

Trajectory and Power Control for UAVs

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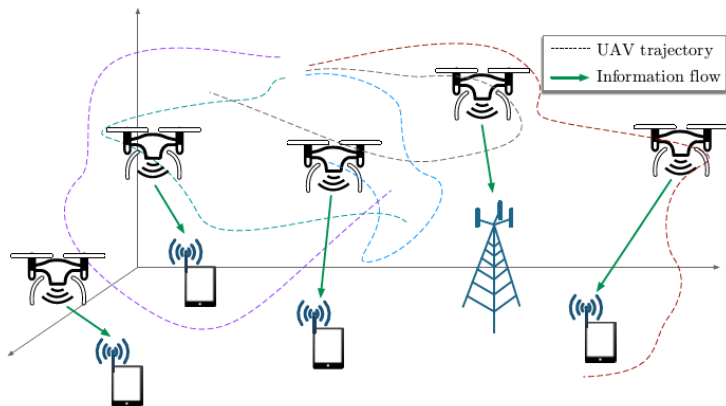
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Outline

- 1 Introduction
- 2 Problem Formulation
- 3 Main Strategies
- 4 Simulation Result
- 5 Conclusion

Introduction



Introduction

- Practical constraints
 - ▶ Transmission interference
 - ▶ Limited speed and altitude
 - ▶ Collision
- TPC Problem
 - ▶ Joint trajectory optimization
 - ▶ Power control

Problem Formulation

- BS position

- ▶ Fixed base station (BS) position: $\mathbf{s} \in \mathbb{R}^{3 \times K}$
- ▶ k -th BS position: $\mathbf{s}[:, k] \in \mathbb{R}^3, \forall k \in \mathcal{K}$, where $\mathcal{K} \triangleq \{1, \dots, K\}$

- UAV location

- ▶ UAV location: $\mathbf{q} \in \mathbb{R}^{3 \times K \times (N+2)}$
- ▶ k -th UAV at time slot n : $\mathbf{q}[:, k, n+1] \in \mathbb{R}^3$ for $n \in \mathcal{N}_1^N$, where $\mathcal{N}_i^j = \{i, \dots, j\}$
- ▶ Initial location: $\mathbf{q}[:, k, 1]$
- ▶ Final location: $\mathbf{q}[:, k, N+2]$

Problem Formulation

- Altitude constraint

- ▶ Minimum and maximum safe altitude for all UAV: H_{\min} and H_{\max}
- ▶ Altitude constraint:

$$H_{\min} \leq \mathbf{q}[3, k, n+1] \leq H_{\max} \quad (1)$$

- Speed constraint

- ▶ Level-flight speed, vertical ascending and descending speed: V_L , V_A and V_D
- ▶ Position constraints:

$$\|\mathbf{q}[1:2, k, n+1] - \mathbf{q}[1:2, k, n]\| \leq V_L T_s \quad (2a)$$

$$-V_D T_s \leq \mathbf{q}[3, k, n+1] - \mathbf{q}[3, k, n] \leq V_A T_s \quad (2b)$$

Problem Formulation

- Collision avoidance

- ▶ Minimum safety distance between any two UAVs: d_{\min}
- ▶ Collision avoidance constraints:

$$\|\mathbf{q}[:, k, n+1] - \mathbf{q}[:, j, n+1]\| \geq d_{\min} \quad (3)$$

- Power constraint

- ▶ Transmission power of UAV: $\mathbf{p} \in \mathbb{R}^{k \times (N+2)}$
- ▶ k -th UAV at time slot n : $\mathbf{p}[k, n+1] \in \mathbb{R}$
- ▶ Maximum transmission power: P_{\max}
- ▶ Power constraint:

$$0 \leq \mathbf{p}[k, n+1] \leq P_{\max} \quad (4)$$

Problem Formulation

- Objective function

- ▶ Channel capacity (bits/second): $R \in \mathbb{R}^{K \times (N+2)}$
- ▶ k -th UAV at time slot n : $R_{[k,n+1]} \in \mathbb{R}$, for $k \in \mathcal{K}$ and $n \in \mathcal{N}_1^N$
- ▶ TPC optimization problem:

$$\begin{aligned} & \max_{\substack{\mathbf{p}_{[:,2:N+1]}, \\ \mathbf{q}_{[:,2:N+1]}}} && \sum_{n=2}^{N+1} \sum_{k=1}^K R_{[k,n]}(\mathbf{p}, \mathbf{q}) \\ & \text{s.t.} && (1), (4) \quad \forall k \in \mathcal{K}, n \in \mathcal{N}_1^N \\ & && (2a), (2b), \quad \forall k \in \mathcal{K}, n \in \mathcal{N}_1^{N+1} \\ & && (3), \quad \forall k, j \in \mathcal{K}, k < j, n \in \mathcal{N}_1^N \end{aligned} \tag{5}$$

Problem Formulation

- $R_{[k,n]}$: Channel capacity
 - ▶ Bandwidth of channel (Hz): B
 - ▶ Signal-to-noise ratio: SNR
 - ▶ Channel capacity:

$$R_{[k,n]} = B \log_2 (1 + \text{SNR}_{[k,n]}) \quad (5.1)$$

- SNR: Signal-to-noise ratio
 - ▶ Channel gain: $G \in \mathbb{R}^{K \times K \times (N+2)}$
 - ▶ Power spectral density of AWGN: N_0
 - ▶ SNR:

$$\text{SNR}_{[k,n]} = \frac{G_{[k,k,n]} \mathbf{p}_{[k,n]}}{BN_0 + \sum_{j=1, j \neq k}^K G_{[j,k,n]} \mathbf{p}_{[j,n]}} \quad (5.2)$$

Problem Formulation

- $G_{[k,k,n]}$: Power gain
 - ▶ Channel gain between UAV and BS: $G \in \mathbb{R}^{K \times K \times (N+2)}$
 - ▶ Channel gain between any UAV and BS at one meter: G_0
 - ▶ Channel gain between j -th UAV and k -th BS at time slot n :

$$G_{[j,k,n+1]} = \frac{G_0}{\|\mathbf{q}_{[:,j,n+1]} - \mathbf{s}_{[:,k]}\|^2} \quad (5.3)$$

Problem Formulation

- Notice

- ▶ UAV should return to its initial position:

$$\mathbf{q}_{[:,k,1]} = \mathbf{q}_{[:,k,N+2]}, \quad \forall k \in \mathcal{K} \quad (6.1)$$

- ▶ Time slot short enough to avoid collision:

$$T_s \leq \frac{d_{\min}}{\sqrt{4V_L^2 + (V_A + V_D)^2}} \quad (6.2)$$

Main Strategies

- Two main difficulties
 - ▶ Large problem dimension
 - ▶ Non-convex optimization

- Two main strategies
 - ▶ Heuristic dimension-reduced method
 - ▶ Successive convex approximation (SCA)

Main Strategies

Fly-hover-fly Strategy

The fly-hover-fly strategy is described as

- UAVs fly to the hovering location.
- UAVs hover over that location.
- UAVs return to the original position from the hovering location.

Assumption 1

*The whole time horizon T is **much longer** than the time UAVs need to fly to the hovering location.*

Main Strategies

Lemma 1

Assume all UAVs return to the initial locations, and the ascending and descending speed are equal, i.e., $V_A = V_D$. Denote one of the optimal solution of TPC problem as \mathbf{q}^ and \mathbf{p}^* , then for all $k \in \mathcal{K}$, $n \in \mathcal{N}_2^{N+2}$,*

$$\mathbf{q}_{[:,k,n]}^* = \mathbf{q}_{[:,k,N+3-n]}^*, \quad \mathbf{p}_{[k,n]}^* = \mathbf{p}_{[k,N+3-n]}^* \quad (6)$$

Main Strategies

Lemma 2

Using the same notations in Lemma 1, for some $M_0 \in \{2, 3, \dots, N+1\}$ and $M \in \{2, 3, \dots, M_0\}$, we have

$$R_{s[n]} \leq R_{s[M]} = R_{s[M+1]} = \dots = R_{s[M_0]}, \quad \forall n \in \mathcal{N}_2^M \quad (7)$$

where M_0 is the time when the UAVs need to return from the hovering location, and M is the time when UAVs arrives the hovering location. In particular, if $V_A = V_D$, $M_0 = N+3-M$.

Main Strategies

• Reformulation 1

$$\begin{aligned}
 & \max_{\substack{\mathbf{p}_{[:,2:M]}, \\ \mathbf{q}_{[:,2:M]}, \\ M \in \mathcal{N}_2^{(N+1)/2}}} & \sum_{n=2}^M \sum_{k=1}^K R_{[k,n]}(\mathbf{p}, \mathbf{q}) \\
 & \quad + \left(\frac{N+1}{2} - M\right) \sum_{k=1}^K R_{[k,M]}(\mathbf{p}, \mathbf{q}) \\
 & \text{s.t.} \quad (1), (2a), (2b), (4), \forall k \in \mathcal{K}, n \in \mathcal{N}_2^M \\
 & \quad (3), \forall k, j \in \mathcal{K}, k < j, n \in \mathcal{N}_2^M
 \end{aligned} \tag{8}$$

• Reformulation 2

$$\begin{aligned}
 & \max_{\substack{\mathbf{p}_{[:,2:M]}, \\ \mathbf{q}_{[:,2:M]}, \\ M \in \mathcal{N}_2^{(N+1)/2}}} & \sum_{n=2}^M \sum_{k=1}^K R_{[k,n]}(\mathbf{p}, \mathbf{q}) \\
 & \quad + \left(\frac{N+1}{2} - M\right) R_s^* \\
 & \text{s.t.} \quad (1), (2a), (2b), (4), \forall k \in \mathcal{K}, n \in \mathcal{N}_2^M \\
 & \quad (3), \forall k, j \in \mathcal{K}, k < j, n \in \mathcal{N}_2^M \\
 & \quad \mathbf{q}_{[:,k,M]} = \mathbf{q} \mathbf{h}_{[:,k]}^*, \mathbf{p}_{[k,M]} = \mathbf{p} \mathbf{h}_{[k]}^*, \forall k \in \mathcal{K}
 \end{aligned} \tag{9}$$

Main Strategies

- **Successive convex approximation.** Find a locally tight convex surrogate function to replace the objective function or constraints.
- Trick. **First-order Taylor's expansion.**

$$\begin{aligned}\frac{x^2}{y} &\geq \frac{2\bar{x}}{\bar{y}}x - \frac{\bar{x}^2}{\bar{y}^2}y, \text{ for fixed } \bar{y} > 0 \\ -\log(1+x) &\geq -\log(1+\bar{x}) - \frac{x-\bar{x}}{1+\bar{x}} \\ x^2 &\geq 2x\bar{x} - \bar{x}^2\end{aligned}$$

- Objective function.

$$\begin{aligned}\bar{R}[k, n](\mathbf{a}, \mathbf{q}) \triangleq & \log \left(1 + \sum_{j=1}^K \frac{G_0(\mathbf{a}[k, j])^2}{BN_0 \mathbf{d}[j, k, n]} \right) \\ & - \log \left(1 + \sum_{j=1, j \neq k}^K \frac{G_0(\mathbf{a}[k, j])^2}{BN_0 \mathbf{d}[j, k, n]} \right)\end{aligned} \quad (11)$$

Main Strategies

- Surrogate function.

$$\begin{aligned} \tilde{R}_{[k,n]}(\mathbf{a}, \mathbf{q}; \mathbf{a}^r, \mathbf{q}^r) \triangleq & \log \left(1 + \frac{G_0}{BN_0} \sum_{j=1}^K \left[\frac{2\mathbf{a}_{[j,n]}^r}{\mathbf{d}_{[j,k,n]}^r} \mathbf{a}_{[j,n]} \right. \right. \\ & \left. \left. - \frac{(\mathbf{a}_{[k,j]}^r)^2}{(\mathbf{d}_{[j,k,n]}^r)^2} \|\mathbf{q}_{[:,j,n]} - \mathbf{s}_{[:,k]}\|^2 \right] \right) - \log(1 + \mathbf{I}_{[k,n]}^r) + \frac{\mathbf{I}_{[k,n]}^r}{1 + \mathbf{I}_{[k,n]}^r} \\ & - \frac{G_0}{BN_0(1 + \mathbf{I}_{[k,n]}^r)} \sum_{j=1, j \neq k}^K \frac{(\mathbf{a}_{[j,n]})^2}{\mathbf{d}_{[j,k,n]}^r + 2(\mathbf{q}_{[:,j,n]}^r - \mathbf{s}_{[:,k]})^T (\mathbf{q}_{[:,j,n]} - \mathbf{q}_{[:,j,n]}^r)} \end{aligned} \quad (12)$$

- Collision avoidance constraint

$$\begin{aligned} 2(\mathbf{q}_{[:,k,n]}^r - \mathbf{q}_{[:,j,n]}^r)^T (\mathbf{q}_{[:,k,n]} - \mathbf{q}_{[:,j,n]}) \geq \\ \left\| \mathbf{q}_{[:,k,n]}^r - \mathbf{q}_{[:,j,n]}^r \right\|^2 + d_{\min}^2 \end{aligned} \quad (13)$$

Main Strategies

Therefore, the convex approximation problem for (9) given M is

$$\begin{aligned} \max_{\mathbf{a}_{[:,2:M]}, \mathbf{q}_{[:,2:M]}} \quad & \sum_{n=2}^M \sum_{k=1}^K \tilde{R}_{[k,n]}(\mathbf{a}, \mathbf{q}; \mathbf{a}^r, \mathbf{q}^r) \\ \text{s.t.} \quad & (1), (2a), (2b), (4), \forall k \in \mathcal{K}, n \in \mathcal{N}_2^M \\ & (13), \forall k, j \in \mathcal{K}, k < j, n \in \mathcal{N}_2^M \\ & \mathbf{q}_{[:,k,M]} = \mathbf{q}_{h[:,k]}^*, \mathbf{p}_{[k,M]} = \mathbf{p}_{h[k]}^*, \forall k \in \mathcal{K} \end{aligned} \tag{14}$$

Main Strategies

Algorithm 1 SCA algorithm for solving problem (9)

- 1: Set iteration index $r = 0$, tolerance $\epsilon > 0$.
 - 2: Initialize $\mathbf{q}_{[:,k,n]}^0$ and $\mathbf{p}_{[k,n]}^0$ for $k \in \mathcal{K}$, $n \in \mathcal{N}_2^M$.
 - 3: Calculate $R^0 = \sum_{k=1}^K \sum_{n=2}^M R_{[k,n]}(\mathbf{p}^0, \mathbf{q}^0)$.
 - 4: **repeat**
 - 5: Calculate $\{\mathbf{a}_{[k,n]}^r, \mathbf{d}_{[j,k,n]}^r, l_{[k,n]}^r\}$ by using $\mathbf{p}_{[k,n]}^r$ and $\mathbf{q}_{[:,k,n]}^r$.
 - 6: Update $\{\mathbf{a}_{[k,n]}^{r+1}, \mathbf{q}_{[:,k,n]}^{r+1}\}$ by solving problem (14) with parameters $\mathbf{a}_{[k,n]}^r$, $\mathbf{d}_{[j,k,n]}^r$, and $l_{[k,n]}^r$.
 - 7: Calculate $\mathbf{p}_{[k,n]}^{r+1}$ by using $\mathbf{a}_{[k,n]}^{r+1}$.
 - 8: Set $r = r + 1$.
 - 9: **until** $\frac{|R^r - R^{r-1}|}{R^{r-1}} \leq \epsilon$
 - 10: **return** $\{\mathbf{p}_{[k,n]}^r, \mathbf{q}_{[:,k,n]}^r\}$
-

Simulation Result

- Simulation Scale

- ▶ # of UAV-BS pairs: $K = 4$
- ▶ # of time slots: $M = 160$

- Parameter Setup

- ▶ $d_{\min} = 20m$
- ▶ $V_L = 20m/s$, $V_A = V_D = 5m/s$
- ▶ $h_{\min} = 100m$, $h_{\max} = 200m$
- ▶ $P_{\max} = 30dbm$
- ▶ Communication bandwidth: $B = 10MHz$
- ▶ Power spectral density of additive white Gaussian noise:
 $N_0 = -160dbm/Hz$

- Initial locations

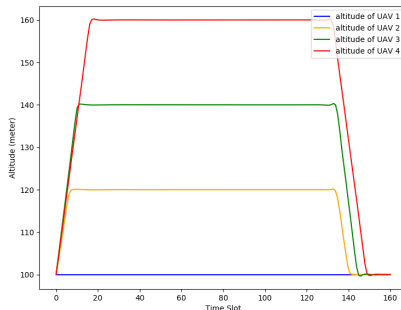
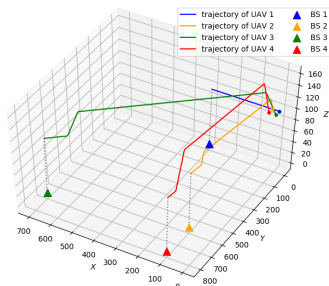
- ▶ UAV: $(0, 0, 100)$, $(30, 0, 100)$, $(0, 30, 100)$, $(30, 30, 100)$
- ▶ Base station: $(300, 0, 0)$, $(100, 600, 0)$, $(700, 700, 0)$, $(100, 800, 0)$

Source Code: <https://github.com/Vito-Swift/EIE3280-CourseProj-TPC>

Simulation Result

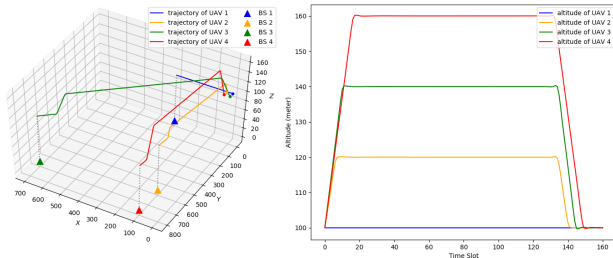
Initial trajectory

- ▶ Step 1: obtain the hovering points $q_{[:,k,M]}^*$ and the transmission powers $p_{[k,M]}^*$
- ▶ Step 2: obtain M^* and the initial trajectory $q_{[:,k,n]}^0$

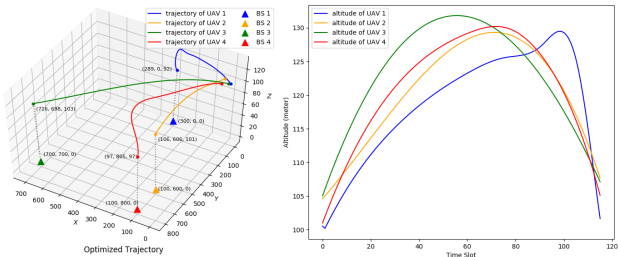


Simulation Result

Initial trajectory

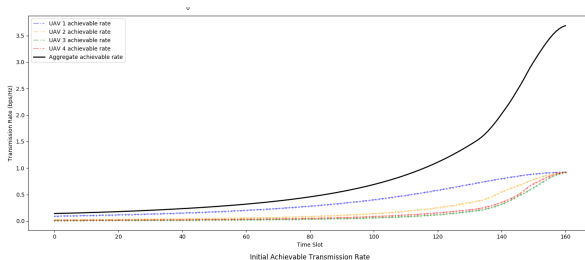


Optimized trajectory

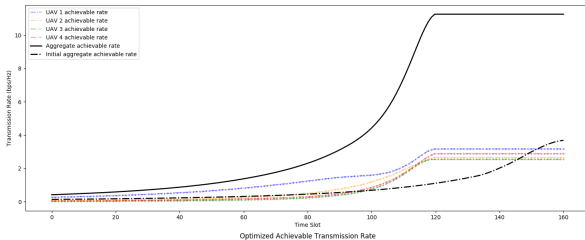


Simulation Result

Initial achievable transmission rate



Optimized achievable transmission rate



Conclusion

- Generally, TPC problem is NP-hard and we apply an efficient SCA-based suboptimal algorithms to optimize the initial trajectory and power control.
- From simulations,
 - ▶ the aggregate transmission rate raised
 - ▶ UAV approaches q^* in a more preferable route
 - ★ shorter traveling duration
 - ★ lower maximum height

Conclusion

- Future Work

- ▶ Perform simulations in more sophisticated scenarios
- ▶ Apply ADMM algorithm to modify our original algorithm, in order to reduce the computation overhead by using parallel computing
- ▶ From one antenna to multiple antennas - extend the current work to scenarios with multi-antenna base stations and UAVs

References

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