbol{s}_{[:,k]})^{\mathrm{T}}(\boldsymbol{q}_{[:,j,n]}-\boldsymbol{q}^r_{[:,j,n]})} \quad (12)$$

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- Collision avoidance constraint

$$2(\boldsymbol{q}^r_{[:,k,n]}-\boldsymbol{q}^r_{[:,j,n]})^{\mathrm{T}}(\boldsymbol{q}_{[:,k,n]}-\boldsymbol{q}_{[:,j,n]}) \geq \left\|\boldsymbol{q}^r_{[:,k,n]}-\boldsymbol{q}^r_{[:,j,n]}\right\|^2 + d^2_{\min} \tag{13}$$

Main Strategies

Therefore, the convex approximation problem for (9) given M is

$$
\begin{aligned}
\max_{\substack{\boldsymbol{a}_{[:,2:M]},\\ \boldsymbol{q}_{[:,:,2:M]}}} \quad & \sum_{n=2}^{M}\sum_{k=1}^{K} \widetilde{R}_{[k,n]}(\boldsymbol{a}, \boldsymbol{q}; \boldsymbol{a}^r, \boldsymbol{q}^r) \\
s.t. \quad & (1), (2a), (2b), (4), \ \forall\, k \in \mathcal{K}, n \in \mathcal{N}_2^M \\
& (13), \ \forall\, k, j \in \mathcal{K}, k < j, n \in \mathcal{N}_2^M \\
& \boldsymbol{q}_{[:,k,M]} = \boldsymbol{q}_{\boldsymbol{h}}{}^{*}_{[:,k]}, \ \boldsymbol{p}_{[k,M]} = \boldsymbol{p}_{\boldsymbol{h}}{}^{*}_{[k]}, \ \forall\, k \in \mathcal{K}
\end{aligned}
\tag{14}
$$

Main Strategies

Algorithm 1 SCA algorithm for solving problem (9)

Set iteration index $r = 0$, tolerance $\epsilon > 0$.
Initialize $\boldsymbol{q}^0_{[:,k,n]}$ and $\boldsymbol{p}^0_{[k,n]}$ for $k \in \mathcal{K}$, $n \in \mathcal{N}_2^M$.
Calculate $R^0 = \sum_{k=1}^{K} \sum_{n=2}^{M} R_{[k,n]}(\boldsymbol{p}^0, \boldsymbol{q}^0)$.
repeat
    Calculate $\left\{\boldsymbol{a}^r_{[k,n]}, \boldsymbol{d}^r_{[j,k,n]}, \boldsymbol{l}^r_{[k,n]}\right\}$ by using $\boldsymbol{p}^r_{[k,n]}$ and $\boldsymbol{q}^r_{[:,k,n]}$.
    Update $\left\{\boldsymbol{a}^{r+1}_{[k,n]}, \boldsymbol{q}^{r+1}_{[:,k,n]}\right\}$ by solving problem (14) with parameters $\boldsymbol{a}^r_{[k,n]}$, $\boldsymbol{d}^r_{[j,k,n]}$, and $\boldsymbol{l}^r_{[k,n]}$.
    Calculate $\boldsymbol{p}^{r+1}_{[k,n]}$ by using $\boldsymbol{a}^{r+1}_{[k,n]}$.
    Set $r = r + 1$.
until $\frac{|R^r - R^{r-1}|}{R^{r-1}} \leq \epsilon$
return $\left\{\boldsymbol{p}^r_{[k,n]}, \boldsymbol{q}^r_{[:,k,n]}\right\}$

Simulation Result

- Simulation Scale
  - # of UAV-BS pairs: $K = 4$
  - # of time slots: $M = 160$

Source Code: *https://github.com/Vito-Swift/EIE3280-CourseProj-TPC*

Simulation Result

- Simulation Scale
  - # of UAV-BS pairs: $K = 4$
  - # of time slots: $M = 160$
- Parameter Setup
  - $d_{\min} = 20m$
  - $V_L = 20m/s$, $V_A = V_D = 5m/s$
  - $h_{\min} = 100m$, $h_{\max} = 200m$
  - $P_{\max} = 30dbm$
  - Communication bandwidth: $B = 10MHz$
  - Power spectral density of addictive white Gaussian noise: $N_0 = -160dbm/Hz$

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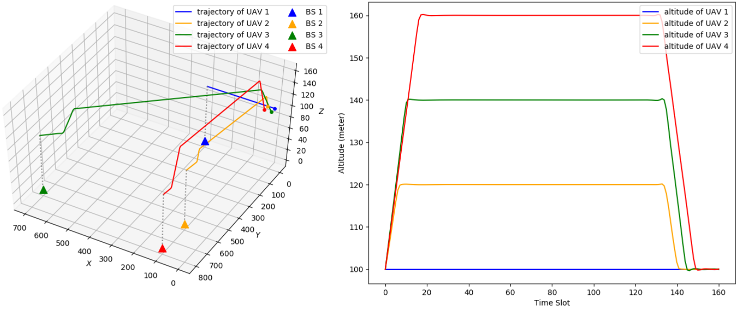
  - Communication bandwidth: $B = 10MHz$
  - Power spectral density of addictive white Gaussian noise: $N_0 = -160dbm/Hz$
- Initial locations
  - UAV: (0, 0, 100), (30, 0, 100), (0, 30, 100), (30, 30, 100)
  - Base station: (300, 0, 0), (100, 600, 0), (700, 700, 0), (100, 800, 0)

Source Code: *https://github.com/Vito-Swift/EIE3280-CourseProj-TPC*

Simulation Result

- Initial trajectory
  - Step 1: obtain the hovering points $q^{*}_{[:,k,M]}$ and the transmission powers $p^{*}_{[k,M]}$
  - Step 2: obtain M^{*} and the initial trajectory $q^{0}_{[:,k,n]}$



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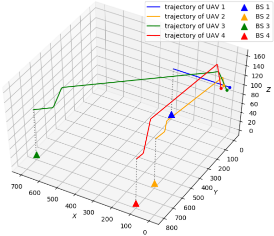
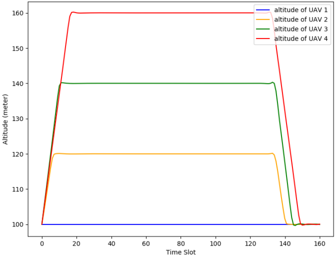
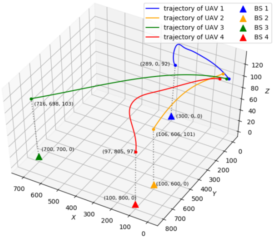
- Initial trajectory

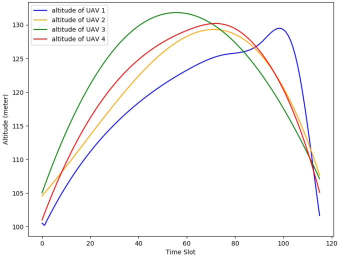




- Optimized trajectory

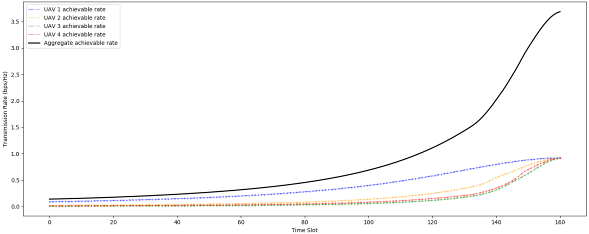


Optimized Trajectory



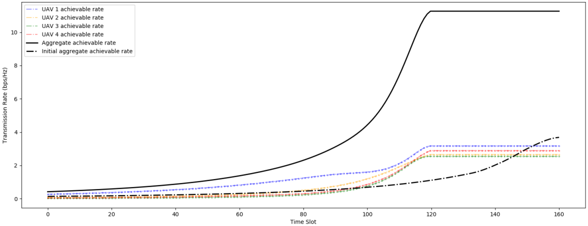
Simulation Result

- Initial achievable transmission rate



Initial Achievable Transmission Rate

- Optimized achievable transmission rate



Optimized Achievable Transmission Rate

Conclusion

- Generally, TPC problem is NP-hard and we apply an efficient SCA-based suboptimal algorithms to optimize the initial trajectory and power control.

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- From simulations,
  - the aggregate transmission rate raised
  - UAV approaches q^* in a more preferable route
    - ★ shorter traveling duration
    - ★ lower maximum height

Conclusion

- Future Work
  - Perform simulations in more sophisticated scenarios
  - Apply ADMM algorithm to modify our original algorithm, in order to reduce the computation overhead by using parallel computing
  - From one antenna to multiple antennas - extend the current work to scenarios with multi-antenna base stations and UAVs

References

[1] C. Shen, T.-H. Chang, J. Gong, Y. Zeng, and R. Zhang, "Multi-uav interference coordination via joint trajectory and power control," *arXiv preprint arXiv:1809.05697*, 2018.

[2] Y. Zeng, R. Zhang, and T. J. Lim, "Wireless communications with unmanned aerial vehicles: Opportunities and challenges," *IEEE Communications Magazine*, vol. 54, no. 5, pp. 36–42, 2016.

[3] Y. Zeng and R. Zhang, "Energy-efficient uav communication with trajectory optimization," *IEEE Transactions on Wireless Communications*, vol. 16, no. 6, pp. 3747–3760, June 2017.

[4] L. Li, T.-H. Chang, and S. Cai, "Uav positioning and power control for two-way wireless relaying," *arXiv preprint arXiv:1904.08280*, 2019.